

$$\begin{array}{ccccccc}
 P_2 & \xrightarrow{\partial_2} & P_1 & \xrightarrow{\partial_1} & P_0 & \xrightarrow{\eta} & M \longrightarrow 0 \\
 & & \downarrow f & \swarrow \text{Im } \partial_1 & \downarrow \exists f & & \parallel \\
 \exists: 0 & \longrightarrow & N & \longrightarrow & E & \longrightarrow & M \longrightarrow 0
 \end{array}$$

$\text{Im } \partial_1 \xrightarrow{f} N$
 $\text{Im } \partial_1 \xrightarrow{f} N$
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$$\begin{array}{ccccc}
 \text{Hom}(P_2, N) & \xleftarrow{\partial_2^*} & \text{Hom}(P_1, N) & \xleftarrow{\partial_1^*} & \text{Hom}(P_0, N) \\
 & & \downarrow f + \text{Im } \partial_1^* & & \\
 & & \uparrow & & \\
 & & \text{Ext}_A^1(M, N) & &
 \end{array}$$

$$\begin{array}{ccccccc}
 0 & \longrightarrow & \text{Im } \partial_1 & \xrightarrow{\eta} & P_0 & \longrightarrow & M \longrightarrow 0 \\
 & & \downarrow f & & \downarrow \exists f & & \parallel \\
 \exists: 0 & \longrightarrow & N & \longrightarrow & E & \longrightarrow & M \longrightarrow 0
 \end{array}$$

$$E = \frac{P_0 \oplus N}{\{(2(x), f(x))\}}$$

$$[\Sigma]_{\sim} \in \text{Ext}_A^1(N', M) \longrightarrow \text{Ext}_A^1(N, M) \longrightarrow \text{Ext}_A^1(M, M')$$

$$N' \xleftarrow{g} N, \quad M \xrightarrow{f} M'$$

$$E \subseteq E' \oplus N$$

$$\parallel$$

$$\{(e', m) \mid f'(e') = g(m)\}$$

$$\Sigma: \begin{array}{ccccccc} 0 & \longrightarrow & M & \longrightarrow & E & \longrightarrow & N \longrightarrow 0 \\ & & \parallel & & \downarrow \perp & & \downarrow g \end{array}$$

$$\Sigma': \begin{array}{ccccccc} 0 & \longrightarrow & M & \longrightarrow & E' & \xrightarrow{f'} & N' \longrightarrow 0 \end{array}$$

$$\Sigma: \begin{array}{ccccccc} 0 & \longrightarrow & M & \longrightarrow & E & \longrightarrow & N \longrightarrow 0 \end{array}$$

$$\Sigma'': \begin{array}{ccccccc} & & \downarrow f & & \downarrow \perp & & \parallel \\ 0 & \longrightarrow & M' & \longrightarrow & E'' & \longrightarrow & N \longrightarrow 0 \end{array}$$

$$0 \longrightarrow I_A \xrightarrow{\iota} A \xrightarrow{\pi} A/I_A \longrightarrow 0$$

— IF PROJ. DIM $A/I \leq 1$, THEN

$$\text{Ext}_A^2(A, N) \overset{\cong}{=} \text{Ext}_A^2(A/I, N) \longleftarrow \boxed{\text{Ext}_A^1(I, N)} \longleftarrow \text{Ext}_A^1(A, N)$$

$\overset{\cong}{\parallel} 0$ $\overset{\cong}{\parallel} 0$ $\overset{\cong}{\parallel} 0$

PROJ. DIM. $A=0$

$\forall N_A$

$$\Rightarrow \text{Ext}_A^1(I, -) \equiv 0$$

\Rightarrow

PROJ. DIM $I = 0$

\Rightarrow

I PROJECTIVE