RATE-INDEPENDENT DISSIPATION IN PHASE-FIELD MODELING OF EVOLVING MICROSTRUCTURE

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1. Introduction and description of the model

Shape memory alloys (SMA) are known for the shape memory effect and pseudoelasticity that are associated with the martensitic phase transformation. The material can exist in different phases (austenite, martensite). During the phase transformation the interfaces are created and a part of the energy is stored into them.

Recently, we have developed a rate-dependent phase-field model for evolution of the microstructure. This model belongs to the class of usual viscous-type models, when one phase may transform into another with arbitrarily small thermodynamic driving force. However, experimental results show that for SMA some critical driving force has to be reached for the initiation of the transformation. This is related to the observations that the hysteresis, e.g. in the stress-strain response, does not tend to zero as the rate of loading tends to zero. Thus, the usual viscous models are not capable of describing rate-independent dissipative effects.

The current phase-field model [3] was modified such that it takes into account these phenomena by including a non-smooth mixed-type dissipation potential that combines the viscous and rate-independent contributions, see [4]. The finite element implementation of the model handles the physical constraints on the order parameter together with the non-smooth dissipation potential by a single Lagrange multiplier using the augmented Lagrangian method [2]. By employing this method the originally non-smooth minimization problem is transformed into a smooth saddle-point problem that is convenient to be solved numerically.

2. Results

The main purpose of the numerical examples is to show the differences between the usual viscous dissipation and the rate-independent dissipation. The first example shows the fundamental qualitative difference. Initially, at $t = 0$, a random distribution of two variants of martensite is prescribed in an unconstrained domain. Then, an evolution of the system is simulated until a steady state is attained. Figure 1 shows snapshots of microstructure evolution simulated for the viscous dissipation ($f_\mathrm{c} = 0$) and mixed-type dissipation ($f_\mathrm{c} = 1\,\mathrm{MPa}$), both starting from the same initial distribution. The system evolves such that the total free energy is decreasing, see Figure 2. For a viscous dissipation, the steady state is a pure single variant and the total free energy attains zero. For the mixed-type dissipation, microstructure evolution stops, the steady state is a non-trivial frozen microstructure and the free energy stays positive.

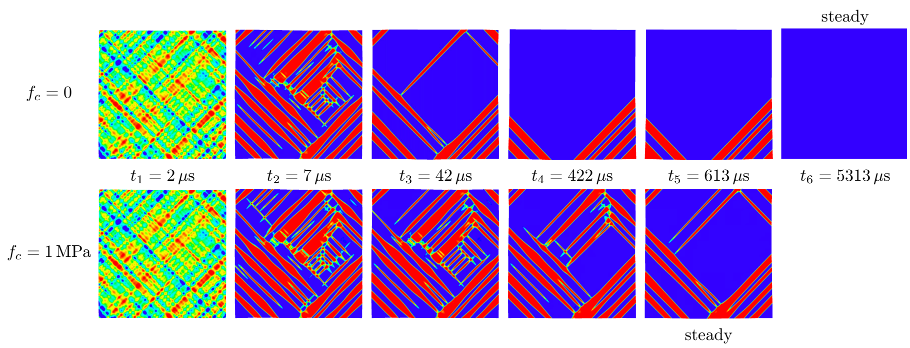


Figure 1: Qualitative difference in the microstructure evolution between the viscous dissipation (top) and mixed-type dissipation (bottom).

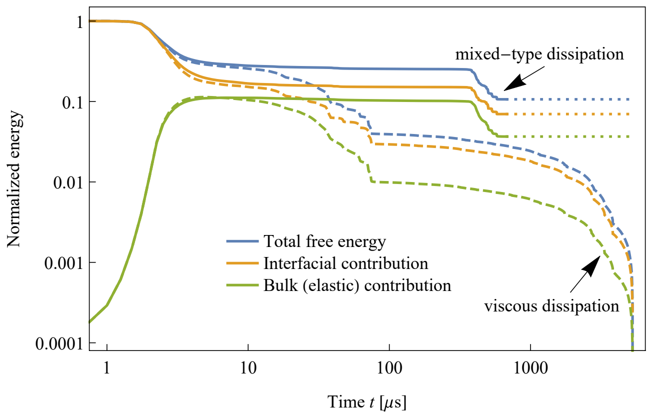


Figure 2: Dependence of free energy on time during microstructure evolution.

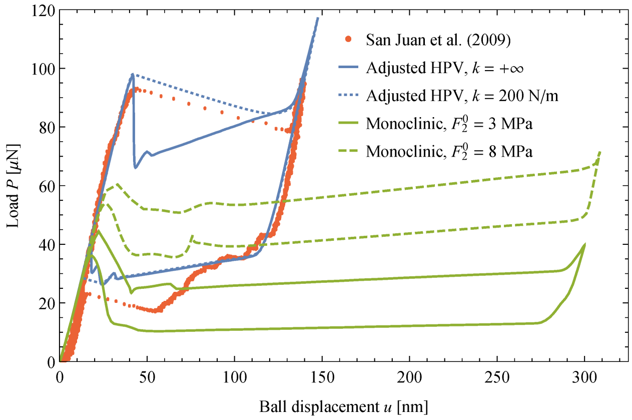
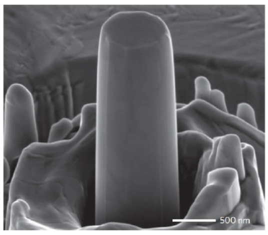


Figure 3: Hysteresis stress-strain curve recorded by San Juan et al. (2009) in the compression experiment.

The second example simulates the experiment by San Juan et al. (2009) in which the micro-pillar is compressed by a nano-indentation device, then released and the corresponding stress-strain curve is recorded, see red points in Figure 3. It shows that the hysteresis curve creates a non-zero area which can not be described by the viscous dissipation. Presented mixed-type dissipation captures the observed curve quite well. Initially, the micro-pillar is in austenite phase and during the compression is being transformed into martensite, see Figure 4.



Reproduced from [1].

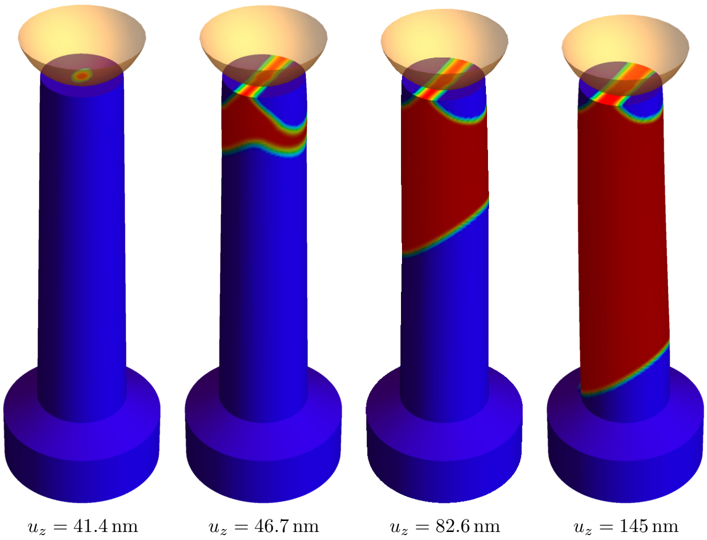


Figure 4: Snapshots of the transformation pattern during the micro-pillar compression (austenite is blue, martensite is red).

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