$Q: 1 \to 2 \to 3 \to 4$

$$P(1) = (k \xrightarrow{1} k \xrightarrow{1} k \xrightarrow{1} k) = I(4)$$

$$P(2) = (0 \to k \xrightarrow{1} k \xrightarrow{1} k) \xrightarrow{c^-} (k \xrightarrow{1} k \xrightarrow{1} k \to 0) = I(3)$$

$$P(3) = (0 \to 0 \to k \xrightarrow{1} k) \xrightarrow{c^-} (0 \to k \xrightarrow{1} k \to 0) \xrightarrow{C^-} (k \xrightarrow{1} k \to 0 \to 0) = I(2)$$

$$S(4) = P(4) = (0 \to 0 \to 0 \to k) \xrightarrow{C^-} \underset{\overset{\shortparallel}{S(3)}}{(0 \to 0 \to k \to 0)} \xrightarrow{C^-} \underset{\overset{\shortparallel}{S(2)}}{(0 \to k \to 0 \to 0)} \xrightarrow{C^-} \underset{\overset{\shortparallel}{S(1)}}{(k \to 0 \to 0 \to 0)} = I(1)$$

1111

$0111 \xrightarrow{c^{-1}} 1110$

$0011 \xrightarrow{c^{-1}} 0110 \xrightarrow{c^{-1}} 1100$

$0001 \xrightarrow{c^{-1}} 0010 \xrightarrow{c^{-1}} 0100 \xrightarrow{c^{-1}} 1000$

THESE ARE ALL POSITIVE ROOTS FOR A_4

$Q = (2 \to 1 \leftarrow 3)$

$$P(2) = (k \xrightarrow{1} k \leftarrow 0) \xmapsto{\quad C^- \quad} (0 \to 0 \leftarrow k) = S(3) = I(3)$$

$$S(1) = P(1) = (0 \to k \leftarrow 0) \xmapsto{\quad C^- \quad} (k \xrightarrow{1} k \xleftarrow{1} k) = I(1)$$

$$P(3) = (0 \to k \xleftarrow{1} k) \xmapsto{\quad C^- \quad} (k \to 0 \leftarrow 0) = S(2) = I(2)$$

$$110 \xmapsto{\; C^{-1} \;} 001$$

$$010 \xmapsto{\; C^{-1} \;} 111$$

$$011 \xmapsto{\; C^{-1} \;} 100$$

ALL POSITIVE ROOTS OF A_3 HERE!

Q OF TYPE D_4, E.G.

$$Q: \begin{tikzcd}[sep=small] 2 \arrow[dr] & 3 \arrow[d] & 4 \arrow[dl] \\ & 1 & \end{tikzcd}$$

$$P(4) = \left(\begin{tikzcd}[sep=small] 0 \arrow[dr] & 0 \arrow[d] & K \arrow[dl, "1"] \\ & K & \end{tikzcd}\right) \xrightarrow{C^-} \left(\begin{tikzcd}[sep=small] K \arrow[dr, "1"'] & K \arrow[d, "1"] & 0 \arrow[dl] \\ & K & \end{tikzcd}\right) \xrightarrow{C^-} \left(\begin{tikzcd}[sep=small] 0 \arrow[dr] & 0 \arrow[d] & K \arrow[dl] \\ & 0 & \end{tikzcd}\right) = S(4) = I(4)$$

$$P(3) = \left(\begin{tikzcd}[sep=small] 0 \arrow[dr, "0"'] & K \arrow[d, "1"] & 0 \arrow[dl, "0"] \\ & K & \end{tikzcd}\right) \xrightarrow{C^-} \left(\begin{tikzcd}[sep=small] K \arrow[dr, "1"'] & 0 \arrow[d] & K \arrow[dl, "1"] \\ & K & \end{tikzcd}\right) \xrightarrow{C^-} S(3) = I(3)$$

$$P(2) = \left(\begin{tikzcd}[sep=small] K \arrow[dr, "1"'] & 0 \arrow[d] & 0 \arrow[dl] \\ & K & \end{tikzcd}\right) \xrightarrow{C^-} \left(\begin{tikzcd}[sep=small] 0 \arrow[dr, "0"'] & K \arrow[d, "1"] & K \arrow[dl, "1"] \\ & K & \end{tikzcd}\right) \xrightarrow{C^-} S(2) = I(2)$$

$$S(1) = P(1) = \left(\begin{tikzcd}[sep=small] 0 \arrow[dr] & 0 \arrow[d] & 0 \arrow[dl] \\ & K & \end{tikzcd}\right) \xrightarrow{C^-} \left(\begin{tikzcd}[sep=small] K \arrow[dr, "\binom{0}{1}"'] & K \arrow[d, "\binom{1}{0}"] & K \arrow[dl, "\binom{1}{1}"] \\ & K^2 & \end{tikzcd}\right) \xrightarrow{C^-} \left(\begin{tikzcd}[sep=small] K \arrow[dr, "1"'] & K \arrow[d, "1"] & K \arrow[dl, "1"] \\ & K & \end{tikzcd}\right) = I(1)$$

$$\begin{smallmatrix} 0 & 0 & 1 \\ & 1 & \end{smallmatrix} \xrightarrow{C^{-1}} \begin{smallmatrix} 1 & 1 & 0 \\ & 1 & \end{smallmatrix} \xrightarrow{C^{-1}} \begin{smallmatrix} 0 & 0 & 1 \\ & 0 & \end{smallmatrix}$$

$$\begin{smallmatrix} 0 & 1 & 0 \\ & 1 & \end{smallmatrix} \xrightarrow{C^{-1}} \begin{smallmatrix} 1 & 0 & 1 \\ & 1 & \end{smallmatrix} \xrightarrow{C^{-1}} \begin{smallmatrix} 0 & 1 & 0 \\ & 0 & \end{smallmatrix}$$

$$\begin{smallmatrix} 1 & 0 & 0 \\ & 1 & \end{smallmatrix} \xrightarrow{C^{-1}} \begin{smallmatrix} 0 & 1 & 1 \\ & 1 & \end{smallmatrix} \xrightarrow{C^{-1}} \begin{smallmatrix} 1 & 0 & 0 \\ & 0 & \end{smallmatrix}$$

$$\begin{smallmatrix} 0 & 0 & 0 \\ & 1 & \end{smallmatrix} \xrightarrow{C^{-1}} \begin{smallmatrix} 1 & 1 & 1 \\ & 2 & \end{smallmatrix} \xrightarrow{C^{-1}} \begin{smallmatrix} 1 & 1 & 1 \\ & 1 & \end{smallmatrix}$$

THESE ARE ALL POSITIVE ROOTS FOR D_4!

$\tilde{A}_0$: $\quad Q : \; \bullet \circlearrowleft x$

- FIN. DIM. REP. OF Q $=$ FIN. DIM. $K[x]$-MODULE

$$f \circlearrowright M$$

- IF $K = \overline{K}$ $\rightsquigarrow$ IND. FIN. DIM. REPRESENTATIONS ARE UP TO ISO. PRECISELY $J_n(\lambda) \circlearrowright K^n$

$$J_n(\lambda) = \begin{pmatrix} \lambda & 1 & & \\ & \ddots & \ddots & \\ & & \ddots & 1 \\ & & & \lambda \end{pmatrix}$$

$\tilde{A}_n$ WITH CYCLIC ORIENTATION: $\quad Q: \; 1 \to 2 \to 3 \to \cdots \to 1$ $\quad$ KQ — HEREDITARY ORDER

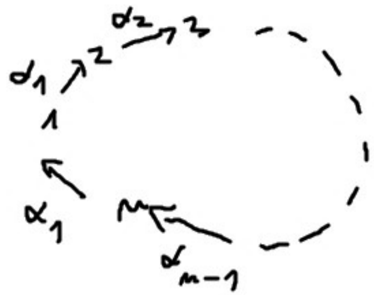
$\rightsquigarrow$ HAVE REPRES. (ALL. INDECOMP)

$$K^n \xrightarrow{1} K^n \xrightarrow{J_n(\lambda)} K^n \xrightarrow{1} \cdots \xrightarrow{1} K^n = M_{\lambda,n}$$

$\rightsquigarrow$ SOME MORE!

<u>REM</u>: $\lambda \neq 0$, $n = 1$: $K \xrightarrow{\lambda} K \xrightarrow{1} K \xrightarrow{1} \cdots$ IS SIMPLE!

THE MORE REP's FOR

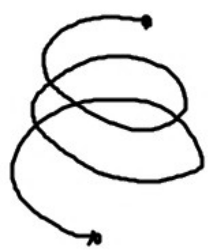


$$\underset{m}{\bullet} \xrightarrow{\alpha_{\bar m}} \underset{\bar m+1}{\bullet} \xrightarrow{\alpha_{\bar m+1}} \bullet \xrightarrow{\alpha_{\bar m+2}} \bullet \; - - - - - - - \; \underset{n}{\bullet} \xrightarrow{\alpha_{\bar n}} \underset{n+1}{\bullet}$$

(INDICES MODULO n)

$1 \to 2 \to 1$

$$K\times K \underset{\binom{1}{0}}{\overset{(0\,1)}{\rightleftarrows}} K$$



— EXPL: $1 \rightleftarrows 2$

$$K \rightleftarrows 0 \;\Big|\; 0 \rightleftarrows K \;\Big|\; K \underset{0}{\overset{1}{\rightleftarrows}} K \;\Big|\; K \underset{1}{\overset{0}{\rightleftarrows}} K \;\Big|\; K \underset{(0\,1)}{\overset{\binom{1}{0}}{\rightleftarrows}} K\times K \;\Big|\; \cdots$$

$\bullet\, 1$ | $\bullet\, 2$ | $1 \to 2$ | $2 \to 1$ | $2 \to 1 \to 2$