- T TĚLESO, $\operatorname{char} T \neq 2$
- $f = ax^2 + bx + c \in T[x]$, $a \neq 0$

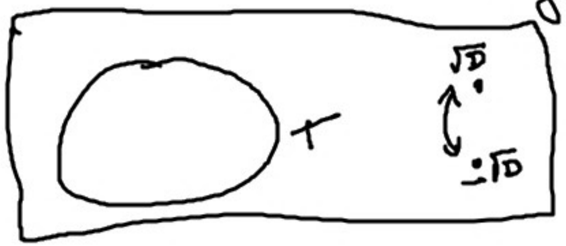
$$\Longrightarrow \quad x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2}$$

- ROZKLAD. NADTĚLESO f NAD T JE $S = T\left(\sqrt{b^2-4ac}\right) = T(\sqrt{D})$

  - POLYNOM $x^2 - D$ MÁ V T KOŘEN $(\sqrt{D} \in T)$ $\leadsto S = T$, $\mathrm{Gal}(S/T) = \{id\}$
  - $\sqrt{D} \notin T \leadsto m_{\sqrt{D},T} = x^2 - D \leadsto [S:T] = 2$, BÁZE S NAD T JE $\{1, \sqrt{D}\}$

$$\leadsto \mathrm{Gal}(S/T) = \{id, \text{"}\sqrt{D} \mapsto -\sqrt{D}\text{"}\} \cong \mathbb{Z}_2$$

VIZTE L2.2



- GALOISOVA GRUPA
  IREDUCIBILNÍHO KUBICKÉHO POLYNOMU $f \in T[x]$, CHAR $T = 0$
- $S :=$ ROZKLADOVÉ NADTĚLESO

$\rightsquigarrow$ $\mathrm{NSD}(f, f') = 1$ V $T[x]$ $\rightsquigarrow$ f MÁ V S 3 RŮZNÉ KOŘENY

$\rightsquigarrow$ $\mathrm{Gal}(S/T) \hookrightarrow S_3$, $|\mathrm{Gal}(S/T)| \geq 3$ (T2.5, část 2)

LAGRANGE $\Longrightarrow$

$$|\mathrm{Gal}(S/T)| = \begin{cases} 3 & \ldots\ \mathrm{Gal}(S/T) \cong \underbrace{\{id, (123), (132)\}}_{\cong \mathbb{Z}_3} \subseteq S_3 \\ 6 & \ldots\ \mathrm{Gal}(S/T) \cong S_3 \end{cases}$$

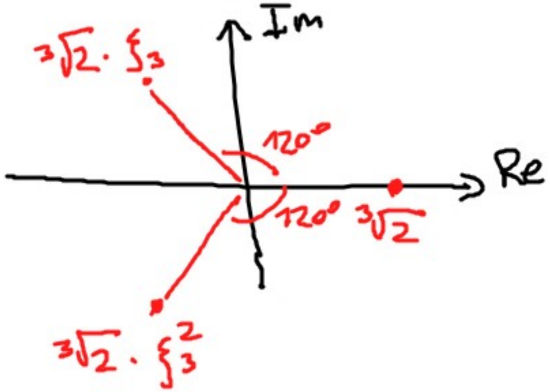
– PŘ: $Gal(x^3-2 / \mathbb{Q})$

– ROZKLAD. NADTĚLESO

$$\mathbb{Q} \leq \mathbb{Q}(\sqrt[3]{2}) \leq \mathbb{Q}(\xi_3, \sqrt[3]{2}) = S$$

$\wedge\!|$

$\mathbb{R}$



$$\boxed{\xi_3 = e^{\frac{2\pi i}{3}}}$$

$m_{\xi_3,\mathbb{Q}} =$

$m_{\xi_3,\mathbb{Q}(\sqrt[3]{2})} =$

$$\frac{x^3-1}{x-1} = x^2+x+1$$

$$[\mathbb{Q}(\sqrt[3]{2}):\mathbb{Q}] = 3, \quad [S:\mathbb{Q}(\sqrt[3]{2})] = 2 \Rightarrow [S:\mathbb{Q}] = 6$$

– UKÁŽEME, ŽE $Gal(S/\mathbb{Q}) \cong S_3$

– VEZMEME $\pi \in S$, $\pi(1) = k$ $\rightsquigarrow$ $\exists$ Q-ISO. $\mathbb{Q}(\sqrt[3]{2}) \xrightarrow{\varphi_0} \mathbb{Q}(\sqrt[3]{2}\cdot\xi_3^k)$

(V2.1, L2.2)

$\wedge\!|$ $\wedge\!|$

$\exists\varphi_1: \xi_3 \longmapsto \xi_3$

$\exists\varphi_2: \xi_3 \longmapsto \xi_3^2$

(L.2.2) $\mathbb{Q}(\sqrt[3]{2}, \xi_3) \xrightarrow{\exists\varphi} \mathbb{Q}(\sqrt[3]{2}, \xi_3)$

KOŘEN. NADTĚLESO $x^2+x+1 / \mathbb{Q}(\sqrt[3]{2})$

– PŘ: $\mathrm{Gal}\left(x^3 - \frac{3}{4}x - \frac{1}{8} / \mathbb{Q}\right)$

– PŘIPOMENUTÍ: $\cos(3\alpha) = 4(\cos\alpha)^3 - 3$

$\alpha = 20°$: $\frac{1}{2} = 4(\cos 20°)^3 - 3 \rightsquigarrow (\cos 20°)^3 - \frac{3}{4}(\cos 20°) - \frac{1}{8} = 0$

$\alpha = 140°$: $(\cos 140°)^3 - \frac{3}{4}(\cos 140°) - \frac{1}{8} = 0$

$\alpha = 260°$: – – – – – –

– ROZK. NADTĚLESO $S = \mathbb{Q}(\cos 20°, \cos 140°, \cos 260°) \supseteq \mathbb{Q}$

$\left(f = (x - \cos 20°)(x - \cos 140°)(x - \cos 260°)\right)$

– OBECNĚ: $n \geq 1 \rightsquigarrow \exists\, t_n \in \mathbb{Q}[x] : \cos(n\alpha) = t_n(\cos\alpha)$

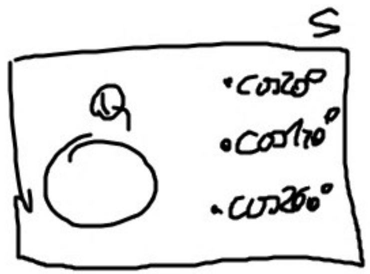
(ČEBYŠEVOVY POLYNOMY)

https://mathworld.wolfram.com/Multiple-AngleFormulas.html

https://en.wikipedia.org/wiki/Chebyshev_polynomials

$\rightsquigarrow \cos 140° = t_7(\cos 20°), \quad \cos 260° = t_7(\cos 140°)$

$\Rightarrow S = \mathbb{Q}(\cos 20°), \quad \mathrm{Gal}(S/\mathbb{Q}) \cong \mathbb{Z}_3$



Galoisova grupa $x^n - a$ obecně

Tvrzení 2.6, část 3.
Buď $\mathbb{Q} \leq T \leq \mathbb{C}$ a $n \geq 1$ a $a \in T$. Pak $\mathbf{Gal}(x^n - a/T)$ je metabelovská grupa.

– PŘ:

- $Gal(x^3-2) \cong S_3$
- $Gal(x^4-2) \cong D_8$
- $Gal(x^5-2) \cong ?$ (DÚ)

$\boxed{T=\mathbb{Q}}$