Homework #3

1. Consider an incompressible fluid whose Cauchy stress tensor is described by a relation

$$\mathbf{T} = -p\mathbf{I} + \mathbf{S}, \quad (1 + |\mathbf{S}|^2)^{r-2}\mathbf{S} = \mathbf{D}.$$

Study a simple shear flow between two infinite parallel plates located in y = -1 and y = 1. At both boundaries assume no slip boundary condition. Find the solution u(y) using the following viscosity in the calculation: $\mu_0 = 1$ Pa s. Plot the result corresponding to r = -2, 2, 10 (normalize all graphs such that the maximum velocity is equal to one).

2. Prove that a response of a generalized Bingham fluid

$$\begin{split} |\mathbf{S}| &\leq \tau^* \Leftrightarrow \mathbf{D} = \mathbf{0} \\ |\mathbf{S}| &> \tau^* \Leftrightarrow \mathbf{S} = \tau^* \frac{\mathbf{D}}{|\mathbf{D}|} + 2\mu(|\mathbf{D}|^2)\mathbf{D} \end{split}$$

is equivalent to the constitutive relation

$$2\mu(|\mathbf{D}|^2)\mathbf{D} = \frac{(|\mathbf{S}| - \tau^*)^+}{|\mathbf{S}|}\mathbf{S},$$

where $x^+ := \max\{0, x\}, \, \tau^* > 0 \text{ and } \mu(\cdot) : \mathbb{R}^+_0 \to \mathbb{R}^+.$