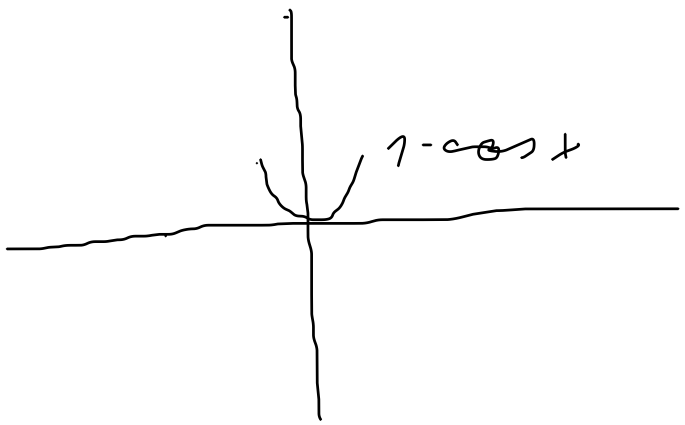
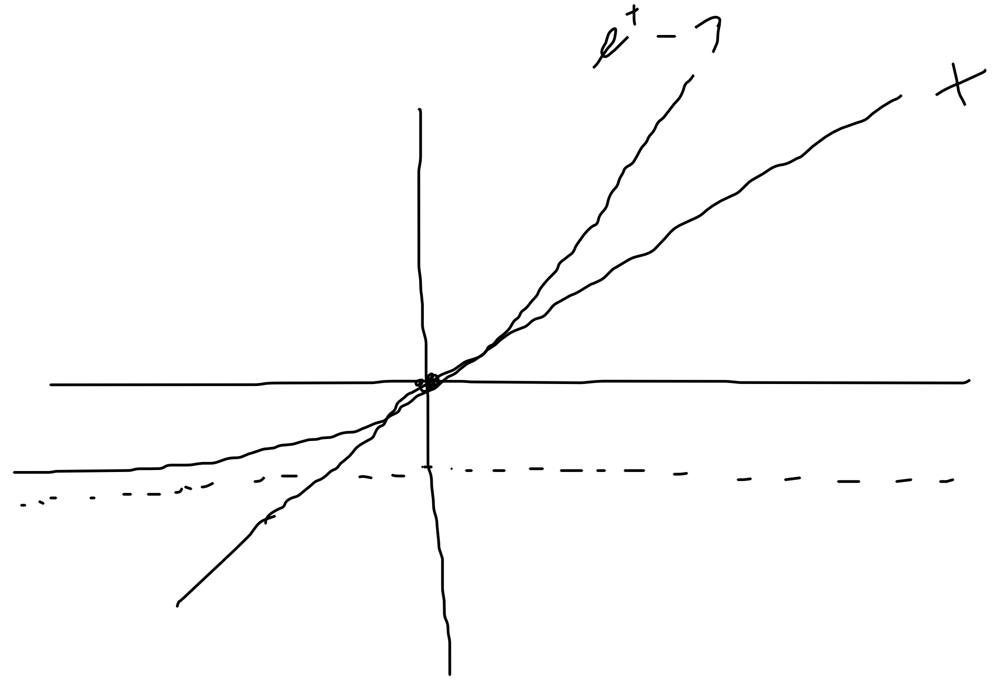
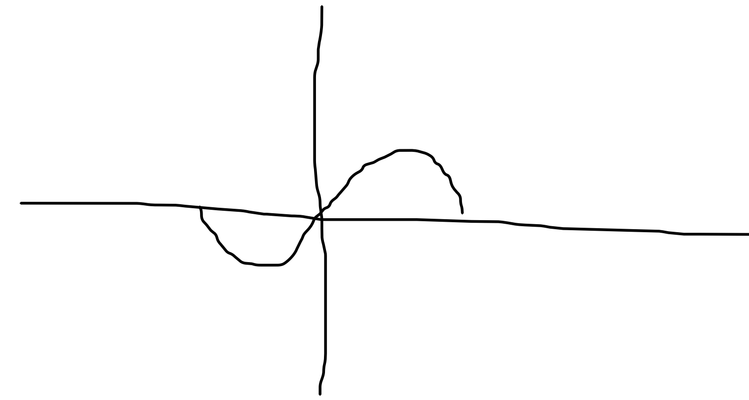
Známé limity:

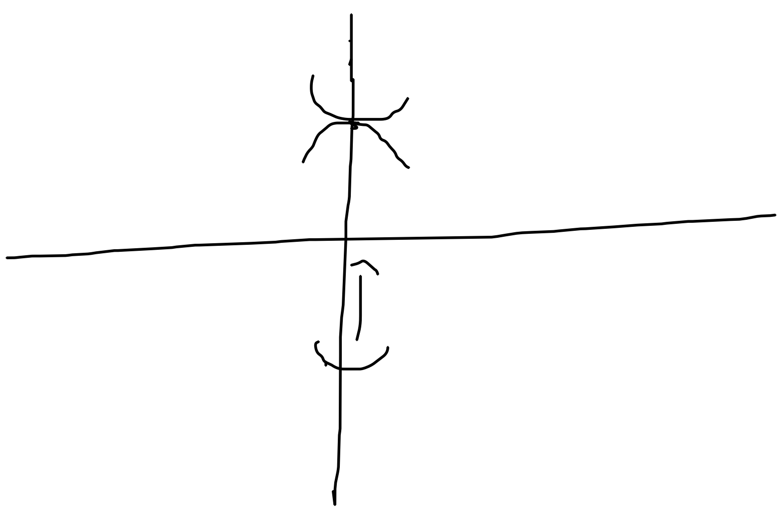
$$\lim_{x\to 0} \frac{e^x - 1}{x} = 1, \quad \lim_{x\to 0} \frac{\sin x}{x} = 1, \quad \lim_{x\to 0} \frac{\ln(1+x)}{x} = 1,$$

$$\lim_{x\to 0} \frac{1-\cos x}{x^2} = \frac{1}{2}.$$









1. $\lim\limits_{x\to 0} \dfrac{\sin x}{\ln(x+1)} = \lim\limits_{x\to 0} \dfrac{\sin x}{x} \cdot \dfrac{x}{\ln(x+1)} \overset{AL}{=} \lim\limits_{x\to 0} \dfrac{\sin x}{x} \lim\limits_{x\to 0} \dfrac{x}{\ln(x+1)}$

$= 1 \cdot 1 = 1.$

2. $\lim\limits_{x\to 2} \left(\dfrac{1}{x(x-2)^2} - \dfrac{1}{\underline{x^2-3x+2}} \right) =$

$= \lim\limits_{x\to 2} \left(\dfrac{1}{x(x-2)^2} - \dfrac{1}{(x-2)(x-1)} \right) =$ $\qquad (x-2)^2 = x^2-4x+4$

$= \lim\limits_{x\to 2} \dfrac{x-1-(x-2)x}{(x-2)^2 x(x-1)} = \lim\limits_{x\to 2} \dfrac{-x^2+3x-1}{(x-2)^2 x(x-1)} =$

$\overset{AL}{=} \lim\limits_{x\to 2} \underbrace{\dfrac{1}{(x-2)^2}}_{\to\,\infty} \lim\limits_{x\to 2} \underbrace{\dfrac{-x^2+3x-1}{x(x-1)}}_{\to\,\frac{1}{2}} = \infty$

__Vezi__: $\lim a_n = 0$, $\boxed{a_n > 0}\ \forall n \Rightarrow \lim\limits_{n\to\infty} \dfrac{1}{a_n} = \infty$

neck $\varepsilon > 0$, vedem $\qquad \dfrac{1}{(x_0-2)^2} = \dfrac{1}{\varepsilon} + 1$

$\dots, \quad x_0 = \dots$

$\forall x \in (2-x_0, 2+x_0) \setminus \{2\} : \dfrac{1}{(x-2)^2} > \dfrac{1}{\varepsilon}$

$\forall \varepsilon > 0\ \exists \delta > 0\ \forall z \in B(2,\delta) : f(z) > \dfrac{1}{\varepsilon}$

3. $\lim\limits_{x\to 0} \dfrac{(1+mx)^n - (1+nx)^m}{x^2}$

$$= \lim_{x\to 0} \frac{\cancel{1} + \cancel{\binom{n}{1} mx} + \binom{n}{2}(mx)^2 + \dots - \left(\cancel{1} + \cancel{\binom{m}{1} nx} + \binom{m}{2}(nx)^2 + \dots\right)}{x^2}$$

$$= \lim_{x\to 0} \left[\left(\binom{n}{2} m - \binom{m}{2} n\right) + \underbrace{\frac{C_1^1 x^3 + C_2^1 x^4 \dots - C_1^2 x^3 - C_2^2 x^4 \dots}{x^2}}_{\to 0} \right]$$

C_i^j jsou konst, nezávislé na x, tedy celý výraz $\to 0$

$$\overset{AL}{=} \frac{n!}{(n-2)!\,2} m - \frac{m!}{(m-2)!\,2} n = \frac{nm(n-1)}{2} - \frac{mn(m-1)}{2}$$

$$= \frac{mn}{2}\big((n-1)-(m-1)\big) = \frac{mn}{2}(n-m)$$

$? = \binom{n}{2} m - \binom{m}{2} n$ z AL a protože zbytek členů je tvaru $C x^k$ kde $k>0$ a tedy jde k nule.

$$\frac{\cdots}{\cdots} \xrightarrow{x\to a} \frac{2}{1+\frac{1}{2}}$$