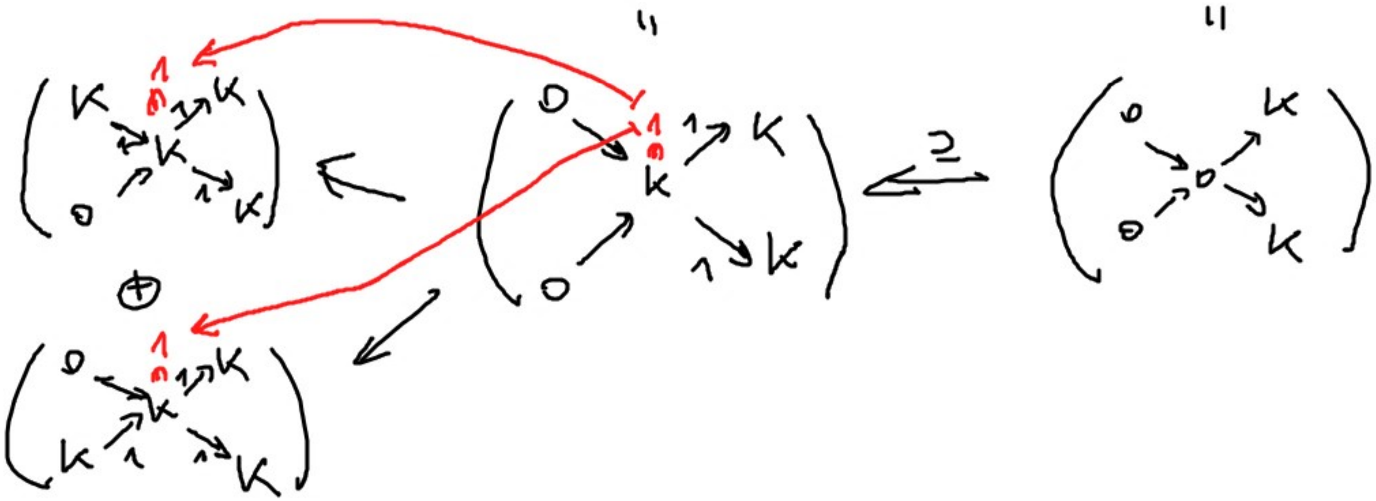
— EXAMPLE

$$\begin{tikzcd}
4 \arrow[dr, "\gamma"] & & 1 \\
& 3 \arrow[ur, "\alpha"] \arrow[dr, "\beta"] & \\
5 \arrow[ur, "\delta"] & & 2
\end{tikzcd}
\qquad i = 3$$

$$P(4) \oplus P(5) \xleftarrow{\ \tau(3) = \begin{pmatrix} \gamma^* \\ \delta^* \end{pmatrix}\ } P(3) \xleftarrow{\ \sigma(3) = (\alpha^* \mid \beta^*)\ } P(1) \oplus P(2)$$



- Q FINITE ACYCLIC, CONNECTED
- WE HAVE $\forall$ ARROW $\alpha: i \to j$ OBTAINED IRREDUCIBLE MORPHISMS

$$\begin{tikzcd}[column sep=small]
 & P(i) \arrow[dr, "\alpha_*"] & & C^{-}P(i) \arrow[dr, "C^{-}\alpha_*"] & & \cdots \\
P(j) \arrow[ur, "\alpha^*"] & & C^{-}P(j) \arrow[ur, "C^{-}\alpha^*"] & & C^{-2}P(j) \arrow[ur] &
\end{tikzcd}$$

WE CAN GO ON LIKE THAT UNTIL $C^{-n}P(i) = 0$ OR $C^{-n}P(j) = 0$
(WE CAN GO ON FOREVER IF Q IS NOT OF DYNKIN TYPE)

- E.G.: $Q = (3 \xrightarrow{\beta} 2 \xrightarrow{\alpha} 1)$

$$\begin{tikzcd}[column sep=small]
 & & P(3) \arrow[dr, "\beta_*"] & & \\
 & P(2) \arrow[ur, "\beta^*"] \arrow[dr, "\alpha_*"] & & C^{-}P(2) \arrow[dr, "\alpha_*"] & \\
P(1) \arrow[ur, "\alpha^*"] & & C^{-}P(1) \arrow[ur, "\alpha^*"] & & C^{-2}P(1)
\end{tikzcd}$$

$C^{-}P(1) = S(2)$, $\quad C^{-2}P(1) = S(3)$

- E.G.: $Q = (2 \overset{\alpha}{\underset{\beta}{\rightrightarrows}} 1)$

$$P(1) \overset{\alpha^*}{\underset{\beta^*}{\rightrightarrows}} P(2) \overset{\alpha_*}{\underset{\beta_*}{\rightrightarrows}} C^{-}P(1) \overset{\alpha^*}{\underset{\beta^*}{\rightrightarrows}} C^{-}P(2) \rightrightarrows C^{-2}P(1) \cdots$$

- P7.3.4: $X = C^{-n}P(i)$, $Y = C^{-m}P(j)$, THEN $\mathrm{Irr}(X,Y) \cong$

- **P7.3.4**: Q FIN. ACYCLIC, $X = C^{-n}P(i)$, $Y = C^{-s}P(j)$, BOTH $X, Y \neq 0$.

THEN

$$\mathrm{Irr}(X,Y) = \frac{\mathrm{Rad}(X,Y)}{\mathrm{Rad}^2(X,Y)} = \begin{cases} k\cdot\{\alpha^* \mid \alpha: j \to i\} \cong kQ_1(j,i) & \ldots\ n=s \\ k\cdot\{\alpha_* \mid \alpha: i \to j\} \cong kQ_1(i,j) & \ldots\ n=s+1 \\ 0 & \ldots\ \text{OTHERWISE} \end{cases}$$

"PF": USE L7.1.3 & REFLECTIONS

- **COR**: (USE HARADA-SAI)

ALL $\alpha^*: C^{-n}P(i) \to C^{-n}P(j)$, $\alpha_*: C^{-n}P(j) \to C^{-n-1}P(i)$

$(\alpha: j \to i)$

GENERATE MORPHISMS BETWEEN PREPROJECTIVES

(SEE NEXT WHITEBOARD ...)

- Q FIN. ACYCLIC QUIVER

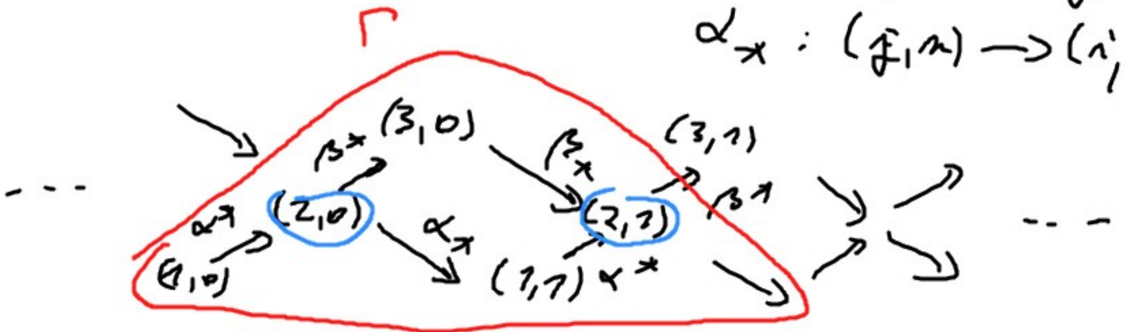
$\rightsquigarrow \mathbb{Z}Q$: ANOTHER QUIVER, $(\mathbb{Z}Q)_0 = (i, n)$, $i \in Q_0$, $n \in \mathbb{Z}$

ARROWS: IF $\alpha : j \to i$, $n \in \mathbb{Z}$ $\rightsquigarrow$ $\alpha^* : (i,n) \to (j,n)$

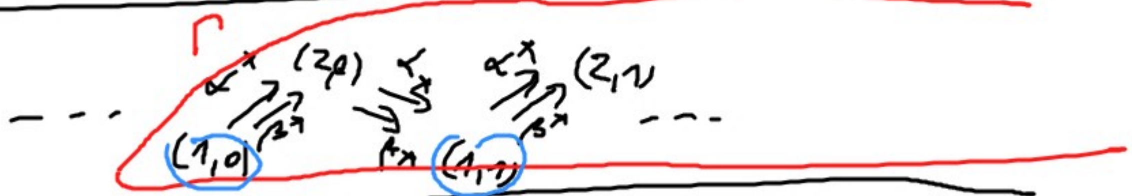
$\alpha_* : (j,n) \to (i,n+1)$

- EXAMPLE: $Q = (3 \to 2 \to 1)$

$\rightsquigarrow \mathbb{Z}Q$:



- EXAMPLE: $Q = (2 \overset{\alpha}{\underset{\beta}{\rightrightarrows}} 1)$



- RELATIONS: IF $i \in Q_0$, $n \in \mathbb{Z}$ SUCH THAT $(i,n), (i,n+1) \in \Gamma$,

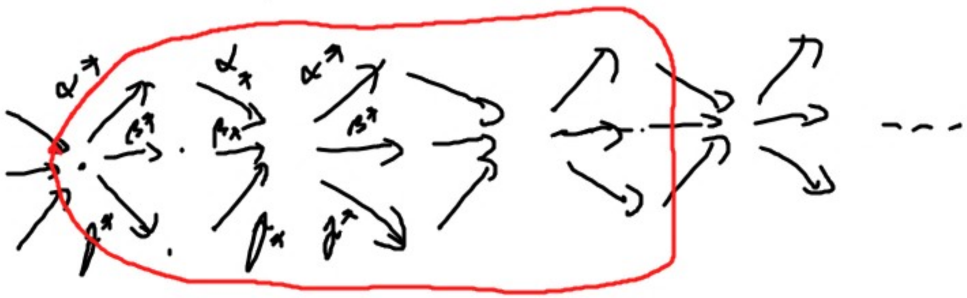
THEN "$\sum \alpha^2 = 0$". PRECISELY $\sum_{i \xrightarrow{\alpha} j} \alpha_* \alpha^* + \sum \beta^* \beta_* = 0$ IN Γ.

- REASON: $\exists$ S.E.S. $0 \to C^{-n}P(i) \longrightarrow \bigoplus_{\alpha : j \to i} C^{-n}P(j) \oplus \bigoplus_{\beta : i \to j} C^{-n-1}P(j) \longrightarrow C^{-n-1}P(i) \to 0$

- EXAMPLE: $\quad Q:$

$$\begin{tikzcd} & 2 \arrow[dl, "\alpha"'] \\ 1 & 3 \arrow[l, "\beta"'] \\ & 4 \arrow[ul, "\gamma"] \end{tikzcd}$$

$\mathbb{Z}Q:$



Γ

$\mathcal{P}:$

$$\begin{tikzcd}[column sep=small] & P(2) \arrow[dr, "\alpha_*"] & & C^{-}P(2) \arrow[dr] & & C^{-2}P(2) \\ P(1) \arrow[ur, "\alpha^*"] \arrow[r, "\beta^*"] \arrow[dr, "\gamma^*"'] & P(3) \arrow[r, "\beta_*"] & C^{-}P(1) \arrow[ur, "\alpha^*"] \arrow[r] \arrow[dr] & C^{-}P(3) \arrow[r] & C^{-2}P(1) \arrow[ur] \arrow[r] \arrow[dr] & C^{-2}P(3) \\ & P(4) \arrow[ur, "\gamma_*"'] & & C^{-}P(4) \arrow[ur] & & C^{-2}P(4) \end{tikzcd}$$

- DIMENSION VECTORS

$$\begin{tikzcd}[column sep=small] & 1\,\substack{1\\0\\0} \arrow[dr] & & 1\,\substack{0\\1\\1} \arrow[dr] & & 0\,\substack{1\\0\\0} \\ 1\,\substack{0\\0\\0} \arrow[ur] \arrow[r] \arrow[dr] & 1\,\substack{0\\1\\0} \arrow[r] & 2\,\substack{1\\1\\1} \arrow[ur] \arrow[r] \arrow[dr] & 1\,\substack{1\\0\\1} \arrow[r] & 1\,\substack{1\\1\\1} \arrow[ur] \arrow[r] \arrow[dr] & 0\,\substack{0\\1\\0} \\ & 1\,\substack{0\\0\\1} \arrow[ur] & & 1\,\substack{1\\1\\0} \arrow[ur] & & 0\,\substack{0\\0\\1} \end{tikzcd}$$