2. Lineární zobrazení

Je $L : U \to V$ lineární zobrazení? Pokud ano, jak vypadá $\mathrm{Ker}(L)$ a $\mathrm{Im}(L)$?

1. $U = \mathbf{R}^3$, $V = \mathbf{R}^4$;

a) $L(u, v, w) = [u + w, w - v + 7u, u, u]$,
b) $L(u, v, w) = [u^2, v, 0, 0]$,
c) $L(u, v, w) = [0, 0, 0, 0]$,
d) $L(u, v, w) = [u, v - 1, 4w, u + w]$,
e) $L(u, v, w) = [u, v, w, w]$,
f) $L(u, v, w) = [u + v, u - v, w, 10u]$.

2. $U = C(\mathbf{R})$, $V = C(\mathbf{R})$;

a) $L(f)(x) = f(x + 1) - f(x)$,
b) $L(f)(x) = \int_0^x f(t)\,dt$,
c) $L(f)(x) = f(x^2)$,
d) $L(f)(x) = \int_{x-1}^{x+1} f(t)\,dt$.

3. $U = C^1(\mathbf{R})$, $V = C(\mathbf{R})$;

a) $L(f)(x) = f'(x)$,
b) $L(f)(x) = f(x) - f'(x)$,
c) $L(f)(x) = \int_0^x f'(t)\,dt$.