Representation Theory of Finite-Dimensional Algebras
NMAG442

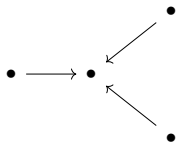
Exercise session 4—April 6, 2023

We work over an algebraically closed field k and with finite-dimensional modules.

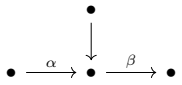
Injective, socle representations and hereditary algerbas.

Exercise 1. In each of the following examples, describe the the indecomposable injectives and their quotients by their socle.

a) Q:



b) Q:

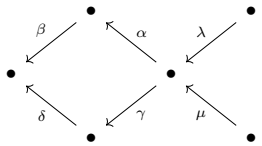


where $\alpha\beta = 0$

c) Q:

where $I = \mathrm{rad}^2 kQ$

d) Q:



where $\mu\alpha = 0, \mu\gamma = 0, \lambda\alpha = 0, \alpha\beta = \gamma\delta$

Exercise 2. Let $a \in Q_0$ be a point in a finite quiver $Q = (Q_0, Q_1)$.

a) Show that the projective kQ-module $P(a)$ is simple if and only if a is a sink.

b) Show that the injective kQ-module $I(a)$ is simple if and only if a is a source.

c) Characterise the points $a \in Q_0$ such that $\mathrm{rad}P(a)$ is simple.

d) Characterise the points $a \in Q_0$ such that $I(a)/S(a)$ is simple.

Exercise 3. Construct an hereditary matrix algebra such that its quiver is equal to a), b) and c) of *Exercise* 1. For the cases b) and c) consider the quiver without relations.

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