

## Annotation for the 12<sup>th</sup> week

We will learn how to sketch the graph of a function. In the lecture you will see graph of

$$f(x) = \log\left(x + \frac{1}{x}\right).$$

On the tutorial we will deal either with

$$f(x) = x + \frac{x}{\sqrt{x^2 - 1}}.$$

Then, you can try it by yourself with the following example

$$f(x) = (x^2 - 3)e^{-x}.$$

Further examples to practise:

**Exercise (Warm-up).**

- |                                  |                          |  |
|----------------------------------|--------------------------|--|
| (a) $f(x) = \sqrt{x} - x.$       | (d) $f(x) = xe^x.$       | (g) $f(x) = \arctan \frac{\sqrt{3}}{x^2}.$ |
| (b) $f(x) = 8x + \frac{4}{x^2}.$ | (e) $f(x) = xe^{-x^2}.$  | (h) $f(x) = x\sqrt{1 - x^2}.$              |
| (c) $f(x) = x(x + 2)^3.$         | (f) $f(x) = x \log^2 x.$ | (i) $f(x) = \frac{2x}{1 - x^2}.$           |

**Exercise (Exam).**

- |                                  |  |  |
|----------------------------------|--|--|
| (a) $f(x) = \frac{x}{1+x^4}.$    | (g) $f(x) = \arctan(x^2 - 1).$                 | (m) $f(x) = \sqrt{\frac{x}{x+1}}.$         |
| (b) $f(x) = \frac{x^3}{1+x^2}.$  | (h) $f(x) = \frac{x}{\log x}.$                 | (n) $f(x) = e^{\sin x}.$                   |
| (c) $f(x) = (x - 1)e^{-2x+3}.$   | (i) $f(x) = (x + 2)e^{\frac{1}{x}}.$           | (o) $f(x) = \sqrt{\frac{x^2}{x^3+1}}.$     |
| (d) $f(x) = \log(x^2 + 4x + 4).$ | (j) $f(x) = \arctan \frac{2x}{1-x^2}.$         | (p) $f(x) = \log(\sin x).$                 |
| (e) $f(x) = \log(x^2 + 2x).$     | (k) $f(x) = \sqrt{x^2 + x - 6}.$               | (q) $f(x) = \sin^2 x - 2 \sin x.$          |
| (f) $f(x) = \frac{2x}{1-x^2}.$   | (l) $f(x) = \log\left(x - \frac{1}{x}\right).$ | (r) $f(x) = x + \sqrt{\frac{x^2}{x^2-1}}.$ |

**We want:**

- The domain and continuity of  $f$ .
- Points of intersections with axes.
- Symmetries: oddness, evenness, periodicity.
- Limits at the “endpoints of the domain”.
- Asymptotes of the function.
- The intervals of monotonicity; local and global extrema.
- The range of  $f$ .
- The intervals of concavity or convexity.
- The inflection points.
- The sketch of the graph of  $f$ .