

## Annotation for the 8<sup>th</sup> week

We will start with the midterm, which takes 60 minutes. Then, we learn how to deal with a new type of problems. For example, the simple limit like

$$\lim_{x \rightarrow 0} \frac{1 - \cos \frac{1}{4}x^2}{x^4},$$

or a little bit harder one

$$\lim_{x \rightarrow -\infty} \frac{\log(1 + e^x)}{\sin \frac{1}{x}}$$

and the exam-like question would be

$$\lim_{x \rightarrow -1} \frac{e^{x^3-x} - \cos(\log x^2)}{\arcsin(x^2 - 1)}.$$

To solve these problems we will use two ingredients. First, there are the following well-known limits:

$$\begin{aligned} \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1, \quad \lim_{x \rightarrow 0} \frac{e^x - 1}{x} = 1, \quad \lim_{x \rightarrow 1} \frac{\log x}{x - 1} = 1, \\ \lim_{x \rightarrow 0} \frac{\tan x}{x} = 1, \quad \lim_{x \rightarrow 0} \frac{\arcsin x}{x} = 1, \quad \lim_{x \rightarrow 0} \frac{\arctan x}{x} = 1, \quad \lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2} = \frac{1}{2}. \end{aligned}$$

Second, we transform more complicated limits as above to the basic ones using the theorem about limit of a composition with (I) variant.