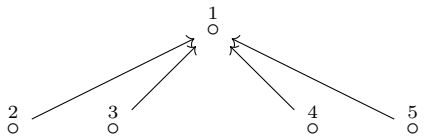
NMAG442 Representation Theory of Finite-Dimensional Algebras

Excercise session 6—May 7, 2020

This exercise session is concerned mainly with representations of Euclidean (extended Dynkin) quivers and wild phenomena. The main references are [2] and [1]. There ia also an easy exercise on admissible orderings of vertices of quivers.

Representations of Euclidean quivers

Exercise 1 (Remark 2 in §9 in [1]). Find regular simple representations (representations with no regular subrepresentation) of the 4-subspace quiver:

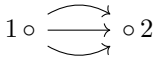


They are of dimension vectors $(1, a, b, c, d)$ (two of a, b, c, d are equal to 0 and two to 1) and $(2, 1, 1, 1)$.

Exercise 2. Compute the Coxeter transformation for the Euclidean quiver of type $\tilde{A}_3$ with a chosen orientation, and find dimension vectors of preprojective representations over this quiver. Describe some of its preprojective representations.

Wild phenomena

Exercise 3 (Inspired by section 10.2 in [2]). Exhibit $1 - \langle \alpha, \alpha \rangle$-dimensional families of bricks (representations with only trivial endomorphisms) for dimension vectors α of $(1, 1), (1, 3)$ and $\{(n, n+1)\}_{n \geq 1}$ of the quiver $K(3)$:



(Hints: All the families may be paramterized by affine spaces of respective dimensions, for instance. You may use the result of exercise 1 from the 3rd exercise session.)

References

[1] Crawley-Boevey, W. Lectures on representations of quivers.

[2] Krause, H. Representations of quivers via reflection functors. *arXiv preprint arXiv:0804.1428* (2008).

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