Investigation of a function - Manual

- Beginning
  - Check f once more.
  - The domain and continuity of f.
  - Points of intersections with axes.
  - Symmetries: oddness, evenness, periodicity.

  Due to $\mathcal{D}_f$, it is often clear that it cannot be e.g. an even function. On the other hand, if f is even or odd we can restrict our investigation on $\mathcal{D}_f \cap [0, +\infty)$ and then extend the graph on the rest of the domain. If f is periodic, we can similarly work just on some fixed interval of the length of the period. It may substantially simplify further computations.

- Limits
  - Limits at the "endpoints of the domain".
  - Asymptotes of the function. We solve $\lim_{x\to+\infty} \frac{f(x)}{x} = k$ and (if $k \in \mathbb{R}$) $\lim_{x\to+\infty}(f(x)-kx) = q$, asymptote at $+\infty$ then is $y = kx + q$. The same at $-\infty$.
  - We sketch all available information.

  Limits are usually quite straightforward, do not forget to find one-sided limits at the points excluded from the domain. If there is an asymptote at $\pm\infty$, it says that the graph of the function adheres to this line for x approaching $\pm\infty$. There might be 0, 1, or two asymptotes. Sketch of the graph at this moment is useful; it says that the function needs to decrease/increase somewhere, there needs to be certain type of extrema.

- First derivative
  - Straightforward computation of f'. Determine $\mathcal{D}_{f'}$.
  - Zeros of f'; when it is positive or negative.
  - The intervals of monotonicity; local and global extrema.
  - Determine $\mathcal{R}_f$.
  - Put everything into the sketch of the graph. Does it make sense?

  You do not need to compute one-sided derivatives at $\mathcal{D}_f \setminus \mathcal{D}_{f'}$.

- Second derivative
  - Straightforward computation of f''. Determine $\mathcal{D}_{f''}$.
  - Zeros of f''; when it is positive or negative.
  - The intervals of concavity or convexity.
  - The inflection points.

  We ignore one-sided second derivatives at $\mathcal{D}_{f'} \setminus \mathcal{D}_{f''}$. Often, part of f'' is positive for free.

- Conclusion
  - Prepare enough space for the graph. Then, draw asymptotes and all important points, i.e. points of intersections, extrema, and inflection points.
  - The sketch of the graph of f.

  In particular, the change from convexity to concavity should be visible. Also, if you used some symmetricity argument, do not forget to make the sketch on the whole domain, not the restricted one.