Homework #5 **Deadline: May 13, 2020, 12:00.**

1. Using the balance of mass, compute the material time derivative of gradient of density, i.e.
$$\frac{\mathrm{d}}{\mathrm{d}t}(\nabla\varrho).$$

2. Assuming that $\mathbb{F}_{\kappa_{p(t)}}$ transforms under the change of transformer in the same way as the deformation gradient $\mathbb{F}$, decide whether the following dissipations ξ are objective:

   (i) $\xi = \mathbb{D}_{\kappa_{p(t)}}\mathbb{B}_{\kappa_{p(t)}} \cdot \mathbb{D}_{\kappa_{p(t)}}$

   (ii) $\xi = \mathbb{D}_{\kappa_{p(t)}}\mathbb{C}_{\kappa_{p(t)}} \cdot \mathbb{D}_{\kappa_{p(t)}}$

3. Consider a two-dimensional simple shear flow between two plates ($y = 0$ and $y = h$) of the viscoelastic fluid described by the Oldroyd-B model
$$\begin{aligned}\mathbb{T} &= -p\mathbb{I} + 2\mu_2\mathbb{D} + G(\mathbb{B} - \mathbb{I}),\\ \overset{\triangledown}{\mathbb{B}} &+ \frac{1}{\tau}(\mathbb{B} - \mathbb{I}) = \mathbb{O}.\end{aligned}$$
   Find the steady solution of the Couette problem with the boundary conditions $\mathbf{v} = (0, 0)$ at $y = 0$ and $\mathbf{v} = (u_0, 0)$ at $y = h$, i.e. find velocity $\mathbf{v}$, the tensor $\mathbb{B}$ and the Cauchy stress $\mathbb{T}$. What non-Newtonian phenomena does the model capture?