BOOKLET OF

THE 50, 70, 80,... ∞ CONFERENCE IN MATHEMATICS

in honor of Professor Lars-Erik Persson, on the occasion of his 80 years anniversary

Karlstad, Sweden

August 19-23, 2024

THE ARCTIC UNIVERSITY OF NORWAY • UIT •

KARLSTADS UNIVERSITET

FACULTY OF ELECTRICAL ENGINEERING CTU IN PRAGUE

Welcome to the conference!

This event is organised in honor of Professor Lars-Erik Persson, on the occasion of his 80 years anniversary.

The conference is devoted to recent developments in Functional and Fourier Analysis, Interpolation, Homogenization and PDEs, Inequalities, Convexity Theory and Engineering Mathematics. The main reason of this variety of topics is that Lars-Erik Persson has in different periods of his life worked in these areas, as a professor in Luleå, Uppsala, Narvik, Astana, Lund and Karlstad. He has been a supervisor for 70 students with PhD exams and has authored several books. These books, among others, will be on display during the conference at the SpringerNature table, and it will also feature a preview of Lars-Erik's newest book to be soon published.

Many friends, international collaborators and PhD students enjoyed discussing and doing mathematics in Lars-Erik's house the famous Hotel Infinity, in Luleå for many years. Moreover, Lars-Erik has been an especially successful participant at the Swedish cross-country ski competition Vasaloppet (90 km skiing) being one of around 50 persons, in its long history who reached the finish line for 50 years, or more.

We hope that this conference will be not only a nice opportunity to celebrate an important jubilee of Lars-Erik Persson with many mathematical friends but also forum for the fostering of new ideas and establishing or continuing of new collaborative researches. It will also be a prospect to enjoy of the visit of Karlstad, one of the nice cities of Sweden, and surroundings.

Venue

The conference will be held at University of Karlstad in House 21, Eva Eriksson-ssalen, right in front the big Egg, starting with Tuesday morning. The University can be reached by bus number 1 from the city center, main square.

Organising committee

- **Sorina Barza** - Karlstad University
- **Harpal Singh** - UiT The Arctic University of Norway
- **Hana Turčinová** - Czech Technical University in Prague

Participants

Lars-Erik Persson - celebrity - UiT The Arctic University of Norway and Karlstad University
Sorina Barza - organizer - Karlstad University
Harpal Singh - organizer - UiT The Arctic University of Norway
Hana Turčinová - organizer - Czech Technical University in Prague and Charles University, Prague
Francesco Altomare - invited speaker - University of Bari
Björn Birnir - invited speaker - University of California, Santa Barbara
María Carro - invited speaker - Complutense University of Madrid
Fernando Cobos - invited speaker - Complutense University of Madrid
Hans-Georg Feichtinger - invited speaker - University of Vienna
Amiran Gogatishvili - invited speaker - Czech Academy of Sciences, Prague
Frank Hansen - invited speaker - University of Copenhagen
Sten Kaijser - invited speaker - Uppsala University
Victor Lie - invited speaker - Purdue University
Martin Lind - invited speaker - Karlstad University
Constantin Niculescu - invited speaker - Institute of Mathematics of Romanian Academy, Bucharest
Ryskul Oinarov - invited speaker - L.N. Gumilyov Eurasian National University, Astana
Andrey Piatnitski - invited speaker - UiT The Arctic University of Norway
Luboš Pick - invited speaker - Charles University, Prague
Maria Alessandra Ragusa - invited speaker - University of Catania
Natasha Samko - invited speaker - UiT The Arctic University of Norway
Yoshihiro Sawano - invited speaker - Chuo University, Tokyo
Javier Soria - invited speaker - Complutense University of Madrid
Sanja Varošanec - invited speaker - University of Zagreb
Ferenc Weisz - invited speaker - University of Budapest
Akbota Abylayeva - L.N. Gumilyov Eurasian National University, Astana
Joachim Jørgensen Ågotnes - UiT The Arctic University of Norway
Gorana Aras-Gazić - University of Zagreb
Davit Baramidze - The University of Georgia, Tbilisi, and UiT The Arctic University of Norway
Eduardo Brandani da Silva - Maringa State University
Onorita Elena Buican - National College "Octavian Goga", Sibiu, Romania
Héctor Camilo Chaparro Gutiérrez - Universidad de Cartagena, Colombia
Gregory Aleksandrovich Chechkin
Rune Dalmo - UiT The Arctic University of Norway
Lubomir Dechewsky - UiT The Arctic University of Norway
Roland Duduchava - The University of Georgia
Luz M. Fernández-Cabrera - Complutense University of Madrid
Dorothee D. Haroske - Friedrich Schiller University Jena
Anders Heyden - Lund University
Pankaj Jain - South Asian University, New Delhi

Nikos Kavallaris - Karlstad University
Kristina Krulić Himmelreich - University of Zagreb
Martin Křepela - Czech Technical University in Prague
Abdulhamit Kucukaslan - Ankara Yildirim Beyazit University
Staffan Lundberg - UiT The Arctic University of Norway
Anca Marcoci - Technical University of Civil Constructions, Bucharest
Liviu-Gabriel Marcoci - Technical University of Civil Constructions, Bucharest
Dorothy Mazlum - Springer Nature Switzerland AG
Adrian Muntean - Karlstad University
Madi Muratbekov - L.N. Gumilyov Eurasian National University, Astana
Khanh Nguyen - Karlstad University
Thu Huong Nguyen - University of Oslo
Carmen-Adina Oancea - National College "Octavian Goga", Sibiu, Romania
Walter A. Ortiz - Autonomous University of Barcelona
Alejandro Santacruz Hidalgo - University of Western Ontario
Mario Setta - Karlstad University
Ksenija Smoljak Kalamir - University of Zagreb
Kristoffer Tangrand - UiT The Arctic University of Norway
George Tephnadze - The University of Georgia, Tbilisi
Giorgi Tutberidze - The University of Georgia, Tbilisi, and Ivane Javakhishvili Tbilisi State University
Milela Vinerean Bernhoff - Karlstad University
Eddie Wadbro - Karlstad University
Yosief Wondmagegne - Karlstad University

Program

20 August, Tuesday

09.00–09.20 *opening*

09.30–10.20 Hans-Georg Feichtinger
10.25–11.15 Victor Lie

11.15–11.30 *coffee break*

11.30–12.30 short talks: Alejandro Santacruz Hidalgo
Dorothee D. Haroske
Martin Křepela

12.30–13.30 *lunch*

13.30–14.20 Luboš Pick
14.25–15.15 Amiran Gogatishvili
15.20–16.10 Sanja Varošanec

16.10–16.25 *coffee break*

16.25–18.25 short talks: Abdulhamit Kucukaslan
Pankaj Jain
Eduardo Brandani da Silva
Joachim Jørgensen Ågotnes
Staffan Lundberg
Hana Turčinová

19.00 *Festive dinner*

21 August, Wednesday

09.30–10.20 Javier Soria (online)
10.25–11.15 María Carro (online)

11.15–11.30 *coffee break*

11.30–12.30 short talks: Eddie Wadbro
Nikos Kavallaris
Gregory Aleksandrovich Chechkin

12.30–13.30 *lunch*

13.30–14.20 Maria Alessandra Ragusa

14.25–16.05 short talks: Davit Baramidze
Giorgi Tutberidze
Anca Marcoci
Liviu-Gabriel Marcoci
Kristina Krulić Himmelreich

16.05–16.20 *coffee break*

16.20–17.15 short talks: Ksenija Smoljak Kalamir
Gorana Aras-Gazić
Madi Muratbekov

22 August, Thursday

09.30–10.20 Ferenc Weisz
10.25–11.15 Natasha Samko

11.15–11.30 *coffee break*

11.30–12.30 short talks: Walter A. Ortiz
Roland Duduchava
George Tephnadze

12.30–13.30 *lunch*

13.30–14.20 Ryskul Oinarov
14.25–15.15 Björn Birnir

15.15–15.30 *coffee break*

15.30–16.20 Fernando Cobos

18.00–21.00 *Excursion by boat on Vännern & Dinner*

23 August, Friday

9.30–10.20 Martin Lind
10.25–11.15 Yoshihiro Sawano

11.15–11.30 *coffee break*

11.30–12.30 short talks: Héctor Camilo Chaparro Gutiérrez
Onorita Elena Buican & Carmen-Adina Oancea
Rune Dalmo (online)

12.30–13.30 *lunch*

13.30–14.20 Andrey Piatnitski
14.25–15.15 Frank Hansen

15.15–15.30 *coffee break*

15.30–16.20 Sten Kaijser

16.25 *closing*

Participation canceled: Francesco Altomare, Constantin Niculescu

Invited lectures

Francesco Altomare[1]
University of Bari

On the convergence of sequences of positive linear operators towards composition operators

Abstract: The talk will be devoted to discuss some sufficient conditions under which arbitrary sequences of positive linear operators acting on a given function space E defined on a metric space X, converge towards the composition operator associated with a given mapping $\varphi : X \to X$, i.e., to the operator $T(f) := f \circ \varphi$ $(f \in E)$. The mode of converge in E is understood as the uniform convergence on a given subset of X or the uniform convergence on compact subset of X.

A possible interest for such problem area arises, for instance, from some relevant results obtained very recently in [1, Section 2 and Section 5] and in [3] where the reader can find several applications concerning the limit behaviours of sequences of operators associated with generalized means as well as other ones which are mainly of interest in Approximation Theory (see also [2] and [4, Sections 4-6].)

The developed approach also allows to obtain a unifying reassessment of two milestones of the approximation theory by positive linear operators, namely, Korovkin's theorem and Feller's theorem. In addition, these theorems are extended to the case where the limit operator is a composition operator as well.

Finally some illustrative examples will be presented and, in particular, some approximation properties of Bernstein-Schnabl operators in the framework of convex bounded subsets of Banach spaces will be shown.

Bibliography

[1] F. Altomare, On positive linear functionals and operators associated with generalized means, J. Math. Anal. Appl. 502 (2021), no. 2, Paper No. 125278, 20 pp.

[2] F. Altomare, Korovkin-type theorems and local approximation problems, Expo. Math. 40 (2022), no. 4, 1229-1243.

[3] F. Altomare, Local Korovkin-type approximation problems for bounded function spaces, Rev. R. Acad. Cienc. Exactas Fís. Nat. Ser. A Mat. RACSAM, (2024) 118:88.

[4] D. Popa, An operator version of the Korovkin theorem, J. Math. Anal. Appl. 515 (2022), no. 1, paper no. 126375, 16 pp.

[1] Participation canceled.

Björn Birnir
University of California, Santa Barbara
Prediction of Accelerated Melting of the Northeast Greenland Ice Sheet
Abstract: Due to its responsiveness to changes in the marine environment, G. Rose suggested in 2005 that the capelin, a small pelagic fish that is key to the ecology and fisheries of the North Atlantic, could be seen as a "canary in the coalmine" to detect signals of changes in the Arctic and sub-Arctic Ocean. We describe the historical data that make possible a quantitative assessment of the geographical shift capelin migrations and spawning grounds undergo, with increasing temperature, and the time it takes to make these shifts long-lasting. Then we introduce recent data that make these quantitative measurements more accurate and predictive.

Direct measurements made in the fall expeditions of Iceland's Marine and Fresh- water Research Institute along the East Coast of Greenland, and the Copernicus database of the European Union, are used to examine the evolution of the returning Atlantic water (from Svalbard) that is forming a warmer and saltier boundary current under the colder and fresher East Greenland polar current. The returning Atlantic water (RAW) has a temperature range (1 to 4 degrees Centigrade) suitable for feeding migrations of the capelin. This current is reaching further north along the coast of Northeast Greenland and we use simulated data from Copernicus to monitor this evolution. We calibrate the Copernicus data with the direct measurements made by the Marine and Freshwater Research Institute (MFRI), in Iceland.

A trend emerges, both in the direct measurements and in Copernicus data, showing that the returning Atlantic water (RAW) current may reach Greenland's major northeastern glacier streams, draining the bulk of the Northeastern Greenland Ice Sheet (GrIS) in the near future. We use both the capelin data and the Copernicus simulations to predict that this may happen in less than 12 years.

María Carro
Complutense University of Madrid
Boundary value problems and Muckenhoupt weights
Abstract: We shall present several classical problems concerning the Laplacian on Graph Lipschitz domains in the plane, which can be solved with the theory of Muckenhoupt weights.

This is a Joint work with Teresa Luque (Universidad Complutense de Madrid) and Virginia Naibo (Kansas State University).

Fernando Cobos
Complutense University of Madrid
Convexity estimates for the outer measure of bilinear operators
Abstract: The classical Riesz-Thorin theorem for linear operators between L_p-spaces yields the convexity inequality

$$\|T_{L_p,L_q}\| \leq \|T_{L_{p_0},L_{q_0}}\|^{1-\theta}\|T_{L_{p_1},L_{q_1}}\|^{\theta}$$

provided that $1/p = (1-\theta)/p_0 + \theta/p_1, 1/q = (1-\theta)/q_0 + \theta/q_1$ and $0 < \theta < 1$. This inequality can be extended to compatible couples of Banach spaces and the interpolation spaces obtained by the real (or the complex) method.

A similar estimate holds for the measure of non-compactness $\omega(T)$ of a linear operator T interpolated by the real method. In a more general way, a convexity inequality also holds for the outher measure $\gamma_{\mathcal{I}}$ associated to an ideal $\mathcal{I}$ of linear operators.

Recently a lot of work is being done for establishing convexity inequalities for measures related to ideals of bilinear operators (see, for example, [5, 1, 4]).

In this talk after revising a number of old results, we will describe how to extend $\gamma_{\mathcal{I}}$ to a measure $\gamma_{\mathfrak{I}}$ for bilinear operators R and the convexity inequality that holds for the measure $\gamma_{\mathfrak{I}}(T)$ of a bilinear operator R interpolated by the real method. These last results are taken from joint papers with Fernández-Cabrera and Martínez [2, 3].

References

[1] B.F. Besoy, F. Cobos, *Interpolation of the measure of non-compactness of bilinear operators among quasi-Banach spaces*, J. Approx. Theory **243** (2019) 25–44.
[2] F. Cobos, L.M. Fernández-Cabrera, A. Martínez, *Interpolation of closed ideals of bilinear operators*, Acta Math. Sinica (English Ser.) (to appear).
[3] F. Cobos, L.M. Fernández-Cabrera, A. Martínez, *Convexity estimates for the outer measure of bilinear operators*, Preprint (2024).
[4] A. Manzano, P. Rueda, E.A. Sánchez-Pérez, *Closed surjective ideals of multilinear operators and interpolation*, Banach J. Math. **15** (2021) Article number 27.
[5] M. Mastyło, E.B. Silva, *Interpolation of the measure of non-compactness of bilinear operators*, Trans. Amer. Math. Soc. **370** (2018) 8979-8997.

Hans-Georg Feichtinger
University of Vienna
How Time-Frequency Analysis arose out of Fourier Analysis - The Role of Function Spaces in Gabor Analysis
Abstract: Classical Fourier Analysis, a foundational aspect of Functional Analysis, heavily relies on the properties of function spaces defined by integrability conditions. These function spaces are instrumental in the progression of classical Fourier methods. Abstract Harmonic Analysis unifies results first obtained for periodic and then for square integrable functions on Euclidean spaces. By starting with a locally compact Abelian group, endowed with a Haar measure,

one can introduce the corresponding notion of Fourier transforms. In the case of finite Abelian groups, this yields the classical Discrete Fourier Transform (DFT) and its efficient computation through the Fast Fourier Transform (FFT).

From a practical standpoint, such theoretical considerations have had limited direct impact on the development of modern signal processing algorithms, such as the MP3 compression scheme widely used for audio signals. These signals are not truly L^2-functions but instead have a (discrete-time) short-time Fourier transform (STFT), based on the digital version of the signal, sampled at 44,100 samples per second. This scenario necessitates different function spaces. Collaboration with engineers (e.g. in the area of mobile communication) has reconfirmed a gap in the understanding of the necessary mathematical framework.

Bridging this gap has led to the development of new mathematical methods over the past 40 years, creating a dynamic field of mathematical analysis, known as Time-Frequency Analysis, with Gabor Analysis as a central component. This field utilizes series expansions of non-periodic signals (such as music) into localized Fourier series expansions. The relevant function spaces in this context are modulation spaces, with "Feichtinger's algebra" and its dual, known as the space of "mild distributions", being the most pertinent. Among others, one can discuss new and interesting approximation theoretic questions relevant for applications.

Amiran Gogatishvili
Czech Academy of Sciences, Prague

After 100 years of Hardy inequality still about Hardy inequality

Abstract: The main objective of this talk is to provide a comprehensive survey of Hardy inequalities. In the second part of my talk I will present new results about Hardy inequalities obtained in the last two years.

References

[1] A. Gogatishvili, L. Pick. The two-weight Hardy inequality: a new elementary and universal proof, Proc. Amer. Math. Soc., 2023, in press. DOI: https://doi.org/10.1090/proc/16409

[2] A. Gogatishvili, L. Pick and T. Ünver. Weighted inequalities for discrete iterated kernel operators, Math. Nachr. 295, (2022), no. 11, 2171-2196,

[3] A. Gogatishvili, Z. Mihula, L. Pick, H. Turčinová and T. Ünver. Weighted inequalities for a superposition of the Copson operator and the Hardy operator, J. Fourier Anal. Appl 28, (2022), no. 2, Paper No. 24, 24 pp.

[4] A. Gogatishvili, L. Pick, and T. Ünver. Weighted inequalities involving Hardy and Copson operators. J. Funct. Anal., 283(12): Paper No. 109719, 50, 2022.

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Frank Hansen
University of Copenhagen
Geometric properties for a class of deformed trace functions
Abstract: Numerical inequalities play an important role in both theoretical mathematics and in applications. In the context of non-commutative mathematics it may be of great importance to extend classical notions as inequalities and convexity (concavity) from the numerical domain to that of matrices or operators on a Hilbert space. Lars-Erik Persson reformulated the classical proof of Hardy's inequality to reveal that it is in fact a reflection of convexity. This idea inspired the speaker to extend Hardy's inequality from numbers to operators on a Hilbert space, albeit with a more narrow parameter range.

In the present talk we explore a class of deformed trace functions and determine their geometric properties. As with Hardy's inequality the parameter ranges are reduced in comparison with the classical numerical cases, but allow for general statements about matrices. We obtain corollary statements that may be compared with earlier results of Epstein, Hiai, Carlen and Lieb.

Sten Kaijser
Uppsala University
My young friend Lars-Erik Persson
Abstract:
50 years ago Lars-Erik and I both started in the same Vasalopp.
42 years ago we were both at a conference in Lund about interpolation (of operators and Banach spaces).
32 years ago Lars-Erik organized a wonderful Scandinavian Conference.
30 years ago the first Graduate student of Lars-Erik received his PhD.
27 years ago Lars-Erik and I participated in Ski og Matematikk in Rondane.

It is often said that mathematics is a science for young people, but not many realize that a mathematician can be a young man as long as he lives.

I will talk about achievements by Lars-Erik Persson as a researcher, supervisor, teacher, author of books and perhaps also as a skirunner. I also hope to present an inequality that perhaps not even Lars-Erik Persson knows.

Victor Lie
Purdue University
The LGC method: Recent progress on several problems in harmonic analysis
Abstract: Building on the (Rank I) LGC-methodology introduced by the speaker and on the novel perspective employed in the time-frequency discretization of the non-resonant bilinear Hilbert–Carleson operator (joint work with C. Benea, F. Bernicot and M. Vitturi), we develop a new, versatile approach—referred to as Rank II LGC—that has as a consequence the resolution of the following three problems:

- (joint with my postdoc Bingyang Hu) the boundedness of the trilinear Hilbert transform along the moment curve:

$$T_C(f_1, f_2, f_3)(x) := \text{p.v.} \int_{\mathbb{R}} f_1(x-t) f_2(x+t^2) f_3(x+t^3) \frac{dt}{t}, \quad x \in \mathbb{R}\,.$$

- (joint with C. Benea and F. Bernicot) the boundedness of the hybrid trilinear Hilbert transform:

$$T_H(f_1, f_2, f_3)(x) := \text{p.v.} \int_{\mathbb{R}} f_1(x-t) f_2(x+t) f_3(x+t^3) \frac{dt}{t}, \quad x \in \mathbb{R}\,.$$

- (joint with my graduate student Martin Hsu) the boundedness of the 2D non-resonant curved Carleson–Radon transform:

$$CR(f)(x,y) := \sup_{a \in \mathbb{R}} \left| \text{p.v.} \int_{\mathbb{R}} f(x-t, y-t^2)\, \frac{e^{a\,i\,t^3}}{t}\, dt \right|, \quad (x,y) \in \mathbb{R}^2\,.$$

One of the main difficulties in approaching all of the above problems is the lack of absolute summability for the associated (Rank I) LGC-derived discretized model. In order to overcome this, we design a so-called *correlative* time-frequency model whose control is achieved via the following interdependent elements:

- a sparse-unform decomposition of the input function(s) adapted to an appropriate time-frequency foliation of the phase-space,
- a structural analysis of suitable maximal "joint Fourier coefficients", and
- a level set analysis with respect to the time-frequency correlation set.

Martin Lind
Karlstad University
Limit points and discrepancy of the fractional parts of b^n/n
Abstract: In [2], I determined the exact structure of certain prime subsequences of

$$\left\{\frac{b^n}{n} \pmod 1 : n \in \mathbb{N}\right\} \qquad (b \in \mathbb{N}). \tag{1}$$

This result was used to

- prove denseness of (1) in $[0,1]$;
- improve a discrepancy estimate of Cilleruelo, Kumchev, Luca, Rué and Shparlinski [1].

In the talk, I will discuss uniform distribution modulo 1, discrepancy, as well as (1) and related sequences.

References

[1] J. Cilleruelo, A. Kumchev, F. Luca, J. Rué and I. E. Shparlinski, *On the fractional parts of a^n/n*, Bull. London Math. Soc. 45 (2013), no. 2, 249–256.
[2] M. Lind, *Limit points and discrepancy of the fractional parts of b^n/n*, Acta Arith., to appear.

Constantin Niculescu[2]
Institute of Mathematics of Romanian Academy, Bucharest
Korovkin-type theorems for a class of weakly nonlinear and monotone operators
Abstract: Korovkin's theorem (1953) provides a very simple test of convergence to the identity for any sequence $(T_n)_n$ of positive linear operators that map $C([0,1])$ into itself. Precisely, if

$$\lim_{n\to\infty} T_n(f) = f \quad \text{uniformly on } [0,1]$$

for each of the functions 1, x and x^2, then this holds for all functions $f \in C([0,1])$. The same works if $C([0,1])$ is replaced by $C_{2\pi}(\mathbb{R})$ and the triplet of test functions is replaced by 1, $\cos$ and $\sin$.

Our talk is aimed to report recent work [3] – [7] with Sorin G. Gal (from the University of Oradea). We focus on the extension of Korovkin's theorem to a large class of nonlinear operators which are genuine in the context of Choquet's theory of integration [1], [2]. A sample of our results is as follows:

Theorem 1. (See [3], [5]) *Suppose that X is a locally compact subset of the Euclidean space $\mathbb{R}^N$ and E is a vector sublattice of $\mathcal{F}(X)$ that contains the test functions 1, $\pm \mathrm{pr}_1, ..., \pm \mathrm{pr}_N$ and $\sum_{k=1}^N \mathrm{pr}_k^2$.*

(i) If $(T_n)_n$ is a sequence of monotone and sublinear operators from E into E such that

$$\lim_{n\to\infty} T_n(f) = f \quad \textit{uniformly on the compact subsets of } X$$

[2]Participation canceled.

for each of the $2N+2$ *aforementioned test functions, then this property also holds for all nonnegative functions* f *in* $E \cap C_b(X)$.

(ii) *If, in addition, each operator* T_n *is comonotone additive, then* $(T_n(f))_n$ *converges to* f *uniformly on the compact subsets of* X, *for every* $f \in E \cap C_b(X)$.

Notice that in both cases (i) *and* (ii) *the family of testing functions can be reduced to* $1, -\mathrm{pr}_1, ..., -\mathrm{pr}_N$ *and* $\sum_{k=1}^{N} \mathrm{pr}_k^2$ *when* K *is included in the positive cone of* $\mathbb{R}^N$. *Also, the convergence of* $(T_n(f))_n$ *to* f *is uniform on* X *when* $f \in E$ *is uniformly continuous and bounded on* X.

References

[1] G. Choquet, *Theory of capacities*, Annales de l' Institut Fourier **5** (1954), 131–295.

[2] D. Denneberg, *Non-Additive Measure and Integral*, Kluwer Academic Publisher, Dordrecht, 1994.

[3] S. G. Gal and C. P. Niculescu, *A nonlinear extension of Korovkin's theorem*, Mediterr. J. Math. **17** (2020), Article no. 145.

[4] S. G. Gal and C. P. Niculescu, *Choquet operators associated to vector capacities*, J. Math. Anal. Appl. **500** (2021), article no. 125153.

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[6] S. G. Gal and C. P. Niculescu, *Korovkin type theorems for weakly nonlinear and monotone operators*, Mediterr. J. Math. **20** (2023), issue 2, article 56.

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Ryskul Oinarov

L.N. Gumilyov Eurasian National University, Astana

Professor Lars-Erik Persson is the man who opened doors for Kazakhstanis to achieve their PhDs abroad

Abstract: The presentation will outline the history of the beginning of the work on training joint PhD students. The research topics of each of the joint students, their publications and achievements will also be highlighted. Our scientific contact has not been interrupted so that we will present publications up to today.

Andrey Piatnitski
UiT The Arctic University of Norway
TBA
Abstract: TBA

Luboš Pick
Charles University, Prague
The supremal role of the supremal operators
Abstract: The purpose of this talk will be to point out certain fascinating intrinsic features of operators involving suprema.

The pivotal instance of such operator associates a given measurable function $f\colon (0,\infty) \to \mathbb{R}$ with a new one, defined as $\sup_{s\in(t,\infty)} |f(s)|w(s)$, in which w is a prescribed weight (nonnegative measurable function). Possible modifications include various extra operations such as taking the non-increasing rearrangement of a function or integral means. These operators surfaced in late 1990's and early 2000's in connection with the hunt for Calderón-type estimates for fractional maximal operators, but soon afterwards in turned out that their influence in analysis is ubiquitous.

My intention will be to present a short survey of the history, motivation, fields of applications of the supremal operators, and their relations to other topics. The talk will be closely related to that of Amiran, who will give a similar survey for Hardy-type integral operators.

Maria Alessandra Ragusa
University of Catania
Lars-Erik Persson and regularity problems: a look at the past, a look at the future
Abstract: I show the advances on the regularity problem and present recent results related to minimizers

$$u(x) : \Omega \to \mathbb{R}^n$$

of quadratic and non quadratic growth functionals of the following type

$$\int_\Omega A(x, u, Du)dx,$$

where $\Omega \subset \mathbb{R}^m$ is a bounded domain. About the dependence on the variable x is supposed that $A(\cdot, u, p)$ is in the vanishing mean oscillation class, as a function of x. Then, is pointed out that the continuity of $A(x, u, p)$, with respect to x, is not assumed.

Natasha Samko
University of Tromsø
Weighted singular operators and their commutators in generalized Morrey spaces beyond Muckenhoupt range, and applications to elliptic PDEs

Abstract: We discuss weighted norm estimates for commutators with BMO coefficients of singular operators in local generalized Morrey spaces for a class of radial weights. As a consequence of these estimates, we obtain norm inequalities for such commutators in the generalized Stummel-Morrey spaces.

The existence of singular integrals on functions in spaces under consideration is not clear a priori, even in the almost everywhere sense, and needs justification. We find a general condition on the weight w, function φ and exponent p, under which the singular operators and their commutators are defined almost everywhere on the whole weighted local Morrey space. Note that such a justification of a well-posedness of singular operators is essential because Morrey spaces are not separable and singular operators cannot be treated just as a continuation from a dense set.

The obtained estimates are applied to prove interior regularity for solutions of elliptic PDEs in the frameworks of the corresponding weighted Sobolev spaces based on the local generalized Morrey spaces or Stummel-Morrey spaces. To this end also conditions for the applicability of the representation formula, for the second-order derivatives of solutions to elliptic PDEs, are found for the case of such weighted spaces. In both results, for commutators and applications, we admit weights beyond the Muckenhoupt range.

Yoshihiro Sawano
Chuo University, Tokyo
A substitute for the L^1-norm

Abstract: When we consider the Sobolev embedding, we use the inequality

$$\|f\|_{L^{\frac{n}{n-1}}} \leq \|\nabla f\|_{L^1}$$

for $f \in C_{\mathrm{c}}^{\infty}$. We are oriented to the generalization of this inequality:

i) weighted case,

ii) a passage of the differential order 1 to general order α

iii) a passage to other function spaces.

This attempt was initiated by Armin Schikorra, Daniel Spector, and Jean Van Schaftingen.

This work is based mainly on the joint work with Denny Ivanal Hakim.

Javier Soria
Complutense University of Madrid
Least-estimates: probing effective results. Spectral studies on networks.
Abstract: We study the least doubling constant $C(G)$ among all possible doubling measures defined on a graph G. In particular, for a path graph $G = L_n$, we show that $1 + \cos\left(\frac{\pi}{n+1}\right) \leq C(L_n) \leq 3$, with equality on the lower bound if and only if $n \leq 8$.

For a general G, $C(G)$ can be estimated from below by $1 + r(G)$, where $r(G)$ is the spectral radius of the adjacency matrix of G, and we then study when both quantities coincide.

Finally, we give a complete characterization of graphs with doubling constant smaller than 3, in the spirit of Smith graphs.

This is a joint work with Estibalitz Durand-Cartagena (UNED) and Pedro Tradacete (CSIC).

Sanja Varošanec
University of Zagreb
Recent Progress in the study of continuous forms of inequalities
Abstract: The main goal of this lecture is to present results related to continuous forms of some classical inequalities and their refinements.

Some classical inequalities such as the Hölder inequality and the Minkowski inequality for finitely many functions were generalized to hold for continuously (infinitely) many functions decades ago. For example, the continuous form of the Hölder inequality is given in the following theorem:

Theorem *Let u and v be weight functions on the measure spaces (X, μ) and (Y, ν), respectively, such that $\int_X u(x)\, d\mu(x) = 1$. Let f be a positive function on $X \times Y$ and measurable with respect to the measure $\mu \times \nu$. Then*

$$\int_Y \exp\left(\int_X \log f(x,y) u(x)\, d\mu(x)\right) v(y)\, d\nu(y) \leq \exp\left(\int_X \log\left(\int_Y f(x,y) v(y)\, d\nu(y)\right) u(x)\, d\mu(x)\right).$$

In the last decade, new knowledge has been gained about the continuous forms of inequalities. We will present new results related to the Popoviciu inequality, the Bellman, the Gauss-Pólya, and other inequalities. Refinements of the above-mentioned continuous inequalities will also be presented.

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Ferenc Weisz
University of Budapest

Hardy spaces and applications for trigonometric and Walsh-Fourier series

Abstract: We introduce one- and higher dimensional martingale and classical Hardy spaces and consider trigonometric and Walsh-Fourier series. We state that the maximal operator of the Fejér or Cesàro means of a higher dimensional function is bounded from the corresponding Hardy space to the Lebesgue space. This implies some almost everywhere convergence of the Cesàro means. We characterize the set of convergence as different types of Lebesgue points.

Short talks

Joachim Jørgensen Ågotnes
UiT The Arctic University of Norway

Mechanical properties of Fused Filament Deposition 3D printing

Abstract: Additive manufacturing (AM) has become a popular method of production due to the commercialization of 3D printers. The most common 3D fabrication machine is the Fused Filament Deposition (FDM) machine where a thread of thermoplastic is heated up and extruded through a nozzle, constructing a 3D model layer by layer. As the fabrication method has evolved from producing art objects to functional pieces, the mechanical properties of FDM printing have become more important to understand. However, there is not yet a unified agreed-upon standard that strictly complies with 3D-printed specimens.

The mechanical properties presented have been conducted through tensile testing and verified using Classical Laminate Theory (CLT). Here we investigate the mechanical properties and the impact of the density and raster angles using the *line* infill configuration using the standard ISO 1926:2009. This type of engineering research is strongly supported by mathematical modeling (homogenization theory) as described e.g. in [1] by Lars-Erik Persson with co-authors.

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Gorana Aras-Gazić
University of Zagreb

Generalized Levinson's Inequality for Divided Differences and Averages of 3-convex Functions

Abstract: By employing the generalization of Levinson's inequality, we derive generalized form applicaple for averages of 3-convex functions. The obtained result extends Wulbert's theorem, which asserts that if f is convex on (a, b), then the integral arithmetic mean F is convex on $(a, b) \times (a, b)$. Furthermore, we establish a generalized Levinson's inequality for divided differences. Consequently, we obtain convexities for functions of higher order.

This talk is based on joint work with J. Pečarić and A. Vukelić.

Davit Baramidze
The University of Georgia, Tbilisi, and UiT, Narvik

Some new restricted maximal operators of Fejér means of Walsh-Fourier series

Abstract: The theory of Fourier series with respect to Walsh system deals with decomposition of a function into rectangular waves (for details see the book [2]).

This talk is devoted to characterize maximal subspace of natural numbers such that restricted maximal operator of Walsh-Fejér means on this subspace

is bounded from the martingale Hardy space $H_{1/2}$ to the Lebesgue space $L_{1/2}$ (for details see the book [1]).

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Eduardo Brandani da Silva
Maringa State University

s-Numbers of Multilinear Operators

Abstract: In this work, we investigate s-number variants in the context of multilinear operators. The notion of an s-scale of k-linear operators is defined. In particular, we deal with multilinear variants of approximation numbers, Gelfand, Hilbert, Kolmogorov, and Weyl numbers. We explore if the multilinear case inherits the fundamental properties of s-numbers, which are important for linear operators. We prove relations between some s-numbers of k-linear operators with their classical s-numbers for a generalized adjoint operator of a multilinear operator.

Onorita-Elena Buican & Carmen-Adina Oancea
National College "Octavian Goga", Sibiu, Romania

Some methods and techniques of teaching mathematics in middle school

Abstract: The study of mathematics in compulsory education in Romania aims to provide all students with the formation of basic problem-solving skills involving both algebraic calculus and geometric reasoning. Learning mathematics in middle school aims to raise awareness of the nature of mathematics as an activity of describing and solving problems, using a unitary language, which makes it a discipline, closely related to society through its relevance in everyday life and through its role in Natural Sciences, in Economic Sciences, in Technologies, even in Social Sciences.

The paper further highlights examples on how to integrate different subjects as Biology, Physics, Astronomy, Economy or Music during the Mathematics lessons.

Héctor Camilo Chaparro Gutiérrez
Universidad de Cartagena, Colombia

On a Two Parameter Riesz Type Variation

Abstract: Bounded variation functions are fascinating mathematical objects, which were introduced by Camille Jordan [2] in his article *Sur les séries de Fourier* (1881).

This type of functions has had an impact on various branches of mathematics, such as Real Analysis, Numerical Analysis, Functional Analysis, Measure The-

ory, as well as in applied areas such as image processing and computer graphics generation, signal processing, and financial mathematics. The interested reader may consult the excellent text [1] for more information on this topic.

The classical spaces of bounded variation, $BV([a,b])$, are defined as follows. Recall that a partition P of the interval $[a,b]$ is a finite set $P = \{t_0, t_1, \ldots, t_m\}$ (with $m \in \mathbb{N}$ variable) such that:

$$a = t_0 < t_1 < t_2 < \cdots < t_m = b.$$

We denote by $P([a,b])$ the set of all partitions of the interval $[a,b]$. The number

$$\mathrm{Var}(f,P) := \sum_{j=1}^{m} |f(t_j) - f(t_{j-1})|,$$

is called the (Jordan) variation of f in $[a,b]$ with respect to P. The quantity

$$\mathrm{Var}(f) := \sup_{P\in P([a,b])} \mathrm{Var}(f,P) = \sup_{P\in P([a,b])} \sum_{j=1}^{m} |f(t_j) - f(t_{j-1})|,$$

is called the total (Jordan) variation of f in $[a,b]$.

If $\mathrm{Var}(f) < \infty$, we say that f is a function of bounded variation on $[a,b]$, and we write $f \in BV([a,b])$.

The concept of bounded variation has been generalized over time. For example, given $p \geq 1$, the Wiener variation [3] (1924) is defined as

$$\mathrm{Var}_p(f,P) := \sum_{j=1}^{m} |f(t_j) - f(t_{j-1})|^p.$$

The quantity

$$\mathrm{Var}_p(f) := \sup_{P\in P([a,b])} \mathrm{Var}_p(f,P) = \sup_{P\in P([a,b])} \sum_{j=1}^{m} |f(t_j) - f(t_{j-1})|^p,$$

is called the total (Wiener) variation of f in $[a,b]$.

If $\mathrm{Var}_p(f) < \infty$, we say that f is a function of p-bounded variation on $[a,b]$, and we write $f \in BV_p([a,b])$.

Given $p \geq 1$, the Riesz variation is defined as

$$\mathrm{Var}_p^R(f) := \sup_{P\in P([a,b])} \sum_{j=1}^{m} \frac{|f(t_j) - f(t_{j-1})|^p}{(t_j - t_{j-1})^{p-1}},$$

and is called the total (Riesz) variation of f in $[a,b]$.

If $\mathrm{Var}_p^R(f) < \infty$, we say that f is a function of p-bounded variation in the Riesz sense on $[a,b]$, and we write $f \in RBV_p([a,b])$.

In this talk, we will discuss the variations described above, and also a new biparametric variation similar to the Riesz variation, given by

$$\mathrm{Var}_{(p,q)}^R(f) = \sup_{P\in P([0,1])} \sum_{j=1}^{m} \frac{|f(t_j) - f(t_{j-1})|^p}{(t_j - t_{j-1})^{q-1}},$$

which we will call the total (Riesz) (p,q)-variation of f on $[0,1]$. This is a joint work with Prof. René Castillo (Universidad Nacional de Colombia, Bogotá,

Colombia) and Prof. Oscar Mauricio Guzmán (Universidad de América, Bogotá, Colombia).

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Gregory Aleksandrovich Chechkin

On Higher Integrability of Solutions to Zaremba problem[3]

Abstract: Consider a bounded Lipschitz domain $D \subset \mathbb{R}^d$. We define the Sobolev space of functions $W_2^1(D,F)$, where $F \subset \partial D$ is a closed set, as a completion of functions infinitely differentiable in the closure of D and equal to zero in a neighborhood of F, by the norm $\| u \|_{W_2^1(D,F)} = \left(\int\limits_D v^2\,dx + \int\limits_D |\nabla v|^2\,dx \right)^{1/2}$. We consider the Zaremba problem

$$\Delta u = l \quad \text{in} \quad D, \qquad u = 0 \quad \text{on} \quad F, \qquad \frac{\partial u}{\partial \nu} = 0 \quad \text{on} \quad G, \tag{1}$$

where $G = \partial D \setminus F$, and $\frac{\partial u}{\partial \nu}$ is the outward normal derivative of the function u, and l is a linear functional on the space $W_2^1(D,F)$.

A variational solution to problem (1) is a function $u \in W_2^1(D,F)$ subject the identity $\int\limits_D \nabla u \cdot \nabla \varphi\,dx = \int\limits_D f \cdot \nabla \varphi\,dx$, for any $\varphi \in W_2^1(D,F)$. Here $f = (f_1, \dots, f_d)$, $f_i \in L_2(D)$, appears in the representation of the functional l due to the Hahn-Banach theorem.

Theorem. *If* $f \in \left(L_{2+\delta_0}(D) \right)^d$, *where* $\delta_0 > 0$, *then there are positive constants* $\delta(n,\delta_0) < \delta_0$ *and* C *such that for solution of problem* (1) *the following estimate holds:*

$$\int\limits_D |\nabla u|^{2+\delta} dx \le C \int\limits_D |f|^{2+\delta}\,dx,$$

where C *depends only on* δ_0, *space dimension* d *and also constants* L *involved in the definition of the Lipschitz property of the domain* D.

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Rune Dalmo
UiT The Arctic University of Norway
Old and new on the Peetre K-functional and its relations to real interpolation theory, quasi-monotone functions and wavelets
Abstract: The Peetre K-functional is a key object in the development of the real method of interpolation. As a new basis for further development we present both old and new concerning the K-functional and its relation to quasi-monotone functions, various index numbers and interpolation theory. In particular, we generalize for the K-functional some index conditions known from quasi-monotone functions and generalize the parameter method for the real interpolation space with a parameter function. We describe relations between the Peetre K-functional, interpolation theory and wavelets from an engineering point of view as well as from a mathematical point of view.

This talk is based on joint work with Lars-Erik Persson and Natasha Samko.

Roland Duduchava
The University of Georgia
Convolution integro-dIfferential equations on Lie groups in Generic Bessel Potential Spaces
Abstract: The purpose of the presentation is to discuss the convolution integro-differential equations on Lie groups and their applications to some equations of Mathematical Physics. In this framework we suggest to underline the role of Generic Bessel potential spaces (GBPS) to the structure of underlying Lie group. Definition of GBPS are based on generic differential operators from the Lie algebra of the Lie group. Such generic Bessel potential spaces are adapted better to the investigation of integro-differential (of pseudo-differential) operators on Lie groups.

We concentrate investigation on a Lie groups $\{G, x\circ y\}$ with the group operation $x \circ y$, which are homeomorphic to the Lie group $\{\mathbb{R}^n, x \circ y = x + y\}$. Then on $\{G, x\circ y\}$ we have uniquelly defined Haar measure $d_G\mu$, the Fourier transform $\mathcal{F}_G$, its inverse $\mathcal{F}_G^{-1}$ and generic differential operators $\mathfrak{D}_1, \dots, \mathfrak{D}_n$, generated by the vector fields from the corresponding Lie algebra. The dual group is then $\widehat{G} = \mathbb{R}^n$ and Convolution operators are

$$\boldsymbol{W}^0_{a,G} := \mathcal{F}_G^{-1} a \mathcal{F}_G \ : \ \mathbb{S}(G) \to \mathbb{S}'(G), \tag{2}$$

where the symbol $a(\xi)$ is a distribution on the dual group $a \in \mathbb{S}'(\widehat{G}) = \mathbb{S}'(\widehat{\mathbb{R}^n})$, $\mathbb{S}(G)$ is the Schwartz spaces of fast decaying smooth functions and $\mathbb{S}'(G)$ is the spaces of distributions.

We will expose several examples of Lie groups and corresponding GBPS. Then we concentrate on the investigation of boundary value problems (BVPs) for the Laplace-Beltrami equation on a hypersurface $\mathcal{C}$ with the Lipschitz boundary, containing a finite number of angular points (knots). The Dirichlet, Neumann and mixed type BVPs are considered in two different non-classical setting: A) Solutions are sought in the classical Bessel potential spaces $\mathbb{H}^s_p(\mathcal{C})$, $1 < p < \infty$,

$s > 1/p$; A) Solutions are sought in the generic Bessel potential spaces with weight $\mathbb{GH}^s_p(\mathcal{C}, \rho)$. By the localization the problem is reduced to the investigation of Model Dirichlet, Neumann and mixed BVPs for the Laplace equation in a planar angular domains, also in cases of double angles. Explicit criteria for the Fredholm property and the unique solvability of the initial BVPs in both cases are obtained and, for the Generic Bessel potential spaces also singularities of solutions at knots of the mentioned BVPs are indicated explicitely.

The first part part of the presentation is based on joint results with M. Ruzhanski, D, Cardona, A. Hendrix (Ghent) and the second part-on joint work with M. Caava, M. Tutberidze (Tbilisi).

Dorothee D. Haroske
Friedrich Schiller University Jena

Generalised Morrey smoothness spaces

Abstract: In the recent past, smoothness spaces of Besov or Triebel-Lizorkin type, built upon Morrey spaces $\mathcal{M}_{u,p}(\mathbb{R}^d)$, $0 < p \le u < \infty$, have been studied intensively. These scales $\mathcal{N}^s_{u,p,q}(\mathbb{R}^d)$ and $\mathcal{E}^s_{u,p,q}(\mathbb{R}^d)$, $s \in \mathbb{R}$, $0 < q \le \infty$, have become popular in connection with applications in PDE, but are also interesting for their own sake. They generalise the well-known scales of Besov and Triebel-Lizorkin spaces, $B^s_{p,q}(\mathbb{R}^d)$ and $F^s_{p,q}(\mathbb{R}^d)$, since $\mathcal{M}_{p,p}(\mathbb{R}^d) = L_p(\mathbb{R}^d)$, $0 < p < \infty$. Following a similar intention, the Besov-type and Triebel-Lizorkin-type spaces $B^{s,\tau}_{p,q}(\mathbb{R}^d)$ and $F^{s.\tau}_{p,q}(\mathbb{R}^d)$, $\tau \ge 0$, were introduced and systematically studied in the last years.

Now we follow an idea of Mizuhara and Nakai who introduced in the beginning of the 1990's generalised Morrey spaces $\mathcal{M}_{\varphi,p}(\mathbb{R}^d)$, where $\varphi : (0,\infty) \to (0,\infty)$ stands for a function belonging to a so-called $\mathcal{G}_p$ class, $0 < p < \infty$, and the special setting $\varphi(t) \sim t^{d/u}$, $0 < t < \infty$, covers the case $\mathcal{M}_{u,p}(\mathbb{R}^d)$. In a parallel way one can generalise the spaces $B^{s,\tau}_{p,q}(\mathbb{R}^d)$ and $F^{s.\tau}_{p,q}(\mathbb{R}^d)$ with the help of such a function φ.

We studied basic properties of these new scales of spaces as well as embeddings, decompositions, and local singularity behaviour of distributions in such spaces. One might think, at first glance, that replacing one index (u or τ) by some function $\varphi \in \mathcal{G}_p$ will presumably make the situation more complicated, rather than more transparent and comprehensible. And we admit, that this assumption is surely true in view of the proofs and their technicalities. However, looking at our first results, it turned out that some peculiarities of our recent findings can now be much better classified in the new setting. Roughly speaking, the number of cases to be distinguished when studying embeddings, growth envelopes etc., is reduced to some qualitative limit behaviour of the function φ. We give some survey of our recent results in this direction.

This is joint work with Zhen Liu (Jena), Susana Moura (Coimbra), and Leszek Skrzypczak (Poznań).

Pankaj Jain
South Asian University, New Delhi
On duality principle of Saywer type
Abstract: The duality principle of Sawyer deals with the duality of weighted Lebesgue spaces L^p_v for non-negative non-increasing functions. In this talk, we shall discuss the corresponding duality when the function g is replaced by the integral average $\frac{1}{x}\int_0^x g(t)dt$ or its adjoint $\int_x^\infty \frac{g(t)}{t}dt$. We shall also touch upon the Sawyer's duality in the framework of grand Lebesgue spaces $L^{p)}$ and moreover some applications will be pointed out.

Nikos Kavallaris
Karlstad University
Convergence to steady-states of the normalized Ricci flow
Abstract: In this talk we show convergence of global-in-time solutions towards steady-states, using a gradient inequality of Łojasiewicz type. Our technique infers an alternative proof of the convergence results presented in [1, 2]; it also applies to general two-dimensional surfaces and not only to the unit sphere as it happens with the geometric approach developed in [1, 2]. As a byproduct of our analytical (PDE) approach, we obtain the exponential rate of convergence towards a steady-state in case it occurs as a nondegenerate critical point of a related energy functional, cf. [3]. This is a joint work with Takashi Suzuki (Osaka University, Japan).

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Martin Křepela
Czech Technical University in Prague
Homogeneous rearrangement-invariant norms
Abstract: The talk concerns an ongoing joint work with Javier Soria and Santiago Boza. The concept of a homogeneous function norm will be introduced in the framework of rearrangement-invariant spaces, discussing typical examples and counterexamples of homogeneous r.i. norms and their further properties.

Kristina Krulić Himmelreich
University of Zagreb
Hardy-type inequalities generalized via Montgomery identity
Abstract: In this talk, we give generalization of Hardy's type inequalities by using the Green function and the Montgomery identity. We lean on the idea of the generalization of the Hardy inequality that includes measure spaces with positive σ-finite measures. We provide the result concerning the n-convexity property of the function and establish the connection between new and known result. In order to get upper bounds for the identities related to generalizations of the Hardy's inequality, we obtain Grüss and Ostrowski-type inequalities.

This presentation is based on a joint work with Dora Pokaz, Josip Pečarić and Marjan Praljak.

Abdulhamit Kucukaslan
Ankara Yildirim Beyazit University
Generalized Fractional Integrals in the Vanishing Generalized Weighted Local and Global Morrey Spaces
Abstract: In this study, we prove the boundedness of generalized fractional integral operators I_ρ in the vanishing generalized weighted Morrey-type spaces, such as vanishing generalized weighted local Morrey spaces $\mathcal{VM}^{\{x_0\}}_{p,\varphi}(\mathbb{R}^n, w^p)$ and vanishing generalized weighted global Morrey spaces $\mathcal{VM}_{p,\varphi^{\frac{1}{p}}}(\mathbb{R}^n, w)$ by using weighted L_p estimates over balls.

In more detail, we obtain the Spanne-type boundedness of the generalized fractional integral operators I_ρ from the vanishing generalized weighted local Morrey spaces $\mathcal{VM}^{\{x_0\}}_{p,\varphi_1}(\mathbb{R}^n, w^p)$ to another one $\mathcal{VM}^{\{x_0\}}_{q,\varphi_2}(\mathbb{R}^n, w^q)$ with $w^q \in A_{1+\frac{q}{p'}}$ for $1 < p < q < \infty$, and from the vanishing generalized weighted local Morrey spaces $\mathcal{VM}^{\{x_0\}}_{1,\varphi_1}(\mathbb{R}^n, w)$ to the vanishing generalized weighted weak local Morrey spaces $\mathcal{VWM}^{\{x_0\}}_{q,\varphi_2}(\mathbb{R}^n, w^q)$ with $w \in A_{1,q}$ for $p = 1, 1 < q < \infty$, where $\varphi_1, \varphi_2 \in \mathfrak{M}_{\text{loc}}$ class. We also prove the Adams-type boundedness of the generalized fractional integral operators I_ρ from the vanishing generalized weighted global Morrey spaces $\mathcal{VM}_{p,\varphi^{\frac{1}{p}}}(\mathbb{R}^n, w)$ to $\mathcal{VM}_{q,\varphi^{\frac{1}{q}}}(\mathbb{R}^n, w)$ with $w \in A_{p,q}$ for $1 < p < q < \infty$ and from the vanishing generalized weighted global Morrey spaces $\mathcal{VM}_{1,\varphi}(\mathbb{R}^n, w)$ to the vanishing generalized weighted weak global Morrey spaces $\mathcal{WM}_{q,\varphi^{\frac{1}{q}}}(\mathbb{R}^n, w)$ with $w \in A_{1,q}$ for $p = 1, 1 < q < \infty$, where $\varphi \in \mathfrak{M}_{\text{glob}}$ class. The our all weight functions belong to Muckenhoupt-Weeden classes $A_{p,q}$.

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Staffan Lundberg
UiT The Arctic University of Norway
Note on necessary conditions for the boundedness of a class of sublinear operators between function spaces with homogeneity properties
Abstract: The goal of this note is to attract attention of researches to a certain trick, which allows to easily obtain necessary condition on spaces and an operator for the bound- edness of the operator from one space to another. This trick is known in some special cases but up to my knowledge never before was presented in a general form. This trick is based on homogeneity properties of both the domain and target spaces and operator and is known in the case of the Riesz potential operator for which the action $I^\alpha : L^p(\mathbb{R}^n) \to L^q(\mathbb{R}^n)$ is possible only if $\frac{1}{p} - \frac{1}{q} = \frac{\alpha}{n}$.

We demonstrate here that such kind of necessary conditions may be easily obtained for a big variety of spaces and operators which possess certain homogeneity properties. We show this in a general form for abstract spaces and operators defined on $\mathbb{R}^n$, which is then illustrated in a big number of examples.

Such kind of results maybe also presented in terms of groups instead of $\mathbb{R}^n$, which are equipped with the corresponding dilation operator, but we do not touch it here.

Anca Marcoci
Technical University of Civil Constructions, Bucharest
Factorizations of Weighted Hardy Inequalities
Abstract: For weights that meet the requirements that guarantee the boundedness of the Hardy's integral operator between weighted Lebesgue spaces, we prove factorizations of weighted Lebesgue, Cesàro, and Copson spaces.

Liviu-Gabriel Marcoci
Technical University of Civil Constructions, Bucharest
On duality between some Banach spaces of analytic matrices
Abstract: In this talk, we will present some duality results between Bergman and Bloch spaces.

Madi Muratbekov
L.N. Gumilyov Eurasian National University, Astana
TBA
Abstract: TBA

Walter A. Ortiz
Universitat Autònoma de Barcelona
A note on rearrangement Poincaré inequalities and the doubling condition
Abstract: We introduce Poincaré type inequalities based on rearrangement invariant spaces in the setting of metric measure spaces and analyze when they imply the doubling condition on the underline measure. This work is join Joaquim Martín from Universitat Autónoma de Barcelona (Spain).

Alejandro Santacruz Hidalgo
University of Western Ontario
Monotonicity in Kernel operators and abstract Hardy inequalities
Abstract: We consider operators of the form $f(x) \mapsto \int_U k(x,y)f(y)d\mu(y)$ where (U,μ) is a σ-finite measure space and the kernel nonnegative kernel $k(x,y)$ satisfies a monotonicity condition of its level sets. Examples of these types of operators include Cesaro operators, Hardy averaging operators, and Abstract Hardy operators. We will see that these operators induce a totally ordered subset of the σ-algebra called an ordered core, this will allow us to introduce a definition of monotone functions compatible with the ordered core. We extend the greatest core decreasing minorant, previously only available for functions over the real line, to this new setting. This will allow us to transfer the monotonicity condition from the kernel in certain weighted norm inequalities. As an application of the customization and flexibility provided by this novel definition of monotone functions, we will show new results for Hardy inequalities in metric measure spaces, as well as a new characterization of the boundedness for the Abstract Hardy operator from $L^1 \to L^q$. This talk is based on joint work with Gord Sinnamon.

Harpal Singh[4]
UiT The Arctic University of Norway
A hybrid system for monitoring of remote civil engineering infrastructure
Abstract: Extreme Arctic conditions have adverse effect on the civil engineering infrastructure. Moreover, due to global warming earth is undergoing through a rapid climate change. As a result, storms are getting bigger and the waves in the ocean are becoming more violent. Such severe storms under extreme arctic environment could lead to further degradation of the infrastructure. It requires a lot of logistics to carry out a planned inspection activity of a structure in remote area in arctic regions. Reduced performance of structure or its failure can cause prolonged downtime that could have severe economic effects in a region. Structural health monitoring (SHM) is a technique used for monitoring the health of infrastructure. In this talk I present a new hybrid system based on drones, machine learning and artificial intelligence for SHM of remote civil engineering infrastructure to enhance safety and reduce structures downtime. Moreover, the

[4]The talk rescheduled.

new developments in Fourier Series [1], nonseparable spaces [2] and unbounded orthonormal systems [3] that could improve the signal processing algorithms for damage detection in infrastructure are discussed. A new case study of SHM for Herøysund Bridge is presented.

References

[1] Persson, L. E., Tephnadze, G., Weisz, F. (2022). *Martingale Hardy spaces and summability of one-dimensional Vilenkin-Fourier series.* Springer Nature.

[2] Samko, N., Singh, H. (2023). *A note on contributions concerning nonseparable spaces with respect to signal processing within Bayesian frameworks.* Mathematical Methods in the Applied Sciences, 46(1), 1178-1184.

[3] Akishev, G., Persson, L. E., Singh, H. (2021). *Some New Fourier and Jackson-Nikol'skii Type Inequalities In Unbounded Orthonormal Systems.* Constructive Mathematical Analysis, 4(3), 291-304.

Ksenija Smoljak Kalamir
University of Zagreb

On bounds for generalizations of Steffensen's inequality using Lidstone interpolation

Abstract: Steffensen's inequality is the subject of research in many scientific papers. The research includes not only its refinements and generalizations but also its connection with other classical inequalities. Many generalizations of Steffensen's inequality have been obtained using interpolation polynomials. In this talk, we will consider the generalizations of Steffensen's inequality ob- tained using Lidstone's interpolation polynomial. For these generalizations, we will prove new bounds using the weighted Hermite-Hadamard inequality.

Joint work with Josip Pečarić and Anamarija Perušić Pribanić.

George Tephnadze
The University of Georgia, Tbilisi

Almost everywhere convergence of partial sums and certain summability methods of trigonometric and Vilenkin systems

Abstract: The classical theory of Fourier series deals with decomposition of a function into sinusoidal waves. Unlike these continuous waves the Vilenkin (Walsh) functions are rectangular waves (for details see the book [2]). There are many similarities between these theories, but there exist differences also. Much of these can be explained by modern abstract harmonic analysis, combined with martingale theory.

This talk is devoted to investigating tools which are used to study almost everywhere convergence of the partial sums of trigonometric and Vilenkin systems. In particular, these methods combined with martingale theory helps to give a simpler proof of an analogy of the famous Carleson-Hunt theorem for Fourier series with respect to the Vilenkin system (for details see [2]). We also characterize some subsequences of natural numbers such that partial sums of any integrable

function with such indices converges almost everywhere to this function (for details see [3]).

References

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[2] G. Tephnadze, Almost everywhere convergence of subsequences of partial sums of Vilenkin-Fourier series of integrable functions, *(submitted).*

[3] L. E. Persson, G. Tephnadze and F. Weisz, *Martingale Hardy Spaces and Summability of Vilenkin-Fourier Series*, Birkhäuser/Springer, 2022.

Hana Turčinová

Czech Technical University in Prague and Charles University

Zero traces in Hotel Infinity

Abstract: This talk is dedicated to the geometry of domain $\Omega \in \mathbb{R}^N$, for which it holds that a function $u\colon \Omega \to \mathbb{R}$ satisfying $u/d \in L_a^{1,\infty}(\Omega)$ and $|\nabla u| \in L^p$ is necessarily in the space $W_0^{1,p}(\Omega)$, where $d(x)$ is the distance function from the boundary of Ω. This is part of the results of paper [1], which I partly wrote in the LEP's Hotel Infinity in between inspirational trips in snow traces.

References

[1] A. Nekvinda and H. Turčinová. Characterization of functions with zero traces via the distance function and Lorentz spaces. J. Math. Anal. Appl. 529, no. 1, Paper no. 127567, 2024.

Giorgi Tutberidze

The University of Georgia, Tbilisi, and Ivane Javakhishvili Tbilisi State University

Unconditional convergence of general Fourier series

Abstract: S. Banach proved that for any function, even $f(x) = 1$, where $[0, 1]$, the convergence of its Fourier series with respect to the general orthonormal systems (ONS) is not guaranteed. In this presentation, we find conditions for the functions φ_n of an ONS (φ_n), under which the Fourier series of functions $f \in Lip_1$ are unconditionally convergent almost everywhere. Our research has concluded that the conditions do exist and are the best possible in a certain sense. We also prove that any ONS contains a subsystem such that the Fourier series of any function $f \in Lip_1$ are unconditionally convergent a.e. on $[0, 1]$. Additionally, we have shown that the solutions for these types of problems for the general ONS are trivial for classical ONS (trigonometric, Haar, and Walsh systems).

Eddie Wadbro
Karlstad University

Accomplishing a true wave-focusing acoustic black hole through topology optimization

Abstract: The acoustic black hole, a device that progressively reduces wave propagation velocity and increases amplitude, can be used for wave damping, weak signal sensing, and wave focusing. While transverse elastic waves in beams and plates have been extensively studied over the last 20 years, acoustic waves in fluids have received significantly less attention. Unfortunately, the few types of layouts that have been suggested in the literature turn out to essentially lack the wave-focusing effect, despite their often-good damping properties.

We demonstrate that density-based topology optimization can design a waveguide interior for a strong and robust broadband wave focusing. A complicating factor is that viscothermal effects turn out to be significant in this type of device and need to be considered in the optimization. Instead of using computationally demanding linearized, compressible Navier–Stokes equations, we model wave propagation with the Helmholtz equation in the bulk volume coupled with a Wentzell type generalized impedance boundary condition at solid walls. This boundary condition, potentially present at each element interface in the computational mesh, is scaled by the square of the design variable jump across the interface. Consequently, in the limit of zero–one-valued design variables, the viscothermal losses will only be present at solid–air interfaces. Employing this strategy, we designed a waveguide device (length 255 mm, radius 115 mm) that shows strong wave focusing towards a region at the device end for a broad band of frequencies (400–1000 Hz).

Co-authors: Martin Berggren, Linus Hägg, and Abbas Mousavi