# Tilting theory in the context of Grothendieck derivators

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# Outline

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# BGP reflections for Dynkin $A_3$ quivers

## Fact (Happel)

Let $k$ be a field. Then $\mathrm{D}(k(\bullet \leftarrow \bullet \rightarrow \bullet)) \simeq \mathrm{D}(k(\bullet \rightarrow \bullet \leftarrow \bullet))$.

## Proof

- Bernstein-Gelfand-Ponomarev reflection functors:

$$s^- : \operatorname{rep}_k(\bullet \leftarrow \bullet \rightarrow \bullet) \longrightarrow \operatorname{rep}_k(\bullet \rightarrow \bullet \leftarrow \bullet)$$

- Then $\mathbf{L}s^- \cong T \otimes^{\mathbf{L}} - : \mathrm{D}(k(\bullet \leftarrow \bullet \rightarrow \bullet)) \xrightarrow{\simeq} \mathrm{D}(k(\bullet \rightarrow \bullet \leftarrow \bullet))$.

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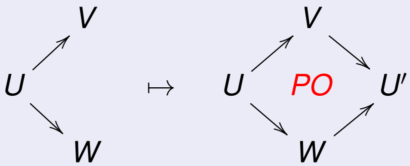


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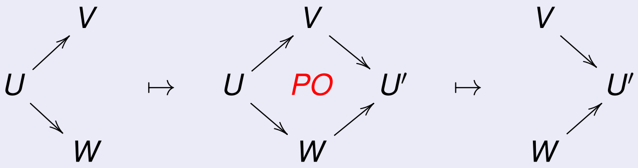


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- We first need to construct the reflection functor $s^-$ for modules/complexes and then derive it.
- We cannot construct the equivalence right away at the level of the derived categories because we cannot construct the pushout there.
- Or can we?

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# Expressing (co)limits abstractly

- Let $\mathcal{C}$ be a cocomplete category and $I \in \mathcal{C}at$ a small category. Then we have

$$\mathcal{C}^I \underset{\text{const}}{\overset{\text{colim}}{\rightleftarrows}} \mathcal{C}.$$

- If say $\mathcal{C}$ is a category of complexes and $W$ the class of quasi-isomorphisms, we just derive the adjoint pair of functors!

$$\mathcal{C}^I[W_I^{-1}] \underset{\text{const}}{\overset{\text{hocolim}}{\rightleftarrows}} \mathcal{C}[W^{-1}],$$

where $W_I$ are the morphisms which are componentwise quasi-isomorphisms and hocolim $= \mathbf{L}$colim.

- One should work with $\mathcal{C}^I[W_I^{-1}]$ rather than $\mathcal{C}[W^{-1}]^I$.
- More explicitly: $\mathrm{D}(\mathrm{Mod}R^I)$ rather than $\mathrm{D}(\mathrm{Mod}R)^I$.

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- Now we obtain $\mathbf{L}s^{-}$ as:

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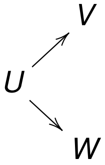


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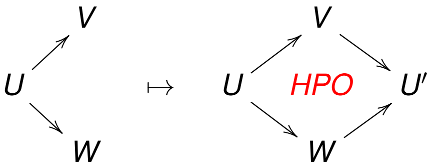


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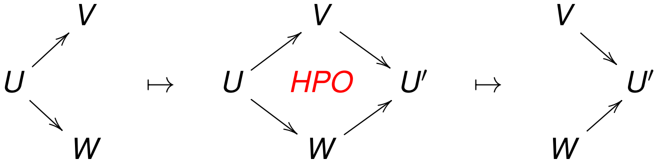


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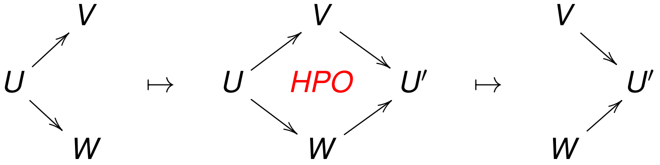


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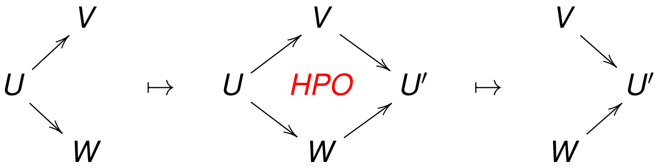


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# Definition

## Idea (Grothendieck, Heller, Franke, others)

Given $(\mathcal{C}, W)$, the category of $I$-shaped diagrams in the homotopy category $\mathcal{C}[W^{-1}]$ contains too little information. We need to remember $\mathcal{C}^I[W_I^{-1}]$ instead, i.e. the homotopy category of $I$-shaped diagrams.

## Definition

A prederivator is a strict 2-functor $\mathscr{D}\colon \mathcal{C}at^{\mathrm{op}} \to \mathrm{CAT}$:

$$\mathscr{D}\colon \quad I \underset{g}{\overset{f}{\rightrightarrows}} J \ (\Downarrow \eta) \quad \longmapsto \quad \mathscr{D}(I) \underset{g^*}{\overset{f^*}{\leftleftarrows}} \mathscr{D}(J) \ (\Downarrow \eta^*)$$

A derivator is a prederivator satisfying certain simple category-theoretic axioms to allow for a well behaved calculus of homotopy Kan extensions.

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# Relevance of representation theory

- Let $k$ be a field. Then the corresponding derivator $\mathscr{D}_k$ is given by

$$\mathscr{D}_k(I) = \mathrm{D}(\mathrm{Mod}\, kI).$$

- Although $\mathscr{D}_k$ enhances the rather uninteresting category $\mathrm{D}(\mathrm{Mod}\, k)$, the derivator itself is very interesting.
- In some sense, the main goal of representation theory is to understand this derivator in detail.
- There is more: Representation theoretic concepts (Auslander-Reiten theory, reflection functors, tilting modules) are very useful in studying general derivators.

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$$\mathscr{D}_k(I) = \mathrm{D}(\mathrm{Mod}\, kI).$$

- Although $\mathscr{D}_k$ enhances the rather uninteresting category $\mathrm{D}(\mathrm{Mod}\, k)$, the derivator itself is very interesting.
- In some sense, the main goal of representation theory is to understand this derivator in detail.
- There is more: Representation theoretic concepts (Auslander-Reiten theory, reflection functors, tilting modules) are very useful in studying general derivators.

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# Surprise #1: homotopy theory sneaks in

- Let $\mathscr{T}$ be the derivator of topological spaces, i.e. $\mathcal{C} = \mathcal{T}op$, $W =$ weak equivalences, and $\mathscr{T}(I) = \mathcal{T}op^I[W_I^{-1}]$.
- Then $\mathscr{T}$ is a universal derivator (Cisinski, Heller). Roughly speaking, given a derivator $\mathscr{D}$ and $X \in \mathscr{D}(*)$, there are a canonical functors

$$\text{“} - \otimes X\text{''}\colon \mathscr{T}(J) \to \mathscr{D}(J), \quad \text{pt.} \mapsto X.$$

- Even more holds. Every derivator is a module over $\mathscr{T}$:

$$\otimes\colon \mathscr{T} \times \mathscr{D} \longrightarrow \mathscr{D}$$

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# Stability

- The derivators $\mathscr{D}$ enhancing derived categories satisfy more:
  1. the base category $\mathscr{D}(*)$ is pointed
  2. homotopy pullbacks = homotopy pushouts (recall the example with reflections again!) back to reflections
- Such derivators are called stable.
- A topological example: The derivator $\mathscr{S}$ of topological spectra.
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# Surprise #2: canonical triangulation

## Theorem (Franke, Maltsiniotis, Groth)

*A stable derivator admits a canonical additive structure, i.e. we actually have a 2-functor*

$$\mathscr{D}\colon \mathcal{C}at^{\mathrm{op}} \to \mathrm{ADD}.$$

*Under an additional mild hypothesis, we even have a canonical triangulated structure:*

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## Remark

Unlike for standalone triangulated categories, the triangulation on a derivator is not an additional structure. It is only a shadow of universal constructions inherent to the derivator.

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# Outline

# Results

## Theorem (Groth & Š., 2013)

*Let $Q, Q'$ be two finite oriented trees with the same underlying graph. Then*

$$\mathscr{D}(Q) \simeq \mathscr{D}(Q')$$

*for any stable derivator $\mathscr{D}$. Moreover, this equivalence can be taken of the form*

$$T \otimes_{[Q]} - : \mathscr{D}(Q) \to \mathscr{D}(Q')$$

*for a suitable spectral bimodule $T \in \mathscr{S}(Q' \times Q)$.*

Other (intended) results and applications:

- A conceptual explanation of May's axioms for tensor triang. cat.
- Stable derivators are also enhancements of various versions of "higher triangulated" categories.
- Equivalences for all quivers without oriented cycles.
- Ambitious: abstract representation theory.

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