- 1. The Navier-Stokes equations are obtained by inserting the constitutive equations for the Navier-Stokes fluids (compressible, incompressible non-homogeneous, incompressible homogeneous) into the balance equations for mass and linear momentum. Perform this operation for all three cases and distinguish the situations μ, λ are depending on ϱ or being constant, simplify whenever possible.
- 2. Show that if \mathbf{v} is smooth enough, div $\mathbf{v} = 0$ in Ω and $\mathbf{v} = \mathbf{0}$ on $\partial \Omega$ then the following identity holds

$$2\|\mathbf{D}(\mathbf{v})\|_2^2 = \|\nabla\mathbf{v}\|_2^2,$$

where $\mathbf{D}(\mathbf{v}) = (\nabla \mathbf{v} + (\nabla \mathbf{v})^{\mathrm{T}})/2.$

3. The following table shows the typical values of the viscosities in the units cP.

Fluid	Viscosity [cP]
air $(18 \ ^{\circ}C)$	0.02638
gasoline	0.5
water (18 $^{\circ}C$)	1
milk	3
olive oil	84
motor oil SAE50	540
honey	2000 - 3000
ketchup	50000 - 70000
peanut butter	150000-250000
asphalt / tar	3×10^{10}
Earth lower mantel	3×10^{25} .

What is cP? Is it a unit of kinematic or dynamic viscosity? How is it related to a standard unit? After who is the unit named? Write three pieces of information about this scientist.