Representation Theory of Finite-Dimensional Algebras
NMAG442

Exercise session 5—April 20, 2023

We work over an algebraically closed field k and with finite-dimensional modules.

Euler characteristic, Coxeter transformation, roots and admissible ordering.

Exercise 1. Compute the Euler characteristic and the Coxeter matrix of the algebra kQ where Q is the kronecker quiver.

Exercise 2. Let Q be the quiver

$$1 \longleftarrow 3 \longrightarrow 2.$$

a) Express the reflections $\sigma_1, \sigma_2, \sigma_3$ by their matrices with respect to the canonical basis.

b) Compute the Coxeter trasformation of Q via reflections.

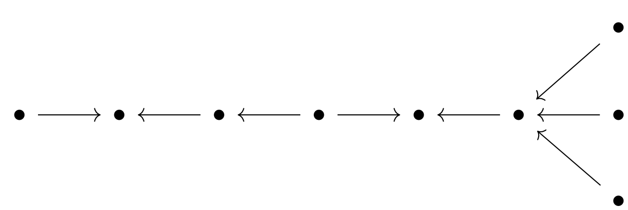
c) Verify that the matrix obtained in point b) is the Coxeter matrix.

Exercise 3. Consider the underlying graph of the quiver A_n, where $n \geq 1$. Show that the positive roots of the associated quadratic form are just the vectors $e_1, \cdots, e_n$ and $e_i + e_{i+1} + \cdots + e_j$ for $1 \leq i < j \leq n$. Then observe that Q affords $\frac{n(n+1)}{2}$ positive roots.

Exercise 4. Label the vertices of the following quivers and find an admissible ordering thereof.

$a)$ $$\bullet \longrightarrow \bullet \longrightarrow \bullet \longrightarrow \bullet \longleftarrow \bullet \longleftarrow \bullet \longleftarrow \bullet \longrightarrow \bullet$$

$b)$



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