

# Universal Algebra 1 - Homework 5

Deadline 10.01.2019, 10:40

1. Let  $\mathbf{L} = (\{0, 1, 2\}, \wedge, \vee)$  be the three-element lattice. Find a monotone idempotent operation that is not in  $\text{Clo}(\mathbf{L})$ .
2. Let  $p$  be a fixed prime number and  $\mathbf{A} = (\mathbb{Z}_p, m)$  with  $m(x, y, z) = x - y + z$ . Recall that a relation  $R \subseteq \mathbb{Z}_p^n$  is called affine if it is closed under affine combinations. Prove that  $R \in \text{Inv}(\mathbf{A})$  if and only if  $R$  is affine.
3. Let  $\mathbf{G} = \text{Sym}(3)$  be the symmetric group on 3 elements. Let  $f(x, y, z)$  be the ternary operation defined by  $f(x, y, z) = x$  if  $y = z$  or  $x = z$ , and  $f(x, y, z) = z$  otherwise. Prove that  $f \notin \text{Clo}(\mathbf{G})$ .