

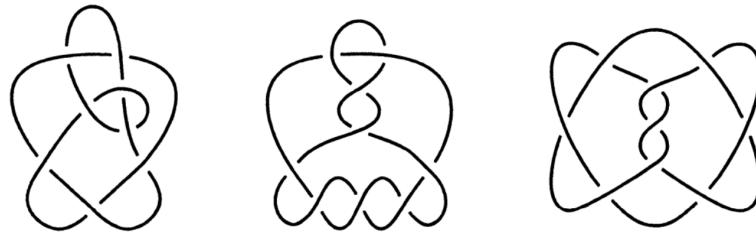
# Algebraic Invariants in Knot Theory

## Practicals 4

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**Exercise 1 (4.6.4 rev).** Are the following knots 3-colorable?



**Exercise 2.** Determine the smallest prime  $p$  such that

- (i) The knots  $\mathbf{3}_1$ ,  $\mathbf{4}_1$ ,  $\mathbf{5}_1$ ,  $\mathbf{5}_2$  are non-trivially  $p$ -colorable.
- (ii) The following knot is non-trivially  $p$ -colorable.



**Exercise 3.** Let  $G$  be a group, and consider the conjugation quandle  $\text{Conj}(G)$ .

- (i) Find and prove a group-theoretic condition in the language of  $G$  that characterizes the  $\text{Conj}(G)$ -colorability of the knot  $\mathbf{3}_1$ .
- (ii) Determine the smallest prime  $p$  such that the knot  $\mathbf{3}_1$  is non-trivially  $\text{Conj}(G)$ -colorable for  $G = \text{GL}(2, p)$  and  $G = \text{SL}(2, p)$ .