

Annotation for the 6th week

In the first circa 60 minutes, you will again solve limits of sequences by yourself. We can discuss ideas you are not sure about.

Next, we will move towards a new topic - functions of a real variable. We will probably quickly review some notions from the lecture, e.g. image, preimage, periodicity, and monotonicity.

Exercise. Let us consider the following function

$$f(x) = \begin{cases} x^2, & x > 0 \\ 1 - x^2, & x \leq 0 \end{cases}.$$

- (1) What is \mathcal{D}_f ? What is \mathcal{R}_f ?
- (2) Is f injective, surjective (=onto) or bijective? Is f even or odd? Is it periodic?
- (3) Is f (non-)increasing or (non-)decreasing on some intervals?
- (4) Find the image of $A_1 = [-2, 0)$, $A_2 = (-1, 1)$, $A_3 = [0, \frac{1}{2}]$ under f .
- (5) Find the preimage of $B_1 = (2, +\infty)$, $B_2 = \{1\}$, $B_3 = [\frac{1}{4}, 1]$ under f .
- (6) What are $\lim_{x \rightarrow +\infty} f(x)$, $\lim_{x \rightarrow -\infty} f(x)$?
- (7) Is f continuous?

Then we will think about the concept of a limit of function. Some ideas are basically the same as in the previous chapter, e.g.

$$\lim_{x \rightarrow +\infty} x^2, \lim_{x \rightarrow +\infty} e^x, \lim_{x \rightarrow +\infty} \frac{2^x + x + \log x}{2 \cdot 2^x + x^2}, \lim_{x \rightarrow -\infty} e^x, \lim_{x \rightarrow -\infty} \frac{x^3 - x + 1}{x^3 - x^2 + 1}, \lim_{x \rightarrow -\infty} \frac{1}{x}.$$

Others are kind of new. You can try to think about what should be results of the following examples

$$\begin{aligned} & \lim_{x \rightarrow 0} x^2, \quad \lim_{x \rightarrow 0} e^x, \quad \lim_{x \rightarrow \frac{\pi}{2}} \sin x, \\ & \lim_{x \rightarrow 0} \sqrt{x}, \quad \lim_{x \rightarrow 0_+} \sqrt{x}, \quad \lim_{x \rightarrow 0} \sqrt[3]{x}, \\ & \lim_{x \rightarrow 0} \frac{1}{x}, \quad \lim_{x \rightarrow 0_+} \frac{1}{x}, \quad \lim_{x \rightarrow 0} \frac{1}{x^2}. \end{aligned}$$