3D simulation of a wheel tracker test of asphalt concrete described by the Burgers model

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1 Introduction

Asphalt concrete is important for its usage in the construction of the roads, highways or runways. When the cars are running over the surface of the road, the material is being repeatedly compressed. Therefore, it is important to study how the response of the material depends on the applied load and its speed.

This contribution reflects joint research with V. Průša, J. Málek and J.M. Krishnan who studied the response of the asphalt concrete in a wheel tracker test. The experiment was performed for asphalt concrete confined with 200 kPa and containing 2% of air voids. In this abstract we present a numerical simulation of this experiment described by th Burgers model.

2 Description of the experiment

In the experiment, that is being simulated, the sample of the shape of a brick with the dimensions 30 cm × 13.8 cm × 5 cm is subject to the applied stress that starts at $t = 0$ at the top of the brick according to Figure 1 and it moves to the right and then back (this is one cycle) with a constant velocity. The dimensions of the contact area are equal to 2.5 cm × 6 cm and 1 000 cycles are performed. The experiment is performed with two different applied stresses 540 kPa and 800 kPa moving with two different speeds 1 km/h and 10 km/h. The experimental setup is depicted in Figure 1, the dashed line shows the trajectory of the applied stress.

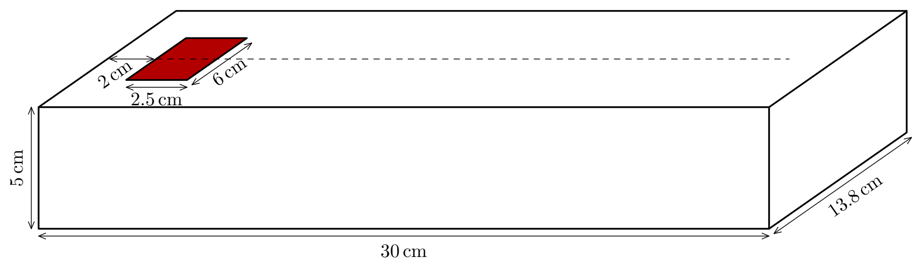

Figure 1: Schematic description of the experimental setup.

3 Mathematical model

Asphalt concrete is simulated by the viscoelastic fluid-like Burgers model that is capable of capturing two different relaxation mechanisms that appear in asphalt concrete. It is assumed that the density ρ of the material is constant, then the balance of mass and balance of linear and angular momentum are in the form

$$\operatorname{div} \mathbf{v} = 0, \tag{1}$$

$$\rho \left(\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v} \right) = \operatorname{div} \mathbf{T}, \quad \mathbf{T} = \mathbf{T}^{\mathrm{T}}, \tag{2}$$

where $\mathbf{v}$ is the fluid velocity. Next, $\mathbf{T}$ is the symmetric Cauchy stress tensor in the form

$$\mathbf{T} = -p\mathbf{I} + \mu_{\mathrm{s}} \left(\nabla \mathbf{v} + (\nabla \mathbf{v})^{\mathrm{T}} \right) + G_1(\mathbf{B}_1 - \mathbf{I}) + G_2(\mathbf{B}_2 - \mathbf{I}), \tag{3}$$

where μ_{s} is the solvent viscosity, G_1 and G_2 are the elastic moduli and $\mathbf{B}_1, \mathbf{B}_2$ are the additional stress tensors. They satisfy the set of evolution differential equations (fully coupled through the velocity field $\mathbf{v}$)

$$\overset{\triangledown}{\mathbf{B}}_1 + \frac{1}{\tau_1}(\mathbf{B}_1 - \mathbf{I}) = 0, \tag{4}$$

$$\overset{\triangledown}{\mathbf{B}}_2 + \frac{1}{\tau_2}(\mathbf{B}_2 - \mathbf{I}) = 0, \tag{5}$$

where $\overset{\triangledown}{\mathbf{B}} = \frac{\partial \mathbf{B}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{B} - (\nabla \mathbf{v})\mathbf{B} - \mathbf{B}(\nabla \mathbf{v})^{\mathrm{T}}$ is the objective upper convected Oldroyd derivative and τ_1, τ_2 are two relaxation times describing two different relaxation mechanisms of the material.

It is worth mentioning that the Burgers model can be derived using the framework of thermomechanics of continuum based on two notions that assure that the second law of thermodynamics is automatically satisfied. The first notion is the principle of the maximum rate of entropy production and the other one is the natural configuration that splits the total deformation into the dissipative part and the purely elastic part that corresponds to that of the compressible neo-Hookean solid. For more details see [1].

We obtain the material parameters $\mu_{\mathrm{s}}, G_1, G_2, \tau_1, \tau_2$ by comparing the predictions of the model to the simple compression experiment. The fitting procedure is based on the minimization of the difference between the measured experimental data and the numerical simulation, for more details see [2, 3]. Fitted material parameters $\mu_{\mathrm{s}} = 2.81\,\mathrm{MPa\ s}$, $G_1 = 52.4\,\mathrm{MPa}$, $G_2 = 9.81\,\mathrm{MPa}$, $\tau_1 = 101.3\,\mathrm{s}$, $\tau_2 = 109.3\,\mathrm{s}$ are then used in the simulation of the wheel tracker test.

4 Numerical implementation

In the present experiment the top boundary of the asphalt concrete brick was deforming. In order to simulate it, the arbitrary Lagrangian-Eulerian (ALE) method is employed. By using a new unknown arbitrary deformation $\hat{\mathbf{u}}$ the standard weak formulation in deforming Eulerian domain Ω_x is transformed to a fixed ALE domain Ω_χ. It is assumed that all points on the boundaries are material points, i.e. the time derivative of $\hat{\mathbf{u}}$ is equal to the fluid velocity $\mathbf{v}$. For more details on the transformation to the ALE domain see [4, 5], where this method is applied in two spacial dimensions.

In three dimensional space the weak formulation in $\Omega_\chi \in \mathbb{R}^3$ is in the form

$$\hat{\mathbf{F}} = \mathbf{I} + \nabla_\chi \hat{\mathbf{u}}, \quad \hat{J} = \det \hat{\mathbf{F}}, \quad \int_{\Omega_\chi} \hat{J} \operatorname{tr} \left((\nabla_\chi \mathbf{v}) \hat{\mathbf{F}}^{-1} \right) q \, \mathrm{d}\chi = 0,$$

$$\int_{\Omega_\chi} \hat{J}\rho \left[\frac{\partial \mathbf{v}}{\partial t} + (\nabla_\chi \mathbf{v}) \left(\hat{\mathbf{F}}^{-1} \left(\mathbf{v} - \frac{\partial \hat{\mathbf{u}}}{\partial t} \right) \right) \right] \cdot \mathbf{q} \, \mathrm{d}\chi + \int_{\Omega_\chi} \hat{J} \hat{\mathbf{T}} \hat{\mathbf{F}}^{-\mathrm{T}} \cdot \nabla \mathbf{q} \, \mathrm{d}\chi = \int_{\partial \Omega_\chi} \hat{\mathbf{t}}_n \cdot \mathbf{q} \, \mathrm{d}S_\chi,$$

$$\hat{\mathbf{T}} = -p\mathbf{I} + \mu_{\mathrm{s}} \left((\nabla_\chi \mathbf{v}) \hat{\mathbf{F}}^{-1} + \hat{\mathbf{F}}^{-\mathrm{T}} (\nabla_\chi \mathbf{v})^{\mathrm{T}} \right) + G_1(\mathbf{B}_1 - \mathbf{I}) + G_2(\mathbf{B}_2 - \mathbf{I}),$$

$$\int_{\Omega_\chi} \hat{J}\left[\frac{\partial \mathbf{B}_i}{\partial t} + (\nabla_\chi \mathbf{B}_i)\left(\hat{\mathbf{F}}^{-1}\left(\mathbf{v} - \frac{\partial \hat{\mathbf{u}}}{\partial t}\right)\right) - (\nabla_\chi \mathbf{v})\hat{\mathbf{F}}^{-1}\mathbf{B}_i - \mathbf{B}_i\hat{\mathbf{F}}^{-\mathrm{T}}(\nabla_\chi \mathbf{v})^{\mathrm{T}} + \frac{1}{\tau_i}(\mathbf{B}_i - \mathbf{I})\right] \cdot \mathbf{Q}_i \,\mathrm{d}\chi = 0,\ i = 1,2,$$

$$\int_{\Omega_\chi} \nabla_\chi \hat{\mathbf{u}} \cdot \nabla \mathbf{r} \,\mathrm{d}\chi = 0,$$

which holds for all admissible test functions $q, \mathbf{q}, \mathbf{Q}_1, \mathbf{Q}_2$ and $\mathbf{r}$. The vector $\hat{\mathbf{t}}_n$ is used for prescribing the time-dependent Neumann boundary condition, i.e. for prescribing the moving compression on the top side of the domain. All other sides of the domain are fixed, i.e. described by zero Dirichlet boundary conditions for $\mathbf{v}$ and $\hat{\mathbf{u}}$.

The numerical implementation is based on this weak formulation and is has been performed using *AceGen/AceFEM* system [6, 7]. *AceGen* is a code generation system and *AceFEM* is a finite element environment that uses the generated code. The system provides the automatic differentiation to compute the exact tangent matrix from the residuum which implies the efficient and robust implementation of the Newton solver.

Due to the symmetry of the problem with respect to the dashed line in Figure 1 only one half is computed. The symmetric part of Ω_χ is discretized by regular hexahedra, pressure p, parts of the Cauchy stress tensor $\mathbf{Q}_1$ and $\mathbf{Q}_2$ are approximated by piecewise discontinuous linear $\mathrm{P1}^{\mathrm{disc}}$ elements, the velocity $\mathbf{v}$ is approximated by piecewise triquadratic H2 elements, and in order to decrease the size of the problem the arbitrary deformation $\hat{\mathbf{u}}$ is approximated by piecewise trilinear H1 elements. The time derivatives are approximated by backward Euler method, nonlinearities are solved with the Newton method and the consequent set of linear equations are solved with the iterative CGS solver with a LU decomposition used as a constant preconditioner (MKL Pardiso). The linear iterations are stopped when the relative residuum reaches 10^{-4} and the stopping condition for the Newton iterations is 10^{-9}.

5 Results

The results are computed on the mesh containing 1 680 hexahedra and described by 88 052 degrees of freedom. The problem is calculated parallely with 24 threads on the system with two Intel Xeon E5-2620 v2, the typical time of the assembly of the residuum and the tangent matrix is 0.85 s, LU decomposition that is needed only once takes 5.8 s and because the solution between two Newton iterations is not changing a lot, usually it is enough to perform only two CGS iterations which take 0.13 s. One compression cycle is approximated by 200 time steps, hence all together 200 000 time steps has to be performed to compute the whole simulation.

Figure 2 shows the snapshot in the 1000^{th} cycle at the time when the 800 kPa compression is at the top in the middle going to the left with the lower speed 1 km/h. The pressure is localized mainly under the compression area. The cumulated deformation of the domain is very small and thus the dependence of the deformation on the cycle number Figure 3a) and also coordinate x Figure 3b) are plotted. The graph in 3b) shows that the upper side is mostly depreciated at $x \doteq 16\,\mathrm{cm}$ which is on the right from the middle and which shows the inertia of the material. The graph in 3a) shows that the deformation of the material is bigger when it is pressed with higher stress or when it moves with a slower speed.

In the next step of our research the simulation will be compared to the experimental results.

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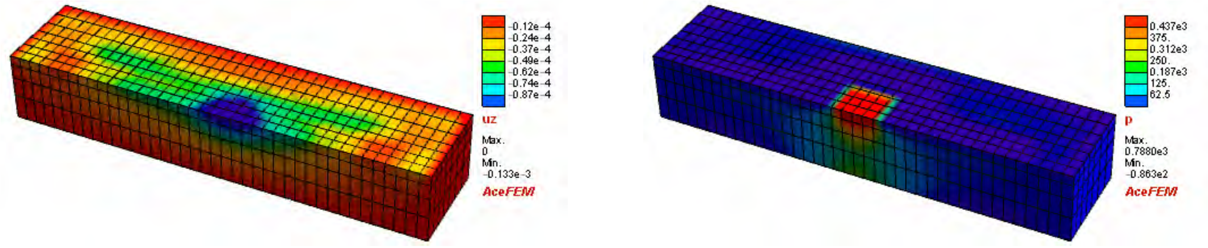

Figure 2: A snapshot of the asphalt concrete pressed with 800 kPa and the speed 1 km/h in cycle 1000 going to the left, displacement u_z [m] (left) and pressure p [kPa] (right).

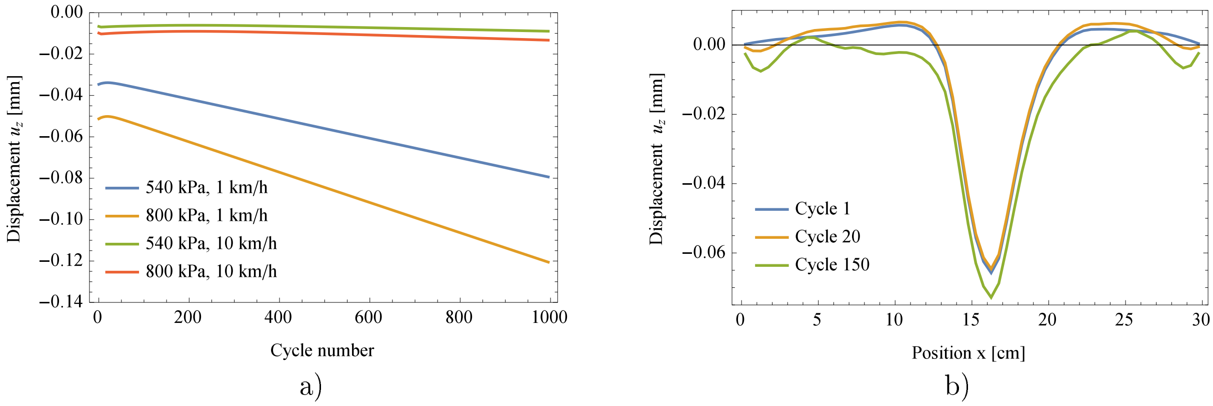

Figure 3: Graph of the dependence of the displacement u_z on the number of the cycle measured at the top in the middle for different speeds and applied pressures (left), and the dependence of the displacement u_z on the position x obtained for the applied pressure 800 kPa and the speed 1 km/h (right).

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