Homework #2 **Deadline: March 12, 2019, 13:10.**

1. Consider an incompressible Barus fluid in $\Omega = \mathbb{R}^2 \times (0, d)$ described by

$$\begin{aligned} \operatorname{div} \mathbf{v} &= 0, \\ \varrho \left(\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v} \right) &= \operatorname{div} \mathbf{T} + \varrho \mathbf{b}, \\ \mathbf{T} &= -p\mathbf{I} + 2\mu_0 \exp(\alpha p)\mathbf{D}, \end{aligned}$$

where $\mu_0 = 1$ and $\varrho \mathbf{b} = (0, 0, -\varrho g)^{\mathrm{T}}$, $\varrho, g = const > 0$. This fluid is flowing between two parallel plates located at $z = 0$ and $z = d$. Let further the following boundary conditions are prescribed: $\mathbf{v}\big|_{z=0} = (0, 0, 0)^{\mathrm{T}}$, $\mathbf{v}\big|_{z=d} = (v^*, 0, 0)^{\mathrm{T}}, v^* > 0$ and $p\big|_{z=d} = p_0$.

   a) Show that

$$p(z) = p_0 + \varrho g(d - z)$$

   and

$$\mathbf{v} = \left(v^* \frac{\exp(\alpha \varrho g z) - 1}{\exp(\alpha \varrho g d) - 1}, 0, 0 \right)^{\mathrm{T}}$$

   solve the problem.

   b) Find the solution in the form of steady simple shear flow for the Navier-Stokes fluid in the same geometry and for the same data.

2. Whose name is "hidden" in the unit of dynamic viscosity Poise? Provide few interesting facts about this scientist – what was his area of interest? What is the relation between units P, cP, Pa.s?