More invariants and distinguishing mirror images

Definition 1: By K^* we denote the mirror image of K. And similarly by D^* we denote the mirror diagram of knot diagram D.

Definition 2: By M_D we denote the Seifert matrix of the diagram D as constructed in the previous lectures.

Definition 3: Recall that signature of a symmetric matrix A (think of it as a quadratic form) is $\sigma(A) = p - n$, where p is the number of positive entries in its diagnoal form and n the number of negative entries in its diagonal form. Also we will denote $z(A)$ the nullity of A. (This can be also viewed as the number of zeroes in diagonal form)

Definition 4: For diagram D of a knot K we denote $\sigma(D) = \sigma(M_D + M_D^T)$ and $z(D) = z(M_D + M_D^T)$. It turns out these are also knot invariants, hence we will use them in notation with knots as $\sigma(K)$ and $z(K)$.

Exercise 1: Show that Alexander polynomial cannot distinguish mirror image of a knot.

Definition 5: Recall the S-equivalence operations Λ_1, Λ_2

$$\Lambda_1 \colon M_1 \mapsto P M_1 P^T$$

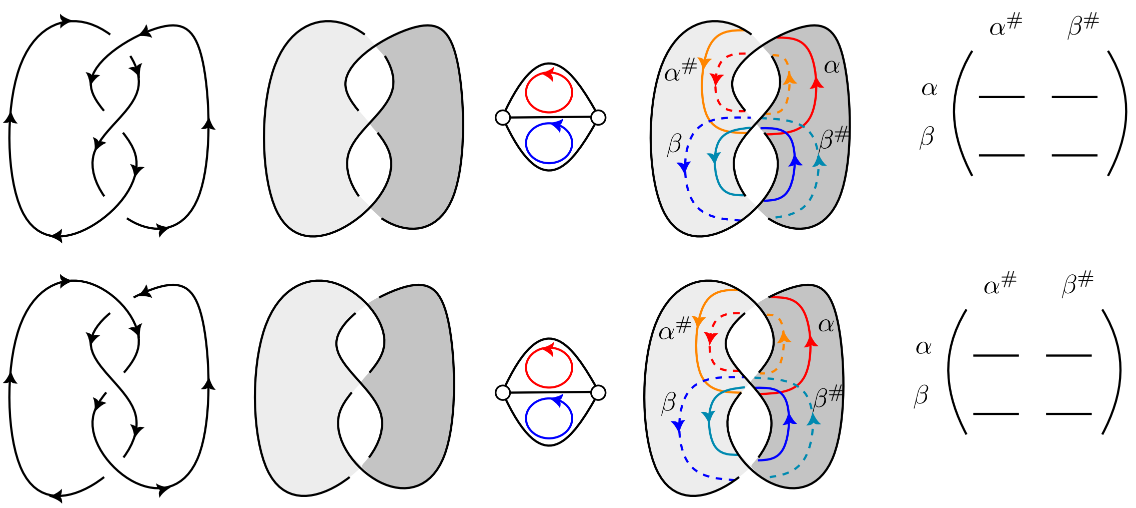
$$\Lambda_2 \colon M_1 \mapsto \begin{pmatrix} & & & * & 0 \\ & M_1 & & \vdots & \vdots \\ & & & * & 0 \\ 0 & \dots & 0 & 0 & 1 \\ 0 & \dots & 0 & 0 & 0 \end{pmatrix} \text{ or } \begin{pmatrix} & & & 0 & 0 \\ & M_1 & & \vdots & \vdots \\ & & & 0 & 0 \\ * & \dots & * & 0 & 0 \\ 0 & \dots & 0 & 1 & 0 \end{pmatrix}$$

Where P is integer matrix with determinant ± 1.

And recall that matrices A and B are said to be S-equivalent if one can be obtained from the other using finite number of Λ_1 and Λ_2 operations and their inverses.

Exercise 2: Show that if A is S-equivalent to B, then $\sigma(A + A^T) = \sigma(B + B^T)$ and $z(A + A^T) = z(B + B^T)$. From this we can conclude that σ and z are indeed knot invariants.

Exercise 3: Using the following diagrams of the two trefoils, fill in their Seifert matrix and calculate their signature. If you have time, calculate it for the figure eight knot.



Exercise 4: Show that $\sigma(K^*) = -\sigma(K)$