Consider the following set of axioms:

Axiom 1: $x1 = mult(x1, unit())$
Axiom 2: $x1 = mult(i(x2), mult(x2, x1))$
Axiom 3: $x1 = mult(unit(), x1)$
Axiom 4: $x1 = mult(rd(x1, x2), x2)$
Axiom 5: $x1 = rd(mult(x1, x2), x2)$
Axiom 6: $mult(x1, mult(x2, mult(x1, x3))) = mult(mult(x1, mult(x2, x1)), x3)$
Axiom 7: $mult(x1, i(x1)) = unit()$
Axiom 8: $mult(mult(x1, x2), x3) = mult(mult(x1, mult(x2, x3)), asoc(x1, x2, x3))$
Axiom 9: $mult(i(x1), x1) = unit()$
Axiom 10: $mult(i(x1), mult(x2, x1)) = op_t(x2, x1)$
Axiom 11: $mult(i(x1), i(x2)) = i(mult(x1, x2))$
Axiom 12: $mult(i(mult(x1, x2)), mult(x1, mult(x2, x3))) = op_l(x3, x2, x1)$
Axiom 13: $op_t(op_t(x1, x2), x3) = op_t(op_t(x1, x3), x2)$
Axiom 14: $op_t(op_r(x1, x2, x3), x4) = op_r(op_t(x1, x4), x2, x3)$
Axiom 15: $op_t(op_l(x1, x2, x3), x4) = op_l(op_t(x1, x4), x2, x3)$
Axiom 16: $op_r(x1, x2, x3) = rd(mult(mult(x1, x2), x3), mult(x2, x3))$
Axiom 17: $op_r(op_r(x1, x2, x3), x4, x5) = op_r(op_r(x1, x4, x5), x2, x3)$
Axiom 18: $op_r(op_l(x1, x2, x3), x4, x5) = op_l(op_r(x1, x4, x5), x2, x3)$
Axiom 19: $op_l(op_l(x1, x2, x3), x4, x5) = op_l(op_l(x1, x4, x5), x2, x3)$

This theorem holds true:

Theorem 1: $unit() = asoc(asoc(a(), b(), c()), d(), e())$

Proof:

Lemma 1: $rd(x1, unit()) = x1$

$$\begin{aligned}
& \underbrace{rd(x1, unit())} \\
= \quad & \text{by Axiom 1 LR with } \{x1 \leftarrow rd(x1, unit())\} \\
& \overbrace{\underbrace{mult(rd(x1, unit()), unit())}} \\
= \quad & \text{by Axiom 4 RL with } \{x2 \leftarrow unit(), x1 \leftarrow x1\} \\
& \overbrace{x1}
\end{aligned}$$

Lemma 2: $rd(x1, x1) = unit()$

$$\begin{aligned}
& rd(\underbrace{x1}, x1) \\
= \quad & \text{by Axiom 3 LR with } \{x1 \leftarrow x1\} \\
& \underbrace{rd(\overbrace{mult(unit(), x1)}, x1)} \\
= \quad & \text{by Axiom 5 RL with } \{x2 \leftarrow x1, x1 \leftarrow unit()\} \\
& \overbrace{unit()}
\end{aligned}$$

Lemma 3: $i(i(x1)) = x1$

$$\begin{aligned}
& \underbrace{i(i(x1))} \\
= \quad & \text{by Axiom 1 LR with } \{x1 \leftarrow i(i(x1))\} \\
& \overbrace{mult(i(i(x1)), \underbrace{unit()})} \\
= \quad & \text{by Axiom 9 RL with } \{x1 \leftarrow x1\} \\
& \underbrace{mult(i(i(x1)), \overbrace{mult(i(x1), x1)})} \quad {}^{1} \\
= \quad & \text{by Axiom 2 RL with } \{x2 \leftarrow i(x1), x1 \leftarrow x1\} \\
& \overbrace{x1}
\end{aligned}$$

Lemma 4: $op_t(x1, x1) = x1$

$$\begin{aligned}
& \underbrace{op_t(x1, x1)} \\
= \quad & \text{by Axiom 10 RL with } \{x2 \leftarrow x1, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{mult(i(x1), mult(x1, x1))}} \\
= \quad & \text{by Axiom 2 RL with } \{x2 \leftarrow x1, x1 \leftarrow x1\}
\end{aligned}$$

Lemma 6: $op_r(x1, x2, unit()) = x1$

$$
\begin{aligned}
& \underbrace{op_r(x1, x2, unit())} \\
= & \quad \text{by Axiom 16 LR with } \{x3 \leftarrow unit(), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{rd(\underbrace{mult(mult(x1, x2), unit())}, mult(x2, unit()))} \\
= & \quad \text{by Axiom 1 RL with } \{x1 \leftarrow mult(x1, x2)\} \\
& rd(\overbrace{mult(x1, x2)}, \underbrace{mult(x2, unit())}) \\
= & \quad \text{by Axiom 1 RL with } \{x1 \leftarrow x2\} \\
& \underbrace{rd(mult(x1, x2), \overbrace{x2})} \\
= & \quad \text{by Axiom 5 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{x1}
\end{aligned}
$$

Lemma 7: $op_t(i(x1), x1) = i(x1)$

$$
\begin{aligned}
& \underbrace{op_t(i(x1), x1)} \\
= & \quad \text{by Axiom 10 RL with } \{x2 \leftarrow i(x1), x1 \leftarrow x1\} \\
& \overbrace{mult(i(x1), \underbrace{mult(i(x1), x1)})} \\
= & \quad \text{by Axiom 9 LR with } \{x1 \leftarrow x1\} \\
& \underbrace{mult(i(x1), \overbrace{unit()})} \\
= & \quad \text{by Axiom 1 RL with } \{x1 \leftarrow i(x1)\} \\
& \overbrace{i(x1)}
\end{aligned}
$$

Lemma 8: $x1 = op_t(x1, i(x1))$

$$
\begin{aligned}
& \underbrace{x1} \\
= & \quad \text{by Lemma 3 RL with } \{x1 \leftarrow x1\} \\
& \overbrace{\underbrace{i(i(x1))}} \\
= & \quad \text{by Lemma 7 RL with } \{x1 \leftarrow i(x1)\} \\
& \overbrace{op_t(\underbrace{i(i(x1))}, i(x1))} \\
= & \quad \text{by Lemma 3 LR with } \{x1 \leftarrow x1\} \\
& op_t(\overbrace{x1}, i(x1))
\end{aligned}
$$

Lemma 9: $rd(x1, mult(x2, x1)) = i(x2)$

$$
\begin{aligned}
& rd(\underbrace{x1}, mult(x2, x1)) \\
= & \quad \text{by Axiom 2 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{rd(\overbrace{mult(i(x2), mult(x2, x1))}, mult(x2, x1))} \\
= & \quad \text{by Axiom 5 RL with } \{x2 \leftarrow mult(x2, x1), x1 \leftarrow i(x2)\} \\
& \overbrace{i(x2)}
\end{aligned}
$$

Lemma 10: $i(rd(x1, x2)) = rd(x2, x1)$

$$
\begin{aligned}
& \underbrace{i(rd(x1, x2))} \\
= & \quad \text{by Lemma 9 RL with } \{x2 \leftarrow rd(x1, x2), x1 \leftarrow x2\} \\
& \overbrace{rd(x2, \underbrace{mult(rd(x1, x2), x2)})} \\
= & \quad \text{by Axiom 4 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& rd(x2, \overbrace{x1}^{})
\end{aligned}
$$

Lemma 11: $op_t(op_t(x1, x2), x1) = op_t(x1, x2)$

$$
\begin{aligned}
& \underbrace{op_t(op_t(x1, x2), x1)} \\
= & \quad \text{by Axiom 13 RL with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x1\} \\
& \overbrace{op_t(\underbrace{op_t(x1, x1)}, x2)} \\
= & \quad \text{by Lemma 4 LR with } \{x1 \leftarrow x1\} \\
& op_t(\overbrace{x1}^{}, x2)
\end{aligned}
$$

Lemma 12: $mult(x1, mult(i(x1), x2)) = x2$

$$
\begin{aligned}
& mult(\underbrace{x1}, mult(i(x1), x2)) \\
= & \quad \text{by Lemma 3 RL with } \{x1 \leftarrow x1\} \\
& \underbrace{mult(\overbrace{i(i(x1))}, mult(i(x1), x2))} \\
= & \quad \text{by Axiom 2 RL with } \{x2 \leftarrow i(x1), x1 \leftarrow x2\} \\
& \overbrace{x2}^{}
\end{aligned}
$$

Lemma 13: $mult(x1, op_t(x2, x1)) = mult(x2, x1)$

$$
\begin{aligned}
& mult(x1, \underbrace{op_t(x2, x1)}) \\
= & \quad \text{by Axiom 10 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{mult(x1, \overbrace{mult(i(x1), mult(x2, x1))})} \\
= & \quad \text{by Lemma 12 LR with } \{x2 \leftarrow mult(x2, x1), x1 \leftarrow x1\} \\
& \overbrace{mult(x2, x1)}^{}
\end{aligned}
$$

Lemma 14: $op_t(rd(x1, x2), x2) = mult(i(x2), x1)$

$$
\begin{aligned}
& \underbrace{op_t(rd(x1, x2), x2)} \\
= & \quad \text{by Axiom 10 RL with } \{x2 \leftarrow rd(x1, x2), x1 \leftarrow x2\} \\
& \overbrace{mult(i(x2), \underbrace{mult(rd(x1, x2), x2)})} \\
= & \quad \text{by Axiom 4 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& mult(i(x2), \overbrace{x1}^{})
\end{aligned}
$$

Lemma 15: $op_t(i(x1), i(x2)) = i(op_t(x1, x2))$

$$
\begin{aligned}
& \underbrace{op_t(i(x1), i(x2))} \\
= \quad & \text{by Axiom 10 RL with } \{x2 \leftarrow i(x1), x1 \leftarrow i(x2)\} \\
& \overbrace{mult(i(i(x2)), \underbrace{mult(i(x1), i(x2))})} \\
= \quad & \text{by Axiom 11 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{mult(i(i(x2)), \overbrace{i(mult(x1, x2))})} \\
= \quad & \text{by Axiom 11 LR with } \{x2 \leftarrow mult(x1, x2), x1 \leftarrow i(x2)\} \\
& \overbrace{i(\underbrace{mult(i(x2), mult(x1, x2))})} \\
= \quad & \text{by Axiom 10 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& i(\overbrace{op_t(x1, x2)})
\end{aligned}
$$

Lemma 16: $i(mult(i(x1), x2)) = mult(x1, i(x2))$

$$
\begin{aligned}
& \underbrace{i(mult(i(x1), x2))} \\
= \quad & \text{by Axiom 11 RL with } \{x2 \leftarrow x2, x1 \leftarrow i(x1)\} \\
& \overbrace{mult(\underbrace{i(i(x1))}, i(x2))} \\
= \quad & \text{by Lemma 3 LR with } \{x1 \leftarrow x1\} \\
& mult(\overbrace{x1}, i(x2))
\end{aligned}
$$

Lemma 17: $i(mult(x1, i(x2))) = mult(i(x1), x2)$

$$
\begin{aligned}
& \underbrace{i(mult(x1, i(x2)))} \\
= \quad & \text{by Axiom 11 RL with } \{x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& \overbrace{mult(i(x1), \underbrace{i(i(x2))})} \\
= \quad & \text{by Lemma 3 LR with } \{x1 \leftarrow x2\} \\
& mult(i(x1), \overbrace{x2})
\end{aligned}
$$

Lemma 18: $i(op_t(i(x1), x2)) = op_t(x1, i(x2))$

$$
\begin{aligned}
& \underbrace{i(op_t(i(x1), x2))} \\
= \quad & \text{by Lemma 15 RL with } \{x2 \leftarrow x2, x1 \leftarrow i(x1)\} \\
& \overbrace{op_t(\underbrace{i(i(x1))}, i(x2))} \\
= \quad & \text{by Lemma 3 LR with } \{x1 \leftarrow x1\} \\
& op_t(\overbrace{x1}, i(x2))
\end{aligned}
$$

Lemma 19: $i(op_t(x1, i(x2))) = op_t(i(x1), x2)$

$$
\begin{aligned}
& \underbrace{i(op_t(x1, i(x2)))} \\
= \quad & \text{by Lemma 15 RL with } \{x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& \overbrace{op_t(i(x1), \underbrace{i(i(x2))})} \\
= \quad & \text{by Lemma 3 LR with } \{x1 \leftarrow x2\} \\
& op_t(i(x1), \overbrace{x2})
\end{aligned}
$$

Lemma 20: $mult(x1, i(mult(x1, x2))) = i(x2)$

$$
\begin{aligned}
& mult(x1, \underbrace{i(mult(x1, x2))}) \\
= & \quad \text{by Axiom 11 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{mult(x1, \overbrace{mult(i(x1), i(x2))})} \\
= & \quad \text{by Lemma 12 LR with } \{x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& \overbrace{i(x2)}
\end{aligned}
$$

Lemma 21: $mult(rd(i(x1), x2), x1) = i(x2)$

$$
\begin{aligned}
& mult(\underbrace{rd(i(x1), x2)}, x1) \\
= & \quad \text{by Lemma 10 RL with } \{x2 \leftarrow i(x1), x1 \leftarrow x2\} \\
& \underbrace{mult(\overbrace{i(rd(x2, i(x1)))}, x1)} \\
= & \quad \text{by Lemma 17 RL with } \{x2 \leftarrow x1, x1 \leftarrow rd(x2, i(x1))\} \\
& \overbrace{i(\underbrace{mult(rd(x2, i(x1)), i(x1))})} \\
= & \quad \text{by Axiom 4 RL with } \{x2 \leftarrow i(x1), x1 \leftarrow x2\} \\
& i(\overbrace{x2}) \\
\end{aligned}
$$

Lemma 22: $mult(rd(x1, x2), i(x1)) = i(x2)$

$$
\begin{aligned}
& mult(rd(x1, x2), i(\underbrace{x1})) \\
= & \quad \text{by Axiom 4 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{mult(rd(x1, x2), i(\overbrace{mult(rd(x1, x2), x2)}))} \\
= & \quad \text{by Lemma 20 LR with } \{x2 \leftarrow x2, x1 \leftarrow rd(x1, x2)\} \\
& \overbrace{i(x2)}
\end{aligned}
$$

Lemma 23: $rd(i(x1), i(x2)) = rd(x2, x1)$

$$
\begin{aligned}
& rd(\underbrace{i(x1)}, i(x2)) \\
= & \quad \text{by Lemma 22 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \underbrace{rd(\overbrace{mult(rd(x2, x1), i(x2))}, i(x2))} \\
= & \quad \text{by Axiom 5 RL with } \{x2 \leftarrow i(x2), x1 \leftarrow rd(x2, x1)\} \\
& \overbrace{rd(x2, x1)}
\end{aligned}
$$

Lemma 24: $mult(mult(x1, x1), x2) = mult(x1, mult(x1, x2))$

$$
\begin{aligned}
& mult(mult(x1, \underbrace{x1}), x2) \\
= & \quad \text{by Axiom 3 LR with } \{x1 \leftarrow x1\} \\
& \underbrace{mult(mult(x1, \overbrace{mult(unit(), x1)}), x2)} \\
= & \quad \text{by Axiom 6 RL with } \{x3 \leftarrow x2, x2 \leftarrow unit(), x1 \leftarrow x1\} \\
& \overbrace{mult(x1, \underbrace{mult(unit(), mult(x1, x2))})} \\
= & \quad \text{by Axiom 3 RL with } \{x1 \leftarrow mult(x1, x2)\} \\
& mult(x1, \overbrace{mult(x1, x2)}) \\
\end{aligned}
$$

Lemma 25: $op_l(x1, x2, x2) = x1$

$$
\begin{aligned}
& \underbrace{op_l(x1, x2, x2)} \\
= \quad & \text{by Axiom 12 RL with } \{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x2\} \\
& \overbrace{mult(i(mult(x2, x2)), \underbrace{mult(x2, mult(x2, x1))})} \\
= \quad & \text{by Lemma 24 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \underbrace{mult(i(mult(x2, x2)), \overbrace{mult(mult(x2, x2), x1)})} \\
= \quad & \text{by Axiom 2 RL with } \{x2 \leftarrow mult(x2, x2), x1 \leftarrow x1\} \\
& \overbrace{x1}
\end{aligned}
$$

Lemma 26: $op_r(x1, x1, x2) = x1$

$$
\begin{aligned}
& \underbrace{op_r(x1, x1, x2)} \\
= \quad & \text{by Axiom 16 LR with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x1\} \\
& \overbrace{rd(\underbrace{mult(mult(x1, x1), x2)}, mult(x1, x2))} \\
= \quad & \text{by Lemma 24 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{rd(\overbrace{mult(x1, mult(x1, x2))}, mult(x1, x2))} \\
= \quad & \text{by Axiom 5 RL with } \{x2 \leftarrow mult(x1, x2), x1 \leftarrow x1\} \\
& \overbrace{x1}
\end{aligned}
$$

Lemma 27: $mult(op_t(x1, x2), x1) = mult(x1, op_t(x1, x2))$

$$
\begin{aligned}
& \underbrace{mult(op_t(x1, x2), x1)} \\
= \quad & \text{by Lemma 13 RL with } \{x2 \leftarrow op_t(x1, x2), x1 \leftarrow x1\} \\
& \overbrace{mult(x1, \underbrace{op_t(op_t(x1, x2), x1)})} \\
= \quad & \text{by Lemma 11 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& mult(x1, \overbrace{op_t(x1, x2)})
\end{aligned}
$$

Lemma 28: $op_r(op_t(x1, x2), x1, x3) = op_t(x1, x2)$

$$
\begin{aligned}
& \underbrace{op_r(op_t(x1, x2), x1, x3)} \\
= \quad & \text{by Axiom 14 RL with } \{x4 \leftarrow x2, x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow x1\} \\
& \overbrace{op_t(\underbrace{op_r(x1, x1, x3)}, x2)} \\
= \quad & \text{by Lemma 26 LR with } \{x2 \leftarrow x3, x1 \leftarrow x1\} \\
& op_t(\overbrace{x1}, x2)
\end{aligned}
$$

Lemma 29: $rd(mult(x1, x2), op_t(x1, x2)) = x2$

$$
\begin{aligned}
& rd(mult(x1, x2), \underbrace{op_t(x1, x2)}) \\
= \quad & \text{by Axiom 10 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \underbrace{rd(mult(x1, x2), \overbrace{mult(i(x2), mult(x1, x2))})} \\
= \quad & \text{by Lemma 9 LR with } \{x2 \leftarrow i(x2), x1 \leftarrow mult(x1, x2)\} \\
& \underbrace{\overbrace{i(i(x2))}} \\
= \quad & \text{by Lemma 3 LR with } \{x1 \leftarrow x2\} \\
& \overbrace{x2}
\end{aligned}
$$

Lemma 30: $mult(x1, mult(x2, i(x1))) = op_t(x2, i(x1))$

$$
\begin{aligned}
& mult(\underbrace{x1}, mult(x2, i(x1))) \\
= \quad & \text{by Lemma 3 RL with } \{x1 \leftarrow x1\} \\
& \underbrace{mult(\overbrace{i(i(x1))}, mult(x2, i(x1)))} \\
= \quad & \text{by Axiom 10 LR with } \{x2 \leftarrow x2, x1 \leftarrow i(x1)\} \\
& \overbrace{op_t(x2, i(x1))}
\end{aligned}
$$

Lemma 31: $mult(x1, i(mult(x2, x1))) = i(op_t(x2, x1))$

$$
\begin{aligned}
& \underbrace{mult(x1, i(mult(x2, x1)))} \\
= \quad & \text{by Lemma 16 RL with } \{x2 \leftarrow mult(x2, x1), x1 \leftarrow x1\} \\
& \overbrace{i(\underbrace{mult(i(x1), mult(x2, x1))})} \\
= \quad & \text{by Axiom 10 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& i(\overbrace{op_t(x2, x1)})
\end{aligned}
$$

Lemma 32: $op_t(rd(i(x2), x1), x2) = i(mult(i(i(x2)), x1))$

$$
\begin{aligned}
& op_t(\underbrace{rd(i(x2), x1)}, x2) \\
= \quad & \text{by Lemma 10 RL with } \{x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& \underbrace{op_t(\overbrace{i(rd(x1, i(x2)))}, x2)} \\
= \quad & \text{by Lemma 19 RL with } \{x2 \leftarrow x2, x1 \leftarrow rd(x1, i(x2))\} \\
& \overbrace{i(\underbrace{op_t(rd(x1, i(x2)), i(x2))})} \\
= \quad & \text{by Lemma 14 LR with } \{x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& i(\overbrace{mult(i(i(x2)), x1)})
\end{aligned}
$$

Lemma 33: $op_t(rd(i(x1), x2), x1) = i(mult(x1, x2))$

$$
\begin{aligned}
& \underbrace{op_t(rd(i(x1), x2), x1)} \\
= \quad & \text{by Lemma 32 LR with } \{x1 \leftarrow x2, x2 \leftarrow x1\} \\
& \underbrace{\overbrace{i(mult(i(i(x1)), x2))}} \\
= \quad & \text{by Lemma 16 LR with } \{x2 \leftarrow x2, x1 \leftarrow i(x1)\} \\
& \underbrace{\overbrace{mult(i(x1), i(x2))}} \\
= \quad & \text{by Axiom 11 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{i(mult(x1, x2))}
\end{aligned}
$$

Lemma 34: $op_t(rd(x1, x2), i(x1)) = mult(x1, i(x2))$

$$
\begin{aligned}
& \underbrace{op_t(rd(x1, x2), i(x1))} \\
= \quad & \text{by Axiom 10 RL with } \{x2 \leftarrow rd(x1, x2), x1 \leftarrow i(x1)\} \\
& \overbrace{mult(i(i(x1)), \underbrace{mult(rd(x1, x2), i(x1))})} \\
= \quad & \text{by Lemma 22 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{mult(i(i(x1)), \overbrace{i(x2)})} \\
= \quad & \text{by Axiom 11 LR with } \{x2 \leftarrow x2, x1 \leftarrow i(x1)\} \\
& \underbrace{\overbrace{i(mult(i(x1), x2))}} \\
= \quad & \text{by Lemma 16 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{mult(x1, i(x2))}
\end{aligned}
$$

Lemma 35: $op_t(i(op_t(x1, x2)), x1) = i(op_t(x1, x2))$

$$
\begin{aligned}
& \underbrace{op_t(i(op_t(x1, x2)), x1)} \\
= \quad & \text{by Lemma 19 RL with } \{x2 \leftarrow x1, x1 \leftarrow op_t(x1, x2)\} \\
& \underbrace{\overbrace{i(op_t(op_t(x1, x2), i(x1)))}} \\
= \quad & \text{by Lemma 15 RL with } \{x2 \leftarrow i(x1), x1 \leftarrow op_t(x1, x2)\} \\
& \overbrace{op_t(\underbrace{i(op_t(x1, x2))}, i(i(x1)))} \\
= \quad & \text{by Lemma 15 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{op_t(\overbrace{op_t(i(x1), i(x2))}, i(i(x1)))} \\
= \quad & \text{by Axiom 13 RL with } \{x3 \leftarrow i(x2), x2 \leftarrow i(i(x1)), x1 \leftarrow i(x1)\} \\
& \overbrace{op_t(\underbrace{op_t(i(x1), i(i(x1)))}, i(x2))} \\
= \quad & \text{by Lemma 8 RL with } \{x1 \leftarrow i(x1)\} \\
& \underbrace{op_t(\overbrace{i(x1)}, i(x2))} \\
= \quad & \text{by Lemma 15 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{i(op_t(x1, x2))}
\end{aligned}
$$

Lemma 36: $rd(op_t(x1,x2),mult(x1,x2)) = i(x2)$

$$
\begin{aligned}
& \underbrace{rd(op_t(x1,x2),mult(x1,x2))} \\
= & \quad \text{by Axiom 10 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \underbrace{rd(\overbrace{mult(i(x2),mult(x1,x2))},mult(x1,x2))} \\
= & \quad \text{by Axiom 5 RL with } \{x2 \leftarrow mult(x1,x2), x1 \leftarrow i(x2)\} \\
& \overbrace{i(x2)}
\end{aligned}
$$

Lemma 37: $rd(i(x1),x2) = rd(i(x2),x1)$

$$
\begin{aligned}
& \underbrace{rd(i(x1),x2)} \\
= & \quad \text{by Axiom 5 LR with } \{x2 \leftarrow x1, x1 \leftarrow rd(i(x1),x2)\} \\
& \overbrace{rd(\underbrace{mult(rd(i(x1),x2),x1)},x1)} \\
= & \quad \text{by Lemma 21 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& rd(\overbrace{i(x2)},x1)
\end{aligned}
$$

Lemma 38: $rd(x2,i(x1)) = rd(x1,i(x2))$

$$
\begin{aligned}
& \underbrace{rd(x2,i(x1))} \\
= & \quad \text{by Lemma 10 RL with } \{x2 \leftarrow x2, x1 \leftarrow i(x1)\} \\
& \overbrace{\underbrace{i(rd(i(x1),x2))}} \\
= & \quad \text{by Lemma 9 RL with } \{x2 \leftarrow rd(i(x1),x2), x1 \leftarrow x1\} \\
& \overbrace{rd(x1,\underbrace{mult(rd(i(x1),x2),x1)})} \\
= & \quad \text{by Lemma 21 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& rd(x1,\overbrace{i(x2)})
\end{aligned}
$$

Lemma 39: $rd(rd(x1,x2),i(x3)) = rd(x3,rd(x2,x1))$

$$
\begin{aligned}
& rd(\underbrace{rd(x1,x2)},i(x3)) \\
= & \quad \text{by Lemma 10 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \underbrace{rd(\overbrace{i(rd(x2,x1))},i(x3))} \\
= & \quad \text{by Lemma 23 LR with } \{x2 \leftarrow x3, x1 \leftarrow rd(x2,x1)\} \\
& \overbrace{rd(x3,rd(x2,x1))}
\end{aligned}
$$

Lemma 40: $rd(i(x1),rd(x2,x3)) = rd(rd(x3,x2),x1)$

$$
\begin{aligned}
& rd(i(x1),\underbrace{rd(x2,x3)}) \\
= & \quad \text{by Lemma 10 RL with } \{x2 \leftarrow x2, x1 \leftarrow x3\} \\
& \underbrace{rd(i(x1),\overbrace{i(rd(x3,x2))})} \\
= & \quad \text{by Lemma 23 LR with } \{x2 \leftarrow rd(x3,x2), x1 \leftarrow x1\} \\
& \overbrace{rd(rd(x3,x2),x1)}
\end{aligned}
$$

Lemma 41: $rd(mult(x1, mult(x1, x2)), x2) = mult(x1, x1)$

$$
\begin{aligned}
& \overbrace{rd(\underbrace{mult(x1, mult(x1, x2))}, x2)} \\
= \quad & \text{by Lemma 24 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{rd(\overbrace{mult(mult(x1, x1), x2)}, x2)}} \\
= \quad & \text{by Axiom 5 RL with } \{x2 \leftarrow x2, x1 \leftarrow mult(x1, x1)\} \\
& \overbrace{mult(x1, x1)}
\end{aligned}
$$

Lemma 42: $rd(x1, i(x1)) = mult(x1, x1)$

$$
\begin{aligned}
& rd(\underbrace{x1}, i(x1)) \\
= \quad & \text{by Axiom 1 LR with } \{x1 \leftarrow x1\} \\
& rd(\overbrace{mult(x1, \underbrace{unit()})}, i(x1)) \\
= \quad & \text{by Axiom 7 RL with } \{x1 \leftarrow x1\} \\
& \underbrace{rd(mult(x1, \overbrace{mult(x1, i(x1))}), i(x1))} \\
= \quad & \text{by Lemma 41 LR with } \{x2 \leftarrow i(x1), x1 \leftarrow x1\} \\
& \overbrace{mult(x1, x1)}
\end{aligned}
$$

Lemma 43: $i(mult(x1, x1)) = rd(i(x1), x1)$

$$
\begin{aligned}
& \underbrace{i(mult(x1, x1))} \\
= \quad & \text{by Axiom 11 RL with } \{x2 \leftarrow x1, x1 \leftarrow x1\} \\
& \underbrace{\overbrace{mult(i(x1), i(x1))}} \\
= \quad & \text{by Lemma 41 RL with } \{x2 \leftarrow x1, x1 \leftarrow i(x1)\} \\
& \overbrace{rd(mult(i(x1), \underbrace{mult(i(x1), x1)}), x1)} \\
= \quad & \text{by Axiom 9 LR with } \{x1 \leftarrow x1\} \\
& rd(\underbrace{mult(i(x1), \overbrace{unit()})}, x1) \\
= \quad & \text{by Axiom 1 RL with } \{x1 \leftarrow i(x1)\} \\
& rd(\overbrace{i(x1)}, x1)
\end{aligned}
$$

Lemma 44: $i(mult(rd(x1, x2), x3)) = mult(rd(x2, x1), i(x3))$

$$
\begin{aligned}
& \underbrace{i(mult(rd(x1, x2), x3))} \\
= \quad & \text{by Axiom 11 RL with } \{x2 \leftarrow x3, x1 \leftarrow rd(x1, x2)\} \\
& \overbrace{mult(\underbrace{i(rd(x1, x2))}, i(x3))} \\
= \quad & \text{by Lemma 10 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& mult(\overbrace{rd(x2, x1)}, i(x3))
\end{aligned}
$$

Lemma 45: $i(mult(x1, rd(x2, x3))) = mult(i(x1), rd(x3, x2))$

$$
\begin{aligned}
& \underbrace{i(mult(x1, rd(x2, x3)))} \\
= \quad & \text{by Axiom 11 RL with } \{x2 \leftarrow rd(x2, x3), x1 \leftarrow x1\} \\
& \overbrace{mult(i(x1), \underbrace{i(rd(x2, x3))})} \\
= \quad & \text{by Lemma 10 LR with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& mult(i(x1), \overbrace{rd(x3, x2)})
\end{aligned}
$$

Lemma 46: $i(op_t(rd(x1, x2), x3)) = op_t(rd(x2, x1), i(x3))$

$$
\begin{aligned}
& \underbrace{i(op_t(rd(x1, x2), x3))} \\
= \quad & \text{by Lemma 15 RL with } \{x2 \leftarrow x3, x1 \leftarrow rd(x1, x2)\} \\
& \overbrace{op_t(\underbrace{i(rd(x1, x2))}, i(x3))} \\
= \quad & \text{by Lemma 10 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_t(\overbrace{rd(x2, x1)}, i(x3))
\end{aligned}
$$

Lemma 47: $i(op_t(x1, rd(x2, x3))) = op_t(i(x1), rd(x3, x2))$

$$
\begin{aligned}
& \underbrace{i(op_t(x1, rd(x2, x3)))} \\
= \quad & \text{by Lemma 15 RL with } \{x2 \leftarrow rd(x2, x3), x1 \leftarrow x1\} \\
& \overbrace{op_t(i(x1), \underbrace{i(rd(x2, x3))})} \\
= \quad & \text{by Lemma 10 LR with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& op_t(i(x1), \overbrace{rd(x3, x2)})
\end{aligned}
$$

Lemma 48: $op_t(x1, i(op_t(x1, x2))) = x1$

$$
\begin{aligned}
& \underbrace{op_t(x1, i(op_t(x1, x2)))} \\
= \quad & \text{by Lemma 18 RL with } \{x2 \leftarrow op_t(x1, x2), x1 \leftarrow x1\} \\
& \overbrace{\underbrace{i(op_t(i(x1), op_t(x1, x2)))}} \\
= \quad & \text{by Lemma 31 RL with } \{x2 \leftarrow i(x1), x1 \leftarrow op_t(x1, x2)\} \\
& \overbrace{mult(op_t(x1, x2), i(mult(i(x1), op_t(\underbrace{x1}, x2))))} \\
= \quad & \text{by Lemma 8 LR with } \{x1 \leftarrow x1\} \\
& mult(op_t(x1, x2), i(mult(i(x1), \underbrace{op_t(\overbrace{op_t(x1, i(x1))}, x2)}))) \\
= \quad & \text{by Axiom 13 LR with } \{x3 \leftarrow x2, x2 \leftarrow i(x1), x1 \leftarrow x1\} \\
& mult(op_t(x1, x2), i(\underbrace{mult(i(x1), \overbrace{op_t(op_t(x1, x2), i(x1))})})) \\
= \quad & \text{by Lemma 13 LR with } \{x2 \leftarrow op_t(x1, x2), x1 \leftarrow i(x1)\} \\
& \underbrace{mult(op_t(x1, x2), i(\overbrace{mult(op_t(x1, x2), i(x1))}))} \\
= \quad & \text{by Lemma 20 LR with } \{x2 \leftarrow i(x1), x1 \leftarrow op_t(x1, x2)\} \\
& \overbrace{\underbrace{i(i(x1))}} \\
= \quad & \text{by Lemma 3 LR with } \{x1 \leftarrow x1\} \\
& \overbrace{x1}
\end{aligned}
$$

Lemma 49: $rd(i(x1), x1) = rd(x2, mult(x1, mult(x1, x2)))$

$$
\begin{aligned}
& \overbrace{rd(i(x1), x1)} \\
= \quad & \text{by Lemma 43 RL with } \{x1 \leftarrow x1\} \\
& \overbrace{i(mult(x1, x1))} \\
= \quad & \text{by Lemma 9 RL with } \{x2 \leftarrow mult(x1, x1), x1 \leftarrow x2\} \\
& \overbrace{rd(x2, \underbrace{mult(mult(x1, x1), x2)})} \\
= \quad & \text{by Lemma 24 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& rd(x2, \overbrace{mult(x1, mult(x1, x2))})
\end{aligned}
$$

Lemma 50: $mult(x1, op_t(op_t(x2, x1), x3)) = mult(op_t(x2, x3), x1)$

$$
\begin{aligned}
& mult(x1, \underbrace{op_t(op_t(x2, x1), x3)}) \\
= \quad & \text{by Axiom 13 RL with } \{x3 \leftarrow x1, x2 \leftarrow x3, x1 \leftarrow x2\} \\
& \underbrace{mult(x1, \overbrace{op_t(op_t(x2, x3), x1)})} \\
= \quad & \text{by Lemma 13 LR with } \{x2 \leftarrow op_t(x2, x3), x1 \leftarrow x1\} \\
& \overbrace{mult(op_t(x2, x3), x1)}
\end{aligned}
$$

Lemma 51: $op_r(rd(x1, x2), rd(x2, x1), x3) = rd(x1, x2)$

$$
\begin{aligned}
& \underbrace{op_r(rd(x1, x2), rd(x2, x1), x3)} \\
= \quad & \text{by Axiom 16 LR with } \{x3 \leftarrow x3, x2 \leftarrow rd(x2, x1), x1 \leftarrow rd(x1, x2)\} \\
& \overbrace{rd(mult(mult(rd(x1, x2), \underbrace{rd(x2, x1)}), x3), mult(rd(x2, x1), x3))} \\
= \quad & \text{by Lemma 10 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& rd(mult(\underbrace{mult(rd(x1, x2), \overbrace{i(rd(x1, x2))})}, x3), mult(rd(x2, x1), x3)) \\
= \quad & \text{by Axiom 7 LR with } \{x1 \leftarrow rd(x1, x2)\} \\
& rd(\underbrace{mult(\overbrace{unit()}, x3)}, mult(rd(x2, x1), x3)) \\
= \quad & \text{by Axiom 3 RL with } \{x1 \leftarrow x3\} \\
& \underbrace{rd(\overbrace{\ x3\ }, mult(rd(x2, x1), x3))} \\
= \quad & \text{by Lemma 9 LR with } \{x2 \leftarrow rd(x2, x1), x1 \leftarrow x3\} \\
& \underbrace{\overbrace{i(rd(x2, x1))}} \\
= \quad & \text{by Lemma 10 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \overbrace{rd(x1, x2)}
\end{aligned}
$$

Lemma 52: $op_r(x1, i(x1), x2) = x1$

$$
\begin{aligned}
& op_r(x1, \underbrace{i(x1)}, x2) \\
= & \quad \text{by Lemma 9 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& op_r(\underbrace{x1}, \overbrace{rd(x2, mult(x1, x2))}, x2) \\
= & \quad \text{by Axiom 5 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{op_r(\overbrace{rd(mult(x1, x2), x2)}, rd(x2, mult(x1, x2)), x2)} \\
= & \quad \text{by Lemma 51 LR with } \{x3 \leftarrow x2, x2 \leftarrow x2, x1 \leftarrow mult(x1, x2)\} \\
& \underbrace{\overbrace{rd(mult(x1, x2), x2)}} \\
= & \quad \text{by Axiom 5 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{x1}
\end{aligned}
$$

Lemma 53: $rd(x1, x2) = op_t(rd(x1, x2), mult(i(x1), x2))$

$$
\begin{aligned}
& \underbrace{rd(x1, x2)} \\
= & \quad \text{by Lemma 48 RL with } \{x2 \leftarrow i(x1), x1 \leftarrow rd(x1, x2)\} \\
& \overbrace{op_t(rd(x1, x2), i(\underbrace{op_t(rd(x1, x2), i(x1))}))} \\
= & \quad \text{by Lemma 34 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_t(rd(x1, x2), \underbrace{i(\overbrace{mult(x1, i(x2))})}) \\
= & \quad \text{by Lemma 17 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_t(rd(x1, x2), \overbrace{mult(i(x1), x2)})
\end{aligned}
$$

Lemma 54: $mult(x1, mult(rd(x2, x1), mult(x1, x3))) = mult(mult(x1, x2), x3)$

$$
\begin{aligned}
& \underbrace{mult(x1, mult(rd(x2, x1), mult(x1, x3)))} \\
= & \quad \text{by Axiom 6 LR with } \{x3 \leftarrow x3, x2 \leftarrow rd(x2, x1), x1 \leftarrow x1\} \\
& \overbrace{mult(mult(x1, \underbrace{mult(rd(x2, x1), x1)}), x3)} \\
= & \quad \text{by Axiom 4 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& mult(mult(x1, \overbrace{x2}), x3)
\end{aligned}
$$

Lemma 55: $mult(mult(x1, x2), i(x1)) = mult(x1, rd(x2, x1))$

$$
\begin{aligned}
& \underbrace{mult(mult(x1, x2), i(x1))} \\
= & \quad \text{by Lemma 54 RL with } \{x3 \leftarrow i(x1), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{mult(x1, mult(rd(x2, x1), \underbrace{mult(x1, i(x1))}))} \\
= & \quad \text{by Axiom 7 LR with } \{x1 \leftarrow x1\} \\
& mult(x1, \underbrace{mult(rd(x2, x1), \overbrace{unit()})}) \\
= & \quad \text{by Axiom 1 RL with } \{x1 \leftarrow rd(x2, x1)\} \\
& mult(x1, \overbrace{rd(x2, x1)})
\end{aligned}
$$

Lemma 56: $mult(i(mult(x1, x2)), x1) = mult(i(x1), rd(x1, x2))$

$$
\begin{aligned}
& \overbrace{mult(i(mult(x1,x2)),x1)} \\
= \quad & \text{by Lemma 17 RL with } \{x2 \leftarrow x1, x1 \leftarrow mult(x1,x2)\} \\
& \overbrace{i(mult(mult(x1,x2),i(x1)))} \\
= \quad & \text{by Axiom 11 RL with } \{x2 \leftarrow i(x1), x1 \leftarrow mult(x1,x2)\} \\
& \overbrace{mult(\underbrace{i(mult(x1,x2))}, i(i(x1)))} \\
= \quad & \text{by Axiom 11 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{mult(\overbrace{mult(i(x1),i(x2))}, i(i(x1)))} \\
= \quad & \text{by Lemma 55 LR with } \{x2 \leftarrow i(x2), x1 \leftarrow i(x1)\} \\
& \overbrace{mult(i(x1), \underbrace{rd(i(x2),i(x1))})} \\
= \quad & \text{by Lemma 23 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& mult(i(x1), \overbrace{rd(x1,x2)})
\end{aligned}
$$

Lemma 57: $rd(x1, mult(i(x1), rd(x1, x2))) = mult(x1, x2)$

$$
\begin{aligned}
& rd(x1, \underbrace{mult(i(x1), rd(x1,x2))}) \\
= \quad & \text{by Lemma 45 RL with } \{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{rd(x1, \overbrace{i(mult(x1, rd(x2,x1))))}} \\
= \quad & \text{by Lemma 38 RL with } \{x1 \leftarrow x1, x2 \leftarrow mult(x1, rd(x2,x1))\} \\
& \overbrace{rd(\underbrace{mult(x1,rd(x2,x1))}, i(x1))} \\
= \quad & \text{by Lemma 55 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{rd(\overbrace{mult(mult(x1,x2),i(x1))}, i(x1))} \\
= \quad & \text{by Axiom 5 RL with } \{x2 \leftarrow i(x1), x1 \leftarrow mult(x1,x2)\} \\
& \overbrace{mult(x1,x2)}
\end{aligned}
$$

Lemma 58: $rd(x1, i(mult(x1, x2))) = mult(x1, mult(x2, x1))$

$$
\begin{aligned}
& rd(x1, \underbrace{i(mult(x1,x2))}) \\
= \quad & \text{by Axiom 11 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& rd(x1, \overbrace{mult(i(x1), \underbrace{i(x2)})}) \\
= \quad & \text{by Lemma 9 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{rd(x1, mult(i(x1), \overbrace{rd(x1, mult(x2,x1))}))} \\
= \quad & \text{by Lemma 57 LR with } \{x2 \leftarrow mult(x2,x1), x1 \leftarrow x1\} \\
& \overbrace{mult(x1, mult(x2,x1))}
\end{aligned}
$$

Lemma 59: $i(mult(x1, mult(x2, x1))) = rd(i(x1), mult(x1, x2))$

$$
\begin{aligned}
& \overbrace{i(mult(x1, mult(x2, x1)))} \\
= \quad & \text{by Axiom 11 RL with } \{x2 \leftarrow mult(x2, x1), x1 \leftarrow x1\} \\
& \overbrace{mult(i(x1), \underbrace{i(mult(x2, x1))})} \\
= \quad & \text{by Axiom 11 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \underbrace{mult(i(x1), \overbrace{mult(i(x2), i(x1))})} \\
= \quad & \text{by Lemma 58 RL with } \{x2 \leftarrow i(x2), x1 \leftarrow i(x1)\} \\
& \overbrace{rd(i(x1), \underbrace{i(mult(i(x1), i(x2)))})} \\
= \quad & \text{by Axiom 11 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{rd(i(x1), \overbrace{i(i(mult(x1, x2)))})} \\
= \quad & \text{by Lemma 23 LR with } \{x2 \leftarrow i(mult(x1, x2)), x1 \leftarrow x1\} \\
& \overbrace{\underbrace{rd(i(mult(x1, x2)), x1)}} \\
= \quad & \text{by Lemma 37 LR with } \{x2 \leftarrow x1, x1 \leftarrow mult(x1, x2)\} \\
& \overbrace{rd(i(x1), mult(x1, x2))}
\end{aligned}
$$

Lemma 60: $mult(x1, mult(i(x2), x1)) = rd(x1, mult(i(x1), x2))$

$$
\begin{aligned}
& \underbrace{mult(x1, mult(i(x2), x1))} \\
= \quad & \text{by Lemma 58 RL with } \{x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& \overbrace{rd(x1, \underbrace{i(mult(x1, i(x2)))})} \\
= \quad & \text{by Lemma 17 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& rd(x1, \overbrace{mult(i(x1), x2)})
\end{aligned}
$$

Lemma 61: $rd(mult(x1, i(x2)), i(x3)) = rd(x3, mult(i(x1), x2))$

$$
\begin{aligned}
& rd(\underbrace{mult(x1, i(x2))}, i(x3)) \\
= \quad & \text{by Lemma 16 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{rd(\overbrace{i(mult(i(x1), x2))}, i(x3))} \\
= \quad & \text{by Lemma 23 LR with } \{x2 \leftarrow x3, x1 \leftarrow mult(i(x1), x2)\} \\
& \overbrace{rd(x3, mult(i(x1), x2))}
\end{aligned}
$$

Lemma 62: $rd(i(x1), mult(x2, i(x3))) = rd(mult(i(x2), x3), x1)$

$$
\begin{aligned}
& rd(i(x1), \underbrace{mult(x2, i(x3))}) \\
= \quad & \text{by Lemma 16 RL with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& \underbrace{rd(i(x1), \overbrace{i(mult(i(x2), x3))})} \\
= \quad & \text{by Lemma 23 LR with } \{x2 \leftarrow mult(i(x2), x3), x1 \leftarrow x1\} \\
& \overbrace{rd(mult(i(x2), x3), x1)}
\end{aligned}
$$

Lemma 63: $rd(mult(i(x1), x2), i(x3)) = rd(x3, mult(x1, i(x2)))$

$$
\begin{aligned}
& rd(\underbrace{mult(i(x1), x2)}, i(x3)) \\
= \quad & \text{by Lemma 17 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{rd(\overbrace{i(mult(x1, i(x2)))}, i(x3))} \\
= \quad & \text{by Lemma 23 LR with } \{x2 \leftarrow x3, x1 \leftarrow mult(x1, i(x2))\} \\
& \overbrace{rd(x3, mult(x1, i(x2)))}
\end{aligned}
$$

Lemma 64: $rd(i(x1), mult(i(x2), x3)) = rd(mult(x2, i(x3)), x1)$

$$
\begin{aligned}
& rd(i(x1), \underbrace{mult(i(x2), x3)}) \\
= \quad & \text{by Lemma 17 RL with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& \underbrace{rd(i(x1), \overbrace{i(mult(x2, i(x3)))})} \\
= \quad & \text{by Lemma 23 LR with } \{x2 \leftarrow mult(x2, i(x3)), x1 \leftarrow x1\} \\
& \overbrace{rd(mult(x2, i(x3)), x1)}
\end{aligned}
$$

Lemma 65: $rd(op_t(x1, i(x2)), i(x3)) = rd(x3, op_t(i(x1), x2))$

$$
\begin{aligned}
& rd(\underbrace{op_t(x1, i(x2))}, i(x3)) \\
= \quad & \text{by Lemma 18 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{rd(\overbrace{i(op_t(i(x1), x2))}, i(x3))} \\
= \quad & \text{by Lemma 23 LR with } \{x2 \leftarrow x3, x1 \leftarrow op_t(i(x1), x2)\} \\
& \overbrace{rd(x3, op_t(i(x1), x2))}
\end{aligned}
$$

Lemma 66: $rd(i(x1), op_t(x2, i(x3))) = rd(op_t(i(x2), x3), x1)$

$$
\begin{aligned}
& rd(i(x1), \underbrace{op_t(x2, i(x3))}) \\
= \quad & \text{by Lemma 18 RL with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& \underbrace{rd(i(x1), \overbrace{i(op_t(i(x2), x3))})} \\
= \quad & \text{by Lemma 23 LR with } \{x2 \leftarrow op_t(i(x2), x3), x1 \leftarrow x1\} \\
& \overbrace{rd(op_t(i(x2), x3), x1)}
\end{aligned}
$$

Lemma 67: $rd(op_t(i(x1), x2), i(x3)) = rd(x3, op_t(x1, i(x2)))$

$$
\begin{aligned}
& rd(\underbrace{op_t(i(x1), x2)}, i(x3)) \\
= \quad & \text{by Lemma 19 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{rd(\overbrace{i(op_t(x1, i(x2)))}, i(x3))} \\
= \quad & \text{by Lemma 23 LR with } \{x2 \leftarrow x3, x1 \leftarrow op_t(x1, i(x2))\} \\
& \overbrace{rd(x3, op_t(x1, i(x2)))}
\end{aligned}
$$

Lemma 68: $rd(i(x1), op_t(i(x2), x3)) = rd(op_t(x2, i(x3)), x1)$

$$
\begin{aligned}
& rd(i(x1), \underbrace{op_t(i(x2), x3)}) \\
= & \quad \text{by Lemma 19 RL with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& \underbrace{rd(i(x1), \overbrace{i(op_t(x2, i(x3)))})} \\
= & \quad \text{by Lemma 23 LR with } \{x2 \leftarrow op_t(x2, i(x3)), x1 \leftarrow x1\} \\
& \overbrace{rd(op_t(x2, i(x3)), x1)}
\end{aligned}
$$

Lemma 69: $mult(i(x1), rd(x2, i(x1))) = mult(mult(i(x1), x2), x1)$

$$
\begin{aligned}
& mult(i(x1), \underbrace{rd(x2, i(x1))}) \\
= & \quad \text{by Axiom 1 LR with } \{x1 \leftarrow rd(x2, i(x1))\} \\
& mult(i(x1), \overbrace{mult(rd(x2, i(x1)), \underbrace{unit()})}) \\
= & \quad \text{by Axiom 9 RL with } \{x1 \leftarrow x1\} \\
& \underbrace{mult(i(x1), mult(rd(x2, i(x1)), \overbrace{mult(i(x1), x1)}))} \\
= & \quad \text{by Lemma 54 LR with } \{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow i(x1)\} \\
& \overbrace{mult(mult(i(x1), x2), x1)}
\end{aligned}
$$

Lemma 70: $mult(x1, mult(x1, rd(x2, x1))) = op_t(mult(x1, x2), i(x1))$

$$
\begin{aligned}
& mult(\underbrace{x1}, mult(x1, rd(x2, x1))) \\
= & \quad \text{by Lemma 3 RL with } \{x1 \leftarrow x1\} \\
& mult(\overbrace{i(i(x1))}, \underbrace{mult(x1, rd(x2, x1))}) \\
= & \quad \text{by Lemma 55 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{mult(i(i(x1)), \overbrace{mult(mult(x1, x2), i(x1))})} \\
= & \quad \text{by Axiom 10 LR with } \{x2 \leftarrow mult(x1, x2), x1 \leftarrow i(x1)\} \\
& \overbrace{op_t(mult(x1, x2), i(x1))}
\end{aligned}
$$

Lemma 71: $mult(i(x1), rd(x1, i(x2))) = mult(mult(i(x1), x2), x1)$

$$
\begin{aligned}
& \underbrace{mult(i(x1), rd(x1, i(x2)))} \\
= & \quad \text{by Lemma 56 RL with } \{x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& \overbrace{mult(\underbrace{i(mult(x1, i(x2)))}, x1)} \\
= & \quad \text{by Lemma 17 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& mult(\overbrace{mult(i(x1), x2)}, x1)
\end{aligned}
$$

Lemma 72: $mult(x1, mult(op_t(x2, x1), x1)) = rd(x1, i(mult(x2, x1)))$

$$
\begin{aligned}
& \underbrace{mult(x1, mult(op_t(x2, x1), x1))} \\
= & \quad \text{by Lemma 58 RL with } \{x2 \leftarrow op_t(x2, x1), x1 \leftarrow x1\} \\
& \overbrace{rd(x1, i(\underbrace{mult(x1, op_t(x2, x1))}))} \\
= & \quad \text{by Lemma 13 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& rd(x1, i(\overbrace{mult(x2, x1)}))
\end{aligned}
$$

Lemma 73: $mult(i(x1), mult(x2, i(x1))) = rd(mult(i(x1), x2), x1)$

$$
\begin{aligned}
& \underbrace{mult(i(x1), mult(x2, i(x1)))} \\
= & \quad \text{by Lemma 58 RL with } \{x2 \leftarrow x2, x1 \leftarrow i(x1)\} \\
& \overbrace{\underbrace{rd(i(x1), i(mult(i(x1), x2)))}} \\
= & \quad \text{by Lemma 23 LR with } \{x2 \leftarrow mult(i(x1), x2), x1 \leftarrow x1\} \\
& \overbrace{rd(mult(i(x1), x2), x1)}
\end{aligned}
$$

Lemma 74: $mult(i(x2), x1) = op_t(mult(i(x2), x1), rd(x1, x2))$

$$
\begin{aligned}
& \underbrace{mult(i(x2), x1)} \\
= & \quad \text{by Lemma 14 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_t(rd(x1, x2), x2)}} \\
= & \quad \text{by Lemma 11 RL with } \{x2 \leftarrow x2, x1 \leftarrow rd(x1, x2)\} \\
& \overbrace{op_t(\underbrace{op_t(rd(x1, x2), x2)}, rd(x1, x2))} \\
= & \quad \text{by Lemma 14 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_t(\overbrace{mult(i(x2), x1)}, rd(x1, x2))
\end{aligned}
$$

Lemma 75: $mult(x1, x2) = op_t(mult(x1, x2), rd(x2, i(x1)))$

$$
\begin{aligned}
& mult(\underbrace{x1}, x2) \\
= & \quad \text{by Lemma 3 RL with } \{x1 \leftarrow x1\} \\
& \underbrace{mult(\overbrace{i(i(x1))}, x2)} \\
= & \quad \text{by Lemma 74 LR with } \{x1 \leftarrow x2, x2 \leftarrow i(x1)\} \\
& \overbrace{op_t(mult(\underbrace{i(i(x1))}, x2), rd(x2, i(x1)))} \\
= & \quad \text{by Lemma 3 LR with } \{x1 \leftarrow x1\} \\
& op_t(mult(\overbrace{x1}, x2), rd(x2, i(x1)))
\end{aligned}
$$

Lemma 76: $rd(mult(x1, mult(x2, mult(x1, x3))), x3) = mult(x1, mult(x2, x1))$

$$
\begin{aligned}
& rd(\underbrace{mult(x1, mult(x2, mult(x1, x3)))}, x3) \\
= & \quad \text{by Axiom 6 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{rd(\overbrace{mult(mult(x1, mult(x2, x1)), x3)}, x3)} \\
= & \quad \text{by Axiom 5 RL with } \{x2 \leftarrow x3, x1 \leftarrow mult(x1, mult(x2, x1))\} \\
& \overbrace{mult(x1, mult(x2, x1))}
\end{aligned}
$$

Lemma 77: $rd(x1, op_t(x2, mult(i(x2), x1))) = mult(i(x2), x1)$

$$
\begin{aligned}
& rd(\underbrace{x1}, op_t(x2, mult(i(x2), x1))) \\
= \quad & \text{by Lemma 12 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \underbrace{rd(\overbrace{mult(x2, mult(i(x2), x1))}, op_t(x2, mult(i(x2), x1)))} \\
= \quad & \text{by Lemma 29 LR with } \{x2 \leftarrow mult(i(x2), x1), x1 \leftarrow x2\} \\
& \overbrace{mult(i(x2), x1)}
\end{aligned}
$$

Lemma 78: $op_r(i(x1), x2, mult(x1, i(x2))) = op_t(i(x1), x2)$

$$
\begin{aligned}
& \underbrace{op_r(i(x1), x2, mult(x1, i(x2)))} \\
= \quad & \text{by Axiom 16 LR with } \{x3 \leftarrow mult(x1, i(x2)), x2 \leftarrow x2, x1 \leftarrow i(x1)\} \\
& \overbrace{rd(mult(mult(i(x1), x2), \underbrace{mult(x1, i(x2))}), mult(x2, mult(x1, i(x2))))} \\
= \quad & \text{by Lemma 16 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& rd(\underbrace{mult(mult(i(x1), x2), \overbrace{i(mult(i(x1), x2))})}, mult(x2, mult(x1, i(x2)))) \\
= \quad & \text{by Axiom 7 LR with } \{x1 \leftarrow mult(i(x1), x2)\} \\
& \underbrace{rd(\overbrace{unit()}, mult(x2, mult(x1, i(x2))))} \\
= \quad & \text{by Lemma 5 LR with } \{x1 \leftarrow mult(x2, mult(x1, i(x2)))\} \\
& \overbrace{i(\underbrace{mult(x2, mult(x1, i(x2)))})} \\
= \quad & \text{by Lemma 30 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \underbrace{i(\overbrace{op_t(x1, i(x2))})} \\
= \quad & \text{by Lemma 19 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_t(i(x1), x2)}
\end{aligned}
$$

Lemma 79: $op_r(x1, x2, i(mult(x1, x2))) = op_t(x1, x2)$

$$
\begin{aligned}
& op_r(\underbrace{x1}, x2, i(mult(x1, x2))) \\
= \quad & \text{by Lemma 3 RL with } \{x1 \leftarrow x1\} \\
& op_r(\overbrace{i(i(x1))}, x2, \underbrace{i(mult(x1, x2))}) \\
= \quad & \text{by Axiom 11 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{op_r(i(i(x1)), x2, \overbrace{mult(i(x1), i(x2))})} \\
= \quad & \text{by Lemma 78 LR with } \{x2 \leftarrow x2, x1 \leftarrow i(x1)\} \\
& \overbrace{op_t(\underbrace{i(i(x1))}, x2)} \\
= \quad & \text{by Lemma 3 LR with } \{x1 \leftarrow x1\} \\
& op_t(\overbrace{x1}, x2)
\end{aligned}
$$

Lemma 80: $op_t(rd(x2, i(i(x1))), x1) = op_r(i(rd(i(i(x1)), x2)), x1, i(x2))$

$$
\begin{aligned}
& op_t(\underbrace{rd(x2, i(i(x1)))}, x1) \\
= & \quad \text{by Lemma 10 RL with } \{x2 \leftarrow x2, x1 \leftarrow i(i(x1))\} \\
& \underbrace{op_t(\overbrace{i(rd(i(i(x1)), x2))}, x1)} \\
= & \quad \text{by Lemma 78 RL with } \{x2 \leftarrow x1, x1 \leftarrow rd(i(i(x1)), x2)\} \\
& \overbrace{op_r(i(rd(i(i(x1)), x2)), x1, \underbrace{mult(rd(i(i(x1)), x2), i(x1))})} \\
= & \quad \text{by Lemma 21 LR with } \{x2 \leftarrow x2, x1 \leftarrow i(x1)\} \\
& op_r(i(rd(i(i(x1)), x2)), x1, \overbrace{i(x2)})
\end{aligned}
$$

Lemma 81: $mult(i(x1), x2) = op_r(i(rd(i(i(x1)), x2)), x1, i(x2))$

$$
\begin{aligned}
& \underbrace{mult(i(x1), x2)} \\
= & \quad \text{by Lemma 14 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \overbrace{op_t(rd(x2, \underbrace{x1}), x1)} \\
= & \quad \text{by Lemma 3 RL with } \{x1 \leftarrow x1\} \\
& \underbrace{op_t(rd(x2, \overbrace{i(i(x1))}), x1)} \\
= & \quad \text{by Lemma 80 LR with } \{x1 \leftarrow x1, x2 \leftarrow x2\} \\
& \overbrace{op_r(i(rd(i(i(x1)), x2)), x1, i(x2))}
\end{aligned}
$$

Lemma 82: $mult(i(x1), x2) = op_r(rd(x2, x1), x1, i(x2))$

$$
\begin{aligned}
& \underbrace{mult(i(x1), x2)} \\
= & \quad \text{by Lemma 81 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_r(\underbrace{i(rd(i(i(x1)), x2))}, x1, i(x2))} \\
= & \quad \text{by Lemma 10 LR with } \{x2 \leftarrow x2, x1 \leftarrow i(i(x1))\} \\
& op_r(\overbrace{rd(x2, \underbrace{i(i(x1))})}, x1, i(x2)) \\
= & \quad \text{by Lemma 3 LR with } \{x1 \leftarrow x1\} \\
& op_r(rd(x2, \overbrace{x1}), x1, i(x2))
\end{aligned}
$$

Lemma 83: $rd(i(x1), mult(x1, rd(x2, x1))) = i(mult(x1, x2))$

$$
\begin{aligned}
& rd(i(x1), \underbrace{mult(x1, rd(x2, x1))}) \\
= & \quad \text{by Lemma 55 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{rd(i(x1), \overbrace{mult(mult(x1, x2), i(x1))})} \\
= & \quad \text{by Lemma 9 LR with } \{x2 \leftarrow mult(x1, x2), x1 \leftarrow i(x1)\} \\
& \overbrace{i(mult(x1, x2))}
\end{aligned}
$$

Lemma 84: $mult(x1, rd(mult(i(x1), x2), x1)) = mult(x2, i(x1))$

$$
\begin{aligned}
& \overbrace{mult(x1, rd(mult(i(x1), x2), x1))} \\
= & \quad \text{by Lemma 55 RL with } \{x2 \leftarrow mult(i(x1), x2), x1 \leftarrow x1\} \\
& \overbrace{mult(\underbrace{mult(x1, mult(i(x1), x2))}, i(x1))} \\
= & \quad \text{by Lemma 12 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& mult(\overbrace{x2}, i(x1))
\end{aligned}
$$

Lemma 85: $rd(mult(i(x1), rd(x1, x2)), x1) = i(mult(x1, x2))$

$$
\begin{aligned}
& rd(\underbrace{mult(i(x1), rd(x1, x2))}, x1) \\
= & \quad \text{by Lemma 56 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{rd(\overbrace{mult(i(mult(x1, x2)), x1)}, x1)} \\
= & \quad \text{by Axiom 5 RL with } \{x2 \leftarrow x1, x1 \leftarrow i(mult(x1, x2))\} \\
& \overbrace{i(mult(x1, x2))}
\end{aligned}
$$

Lemma 86: $mult(x1, mult(mult(i(x1), x2), x1)) = rd(x1, i(x2))$

$$
\begin{aligned}
& \underbrace{mult(x1, mult(mult(i(x1), x2), x1))} \\
= & \quad \text{by Lemma 58 RL with } \{x2 \leftarrow mult(i(x1), x2), x1 \leftarrow x1\} \\
& \overbrace{rd(x1, i(\underbrace{mult(x1, mult(i(x1), x2))}))} \\
= & \quad \text{by Lemma 12 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& rd(x1, i(\overbrace{x2}))
\end{aligned}
$$

Lemma 87: $i(mult(x1, mult(i(x2), x3))) = mult(i(x1), mult(x2, i(x3)))$

$$
\begin{aligned}
& \underbrace{i(mult(x1, mult(i(x2), x3)))} \\
= & \quad \text{by Axiom 11 RL with } \{x2 \leftarrow mult(i(x2), x3), x1 \leftarrow x1\} \\
& \overbrace{mult(i(x1), \underbrace{i(mult(i(x2), x3))})} \\
= & \quad \text{by Lemma 16 LR with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& mult(i(x1), \overbrace{mult(x2, i(x3))})
\end{aligned}
$$

Lemma 88: $i(mult(mult(x1, i(x2)), x3)) = mult(mult(i(x1), x2), i(x3))$

$$
\begin{aligned}
& i(mult(\underbrace{mult(x1, i(x2))}, x3)) \\
= & \quad \text{by Lemma 16 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{i(mult(\overbrace{i(mult(i(x1), x2))}, x3))} \\
= & \quad \text{by Lemma 16 LR with } \{x2 \leftarrow x3, x1 \leftarrow mult(i(x1), x2)\} \\
& \overbrace{mult(mult(i(x1), x2), i(x3))}
\end{aligned}
$$

Lemma 89: $i(op_t(mult(x1, i(x2)), x3)) = op_t(mult(i(x1), x2), i(x3))$

$$
\begin{aligned}
& \overbrace{i(op_t(mult(x1, i(x2)), x3))} \\
= & \quad \text{by Lemma 15 RL with } \{x2 \leftarrow x3, x1 \leftarrow mult(x1, i(x2))\} \\
& \overbrace{op_t(\underbrace{i(mult(x1, i(x2)))}, i(x3))} \\
= & \quad \text{by Lemma 17 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_t(\overbrace{mult(i(x1), x2)}, i(x3))
\end{aligned}
$$

Lemma 90: $i(op_t(x1, op_t(i(x2), x3))) = op_t(i(x1), op_t(x2, i(x3)))$

$$
\begin{aligned}
& \overbrace{i(op_t(x1, op_t(i(x2), x3)))} \\
= & \quad \text{by Lemma 15 RL with } \{x2 \leftarrow op_t(i(x2), x3), x1 \leftarrow x1\} \\
& \overbrace{op_t(i(x1), \underbrace{i(op_t(i(x2), x3))})} \\
= & \quad \text{by Lemma 18 LR with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& op_t(i(x1), \overbrace{op_t(x2, i(x3))})
\end{aligned}
$$

Lemma 91: $mult(op_r(x1, x2, x3), mult(x2, x3)) = mult(mult(x1, x2), x3)$

$$
\begin{aligned}
& mult(\underbrace{op_r(x1, x2, x3)}, mult(x2, x3)) \\
= & \quad \text{by Axiom 16 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{mult(\overbrace{rd(mult(mult(x1, x2), x3), mult(x2, x3))}, mult(x2, x3))} \\
= & \quad \text{by Axiom 4 RL with } \{x2 \leftarrow mult(x2, x3), x1 \leftarrow mult(mult(x1, x2), x3)\} \\
& \overbrace{mult(mult(x1, x2), x3)}
\end{aligned}
$$

Lemma 92: $mult(mult(x1, x2), i(x2)) = op_r(x1, x2, i(x2))$

$$
\begin{aligned}
& \underbrace{mult(mult(x1, x2), i(x2))} \\
= & \quad \text{by Lemma 91 RL with } \{x3 \leftarrow i(x2), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{mult(op_r(x1, x2, i(x2)), \underbrace{mult(x2, i(x2))})} \\
= & \quad \text{by Axiom 7 LR with } \{x1 \leftarrow x2\} \\
& \underbrace{mult(op_r(x1, x2, i(x2)), \overbrace{unit()})} \\
= & \quad \text{by Axiom 1 RL with } \{x1 \leftarrow op_r(x1, x2, i(x2))\} \\
& \overbrace{op_r(x1, x2, i(x2))}
\end{aligned}
$$

Lemma 93: $mult(mult(x1, i(x2)), x2) = op_r(x1, i(x2), x2)$

$$
\begin{aligned}
& \underbrace{mult(mult(x1, i(x2)), x2)} \\
= \quad & \text{by Lemma 91 RL with } \{x3 \leftarrow x2, x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& \overbrace{mult(op_r(x1, i(x2), x2), \underbrace{mult(i(x2), x2)})} \\
= \quad & \text{by Axiom 9 LR with } \{x1 \leftarrow x2\} \\
& \underbrace{mult(op_r(x1, i(x2), x2), \overbrace{unit()})} \\
= \quad & \text{by Axiom 1 RL with } \{x1 \leftarrow op_r(x1, i(x2), x2)\} \\
& \overbrace{op_r(x1, i(x2), x2)}
\end{aligned}
$$

Lemma 94: $op_t(x1, x2) = mult(mult(x1, op_t(x1, x2)), i(x1))$

$$
\begin{aligned}
& \underbrace{op_t(x1, x2)} \\
= \quad & \text{by Lemma 28 RL with } \{x3 \leftarrow i(x1), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{\overbrace{op_r(op_t(x1, x2), x1, i(x1))}} \\
= \quad & \text{by Lemma 92 RL with } \{x2 \leftarrow x1, x1 \leftarrow op_t(x1, x2)\} \\
& \overbrace{mult(\underbrace{mult(op_t(x1, x2), x1)}, i(x1))} \\
= \quad & \text{by Lemma 27 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& mult(\overbrace{mult(x1, op_t(x1, x2))}, i(x1))
\end{aligned}
$$

Lemma 95: $mult(x1, rd(op_t(x1, x2), x1)) = op_t(x1, x2)$

$$
\begin{aligned}
& \underbrace{mult(x1, rd(op_t(x1, x2), x1))} \\
= \quad & \text{by Lemma 55 RL with } \{x2 \leftarrow op_t(x1, x2), x1 \leftarrow x1\} \\
& \underbrace{\overbrace{mult(mult(x1, op_t(x1, x2)), i(x1))}} \\
= \quad & \text{by Lemma 94 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, x2)}
\end{aligned}
$$

Lemma 96: $mult(i(x1), op_t(x1, x2)) = rd(op_t(x1, x2), x1)$

$$
\begin{aligned}
& mult(i(x1), \underbrace{op_t(x1, x2)}) \\
= \quad & \text{by Lemma 95 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{mult(i(x1), \overbrace{mult(x1, rd(op_t(x1, x2), x1))})} \\
= \quad & \text{by Axiom 2 RL with } \{x2 \leftarrow x1, x1 \leftarrow rd(op_t(x1, x2), x1)\} \\
& \overbrace{rd(op_t(x1, x2), x1)}
\end{aligned}
$$

Lemma 97: $mult(op_t(x1, x2), i(x1)) = rd(op_t(x1, x2), x1)$

$$
\begin{aligned}
& \underbrace{mult(op_t(x1, x2), i(x1))} \\
= \quad & \text{by Lemma 13 RL with } \{x2 \leftarrow op_t(x1, x2), x1 \leftarrow i(x1)\} \\
& \overbrace{mult(i(x1), \underbrace{op_t(op_t(x1, x2), i(x1))})} \\
= \quad & \text{by Axiom 13 RL with } \{x3 \leftarrow x2, x2 \leftarrow i(x1), x1 \leftarrow x1\} \\
& mult(i(x1), \overbrace{op_t(\underbrace{op_t(x1, i(x1))}, x2)}) \\
= \quad & \text{by Lemma 8 RL with } \{x1 \leftarrow x1\} \\
& \underbrace{mult(i(x1), op_t(\overbrace{x1}, x2))} \\
= \quad & \text{by Lemma 96 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{rd(op_t(x1, x2), x1)}
\end{aligned}
$$

Lemma 98: $rd(x1, op_t(x1, x2)) = mult(x1, i(op_t(x1, x2)))$

$$
\begin{aligned}
& \underbrace{rd(x1, op_t(x1, x2))} \\
= \quad & \text{by Lemma 10 RL with } \{x2 \leftarrow x1, x1 \leftarrow op_t(x1, x2)\} \\
& \overbrace{i(\underbrace{rd(op_t(x1, x2), x1)})} \\
= \quad & \text{by Lemma 20 RL with } \{x2 \leftarrow rd(op_t(x1, x2), x1), x1 \leftarrow x1\} \\
& \overbrace{mult(x1, i(\underbrace{mult(x1, rd(op_t(x1, x2), x1))}))} \\
= \quad & \text{by Lemma 95 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& mult(x1, i(\overbrace{op_t(x1, x2)}))
\end{aligned}
$$

Lemma 99: $mult(i(op_t(x1, x2)), x1) = rd(x1, op_t(x1, x2))$

$$
\begin{aligned}
& \underbrace{mult(i(op_t(x1, x2)), x1)} \\
= \quad & \text{by Lemma 13 RL with } \{x2 \leftarrow i(op_t(x1, x2)), x1 \leftarrow x1\} \\
& \overbrace{mult(x1, \underbrace{op_t(i(op_t(x1, x2)), x1)})} \\
= \quad & \text{by Lemma 35 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{mult(x1, \overbrace{i(op_t(x1, x2))})} \\
= \quad & \text{by Lemma 98 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{rd(x1, op_t(x1, x2))}
\end{aligned}
$$

Lemma 100: $mult(i(mult(x1, x2)), x2) = op_r(i(x1), i(x2), x2)$

$$
\begin{aligned}
& \overbrace{mult(i(mult(x1, x2)), x2)} \\
= \quad & \text{by Lemma 17 RL with } \{x2 \leftarrow x2, x1 \leftarrow mult(x1, x2)\} \\
& \overbrace{i(\underbrace{mult(mult(x1, x2), i(x2))}_{})} \\
= \quad & \text{by Axiom 11 RL with } \{x2 \leftarrow i(x2), x1 \leftarrow mult(x1, x2)\} \\
& \overbrace{mult(\underbrace{i(mult(x1, x2))}_{}, i(i(x2)))} \\
= \quad & \text{by Axiom 11 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{mult(\overbrace{mult(i(x1), i(x2))}^{}, i(i(x2)))}_{} \\
= \quad & \text{by Lemma 92 LR with } \{x2 \leftarrow i(x2), x1 \leftarrow i(x1)\} \\
& \overbrace{op_r(i(x1), i(x2), \underbrace{i(i(x2))}_{})} \\
= \quad & \text{by Lemma 3 LR with } \{x1 \leftarrow x2\} \\
& op_r(i(x1), i(x2), \overbrace{x2}^{})
\end{aligned}
$$

Lemma 101: $op_r(i(x1), i(x2), x2) = i(op_r(x1, x2, i(x2)))$

$$
\begin{aligned}
& \underbrace{op_r(i(x1), i(x2), x2)}_{} \\
= \quad & \text{by Lemma 100 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{mult(i(mult(x1, x2)), x2)}_{}} \\
= \quad & \text{by Lemma 17 RL with } \{x2 \leftarrow x2, x1 \leftarrow mult(x1, x2)\} \\
& \overbrace{i(\underbrace{mult(mult(x1, x2), i(x2))}_{})} \\
= \quad & \text{by Lemma 92 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& i(\overbrace{op_r(x1, x2, i(x2))}^{})
\end{aligned}
$$

Lemma 102: $rd(x1, op_t(x1, i(x2))) = mult(x1, op_t(i(x1), x2))$

$$
\begin{aligned}
& \underbrace{rd(x1, op_t(x1, i(x2)))}_{} \\
= \quad & \text{by Lemma 67 RL with } \{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{rd(op_t(i(x1), x2), i(x1))}_{}} \\
= \quad & \text{by Lemma 12 RL with } \{x2 \leftarrow rd(op_t(i(x1), x2), i(x1)), x1 \leftarrow x1\} \\
& \overbrace{mult(x1, \underbrace{mult(i(x1), rd(op_t(i(x1), x2), i(x1)))}_{})} \\
= \quad & \text{by Lemma 95 LR with } \{x2 \leftarrow x2, x1 \leftarrow i(x1)\} \\
& mult(x1, \overbrace{op_t(i(x1), x2)}^{})
\end{aligned}
$$

Lemma 103: $mult(x1, op_t(x1, i(x2))) = rd(x1, op_t(i(x1), x2))$

$mult(x1, \underbrace{op_t(x1, i(x2))})$

$=$ by Lemma 18 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$mult(x1, \overbrace{i(\underbrace{op_t(i(x1), x2)})})$

$=$ by Lemma 21 RL with $\{x2 \leftarrow op_t(i(x1), x2), x1 \leftarrow x1\}$

$mult(x1, \overbrace{mult(\underbrace{rd(i(x1), op_t(i(x1), x2))}, x1)})$

$=$ by Lemma 98 LR with $\{x2 \leftarrow x2, x1 \leftarrow i(x1)\}$

$\underbrace{mult(x1, mult(\overbrace{mult(i(x1), i(op_t(i(x1), x2)))}, x1))}$

$=$ by Lemma 86 LR with $\{x2 \leftarrow i(op_t(i(x1), x2)), x1 \leftarrow x1\}$

$\overbrace{rd(x1, \underbrace{i(i(op_t(i(x1), x2)))})}$

$=$ by Lemma 3 LR with $\{x1 \leftarrow op_t(i(x1), x2)\}$

$rd(x1, \overbrace{op_t(i(x1), x2)})$

Lemma 104: $op_r(rd(x1, x2), x2, i(x2)) = mult(x1, i(x2))$

$\underbrace{op_r(rd(x1, x2), x2, i(x2))}$

$=$ by Lemma 92 RL with $\{x2 \leftarrow x2, x1 \leftarrow rd(x1, x2)\}$

$\overbrace{mult(\underbrace{mult(rd(x1, x2), x2)}, i(x2))}$

$=$ by Axiom 4 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$mult(\overbrace{x1}, i(x2))$

Lemma 105: $op_r(rd(x1, x2), i(x1), x1) = mult(i(x2), i(i(x1)))$

$op_r(rd(x1, x2), i(x1), \underbrace{x1})$

$=$ by Lemma 3 RL with $\{x1 \leftarrow x1\}$

$\underbrace{op_r(rd(x1, x2), i(x1), \overbrace{i(i(x1))})}$

$=$ by Lemma 92 RL with $\{x2 \leftarrow i(x1), x1 \leftarrow rd(x1, x2)\}$

$\overbrace{mult(\underbrace{mult(rd(x1, x2), i(x1))}, i(i(x1)))}$

$=$ by Lemma 22 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$mult(\overbrace{i(x2)}, i(i(x1)))$

Lemma 106: $mult(i(x1), x2) = rd(x2, op_r(x1, i(x2), x2))$

$\underbrace{mult(i(x1), x2)}$

$=$ by Lemma 17 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{\underbrace{i(mult(x1, i(x2)))}}$

$=$ by Lemma 9 RL with $\{x2 \leftarrow mult(x1, i(x2)), x1 \leftarrow x2\}$

$\overbrace{rd(x2, \underbrace{mult(mult(x1, i(x2)), x2)})}$

$=$ by Lemma 93 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$rd(x2, \overbrace{op_r(x1, i(x2), x2)})$

Lemma 107: $mult(x1, asoc(x1, x2, i(x2))) = op_r(x1, x2, i(x2))$

$$
\begin{aligned}
& mult(\underbrace{x1}, asoc(x1, x2, i(x2))) \\
= & \quad \text{by Axiom 1 LR with } \{x1 \leftarrow x1\} \\
& mult(\overbrace{mult(x1, \underbrace{unit()})}, asoc(x1, x2, i(x2))) \\
= & \quad \text{by Axiom 7 RL with } \{x1 \leftarrow x2\} \\
& \underbrace{mult(mult(x1, \overbrace{mult(x2, i(x2))}), asoc(x1, x2, i(x2)))} \\
= & \quad \text{by Axiom 8 RL with } \{x3 \leftarrow i(x2), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{\overbrace{mult(mult(x1, x2), i(x2))}} \\
= & \quad \text{by Lemma 92 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, x2, i(x2))}
\end{aligned}
$$

Lemma 108: $mult(i(mult(x1, mult(x2, x3))), mult(mult(x1, x2), x3)) = asoc(x1, x2, x3)$

$$
\begin{aligned}
& mult(i(mult(x1, mult(x2, x3))), \underbrace{mult(mult(x1, x2), x3)}) \\
= & \quad \text{by Axiom 8 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{mult(i(mult(x1, mult(x2, x3))), \overbrace{mult(mult(x1, mult(x2, x3)), asoc(x1, x2, x3))})} \\
= & \quad \text{by Axiom 2 RL with } \{x2 \leftarrow mult(x1, mult(x2, x3)), x1 \leftarrow asoc(x1, x2, x3)\} \\
& \overbrace{asoc(x1, x2, x3)}
\end{aligned}
$$

Lemma 109: $mult(x1, rd(op_t(x2, x1), x1)) = op_r(x2, x1, i(x1))$

$$
\begin{aligned}
& \underbrace{mult(x1, rd(op_t(x2, x1), x1))} \\
= & \quad \text{by Lemma 55 RL with } \{x2 \leftarrow op_t(x2, x1), x1 \leftarrow x1\} \\
& \overbrace{mult(\underbrace{mult(x1, op_t(x2, x1))}, i(x1))} \\
= & \quad \text{by Lemma 13 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{mult(\overbrace{mult(x2, x1)}, i(x1))} \\
= & \quad \text{by Lemma 92 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \overbrace{op_r(x2, x1, i(x1))}
\end{aligned}
$$

Lemma 110: $mult(x1, op_r(x2, x1, i(x1))) = op_t(mult(x2, x1), i(x1))$

$$
\begin{aligned}
& mult(\underbrace{x1}, op_r(x2, x1, i(x1))) \\
= & \quad \text{by Lemma 3 RL with } \{x1 \leftarrow x1\} \\
& mult(\overbrace{i(i(x1))}, \underbrace{op_r(x2, x1, i(x1))}) \\
= & \quad \text{by Lemma 92 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \underbrace{mult(i(i(x1)), \overbrace{mult(mult(x2, x1), i(x1))})} \\
= & \quad \text{by Axiom 10 LR with } \{x2 \leftarrow mult(x2, x1), x1 \leftarrow i(x1)\} \\
& \overbrace{op_t(mult(x2, x1), i(x1))}
\end{aligned}
$$

Lemma 111: $mult(i(x1), op_t(i(x1), x2)) = rd(op_t(i(x1), x2), x1)$

$$
\begin{aligned}
& mult(i(x1), \underbrace{op_t(i(x1), x2)}) \\
= & \quad \text{by Axiom 4 LR with } \{x2 \leftarrow i(x1), x1 \leftarrow op_t(i(x1), x2)\} \\
& \underbrace{mult(i(x1), \overbrace{mult(rd(op_t(i(x1), x2), i(x1)), i(x1))})} \\
= & \quad \text{by Lemma 58 RL with } \{x2 \leftarrow rd(op_t(i(x1), x2), i(x1)), x1 \leftarrow i(x1)\} \\
& \overbrace{rd(i(x1), \underbrace{i(\underbrace{mult(i(x1), \overbrace{rd(op_t(i(x1), x2), i(x1))})})})} \\
= & \quad \text{by Lemma 95 LR with } \{x2 \leftarrow x2, x1 \leftarrow i(x1)\} \\
& rd(i(x1), \underbrace{i(\overbrace{op_t(i(x1), x2)})}) \\
= & \quad \text{by Lemma 18 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{rd(i(x1), \overbrace{op_t(x1, i(x2))})} \\
= & \quad \text{by Lemma 66 LR with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x1\} \\
& \overbrace{rd(op_t(i(x1), x2), x1)}
\end{aligned}
$$

Lemma 112: $mult(op_r(x1, x2, x3), i(x1)) = rd(op_r(x1, x2, x3), x1)$

$$
\begin{aligned}
& \underbrace{mult(op_r(x1, x2, x3), i(x1))} \\
= & \quad \text{by Axiom 5 LR with } \{x2 \leftarrow x1, x1 \leftarrow mult(op_r(x1, x2, x3), i(x1))\} \\
& \overbrace{rd(\underbrace{mult(mult(op_r(x1, x2, x3), i(x1)), x1)}, x1)} \\
= & \quad \text{by Lemma 93 LR with } \{x2 \leftarrow x1, x1 \leftarrow op_r(x1, x2, x3)\} \\
& rd(\overbrace{\underbrace{op_r(op_r(x1, x2, x3), i(x1), x1)}}, x1) \\
= & \quad \text{by Axiom 17 RL with } \{x5 \leftarrow x3, x4 \leftarrow x2, x3 \leftarrow x1, x2 \leftarrow i(x1), x1 \leftarrow x1\} \\
& rd(\overbrace{op_r(\underbrace{op_r(x1, i(x1), x1)}, x2, x3)}, x1) \\
= & \quad \text{by Lemma 52 LR with } \{x2 \leftarrow x1, x1 \leftarrow x1\} \\
& rd(op_r(\overbrace{x1}, x2, x3), x1)
\end{aligned}
$$

Lemma 113: $mult(i(op_r(x1, x2, x3)), x1) = rd(x1, op_r(x1, x2, x3))$

$$
\begin{aligned}
& \underbrace{mult(i(op_r(x1, x2, x3)), x1)} \\
= & \quad \text{by Lemma 106 LR with } \{x2 \leftarrow x1, x1 \leftarrow op_r(x1, x2, x3)\} \\
& \overbrace{rd(x1, \underbrace{op_r(op_r(x1, x2, x3), i(x1), x1)})} \\
= & \quad \text{by Axiom 17 RL with } \{x5 \leftarrow x3, x4 \leftarrow x2, x3 \leftarrow x1, x2 \leftarrow i(x1), x1 \leftarrow x1\} \\
& rd(x1, \overbrace{op_r(\underbrace{op_r(x1, i(x1), x1)}, x2, x3)}) \\
= & \quad \text{by Lemma 52 LR with } \{x2 \leftarrow x1, x1 \leftarrow x1\} \\
& rd(x1, op_r(\overbrace{x1}, x2, x3))
\end{aligned}
$$

Lemma 114: $rd(x1, i(op_r(x1, x2, x3))) = mult(op_r(x1, x2, x3), x1)$

$$
\begin{aligned}
& \underbrace{rd(x1, i(op_r(x1, x2, x3)))} \\
= & \quad \text{by Lemma 38 RL with } \{x1 \leftarrow x1, x2 \leftarrow op_r(x1, x2, x3)\} \\
& \overbrace{\underbrace{rd(op_r(x1, x2, x3), i(x1))}} \\
= & \quad \text{by Lemma 86 RL with } \{x2 \leftarrow x1, x1 \leftarrow op_r(x1, x2, x3)\} \\
& \overbrace{mult(op_r(x1, x2, x3), mult(\underbrace{mult(i(op_r(x1, x2, x3)), x1)}, op_r(x1, x2, x3)))} \\
= & \quad \text{by Lemma 113 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& mult(op_r(x1, x2, x3), \underbrace{mult(\overbrace{rd(x1, op_r(x1, x2, x3))}, op_r(x1, x2, x3)))} \\
= & \quad \text{by Axiom 4 RL with } \{x2 \leftarrow op_r(x1, x2, x3), x1 \leftarrow x1\} \\
& mult(op_r(x1, x2, x3), \overbrace{x1}^{})
\end{aligned}
$$

Lemma 115: $mult(x1, i(op_r(x1, x2, x3))) = rd(x1, op_r(x1, x2, x3))$

$$
\begin{aligned}
& \underbrace{mult(x1, i(op_r(x1, x2, x3)))} \\
= & \quad \text{by Lemma 57 RL with } \{x2 \leftarrow i(op_r(x1, x2, x3)), x1 \leftarrow x1\} \\
& \overbrace{rd(x1, mult(i(x1), \underbrace{rd(x1, i(op_r(x1, x2, x3)))}))} \\
= & \quad \text{by Lemma 114 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& rd(x1, \underbrace{mult(i(x1), \overbrace{mult(op_r(x1, x2, x3), x1)})}) \\
= & \quad \text{by Axiom 10 LR with } \{x2 \leftarrow op_r(x1, x2, x3), x1 \leftarrow x1\} \\
& rd(x1, \overbrace{\underbrace{op_t(op_r(x1, x2, x3), x1)}}) \\
= & \quad \text{by Axiom 14 LR with } \{x4 \leftarrow x1, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& rd(x1, \overbrace{op_r(\underbrace{op_t(x1, x1)}, x2, x3)}) \\
= & \quad \text{by Lemma 4 LR with } \{x1 \leftarrow x1\} \\
& rd(x1, op_r(\overbrace{x1}^{}, x2, x3))
\end{aligned}
$$

Lemma 116: $mult(i(x1), op_r(x1, x2, x3)) = rd(mult(i(x1), mult(op_r(x1, x2, x3), x1)), x1)$

$$
\begin{aligned}
& \underbrace{mult(i(x1), op_r(x1, x2, x3))} \\
= & \quad \text{by Lemma 17 RL with } \{x2 \leftarrow op_r(x1, x2, x3), x1 \leftarrow x1\} \\
& \overbrace{\underbrace{i(mult(x1, i(op_r(x1, x2, x3))))}} \\
= & \quad \text{by Lemma 85 RL with } \{x2 \leftarrow i(op_r(x1, x2, x3)), x1 \leftarrow x1\} \\
& \overbrace{rd(mult(i(x1), \underbrace{rd(x1, i(op_r(x1, x2, x3)))}), x1)} \\
= & \quad \text{by Lemma 114 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& rd(mult(i(x1), \overbrace{mult(op_r(x1, x2, x3), x1)}), x1)
\end{aligned}
$$

Lemma 117: $mult(i(x1), op_r(x1, x2, x3)) = rd(op_r(x1, x2, x3), x1)$

$$
\begin{aligned}
& \overbrace{mult(i(x1), op_r(x1, x2, x3))} \\
= \quad & \text{by Lemma 116 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{rd(\underbrace{mult(i(x1), mult(op_r(x1, x2, x3), x1))}, x1)} \\
= \quad & \text{by Axiom 10 LR with } \{x2 \leftarrow op_r(x1, x2, x3), x1 \leftarrow x1\} \\
& rd(\overbrace{\underbrace{op_t(op_r(x1, x2, x3), x1)}}, x1) \\
= \quad & \text{by Axiom 14 LR with } \{x4 \leftarrow x1, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& rd(\overbrace{op_r(\underbrace{op_t(x1, x1)}, x2, x3)}, x1) \\
= \quad & \text{by Lemma 4 LR with } \{x1 \leftarrow x1\} \\
& rd(op_r(\overbrace{x1}, x2, x3), x1)
\end{aligned}
$$

Lemma 118: $mult(op_r(x1, x2, x3), x1) = mult(x1, mult(rd(op_r(x1, x2, x3), x1), x1))$

$$
\begin{aligned}
& \underbrace{mult(op_r(x1, x2, x3), x1)} \\
= \quad & \text{by Lemma 114 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{\overbrace{rd(x1, i(op_r(x1, x2, x3)))}} \\
= \quad & \text{by Lemma 86 RL with } \{x2 \leftarrow op_r(x1, x2, x3), x1 \leftarrow x1\} \\
& \overbrace{mult(x1, mult(\underbrace{mult(i(x1), op_r(x1, x2, x3))}, x1))} \\
= \quad & \text{by Lemma 117 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& mult(x1, mult(\overbrace{rd(op_r(x1, x2, x3), x1)}, x1))
\end{aligned}
$$

Lemma 119: $mult(op_r(x1, x2, x3), x1) = mult(x1, op_r(x1, x2, x3))$

$$
\begin{aligned}
& \underbrace{mult(op_r(x1, x2, x3), x1)} \\
= \quad & \text{by Lemma 118 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{mult(x1, \underbrace{mult(rd(op_r(x1, x2, x3), x1), x1)})} \\
= \quad & \text{by Axiom 4 RL with } \{x2 \leftarrow x1, x1 \leftarrow op_r(x1, x2, x3)\} \\
& mult(x1, \overbrace{op_r(x1, x2, x3)})
\end{aligned}
$$

Lemma 120: $op_t(x1, op_r(x1, x2, x3)) = x1$

$$
\begin{aligned}
& \underbrace{op_t(x1, op_r(x1, x2, x3))} \\
= \quad & \text{by Axiom 10 RL with } \{x2 \leftarrow x1, x1 \leftarrow op_r(x1, x2, x3)\} \\
& \overbrace{mult(i(op_r(x1, x2, x3)), \underbrace{mult(x1, op_r(x1, x2, x3))})} \\
= \quad & \text{by Lemma 119 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{mult(i(op_r(x1, x2, x3)), \overbrace{mult(op_r(x1, x2, x3), x1)})} \\
= \quad & \text{by Axiom 2 RL with } \{x2 \leftarrow op_r(x1, x2, x3), x1 \leftarrow x1\} \\
& \overbrace{x1}
\end{aligned}
$$

Lemma 121: $rd(x1, i(op_r(x1, x2, x3))) = mult(x1, op_r(x1, x2, x3))$

$$
\begin{aligned}
& \underbrace{rd(x1, i(op_r(x1, x2, x3)))} \\
= & \quad \text{by Lemma 114 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{mult(op_r(x1, x2, x3), x1)}} \\
= & \quad \text{by Lemma 119 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{mult(x1, op_r(x1, x2, x3))}
\end{aligned}
$$

Lemma 122: $rd(i(x1), op_r(x2, x1, i(x1))) = i(mult(x2, x1))$

$$
\begin{aligned}
& rd(i(x1), \underbrace{op_r(x2, x1, i(x1))}) \\
= & \quad \text{by Lemma 92 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \underbrace{rd(i(x1), \overbrace{mult(mult(x2, x1), i(x1))})} \\
= & \quad \text{by Lemma 9 LR with } \{x2 \leftarrow mult(x2, x1), x1 \leftarrow i(x1)\} \\
& \overbrace{i(mult(x2, x1))}
\end{aligned}
$$

Lemma 123: $mult(x1, rd(op_r(x1, x2, x3), x1)) = op_r(x1, x2, x3)$

$$
\begin{aligned}
& mult(\underbrace{x1}, rd(op_r(x1, x2, x3), x1)) \\
= & \quad \text{by Lemma 3 RL with } \{x1 \leftarrow x1\} \\
& mult(\overbrace{i(i(x1))}, \underbrace{rd(op_r(x1, x2, x3), x1)}) \\
= & \quad \text{by Lemma 112 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{mult(i(i(x1)), \overbrace{mult(op_r(x1, x2, x3), i(x1))})} \\
= & \quad \text{by Axiom 10 LR with } \{x2 \leftarrow op_r(x1, x2, x3), x1 \leftarrow i(x1)\} \\
& \overbrace{\underbrace{op_t(op_r(x1, x2, x3), i(x1))}} \\
= & \quad \text{by Axiom 14 LR with } \{x4 \leftarrow i(x1), x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_r(\underbrace{op_t(x1, i(x1))}, x2, x3)} \\
= & \quad \text{by Lemma 8 RL with } \{x1 \leftarrow x1\} \\
& op_r(\overbrace{x1}, x2, x3)
\end{aligned}
$$

Lemma 124: $mult(mult(x1, op_t(x1, x2)), x3) = mult(op_t(x1, x2), mult(x1, x3))$

$$
\begin{aligned}
& mult(\underbrace{mult(x1, op_t(x1, x2))}, x3) \\
= & \quad \text{by Lemma 27 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{mult(\overbrace{mult(op_t(x1, x2), x1)}, x3)} \\
= & \quad \text{by Lemma 91 RL with } \{x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow op_t(x1, x2)\} \\
& \overbrace{mult(\underbrace{op_r(op_t(x1, x2), x1, x3)}, mult(x1, x3))} \\
= & \quad \text{by Lemma 28 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& mult(\overbrace{op_t(x1, x2)}, mult(x1, x3))
\end{aligned}
$$

Lemma 125: $op_r(mult(x1,x2),i(x1),x1) = mult(mult(x1,rd(x2,x1)),i(i(x1)))$

$$
\begin{aligned}
& op_r(mult(x1,x2),i(x1),\underbrace{x1}) \\
= \quad & \text{by Lemma 3 RL with } \{x1 \leftarrow x1\} \\
& \underbrace{op_r(mult(x1,x2),i(x1),\overbrace{i(i(x1))})} \\
= \quad & \text{by Lemma 92 RL with } \{x2 \leftarrow i(x1), x1 \leftarrow mult(x1,x2)\} \\
& \overbrace{mult(\underbrace{mult(mult(x1,x2),i(x1))},i(i(x1)))} \\
= \quad & \text{by Lemma 55 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& mult(\overbrace{mult(x1,rd(x2,x1))},i(i(x1)))
\end{aligned}
$$

Lemma 126: $op_r(mult(x1,x2),i(x2),x2) = mult(op_r(x1,x2,i(x2)),i(i(x2)))$

$$
\begin{aligned}
& op_r(mult(x1,x2),i(x2),\underbrace{x2}) \\
= \quad & \text{by Lemma 3 RL with } \{x1 \leftarrow x2\} \\
& \underbrace{op_r(mult(x1,x2),i(x2),\overbrace{i(i(x2))})} \\
= \quad & \text{by Lemma 92 RL with } \{x2 \leftarrow i(x2), x1 \leftarrow mult(x1,x2)\} \\
& \overbrace{mult(\underbrace{mult(mult(x1,x2),i(x2))},i(i(x2)))} \\
= \quad & \text{by Lemma 92 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& mult(\overbrace{op_r(x1,x2,i(x2))},i(i(x2)))
\end{aligned}
$$

Lemma 127: $mult(op_r(x1,x2,i(x2)),x2) = op_r(mult(x1,x2),i(x2),x2)$

$$
\begin{aligned}
& mult(op_r(x1,x2,i(x2)),\underbrace{x2}) \\
= \quad & \text{by Lemma 3 RL with } \{x1 \leftarrow x2\} \\
& \underbrace{mult(op_r(x1,x2,i(x2)),\overbrace{i(i(x2))})} \\
= \quad & \text{by Lemma 126 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_r(mult(x1,x2),i(x2),x2)}
\end{aligned}
$$

Lemma 128: $rd(mult(x1,i(op_t(x1,x2))),i(x1)) = i(mult(i(x1),rd(op_t(x1,x2),x1)))$

$$
\begin{aligned}
& \underbrace{rd(mult(x1,i(op_t(x1,x2))),i(x1))} \\
= \quad & \text{by Lemma 64 RL with } \{x3 \leftarrow op_t(x1,x2), x2 \leftarrow x1, x1 \leftarrow i(x1)\} \\
& \overbrace{\underbrace{rd(i(i(x1)),mult(i(x1),op_t(x1,x2)))}} \\
= \quad & \text{by Lemma 59 RL with } \{x2 \leftarrow op_t(x1,x2), x1 \leftarrow i(x1)\} \\
& \overbrace{i(mult(i(x1),\underbrace{mult(op_t(x1,x2),i(x1))}))} \\
= \quad & \text{by Lemma 97 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& i(mult(i(x1),\overbrace{rd(op_t(x1,x2),x1)}))
\end{aligned}
$$

Lemma 129: $rd(x1, rd(op_t(x1, x2), x1)) = i(mult(i(x1), rd(op_t(x1, x2), x1)))$

$$
\begin{aligned}
& \underbrace{rd(x1, \underbrace{rd(op_t(x1, x2), x1)}}) \\
= & \quad \text{by Lemma 96 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{rd(x1, \overbrace{mult(i(x1), op_t(x1, x2))})} \\
= & \quad \text{by Lemma 61 RL with } \{x3 \leftarrow x1, x2 \leftarrow op_t(x1, x2), x1 \leftarrow x1\} \\
& \underbrace{\overbrace{rd(mult(x1, i(op_t(x1, x2))), i(x1))}} \\
= & \quad \text{by Lemma 128 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{i(mult(i(x1), rd(op_t(x1, x2), x1)))}
\end{aligned}
$$

Lemma 130: $rd(x1, rd(op_t(x1, x2), x1)) = mult(x1, rd(x1, op_t(x1, x2)))$

$$
\begin{aligned}
& \underbrace{rd(x1, rd(op_t(x1, x2), x1))} \\
= & \quad \text{by Lemma 129 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{\overbrace{i(mult(i(x1), rd(op_t(x1, x2), x1)))}} \\
= & \quad \text{by Lemma 16 LR with } \{x2 \leftarrow rd(op_t(x1, x2), x1), x1 \leftarrow x1\} \\
& \overbrace{mult(x1, \underbrace{i(rd(op_t(x1, x2), x1))})} \\
= & \quad \text{by Lemma 10 LR with } \{x2 \leftarrow x1, x1 \leftarrow op_t(x1, x2)\} \\
& mult(x1, \overbrace{rd(x1, op_t(x1, x2))})
\end{aligned}
$$

Lemma 131: $rd(i(x1), op_r(x1, x2, x3)) = i(mult(x1, op_r(x1, x2, x3)))$

$$
\begin{aligned}
& \underbrace{rd(i(x1), op_r(x1, x2, x3))} \\
= & \quad \text{by Lemma 37 RL with } \{x2 \leftarrow x1, x1 \leftarrow op_r(x1, x2, x3)\} \\
& \underbrace{\overbrace{rd(i(op_r(x1, x2, x3)), x1)}} \\
= & \quad \text{by Lemma 10 RL with } \{x2 \leftarrow i(op_r(x1, x2, x3)), x1 \leftarrow x1\} \\
& \overbrace{i(\underbrace{rd(x1, i(op_r(x1, x2, x3)))})} \\
= & \quad \text{by Lemma 121 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& i(\overbrace{mult(x1, op_r(x1, x2, x3))})
\end{aligned}
$$

Lemma 132: $mult(mult(x1, x2), op_l(x3, x2, x1)) = mult(x1, mult(x2, x3))$

$$
\begin{aligned}
& mult(mult(x1, x2), \underbrace{op_l(x3, x2, x1)}) \\
= & \quad \text{by Axiom 12 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{mult(mult(x1, x2), \overbrace{mult(i(mult(x1, x2)), mult(x1, mult(x2, x3)))})} \\
= & \quad \text{by Lemma 12 LR with } \{x2 \leftarrow mult(x1, mult(x2, x3)), x1 \leftarrow mult(x1, x2)\} \\
& \overbrace{mult(x1, mult(x2, x3))}
\end{aligned}
$$

Lemma 133: $mult(i(op_t(x1,x2)),mult(x1,x2)) = op_t(x2,op_t(x1,x2))$

$$
\begin{aligned}
& mult(i(op_t(x1,x2)),\underbrace{mult(x1,x2)}) \\
= & \quad \text{by Lemma 13 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \underbrace{mult(i(op_t(x1,x2)),\overbrace{mult(x2,op_t(x1,x2))})} \\
= & \quad \text{by Axiom 10 LR with } \{x2 \leftarrow x2, x1 \leftarrow op_t(x1,x2)\} \\
& \overbrace{op_t(x2,op_t(x1,x2))}
\end{aligned}
$$

Lemma 134: $mult(x1,mult(x1,x2)) = op_t(mult(x1,mult(x2,x1)),i(x1))$

$$
\begin{aligned}
& mult(x1,\underbrace{mult(x1,x2)}) \\
= & \quad \text{by Lemma 3 RL with } \{x1 \leftarrow mult(x1,x2)\} \\
& \underbrace{mult(x1,\overbrace{i(i(mult(x1,x2)))})} \\
= & \quad \text{by Lemma 34 RL with } \{x2 \leftarrow i(mult(x1,x2)), x1 \leftarrow x1\} \\
& \overbrace{op_t(\underbrace{rd(x1,i(mult(x1,x2)))},i(x1))} \\
= & \quad \text{by Lemma 58 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_t(\overbrace{mult(x1,mult(x2,x1))},i(x1))
\end{aligned}
$$

Lemma 135: $mult(mult(x1,x2),i(x1)) = rd(i(i(mult(x1,x2))),mult(i(mult(x1,x2)),mult(x1,mult(x2,x1))))$

$$
\begin{aligned}
& \underbrace{mult(mult(x1,x2),i(x1))} \\
= & \quad \text{by Lemma 16 RL with } \{x2 \leftarrow x1, x1 \leftarrow mult(x1,x2)\} \\
& \overbrace{\underbrace{i(mult(i(mult(x1,x2)),x1))}} \\
= & \quad \text{by Lemma 83 RL with } \{x2 \leftarrow x1, x1 \leftarrow i(mult(x1,x2))\} \\
& \overbrace{rd(i(i(mult(x1,x2))),mult(i(mult(x1,x2)),\underbrace{rd(x1,i(mult(x1,x2)))}))} \\
= & \quad \text{by Lemma 58 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& rd(i(i(mult(x1,x2))),mult(i(mult(x1,x2)),\overbrace{mult(x1,mult(x2,x1))}))
\end{aligned}
$$

Lemma 136: $mult(x1,rd(x2,x1)) = rd(mult(mult(x1,x2),i(mult(x1,mult(x2,x1)))),i(mult(x1,x2)))$

$$
\begin{aligned}
& \underbrace{mult(x1,rd(x2,x1))} \\
= & \quad \text{by Lemma 55 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{mult(mult(x1,x2),i(x1))}} \\
= & \quad \text{by Lemma 135 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{rd(i(i(mult(x1,x2))),mult(i(mult(x1,x2)),mult(x1,mult(x2,x1))))}} \\
= & \quad \text{by Lemma 64 LR with } \{x3 \leftarrow mult(x1,mult(x2,x1)), x2 \leftarrow mult(x1,x2), x1 \leftarrow i(mult(x1,x2))\} \\
& \overbrace{rd(mult(mult(x1,x2),i(mult(x1,mult(x2,x1)))),i(mult(x1,x2)))}
\end{aligned}
$$

Lemma 137: $mult(x1, mult(x2, op_l(x3, x1, x2))) = mult(mult(x1, x2), x3)$

$$
\begin{aligned}
& mult(x1, mult(x2, \underbrace{op_l(x3, x1, x2)})) \\
= & \quad \text{by Axiom 12 RL with } \{x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow x2\} \\
& mult(x1, \underbrace{mult(x2, \overbrace{mult(i(mult(x2, x1)), mult(x2, mult(x1, x3)))})}) \\
= & \quad \text{by Axiom 6 LR with } \{x3 \leftarrow mult(x1, x3), x2 \leftarrow i(mult(x2, x1)), x1 \leftarrow x2\} \\
& \underbrace{mult(x1, \overbrace{mult(mult(x2, mult(i(mult(x2, x1)), x2)), mult(x1, x3))})} \\
= & \quad \text{by Axiom 6 LR with } \{x3 \leftarrow x3, x2 \leftarrow mult(x2, mult(i(mult(x2, x1)), x2)), x1 \leftarrow x1\} \\
& \overbrace{mult(mult(x1, \underbrace{mult(mult(x2, mult(i(mult(x2, x1)), x2)), x1)}), x3)} \\
= & \quad \text{by Axiom 6 RL with } \{x3 \leftarrow x1, x2 \leftarrow i(mult(x2, x1)), x1 \leftarrow x2\} \\
& mult(mult(x1, \overbrace{mult(x2, \underbrace{mult(i(mult(x2, x1)), mult(x2, x1))})}), x3) \\
= & \quad \text{by Axiom 9 LR with } \{x1 \leftarrow mult(x2, x1)\} \\
& mult(mult(x1, \underbrace{mult(x2, \overbrace{unit()})}), x3) \\
= & \quad \text{by Axiom 1 RL with } \{x1 \leftarrow x2\} \\
& mult(mult(x1, \overbrace{x2}), x3)
\end{aligned}
$$

Lemma 138: $op_l(op_l(x1, x2, x3), x3, x2) = x1$

$$
\begin{aligned}
& \underbrace{op_l(op_l(x1, x2, x3), x3, x2)} \\
= & \quad \text{by Axiom 12 RL with } \{x3 \leftarrow op_l(x1, x2, x3), x2 \leftarrow x3, x1 \leftarrow x2\} \\
& \overbrace{mult(i(mult(x2, x3)), \underbrace{mult(x2, mult(x3, op_l(x1, x2, x3)))})} \\
= & \quad \text{by Lemma 137 LR with } \{x3 \leftarrow x1, x2 \leftarrow x3, x1 \leftarrow x2\} \\
& \underbrace{mult(i(mult(x2, x3)), \overbrace{mult(mult(x2, x3), x1)})} \\
= & \quad \text{by Axiom 2 RL with } \{x2 \leftarrow mult(x2, x3), x1 \leftarrow x1\} \\
& \overbrace{x1}
\end{aligned}
$$

Lemma 139: $mult(i(x1), op_r(x2, x1, i(x1))) = rd(op_t(x2, x1), x1)$

$$
\begin{aligned}
& mult(i(x1), \underbrace{op_r(x2, x1, i(x1))}) \\
= & \quad \text{by Lemma 92 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \underbrace{mult(i(x1), \overbrace{mult(mult(x2, x1), i(x1))})} \\
= & \quad \text{by Lemma 73 LR with } \{x2 \leftarrow mult(x2, x1), x1 \leftarrow x1\} \\
& \overbrace{rd(\underbrace{mult(i(x1), mult(x2, x1))}, x1)} \\
= & \quad \text{by Axiom 10 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& rd(\overbrace{op_t(x2, x1)}, x1)
\end{aligned}
$$

Lemma 140: $rd(x1, rd(op_t(i(x2), x1), x2)) = mult(x2, mult(x1, x2))$

$$
\begin{aligned}
& \underbrace{rd(x1, rd(op_t(i(x2), x1), x2))} \\
= \quad & \text{by Lemma 39 RL with } \{x3 \leftarrow x1, x2 \leftarrow op_t(i(x2), x1), x1 \leftarrow x2\} \\
& \overbrace{rd(\underbrace{rd(x2, op_t(i(x2), x1))}, i(x1))} \\
= \quad & \text{by Lemma 103 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& rd(\overbrace{mult(x2, \underbrace{op_t(x2, i(x1))})}, i(x1)) \\
= \quad & \text{by Axiom 10 RL with } \{x2 \leftarrow x2, x1 \leftarrow i(x1)\} \\
& \underbrace{rd(mult(x2, \overbrace{mult(i(i(x1)), mult(x2, i(x1)))}), i(x1))} \\
= \quad & \text{by Lemma 76 LR with } \{x3 \leftarrow i(x1), x2 \leftarrow i(i(x1)), x1 \leftarrow x2\} \\
& \overbrace{\underbrace{mult(x2, mult(i(i(x1)), x2))}} \\
= \quad & \text{by Lemma 60 LR with } \{x2 \leftarrow i(x1), x1 \leftarrow x2\} \\
& \overbrace{rd(x2, \underbrace{mult(i(x2), i(x1))})} \\
= \quad & \text{by Axiom 11 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \underbrace{rd(x2, \overbrace{i(mult(x2, x1))})} \\
= \quad & \text{by Lemma 58 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \overbrace{mult(x2, mult(x1, x2))}
\end{aligned}
$$

Lemma 141: $rd(op_r(x1, x2, i(x2)), x1) = asoc(x1, x2, i(x2))$

$$
\begin{aligned}
& \underbrace{rd(op_r(x1, x2, i(x2)), x1)} \\
= \quad & \text{by Lemma 117 RL with } \{x3 \leftarrow i(x2), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{mult(i(x1), \underbrace{op_r(x1, x2, i(x2))})} \\
= \quad & \text{by Lemma 107 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{mult(i(x1), \overbrace{mult(x1, asoc(x1, x2, i(x2)))})} \\
= \quad & \text{by Axiom 2 RL with } \{x2 \leftarrow x1, x1 \leftarrow asoc(x1, x2, i(x2))\} \\
& \overbrace{asoc(x1, x2, i(x2))}
\end{aligned}
$$

Lemma 142: $asoc(x1, i(x2), x2) = rd(op_r(x1, i(x2), x2), x1)$

$$
\begin{aligned}
& asoc(x1, i(x2), \underbrace{x2}) \\
= \quad & \text{by Lemma 3 RL with } \{x1 \leftarrow x2\} \\
& \underbrace{asoc(x1, i(x2), \overbrace{i(i(x2))})} \\
= \quad & \text{by Lemma 141 RL with } \{x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& \overbrace{rd(op_r(x1, i(x2), \underbrace{i(i(x2))}), x1)} \\
= \quad & \text{by Lemma 3 LR with } \{x1 \leftarrow x2\} \\
& rd(op_r(x1, i(x2), \overbrace{x2}), x1)
\end{aligned}
$$

Lemma 143: $rd(x1, op_r(x1, i(x2), x2)) = asoc(i(x1), x2, i(x2))$

$$
\begin{aligned}
& rd(x1, op_r(\underbrace{x1}, i(x2), x2)) \\
= \quad & \text{by Lemma 3 RL with } \{x1 \leftarrow x1\} \\
& rd(x1, \underbrace{op_r(\overbrace{i(i(x1))}, i(x2), x2)}) \\
= \quad & \text{by Lemma 101 LR with } \{x2 \leftarrow x2, x1 \leftarrow i(x1)\} \\
& \underbrace{rd(x1, \overbrace{i(op_r(i(x1), x2, i(x2)))})} \\
= \quad & \text{by Lemma 38 RL with } \{x1 \leftarrow x1, x2 \leftarrow op_r(i(x1), x2, i(x2))\} \\
& \overbrace{\underbrace{rd(op_r(i(x1), x2, i(x2)), i(x1))}} \\
= \quad & \text{by Lemma 141 LR with } \{x2 \leftarrow x2, x1 \leftarrow i(x1)\} \\
& \overbrace{asoc(i(x1), x2, i(x2))}
\end{aligned}
$$

Lemma 144: $asoc(i(x1), i(x2), x2) = rd(x1, op_r(x1, x2, i(x2)))$

$$
\begin{aligned}
& asoc(i(x1), i(x2), \underbrace{x2}) \\
= \quad & \text{by Lemma 3 RL with } \{x1 \leftarrow x2\} \\
& \underbrace{asoc(i(x1), i(x2), \overbrace{i(i(x2))})} \\
= \quad & \text{by Lemma 143 RL with } \{x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& \overbrace{rd(x1, op_r(x1, \underbrace{i(i(x2))}, i(x2)))} \\
= \quad & \text{by Lemma 3 LR with } \{x1 \leftarrow x2\} \\
& rd(x1, op_r(x1, \overbrace{x2}, i(x2)))
\end{aligned}
$$

Lemma 145: $asoc(i(x1), i(x2), x2) = i(asoc(x1, x2, i(x2)))$

$$
\begin{aligned}
& \underbrace{asoc(i(x1), i(x2), x2)} \\
= \quad & \text{by Lemma 144 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{\overbrace{rd(x1, op_r(x1, x2, i(x2)))}} \\
= \quad & \text{by Lemma 10 RL with } \{x2 \leftarrow x1, x1 \leftarrow op_r(x1, x2, i(x2))\} \\
& \overbrace{i(\underbrace{rd(op_r(x1, x2, i(x2)), x1)})} \\
= \quad & \text{by Lemma 141 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& i(\overbrace{asoc(x1, x2, i(x2))})
\end{aligned}
$$

Lemma 146: $rd(x1, mult(x1, op_r(x1, x2, x3))) = i(op_r(x1, x2, x3))$

$$
\begin{aligned}
& rd(x1, \underbrace{mult(x1, op_r(x1, x2, x3))}) \\
= \quad & \text{by Lemma 119 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{rd(x1, \overbrace{mult(op_r(x1, x2, x3), x1)})} \\
= \quad & \text{by Lemma 9 LR with } \{x2 \leftarrow op_r(x1, x2, x3), x1 \leftarrow x1\} \\
& \overbrace{i(op_r(x1, x2, x3))}
\end{aligned}
$$

Lemma 147: $op_r(mult(x1,x2),rd(i(x1),x2),x3) = mult(x1,x2)$

$$
\begin{aligned}
& op_r(\underbrace{mult(x1,x2)}, rd(i(x1),x2),x3) \\
= & \quad \text{by Lemma 3 RL with } \{x1 \leftarrow mult(x1,x2)\} \\
& op_r(\overbrace{i(\underbrace{i(mult(x1,x2))}}), rd(i(x1),x2),x3) \\
= & \quad \text{by Lemma 33 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_r(i(\overbrace{op_t(rd(i(x1),x2),x1)}), \underbrace{rd(i(x1),x2)},x3) \\
= & \quad \text{by Lemma 3 RL with } \{x1 \leftarrow rd(i(x1),x2)\} \\
& op_r(\underbrace{i(op_t(rd(i(x1),x2),x1))}, \overbrace{i(i(rd(i(x1),x2)))},x3) \\
= & \quad \text{by Lemma 15 RL with } \{x2 \leftarrow x1, x1 \leftarrow rd(i(x1),x2)\} \\
& \underbrace{op_r(\overbrace{op_t(i(rd(i(x1),x2)),i(x1))}, i(i(rd(i(x1),x2))),x3)} \\
= & \quad \text{by Axiom 14 RL with } \{x4 \leftarrow i(x1), x3 \leftarrow x3, x2 \leftarrow i(i(rd(i(x1),x2))), x1 \leftarrow i(rd(i(x1),x2))\} \\
& \overbrace{op_t(\underbrace{op_r(i(rd(i(x1),x2)),i(i(rd(i(x1),x2))),x3)}, i(x1))} \\
= & \quad \text{by Lemma 52 LR with } \{x2 \leftarrow x3, x1 \leftarrow i(rd(i(x1),x2))\} \\
& \underbrace{op_t(\overbrace{i(rd(i(x1),x2))}, i(x1))} \\
= & \quad \text{by Lemma 15 LR with } \{x2 \leftarrow x1, x1 \leftarrow rd(i(x1),x2)\} \\
& \overbrace{\underbrace{i(op_t(rd(i(x1),x2),x1))}} \\
= & \quad \text{by Lemma 46 LR with } \{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow i(x1)\} \\
& \overbrace{\underbrace{op_t(rd(x2,i(x1)),i(x1))}} \\
= & \quad \text{by Lemma 14 LR with } \{x2 \leftarrow i(x1), x1 \leftarrow x2\} \\
& \overbrace{mult(\underbrace{i(i(x1))},x2)} \\
= & \quad \text{by Lemma 3 LR with } \{x1 \leftarrow x1\} \\
& mult(\overbrace{x1},x2)
\end{aligned}
$$

Lemma 148: $i(x2) = op_r(i(x2),rd(i(x1),i(mult(x1,x2))),x3)$

$$
\begin{aligned}
& \underbrace{i(x2)} \\
= & \quad \text{by Lemma 20 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{mult(x1,i(mult(x1,x2)))}} \\
= & \quad \text{by Lemma 147 RL with } \{x3 \leftarrow x3, x2 \leftarrow i(mult(x1,x2)), x1 \leftarrow x1\} \\
& \overbrace{op_r(\underbrace{mult(x1,i(mult(x1,x2)))}, rd(i(x1),i(mult(x1,x2))),x3)} \\
= & \quad \text{by Lemma 20 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_r(\overbrace{i(x2)}, rd(i(x1),i(mult(x1,x2))),x3)
\end{aligned}
$$

Lemma 149: $i(x2) = op_r(i(x2), rd(i(rd(x1, x2)), i(x1)), x3)$

$$
\begin{aligned}
& \underbrace{i(x2)} \\
= \quad & \text{by Lemma 22 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{mult(rd(x1, x2), i(x1))}} \\
= \quad & \text{by Lemma 147 RL with } \{x3 \leftarrow x3, x2 \leftarrow i(x1), x1 \leftarrow rd(x1, x2)\} \\
& \overbrace{op_r(\underbrace{mult(rd(x1, x2), i(x1))}, rd(i(rd(x1, x2)), i(x1)), x3)} \\
= \quad & \text{by Lemma 22 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_r(\overbrace{i(x2)}, rd(i(rd(x1, x2)), i(x1)), x3)
\end{aligned}
$$

Lemma 150: $i(mult(i(x1), rd(mult(x2, x1), x2))) = rd(i(rd(mult(x2, x1), x2)), i(x1))$

$$
\begin{aligned}
& \underbrace{i(mult(i(x1), rd(mult(x2, x1), x2)))} \\
= \quad & \text{by Lemma 122 RL with } \{x2 \leftarrow i(x1), x1 \leftarrow rd(mult(x2, x1), x2)\} \\
& \overbrace{rd(i(rd(mult(x2, x1), x2)), op_r(i(x1), \underbrace{rd(mult(x2, x1), x2)}, i(rd(mult(x2, x1), x2))))} \\
= \quad & \text{by Lemma 23 RL with } \{x2 \leftarrow mult(x2, x1), x1 \leftarrow x2\} \\
& rd(i(rd(mult(x2, x1), x2)), \underbrace{op_r(i(x1), \overbrace{rd(i(x2), i(mult(x2, x1)))}, i(rd(mult(x2, x1), x2)))}) \\
= \quad & \text{by Lemma 148 RL with } \{x3 \leftarrow i(rd(mult(x2, x1), x2)), x1 \leftarrow x2, x2 \leftarrow x1\} \\
& rd(i(rd(mult(x2, x1), x2)), \overbrace{i(x1)})
\end{aligned}
$$

Lemma 151: $mult(x1, rd(x2, mult(x2, x1))) = rd(i(rd(mult(x2, x1), x2)), i(x1))$

$$
\begin{aligned}
& mult(x1, \underbrace{rd(x2, mult(x2, x1))}) \\
= \quad & \text{by Lemma 10 RL with } \{x2 \leftarrow x2, x1 \leftarrow mult(x2, x1)\} \\
& \underbrace{mult(x1, \overbrace{i(rd(mult(x2, x1), x2))})} \\
= \quad & \text{by Lemma 16 RL with } \{x2 \leftarrow rd(mult(x2, x1), x2), x1 \leftarrow x1\} \\
& \overbrace{\underbrace{i(mult(i(x1), rd(mult(x2, x1), x2)))}} \\
= \quad & \text{by Lemma 150 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{rd(i(rd(mult(x2, x1), x2)), i(x1))}
\end{aligned}
$$

Lemma 152: $i(mult(i(x1), rd(x2, rd(x2, x1)))) = rd(i(rd(x2, rd(x2, x1))), i(x1))$

$$
\begin{aligned}
& \underbrace{i(mult(i(x1), rd(x2, rd(x2, x1))))} \\
= \quad & \text{by Lemma 122 RL with } \{x2 \leftarrow i(x1), x1 \leftarrow rd(x2, rd(x2, x1))\} \\
& \overbrace{rd(i(rd(x2, rd(x2, x1))), op_r(i(x1), \underbrace{rd(x2, rd(x2, x1))}, i(rd(x2, rd(x2, x1)))))} \\
= \quad & \text{by Lemma 23 RL with } \{x2 \leftarrow x2, x1 \leftarrow rd(x2, x1)\} \\
& rd(i(rd(x2, rd(x2, x1))), \underbrace{op_r(i(x1), \overbrace{rd(i(rd(x2, x1)), i(x2))}, i(rd(x2, rd(x2, x1))))}) \\
= \quad & \text{by Lemma 149 RL with } \{x3 \leftarrow i(rd(x2, rd(x2, x1))), x1 \leftarrow x2, x2 \leftarrow x1\} \\
& rd(i(rd(x2, rd(x2, x1))), \overbrace{i(x1)})
\end{aligned}
$$

Lemma 153: $mult(x1, rd(rd(x2, x1), x2)) = rd(i(rd(x2, rd(x2, x1))), i(x1))$

$$
\begin{aligned}
& mult(x1, \underbrace{rd(rd(x2, x1), x2)}) \\
= & \quad \text{by Lemma 10 RL with } \{x2 \leftarrow rd(x2, x1), x1 \leftarrow x2\} \\
& \underbrace{mult(x1, \overbrace{i(rd(x2, rd(x2, x1)))})} \\
= & \quad \text{by Lemma 16 RL with } \{x2 \leftarrow rd(x2, rd(x2, x1)), x1 \leftarrow x1\} \\
& \overbrace{\underbrace{i(mult(i(x1), rd(x2, rd(x2, x1))))}} \\
= & \quad \text{by Lemma 152 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{rd(i(rd(x2, rd(x2, x1))), i(x1))}
\end{aligned}
$$

Lemma 154: $op_t(x1, rd(i(x1), op_r(x1, x2, x3))) = x1$

$$
\begin{aligned}
& op_t(x1, \underbrace{rd(i(x1), op_r(x1, x2, x3))}) \\
= & \quad \text{by Lemma 131 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{op_t(x1, \overbrace{i(mult(x1, op_r(x1, x2, x3)))})} \\
= & \quad \text{by Lemma 18 RL with } \{x2 \leftarrow mult(x1, op_r(x1, x2, x3)), x1 \leftarrow x1\} \\
& \overbrace{i(op_t(i(x1), \underbrace{mult(x1, op_r(x1, x2, x3))}))} \\
= & \quad \text{by Lemma 119 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& i(\underbrace{op_t(i(x1), \overbrace{mult(op_r(x1, x2, x3), x1)})}) \\
= & \quad \text{by Axiom 10 RL with } \{x2 \leftarrow i(x1), x1 \leftarrow mult(op_r(x1, x2, x3), x1)\} \\
& i(\overbrace{mult(i(mult(op_r(x1, x2, x3), x1)), \underbrace{mult(i(x1), mult(op_r(x1, x2, x3), x1))})}) \\
= & \quad \text{by Axiom 10 LR with } \{x2 \leftarrow op_r(x1, x2, x3), x1 \leftarrow x1\} \\
& i(mult(i(\underbrace{mult(op_r(x1, x2, x3), x1)}), \overbrace{op_t(op_r(x1, x2, x3), x1)})) \\
= & \quad \text{by Lemma 119 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& i(mult(\underbrace{i(\overbrace{mult(x1, op_r(x1, x2, x3))})}, op_t(op_r(x1, x2, x3), x1))) \\
= & \quad \text{by Lemma 131 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& i(mult(\overbrace{rd(i(x1), op_r(x1, x2, x3))}, \underbrace{op_t(op_r(x1, x2, x3), x1)})) \\
= & \quad \text{by Axiom 14 LR with } \{x4 \leftarrow x1, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& i(mult(rd(i(x1), op_r(x1, x2, x3)), \overbrace{op_r(\underbrace{op_t(x1, x1)}, x2, x3)})) \\
= & \quad \text{by Lemma 4 LR with } \{x1 \leftarrow x1\} \\
& i(\underbrace{mult(rd(i(x1), op_r(x1, x2, x3)), op_r(\overbrace{x1}, x2, x3))}) \\
= & \quad \text{by Axiom 4 RL with } \{x2 \leftarrow op_r(x1, x2, x3), x1 \leftarrow i(x1)\} \\
& \underbrace{i(\overbrace{i(x1)})} \\
= & \quad \text{by Lemma 3 LR with } \{x1 \leftarrow x1\} \\
& \overbrace{x1}
\end{aligned}
$$

Lemma 155: $mult(x1, op_r(op_t(x2, x1), x3, x4)) = mult(op_r(x2, x3, x4), x1)$

$$
\begin{aligned}
& \underbrace{mult(x1, \overbrace{op_r(op_t(x2, x1), x3, x4)}^{})}_{} \\
= & \quad \text{by Axiom 14 RL with } \{x4 \leftarrow x1, x3 \leftarrow x4, x2 \leftarrow x3, x1 \leftarrow x2\} \\
& \underbrace{mult(x1, \overbrace{op_t(op_r(x2, x3, x4), x1)}^{})}_{} \\
= & \quad \text{by Lemma 13 LR with } \{x2 \leftarrow op_r(x2, x3, x4), x1 \leftarrow x1\} \\
& \overbrace{mult(op_r(x2, x3, x4), x1)}^{}
\end{aligned}
$$

Lemma 156: $mult(op_r(x1, op_t(x1, x2), x3), x2) = mult(x1, x2)$

$$
\begin{aligned}
& \underbrace{mult(op_r(x1, op_t(x1, x2), x3), x2)}_{} \\
= & \quad \text{by Lemma 155 RL with } \{x4 \leftarrow x3, x3 \leftarrow op_t(x1, x2), x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \overbrace{mult(x2, \underbrace{op_r(op_t(x1, x2), op_t(x1, x2), x3)}_{})}^{} \\
= & \quad \text{by Lemma 26 LR with } \{x2 \leftarrow x3, x1 \leftarrow op_t(x1, x2)\} \\
& \underbrace{mult(x2, \overbrace{op_t(x1, x2)}^{})}_{} \\
= & \quad \text{by Lemma 13 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \overbrace{mult(x1, x2)}^{}
\end{aligned}
$$

Lemma 157: $op_r(x1, op_t(x1, x2), x3) = x1$

$$
\begin{aligned}
& \underbrace{op_r(x1, op_t(x1, x2), x3)}_{} \\
= & \quad \text{by Axiom 5 LR with } \{x2 \leftarrow x2, x1 \leftarrow op_r(x1, op_t(x1, x2), x3)\} \\
& \overbrace{rd(\underbrace{mult(op_r(x1, op_t(x1, x2), x3), x2)}_{}, x2)}^{} \\
= & \quad \text{by Lemma 156 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{rd(\overbrace{mult(x1, x2)}^{}, x2)}_{} \\
= & \quad \text{by Axiom 5 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{x1}^{}
\end{aligned}
$$

Lemma 158: $mult(x1, asoc(x1, x2, x1)) = op_l(x1, x1, x2)$

$$
\begin{aligned}
& \underbrace{mult(x1, asoc(x1, x2, x1))}_{} \\
= & \quad \text{by Lemma 138 RL with } \{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow mult(x1, asoc(x1, x2, x1))\} \\
& \overbrace{op_l(\underbrace{op_l(mult(x1, \overbrace{asoc(x1, x2, x1)}^{})), x2, x1)}_{}, x1, x2)}^{} \\
= & \quad \text{by Axiom 12 RL with } \{x3 \leftarrow mult(x1, asoc(x1, x2, x1)), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_l(\overbrace{mult(i(mult(x1, x2)), \underbrace{mult(x1, mult(x2, mult(x1, asoc(x1, x2, x1))))}_{})}^{}, x1, x2) \\
= & \quad \text{by Axiom 6 LR with } \{x3 \leftarrow asoc(x1, x2, x1), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_l(mult(i(mult(x1, x2)), \overbrace{\underbrace{mult(mult(x1, mult(x2, x1)), asoc(x1, x2, x1))}_{}}^{}), x1, x2) \\
= & \quad \text{by Axiom 8 RL with } \{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_l(\underbrace{mult(i(mult(x1, x2)), \overbrace{mult(mult(x1, x2), x1)}^{})}_{}, x1, x2) \\
= & \quad \text{by Axiom 2 RL with } \{x2 \leftarrow mult(x1, x2), x1 \leftarrow x1\} \\
& op_l(\overbrace{x1}^{}, x1, x2)
\end{aligned}
$$

Lemma 159: $mult(op_r(i(x1), x2, x3), x1) = mult(x1, op_r(i(x1), x2, x3))$

$$
\begin{aligned}
& \underbrace{mult(op_r(i(x1), x2, x3), x1)} \\
= \quad & \text{by Lemma 155 RL with } \{x4 \leftarrow x3, x3 \leftarrow x2, x2 \leftarrow i(x1), x1 \leftarrow x1\} \\
& \overbrace{mult(x1, op_r(\underbrace{op_t(i(x1), x1)}, x2, x3))} \\
= \quad & \text{by Lemma 7 LR with } \{x1 \leftarrow x1\} \\
& mult(x1, op_r(\overbrace{i(x1)}, x2, x3))
\end{aligned}
$$

Lemma 160: $op_t(x1, i(op_r(x1, x2, x3))) = x1$

$$
\begin{aligned}
& op_t(x1, i(op_r(\underbrace{x1}, x2, x3))) \\
= \quad & \text{by Lemma 3 RL with } \{x1 \leftarrow x1\} \\
& \underbrace{op_t(x1, i(op_r(\overbrace{i(i(x1))}, x2, x3)))} \\
= \quad & \text{by Lemma 18 RL with } \{x2 \leftarrow op_r(i(i(x1)), x2, x3), x1 \leftarrow x1\} \\
& \overbrace{i(\underbrace{op_t(i(x1), op_r(i(i(x1)), x2, x3))})} \\
= \quad & \text{by Axiom 10 RL with } \{x2 \leftarrow i(x1), x1 \leftarrow op_r(i(i(x1)), x2, x3)\} \\
& i(\overbrace{mult(i(op_r(i(i(x1)), x2, x3)), \underbrace{mult(i(x1), op_r(i(i(x1)), x2, x3))})}) \\
= \quad & \text{by Lemma 159 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow i(x1)\} \\
& i(\underbrace{mult(i(op_r(i(i(x1)), x2, x3)), \overbrace{mult(op_r(i(i(x1)), x2, x3), i(x1))})}) \\
= \quad & \text{by Axiom 2 RL with } \{x2 \leftarrow op_r(i(i(x1)), x2, x3), x1 \leftarrow i(x1)\} \\
& \underbrace{i(\overbrace{i(x1)})} \\
= \quad & \text{by Lemma 3 LR with } \{x1 \leftarrow x1\} \\
& \overbrace{x1}
\end{aligned}
$$

Lemma 161: $mult(op_t(x1, x2), mult(x1, x3)) = mult(x1, mult(op_t(x1, x2), x3))$

$$
\begin{aligned}
& \underbrace{mult(op_t(x1, x2), mult(x1, x3))} \\
= \quad & \text{by Lemma 124 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{\overbrace{mult(mult(x1, op_t(x1, x2)), x3)}} \\
= \quad & \text{by Lemma 91 RL with } \{x3 \leftarrow x3, x2 \leftarrow op_t(x1, x2), x1 \leftarrow x1\} \\
& \overbrace{mult(\underbrace{op_r(x1, op_t(x1, x2), x3)}, mult(op_t(x1, x2), x3))} \\
= \quad & \text{by Lemma 157 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& mult(\overbrace{x1}, mult(op_t(x1, x2), x3))
\end{aligned}
$$

Lemma 162: $mult(mult(x1, op_t(x1, x2)), x3) = mult(x1, mult(op_t(x1, x2), x3))$

$$
\begin{aligned}
& \underbrace{\overbrace{mult(mult(x1, op_t(x1, x2)), x3)}} \\
= \quad & \text{by Lemma 124 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{\overbrace{mult(op_t(x1, x2), mult(x1, x3))}} \\
= \quad & \text{by Lemma 161 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{mult(x1, mult(op_t(x1, x2), x3))}
\end{aligned}
$$

Lemma 163: $op_l(x1, op_t(x2, x3), x2) = x1$

$$
\begin{aligned}
& \underbrace{op_l(x1, op_t(x2, x3), x2)} \\
= & \quad \text{by Axiom 12 RL with } \{x3 \leftarrow x1, x2 \leftarrow op_t(x2, x3), x1 \leftarrow x2\} \\
& \overbrace{mult(i(mult(x2, op_t(x2, x3))), \underbrace{mult(x2, mult(op_t(x2, x3), x1))})} \\
= & \quad \text{by Lemma 162 RL with } \{x3 \leftarrow x1, x2 \leftarrow x3, x1 \leftarrow x2\} \\
& \underbrace{mult(i(mult(x2, op_t(x2, x3))), \overbrace{mult(mult(x2, op_t(x2, x3)), x1)})} \\
= & \quad \text{by Axiom 2 RL with } \{x2 \leftarrow mult(x2, op_t(x2, x3)), x1 \leftarrow x1\} \\
& \overbrace{x1}
\end{aligned}
$$

Lemma 164: $op_l(x1, mult(i(x2), x3), rd(x3, x2)) = x1$

$$
\begin{aligned}
& op_l(x1, \underbrace{mult(i(x2), x3)}, rd(x3, x2)) \\
= & \quad \text{by Lemma 14 RL with } \{x2 \leftarrow x2, x1 \leftarrow x3\} \\
& \underbrace{op_l(x1, \overbrace{op_t(rd(x3, x2), x2)}, rd(x3, x2))} \\
= & \quad \text{by Lemma 163 LR with } \{x3 \leftarrow x2, x2 \leftarrow rd(x3, x2), x1 \leftarrow x1\} \\
& \overbrace{x1}
\end{aligned}
$$

Lemma 165: $op_l(x1, x2, op_r(x2, x3, i(x3))) = x1$

$$
\begin{aligned}
& op_l(\underbrace{x1}, x2, op_r(x2, x3, i(x3))) \\
= & \quad \text{by Lemma 164 RL with } \{x3 \leftarrow mult(rd(i(x3), op_r(x2, x3, i(x3))), op_r(x2, x3, i(x3))), x2 \leftarrow rd(i(x3), op_r(x2, x3, i(x3))), x1 \leftarrow \\
& op_l(\overbrace{op_l(x1, \underbrace{mult(i(rd(i(x3), op_r(x2, x3, i(x3)))), mult(rd(i(x3), op_r(x2, x3, i(x3))), op_r(x2, x3, i(x3))))}, rd(mult(rd(i(x3), o} \\
= & \quad \text{by Axiom 2 RL with } \{x2 \leftarrow rd(i(x3), op_r(x2, x3, i(x3))), x1 \leftarrow op_r(x2, x3, i(x3))\} \\
& op_l(op_l(x1, \overbrace{op_r(x2, x3, i(x3))}, rd(\underbrace{mult(rd(i(x3), op_r(x2, x3, i(x3))), op_r(x2, x3, i(x3)))}, rd(i(x3), op_r(x2, x3, i(x3))))), x2, op \\
= & \quad \text{by Axiom 4 RL with } \{x2 \leftarrow op_r(x2, x3, i(x3)), x1 \leftarrow i(x3)\} \\
& op_l(op_l(x1, op_r(x2, x3, i(x3)), rd(\overbrace{i(x3)}, \underbrace{rd(i(x3), op_r(x2, x3, i(x3)))})), x2, op_r(x2, x3, i(x3))) \\
= & \quad \text{by Lemma 122 LR with } \{x2 \leftarrow x2, x1 \leftarrow x3\} \\
& op_l(op_l(x1, op_r(x2, x3, i(x3)), \underbrace{rd(i(x3), \overbrace{i(mult(x2, x3))})}), x2, op_r(x2, x3, i(x3))) \\
= & \quad \text{by Lemma 23 LR with } \{x2 \leftarrow mult(x2, x3), x1 \leftarrow x3\} \\
& op_l(op_l(x1, op_r(x2, x3, i(x3)), \overbrace{\underbrace{rd(mult(x2, x3), x3)}}), x2, op_r(x2, x3, i(x3))) \\
= & \quad \text{by Axiom 5 RL with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& \underbrace{op_l(op_l(x1, op_r(x2, x3, i(x3)), \overbrace{x2}), x2, op_r(x2, x3, i(x3)))} \\
= & \quad \text{by Lemma 138 LR with } \{x3 \leftarrow x2, x2 \leftarrow op_r(x2, x3, i(x3)), x1 \leftarrow x1\} \\
& \overbrace{x1}
\end{aligned}
$$

Lemma 166: $mult(op_r(x1, i(op_t(x1, x2)), x3), x2) = mult(x1, x2)$

$$
\begin{aligned}
& \underbrace{mult(op_r(x1, i(op_t(x1, x2)), x3), x2)} \\
= \quad & \text{by Lemma 155 RL with } \{x4 \leftarrow x3, x3 \leftarrow i(op_t(x1, x2)), x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \overbrace{mult(x2, \underbrace{op_r(op_t(x1, x2), i(op_t(x1, x2)), x3)})} \\
= \quad & \text{by Lemma 52 LR with } \{x2 \leftarrow x3, x1 \leftarrow op_t(x1, x2)\} \\
& \underbrace{mult(x2, \overbrace{op_t(x1, x2)})} \\
= \quad & \text{by Lemma 13 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \overbrace{mult(x1, x2)}
\end{aligned}
$$

Lemma 167: $mult(x1, op_l(op_t(x2, x1), x3, x4)) = mult(op_l(x2, x3, x4), x1)$

$$
\begin{aligned}
& mult(x1, \underbrace{op_l(op_t(x2, x1), x3, x4)}) \\
= \quad & \text{by Axiom 15 RL with } \{x4 \leftarrow x1, x3 \leftarrow x4, x2 \leftarrow x3, x1 \leftarrow x2\} \\
& \underbrace{mult(x1, \overbrace{op_t(op_l(x2, x3, x4), x1)})} \\
= \quad & \text{by Lemma 13 LR with } \{x2 \leftarrow op_l(x2, x3, x4), x1 \leftarrow x1\} \\
& \overbrace{mult(op_l(x2, x3, x4), x1)}
\end{aligned}
$$

Lemma 168: $mult(op_l(x1, x2, x3), x1) = mult(x1, op_l(x1, x2, x3))$

$$
\begin{aligned}
& \underbrace{mult(op_l(x1, x2, x3), x1)} \\
= \quad & \text{by Lemma 167 RL with } \{x4 \leftarrow x3, x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x1\} \\
& \overbrace{mult(x1, op_l(\underbrace{op_t(x1, x1)}, x2, x3))} \\
= \quad & \text{by Lemma 4 LR with } \{x1 \leftarrow x1\} \\
& mult(x1, op_l(\overbrace{x1}, x2, x3))
\end{aligned}
$$

Lemma 169: $op_t(x1, op_l(x1, x2, x3)) = x1$

$$
\begin{aligned}
& \underbrace{op_t(x1, op_l(x1, x2, x3))} \\
= \quad & \text{by Axiom 10 RL with } \{x2 \leftarrow x1, x1 \leftarrow op_l(x1, x2, x3)\} \\
& \overbrace{mult(i(op_l(x1, x2, x3)), \underbrace{mult(x1, op_l(x1, x2, x3))})} \\
= \quad & \text{by Lemma 168 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{mult(i(op_l(x1, x2, x3)), \overbrace{mult(op_l(x1, x2, x3), x1)})} \\
= \quad & \text{by Axiom 2 RL with } \{x2 \leftarrow op_l(x1, x2, x3), x1 \leftarrow x1\} \\
& \overbrace{x1}
\end{aligned}
$$

Lemma 170: $mult(x1, rd(op_l(x1, x2, x3), x1)) = op_l(x1, x2, x3)$

$$
\begin{aligned}
& \underbrace{mult(x1, rd(op_l(x1, x2, x3), x1))} \\
= \quad & \text{by Lemma 55 RL with } \{x2 \leftarrow op_l(x1, x2, x3), x1 \leftarrow x1\} \\
& \overbrace{mult(\underbrace{mult(x1, op_l(x1, x2, x3))}, i(x1))} \\
= \quad & \text{by Lemma 168 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{mult(\overbrace{mult(op_l(x1, x2, x3), x1)}, i(x1))} \\
= \quad & \text{by Lemma 92 LR with } \{x2 \leftarrow x1, x1 \leftarrow op_l(x1, x2, x3)\} \\
& \overbrace{\underbrace{op_r(op_l(x1, x2, x3), x1, i(x1))}} \\
= \quad & \text{by Axiom 18 LR with } \{x5 \leftarrow i(x1), x4 \leftarrow x1, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(\underbrace{op_r(x1, x1, i(x1))}, x2, x3)} \\
= \quad & \text{by Lemma 26 LR with } \{x2 \leftarrow i(x1), x1 \leftarrow x1\} \\
& op_l(\overbrace{x1}, x2, x3)
\end{aligned}
$$

Lemma 171: $mult(i(x1), op_l(x1, x2, x3)) = rd(op_l(x1, x2, x3), x1)$

$$
\begin{aligned}
& mult(i(x1), \underbrace{op_l(x1, x2, x3)}) \\
= \quad & \text{by Lemma 170 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{mult(i(x1), \overbrace{mult(x1, rd(op_l(x1, x2, x3), x1))})} \\
= \quad & \text{by Axiom 2 RL with } \{x2 \leftarrow x1, x1 \leftarrow rd(op_l(x1, x2, x3), x1)\} \\
& \overbrace{rd(op_l(x1, x2, x3), x1)}
\end{aligned}
$$

Lemma 172: $rd(op_l(x1, x1, x2), x1) = asoc(x1, x2, x1)$

$$
\begin{aligned}
& \underbrace{rd(op_l(x1, x1, x2), x1)} \\
= \quad & \text{by Lemma 171 RL with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x1\} \\
& \overbrace{mult(i(x1), \underbrace{op_l(x1, x1, x2)})} \\
= \quad & \text{by Lemma 158 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{mult(i(x1), \overbrace{mult(x1, asoc(x1, x2, x1))})} \\
= \quad & \text{by Axiom 2 RL with } \{x2 \leftarrow x1, x1 \leftarrow asoc(x1, x2, x1)\} \\
& \overbrace{asoc(x1, x2, x1)}
\end{aligned}
$$

Lemma 173: $mult(asoc(x1, x2, x1), x1) = op_l(x1, x1, x2)$

$$
\begin{aligned}
& mult(\underbrace{asoc(x1, x2, x1)}, x1) \\
= \quad & \text{by Lemma 172 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{mult(\overbrace{rd(op_l(x1, x1, x2), x1)}, x1)} \\
= \quad & \text{by Axiom 4 RL with } \{x2 \leftarrow x1, x1 \leftarrow op_l(x1, x1, x2)\} \\
& \overbrace{op_l(x1, x1, x2)}
\end{aligned}
$$

Lemma 174: $rd(x1, op_l(x1, x1, x2)) = i(asoc(x1, x2, x1))$

$$
\begin{aligned}
& \underbrace{rd(x1, op_l(x1, x1, x2))} \\
= & \quad \text{by Lemma 10 RL with } \{x2 \leftarrow x1, x1 \leftarrow op_l(x1, x1, x2)\} \\
& \overbrace{i(\underbrace{rd(op_l(x1, x1, x2), x1)})} \\
= & \quad \text{by Lemma 172 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& i(\overbrace{asoc(x1, x2, x1)})
\end{aligned}
$$

Lemma 175: $op_t(op_t(x1, x2), asoc(x1, x3, x1)) = op_t(x1, x2)$

$$
\begin{aligned}
& \underbrace{op_t(op_t(x1, x2), asoc(x1, x3, x1))} \\
= & \quad \text{by Axiom 13 RL with } \{x3 \leftarrow x2, x2 \leftarrow asoc(x1, x3, x1), x1 \leftarrow x1\} \\
& \overbrace{op_t(op_t(x1, \underbrace{asoc(x1, x3, x1)}), x2)} \\
= & \quad \text{by Lemma 172 RL with } \{x2 \leftarrow x3, x1 \leftarrow x1\} \\
& op_t(\underbrace{op_t(x1, \overbrace{rd(op_l(x1, x1, x3), x1)})}, x2) \\
= & \quad \text{by Axiom 10 RL with } \{x2 \leftarrow x1, x1 \leftarrow rd(op_l(x1, x1, x3), x1)\} \\
& op_t(\overbrace{mult(i(rd(op_l(x1, x1, x3), x1)), \underbrace{mult(x1, rd(op_l(x1, x1, x3), x1))})}, x2) \\
= & \quad \text{by Lemma 170 LR with } \{x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow x1\} \\
& op_t(mult(\underbrace{i(rd(op_l(x1, x1, x3), x1))}, \overbrace{op_l(x1, x1, x3)}), x2) \\
= & \quad \text{by Lemma 10 LR with } \{x2 \leftarrow x1, x1 \leftarrow op_l(x1, x1, x3)\} \\
& op_t(\underbrace{mult(\overbrace{rd(x1, op_l(x1, x1, x3))}, op_l(x1, x1, x3))}, x2) \\
= & \quad \text{by Axiom 4 RL with } \{x2 \leftarrow op_l(x1, x1, x3), x1 \leftarrow x1\} \\
& op_t(\overbrace{x1}, x2)
\end{aligned}
$$

Lemma 176: $rd(x1, op_l(x1, x2, x3)) = mult(x1, i(op_l(x1, x2, x3)))$

$$
\begin{aligned}
& \underbrace{rd(x1, op_l(x1, x2, x3))} \\
= & \quad \text{by Lemma 10 RL with } \{x2 \leftarrow x1, x1 \leftarrow op_l(x1, x2, x3)\} \\
& \overbrace{\underbrace{i(rd(op_l(x1, x2, x3), x1))}} \\
= & \quad \text{by Lemma 20 RL with } \{x2 \leftarrow rd(op_l(x1, x2, x3), x1), x1 \leftarrow x1\} \\
& \overbrace{mult(x1, i(\underbrace{mult(x1, rd(op_l(x1, x2, x3), x1))}))} \\
= & \quad \text{by Lemma 170 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& mult(x1, i(\overbrace{op_l(x1, x2, x3)}))
\end{aligned}
$$

Lemma 177: $rd(x1, op_l(x1, x2, x3)) = mult(i(op_l(x1, x2, x3)), x1)$

$$
\begin{aligned}
& rd(\underbrace{x1}, op_l(x1, x2, x3)) \\
= \quad & \text{by Lemma 138 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{rd(\overbrace{op_l(op_l(x1, x2, x3), x3, x2)}, op_l(x1, x2, x3))} \\
= \quad & \text{by Lemma 171 RL with } \{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow op_l(x1, x2, x3)\} \\
& \overbrace{mult(i(op_l(x1, x2, x3)), \underbrace{op_l(op_l(x1, x2, x3), x3, x2)})} \\
= \quad & \text{by Lemma 138 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& mult(i(op_l(x1, x2, x3)), \overbrace{x1})
\end{aligned}
$$

Lemma 178: $mult(op_l(x1, x2, x3), i(x1)) = rd(op_l(x1, x2, x3), x1)$

$$
\begin{aligned}
& mult(op_l(x1, x2, x3), i(\underbrace{x1})) \\
= \quad & \text{by Lemma 138 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{mult(op_l(x1, x2, x3), i(\overbrace{op_l(op_l(x1, x2, x3), x3, x2)}))} \\
= \quad & \text{by Lemma 176 RL with } \{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow op_l(x1, x2, x3)\} \\
& \overbrace{rd(op_l(x1, x2, x3), \underbrace{op_l(op_l(x1, x2, x3), x3, x2)})} \\
= \quad & \text{by Lemma 138 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& rd(op_l(x1, x2, x3), \overbrace{x1})
\end{aligned}
$$

Lemma 179: $op_t(asoc(x1, x2, x1), op_t(x1, x3)) = mult(i(op_t(x1, x3)), mult(op_t(x1, x3), asoc(x1, x2, x1)))$

$$
\begin{aligned}
& op_t(asoc(x1, x2, x1), \underbrace{op_t(x1, x3)}) \\
= \quad & \text{by Lemma 175 RL with } \{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \underbrace{op_t(asoc(x1, x2, x1), \overbrace{op_t(op_t(x1, x3), asoc(x1, x2, x1))})} \\
= \quad & \text{by Lemma 133 RL with } \{x2 \leftarrow asoc(x1, x2, x1), x1 \leftarrow op_t(x1, x3)\} \\
& \overbrace{mult(i(\underbrace{op_t(op_t(x1, x3), asoc(x1, x2, x1))}), mult(op_t(x1, x3), asoc(x1, x2, x1)))} \\
= \quad & \text{by Lemma 175 LR with } \{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x1\} \\
& mult(i(\overbrace{op_t(x1, x3)}), mult(op_t(x1, x3), asoc(x1, x2, x1)))
\end{aligned}
$$

Lemma 180: $op_l(x1, x2, x3) = rd(x1, rd(x1, op_l(x1, x2, x3)))$

$$
\begin{aligned}
& \underbrace{op_l(x1, x2, x3)} \\
= \quad & \text{by Lemma 3 RL with } \{x1 \leftarrow op_l(x1, x2, x3)\} \\
& \underbrace{\overbrace{i(i(op_l(x1, x2, x3)))}} \\
= \quad & \text{by Lemma 9 RL with } \{x2 \leftarrow i(op_l(x1, x2, x3)), x1 \leftarrow x1\} \\
& \overbrace{rd(x1, \underbrace{mult(i(op_l(x1, x2, x3)), x1)})} \\
= \quad & \text{by Lemma 177 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& rd(x1, \overbrace{rd(x1, op_l(x1, x2, x3))})
\end{aligned}
$$

Lemma 181: $rd(x1, i(asoc(x1, x2, x1))) = op_l(x1, x1, x2)$

$$
\begin{aligned}
& \underbrace{rd(x1, i(asoc(x1, x2, x1)))} \\
= & \quad \text{by Lemma 174 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{rd(x1, \overbrace{rd(x1, op_l(x1, x1, x2))})} \\
= & \quad \text{by Lemma 180 RL with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x1, x2)}
\end{aligned}
$$

Lemma 182: $rd(i(x1), asoc(x1, x2, x1)) = i(op_l(x1, x1, x2))$

$$
\begin{aligned}
& \underbrace{rd(i(x1), asoc(x1, x2, x1))} \\
= & \quad \text{by Lemma 37 RL with } \{x2 \leftarrow x1, x1 \leftarrow asoc(x1, x2, x1)\} \\
& \overbrace{\underbrace{rd(i(\overbrace{asoc(x1, x2, x1)}), x1)}} \\
= & \quad \text{by Lemma 10 RL with } \{x2 \leftarrow i(asoc(x1, x2, x1)), x1 \leftarrow x1\} \\
& \overbrace{i(\underbrace{rd(x1, i(asoc(x1, x2, x1)))})} \\
= & \quad \text{by Lemma 181 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& i(\overbrace{op_l(x1, x1, x2)})
\end{aligned}
$$

Lemma 183: $mult(x1, i(asoc(x1, x2, x1))) = rd(x1, rd(op_l(x1, x1, x2), x1))$

$$
\begin{aligned}
& \underbrace{mult(x1, i(asoc(x1, x2, x1)))} \\
= & \quad \text{by Lemma 57 RL with } \{x2 \leftarrow i(asoc(x1, x2, x1)), x1 \leftarrow x1\} \\
& \overbrace{rd(x1, mult(i(x1), \underbrace{rd(x1, i(asoc(x1, x2, x1)))}))} \\
= & \quad \text{by Lemma 181 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& rd(x1, \underbrace{mult(i(x1), \overbrace{op_l(x1, x1, x2)})}) \\
= & \quad \text{by Lemma 171 LR with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x1\} \\
& rd(x1, \overbrace{rd(op_l(x1, x1, x2), x1)})
\end{aligned}
$$

Lemma 184: $mult(i(x1), asoc(x1, x2, x1)) = rd(mult(i(x1), op_l(x1, x1, x2)), x1)$

$$
\begin{aligned}
& \underbrace{mult(i(x1), asoc(x1, x2, x1))} \\
= & \quad \text{by Lemma 17 RL with } \{x2 \leftarrow asoc(x1, x2, x1), x1 \leftarrow x1\} \\
& \overbrace{\underbrace{i(mult(x1, i(\overbrace{asoc(x1, x2, x1)})))}} \\
= & \quad \text{by Lemma 85 RL with } \{x2 \leftarrow i(asoc(x1, x2, x1)), x1 \leftarrow x1\} \\
& \overbrace{rd(mult(i(x1), \underbrace{rd(x1, i(asoc(x1, x2, x1)))}), x1)} \\
= & \quad \text{by Lemma 181 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& rd(mult(i(x1), \overbrace{op_l(x1, x1, x2)}), x1)
\end{aligned}
$$

Lemma 185: $mult(i(x1), asoc(x1, x2, x1)) = rd(asoc(x1, x2, x1), x1)$

$$
\begin{aligned}
& \underbrace{mult(i(x1), asoc(x1, x2, x1))} \\
= \quad & \text{by Lemma 184 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{rd(\underbrace{mult(i(x1), op_l(x1, x1, x2))}, x1)} \\
= \quad & \text{by Lemma 171 LR with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x1\} \\
& rd(\overbrace{\underbrace{rd(op_l(x1, x1, x2), x1)}}, x1) \\
= \quad & \text{by Lemma 172 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& rd(\overbrace{asoc(x1, x2, x1)}, x1)
\end{aligned}
$$

Lemma 186: $i(i(asoc(x1, x2, x1))) = mult(x1, i(rd(x1, asoc(x1, x2, x1))))$

$$
\begin{aligned}
& \underbrace{i(i(asoc(x1, x2, x1)))} \\
= \quad & \text{by Lemma 20 RL with } \{x2 \leftarrow i(asoc(x1, x2, x1)), x1 \leftarrow x1\} \\
& \overbrace{mult(x1, i(\underbrace{mult(x1, i(asoc(x1, x2, x1)))}))} \\
= \quad & \text{by Lemma 183 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& mult(x1, i(\overbrace{rd(x1, \underbrace{rd(op_l(x1, x1, x2), x1)})})) \\
= \quad & \text{by Lemma 172 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& mult(x1, i(rd(x1, \overbrace{asoc(x1, x2, x1)}))))
\end{aligned}
$$

Lemma 187: $asoc(x1, x2, x1) = mult(x1, rd(asoc(x1, x2, x1), x1))$

$$
\begin{aligned}
& \underbrace{asoc(x1, x2, x1)} \\
= \quad & \text{by Lemma 3 RL with } \{x1 \leftarrow asoc(x1, x2, x1)\} \\
& \overbrace{\underbrace{i(i(asoc(x1, x2, x1)))}} \\
= \quad & \text{by Lemma 186 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{mult(x1, \underbrace{i(rd(x1, asoc(x1, x2, x1)))})} \\
= \quad & \text{by Lemma 10 LR with } \{x2 \leftarrow asoc(x1, x2, x1), x1 \leftarrow x1\} \\
& mult(x1, \overbrace{rd(asoc(x1, x2, x1), x1)})
\end{aligned}
$$

Lemma 188: $mult(op_l(i(x1), x2, x3), x1) = mult(x1, op_l(i(x1), x2, x3))$

$$
\begin{aligned}
& \underbrace{mult(op_l(i(x1), x2, x3), x1)} \\
= \quad & \text{by Lemma 167 RL with } \{x4 \leftarrow x3, x3 \leftarrow x2, x2 \leftarrow i(x1), x1 \leftarrow x1\} \\
& \overbrace{mult(x1, op_l(\underbrace{op_t(i(x1), x1)}, x2, x3))} \\
= \quad & \text{by Lemma 7 LR with } \{x1 \leftarrow x1\} \\
& mult(x1, op_l(\overbrace{i(x1)}, x2, x3))
\end{aligned}
$$

Lemma 189: $op_t(x1, op_l(i(x1), x2, x3)) = x1$

$$
\begin{aligned}
& \underbrace{op_t(x1, op_l(i(x1), x2, x3))} \\
= \quad & \text{by Axiom 10 RL with } \{x2 \leftarrow x1, x1 \leftarrow op_l(i(x1), x2, x3)\} \\
& \overbrace{mult(i(op_l(i(x1), x2, x3)), \underbrace{mult(x1, op_l(i(x1), x2, x3))})} \\
= \quad & \text{by Lemma 188 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{mult(i(op_l(i(x1), x2, x3)), \overbrace{mult(op_l(i(x1), x2, x3), x1)})} \\
= \quad & \text{by Axiom 2 RL with } \{x2 \leftarrow op_l(i(x1), x2, x3), x1 \leftarrow x1\} \\
& \overbrace{x1}
\end{aligned}
$$

Lemma 190: $op_t(x1, i(op_l(x1, x2, x3))) = i(i(x1))$

$$
\begin{aligned}
& op_t(x1, i(op_l(\underbrace{x1}, x2, x3))) \\
= \quad & \text{by Lemma 3 RL with } \{x1 \leftarrow x1\} \\
& \underbrace{op_t(x1, i(op_l(\overbrace{i(i(x1))}, x2, x3)))} \\
= \quad & \text{by Lemma 18 RL with } \{x2 \leftarrow op_l(i(i(x1)), x2, x3), x1 \leftarrow x1\} \\
& \overbrace{i(\underbrace{op_t(i(x1), op_l(i(i(x1)), x2, x3))})} \\
= \quad & \text{by Lemma 189 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow i(x1)\} \\
& i(\overbrace{i(x1)})
\end{aligned}
$$

Lemma 191: $op_t(x1, i(op_l(x1, x2, x3))) = x1$

$$
\begin{aligned}
& \underbrace{op_t(x1, i(op_l(x1, x2, x3)))} \\
= \quad & \text{by Lemma 190 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{\overbrace{i(i(x1))}} \\
= \quad & \text{by Lemma 3 LR with } \{x1 \leftarrow x1\} \\
& \overbrace{x1}
\end{aligned}
$$

Lemma 192: $op_t(i(op_l(x1, x2, x3)), x1) = mult(i(x1), mult(x1, i(op_l(x1, x2, x3))))$

$$
\begin{aligned}
& op_t(i(op_l(x1, x2, x3)), \underbrace{x1}) \\
= \quad & \text{by Lemma 191 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{op_t(i(op_l(x1, x2, x3)), \overbrace{op_t(x1, i(op_l(x1, x2, x3)))})} \\
= \quad & \text{by Lemma 133 RL with } \{x2 \leftarrow i(op_l(x1, x2, x3)), x1 \leftarrow x1\} \\
& \overbrace{mult(i(\underbrace{op_t(x1, i(op_l(x1, x2, x3)))}), mult(x1, i(op_l(x1, x2, x3))))} \\
= \quad & \text{by Lemma 191 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& mult(i(\overbrace{x1}), mult(x1, i(op_l(x1, x2, x3))))
\end{aligned}
$$

Lemma 193: $op_t(i(op_l(x1,x2,x3)),x1) = i(op_l(x1,x2,x3))$

$$
\begin{aligned}
& \underbrace{op_t(i(op_l(x1,x2,x3)),x1)} \\
= & \quad \text{by Lemma 192 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{mult(i(x1), mult(x1, i(op_l(x1,x2,x3))))}} \\
= & \quad \text{by Axiom 2 RL with } \{x2 \leftarrow x1, x1 \leftarrow i(op_l(x1,x2,x3))\} \\
& \overbrace{i(op_l(x1,x2,x3))}
\end{aligned}
$$

Lemma 194: $op_t(op_t(rd(x1,x2),x3),i(x1)) = op_t(mult(x1,i(x2)),x3)$

$$
\begin{aligned}
& \underbrace{op_t(op_t(rd(x1,x2),x3),i(x1))} \\
= & \quad \text{by Axiom 13 RL with } \{x3 \leftarrow x3, x2 \leftarrow i(x1), x1 \leftarrow rd(x1,x2)\} \\
& \overbrace{op_t(\underbrace{op_t(rd(x1,x2),i(x1))},x3)} \\
= & \quad \text{by Lemma 34 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_t(\overbrace{mult(x1,i(x2))},x3)
\end{aligned}
$$

Lemma 195: $op_l(x1,i(x2),mult(x2,i(x1))) = mult(mult(i(x2),x1),x2)$

$$
\begin{aligned}
& \underbrace{op_l(x1,i(x2),mult(x2,i(x1)))} \\
= & \quad \text{by Axiom 12 RL with } \{x3 \leftarrow x1, x2 \leftarrow i(x2), x1 \leftarrow mult(x2,i(x1))\} \\
& \overbrace{mult(i(mult(mult(x2,i(x1)),i(x2))),mult(\underbrace{mult(x2,i(x1))},mult(i(x2),x1)))} \\
= & \quad \text{by Lemma 16 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& mult(i(mult(mult(x2,i(x1)),i(x2))),\underbrace{mult(\overbrace{i(mult(i(x2),x1))},mult(i(x2),x1)))} \\
= & \quad \text{by Axiom 9 LR with } \{x1 \leftarrow mult(i(x2),x1)\} \\
& \underbrace{mult(i(mult(mult(x2,i(x1)),i(x2))),\overbrace{unit()})} \\
= & \quad \text{by Axiom 1 RL with } \{x1 \leftarrow i(mult(mult(x2,i(x1)),i(x2)))\} \\
& \overbrace{\underbrace{i(mult(mult(x2,i(x1)),i(x2)))}} \\
= & \quad \text{by Lemma 17 LR with } \{x2 \leftarrow x2, x1 \leftarrow mult(x2,i(x1))\} \\
& \overbrace{mult(\underbrace{i(mult(x2,i(x1)))},x2)} \\
= & \quad \text{by Lemma 17 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& mult(\overbrace{mult(i(x2),x1)},x2)
\end{aligned}
$$

Lemma 196: $op_l(i(mult(x1,x2)),x1,x2) = i(mult(x2,x1))$

$$
\begin{aligned}
& op_l(i(mult(x1,x2)),x1,\underbrace{x2}) \\
= \quad & \text{by Lemma 3 RL with } \{x1 \leftarrow x2\} \\
& op_l(i(mult(x1,x2)),\underbrace{x1},\overbrace{i(i(x2))}) \\
= \quad & \text{by Lemma 3 RL with } \{x1 \leftarrow x1\} \\
& op_l(\underbrace{i(mult(x1,x2))},\overbrace{i(i(x1))},i(i(x2))) \\
= \quad & \text{by Axiom 11 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_l(\overbrace{mult(i(x1),i(x2))},i(i(x1)),\underbrace{i(i(x2))}) \\
= \quad & \text{by Lemma 20 RL with } \{x2 \leftarrow i(x2), x1 \leftarrow i(x1)\} \\
& \underbrace{op_l(mult(i(x1),i(x2)),i(i(x1)),\overbrace{mult(i(x1),i(mult(i(x1),i(x2))))})} \\
= \quad & \text{by Lemma 195 LR with } \{x2 \leftarrow i(x1), x1 \leftarrow mult(i(x1),i(x2))\} \\
& \overbrace{mult(\underbrace{mult(i(i(x1)),mult(i(x1),i(x2)))},i(x1))} \\
= \quad & \text{by Axiom 2 RL with } \{x2 \leftarrow i(x1), x1 \leftarrow i(x2)\} \\
& \underbrace{mult(\overbrace{i(x2)},i(x1))} \\
= \quad & \text{by Axiom 11 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \overbrace{i(mult(x2,x1))}
\end{aligned}
$$

Lemma 197: $op_l(mult(i(x1),x2),x1,i(x2)) = mult(x2,i(x1))$

$$
\begin{aligned}
& op_l(mult(i(x1),x2),x1,i(\underbrace{x2})) \\
= \quad & \text{by Lemma 12 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_l(mult(i(x1),x2),\underbrace{x1},i(\overbrace{mult(x1,mult(i(x1),x2))})) \\
= \quad & \text{by Lemma 3 RL with } \{x1 \leftarrow x1\} \\
& op_l(mult(i(x1),x2),\overbrace{i(i(x1))},\underbrace{i(mult(x1,mult(i(x1),x2)))}) \\
= \quad & \text{by Axiom 11 RL with } \{x2 \leftarrow mult(i(x1),x2), x1 \leftarrow x1\} \\
& \underbrace{op_l(mult(i(x1),x2),i(i(x1)),\overbrace{mult(i(x1),i(mult(i(x1),x2)))})} \\
= \quad & \text{by Lemma 195 LR with } \{x2 \leftarrow i(x1), x1 \leftarrow mult(i(x1),x2)\} \\
& \overbrace{mult(mult(\underbrace{i(i(x1))},mult(i(x1),x2)),i(x1))} \\
= \quad & \text{by Lemma 3 LR with } \{x1 \leftarrow x1\} \\
& \underbrace{mult(mult(\overbrace{x1},mult(i(x1),x2)),i(x1))} \\
= \quad & \text{by Lemma 55 LR with } \{x2 \leftarrow mult(i(x1),x2), x1 \leftarrow x1\} \\
& \overbrace{\underbrace{mult(x1,rd(mult(i(x1),x2),x1))}} \\
= \quad & \text{by Lemma 84 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{mult(x2,i(x1))}
\end{aligned}
$$

Lemma 198: $mult(mult(x1, x2), rd(i(x1), x1)) = mult(x1, mult(rd(x2, x1), i(x1)))$

$$
\begin{aligned}
& \underbrace{mult(mult(x1, x2), rd(i(x1), x1))} \\
= \quad & \text{by Lemma 54 RL with } \{x3 \leftarrow rd(i(x1), x1), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{mult(x1, mult(rd(x2, x1), mult(x1, \underbrace{rd(i(x1), x1)})))} \\
= \quad & \text{by Lemma 43 RL with } \{x1 \leftarrow x1\} \\
& mult(x1, mult(rd(x2, x1), \underbrace{mult(x1, \overbrace{i(mult(x1, x1))})})) \\
= \quad & \text{by Lemma 20 LR with } \{x2 \leftarrow x1, x1 \leftarrow x1\} \\
& mult(x1, mult(rd(x2, x1), \overbrace{i(x1)}))
\end{aligned}
$$

Lemma 199: $mult(mult(x1, x2), rd(i(x1), x1)) = op_t(rd(x2, x1), i(x1))$

$$
\begin{aligned}
& \underbrace{mult(mult(x1, x2), rd(i(x1), x1))} \\
= \quad & \text{by Lemma 198 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{mult(x1, mult(rd(x2, x1), i(x1)))}} \\
= \quad & \text{by Lemma 30 LR with } \{x2 \leftarrow rd(x2, x1), x1 \leftarrow x1\} \\
& \overbrace{op_t(rd(x2, x1), i(x1))}
\end{aligned}
$$

Lemma 200: $op_t(mult(x1, mult(x2, x1)), x1) = mult(x2, mult(x1, x1))$

$$
\begin{aligned}
& op_t(\underbrace{mult(x1, mult(x2, x1))}, x1) \\
= \quad & \text{by Lemma 58 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_t(\overbrace{\underbrace{rd(x1, i(mult(x1, x2)))}}, x1) \\
= \quad & \text{by Lemma 38 RL with } \{x1 \leftarrow x1, x2 \leftarrow mult(x1, x2)\} \\
& op_t(\overbrace{rd(mult(x1, x2), i(x1))}, \underbrace{x1}) \\
= \quad & \text{by Lemma 3 RL with } \{x1 \leftarrow x1\} \\
& \underbrace{op_t(rd(mult(x1, x2), i(x1)), \overbrace{i(i(x1))})} \\
= \quad & \text{by Lemma 199 RL with } \{x2 \leftarrow mult(x1, x2), x1 \leftarrow i(x1)\} \\
& \overbrace{mult(\underbrace{mult(i(x1), mult(x1, x2))}, rd(i(i(x1)), i(x1)))} \\
= \quad & \text{by Axiom 2 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& mult(\overbrace{x2}, \underbrace{rd(i(i(x1)), i(x1))}) \\
= \quad & \text{by Lemma 23 LR with } \{x2 \leftarrow x1, x1 \leftarrow i(x1)\} \\
& mult(x2, \overbrace{\underbrace{rd(x1, i(x1))}}) \\
= \quad & \text{by Lemma 42 LR with } \{x1 \leftarrow x1\} \\
& mult(x2, \overbrace{mult(x1, x1)})
\end{aligned}
$$

Lemma 201: $mult(rd(x1,x2),mult(x2,x2)) = op_t(mult(x2,x1),x2)$

$$
\begin{aligned}
& \underbrace{mult(rd(x1,x2),mult(x2,x2))} \\
= & \quad \text{by Lemma 200 RL with } \{x2 \leftarrow rd(x1,x2), x1 \leftarrow x2\} \\
& \overbrace{op_t(mult(x2,\underbrace{mult(rd(x1,x2),x2)}),x2)} \\
= & \quad \text{by Axiom 4 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_t(mult(x2,\overbrace{x1}),x2)
\end{aligned}
$$

Lemma 202: $rd(x1,i(x1)) = mult(mult(x1,x2),i(op_t(rd(x2,x1),i(x1))))$

$$
\begin{aligned}
& \underbrace{rd(x1,i(x1))} \\
= & \quad \text{by Lemma 10 RL with } \{x2 \leftarrow x1, x1 \leftarrow i(x1)\} \\
& \overbrace{\underbrace{i(rd(i(x1),x1))}} \\
= & \quad \text{by Lemma 20 RL with } \{x2 \leftarrow rd(i(x1),x1), x1 \leftarrow mult(x1,x2)\} \\
& \overbrace{mult(mult(x1,x2),\underbrace{i(mult(mult(x1,x2),rd(i(x1),x1)))})} \\
= & \quad \text{by Lemma 199 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& mult(mult(x1,x2),\overbrace{i(op_t(rd(x2,x1),i(x1)))})
\end{aligned}
$$

Lemma 203: $mult(x1,x1) = mult(mult(x1,x2),op_t(rd(x1,x2),i(i(x1))))$

$$
\begin{aligned}
& \underbrace{mult(x1,x1)} \\
= & \quad \text{by Lemma 42 RL with } \{x1 \leftarrow x1\} \\
& \overbrace{\underbrace{rd(x1,i(x1))}} \\
= & \quad \text{by Lemma 202 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{mult(mult(x1,x2),\underbrace{i(op_t(rd(x2,x1),i(x1)))})} \\
= & \quad \text{by Lemma 46 LR with } \{x3 \leftarrow i(x1), x2 \leftarrow x1, x1 \leftarrow x2\} \\
& mult(mult(x1,x2),\overbrace{op_t(rd(x1,x2),i(i(x1)))})
\end{aligned}
$$

Lemma 204: $mult(mult(x1,x2),op_t(rd(x1,x2),x1)) = mult(x1,x1)$

$$
\begin{aligned}
& mult(mult(x1,x2),op_t(rd(x1,x2),\underbrace{x1})) \\
= & \quad \text{by Lemma 3 RL with } \{x1 \leftarrow x1\} \\
& \underbrace{mult(mult(x1,x2),op_t(rd(x1,x2),\overbrace{i(i(x1))}))} \\
= & \quad \text{by Lemma 203 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{mult(x1,x1)}
\end{aligned}
$$

Lemma 205: $rd(op_t(x1,x2),mult(x2,x2)) = rd(mult(i(x2),x1),x2)$

$$
\begin{aligned}
& rd(op_t(\underbrace{x1},x2),mult(x2,x2)) \\
= & \quad \text{by Lemma 12 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& rd(\underbrace{op_t(\overbrace{mult(x2,mult(i(x2),x1))},x2)},mult(x2,x2)) \\
= & \quad \text{by Lemma 201 RL with } \{x2 \leftarrow x2, x1 \leftarrow mult(i(x2),x1)\} \\
& \underbrace{rd(\overbrace{mult(rd(mult(i(x2),x1),x2),mult(x2,x2))},mult(x2,x2))} \\
= & \quad \text{by Axiom 5 RL with } \{x2 \leftarrow mult(x2,x2), x1 \leftarrow rd(mult(i(x2),x1),x2)\} \\
& \overbrace{rd(mult(i(x2),x1),x2)}
\end{aligned}
$$

Lemma 206: $rd(rd(op_t(x1,x2),x1),x1) = rd(op_t(x1,x2),mult(x1,x1))$

$$
\begin{aligned}
& rd(\underbrace{rd(op_t(x1,x2),x1)},x1) \\
= & \quad \text{by Lemma 96 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{rd(\overbrace{mult(i(x1),op_t(x1,x2))},x1)} \\
= & \quad \text{by Lemma 205 RL with } \{x2 \leftarrow x1, x1 \leftarrow op_t(x1,x2)\} \\
& \overbrace{rd(\underbrace{op_t(op_t(x1,x2),x1)},mult(x1,x1))} \\
= & \quad \text{by Lemma 11 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& rd(\overbrace{op_t(x1,x2)},mult(x1,x1))
\end{aligned}
$$

Lemma 207: $mult(i(x1),rd(x1,op_t(x2,x1))) = op_r(i(x2),i(x1),x1)$

$$
\begin{aligned}
& \underbrace{mult(i(x1),rd(x1,op_t(x2,x1)))} \\
= & \quad \text{by Lemma 56 RL with } \{x2 \leftarrow op_t(x2,x1), x1 \leftarrow x1\} \\
& \overbrace{mult(i(\underbrace{mult(x1,op_t(x2,x1))}),x1)} \\
= & \quad \text{by Lemma 13 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{mult(i(\overbrace{mult(x2,x1)}),x1)} \\
= & \quad \text{by Lemma 100 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \overbrace{op_r(i(x2),i(x1),x1)}
\end{aligned}
$$

Lemma 208: $op_r(i(x1),i(x2),x2) = op_r(i(x1),mult(x1,x2),i(x2))$

$$
\begin{aligned}
& \underbrace{op_r(i(x1),i(x2),x2)} \\
= & \quad \text{by Lemma 100 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{\overbrace{mult(i(mult(x1,x2)),x2)}} \\
= & \quad \text{by Lemma 82 LR with } \{x2 \leftarrow x2, x1 \leftarrow mult(x1,x2)\} \\
& \overbrace{op_r(\underbrace{rd(x2,mult(x1,x2))},mult(x1,x2),i(x2))} \\
= & \quad \text{by Lemma 9 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& op_r(\overbrace{i(x1)},mult(x1,x2),i(x2))
\end{aligned}
$$

Lemma 209: $op_r(i(x1), i(x2), x2) = op_t(i(x1), mult(x1, x2))$

$$
\begin{aligned}
& \underbrace{op_r(i(x1), i(x2), x2)} \\
= & \quad \text{by Lemma 208 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_r(i(x1), mult(x1, x2), \underbrace{i(x2)})} \\
= & \quad \text{by Lemma 20 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{op_r(i(x1), mult(x1, x2), \overbrace{mult(x1, i(mult(x1, x2)))})} \\
= & \quad \text{by Lemma 78 LR with } \{x2 \leftarrow mult(x1, x2), x1 \leftarrow x1\} \\
& \overbrace{op_t(i(x1), mult(x1, x2))}
\end{aligned}
$$

Lemma 210: $mult(x1, op_r(i(x1), x2, i(x2))) = asoc(i(x1), x2, i(x2))$

$$
\begin{aligned}
& mult(x1, \underbrace{op_r(i(x1), x2, i(x2))}) \\
= & \quad \text{by Lemma 107 RL with } \{x2 \leftarrow x2, x1 \leftarrow i(x1)\} \\
& \underbrace{mult(x1, \overbrace{mult(i(x1), asoc(i(x1), x2, i(x2)))})} \\
= & \quad \text{by Lemma 12 LR with } \{x2 \leftarrow asoc(i(x1), x2, i(x2)), x1 \leftarrow x1\} \\
& \overbrace{asoc(i(x1), x2, i(x2))}
\end{aligned}
$$

Lemma 211: $op_r(i(x1), x2, i(x2)) = rd(asoc(i(x1), x2, i(x2)), op_t(x1, op_r(i(x1), x2, i(x2))))$

$$
\begin{aligned}
& \underbrace{op_r(i(x1), x2, i(x2))} \\
= & \quad \text{by Lemma 107 RL with } \{x2 \leftarrow x2, x1 \leftarrow i(x1)\} \\
& \overbrace{\underbrace{mult(i(x1), asoc(i(x1), x2, i(x2)))}} \\
= & \quad \text{by Lemma 77 RL with } \{x2 \leftarrow x1, x1 \leftarrow asoc(i(x1), x2, i(x2))\} \\
& \overbrace{rd(asoc(i(x1), x2, i(x2)), op_t(x1, \underbrace{mult(i(x1), asoc(i(x1), x2, i(x2)))}))} \\
= & \quad \text{by Lemma 107 LR with } \{x2 \leftarrow x2, x1 \leftarrow i(x1)\} \\
& rd(asoc(i(x1), x2, i(x2)), op_t(x1, \overbrace{op_r(i(x1), x2, i(x2))}))
\end{aligned}
$$

Lemma 212: $op_t(rd(x1, i(x2)), x2) = mult(mult(i(x2), x1), rd(x2, i(x2)))$

$$
\begin{aligned}
& op_t(rd(x1, i(x2)), \underbrace{x2}) \\
= & \quad \text{by Lemma 3 RL with } \{x1 \leftarrow x2\} \\
& \underbrace{op_t(rd(x1, i(x2)), \overbrace{i(i(x2))})} \\
= & \quad \text{by Lemma 199 RL with } \{x2 \leftarrow x1, x1 \leftarrow i(x2)\} \\
& \overbrace{mult(mult(i(x2), x1), rd(\underbrace{i(i(x2))}, i(x2)))} \\
= & \quad \text{by Lemma 3 LR with } \{x1 \leftarrow x2\} \\
& mult(mult(i(x2), x1), rd(\overbrace{x2}, i(x2)))
\end{aligned}
$$

Lemma 213: $mult(mult(i(x1), x2), mult(x1, x1)) = op_t(rd(x2, i(x1)), x1)$

$$
\begin{aligned}
& mult(mult(i(x1), x2), \underbrace{mult(x1, x1)}) \\
= & \quad \text{by Lemma 42 RL with } \{x1 \leftarrow x1\} \\
& \underbrace{mult(mult(i(x1), x2), \overbrace{rd(x1, i(x1))})} \\
= & \quad \text{by Lemma 212 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \overbrace{op_t(rd(x2, i(x1)), x1)}
\end{aligned}
$$

Lemma 214: $op_t(mult(x1, i(mult(x2, i(x1)))), x1) = i(mult(x2, mult(i(x1), i(x1))))$

$$
\begin{aligned}
& op_t(\underbrace{mult(x1, i(mult(x2, i(x1))))}, x1) \\
= & \quad \text{by Lemma 16 RL with } \{x2 \leftarrow mult(x2, i(x1)), x1 \leftarrow x1\} \\
& \underbrace{op_t(\overbrace{i(mult(i(x1), mult(x2, i(x1))))}, x1)} \\
= & \quad \text{by Lemma 19 RL with } \{x2 \leftarrow x1, x1 \leftarrow mult(i(x1), mult(x2, i(x1)))\} \\
& \overbrace{i(\underbrace{op_t(mult(i(x1), mult(x2, i(x1))), i(x1))})} \\
= & \quad \text{by Lemma 200 LR with } \{x2 \leftarrow x2, x1 \leftarrow i(x1)\} \\
& i(\overbrace{mult(x2, mult(i(x1), i(x1)))})
\end{aligned}
$$

Lemma 215: $op_t(rd(x1, mult(i(x1), x2)), x1) = i(mult(x2, mult(i(x1), i(x1))))$

$$
\begin{aligned}
& op_t(\underbrace{rd(x1, mult(i(x1), x2))}, x1) \\
= & \quad \text{by Lemma 60 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_t(\overbrace{mult(x1, \underbrace{mult(i(x2), x1)})}, x1) \\
= & \quad \text{by Lemma 17 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \underbrace{op_t(mult(x1, \overbrace{i(mult(x2, i(x1)))}), x1)} \\
= & \quad \text{by Lemma 214 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{i(mult(x2, mult(i(x1), i(x1))))}
\end{aligned}
$$

Lemma 216: $op_t(rd(x1, mult(i(x1), x2)), x1) = mult(i(x2), mult(x1, x1))$

$$
\begin{aligned}
& \underbrace{op_t(rd(x1, mult(i(x1), x2)), x1)} \\
= & \quad \text{by Lemma 215 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{\overbrace{i(mult(x2, mult(i(x1), i(x1))))}} \\
= & \quad \text{by Lemma 87 LR with } \{x3 \leftarrow i(x1), x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \overbrace{mult(i(x2), mult(x1, \underbrace{i(i(x1))}))} \\
= & \quad \text{by Lemma 3 LR with } \{x1 \leftarrow x1\} \\
& mult(i(x2), mult(x1, \overbrace{x1}))
\end{aligned}
$$

Lemma 217: $mult(i(x1), rd(x1, mult(i(x1), x2))) = mult(i(x2), x1)$

$$
\begin{aligned}
& \underbrace{mult(i(x1), rd(x1, mult(i(x1), x2)))} \\
= \quad & \text{by Lemma 56 RL with } \{x2 \leftarrow mult(i(x1), x2), x1 \leftarrow x1\} \\
& \overbrace{mult(i(\underbrace{mult(x1, mult(i(x1), x2))}), x1)} \\
= \quad & \text{by Lemma 12 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& mult(i(\overbrace{x2}), x1)
\end{aligned}
$$

Lemma 218: $rd(x1, op_l(x2, mult(i(x2), x1), x2)) = mult(x1, i(x2))$

$$
\begin{aligned}
& rd(\underbrace{x1}, op_l(x2, mult(i(x2), x1), x2)) \\
= \quad & \text{by Lemma 12 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& rd(\overbrace{mult(x2, mult(i(x2), x1))}, \underbrace{op_l(x2, mult(i(x2), x1), x2)}) \\
= \quad & \text{by Axiom 12 RL with } \{x3 \leftarrow x2, x2 \leftarrow mult(i(x2), x1), x1 \leftarrow x2\} \\
& \underbrace{rd(mult(x2, mult(i(x2), x1)), \overbrace{mult(i(mult(x2, mult(i(x2), x1))), mult(x2, mult(mult(i(x2), x1), x2)))})} \\
= \quad & \text{by Lemma 61 RL with } \{x3 \leftarrow mult(x2, mult(i(x2), x1)), x2 \leftarrow mult(x2, mult(mult(i(x2), x1), x2)), x1 \leftarrow mult(x2, mult(i(x2 \\
& \overbrace{\underbrace{rd(mult(mult(x2, mult(i(x2), x1)), i(mult(x2, mult(mult(i(x2), x1), x2)))), i(mult(x2, mult(i(x2), x1))))}} \\
= \quad & \text{by Lemma 136 RL with } \{x2 \leftarrow mult(i(x2), x1), x1 \leftarrow x2\} \\
& \overbrace{\underbrace{mult(x2, rd(mult(i(x2), x1), x2))}} \\
= \quad & \text{by Lemma 84 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \overbrace{mult(x1, i(x2))}
\end{aligned}
$$

Lemma 219: $rd(op_l(x1, mult(i(x1), x2), x1), x2) = mult(i(x2), x1)$

$$
\begin{aligned}
& \underbrace{rd(op_l(x1, mult(i(x1), x2), x1), x2)} \\
= \quad & \text{by Lemma 10 RL with } \{x2 \leftarrow op_l(x1, mult(i(x1), x2), x1), x1 \leftarrow x2\} \\
& \overbrace{i(\underbrace{rd(x2, op_l(x1, mult(i(x1), x2), x1))})} \\
= \quad & \text{by Lemma 218 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \underbrace{i(\overbrace{mult(x2, i(x1))})} \\
= \quad & \text{by Lemma 17 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \overbrace{mult(i(x2), x1)}
\end{aligned}
$$

Lemma 220: $op_l(mult(x1, x2), x2, i(mult(x2, x1))) = mult(x2, x1)$

$\underbrace{op_l(mult(x1, x2), x2, i(mult(x2, x1)))}$

$=$ by Axiom 12 RL with $\{x3 \leftarrow mult(x1, x2), x2 \leftarrow x2, x1 \leftarrow i(mult(x2, x1))\}$

$\overbrace{mult(i(mult(i(mult(x2, x1)), x2)), mult(i(mult(x2, x1)), \underbrace{mult(x2, mult(x1, x2))}))}$

$=$ by Lemma 58 RL with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$mult(i(mult(i(mult(x2, x1)), x2)), \underbrace{mult(i(mult(x2, x1)), \overbrace{rd(x2, i(mult(x2, x1)))})})$

$=$ by Lemma 55 RL with $\{x2 \leftarrow x2, x1 \leftarrow i(mult(x2, x1))\}$

$\underbrace{mult(i(mult(i(mult(x2, x1)), x2)), \overbrace{mult(mult(i(mult(x2, x1)), x2), i(i(mult(x2, x1))))})}$

$=$ by Axiom 2 RL with $\{x2 \leftarrow mult(i(mult(x2, x1)), x2), x1 \leftarrow i(i(mult(x2, x1)))\}$

$\underbrace{\overbrace{i(i(mult(x2, x1)))}}$

$=$ by Lemma 3 LR with $\{x1 \leftarrow mult(x2, x1)\}$

$\overbrace{mult(x2, x1)}$

Lemma 221: $i(x2) = op_l(mult(i(mult(x1, x2)), x1), x1, i(i(x2)))$

$\underbrace{i(x2)}$

$=$ by Lemma 20 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{\overbrace{mult(x1, i(mult(x1, x2)))}}$

$=$ by Lemma 220 RL with $\{x2 \leftarrow x1, x1 \leftarrow i(mult(x1, x2))\}$

$\overbrace{op_l(mult(i(mult(x1, x2)), x1), x1, i(\underbrace{mult(x1, i(mult(x1, x2)))}))}$

$=$ by Lemma 20 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$op_l(mult(i(mult(x1, x2)), x1), x1, i(\overbrace{i(x2)}))$

Lemma 222: $i(x2) = op_l(mult(i(x1), rd(x1, x2)), x1, x2)$

$\underbrace{i(x2)}$

$=$ by Lemma 221 LR with $\{x1 \leftarrow x1, x2 \leftarrow x2\}$

$\overbrace{op_l(\underbrace{mult(i(mult(x1, x2)), x1)}, x1, i(i(x2)))}$

$=$ by Lemma 56 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$op_l(\overbrace{mult(i(x1), rd(x1, x2))}, x1, \underbrace{i(i(x2))})$

$=$ by Lemma 3 LR with $\{x1 \leftarrow x2\}$

$op_l(mult(i(x1), rd(x1, x2)), x1, \overbrace{x2})$

Lemma 223: $op_l(mult(x1, rd(x2, x1)), i(x1), i(x2)) = x2$

$op_l(\underbrace{mult(x1, rd(x2, x1))}, i(x1), i(x2))$

$=$ by Lemma 55 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$op_l(\overbrace{mult(mult(x1, x2), i(x1))}, i(x1), i(\underbrace{x2}))$

$=$ by Axiom 2 LR with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$\underbrace{op_l(mult(mult(x1, x2), i(x1)), i(x1), i(\overbrace{mult(i(x1), mult(x1, x2))}))}$

$=$ by Lemma 220 LR with $\{x2 \leftarrow i(x1), x1 \leftarrow mult(x1, x2)\}$

$\overbrace{\underbrace{mult(i(x1), mult(x1, x2))}}$

$=$ by Axiom 2 RL with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$\overbrace{x2}$

Lemma 224: $op_l(x1, i(x1), op_l(x1, x2, x3)) = x1$

$op_l(\underbrace{x1}, i(x1), op_l(x1, x2, x3))$

$=$ by Lemma 223 RL with $\{x2 \leftarrow x1, x1 \leftarrow i(op_l(x1, x2, x3))\}$

$op_l(\overbrace{op_l(mult(i(op_l(x1, x2, x3)), rd(x1, i(\underbrace{op_l(x1, x2, x3)}))), i(i(op_l(x1, x2, x3))), i(x1))}, i(x1), op_l(x1, x2, x3))$

$=$ by Lemma 170 RL with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$op_l(op_l(mult(i(op_l(x1, x2, x3)), \underbrace{rd(x1, i(\overbrace{mult(x1, rd(op_l(x1, x2, x3), x1))}))}), i(i(op_l(x1, x2, x3))), i(x1)), i(x1), op_l(x1, x2, x$

$=$ by Lemma 58 LR with $\{x2 \leftarrow rd(op_l(x1, x2, x3), x1), x1 \leftarrow x1\}$

$op_l(op_l(mult(i(op_l(x1, x2, x3)), \overbrace{mult(x1, \underbrace{mult(rd(op_l(x1, x2, x3), x1), x1)})}), i(i(op_l(x1, x2, x3))), i(x1)), i(x1), op_l(x1, x2, x$

$=$ by Axiom 4 RL with $\{x2 \leftarrow x1, x1 \leftarrow op_l(x1, x2, x3)\}$

$op_l(op_l(\underbrace{mult(i(op_l(x1, x2, x3)), mult(x1, \overbrace{op_l(x1, x2, x3)}))}, i(i(op_l(x1, x2, x3))), i(x1)), i(x1), op_l(x1, x2, x3))$

$=$ by Axiom 10 LR with $\{x2 \leftarrow x1, x1 \leftarrow op_l(x1, x2, x3)\}$

$op_l(op_l(\overbrace{op_t(x1, op_l(x1, x2, x3))}, i(i(op_l(x1, x2, x3))), i(x1)), i(x1), op_l(x1, x2, x3))$

$=$ by Lemma 169 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$op_l(op_l(\overbrace{x1}, \underbrace{i(i(op_l(x1, x2, x3)))}), i(x1)), i(x1), op_l(x1, x2, x3))$

$=$ by Lemma 3 LR with $\{x1 \leftarrow op_l(x1, x2, x3)\}$

$\underbrace{op_l(op_l(x1, \overbrace{op_l(x1, x2, x3)}), i(x1)), i(x1), op_l(x1, x2, x3))}$

$=$ by Lemma 138 LR with $\{x3 \leftarrow i(x1), x2 \leftarrow op_l(x1, x2, x3), x1 \leftarrow x1\}$

$\overbrace{x1}$

Lemma 225: $op_l(i(x1), x1, op_l(i(x1), x2, x3)) = i(x1)$

$op_l(i(x1), \underbrace{x1}, op_l(i(x1), x2, x3))$

$=$ by Lemma 3 RL with $\{x1 \leftarrow x1\}$

$\underbrace{op_l(i(x1), \overbrace{i(i(x1))}, op_l(i(x1), x2, x3))}$

$=$ by Lemma 224 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow i(x1)\}$

$\overbrace{i(x1)}$

Lemma 226: $x2 = op_l(mult(mult(i(x1), x2), x1), x1, i(x2))$

$$
\begin{aligned}
& \underbrace{x2} \\
= \quad & \text{by Lemma 12 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{mult(x1, mult(i(x1), x2))}} \\
= \quad & \text{by Lemma 220 RL with } \{x2 \leftarrow x1, x1 \leftarrow mult(i(x1), x2)\} \\
& \overbrace{op_l(mult(mult(i(x1), x2), x1), x1, i(\underbrace{mult(x1, mult(i(x1), x2))}))} \\
= \quad & \text{by Lemma 12 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_l(mult(mult(i(x1), x2), x1), x1, i(\overbrace{x2}))
\end{aligned}
$$

Lemma 227: $op_l(x1, i(x1), mult(x1, x2)) = op_l(x1, x2, x1)$

$$
\begin{aligned}
& op_l(\underbrace{x1}, i(x1), mult(x1, x2)) \\
= \quad & \text{by Lemma 223 RL with } \{x2 \leftarrow x1, x1 \leftarrow i(mult(x1, x2))\} \\
& op_l(\overbrace{op_l(mult(i(mult(x1, x2)), \underbrace{rd(x1, i(mult(x1, x2)))}), i(i(mult(x1, x2))), i(x1))}, i(x1), mult(x1, x2)) \\
= \quad & \text{by Lemma 58 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_l(op_l(\underbrace{mult(i(mult(x1, x2)), \overbrace{mult(x1, mult(x2, x1))})}, i(i(mult(x1, x2))), i(x1)), i(x1), mult(x1, x2)) \\
= \quad & \text{by Axiom 12 LR with } \{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_l(op_l(\overbrace{op_l(x1, x2, x1)}, \underbrace{i(i(mult(x1, x2)))}, i(x1)), i(x1), mult(x1, x2)) \\
= \quad & \text{by Lemma 3 LR with } \{x1 \leftarrow mult(x1, x2)\} \\
& \underbrace{op_l(op_l(op_l(x1, x2, x1), \overbrace{mult(x1, x2)}, i(x1)), i(x1), mult(x1, x2))} \\
= \quad & \text{by Lemma 138 LR with } \{x3 \leftarrow i(x1), x2 \leftarrow mult(x1, x2), x1 \leftarrow op_l(x1, x2, x1)\} \\
& \overbrace{op_l(x1, x2, x1)}
\end{aligned}
$$

Lemma 228: $op_l(x1, asoc(x1, x2, x1), x1) = x1$

$$
\begin{aligned}
& \underbrace{op_l(x1, asoc(x1, x2, x1), x1)} \\
= \quad & \text{by Lemma 227 RL with } \{x2 \leftarrow asoc(x1, x2, x1), x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, i(x1), \underbrace{mult(x1, asoc(x1, x2, x1))})} \\
= \quad & \text{by Lemma 158 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{op_l(x1, i(x1), \overbrace{op_l(x1, x1, x2)})} \\
= \quad & \text{by Lemma 224 LR with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x1\} \\
& \overbrace{x1}
\end{aligned}
$$

Lemma 229: $op_l(x1, mult(i(x1), x2), x1) = op_l(x1, i(x1), x2)$

$$
\begin{aligned}
& \underbrace{op_l(x1, mult(i(x1), x2), x1)} \\
= \quad & \text{by Lemma 227 RL with } \{x2 \leftarrow mult(i(x1), x2), x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, i(x1), \underbrace{mult(x1, mult(i(x1), x2))})} \\
= \quad & \text{by Lemma 12 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_l(x1, i(x1), \overbrace{x2})
\end{aligned}
$$

Lemma 230: $rd(x1, op_l(x2, i(x2), x1)) = mult(x1, i(x2))$

$$
\begin{aligned}
& rd(x1, \underbrace{op_l(x2, i(x2), x1)}) \\
= & \quad \text{by Lemma 229 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \underbrace{rd(x1, \overbrace{op_l(x2, mult(i(x2), x1), x2)})} \\
= & \quad \text{by Lemma 218 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{mult(x1, i(x2))}
\end{aligned}
$$

Lemma 231: $mult(x1, x2) = rd(x1, op_l(i(x2), x2, x1))$

$$
\begin{aligned}
& mult(x1, \underbrace{x2}) \\
= & \quad \text{by Lemma 3 RL with } \{x1 \leftarrow x2\} \\
& \underbrace{mult(x1, \overbrace{i(i(x2))})} \\
= & \quad \text{by Lemma 230 RL with } \{x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& \overbrace{rd(x1, op_l(i(x2), \underbrace{i(i(x2))}, x1))} \\
= & \quad \text{by Lemma 3 LR with } \{x1 \leftarrow x2\} \\
& rd(x1, op_l(i(x2), \overbrace{x2}, x1))
\end{aligned}
$$

Lemma 232: $mult(x1, op_l(i(x2), x2, x1)) = rd(x1, x2)$

$$
\begin{aligned}
& \underbrace{mult(x1, op_l(i(x2), x2, x1))} \\
= & \quad \text{by Lemma 57 RL with } \{x2 \leftarrow op_l(i(x2), x2, x1), x1 \leftarrow x1\} \\
& \overbrace{rd(x1, mult(i(x1), \underbrace{rd(x1, op_l(i(x2), x2, x1))}))} \\
= & \quad \text{by Lemma 231 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& rd(x1, \underbrace{mult(i(x1), \overbrace{mult(x1, x2)})}) \\
= & \quad \text{by Axiom 2 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& rd(x1, \overbrace{x2})
\end{aligned}
$$

Lemma 233: $rd(op_l(x1, i(x1), x2), x2) = mult(i(x2), x1)$

$$
\begin{aligned}
& rd(\underbrace{op_l(x1, i(x1), x2)}, x2) \\
= & \quad \text{by Lemma 229 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{rd(\overbrace{op_l(x1, mult(i(x1), x2), x1)}, x2)} \\
= & \quad \text{by Lemma 219 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{mult(i(x2), x1)}
\end{aligned}
$$

Lemma 234: $op_l(x1, rd(op_l(x1, x2, x3), x1), x1) = x1$

$$
\begin{aligned}
& \underbrace{op_l(x1, rd(op_l(x1, x2, x3), x1), x1)} \\
= & \quad \text{by Lemma 227 RL with } \{x2 \leftarrow rd(op_l(x1, x2, x3), x1), x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, i(x1), \underbrace{mult(x1, rd(op_l(x1, x2, x3), x1))})} \\
= & \quad \text{by Lemma 170 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{op_l(x1, i(x1), \overbrace{op_l(x1, x2, x3)})} \\
= & \quad \text{by Lemma 224 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{x1}
\end{aligned}
$$

Lemma 235: $i(i(x1)) = rd(op_l(i(i(x2)), i(x2), i(x1)), rd(x2, x1))$

$$
\begin{aligned}
& \underbrace{i(i(x1))} \\
= & \quad \text{by Lemma 9 RL with } \{x2 \leftarrow i(x1), x1 \leftarrow op_l(i(i(x2)), i(x2), i(x1))\} \\
& \overbrace{rd(op_l(i(i(x2)), i(x2), i(x1)), \underbrace{mult(i(x1), op_l(i(i(x2)), i(x2), i(x1)))})} \\
= & \quad \text{by Lemma 232 LR with } \{x2 \leftarrow i(x2), x1 \leftarrow i(x1)\} \\
& rd(op_l(i(i(x2)), i(x2), i(x1)), \overbrace{\underbrace{rd(i(x1), i(x2))}}) \\
= & \quad \text{by Lemma 23 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& rd(op_l(i(i(x2)), i(x2), i(x1)), \overbrace{rd(x2, x1)})
\end{aligned}
$$

Lemma 236: $x1 = rd(op_l(x2, i(x2), i(x1)), rd(x2, x1))$

$$
\begin{aligned}
& \underbrace{x1} \\
= & \quad \text{by Lemma 3 RL with } \{x1 \leftarrow x1\} \\
& \overbrace{\underbrace{i(i(x1))}} \\
= & \quad \text{by Lemma 235 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{rd(op_l(\underbrace{i(i(x2))}, i(x2), i(x1)), rd(x2, x1))} \\
= & \quad \text{by Lemma 3 LR with } \{x1 \leftarrow x2\} \\
& rd(op_l(\overbrace{x2}, i(x2), i(x1)), rd(x2, x1))
\end{aligned}
$$

Lemma 237: $mult(x1, op_r(x1, x2, x3)) = mult(rd(op_r(x1, x2, x3), x1), mult(x1, x1))$

$\underbrace{mult(x1, op_r(x1, x2, x3))}$

$=$ by Lemma 119 RL with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{mult(\underbrace{op_r(x1, x2, x3)}, x1)}$

$=$ by Axiom 4 LR with $\{x2 \leftarrow x1, x1 \leftarrow op_r(x1, x2, x3)\}$

$\underbrace{mult(\overbrace{mult(rd(\underbrace{op_r(x1, x2, x3)}, x1), x1)}, x1)}$

$=$ by Lemma 137 RL with $\{x3 \leftarrow x1, x2 \leftarrow x1, x1 \leftarrow rd(op_r(x1, x2, x3), x1)\}$

$\overbrace{mult(rd(op_r(x1, x2, x3), x1), mult(x1, \underbrace{op_l(x1, rd(op_r(x1, x2, x3), x1), x1)}))}$

$=$ by Lemma 227 RL with $\{x2 \leftarrow rd(op_r(x1, x2, x3), x1), x1 \leftarrow x1\}$

$mult(rd(op_r(x1, x2, x3), x1), mult(x1, \overbrace{op_l(x1, i(x1), \underbrace{mult(x1, rd(op_r(x1, x2, x3), x1))})}))$

$=$ by Lemma 123 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$mult(rd(op_r(x1, x2, x3), x1), mult(x1, op_l(\underbrace{x1}, i(x1), \overbrace{op_r(x1, x2, x3)})))$

$=$ by Lemma 223 RL with $\{x2 \leftarrow x1, x1 \leftarrow i(op_r(x1, x2, x3))\}$

$mult(rd(op_r(x1, x2, x3), x1), mult(x1, op_l(\overbrace{op_l(mult(i(op_r(x1, x2, x3)), \underbrace{rd(x1, i(op_r(x1, x2, x3)))}), i(i(op_r(x1, x2, x3))), i(x1$

$=$ by Lemma 121 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$mult(rd(op_r(x1, x2, x3), x1), mult(x1, op_l(op_l(\underbrace{mult(i(op_r(x1, x2, x3)), \overbrace{mult(x1, op_r(x1, x2, x3))})}, i(i(op_r(x1, x2, x3))), i(x1$

$=$ by Axiom 10 LR with $\{x2 \leftarrow x1, x1 \leftarrow op_r(x1, x2, x3)\}$

$mult(rd(op_r(x1, x2, x3), x1), mult(x1, op_l(op_l(\overbrace{op_t(x1, op_r(x1, x2, x3))}, i(i(op_r(x1, x2, x3))), i(x1)), i(x1), op_r(x1, x2, x3))))$

$=$ by Lemma 120 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$mult(rd(op_r(x1, x2, x3), x1), mult(x1, op_l(op_l(\overbrace{x1}, \underbrace{i(i(op_r(x1, x2, x3)))}, i(x1)), i(x1), op_r(x1, x2, x3))))$

$=$ by Lemma 3 LR with $\{x1 \leftarrow op_r(x1, x2, x3)\}$

$mult(rd(op_r(x1, x2, x3), x1), mult(x1, \underbrace{op_l(op_l(x1, \overbrace{op_r(x1, x2, x3)}, i(x1)), i(x1), op_r(x1, x2, x3))}))$

$=$ by Lemma 138 LR with $\{x3 \leftarrow i(x1), x2 \leftarrow op_r(x1, x2, x3), x1 \leftarrow x1\}$

$mult(rd(op_r(x1, x2, x3), x1), mult(x1, \overbrace{x1}))$

Lemma 238: $op_t(mult(x1, op_r(x1, x2, x3)), x1) = mult(x1, op_r(x1, x2, x3))$

$\underbrace{op_t(mult(x1, op_r(x1, x2, x3)), x1)}$

$=$ by Lemma 201 RL with $\{x2 \leftarrow x1, x1 \leftarrow op_r(x1, x2, x3)\}$

$\overbrace{\underbrace{mult(rd(op_r(x1, x2, x3), x1), mult(x1, x1))}}$

$=$ by Lemma 237 RL with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{mult(x1, op_r(x1, x2, x3))}$

Lemma 239: $op_t(x1, mult(x1, op_r(x1, x2, x3))) = mult(i(mult(x1, op_r(x1, x2, x3))), mult(mult(x1, op_r(x1, x2, x3)), x1))$

$op_t(x1, \underbrace{mult(x1, op_r(x1, x2, x3))}$

$=$ by Lemma 238 RL with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{op_t(x1, \overbrace{op_t(mult(x1, op_r(x1, x2, x3)), x1)})}$

$=$ by Lemma 133 RL with $\{x2 \leftarrow x1, x1 \leftarrow mult(x1, op_r(x1, x2, x3))\}$

$\overbrace{mult(i(\underbrace{op_t(mult(x1, op_r(x1, x2, x3)), x1)}), mult(mult(x1, op_r(x1, x2, x3)), x1))}$

$=$ by Lemma 238 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$mult(i(\overbrace{mult(x1, op_r(x1, x2, x3))}), mult(mult(x1, op_r(x1, x2, x3)), x1))$

Lemma 240: $op_t(x1, mult(x1, op_l(x1, x2, x3))) = mult(i(mult(x1, op_l(x1, x2, x3))), mult(mult(x1, op_l(x1, x2, x3)), x1))$

$op_t(x1, \underbrace{mult(x1, op_l(x1, x2, x3))}$

$=$ by Lemma 168 RL with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$op_t(x1, \overbrace{mult(\underbrace{op_l(x1, x2, x3)}, x1)})$

$=$ by Axiom 4 LR with $\{x2 \leftarrow x1, x1 \leftarrow op_l(x1, x2, x3)\}$

$op_t(x1, \underbrace{mult(\overbrace{mult(rd(op_l(x1, x2, x3), x1), x1)}, x1)})$

$=$ by Lemma 137 RL with $\{x3 \leftarrow x1, x2 \leftarrow x1, x1 \leftarrow rd(op_l(x1, x2, x3), x1)\}$

$op_t(x1, \overbrace{mult(rd(op_l(x1, x2, x3), x1), mult(x1, \underbrace{op_l(x1, rd(op_l(x1, x2, x3), x1), x1)}))})$

$=$ by Lemma 234 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$op_t(x1, \underbrace{mult(rd(op_l(x1, x2, x3), x1), mult(x1, \overbrace{x1}))})$

$=$ by Lemma 201 LR with $\{x2 \leftarrow x1, x1 \leftarrow op_l(x1, x2, x3)\}$

$\underbrace{op_t(x1, \overbrace{op_t(mult(x1, op_l(x1, x2, x3)), x1)})}$

$=$ by Lemma 133 RL with $\{x2 \leftarrow x1, x1 \leftarrow mult(x1, op_l(x1, x2, x3))\}$

$\overbrace{mult(i(\underbrace{op_t(mult(x1, op_l(x1, x2, x3)), x1)}), mult(mult(x1, op_l(x1, x2, x3)), x1))}$

$=$ by Lemma 201 RL with $\{x2 \leftarrow x1, x1 \leftarrow op_l(x1, x2, x3)\}$

$mult(i(\overbrace{mult(rd(op_l(x1, x2, x3), x1), mult(x1, \underbrace{x1}))}), mult(mult(x1, op_l(x1, x2, x3)), x1))$

$=$ by Lemma 234 RL with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$mult(i(\underbrace{mult(rd(op_l(x1, x2, x3), x1), mult(x1, \overbrace{op_l(x1, rd(op_l(x1, x2, x3), x1), x1)}))}), mult(mult(x1, op_l(x1, x2, x3)), x1))$

$=$ by Lemma 137 LR with $\{x3 \leftarrow x1, x2 \leftarrow x1, x1 \leftarrow rd(op_l(x1, x2, x3), x1)\}$

$mult(i(\overbrace{mult(\underbrace{mult(rd(op_l(x1, x2, x3), x1), x1)}, x1)}), mult(mult(x1, op_l(x1, x2, x3)), x1))$

$=$ by Axiom 4 RL with $\{x2 \leftarrow x1, x1 \leftarrow op_l(x1, x2, x3)\}$

$mult(i(\underbrace{mult(\overbrace{op_l(x1, x2, x3)}, x1)}), mult(mult(x1, op_l(x1, x2, x3)), x1))$

$=$ by Lemma 168 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$mult(i(\overbrace{mult(x1, op_l(x1, x2, x3))}), mult(mult(x1, op_l(x1, x2, x3)), x1))$

Lemma 241: $op_l(x1, i(x2), mult(x2, x3)) = op_l(x1, x3, x2)$

$op_l(\underbrace{x1}, i(x2), mult(x2, x3))$

$=$ by Axiom 2 LR with $\{x2 \leftarrow x3, x1 \leftarrow x1\}$

$op_l(\overbrace{mult(i(x3), \underbrace{mult(x3, x1)})}, i(x2), mult(x2, x3))$

$=$ by Axiom 2 LR with $\{x2 \leftarrow x2, x1 \leftarrow mult(x3, x1)\}$

$op_l(mult(i(x3), \overbrace{mult(i(x2), \underbrace{mult(x2, mult(x3, x1))})}), i(x2), mult(x2, x3))$

$=$ by Lemma 132 RL with $\{x3 \leftarrow x1, x2 \leftarrow x3, x1 \leftarrow x2\}$

$op_l(mult(i(\underbrace{x3}), mult(i(x2), \overbrace{mult(mult(x2, x3), op_l(x1, x3, x2))})), i(x2), mult(x2, x3))$

$=$ by Axiom 2 LR with $\{x2 \leftarrow x2, x1 \leftarrow x3\}$

$op_l(\underbrace{mult(i(\overbrace{mult(i(x2), mult(x2, x3))}), mult(i(x2), mult(mult(x2, x3), op_l(x1, x3, x2))))}, i(x2), mult(x2, x3))$

$=$ by Axiom 12 LR with $\{x3 \leftarrow op_l(x1, x3, x2), x2 \leftarrow mult(x2, x3), x1 \leftarrow i(x2)\}$

$\underbrace{op_l(\overbrace{op_l(op_l(x1, x3, x2), mult(x2, x3), i(x2))}, i(x2), mult(x2, x3))}$

$=$ by Lemma 138 LR with $\{x3 \leftarrow i(x2), x2 \leftarrow mult(x2, x3), x1 \leftarrow op_l(x1, x3, x2)\}$

$\overbrace{op_l(x1, x3, x2)}$

Lemma 242: $op_l(x1, mult(x2, x3), i(x2)) = op_l(x1, x2, x3)$

$op_l(\underbrace{x1}, mult(x2, x3), i(x2))$

$=$ by Lemma 138 RL with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{op_l(\overbrace{op_l(op_l(x1, x2, x3), x3, x2)}, mult(x2, x3), i(x2))}$

$=$ by Axiom 12 RL with $\{x3 \leftarrow op_l(op_l(x1, x2, x3), x3, x2), x2 \leftarrow mult(x2, x3), x1 \leftarrow i(x2)\}$

$\overbrace{mult(i(\underbrace{mult(i(x2), mult(x2, x3))}), mult(i(x2), mult(mult(x2, x3), op_l(op_l(x1, x2, x3), x3, x2))))}$

$=$ by Axiom 2 RL with $\{x2 \leftarrow x2, x1 \leftarrow x3\}$

$mult(i(\overbrace{x3}), mult(i(x2), \underbrace{mult(mult(x2, x3), op_l(op_l(x1, x2, x3), x3, x2))}))$

$=$ by Lemma 132 LR with $\{x3 \leftarrow op_l(x1, x2, x3), x2 \leftarrow x3, x1 \leftarrow x2\}$

$mult(i(x3), \underbrace{mult(i(x2), \overbrace{mult(x2, mult(x3, op_l(x1, x2, x3)))})})$

$=$ by Axiom 2 RL with $\{x2 \leftarrow x2, x1 \leftarrow mult(x3, op_l(x1, x2, x3))\}$

$\underbrace{mult(i(x3), \overbrace{mult(x3, op_l(x1, x2, x3))})}$

$=$ by Axiom 2 RL with $\{x2 \leftarrow x3, x1 \leftarrow op_l(x1, x2, x3)\}$

$\overbrace{op_l(x1, x2, x3)}$

Lemma 243: $op_l(x1, x2, mult(i(x2), x3)) = op_l(x1, x3, i(x2))$

$op_l(x1, \underbrace{x2}, mult(i(x2), x3))$

$=$ by Lemma 3 RL with $\{x1 \leftarrow x2\}$

$\underbrace{op_l(x1, \overbrace{i(i(x2))}, mult(i(x2), x3))}$

$=$ by Lemma 241 LR with $\{x3 \leftarrow x3, x2 \leftarrow i(x2), x1 \leftarrow x1\}$

$\overbrace{op_l(x1, x3, i(x2))}$

Lemma 244: $op_l(x1, mult(i(x2), x3), x2) = op_l(x1, i(x2), x3)$

$$
\begin{aligned}
& \underbrace{op_l(x1, mult(i(x2), x3), x2)} \\
= & \quad \text{by Lemma 241 RL with } \{x3 \leftarrow mult(i(x2), x3), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, i(x2), \underbrace{mult(x2, mult(i(x2), x3))})} \\
= & \quad \text{by Lemma 12 LR with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& op_l(x1, i(x2), \overbrace{x3}^{})
\end{aligned}
$$

Lemma 245: $op_l(x1, x2, i(mult(x2, x3))) = op_l(x1, i(x3), i(x2))$

$$
\begin{aligned}
& op_l(x1, \underbrace{x2}, i(mult(x2, x3))) \\
= & \quad \text{by Lemma 3 RL with } \{x1 \leftarrow x2\} \\
& op_l(x1, \overbrace{i(i(x2))}, \underbrace{i(mult(x2, x3))}) \\
= & \quad \text{by Axiom 11 RL with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& \underbrace{op_l(x1, i(i(x2)), \overbrace{mult(i(x2), i(x3))})} \\
= & \quad \text{by Lemma 241 LR with } \{x3 \leftarrow i(x3), x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, i(x3), i(x2))}
\end{aligned}
$$

Lemma 246: $op_l(x1, i(x2), mult(x3, x2)) = op_l(x1, op_t(x3, x2), x2)$

$$
\begin{aligned}
& op_l(x1, i(x2), \underbrace{mult(x3, x2)}) \\
= & \quad \text{by Lemma 13 RL with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& \underbrace{op_l(x1, i(x2), \overbrace{mult(x2, op_t(x3, x2))})} \\
= & \quad \text{by Lemma 241 LR with } \{x3 \leftarrow op_t(x3, x2), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, op_t(x3, x2), x2)}
\end{aligned}
$$

Lemma 247: $op_l(x1, i(mult(x2, x3)), x2) = op_l(x1, i(x2), i(x3))$

$$
\begin{aligned}
& \underbrace{op_l(x1, i(mult(x2, x3)), x2)} \\
= & \quad \text{by Lemma 241 RL with } \{x3 \leftarrow i(mult(x2, x3)), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, i(x2), \underbrace{mult(x2, i(mult(x2, x3)))})} \\
= & \quad \text{by Lemma 20 LR with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& op_l(x1, i(x2), \overbrace{i(x3)})
\end{aligned}
$$

Lemma 248: $rd(op_l(x1, x2, i(x1)), x1) = asoc(x1, mult(i(x1), x2), x1)$

$$
\begin{aligned}
& rd(\underbrace{op_l(x1, x2, i(x1))}, x1) \\
= & \quad \text{by Lemma 243 RL with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x1\} \\
& \underbrace{rd(\overbrace{op_l(x1, x1, mult(i(x1), x2))}, x1)} \\
= & \quad \text{by Lemma 172 LR with } \{x2 \leftarrow mult(i(x1), x2), x1 \leftarrow x1\} \\
& \overbrace{asoc(x1, mult(i(x1), x2), x1)}
\end{aligned}
$$

Lemma 249: $op_l(mult(i(x1),x2),x1,x2) = op_t(rd(x2,x1),x2)$

$$
\begin{aligned}
& op_l(mult(i(x1),x2),\underbrace{x1},x2) \\
= \quad & \text{by Lemma 3 RL with } \{x1 \leftarrow x1\} \\
& \underbrace{op_l(mult(i(x1),x2),\overbrace{i(i(x1))},x2)} \\
= \quad & \text{by Axiom 12 RL with } \{x3 \leftarrow mult(i(x1),x2), x2 \leftarrow i(i(x1)), x1 \leftarrow x2\} \\
& \overbrace{mult(i(mult(x2,i(i(x1)))),mult(x2,\underbrace{mult(i(i(x1)),mult(i(x1),x2))})}} \\
= \quad & \text{by Axiom 2 RL with } \{x2 \leftarrow i(x1), x1 \leftarrow x2\} \\
& mult(\underbrace{i(mult(x2,i(i(x1))))},mult(x2,\overbrace{x2})) \\
= \quad & \text{by Lemma 17 LR with } \{x2 \leftarrow i(x1), x1 \leftarrow x2\} \\
& \underbrace{mult(\overbrace{mult(i(x2),i(x1))},mult(x2,x2))} \\
= \quad & \text{by Lemma 213 LR with } \{x2 \leftarrow i(x1), x1 \leftarrow x2\} \\
& \overbrace{op_t(\underbrace{rd(i(x1),i(x2))},x2)} \\
= \quad & \text{by Lemma 23 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_t(\overbrace{rd(x2,x1)},x2)
\end{aligned}
$$

Lemma 250: $op_l(op_t(rd(x1,x2),x1),x1,x2) = mult(i(x2),x1)$

$$
\begin{aligned}
& op_l(\underbrace{op_t(rd(x1,x2),x1)},x1,x2) \\
= \quad & \text{by Lemma 249 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \underbrace{op_l(\overbrace{op_l(mult(i(x2),x1),x2,x1)},x1,x2)} \\
= \quad & \text{by Lemma 138 LR with } \{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow mult(i(x2),x1)\} \\
& \overbrace{mult(i(x2),x1)}
\end{aligned}
$$

Lemma 251: $op_l(op_t(x1,x2),x2,x1) = op_t(x1,mult(x1,x2))$

$op_l(op_t(x1,x2),\underbrace{x2},x1)$

$=$ by Lemma 3 RL with $\{x1 \leftarrow x2\}$

$op_l(op_t(x1,\underbrace{x2}),\overbrace{i(i(x2))},x1)$

$=$ by Lemma 3 RL with $\{x1 \leftarrow x2\}$

$op_l(\underbrace{op_t(x1,\overbrace{i(i(x2))})},i(i(x2)),x1)$

$=$ by Lemma 18 RL with $\{x2 \leftarrow i(x2), x1 \leftarrow x1\}$

$op_l(\overbrace{i(\underbrace{op_t(i(x1),i(x2))})},i(i(x2)),x1)$

$=$ by Lemma 31 RL with $\{x2 \leftarrow i(x1), x1 \leftarrow i(x2)\}$

$\underbrace{op_l(\overbrace{mult(i(x2),i(\underbrace{mult(i(x1),i(x2))}))},i(i(x2)),x1)}$

$=$ by Axiom 12 RL with $\{x3 \leftarrow mult(i(x2),i(mult(i(x1),i(x2)))), x2 \leftarrow i(i(x2)), x1 \leftarrow x1\}$

$\overbrace{mult(i(mult(x1,i(i(x2)))),mult(x1,\underbrace{mult(i(i(x2)),mult(i(x2),i(mult(i(x1),i(x2)))))}))}$

$=$ by Axiom 2 RL with $\{x2 \leftarrow i(x2), x1 \leftarrow i(mult(i(x1),i(x2)))\}$

$mult(\underbrace{i(mult(x1,i(i(x2))))},mult(x1,\overbrace{i(mult(i(x1),i(x2)))}))$

$=$ by Lemma 17 LR with $\{x2 \leftarrow i(x2), x1 \leftarrow x1\}$

$\underbrace{mult(\overbrace{mult(i(x1),i(x2))},mult(x1,i(mult(i(x1),i(x2)))))}$

$=$ by Lemma 30 LR with $\{x2 \leftarrow x1, x1 \leftarrow mult(i(x1),i(x2))\}$

$\overbrace{op_t(x1,\underbrace{i(mult(i(x1),i(x2)))})}$

$=$ by Lemma 16 LR with $\{x2 \leftarrow i(x2), x1 \leftarrow x1\}$

$op_t(x1,\overbrace{mult(x1,\underbrace{i(i(x2))})})$

$=$ by Lemma 3 LR with $\{x1 \leftarrow x2\}$

$op_t(x1,mult(x1,\overbrace{x2}))$

Lemma 252: $op_l(x1,x1,op_l(x1,x2,x3)) = x1$

$op_l(\underbrace{x1},x1,op_l(x1,x2,x3))$

$=$ by Axiom 2 LR with $\{x2 \leftarrow mult(x1,op_l(x1,x2,x3)), x1 \leftarrow x1\}$

$op_l(\underbrace{mult(i(mult(x1,op_l(x1,x2,x3))),\overbrace{mult(mult(x1,op_l(x1,x2,x3)),x1)})},x1,op_l(x1,x2,x3))$

$=$ by Lemma 240 RL with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$op_l(\overbrace{\underbrace{op_t(x1,mult(x1,op_l(x1,x2,x3)))}},x1,op_l(x1,x2,x3))$

$=$ by Lemma 251 RL with $\{x2 \leftarrow op_l(x1,x2,x3), x1 \leftarrow x1\}$

$op_l(\overbrace{op_l(\underbrace{op_t(x1,op_l(x1,x2,x3))},op_l(x1,x2,x3),x1)},x1,op_l(x1,x2,x3))$

$=$ by Lemma 169 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{op_l(op_l(\overbrace{x1},op_l(x1,x2,x3),x1),x1,op_l(x1,x2,x3))}$

$=$ by Lemma 138 LR with $\{x3 \leftarrow x1, x2 \leftarrow op_l(x1,x2,x3), x1 \leftarrow x1\}$

$\overbrace{x1}$

Lemma 253: $asoc(x1, op_l(x1, x2, x3), x1) = mult(i(x1), x1)$

$$
\begin{aligned}
& \overbrace{asoc(x1, op_l(x1, x2, x3), x1)} \\
= \quad & \text{by Lemma 172 RL with } \{x2 \leftarrow op_l(x1, x2, x3), x1 \leftarrow x1\} \\
& \overbrace{rd(op_l(x1, x1, op_l(x1, x2, x3)), x1)} \\
= \quad & \text{by Lemma 171 RL with } \{x3 \leftarrow op_l(x1, x2, x3), x2 \leftarrow x1, x1 \leftarrow x1\} \\
& \overbrace{mult(i(x1), \overbrace{op_l(x1, x1, op_l(x1, x2, x3))})} \\
= \quad & \text{by Lemma 252 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& mult(i(x1), \overbrace{x1})
\end{aligned}
$$

Lemma 254: $op_l(op_t(x1, mult(x1, x2)), x1, x2) = op_t(x1, x2)$

$$
\begin{aligned}
& op_l(\overbrace{op_t(x1, mult(x1, x2))}, x1, x2) \\
= \quad & \text{by Lemma 251 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(\overbrace{op_l(op_t(x1, x2), x2, x1)}, x1, x2)} \\
= \quad & \text{by Lemma 138 LR with } \{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow op_t(x1, x2)\} \\
& \overbrace{op_t(x1, x2)}
\end{aligned}
$$

Lemma 255: $mult(x1, op_t(x2, mult(x2, x1))) = mult(op_l(x2, x1, x2), x1)$

$$
\begin{aligned}
& mult(x1, \overbrace{op_t(x2, mult(x2, x1))}) \\
= \quad & \text{by Lemma 251 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \overbrace{mult(x1, \overbrace{op_l(op_t(x2, x1), x1, x2)})} \\
= \quad & \text{by Lemma 167 LR with } \{x4 \leftarrow x2, x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{mult(op_l(x2, x1, x2), x1)}
\end{aligned}
$$

Lemma 256: $mult(op_l(i(x1), x2, x1), x2) = rd(x2, op_t(x1, x2))$

$\underbrace{mult(op_l(i(x1), x2, x1), x2)}$

$=$ by Lemma 167 RL with $\{x4 \leftarrow x1, x3 \leftarrow x2, x2 \leftarrow i(x1), x1 \leftarrow x2\}$

$\overbrace{mult(x2, \underbrace{op_l(op_t(i(x1), x2), x2, x1)})}$

$=$ by Lemma 241 RL with $\{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow op_t(i(x1), x2)\}$

$mult(x2, \overbrace{op_l(op_t(i(x1), \underbrace{x2}), i(x1), mult(x1, x2))})$

$=$ by Axiom 2 LR with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$mult(x2, \underbrace{op_l(op_t(i(x1), \overbrace{mult(i(x1), mult(x1, x2))}), i(x1), mult(x1, x2))})$

$=$ by Lemma 254 LR with $\{x2 \leftarrow mult(x1, x2), x1 \leftarrow i(x1)\}$

$\underbrace{mult(x2, \overbrace{op_t(i(x1), mult(x1, x2))})}$

$=$ by Lemma 57 RL with $\{x2 \leftarrow op_t(i(x1), mult(x1, x2)), x1 \leftarrow x2\}$

$\overbrace{rd(x2, mult(i(x2), rd(\underbrace{x2}, op_t(i(x1), mult(x1, x2)))))}$

$=$ by Axiom 2 LR with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$rd(x2, mult(i(x2), \underbrace{rd(\overbrace{mult(i(x1), mult(x1, x2))}, op_t(i(x1), mult(x1, x2)))}))$

$=$ by Lemma 29 LR with $\{x2 \leftarrow mult(x1, x2), x1 \leftarrow i(x1)\}$

$rd(x2, \underbrace{mult(i(x2), \overbrace{mult(x1, x2)})})$

$=$ by Axiom 10 LR with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$rd(x2, \overbrace{op_t(x1, x2)})$

Lemma 257: $op_t(x1, mult(i(x1), x2)) = op_l(op_t(x1, x2), x2, i(x1))$

$\underbrace{op_t(x1, mult(i(x1), x2))}$

$=$ by Lemma 254 RL with $\{x2 \leftarrow mult(i(x1), x2), x1 \leftarrow x1\}$

$\overbrace{op_l(op_t(x1, \underbrace{mult(x1, mult(i(x1), x2))}), x1, mult(i(x1), x2))}$

$=$ by Lemma 12 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{op_l(op_t(x1, \overbrace{x2}), x1, mult(i(x1), x2))}$

$=$ by Lemma 243 LR with $\{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow op_t(x1, x2)\}$

$\overbrace{op_l(op_t(x1, x2), x2, i(x1))}$

Lemma 258: $mult(mult(x1, i(x2)), op_t(x2, x1)) = op_l(x1, x2, i(x1))$

$$
\begin{aligned}
& mult(mult(\underbrace{x1}, i(x2)), op_t(x2, x1)) \\
= \quad & \text{by Lemma 29 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& mult(\underbrace{mult(\overbrace{rd(mult(x2, x1), op_t(x2, x1))}, i(x2))}, op_t(x2, x1)) \\
= \quad & \text{by Lemma 44 RL with } \{x3 \leftarrow x2, x2 \leftarrow mult(x2, x1), x1 \leftarrow op_t(x2, x1)\} \\
& mult(\overbrace{i(mult(rd(op_t(x2, x1), mult(x2, x1)), x2))}, \underbrace{op_t(x2, x1)}) \\
= \quad & \text{by Axiom 4 LR with } \{x2 \leftarrow mult(x2, x1), x1 \leftarrow op_t(x2, x1)\} \\
& \underbrace{mult(i(mult(rd(op_t(x2, x1), mult(x2, x1)), x2)), \overbrace{mult(rd(op_t(x2, x1), mult(x2, x1)), mult(x2, x1))})} \\
= \quad & \text{by Axiom 12 LR with } \{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow rd(op_t(x2, x1), mult(x2, x1))\} \\
& \overbrace{op_l(x1, x2, \underbrace{rd(op_t(x2, x1), mult(x2, x1))})} \\
= \quad & \text{by Lemma 36 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& op_l(x1, x2, \overbrace{i(x1)})
\end{aligned}
$$

Lemma 259: $op_l(x1, x1, mult(x2, x1)) = mult(mult(x1, i(mult(x1, mult(x2, x1)))), mult(x2, mult(x1, x1)))$

$$
\begin{aligned}
& \underbrace{op_l(x1, x1, mult(x2, x1))} \\
= \quad & \text{by Lemma 242 RL with } \{x3 \leftarrow mult(x2, x1), x2 \leftarrow x1, x1 \leftarrow x1\} \\
& \underbrace{\overbrace{op_l(x1, mult(x1, mult(x2, x1)), i(x1))}} \\
= \quad & \text{by Lemma 258 RL with } \{x2 \leftarrow mult(x1, mult(x2, x1)), x1 \leftarrow x1\} \\
& \overbrace{mult(mult(x1, i(mult(x1, mult(x2, x1)))), \underbrace{op_t(mult(x1, mult(x2, x1)), x1)})} \\
= \quad & \text{by Lemma 200 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& mult(mult(x1, i(mult(x1, mult(x2, x1)))), \overbrace{mult(x2, mult(x1, x1))})
\end{aligned}
$$

Lemma 260: $op_l(x1, x1, mult(x2, x1)) = op_l(x1, x1, x2)$

$$
\begin{aligned}
& \underbrace{op_l(x1, x1, mult(x2, x1))} \\
= \quad & \text{by Lemma 259 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{mult(\underbrace{mult(x1, i(mult(x1, mult(x2, x1))))}, mult(x2, mult(x1, x1)))} \\
= \quad & \text{by Lemma 20 LR with } \{x2 \leftarrow mult(x2, x1), x1 \leftarrow x1\} \\
& \underbrace{mult(\overbrace{i(mult(x2, x1))}, mult(x2, mult(x1, x1)))} \\
= \quad & \text{by Axiom 12 LR with } \{x3 \leftarrow x1, x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \overbrace{op_l(x1, x1, x2)}
\end{aligned}
$$

Lemma 261: $asoc(x1, mult(x2, x1), x1) = mult(i(x1), op_l(x1, x1, x2))$

$$
\begin{aligned}
& \underbrace{asoc(x1, mult(x2, x1), x1)} \\
= \quad & \text{by Lemma 172 RL with } \{x2 \leftarrow mult(x2, x1), x1 \leftarrow x1\} \\
& \underbrace{\overbrace{rd(op_l(x1, x1, mult(x2, x1)), x1)}} \\
= \quad & \text{by Lemma 171 RL with } \{x3 \leftarrow mult(x2, x1), x2 \leftarrow x1, x1 \leftarrow x1\} \\
& \overbrace{mult(i(x1), \underbrace{op_l(x1, x1, mult(x2, x1))})} \\
= \quad & \text{by Lemma 260 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& mult(i(x1), \overbrace{op_l(x1, x1, x2)})
\end{aligned}
$$

Lemma 262: $asoc(x1, mult(x2, x1), x1) = asoc(x1, x2, x1)$

$$
\begin{aligned}
& \underbrace{asoc(x1, mult(x2, x1), x1)} \\
= \quad & \text{by Lemma 261 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{\overbrace{mult(i(x1), op_l(x1, x1, x2))}} \\
= \quad & \text{by Lemma 171 LR with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x1\} \\
& \underbrace{\overbrace{rd(op_l(x1, x1, x2), x1)}} \\
= \quad & \text{by Lemma 172 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{asoc(x1, x2, x1)}
\end{aligned}
$$

Lemma 263: $asoc(x1, rd(x2, x1), x1) = asoc(x1, x2, x1)$

$$
\begin{aligned}
& \underbrace{asoc(x1, rd(x2, x1), x1)} \\
= \quad & \text{by Lemma 172 RL with } \{x2 \leftarrow rd(x2, x1), x1 \leftarrow x1\} \\
& \underbrace{\overbrace{rd(op_l(x1, x1, rd(x2, x1)), x1)}} \\
= \quad & \text{by Lemma 171 RL with } \{x3 \leftarrow rd(x2, x1), x2 \leftarrow x1, x1 \leftarrow x1\} \\
& \overbrace{mult(i(x1), \underbrace{op_l(x1, x1, rd(x2, x1))})} \\
= \quad & \text{by Lemma 260 RL with } \{x2 \leftarrow rd(x2, x1), x1 \leftarrow x1\} \\
& mult(i(x1), \overbrace{op_l(x1, x1, \underbrace{mult(rd(x2, x1), x1)})}) \\
= \quad & \text{by Axiom 4 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \underbrace{mult(i(x1), op_l(x1, x1, \overbrace{x2}))} \\
= \quad & \text{by Lemma 171 LR with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x1\} \\
& \underbrace{\overbrace{rd(op_l(x1, x1, x2), x1)}} \\
= \quad & \text{by Lemma 172 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{asoc(x1, x2, x1)}
\end{aligned}
$$

Lemma 264: $mult(x1, op_l(x2, x2, x1)) = op_t(mult(x2, x1), x2)$

$$
\begin{aligned}
& mult(\underbrace{x1}, op_l(x2, x2, x1)) \\
= & \quad \text{by Axiom 4 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& mult(\overbrace{mult(rd(x1, x2), x2)}, op_l(x2, x2, \underbrace{x1})) \\
= & \quad \text{by Axiom 4 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& mult(mult(rd(x1, x2), x2), \underbrace{op_l(x2, x2, \overbrace{mult(rd(x1, x2), x2)})}) \\
= & \quad \text{by Lemma 260 LR with } \{x2 \leftarrow rd(x1, x2), x1 \leftarrow x2\} \\
& \underbrace{mult(mult(rd(x1, x2), x2), \overbrace{op_l(x2, x2, rd(x1, x2))})} \\
= & \quad \text{by Lemma 132 LR with } \{x3 \leftarrow x2, x2 \leftarrow x2, x1 \leftarrow rd(x1, x2)\} \\
& \overbrace{\underbrace{mult(rd(x1, x2), mult(x2, x2))}} \\
= & \quad \text{by Lemma 201 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_t(\overbrace{mult(x2, x1)}, x2)}
\end{aligned}
$$

Lemma 265: $mult(i(mult(i(x1), i(x2))), op_t(i(x2), x1)) = i(op_l(i(x1), x2, i(i(x1))))$

$$
\begin{aligned}
& \underbrace{mult(i(mult(i(x1), i(x2))), op_t(i(x2), x1))} \\
= & \quad \text{by Lemma 17 RL with } \{x2 \leftarrow op_t(i(x2), x1), x1 \leftarrow mult(i(x1), i(x2))\} \\
& \overbrace{i(\underbrace{mult(mult(i(x1), i(x2)), \underbrace{i(op_t(i(x2), x1))})})} \\
= & \quad \text{by Lemma 18 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& i(\underbrace{mult(mult(i(x1), i(x2)), \overbrace{op_t(x2, i(x1))})}) \\
= & \quad \text{by Lemma 258 LR with } \{x2 \leftarrow x2, x1 \leftarrow i(x1)\} \\
& i(\overbrace{op_l(i(x1), x2, i(i(x1)))})
\end{aligned}
$$

Lemma 266: $op_l(x1, i(x2), i(x1)) = i(op_l(i(x1), x2, i(i(x1))))$

$$
\begin{aligned}
& \underbrace{op_l(x1, i(x2), i(x1))} \\
= & \quad \text{by Lemma 258 RL with } \{x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& \overbrace{mult(\underbrace{mult(x1, i(i(x2)))}, op_t(i(x2), x1))} \\
= & \quad \text{by Lemma 16 RL with } \{x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& \underbrace{mult(\overbrace{i(mult(i(x1), i(x2)))}, op_t(i(x2), x1))} \\
= & \quad \text{by Lemma 265 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{i(op_l(i(x1), x2, i(i(x1))))}
\end{aligned}
$$

Lemma 267: $i(op_l(i(x1), x2, x1)) = op_l(x1, i(x2), i(x1))$

$$
\begin{aligned}
& i(op_l(i(x1), x2, \underbrace{x1})) \\
= & \quad \text{by Lemma 3 RL with } \{x1 \leftarrow x1\} \\
& \underbrace{i(op_l(i(x1), x2, \overbrace{i(i(x1))}))} \\
= & \quad \text{by Lemma 266 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, i(x2), i(x1))}
\end{aligned}
$$

Lemma 268: $op_l(i(x1), i(x2), x1) = i(op_l(x1, x2, i(x1)))$

$$
\begin{aligned}
& op_l(i(x1), i(x2), \underbrace{x1}) \\
= \quad & \text{by Lemma 3 RL with } \{x1 \leftarrow x1\} \\
& \underbrace{op_l(i(x1), i(x2), \overbrace{i(i(x1))})} \\
= \quad & \text{by Lemma 267 RL with } \{x2 \leftarrow x2, x1 \leftarrow i(x1)\} \\
& \overbrace{i(op_l(\underbrace{i(i(x1))}, x2, i(x1)))} \\
= \quad & \text{by Lemma 3 LR with } \{x1 \leftarrow x1\} \\
& i(op_l(\overbrace{x1}, x2, i(x1)))
\end{aligned}
$$

Lemma 269: $op_l(i(x1), i(x1), x2) = i(op_l(x1, i(i(x1)), i(x2)))$

$$
\begin{aligned}
& \underbrace{op_l(i(x1), i(x1), x2)} \\
= \quad & \text{by Lemma 244 RL with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow i(x1)\} \\
& \overbrace{op_l(i(x1), \underbrace{mult(i(x1), x2)}, x1)} \\
= \quad & \text{by Lemma 3 RL with } \{x1 \leftarrow mult(i(x1), x2)\} \\
& \underbrace{op_l(i(x1), \overbrace{i(i(mult(i(x1), x2)))}, x1)} \\
= \quad & \text{by Lemma 268 LR with } \{x2 \leftarrow i(mult(i(x1), x2)), x1 \leftarrow x1\} \\
& \overbrace{i(\underbrace{op_l(x1, i(mult(i(x1), x2)), i(x1))})} \\
= \quad & \text{by Lemma 247 LR with } \{x3 \leftarrow x2, x2 \leftarrow i(x1), x1 \leftarrow x1\} \\
& i(\overbrace{op_l(x1, i(i(x1)), i(x2))})
\end{aligned}
$$

Lemma 270: $i(op_l(x1, x1, i(x2))) = op_l(i(x1), i(x1), x2)$

$$
\begin{aligned}
& i(op_l(x1, \underbrace{x1}, i(x2))) \\
= \quad & \text{by Lemma 3 RL with } \{x1 \leftarrow x1\} \\
& \underbrace{i(op_l(x1, \overbrace{i(i(x1))}, i(x2)))} \\
= \quad & \text{by Lemma 269 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(i(x1), i(x1), x2)}
\end{aligned}
$$

Lemma 271: $i(asoc(x1, i(x2), x1)) = asoc(i(x1), x2, i(x1))$

$$\underbrace{i(asoc(x1, i(x2), x1))}$$

$=$ by Lemma 174 RL with $\{x2 \leftarrow i(x2), x1 \leftarrow x1\}$

$$\overbrace{\underbrace{rd(x1, op_l(x1, x1, i(x2)))}}$$

$=$ by Lemma 176 LR with $\{x3 \leftarrow i(x2), x2 \leftarrow x1, x1 \leftarrow x1\}$

$$\overbrace{mult(x1, \underbrace{i(op_l(x1, x1, i(x2)))})}$$

$=$ by Lemma 270 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$$mult(x1, \overbrace{\underbrace{op_l(i(x1), i(x1), x2)}})$$

$=$ by Lemma 158 RL with $\{x2 \leftarrow x2, x1 \leftarrow i(x1)\}$

$$\underbrace{mult(x1, \overbrace{mult(i(x1), asoc(i(x1), x2, i(x1)))})}$$

$=$ by Lemma 12 LR with $\{x2 \leftarrow asoc(i(x1), x2, i(x1)), x1 \leftarrow x1\}$

$$\overbrace{asoc(i(x1), x2, i(x1))}$$

Lemma 272: $op_l(x1, x1, op_l(i(x1), x2, x3)) = x1$

$op_l(x1, x1, \underbrace{op_l(i(x1), x2, x3)})$

$=$ by Lemma 3 RL with $\{x1 \leftarrow op_l(i(x1), x2, x3)\}$

$op_l(x1, x1, \overbrace{i(i(op_l(i(x1), x2, x3)))})$

$=$ by Lemma 21 RL with $\{x2 \leftarrow i(op_l(i(x1), x2, x3)), x1 \leftarrow x1\}$

$\underbrace{op_l(x1, x1, \overbrace{mult(rd(i(x1), i(op_l(i(x1), x2, x3))), x1)})}$

$=$ by Lemma 260 LR with $\{x2 \leftarrow rd(i(x1), i(op_l(i(x1), x2, x3))), x1 \leftarrow x1\}$

$\overbrace{op_l(x1, x1, rd(i(x1), \underbrace{i(op_l(i(x1), x2, x3))}))}$

$=$ by Lemma 170 RL with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow i(x1)\}$

$op_l(x1, x1, \underbrace{rd(i(x1), \overbrace{i(mult(i(x1), rd(op_l(i(x1), x2, x3), i(x1))))})})$

$=$ by Lemma 58 LR with $\{x2 \leftarrow rd(op_l(i(x1), x2, x3), i(x1)), x1 \leftarrow i(x1)\}$

$op_l(x1, x1, \overbrace{mult(i(x1), \underbrace{mult(rd(op_l(i(x1), x2, x3), i(x1)), i(x1))})})$

$=$ by Axiom 4 RL with $\{x2 \leftarrow i(x1), x1 \leftarrow op_l(i(x1), x2, x3)\}$

$\underbrace{op_l(x1, x1, mult(i(x1), \overbrace{op_l(i(x1), x2, x3)}))}$

$=$ by Lemma 243 LR with $\{x3 \leftarrow op_l(i(x1), x2, x3), x2 \leftarrow x1, x1 \leftarrow x1\}$

$\overbrace{op_l(x1, \overbrace{op_l(i(x1), x2, x3)}, \underbrace{i(x1)})}$

$=$ by Lemma 225 RL with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{op_l(x1, op_l(i(x1), x2, x3), \overbrace{op_l(i(x1), x1, \overbrace{op_l(i(x1), x2, x3)})})$

$=$ by Axiom 12 RL with $\{x3 \leftarrow x1, x2 \leftarrow op_l(i(x1), x2, x3), x1 \leftarrow op_l(i(x1), x1, op_l(i(x1), x2, x3))\}$

$\overbrace{mult(i(mult(op_l(i(x1), x1, op_l(i(x1), x2, x3)), op_l(i(x1), x2, x3))), mult(op_l(i(x1), x1, op_l(i(x1), x2, x3)), \underbrace{mult(op_l(i(x1), x2}$

$=$ by Lemma 231 LR with $\{x2 \leftarrow x1, x1 \leftarrow op_l(i(x1), x2, x3)\}$

$mult(i(mult(op_l(i(x1), x1, op_l(i(x1), x2, x3)), op_l(i(x1), x2, x3))), \underbrace{mult(op_l(i(x1), x1, op_l(i(x1), x2, x3)), \overbrace{rd(op_l(i(x1), x2, x}$

$=$ by Lemma 55 RL with $\{x2 \leftarrow op_l(i(x1), x2, x3), x1 \leftarrow op_l(i(x1), x1, op_l(i(x1), x2, x3))\}$

$\underbrace{mult(i(mult(op_l(i(x1), x1, op_l(i(x1), x2, x3)), op_l(i(x1), x2, x3))), \overbrace{mult(mult(op_l(i(x1), x1, op_l(i(x1), x2, x3)), op_l(i(x1), x2}$

$=$ by Axiom 2 RL with $\{x2 \leftarrow mult(op_l(i(x1), x1, op_l(i(x1), x2, x3)), op_l(i(x1), x2, x3)), x1 \leftarrow i(op_l(i(x1), x1, op_l(i(x1), x2, x3))$

$\overbrace{i(\underbrace{op_l(i(x1), x1, op_l(i(x1), x2, x3))})}$

$=$ by Lemma 225 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{i(\overbrace{i(x1)})}$

$=$ by Lemma 3 LR with $\{x1 \leftarrow x1\}$

$\overbrace{x1}$

Lemma 273: $op_l(i(x1), x2, x1) = op_l(i(x1), i(x1), mult(x1, rd(x2, x1)))$

$$
\begin{aligned}
& \underbrace{op_l(i(x1), x2, x1)} \\
= \quad & \text{by Lemma 241 RL with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow i(x1)\} \\
& \overbrace{op_l(i(x1), i(x1), mult(x1, x2))} \\
= \quad & \text{by Lemma 260 RL with } \{x2 \leftarrow mult(x1, x2), x1 \leftarrow i(x1)\} \\
& \overbrace{op_l(i(x1), i(x1), \underbrace{mult(mult(x1, x2), i(x1))})} \\
= \quad & \text{by Lemma 55 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_l(i(x1), i(x1), \overbrace{mult(x1, rd(x2, x1))})
\end{aligned}
$$

Lemma 274: $asoc(x1, i(op_r(x1, x2, x3)), x1) = unit()$

$$
\begin{aligned}
& asoc(x1, \underbrace{i(op_r(x1, x2, x3))}, x1) \\
= \quad & \text{by Lemma 21 RL with } \{x2 \leftarrow op_r(x1, x2, x3), x1 \leftarrow x1\} \\
& \underbrace{asoc(x1, \overbrace{mult(rd(i(x1), op_r(x1, x2, x3)), x1)}, x1)} \\
= \quad & \text{by Lemma 262 LR with } \{x2 \leftarrow rd(i(x1), op_r(x1, x2, x3)), x1 \leftarrow x1\} \\
& \overbrace{asoc(x1, \underbrace{rd(i(x1), op_r(x1, x2, x3))}, x1)} \\
= \quad & \text{by Lemma 131 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& asoc(x1, \overbrace{i(mult(x1, op_r(x1, x2, x3)))}, x1) \\
= \quad & \text{by Axiom 11 RL with } \{x2 \leftarrow op_r(x1, x2, x3), x1 \leftarrow x1\} \\
& \underbrace{asoc(x1, \overbrace{mult(i(x1), i(op_r(x1, x2, x3)))}, x1)} \\
= \quad & \text{by Lemma 248 RL with } \{x2 \leftarrow i(op_r(x1, x2, x3)), x1 \leftarrow x1\} \\
& \overbrace{\underbrace{rd(op_l(x1, i(op_r(x1, x2, x3)), i(x1))}, x1)} \\
= \quad & \text{by Lemma 171 RL with } \{x3 \leftarrow i(x1), x2 \leftarrow i(op_r(x1, x2, x3)), x1 \leftarrow x1\} \\
& \overbrace{mult(i(x1), op_l(\underbrace{x1}, i(op_r(x1, x2, x3)), i(x1)))} \\
= \quad & \text{by Lemma 160 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& mult(i(x1), \underbrace{op_l(\overbrace{op_t(x1, i(op_r(x1, x2, x3)))}, i(op_r(x1, x2, x3)), i(x1)))} \\
= \quad & \text{by Lemma 257 RL with } \{x2 \leftarrow i(op_r(x1, x2, x3)), x1 \leftarrow x1\} \\
& mult(i(x1), \overbrace{op_t(x1, \underbrace{mult(i(x1), i(op_r(x1, x2, x3)))})}) \\
= \quad & \text{by Axiom 11 LR with } \{x2 \leftarrow op_r(x1, x2, x3), x1 \leftarrow x1\} \\
& mult(i(x1), op_t(x1, \overbrace{i(mult(x1, op_r(x1, x2, x3)))})) \\
= \quad & \text{by Lemma 131 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& mult(i(x1), \underbrace{op_t(x1, \overbrace{rd(i(x1), op_r(x1, x2, x3))})}) \\
= \quad & \text{by Lemma 154 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{mult(i(x1), \overbrace{x1})} \\
= \quad & \text{by Axiom 9 LR with } \{x1 \leftarrow x1\} \\
& \overbrace{unit()}
\end{aligned}
$$

Lemma 275: $op_l(i(x1), i(mult(x1, x2)), x1) = i(op_l(i(i(x1)), x1, x2))$

$$
\begin{aligned}
& op_l(i(x1), i(mult(x1, x2)), \underbrace{x1}) \\
= \quad & \text{by Lemma 3 RL with } \{x1 \leftarrow x1\} \\
& \underbrace{op_l(i(x1), i(mult(x1, x2)), \overbrace{i(i(x1))})} \\
= \quad & \text{by Lemma 267 RL with } \{x2 \leftarrow mult(x1, x2), x1 \leftarrow i(x1)\} \\
& \overbrace{i(\underbrace{op_l(i(i(x1)), mult(x1, x2), i(x1))})} \\
= \quad & \text{by Lemma 242 LR with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow i(i(x1))\} \\
& i(\overbrace{op_l(i(i(x1)), x1, x2)})
\end{aligned}
$$

Lemma 276: $op_l(i(x1), i(x1), i(x2)) = i(op_l(x1, x1, x2))$

$$
\begin{aligned}
& \underbrace{op_l(i(x1), i(x1), i(x2))} \\
= \quad & \text{by Lemma 247 RL with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow i(x1)\} \\
& \overbrace{\underbrace{op_l(i(x1), i(mult(x1, x2)), x1)}} \\
= \quad & \text{by Lemma 275 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{i(op_l(\underbrace{i(i(x1))}, x1, x2))} \\
= \quad & \text{by Lemma 3 LR with } \{x1 \leftarrow x1\} \\
& i(op_l(\overbrace{x1}, x1, x2))
\end{aligned}
$$

Lemma 277: $op_l(x1, i(i(x1)), i(x2)) = i(op_l(i(x1), i(x1), x2))$

$$
\begin{aligned}
& \underbrace{op_l(x1, i(i(x1)), i(x2))} \\
= \quad & \text{by Lemma 247 RL with } \{x3 \leftarrow x2, x2 \leftarrow i(x1), x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, i(mult(i(x1), x2)), i(x1))}} \\
= \quad & \text{by Lemma 267 RL with } \{x2 \leftarrow mult(i(x1), x2), x1 \leftarrow x1\} \\
& \overbrace{i(\underbrace{op_l(i(x1), mult(i(x1), x2), x1)})} \\
= \quad & \text{by Lemma 244 LR with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow i(x1)\} \\
& i(\overbrace{op_l(i(x1), i(x1), x2)})
\end{aligned}
$$

Lemma 278: $op_l(x1, x1, i(op_l(i(i(x1)), x2, x3))) = i(i(x1))$

$$
\begin{aligned}
& op_l(x1, \underbrace{x1}, i(op_l(i(i(x1)), x2, x3))) \\
= \quad & \text{by Lemma 3 RL with } \{x1 \leftarrow x1\} \\
& \underbrace{op_l(x1, \overbrace{i(i(x1))}, i(op_l(i(i(x1)), x2, x3)))} \\
= \quad & \text{by Lemma 277 LR with } \{x2 \leftarrow op_l(i(i(x1)), x2, x3), x1 \leftarrow x1\} \\
& \overbrace{i(\underbrace{op_l(i(x1), i(x1), op_l(i(i(x1)), x2, x3))})} \\
= \quad & \text{by Lemma 272 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow i(x1)\} \\
& i(\overbrace{i(x1)})
\end{aligned}
$$

Lemma 279: $op_l(x1, x1, i(op_l(x1, x2, x3))) = x1$

$$
\begin{aligned}
& op_l(x1, x1, i(op_l(\underbrace{x1}, x2, x3))) \\
= \quad & \text{by Lemma 3 RL with } \{x1 \leftarrow x1\} \\
& \underbrace{op_l(x1, x1, i(op_l(\overbrace{i(i(x1))}, x2, x3)))} \\
= \quad & \text{by Lemma 278 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{\overbrace{i(i(x1))}} \\
= \quad & \text{by Lemma 3 LR with } \{x1 \leftarrow x1\} \\
& \overbrace{x1}
\end{aligned}
$$

Lemma 280: $op_l(x1, rd(x2, x1), x3) = mult(mult(i(x3), rd(x1, x2)), mult(x3, x2))$

$$
\begin{aligned}
& \underbrace{op_l(x1, rd(x2, x1), x3)} \\
= \quad & \text{by Axiom 12 RL with } \{x3 \leftarrow x1, x2 \leftarrow rd(x2, x1), x1 \leftarrow x3\} \\
& \overbrace{mult(i(mult(x3, rd(x2, x1))), mult(x3, \underbrace{mult(rd(x2, x1), x1)}))} \\
= \quad & \text{by Axiom 4 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& mult(\underbrace{i(mult(x3, rd(x2, x1)))}, mult(x3, \overbrace{x2})) \\
= \quad & \text{by Lemma 45 LR with } \{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x3\} \\
& mult(\overbrace{mult(i(x3), rd(x1, x2))}, mult(x3, x2))
\end{aligned}
$$

Lemma 281: $op_l(x1, x1, x2) = mult(i(x2), op_t(mult(x1, x2), x1))$

$$
\begin{aligned}
& \underbrace{op_l(x1, x1, x2)} \\
= \quad & \text{by Lemma 242 RL with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x1\} \\
& \underbrace{\overbrace{op_l(x1, mult(x1, x2), i(x1))}} \\
= \quad & \text{by Lemma 258 RL with } \{x2 \leftarrow mult(x1, x2), x1 \leftarrow x1\} \\
& \overbrace{mult(\underbrace{mult(x1, i(mult(x1, x2)))}, op_t(mult(x1, x2), x1))} \\
= \quad & \text{by Lemma 20 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& mult(\overbrace{i(x2)}, op_t(mult(x1, x2), x1))
\end{aligned}
$$

Lemma 282: $asoc(i(x1), rd(x1, x2), i(x1)) = mult(x1, i(op_l(x1, x1, x2)))$

$\underbrace{asoc(i(x1), rd(x1, x2), i(x1))}$

$=$ by Lemma 172 RL with $\{x2 \leftarrow rd(x1, x2), x1 \leftarrow i(x1)\}$

$\overbrace{\underbrace{rd(op_l(i(x1), i(x1), rd(x1, x2)), i(x1))}}$

$=$ by Lemma 171 RL with $\{x3 \leftarrow rd(x1, x2), x2 \leftarrow i(x1), x1 \leftarrow i(x1)\}$

$\overbrace{mult(i(i(x1)), \underbrace{op_l(i(x1), i(x1), rd(x1, x2))})}$

$=$ by Lemma 244 RL with $\{x3 \leftarrow rd(x1, x2), x2 \leftarrow x1, x1 \leftarrow i(x1)\}$

$mult(i(i(x1)), \overbrace{op_l(i(x1), \underbrace{mult(i(x1), rd(x1, x2))}, x1)})$

$=$ by Lemma 56 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$mult(i(i(x1)), \underbrace{op_l(i(x1), \overbrace{mult(i(mult(x1, x2)), x1)}, x1)})$

$=$ by Lemma 273 LR with $\{x2 \leftarrow mult(i(mult(x1, x2)), x1), x1 \leftarrow x1\}$

$mult(i(i(x1)), \overbrace{\underbrace{op_l(i(x1), i(x1), mult(x1, rd(mult(i(mult(x1, x2)), x1), x1)))}})$

$=$ by Lemma 241 LR with $\{x3 \leftarrow rd(mult(i(mult(x1, x2)), x1), x1), x2 \leftarrow x1, x1 \leftarrow i(x1)\}$

$mult(i(i(x1)), \overbrace{op_l(i(x1), \underbrace{rd(mult(i(mult(x1, x2)), x1), x1)}, x1)})$

$=$ by Axiom 5 RL with $\{x2 \leftarrow x1, x1 \leftarrow i(mult(x1, x2))\}$

$mult(i(i(x1)), \underbrace{op_l(i(x1), \overbrace{i(mult(x1, x2))}, x1)})$

$=$ by Lemma 247 LR with $\{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow i(x1)\}$

$mult(i(i(x1)), \overbrace{\underbrace{op_l(i(x1), i(x1), i(x2))}})$

$=$ by Lemma 276 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{mult(i(i(x1)), \overbrace{i(op_l(x1, x1, x2))})}$

$=$ by Axiom 11 LR with $\{x2 \leftarrow op_l(x1, x1, x2), x1 \leftarrow i(x1)\}$

$\overbrace{\underbrace{i(mult(i(x1), op_l(x1, x1, x2)))}}$

$=$ by Lemma 16 LR with $\{x2 \leftarrow op_l(x1, x1, x2), x1 \leftarrow x1\}$

$\overbrace{mult(x1, i(op_l(x1, x1, x2)))}$

Lemma 283: $rd(op_t(mult(x1, x2), x1), op_l(x1, x1, x2)) = x2$

$rd(\underbrace{op_t(mult(x1, x2), x1)}, op_l(x1, x1, x2))$

$=$ by Lemma 264 RL with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$\underbrace{rd(\overbrace{mult(x2, op_l(x1, x1, x2))}, op_l(x1, x1, x2))}$

$=$ by Axiom 5 RL with $\{x2 \leftarrow op_l(x1, x1, x2), x1 \leftarrow x2\}$

$\overbrace{x2}$

Lemma 284: $mult(op_l(rd(x1,x2),x1,x2),x1) = rd(x1,mult(i(x1),x2))$

$$
\begin{aligned}
& \overbrace{mult(op_l(rd(x1,x2),x1,x2),x1)} \\
= \quad & \text{by Lemma 167 RL with } \{x4 \leftarrow x2, x3 \leftarrow x1, x2 \leftarrow rd(x1,x2), x1 \leftarrow x1\} \\
& \overbrace{mult(x1,\underbrace{op_l(op_t(rd(x1,x2),x1),x1,x2)})} \\
= \quad & \text{by Lemma 250 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{mult(x1,\overbrace{mult(i(x2),x1)})} \\
= \quad & \text{by Lemma 60 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{rd(x1,mult(i(x1),x2))}
\end{aligned}
$$

Lemma 285: $op_l(x1,mult(x2,x3),x3) = op_l(x1,x2,x3)$

$$
\begin{aligned}
& \underbrace{op_l(x1,mult(x2,x3),x3)} \\
= \quad & \text{by Lemma 241 RL with } \{x3 \leftarrow mult(x2,x3), x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1,i(x3),mult(x3,mult(x2,x3)))}} \\
= \quad & \text{by Axiom 12 RL with } \{x3 \leftarrow x1, x2 \leftarrow i(x3), x1 \leftarrow mult(x3,mult(x2,x3))\} \\
& \overbrace{mult(i(\underbrace{mult(mult(x3,mult(x2,x3)),i(x3))}),mult(mult(x3,mult(x2,x3)),mult(i(x3),x1)))} \\
= \quad & \text{by Axiom 6 RL with } \{x3 \leftarrow i(x3), x2 \leftarrow x2, x1 \leftarrow x3\} \\
& mult(i(\overbrace{mult(x3,mult(x2,mult(x3,i(x3))))}),\underbrace{mult(mult(x3,mult(x2,x3)),mult(i(x3),x1))}) \\
= \quad & \text{by Axiom 6 RL with } \{x3 \leftarrow mult(i(x3),x1), x2 \leftarrow x2, x1 \leftarrow x3\} \\
& mult(i(mult(x3,mult(x2,\underbrace{mult(x3,i(x3))}))),\overbrace{mult(x3,mult(x2,mult(x3,mult(i(x3),x1))))}) \\
= \quad & \text{by Axiom 7 LR with } \{x1 \leftarrow x3\} \\
& mult(i(mult(x3,\underbrace{mult(x2,\overbrace{unit()})})),mult(x3,mult(x2,mult(x3,mult(i(x3),x1))))) \\
= \quad & \text{by Axiom 1 RL with } \{x1 \leftarrow x2\} \\
& \underbrace{mult(i(mult(x3,\overbrace{x2})),mult(x3,mult(x2,mult(x3,mult(i(x3),x1)))))} \\
= \quad & \text{by Axiom 12 LR with } \{x3 \leftarrow mult(x3,mult(i(x3),x1)), x2 \leftarrow x2, x1 \leftarrow x3\} \\
& \overbrace{op_l(\underbrace{mult(x3,mult(i(x3),x1))},x2,x3)} \\
= \quad & \text{by Lemma 12 LR with } \{x2 \leftarrow x1, x1 \leftarrow x3\} \\
& op_l(\overbrace{x1},x2,x3)
\end{aligned}
$$

Lemma 286: $op_l(x1, x2, rd(x3, x2)) = op_l(x1, x2, x3)$

$op_l(x1, x2, rd(x3, \underbrace{x2}))$

$=$ by Axiom 2 LR with $\{x2 \leftarrow x3, x1 \leftarrow x2\}$

$op_l(x1, x2, \underbrace{rd(x3, \overbrace{mult(i(x3), mult(x3, x2))})})$

$=$ by Lemma 60 RL with $\{x2 \leftarrow mult(x3, x2), x1 \leftarrow x3\}$

$\underbrace{op_l(x1, x2, \overbrace{mult(x3, mult(i(mult(x3, x2)), x3))})}$

$=$ by Axiom 12 RL with $\{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow mult(x3, mult(i(mult(x3, x2)), x3))\}$

$\overbrace{mult(i(\underbrace{mult(mult(x3, mult(i(mult(x3, x2)), x3)), x2)}), \underbrace{mult(mult(x3, mult(i(mult(x3, x2)), x3)), mult(x2, x1))})}$

$=$ by Axiom 6 RL with $\{x3 \leftarrow x2, x2 \leftarrow i(mult(x3, x2)), x1 \leftarrow x3\}$

$mult(i(\overbrace{mult(x3, mult(i(mult(x3, x2)), mult(x3, x2)))}), \underbrace{mult(mult(x3, mult(i(mult(x3, x2)), x3)), mult(x2, x1))})$

$=$ by Axiom 6 RL with $\{x3 \leftarrow mult(x2, x1), x2 \leftarrow i(mult(x3, x2)), x1 \leftarrow x3\}$

$mult(i(mult(x3, \underbrace{mult(i(mult(x3, x2)), mult(x3, x2))})), \overbrace{mult(x3, mult(i(mult(x3, x2)), mult(x3, mult(x2, x1))))})$

$=$ by Axiom 9 LR with $\{x1 \leftarrow mult(x3, x2)\}$

$mult(i(\underbrace{mult(x3, \overbrace{unit()})}), mult(x3, mult(i(mult(x3, x2)), mult(x3, mult(x2, x1)))))$

$=$ by Axiom 1 RL with $\{x1 \leftarrow x3\}$

$\underbrace{mult(i(\overbrace{x3}), mult(x3, mult(i(mult(x3, x2)), mult(x3, mult(x2, x1)))))}$

$=$ by Axiom 2 RL with $\{x2 \leftarrow x3, x1 \leftarrow mult(i(mult(x3, x2)), mult(x3, mult(x2, x1)))\}$

$\underbrace{\overbrace{mult(i(mult(x3, x2)), mult(x3, mult(x2, x1)))}}$

$=$ by Axiom 12 LR with $\{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x3\}$

$\overbrace{op_l(x1, x2, x3)}$

Lemma 287: $op_l(x1, x2, op_t(i(x2), x3)) = x1$

$\underbrace{op_l(x1, x2, op_t(i(x2), x3))}$

$=$ by Lemma 286 RL with $\{x3 \leftarrow op_t(i(x2), x3), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_l(\underbrace{x1}, x2, rd(op_t(i(x2), x3), x2))}$

$=$ by Axiom 2 LR with $\{x2 \leftarrow op_t(i(x2), x3), x1 \leftarrow x1\}$

$op_l(\overbrace{mult(i(op_t(i(x2), x3)), \underbrace{mult(op_t(i(x2), x3), x1)})}, x2, rd(op_t(i(x2), x3), x2))$

$=$ by Axiom 2 LR with $\{x2 \leftarrow i(x2), x1 \leftarrow mult(op_t(i(x2), x3), x1)\}$

$op_l(mult(i(op_t(i(x2), x3)), \overbrace{mult(i(i(x2)), \underbrace{mult(i(x2), mult(op_t(i(x2), x3), x1))})}), x2, rd(op_t(i(x2), x3), x2))$

$=$ by Lemma 162 RL with $\{x3 \leftarrow x1, x2 \leftarrow x3, x1 \leftarrow i(x2)\}$

$op_l(mult(i(\underbrace{op_t(i(x2), x3)}), mult(i(i(x2)), \overbrace{mult(mult(i(x2), op_t(i(x2), x3)), x1)})), x2, rd(op_t(i(x2), x3), x2))$

$=$ by Axiom 2 LR with $\{x2 \leftarrow i(x2), x1 \leftarrow op_t(i(x2), x3)\}$

$op_l(\underbrace{mult(i(\overbrace{mult(i(i(x2)), mult(i(x2), op_t(i(x2), x3)))}), mult(i(i(x2)), mult(mult(i(x2), op_t(i(x2), x3)), x1))}), x2, rd(op_t(i(x$

$=$ by Axiom 12 LR with $\{x3 \leftarrow x1, x2 \leftarrow mult(i(x2), op_t(i(x2), x3)), x1 \leftarrow i(i(x2))\}$

$op_l(\overbrace{op_l(x1, mult(i(x2), op_t(i(x2), x3)), \underbrace{i(i(x2))})}, x2, rd(op_t(i(x2), x3), x2))$

$=$ by Lemma 3 LR with $\{x1 \leftarrow x2\}$

$op_l(op_l(x1, \underbrace{mult(i(x2), op_t(i(x2), x3))}, \overbrace{x2}), x2, rd(op_t(i(x2), x3), x2))$

$=$ by Lemma 111 LR with $\{x2 \leftarrow x3, x1 \leftarrow x2\}$

$\underbrace{op_l(op_l(x1, \overbrace{rd(op_t(i(x2), x3), x2)}), x2), x2, rd(op_t(i(x2), x3), x2))}$

$=$ by Lemma 138 LR with $\{x3 \leftarrow x2, x2 \leftarrow rd(op_t(i(x2), x3), x2), x1 \leftarrow x1\}$

$\overbrace{x1}$

Lemma 288: $op_l(x1, i(op_t(x2, x3)), x2) = x1$

$$
\begin{aligned}
& \underbrace{op_l(x1, i(op_t(x2, x3)), x2)} \\
= & \quad \text{by Lemma 286 RL with } \{x3 \leftarrow x2, x2 \leftarrow i(op_t(x2, x3)), x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, i(op_t(x2, x3)), rd(x2, i(\underbrace{op_t(x2, x3)})))} \\
= & \quad \text{by Lemma 95 RL with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& op_l(x1, i(op_t(x2, x3)), \underbrace{rd(x2, i(\overbrace{mult(x2, rd(op_t(x2, x3), x2))}))}) \\
= & \quad \text{by Lemma 58 LR with } \{x2 \leftarrow rd(op_t(x2, x3), x2), x1 \leftarrow x2\} \\
& op_l(x1, i(op_t(x2, x3)), \overbrace{mult(x2, \underbrace{mult(rd(op_t(x2, x3), x2), x2)})}) \\
= & \quad \text{by Axiom 4 RL with } \{x2 \leftarrow x2, x1 \leftarrow op_t(x2, x3)\} \\
& \underbrace{op_l(x1, i(op_t(x2, x3)), mult(x2, \overbrace{op_t(x2, x3)}))} \\
= & \quad \text{by Lemma 246 LR with } \{x3 \leftarrow x2, x2 \leftarrow op_t(x2, x3), x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, \underbrace{op_t(x2, op_t(x2, x3))}, op_t(x2, x3))} \\
= & \quad \text{by Axiom 10 RL with } \{x2 \leftarrow x2, x1 \leftarrow op_t(x2, x3)\} \\
& op_l(x1, \overbrace{mult(i(op_t(x2, x3)), \underbrace{mult(x2, op_t(x2, x3))})}, op_t(x2, x3)) \\
= & \quad \text{by Lemma 27 RL with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& op_l(x1, \underbrace{mult(i(op_t(x2, x3)), \overbrace{mult(op_t(x2, x3), x2)})}, op_t(x2, x3)) \\
= & \quad \text{by Axiom 2 RL with } \{x2 \leftarrow op_t(x2, x3), x1 \leftarrow x2\} \\
& op_l(\underbrace{x1}, \overbrace{x2}, op_t(x2, x3)) \\
= & \quad \text{by Lemma 163 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{op_l(\overbrace{op_l(x1, op_t(x2, x3), x2)}, x2, op_t(x2, x3))} \\
= & \quad \text{by Lemma 138 LR with } \{x3 \leftarrow x2, x2 \leftarrow op_t(x2, x3), x1 \leftarrow x1\} \\
& \overbrace{x1}
\end{aligned}
$$

Lemma 289: $op_l(x1, x2, i(op_t(x2, x3))) = x1$

$$
\begin{aligned}
& op_l(x1, x2, \underbrace{i(op_t(x2, x3))}) \\
= & \quad \text{by Lemma 15 RL with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& \underbrace{op_l(x1, x2, \overbrace{op_t(i(x2), i(x3))})} \\
= & \quad \text{by Lemma 287 LR with } \{x3 \leftarrow i(x3), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{x1}
\end{aligned}
$$

Lemma 290: $x1 = op_l(x1, op_t(x2, x3), i(x2))$

$$
\begin{aligned}
& \overbrace{x1} \\
= \quad & \text{by Lemma 288 RL with } \{x3 \leftarrow i(x3), x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, \underbrace{i(\underbrace{op_t(i(x2), i(x3))}), i(x2)})} \\
= \quad & \text{by Lemma 15 LR with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& op_l(x1, \underbrace{i(\overbrace{i(op_t(x2, x3))})}, i(x2)) \\
= \quad & \text{by Lemma 3 LR with } \{x1 \leftarrow op_t(x2, x3)\} \\
& op_l(x1, \overbrace{op_t(x2, x3)}, i(x2))
\end{aligned}
$$

Lemma 291: $op_l(x1, x2, mult(x3, x2)) = op_l(x1, x2, x3)$

$$
\begin{aligned}
& op_l(x1, \underbrace{x2}, mult(x3, x2)) \\
= \quad & \text{by Axiom 2 LR with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& \underbrace{op_l(x1, \overbrace{mult(i(x3), mult(x3, x2))}, mult(x3, x2))} \\
= \quad & \text{by Lemma 285 LR with } \{x3 \leftarrow mult(x3, x2), x2 \leftarrow i(x3), x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, i(x3), mult(x3, x2))}} \\
= \quad & \text{by Lemma 241 LR with } \{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x2, x3)}
\end{aligned}
$$

Lemma 292: $op_l(x1, rd(x2, x3), x3) = op_l(x1, x2, x3)$

$$
\begin{aligned}
& \underbrace{op_l(x1, rd(x2, x3), x3)} \\
= \quad & \text{by Lemma 285 RL with } \{x3 \leftarrow x3, x2 \leftarrow rd(x2, x3), x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, \underbrace{mult(rd(x2, x3), x3)}, x3)} \\
= \quad & \text{by Axiom 4 RL with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& op_l(x1, \overbrace{x2}, x3)
\end{aligned}
$$

Lemma 293: $x1 = op_l(x1, x2, asoc(x2, x3, i(x3)))$

$$
\begin{aligned}
& \overbrace{x1} \\
= \quad & \text{by Lemma 165 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, x2, op_r(x2, x3, i(x3)))}} \\
= \quad & \text{by Lemma 286 RL with } \{x3 \leftarrow op_r(x2, x3, i(x3)), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x2, \underbrace{rd(op_r(x2, x3, i(x3)), x2)})} \\
= \quad & \text{by Lemma 141 LR with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& op_l(x1, x2, \overbrace{asoc(x2, x3, i(x3))})
\end{aligned}
$$

Lemma 294: $x1 = op_l(x1, x2, rd(x2, op_t(x2, x3)))$

$$
\begin{aligned}
& \overbrace{x1} \\
= \quad & \text{by Lemma 289 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, x2, i(op_t(x2, x3)))}} \\
= \quad & \text{by Lemma 291 RL with } \{x3 \leftarrow i(op_t(x2, x3)), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x2, \underbrace{mult(i(op_t(x2, x3)), x2)})} \\
= \quad & \text{by Lemma 99 LR with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& op_l(x1, x2, \overbrace{rd(x2, op_t(x2, x3))})
\end{aligned}
$$

Lemma 295: $op_l(x1, rd(i(x2), x3), x2) = op_l(x1, i(x3), x2)$

$$
\begin{aligned}
& \underbrace{op_l(x1, rd(i(x2), x3), x2)} \\
= \quad & \text{by Lemma 285 RL with } \{x3 \leftarrow x2, x2 \leftarrow rd(i(x2), x3), x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, \underbrace{mult(rd(i(x2), x3), x2)}, x2)} \\
= \quad & \text{by Lemma 21 LR with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& op_l(x1, \overbrace{i(x3)}, x2)
\end{aligned}
$$

Lemma 296: $op_l(x1, x2, rd(i(x2), x3)) = op_l(x1, x2, i(x3))$

$$
\begin{aligned}
& op_l(x1, x2, \underbrace{rd(i(x2), x3)}) \\
= \quad & \text{by Lemma 37 RL with } \{x2 \leftarrow x2, x1 \leftarrow x3\} \\
& \underbrace{op_l(x1, x2, \overbrace{rd(i(x3), x2)})} \\
= \quad & \text{by Lemma 286 LR with } \{x3 \leftarrow i(x3), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x2, i(x3))}
\end{aligned}
$$

Lemma 297: $x1 = op_l(x1, i(x2), asoc(x2, x3, i(x3)))$

$$
\begin{aligned}
& \overbrace{x1} \\
= \quad & \text{by Lemma 294 LR with } \{x3 \leftarrow mult(x2, x3), x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, i(x2), rd(i(x2), \underbrace{op_t(i(x2), mult(x2, x3))}))} \\
= \quad & \text{by Lemma 209 RL with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& op_l(x1, i(x2), \underbrace{rd(i(x2), \overbrace{op_r(i(x2), i(x3), x3)})}) \\
= \quad & \text{by Lemma 143 LR with } \{x2 \leftarrow x3, x1 \leftarrow i(x2)\} \\
& op_l(x1, i(x2), \overbrace{asoc(\underbrace{i(i(x2))}, x3, i(x3))}) \\
= \quad & \text{by Lemma 3 LR with } \{x1 \leftarrow x2\} \\
& op_l(x1, i(x2), asoc(\overbrace{x2}, x3, i(x3)))
\end{aligned}
$$

Lemma 298: $x1 = op_l(x1, asoc(x2, x3, i(x3)), i(x2))$

$\overbrace{x1}$

$=$ by Lemma 288 RL with $\{x3 \leftarrow mult(x2, x3), x2 \leftarrow i(x2), x1 \leftarrow x1\}$

$\overbrace{op_l(x1, \underbrace{i(op_t(i(x2), mult(x2, x3)))}, i(x2))}$

$=$ by Lemma 29 RL with $\{x2 \leftarrow i(op_t(i(x2), mult(x2, x3))), x1 \leftarrow i(x2)\}$

$op_l(x1, \overbrace{rd(mult(i(x2), \underbrace{i(op_t(i(x2), mult(x2, x3)))}), \underbrace{op_t(i(x2), i(op_t(i(x2), mult(x2, x3))))}}, i(x2))$

$=$ by Lemma 48 LR with $\{x2 \leftarrow mult(x2, x3), x1 \leftarrow i(x2)\}$

$op_l(x1, rd(\underbrace{mult(i(x2), \underbrace{i(op_t(i(x2), mult(x2, x3)))})}, \overbrace{i(x2)}), i(x2))$

$=$ by Lemma 98 RL with $\{x2 \leftarrow mult(x2, x3), x1 \leftarrow i(x2)\}$

$\underbrace{op_l(x1, rd(\overbrace{rd(i(x2), op_t(i(x2), mult(x2, x3)))}, i(x2)), i(x2))}$

$=$ by Lemma 292 LR with $\{x3 \leftarrow i(x2), x2 \leftarrow rd(i(x2), op_t(i(x2), mult(x2, x3))), x1 \leftarrow x1\}$

$\overbrace{op_l(x1, rd(i(x2), \underbrace{op_t(i(x2), mult(x2, x3))}), i(x2))}$

$=$ by Lemma 209 RL with $\{x2 \leftarrow x3, x1 \leftarrow x2\}$

$op_l(x1, \underbrace{rd(i(x2), \overbrace{op_r(i(x2), i(x3), x3)})}, i(x2))$

$=$ by Lemma 143 LR with $\{x2 \leftarrow x3, x1 \leftarrow i(x2)\}$

$op_l(x1, \overbrace{asoc(\underbrace{i(i(x2))}, x3, i(x3))}, i(x2))$

$=$ by Lemma 3 LR with $\{x1 \leftarrow x2\}$

$op_l(x1, asoc(\overbrace{x2}, x3, i(x3)), i(x2))$

Lemma 299: $op_l(x1, rd(x2, x3), i(x2)) = op_l(x1, i(x3), i(x2))$

$\underbrace{op_l(x1, rd(x2, x3), i(x2))}$

$=$ by Lemma 285 RL with $\{x3 \leftarrow i(x2), x2 \leftarrow rd(x2, x3), x1 \leftarrow x1\}$

$\overbrace{op_l(x1, \underbrace{mult(rd(x2, x3), i(x2))}, i(x2))}$

$=$ by Lemma 22 LR with $\{x2 \leftarrow x3, x1 \leftarrow x2\}$

$op_l(x1, \overbrace{i(x3)}, i(x2))$

Lemma 300: $op_l(x1, i(x2), i(x3)) = op_l(x1, mult(i(x2), rd(x2, x3)), x2)$

$\underbrace{op_l(x1, i(x2), i(x3))}$

$=$ by Lemma 247 RL with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{\overbrace{op_l(x1, i(mult(x2, x3)), x2)}}$

$=$ by Lemma 285 RL with $\{x3 \leftarrow x2, x2 \leftarrow i(mult(x2, x3)), x1 \leftarrow x1\}$

$\overbrace{op_l(x1, \underbrace{mult(i(mult(x2, x3)), x2)}, x2)}$

$=$ by Lemma 56 LR with $\{x2 \leftarrow x3, x1 \leftarrow x2\}$

$op_l(x1, \overbrace{mult(i(x2), rd(x2, x3))}, x2)$

Lemma 301: $op_l(x1, i(x2), rd(x2, x3)) = op_l(x1, i(x2), i(x3))$

$$
\begin{aligned}
& \underbrace{op_l(x1, i(x2), rd(x2, x3))} \\
= \quad & \text{by Lemma 244 RL with } \{x3 \leftarrow rd(x2, x3), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, \underbrace{mult(i(x2), rd(x2, x3))}, x2)} \\
= \quad & \text{by Lemma 300 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, i(x2), i(x3))}
\end{aligned}
$$

Lemma 302: $op_l(x1, x2, op_l(x3, i(x3), x2)) = op_l(x1, x3, i(x2))$

$$
\begin{aligned}
& \underbrace{op_l(x1, x2, op_l(x3, i(x3), x2))} \\
= \quad & \text{by Lemma 286 RL with } \{x3 \leftarrow op_l(x3, i(x3), x2), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x2, \underbrace{rd(op_l(x3, i(x3), x2), x2)})} \\
= \quad & \text{by Lemma 233 LR with } \{x2 \leftarrow x2, x1 \leftarrow x3\} \\
& \underbrace{op_l(x1, x2, \overbrace{mult(i(x2), x3)})} \\
= \quad & \text{by Lemma 243 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x3, i(x2))}
\end{aligned}
$$

Lemma 303: $i(op_l(x1, i(x1), x2)) = op_l(i(x1), x1, i(x2))$

$$
\begin{aligned}
& \underbrace{i(op_l(x1, i(x1), x2))} \\
= \quad & \text{by Lemma 222 LR with } \{x1 \leftarrow x2, x2 \leftarrow op_l(x1, i(x1), x2)\} \\
& \overbrace{\underbrace{op_l(mult(i(x2), rd(x2, op_l(x1, i(x1), x2))), x2, op_l(x1, i(x1), x2))}} \\
= \quad & \text{by Lemma 302 LR with } \{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow mult(i(x2), rd(x2, op_l(x1, i(x1), x2)))\} \\
& \overbrace{op_l(mult(i(x2), \underbrace{rd(x2, op_l(x1, i(x1), x2))}), x1, i(x2))} \\
= \quad & \text{by Lemma 230 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& op_l(\underbrace{mult(i(x2), \overbrace{mult(x2, i(x1))})}, x1, i(x2)) \\
= \quad & \text{by Axiom 2 RL with } \{x2 \leftarrow x2, x1 \leftarrow i(x1)\} \\
& op_l(\overbrace{i(x1)}, x1, i(x2))
\end{aligned}
$$

Lemma 304: $op_l(x1, i(x1), i(x2)) = i(op_l(i(x1), x1, x2))$

$$
\begin{aligned}
& op_l(\underbrace{x1}, i(x1), i(x2)) \\
= \quad & \text{by Lemma 3 RL with } \{x1 \leftarrow x1\} \\
& \underbrace{op_l(\overbrace{i(i(x1))}, i(x1), i(x2))} \\
= \quad & \text{by Lemma 303 RL with } \{x2 \leftarrow x2, x1 \leftarrow i(x1)\} \\
& \overbrace{i(op_l(i(x1), \underbrace{i(i(x1))}, x2))} \\
= \quad & \text{by Lemma 3 LR with } \{x1 \leftarrow x1\} \\
& i(op_l(i(x1), \overbrace{x1}, x2))
\end{aligned}
$$

Lemma 305: $op_l(i(x1), x1, mult(i(x1), x2)) = i(op_l(x1, i(x2), i(i(x1))))$

$$
\begin{aligned}
& op_l(i(x1), x1, \underbrace{mult(i(x1), x2)}) \\
= & \quad \text{by Lemma 3 RL with } \{x1 \leftarrow mult(i(x1), x2)\} \\
& \underbrace{op_l(i(x1), x1, \overbrace{i(i(mult(i(x1), x2)))})} \\
= & \quad \text{by Lemma 303 RL with } \{x2 \leftarrow i(mult(i(x1), x2)), x1 \leftarrow x1\} \\
& \overbrace{i(\underbrace{op_l(x1, i(x1), i(\overbrace{mult(i(x1), x2)}))})} \\
= & \quad \text{by Lemma 245 LR with } \{x3 \leftarrow x2, x2 \leftarrow i(x1), x1 \leftarrow x1\} \\
& i(\overbrace{op_l(x1, i(x2), i(i(x1)))})
\end{aligned}
$$

Lemma 306: $op_l(i(x1), x2, i(x1)) = i(op_l(x1, i(x2), x1))$

$$
\begin{aligned}
& \underbrace{op_l(i(x1), x2, i(x1))} \\
= & \quad \text{by Lemma 243 RL with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow i(x1)\} \\
& \overbrace{\underbrace{op_l(i(x1), x1, mult(i(x1), x2))}} \\
= & \quad \text{by Lemma 305 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{i(op_l(x1, i(x2), \underbrace{i(i(x1))}))} \\
= & \quad \text{by Lemma 3 LR with } \{x1 \leftarrow x1\} \\
& i(op_l(x1, i(x2), \overbrace{x1}))
\end{aligned}
$$

Lemma 307: $op_l(i(x1), i(x2), i(x1)) = i(op_l(x1, x2, x1))$

$$
\begin{aligned}
& \underbrace{op_l(i(x1), i(x2), i(x1))} \\
= & \quad \text{by Lemma 245 RL with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow i(x1)\} \\
& \overbrace{\underbrace{op_l(i(x1), x1, i(\overbrace{mult(x1, x2)}))}} \\
= & \quad \text{by Lemma 303 RL with } \{x2 \leftarrow mult(x1, x2), x1 \leftarrow x1\} \\
& \overbrace{i(\underbrace{op_l(x1, i(x1), mult(x1, x2))})} \\
= & \quad \text{by Lemma 241 LR with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x1\} \\
& i(\overbrace{op_l(x1, x2, x1)})
\end{aligned}
$$

Lemma 308: $rd(x1, rd(x2, x1)) = mult(x1, op_l(rd(x1, x2), x2, x1))$

$$
\begin{aligned}
& \underbrace{rd(x1, rd(x2, x1))} \\
= & \quad \text{by Lemma 232 RL with } \{x2 \leftarrow rd(x2, x1), x1 \leftarrow x1\} \\
& \overbrace{mult(x1, \underbrace{op_l(i(\overbrace{rd(x2, x1)}), rd(x2, x1), x1)})} \\
= & \quad \text{by Lemma 292 LR with } \{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow i(rd(x2, x1))\} \\
& mult(x1, \overbrace{op_l(\underbrace{i(rd(x2, x1))}, x2, x1)}) \\
= & \quad \text{by Lemma 10 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& mult(x1, op_l(\overbrace{rd(x1, x2)}, x2, x1))
\end{aligned}
$$

Lemma 309: $mult(mult(x1, x2), rd(x2, x1)) = mult(x1, rd(x2, rd(x1, x2)))$

$\underbrace{mult(mult(x1, x2), rd(x2, x1))}$

$=$ by Lemma 137 RL with $\{x3 \leftarrow rd(x2, x1), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{mult(x1, \underbrace{mult(x2, op_l(rd(x2, x1), x1, x2))})}$

$=$ by Lemma 308 RL with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$mult(x1, \overbrace{rd(x2, rd(x1, x2))})$

Lemma 310: $op_l(i(x1), x1, x2) = mult(i(x2), rd(x2, x1))$

$\underbrace{op_l(i(x1), x1, x2)}$

$=$ by Axiom 12 RL with $\{x3 \leftarrow i(x1), x2 \leftarrow x1, x1 \leftarrow x2\}$

$\overbrace{mult(i(mult(x2, x1)), mult(x2, \underbrace{mult(x1, i(x1))}))}$

$=$ by Axiom 7 LR with $\{x1 \leftarrow x1\}$

$mult(i(mult(x2, x1)), \underbrace{mult(x2, \overbrace{unit()})})$

$=$ by Axiom 1 RL with $\{x1 \leftarrow x2\}$

$\underbrace{mult(i(mult(x2, x1)), \overbrace{x2})}$

$=$ by Lemma 56 LR with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$\overbrace{mult(i(x2), rd(x2, x1))}$

Lemma 311: $op_t(rd(x1, op_l(x1, x2, x3)), x1) = rd(x1, op_l(x1, x2, x3))$

$op_t(\underbrace{rd(x1, op_l(x1, x2, x3))}, x1)$

$=$ by Lemma 176 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{op_t(\overbrace{mult(x1, i(op_l(x1, x2, x3)))}, x1)}$

$=$ by Lemma 264 RL with $\{x2 \leftarrow x1, x1 \leftarrow i(op_l(x1, x2, x3))\}$

$\overbrace{mult(i(op_l(x1, x2, x3)), \underbrace{op_l(x1, x1, i(op_l(x1, x2, x3)))})}$

$=$ by Lemma 279 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{mult(i(op_l(x1, x2, x3)), \overbrace{x1})}$

$=$ by Lemma 177 RL with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{rd(x1, op_l(x1, x2, x3))}$

Lemma 312: $op_l(x1, i(x3), i(x2)) = op_l(x1, i(x3), i(mult(x2, x3)))$

$\underbrace{op_l(x1, i(x3), i(x2))}$

$=$ by Lemma 245 RL with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_l(x1, x2, i(mult(x2, x3)))}}$

$=$ by Lemma 285 RL with $\{x3 \leftarrow i(mult(x2, x3)), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_l(x1, \underbrace{mult(x2, i(mult(x2, x3)))}, i(mult(x2, x3)))}$

$=$ by Lemma 20 LR with $\{x2 \leftarrow x3, x1 \leftarrow x2\}$

$op_l(x1, \overbrace{i(x3)}, i(mult(x2, x3)))$

Lemma 313: $op_l(x1, x2, mult(x3, mult(x3, x2))) = op_l(x1, x2, mult(x3, x3))$

$\underbrace{op_l(x1, x2, mult(x3, mult(x3, x2)))}$

$=$ by Lemma 286 RL with $\{x3 \leftarrow mult(x3, mult(x3, x2)), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_l(x1, x2, \underbrace{rd(mult(x3, mult(x3, x2)), x2)})}$

$=$ by Lemma 41 LR with $\{x2 \leftarrow x2, x1 \leftarrow x3\}$

$op_l(x1, x2, \overbrace{mult(x3, x3)})$

Lemma 314: $mult(i(x1), op_t(x2, rd(x2, x1))) = op_l(rd(x2, x1), x2, x1)$

$mult(i(x1), op_t(\underbrace{x2}, rd(x2, x1)))$

$=$ by Axiom 4 LR with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$mult(i(x1), \underbrace{op_t(\overbrace{mult(rd(x2, x1), x1)}, rd(x2, x1)))}$

$=$ by Lemma 264 RL with $\{x2 \leftarrow rd(x2, x1), x1 \leftarrow x1\}$

$mult(i(x1), \overbrace{mult(x1, \underbrace{op_l(rd(x2, x1), rd(x2, x1), x1)})})$

$=$ by Lemma 292 LR with $\{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow rd(x2, x1)\}$

$\underbrace{mult(i(x1), mult(x1, \overbrace{op_l(rd(x2, x1), x2, x1)}))}$

$=$ by Axiom 2 RL with $\{x2 \leftarrow x1, x1 \leftarrow op_l(rd(x2, x1), x2, x1)\}$

$\overbrace{op_l(rd(x2, x1), x2, x1)}$

Lemma 315: $op_l(op_t(x1,x2),i(x1),i(x2)) = op_l(op_r(op_l(x1,i(x1),i(x2)),i(x2),x2),i(x2),i(x1))$

$\underbrace{op_l(op_t(x1,x2),i(x1),i(x2))}$

$=$ by Axiom 15 RL with $\{x4 \leftarrow x2, x3 \leftarrow i(x2), x2 \leftarrow i(x1), x1 \leftarrow x1\}$

$\overbrace{op_t(\underbrace{op_l(x1,i(x1),i(x2))},x2)}$

$=$ by Lemma 304 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$op_t(\overbrace{i(op_l(i(x1),x1,x2))},\underbrace{x2})$

$=$ by Axiom 2 LR with $\{x2 \leftarrow op_l(i(x1),x1,x2), x1 \leftarrow x2\}$

$\underbrace{op_t(i(op_l(i(x1),x1,x2)),\overbrace{mult(i(op_l(i(x1),x1,x2)),mult(op_l(i(x1),x1,x2),x2))})}$

$=$ by Lemma 251 RL with $\{x2 \leftarrow mult(op_l(i(x1),x1,x2),x2), x1 \leftarrow i(op_l(i(x1),x1,x2))\}$

$\overbrace{op_l(\underbrace{op_t(i(op_l(i(x1),x1,x2)),mult(op_l(i(x1),x1,x2),x2))},mult(op_l(i(x1),x1,x2),x2),i(op_l(i(x1),x1,x2)))}$

$=$ by Lemma 209 RL with $\{x2 \leftarrow x2, x1 \leftarrow op_l(i(x1),x1,x2)\}$

$\underbrace{op_l(\overbrace{op_r(i(op_l(i(x1),x1,x2)),i(x2),x2)},mult(op_l(i(x1),x1,x2),x2),i(op_l(i(x1),x1,x2)))}$

$=$ by Lemma 242 LR with $\{x3 \leftarrow x2, x2 \leftarrow op_l(i(x1),x1,x2), x1 \leftarrow op_r(i(op_l(i(x1),x1,x2)),i(x2),x2)\}$

$\underbrace{\overbrace{op_l(op_r(i(op_l(i(x1),x1,x2)),i(x2),x2),op_l(i(x1),x1,x2),x2)}}$

$=$ by Lemma 292 RL with $\{x3 \leftarrow x2, x2 \leftarrow op_l(i(x1),x1,x2), x1 \leftarrow op_r(i(op_l(i(x1),x1,x2)),i(x2),x2)\}$

$\overbrace{op_l(op_r(i(op_l(i(x1),x1,x2)),i(x2),x2),\underbrace{rd(op_l(i(x1),x1,x2),x2)},x2)}$

$=$ by Lemma 10 RL with $\{x2 \leftarrow op_l(i(x1),x1,x2), x1 \leftarrow x2\}$

$op_l(op_r(i(op_l(i(x1),x1,x2)),i(x2),x2),\overbrace{i(\underbrace{rd(x2,op_l(i(x1),x1,x2))})},x2)$

$=$ by Lemma 231 RL with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$\underbrace{op_l(op_r(i(op_l(i(x1),x1,x2)),i(x2),x2),i(\overbrace{mult(x2,x1)}),x2)}$

$=$ by Lemma 247 LR with $\{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow op_r(i(op_l(i(x1),x1,x2)),i(x2),x2)\}$

$\overbrace{op_l(op_r(\underbrace{i(op_l(i(x1),x1,x2))},i(x2),x2),i(x2),i(x1))}$

$=$ by Lemma 304 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$op_l(op_r(\overbrace{op_l(x1,i(x1),i(x2))},i(x2),x2),i(x2),i(x1))$

Lemma 316: $mult(op_l(x1, i(x1), x2), i(x2)) = rd(op_t(x1, x2), x2)$

$\underbrace{mult(op_l(x1, i(x1), x2), i(x2))}$

$=$ by Lemma 167 RL with $\{x4 \leftarrow x2, x3 \leftarrow i(x1), x2 \leftarrow x1, x1 \leftarrow i(x2)\}$

$\overbrace{mult(i(x2), op_l(op_t(x1, i(x2)), i(x1), \underbrace{x2}))}$

$=$ by Lemma 3 RL with $\{x1 \leftarrow x2\}$

$mult(i(x2), \underbrace{op_l(op_t(x1, i(x2)), i(x1), \overbrace{i(i(x2))})})$

$=$ by Lemma 315 LR with $\{x2 \leftarrow i(x2), x1 \leftarrow x1\}$

$mult(i(x2), \overbrace{op_l(\underbrace{op_r(op_l(x1, i(x1), i(i(x2))), i(i(x2)), i(x2))}, i(i(x2)), i(x1)))}$

$=$ by Axiom 18 LR with $\{x5 \leftarrow i(x2), x4 \leftarrow i(i(x2)), x3 \leftarrow i(i(x2)), x2 \leftarrow i(x1), x1 \leftarrow x1\}$

$mult(i(x2), \underbrace{op_l(\overbrace{op_l(op_r(x1, i(i(x2)), i(x2)), i(x1), i(i(x2)))}, i(i(x2)), i(x1)))}$

$=$ by Lemma 138 LR with $\{x3 \leftarrow i(i(x2)), x2 \leftarrow i(x1), x1 \leftarrow op_r(x1, i(i(x2)), i(x2))\}$

$mult(i(x2), \overbrace{op_r(x1, \underbrace{i(i(x2))}, i(x2))})$

$=$ by Lemma 3 LR with $\{x1 \leftarrow x2\}$

$\underbrace{mult(i(x2), op_r(x1, \overbrace{x2}, i(x2)))}$

$=$ by Lemma 139 LR with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$\overbrace{rd(op_t(x1, x2), x2)}$

Lemma 317: $rd(op_t(mult(x1, x2), x1), x1) = op_r(x2, x1, i(x1))$

$\underbrace{rd(op_t(mult(x1, x2), x1), x1)}$

$=$ by Lemma 316 RL with $\{x2 \leftarrow x1, x1 \leftarrow mult(x1, x2)\}$

$\overbrace{mult(\underbrace{op_l(mult(x1, x2), i(mult(x1, x2)), x1)}, i(x1))}$

$=$ by Lemma 247 LR with $\{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow mult(x1, x2)\}$

$mult(\overbrace{op_l(mult(x1, x2), i(x1), \underbrace{i(x2)})}, i(x1))$

$=$ by Lemma 20 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$mult(\underbrace{op_l(mult(x1, x2), i(x1), \overbrace{mult(x1, i(mult(x1, x2)))})}, i(x1))$

$=$ by Lemma 195 LR with $\{x2 \leftarrow x1, x1 \leftarrow mult(x1, x2)\}$

$mult(\overbrace{mult(\underbrace{mult(i(x1), mult(x1, x2))}, x1)}, i(x1))$

$=$ by Axiom 2 RL with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$\underbrace{mult(mult(\overbrace{x2}, x1), i(x1))}$

$=$ by Lemma 92 LR with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$\overbrace{op_r(x2, x1, i(x1))}$

Lemma 318: $mult(op_r(x1, x2, i(x2)), x2) = op_t(mult(x2, x1), x2)$

$$
\begin{aligned}
& mult(\underbrace{op_r(x1, x2, i(x2))}, x2) \\
= & \quad \text{by Lemma 317 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \underbrace{mult(\overbrace{rd(op_t(mult(x2, x1), x2), x2)}, x2)} \\
= & \quad \text{by Axiom 4 RL with } \{x2 \leftarrow x2, x1 \leftarrow op_t(mult(x2, x1), x2)\} \\
& \overbrace{op_t(mult(x2, x1), x2)}
\end{aligned}
$$

Lemma 319: $rd(op_t(mult(x1, x2), x2), x2) = op_r(op_t(x1, x2), x2, i(x2))$

$$
\begin{aligned}
& rd(op_t(\underbrace{mult(x1, x2)}, x2), x2) \\
= & \quad \text{by Lemma 13 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \underbrace{rd(op_t(\overbrace{mult(x2, op_t(x1, x2))}, x2), x2)} \\
= & \quad \text{by Lemma 317 LR with } \{x2 \leftarrow op_t(x1, x2), x1 \leftarrow x2\} \\
& \overbrace{op_r(op_t(x1, x2), x2, i(x2))}
\end{aligned}
$$

Lemma 320: $op_r(mult(x1, x2), x2, i(x2)) = op_t(mult(x2, x1), x2)$

$$
\begin{aligned}
& \underbrace{op_r(mult(x1, x2), x2, i(x2))} \\
= & \quad \text{by Lemma 109 RL with } \{x2 \leftarrow mult(x1, x2), x1 \leftarrow x2\} \\
& \overbrace{mult(x2, \underbrace{rd(op_t(mult(x1, x2), x2), x2)})} \\
= & \quad \text{by Lemma 319 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{mult(x2, \overbrace{op_r(op_t(x1, x2), x2, i(x2))})} \\
= & \quad \text{by Lemma 155 LR with } \{x4 \leftarrow i(x2), x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \overbrace{\underbrace{mult(op_r(x1, x2, i(x2)), x2)}} \\
= & \quad \text{by Lemma 318 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_t(mult(x2, x1), x2)}
\end{aligned}
$$

Lemma 321: $op_r(x1, i(x2), x2) = op_r(x1, x2, i(x2))$

$$
\begin{aligned}
& op_r(\underbrace{x1}, i(x2), x2) \\
= \quad & \text{by Axiom 4 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{op_r(\overbrace{mult(rd(x1, x2), x2)}, i(x2), x2)} \\
= \quad & \text{by Lemma 127 RL with } \{x2 \leftarrow x2, x1 \leftarrow rd(x1, x2)\} \\
& \overbrace{mult(\underbrace{op_r(rd(x1, x2), x2, i(x2))}, x2)} \\
= \quad & \text{by Lemma 317 RL with } \{x2 \leftarrow rd(x1, x2), x1 \leftarrow x2\} \\
& \underbrace{mult(\overbrace{rd(op_t(mult(x2, rd(x1, x2)), x2), x2)}, x2)} \\
= \quad & \text{by Axiom 4 RL with } \{x2 \leftarrow x2, x1 \leftarrow op_t(mult(x2, rd(x1, x2)), x2)\} \\
& \overbrace{\underbrace{op_t(mult(x2, rd(x1, x2)), x2)}} \\
= \quad & \text{by Lemma 320 RL with } \{x2 \leftarrow x2, x1 \leftarrow rd(x1, x2)\} \\
& \overbrace{op_r(\underbrace{mult(rd(x1, x2), x2)}, x2, i(x2))} \\
= \quad & \text{by Axiom 4 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_r(\overbrace{x1}, x2, i(x2))
\end{aligned}
$$

Lemma 322: $asoc(x1, x2, i(x2)) = asoc(x1, i(x2), x2)$

$$
\begin{aligned}
& \underbrace{asoc(x1, x2, i(x2))} \\
= \quad & \text{by Lemma 141 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{rd(\underbrace{op_r(x1, x2, i(x2))}, x1)} \\
= \quad & \text{by Lemma 321 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{rd(\overbrace{op_r(x1, i(x2), x2)}, x1)} \\
= \quad & \text{by Lemma 142 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{asoc(x1, i(x2), x2)}
\end{aligned}
$$

Lemma 323: $mult(mult(x1, i(x2)), x2) = op_r(x1, x2, i(x2))$

$$
\begin{aligned}
& \underbrace{mult(mult(x1, i(x2)), x2)} \\
= \quad & \text{by Lemma 93 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_r(x1, i(x2), x2)}} \\
= \quad & \text{by Lemma 321 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, x2, i(x2))}
\end{aligned}
$$

Lemma 324: $op_t(mult(x1, i(x2)), x2) = rd(op_t(x1, x2), x2)$

$$
\begin{aligned}
& \overbrace{op_t(mult(x1, i(x2)), x2)} \\
= & \quad \text{by Axiom 10 RL with } \{x2 \leftarrow mult(x1, i(x2)), x1 \leftarrow x2\} \\
& \overbrace{mult(i(x2), \underbrace{mult(mult(x1, i(x2)), x2)}_{})} \\
= & \quad \text{by Lemma 93 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& mult(i(x2), \overbrace{\underbrace{op_r(x1, i(x2), x2)}_{}}) \\
= & \quad \text{by Lemma 321 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{mult(i(x2), \overbrace{op_r(x1, x2, i(x2))}^{})} \\
= & \quad \text{by Lemma 139 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \overbrace{rd(op_t(x1, x2), x2)}
\end{aligned}
$$

Lemma 325: $mult(i(mult(x1, x2)), x2) = op_r(i(x1), x2, i(x2))$

$$
\begin{aligned}
& \underbrace{mult(i(mult(x1, x2)), x2)} \\
= & \quad \text{by Lemma 100 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_r(i(x1), i(x2), x2)}_{}} \\
= & \quad \text{by Lemma 321 LR with } \{x2 \leftarrow x2, x1 \leftarrow i(x1)\} \\
& \overbrace{op_r(i(x1), x2, i(x2))}
\end{aligned}
$$

Lemma 326: $op_t(mult(x1, x2), i(x2)) = rd(x2, op_t(i(x1), x2))$

$$
\begin{aligned}
& \underbrace{op_t(mult(x1, x2), i(x2))} \\
= & \quad \text{by Lemma 110 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \overbrace{mult(x2, \underbrace{op_r(x1, x2, i(x2))}_{})} \\
= & \quad \text{by Lemma 321 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{mult(x2, \overbrace{op_r(x1, i(x2), x2)}^{})} \\
= & \quad \text{by Lemma 57 RL with } \{x2 \leftarrow op_r(x1, i(x2), x2), x1 \leftarrow x2\} \\
& \overbrace{rd(x2, mult(i(x2), \underbrace{rd(x2, op_r(x1, i(x2), x2))}_{}))} \\
= & \quad \text{by Lemma 106 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& rd(x2, \underbrace{mult(i(x2), \overbrace{mult(i(x1), x2)}^{})}) \\
= & \quad \text{by Axiom 10 LR with } \{x2 \leftarrow i(x1), x1 \leftarrow x2\} \\
& rd(x2, \overbrace{op_t(i(x1), x2)})
\end{aligned}
$$

Lemma 327: $rd(i(x1), op_t(x2, i(x1))) = i(mult(x1, op_r(x2, x1, i(x1))))$

$$
\begin{aligned}
& rd(i(x1), \underbrace{op_t(x2, i(x1))}) \\
= & \quad \text{by Lemma 30 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{rd(i(x1), \overbrace{mult(x1, mult(x2, i(x1)))})} \\
= & \quad \text{by Lemma 59 RL with } \{x2 \leftarrow mult(x2, i(x1)), x1 \leftarrow x1\} \\
& \overbrace{i(mult(x1, \underbrace{mult(mult(x2, i(x1)), x1)}))} \\
= & \quad \text{by Lemma 323 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& i(mult(x1, \overbrace{op_r(x2, x1, i(x1))}))
\end{aligned}
$$

Lemma 328: $rd(op_t(i(x2), x1), x1) = i(op_t(mult(x2, x1), i(x1)))$

$$
\begin{aligned}
& \underbrace{rd(op_t(i(x2), x1), x1)} \\
= & \quad \text{by Lemma 66 RL with } \{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{rd(i(x1), op_t(x2, i(x1)))}} \\
= & \quad \text{by Lemma 327 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{i(\underbrace{mult(x1, op_r(x2, x1, i(x1)))})} \\
= & \quad \text{by Lemma 110 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& i(\overbrace{op_t(mult(x2, x1), i(x1))})
\end{aligned}
$$

Lemma 329: $op_t(i(mult(x1, x2)), x2) = rd(op_t(i(x1), x2), x2)$

$$
\begin{aligned}
& \underbrace{op_t(i(mult(x1, x2)), x2)} \\
= & \quad \text{by Lemma 19 RL with } \{x2 \leftarrow x2, x1 \leftarrow mult(x1, x2)\} \\
& \overbrace{\underbrace{i(op_t(mult(x1, x2), i(x2)))}} \\
= & \quad \text{by Lemma 328 RL with } \{x1 \leftarrow x2, x2 \leftarrow x1\} \\
& \overbrace{rd(op_t(i(x1), x2), x2)}
\end{aligned}
$$

Lemma 330: $op_r(rd(x1, x2), x1, i(x1)) = mult(i(x2), x1)$

$$
\begin{aligned}
& \underbrace{op_r(rd(x1, x2), x1, i(x1))} \\
= & \quad \text{by Lemma 321 RL with } \{x2 \leftarrow x1, x1 \leftarrow rd(x1, x2)\} \\
& \overbrace{\underbrace{op_r(rd(x1, x2), i(x1), x1)}} \\
= & \quad \text{by Lemma 105 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{mult(i(x2), i(i(x1)))}} \\
= & \quad \text{by Axiom 11 LR with } \{x2 \leftarrow i(x1), x1 \leftarrow x2\} \\
& \overbrace{\underbrace{i(mult(x2, i(x1)))}} \\
= & \quad \text{by Lemma 17 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \overbrace{mult(i(x2), x1)}
\end{aligned}
$$

Lemma 331: $mult(rd(x2,x1),i(x1)) = rd(i(x1),mult(i(x2),x1))$

$$
\begin{aligned}
& \underbrace{mult(rd(x2,x1),i(x1))} \\
= & \quad \text{by Lemma 44 RL with } \{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{i(mult(rd(x1,x2),x1))}} \\
= & \quad \text{by Lemma 122 RL with } \{x2 \leftarrow rd(x1,x2), x1 \leftarrow x1\} \\
& \overbrace{rd(i(x1),\underbrace{op_r(rd(x1,x2),x1,i(x1))})} \\
= & \quad \text{by Lemma 330 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& rd(i(x1),\overbrace{mult(i(x2),x1)})
\end{aligned}
$$

Lemma 332: $mult(rd(x1,x2),i(x2)) = rd(mult(x1,i(x2)),x2)$

$$
\begin{aligned}
& \underbrace{mult(rd(x1,x2),i(x2))} \\
= & \quad \text{by Lemma 331 LR with } \{x1 \leftarrow x2, x2 \leftarrow x1\} \\
& \overbrace{\underbrace{rd(i(x2),mult(i(x1),x2))}} \\
= & \quad \text{by Lemma 64 LR with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \overbrace{rd(mult(x1,i(x2)),x2)}
\end{aligned}
$$

Lemma 333: $mult(rd(x2,x1),x2) = i(rd(mult(x1,i(x2)),x2))$

$$
\begin{aligned}
& mult(\underbrace{rd(x2,x1)},x2) \\
= & \quad \text{by Lemma 10 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{mult(\overbrace{i(rd(x1,x2))},x2)} \\
= & \quad \text{by Lemma 17 RL with } \{x2 \leftarrow x2, x1 \leftarrow rd(x1,x2)\} \\
& \overbrace{i(\underbrace{mult(rd(x1,x2),i(x2))})} \\
= & \quad \text{by Lemma 332 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& i(\overbrace{rd(mult(x1,i(x2)),x2)})
\end{aligned}
$$

Lemma 334: $rd(x1,mult(x2,i(x1))) = mult(rd(x1,x2),x1)$

$$
\begin{aligned}
& \underbrace{rd(x1,mult(x2,i(x1)))} \\
= & \quad \text{by Lemma 10 RL with } \{x2 \leftarrow x1, x1 \leftarrow mult(x2,i(x1))\} \\
& \overbrace{\underbrace{i(rd(mult(x2,i(x1)),x1))}} \\
= & \quad \text{by Lemma 333 RL with } \{x1 \leftarrow x2, x2 \leftarrow x1\} \\
& \overbrace{mult(rd(x1,x2),x1)}
\end{aligned}
$$

Lemma 335: $rd(x2, mult(i(x1), i(x2))) = mult(rd(x1, i(x2)), x2)$

$$
\begin{aligned}
& \underbrace{rd(x2, mult(i(x1), i(x2)))} \\
= & \quad \text{by Lemma 61 RL with } \{x3 \leftarrow x2, x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& \overbrace{\underbrace{rd(mult(x1, i(i(x2))), i(x2))}} \\
= & \quad \text{by Lemma 332 RL with } \{x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& \overbrace{mult(rd(x1, i(x2)), \underbrace{i(i(x2))})} \\
= & \quad \text{by Lemma 3 LR with } \{x1 \leftarrow x2\} \\
& mult(rd(x1, i(x2)), \overbrace{x2})
\end{aligned}
$$

Lemma 336: $mult(rd(x1, i(x2)), x2) = rd(x2, i(mult(x1, x2)))$

$$
\begin{aligned}
& \underbrace{mult(rd(x1, i(x2)), x2)} \\
= & \quad \text{by Lemma 335 RL with } \{x1 \leftarrow x1, x2 \leftarrow x2\} \\
& \overbrace{rd(x2, \underbrace{mult(i(x1), i(x2))})} \\
= & \quad \text{by Axiom 11 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& rd(x2, \overbrace{i(mult(x1, x2))})
\end{aligned}
$$

Lemma 337: $op_t(rd(x1, i(x2)), x2) = mult(op_t(x1, x2), x2)$

$$
\begin{aligned}
& \underbrace{op_t(rd(x1, i(x2)), x2)} \\
= & \quad \text{by Axiom 10 RL with } \{x2 \leftarrow rd(x1, i(x2)), x1 \leftarrow x2\} \\
& \overbrace{mult(i(x2), \underbrace{mult(rd(x1, i(x2)), x2)})} \\
= & \quad \text{by Lemma 336 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{mult(i(x2), \overbrace{rd(x2, i(mult(x1, x2)))})} \\
= & \quad \text{by Lemma 71 LR with } \{x2 \leftarrow mult(x1, x2), x1 \leftarrow x2\} \\
& \overbrace{mult(\underbrace{mult(i(x2), mult(x1, x2))}, x2)} \\
= & \quad \text{by Axiom 10 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& mult(\overbrace{op_t(x1, x2)}, x2)
\end{aligned}
$$

Lemma 338: $op_t(rd(i(i(x2)), x1), x2) = i(mult(op_t(x1, i(x2)), i(x2)))$

$$
\begin{aligned}
& op_t(\underbrace{rd(i(i(x2)), x1)}, x2) \\
= & \quad \text{by Lemma 10 RL with } \{x2 \leftarrow i(i(x2)), x1 \leftarrow x1\} \\
& \underbrace{op_t(\overbrace{i(rd(x1, i(i(x2))))}, x2)} \\
= & \quad \text{by Lemma 19 RL with } \{x2 \leftarrow x2, x1 \leftarrow rd(x1, i(i(x2)))\} \\
& \overbrace{i(\underbrace{op_t(rd(x1, i(i(x2))), i(x2))})} \\
= & \quad \text{by Lemma 337 LR with } \{x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& i(\overbrace{mult(op_t(x1, i(x2)), i(x2))})
\end{aligned}
$$

Lemma 339: $op_t(rd(x2, x1), x2) = mult(op_t(i(x1), x2), i(i(x2)))$

$$
\begin{aligned}
& op_t(rd(\underbrace{x2}, x1), x2) \\
= \quad & \text{by Lemma 3 RL with } \{x1 \leftarrow x2\} \\
& \underbrace{op_t(rd(\overbrace{i(i(x2))}, x1), x2)} \\
= \quad & \text{by Lemma 338 LR with } \{x1 \leftarrow x1, x2 \leftarrow x2\} \\
& \overbrace{i(mult(\underbrace{op_t(x1, i(x2))}, i(x2)))} \\
= \quad & \text{by Lemma 18 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{i(mult(\overbrace{i(op_t(i(x1), x2))}, i(x2)))} \\
= \quad & \text{by Lemma 16 LR with } \{x2 \leftarrow i(x2), x1 \leftarrow op_t(i(x1), x2)\} \\
& \overbrace{mult(op_t(i(x1), x2), i(i(x2)))}
\end{aligned}
$$

Lemma 340: $mult(op_t(i(x1), x2), x2) = op_t(rd(x2, x1), x2)$

$$
\begin{aligned}
& mult(op_t(i(x1), x2), \underbrace{x2}) \\
= \quad & \text{by Lemma 3 RL with } \{x1 \leftarrow x2\} \\
& \underbrace{mult(op_t(i(x1), x2), \overbrace{i(i(x2))})} \\
= \quad & \text{by Lemma 339 RL with } \{x1 \leftarrow x1, x2 \leftarrow x2\} \\
& \overbrace{op_t(rd(x2, x1), x2)}
\end{aligned}
$$

Lemma 341: $op_t(rd(x1, i(x2)), x1) = mult(op_t(x2, x1), x1)$

$$
\begin{aligned}
& op_t(\underbrace{rd(x1, i(x2))}, x1) \\
= \quad & \text{by Lemma 38 RL with } \{x1 \leftarrow x1, x2 \leftarrow x2\} \\
& \underbrace{op_t(\overbrace{rd(x2, i(x1))}, x1)} \\
= \quad & \text{by Lemma 337 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \overbrace{mult(op_t(x2, x1), x1)}
\end{aligned}
$$

Lemma 342: $rd(op_r(x1, x2, i(x2)), x2) = mult(x1, i(x2))$

$$
\begin{aligned}
& rd(\underbrace{op_r(x1, x2, i(x2))}, x2) \\
= \quad & \text{by Lemma 321 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& rd(\overbrace{\underbrace{op_r(x1, i(x2), x2)}}, x2) \\
= \quad & \text{by Lemma 93 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{rd(\overbrace{mult(mult(x1, i(x2)), x2)}, x2)} \\
= \quad & \text{by Axiom 5 RL with } \{x2 \leftarrow x2, x1 \leftarrow mult(x1, i(x2))\} \\
& \overbrace{mult(x1, i(x2))}
\end{aligned}
$$

Lemma 343: $rd(x1, op_r(x2, x1, i(x1))) = mult(i(x2), x1)$

$$
\begin{aligned}
& rd(x1, \underbrace{op_r(x2, x1, i(x1))}) \\
= & \quad \text{by Lemma 321 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \underbrace{rd(x1, \overbrace{op_r(x2, i(x1), x1)})} \\
= & \quad \text{by Lemma 106 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \overbrace{mult(i(x2), x1)}
\end{aligned}
$$

Lemma 344: $rd(x1, op_t(rd(x1, x2), x1)) = op_t(x2, i(x1))$

$$
\begin{aligned}
& rd(x1, op_t(\underbrace{rd(x1, x2)}, x1)) \\
= & \quad \text{by Lemma 10 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \underbrace{rd(x1, op_t(\overbrace{i(rd(x2, x1))}, x1))} \\
= & \quad \text{by Lemma 65 RL with } \{x3 \leftarrow x1, x2 \leftarrow x1, x1 \leftarrow rd(x2, x1)\} \\
& \overbrace{rd(op_t(\underbrace{rd(x2, x1)}, i(x1)), i(x1))} \\
= & \quad \text{by Axiom 2 LR with } \{x2 \leftarrow x1, x1 \leftarrow rd(x2, x1)\} \\
& \underbrace{rd(op_t(\overbrace{mult(i(x1), mult(x1, rd(x2, x1)))}, i(x1)), i(x1))} \\
= & \quad \text{by Lemma 317 LR with } \{x2 \leftarrow mult(x1, rd(x2, x1)), x1 \leftarrow i(x1)\} \\
& \overbrace{op_r(\underbrace{mult(x1, rd(x2, x1))}, i(x1), i(i(x1)))} \\
= & \quad \text{by Lemma 55 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{op_r(\overbrace{mult(mult(x1, x2), i(x1))}, i(x1), i(i(x1)))} \\
= & \quad \text{by Lemma 320 LR with } \{x2 \leftarrow i(x1), x1 \leftarrow mult(x1, x2)\} \\
& \overbrace{op_t(\underbrace{mult(i(x1), mult(x1, x2))}, i(x1))} \\
= & \quad \text{by Axiom 2 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& op_t(\overbrace{x2}, i(x1))
\end{aligned}
$$

Lemma 345: $rd(op_t(rd(x1, x2), x1), x1) = op_t(i(x2), x1)$

$$
\begin{aligned}
& rd(op_t(rd(\underbrace{x1}, x2), x1), x1) \\
= & \quad \text{by Lemma 3 RL with } \{x1 \leftarrow x1\} \\
& \underbrace{rd(op_t(rd(\overbrace{i(i(x1))}, x2), x1), x1)} \\
= & \quad \text{by Lemma 324 RL with } \{x2 \leftarrow x1, x1 \leftarrow rd(i(i(x1)), x2)\} \\
& \overbrace{op_t(\underbrace{mult(rd(i(i(x1)), x2), i(x1))}, x1)} \\
= & \quad \text{by Lemma 21 LR with } \{x2 \leftarrow x2, x1 \leftarrow i(x1)\} \\
& op_t(\overbrace{i(x2)}, x1)
\end{aligned}
$$

Lemma 346: $mult(rd(x1, rd(x1, x2)), x1) = rd(x1, i(x2))$

$$
\begin{aligned}
& mult(rd(x1, rd(\underbrace{x1}, x2)), x1) \\
= & \quad \text{by Lemma 3 RL with } \{x1 \leftarrow x1\} \\
& \underbrace{mult(rd(x1, rd(\overbrace{i(i(x1))}, x2)), x1)} \\
= & \quad \text{by Lemma 334 RL with } \{x2 \leftarrow rd(i(i(x1)), x2), x1 \leftarrow x1\} \\
& \overbrace{rd(x1, \underbrace{mult(rd(\overbrace{i(i(x1))}, x2), i(x1)))}} \\
= & \quad \text{by Lemma 21 LR with } \{x2 \leftarrow x2, x1 \leftarrow i(x1)\} \\
& rd(x1, \overbrace{i(x2)})
\end{aligned}
$$

Lemma 347: $rd(x1, rd(x1, i(x2))) = rd(rd(x1, x2), x1)$

$$
\begin{aligned}
& rd(x1, \underbrace{rd(x1, i(x2))}) \\
= & \quad \text{by Lemma 346 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{rd(x1, \overbrace{mult(rd(x1, rd(x1, x2)), x1)})} \\
= & \quad \text{by Lemma 9 LR with } \{x2 \leftarrow rd(x1, rd(x1, x2)), x1 \leftarrow x1\} \\
& \overbrace{\underbrace{i(rd(x1, rd(x1, x2)))}} \\
= & \quad \text{by Lemma 10 LR with } \{x2 \leftarrow rd(x1, x2), x1 \leftarrow x1\} \\
& \overbrace{rd(rd(x1, x2), x1)}
\end{aligned}
$$

Lemma 348: $rd(x1, rd(x2, i(x1))) = rd(i(x1), rd(x2, x1))$

$$
\begin{aligned}
& \underbrace{rd(x1, rd(x2, i(x1)))} \\
= & \quad \text{by Lemma 39 RL with } \{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow i(x1)\} \\
& \overbrace{\underbrace{rd(rd(i(x1), x2), i(x1))}} \\
= & \quad \text{by Lemma 347 RL with } \{x2 \leftarrow x2, x1 \leftarrow i(x1)\} \\
& \overbrace{rd(i(x1), \underbrace{rd(i(x1), i(x2)))}} \\
= & \quad \text{by Lemma 23 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& rd(i(x1), \overbrace{rd(x2, x1)})
\end{aligned}
$$

Lemma 349: $op_t(mult(x1, rd(x2, x1)), x1) = op_r(x2, x1, i(x1))$

$$
\begin{aligned}
& \underbrace{op_t(mult(x1, rd(x2, x1)), x1)} \\
= \quad & \text{by Axiom 4 LR with } \{x2 \leftarrow x1, x1 \leftarrow op_t(mult(x1, rd(x2, x1)), x1)\} \\
& \overbrace{mult(\underbrace{rd(op_t(mult(x1, rd(x2, x1)), x1), x1)}, x1)} \\
= \quad & \text{by Lemma 317 LR with } \{x2 \leftarrow rd(x2, x1), x1 \leftarrow x1\} \\
& \underbrace{mult(\overbrace{op_r(rd(x2, x1), x1, i(x1))}, x1)} \\
= \quad & \text{by Lemma 127 LR with } \{x2 \leftarrow x1, x1 \leftarrow rd(x2, x1)\} \\
& \overbrace{op_r(\underbrace{mult(rd(x2, x1), x1)}, i(x1), x1)} \\
= \quad & \text{by Axiom 4 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \underbrace{op_r(\overbrace{x2}, i(x1), x1)} \\
= \quad & \text{by Lemma 321 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \overbrace{op_r(x2, x1, i(x1))}
\end{aligned}
$$

Lemma 350: $mult(op_t(mult(x1, x2), x1), x1) = mult(x2, mult(x1, x1))$

$$
\begin{aligned}
& mult(\underbrace{op_t(mult(x1, x2), x1)}, x1) \\
= \quad & \text{by Lemma 320 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \underbrace{mult(\overbrace{op_r(mult(x2, x1), x1, i(x1))}, x1)} \\
= \quad & \text{by Lemma 318 LR with } \{x2 \leftarrow x1, x1 \leftarrow mult(x2, x1)\} \\
& \overbrace{\underbrace{op_t(mult(x1, mult(x2, x1)), x1)}} \\
= \quad & \text{by Lemma 200 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{mult(x2, mult(x1, x1))}
\end{aligned}
$$

Lemma 351: $op_r(op_r(x1, x2, x2), x2, i(x2)) = x1$

$$
\begin{aligned}
& \underbrace{op_r(op_r(x1, x2, x2), x2, i(x2))} \\
= \quad & \text{by Lemma 317 RL with } \{x2 \leftarrow op_r(x1, x2, x2), x1 \leftarrow x2\} \\
& \overbrace{rd(\underbrace{op_t(mult(x2, op_r(x1, x2, x2)), x2)}, x2)} \\
= \quad & \text{by Axiom 5 LR with } \{x2 \leftarrow x2, x1 \leftarrow op_t(mult(x2, op_r(x1, x2, x2)), x2)\} \\
& rd(\overbrace{rd(\underbrace{mult(op_t(mult(x2, op_r(x1, x2, x2)), x2), x2)}, x2)}, x2) \\
= \quad & \text{by Lemma 350 LR with } \{x2 \leftarrow op_r(x1, x2, x2), x1 \leftarrow x2\} \\
& rd(rd(\overbrace{mult(op_r(x1, x2, x2), mult(x2, x2))}, x2), x2) \\
= \quad & \text{by Lemma 91 LR with } \{x3 \leftarrow x2, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& rd(\underbrace{rd(\overbrace{mult(mult(x1, x2), x2)}, x2)}, x2) \\
= \quad & \text{by Axiom 5 RL with } \{x2 \leftarrow x2, x1 \leftarrow mult(x1, x2)\} \\
& \underbrace{rd(\overbrace{mult(x1, x2)}, x2)} \\
= \quad & \text{by Axiom 5 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{x1}
\end{aligned}
$$

Lemma 352: $mult(op_r(x1, x2, x2), i(x2)) = rd(x1, x2)$

$$
\begin{aligned}
& \underbrace{mult(op_r(x1, x2, x2), i(x2))} \\
= \quad & \text{by Lemma 342 RL with } \{x2 \leftarrow x2, x1 \leftarrow op_r(x1, x2, x2)\} \\
& \overbrace{rd(\underbrace{op_r(op_r(x1, x2, x2), x2, i(x2))}, x2)} \\
= \quad & \text{by Lemma 351 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& rd(\overbrace{x1}, x2)
\end{aligned}
$$

Lemma 353: $rd(x1, rd(x1, x2)) = op_r(x2, x1, x1)$

$$
\begin{aligned}
& \underbrace{rd(x1, rd(x1, x2))} \\
= \quad & \text{by Lemma 39 RL with } \{x3 \leftarrow x1, x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \overbrace{rd(\underbrace{rd(x2, x1)}, i(x1))} \\
= \quad & \text{by Lemma 352 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \underbrace{rd(\overbrace{mult(op_r(x2, x1, x1), i(x1))}, i(x1))} \\
= \quad & \text{by Axiom 5 RL with } \{x2 \leftarrow i(x1), x1 \leftarrow op_r(x2, x1, x1)\} \\
& \overbrace{op_r(x2, x1, x1)}
\end{aligned}
$$

Lemma 354: $rd(rd(x1, x2), x1) = op_r(i(x2), x1, x1)$

$$
\begin{aligned}
& \underbrace{rd(rd(x1, x2), x1)} \\
= \quad & \text{by Lemma 347 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{\overbrace{rd(x1, rd(x1, i(x2)))}} \\
= \quad & \text{by Lemma 353 LR with } \{x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& \overbrace{op_r(i(x2), x1, x1)}
\end{aligned}
$$

Lemma 355: $i(op_r(x1, x2, x2)) = rd(rd(x2, x1), x2)$

$$
\begin{aligned}
& \underbrace{i(op_r(x1, x2, x2))} \\
= \quad & \text{by Lemma 9 RL with } \{x2 \leftarrow op_r(x1, x2, x2), x1 \leftarrow i(x2)\} \\
& \overbrace{rd(i(x2), \underbrace{mult(op_r(x1, x2, x2), i(x2))})} \\
= \quad & \text{by Lemma 352 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{rd(i(x2), \overbrace{rd(x1, x2)})} \\
= \quad & \text{by Lemma 40 LR with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \overbrace{rd(rd(x2, x1), x2)}
\end{aligned}
$$

Lemma 356: $i(op_r(x1, x2, x2)) = op_r(i(x1), x2, x2)$

$$
\begin{aligned}
& \underbrace{i(op_r(x1, x2, x2))} \\
= & \quad \text{by Lemma 355 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{rd(rd(x2, x1), x2)}} \\
= & \quad \text{by Lemma 354 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \overbrace{op_r(i(x1), x2, x2)}
\end{aligned}
$$

Lemma 357: $mult(op_r(x1, x2, x2), x2) = rd(x2, i(x1))$

$$
\begin{aligned}
& mult(\underbrace{op_r(x1, x2, x2)}, x2) \\
= & \quad \text{by Lemma 353 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \underbrace{mult(\overbrace{rd(x2, rd(x2, x1))}, x2)} \\
= & \quad \text{by Lemma 346 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \overbrace{rd(x2, i(x1))}
\end{aligned}
$$

Lemma 358: $rd(rd(x1, i(x2)), x2) = op_r(x1, x2, x2)$

$$
\begin{aligned}
& \underbrace{rd(rd(x1, i(x2)), x2)} \\
= & \quad \text{by Lemma 40 RL with } \{x3 \leftarrow x1, x2 \leftarrow i(x2), x1 \leftarrow x2\} \\
& \overbrace{\underbrace{rd(i(x2), rd(i(x2), x1))}} \\
= & \quad \text{by Axiom 5 LR with } \{x2 \leftarrow i(x2), x1 \leftarrow rd(i(x2), rd(i(x2), x1))\} \\
& \overbrace{rd(\underbrace{mult(rd(i(x2), rd(i(x2), x1)), i(x2))}, i(x2))} \\
= & \quad \text{by Lemma 346 LR with } \{x2 \leftarrow x1, x1 \leftarrow i(x2)\} \\
& rd(\overbrace{\underbrace{rd(i(x2), i(x1))}}, i(x2)) \\
= & \quad \text{by Lemma 23 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \underbrace{rd(\overbrace{rd(x1, x2)}, i(x2))} \\
= & \quad \text{by Lemma 39 LR with } \{x3 \leftarrow x2, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{rd(x2, rd(x2, x1))}} \\
= & \quad \text{by Lemma 353 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \overbrace{op_r(x1, x2, x2)}
\end{aligned}
$$

Lemma 359: $op_r(mult(x1, x2), x2, x2) = rd(x2, i(x1))$

$$
\begin{aligned}
& \underbrace{op_r(mult(x1, x2), x2, x2)} \\
= & \quad \text{by Lemma 353 RL with } \{x2 \leftarrow mult(x1, x2), x1 \leftarrow x2\} \\
& \overbrace{rd(x2, \underbrace{rd(x2, mult(x1, x2))})} \\
= & \quad \text{by Lemma 9 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& rd(x2, \overbrace{i(x1)})
\end{aligned}
$$

Lemma 360: $op_r(x1, i(x2), i(x2)) = rd(rd(x1, x2), i(x2))$

$$
\begin{aligned}
& op_r(\underbrace{x1}, i(x2), i(x2)) \\
= & \quad \text{by Lemma 3 RL with } \{x1 \leftarrow x1\} \\
& \underbrace{op_r(\overbrace{i(i(x1))}, i(x2), i(x2))} \\
= & \quad \text{by Lemma 354 RL with } \{x2 \leftarrow i(x1), x1 \leftarrow i(x2)\} \\
& \overbrace{rd(\underbrace{rd(i(x2), i(x1))}, i(x2))} \\
= & \quad \text{by Lemma 23 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& rd(\overbrace{rd(x1, x2)}, i(x2))
\end{aligned}
$$

Lemma 361: $op_r(x1, i(x2), i(x2)) = op_r(x1, x2, x2)$

$$
\begin{aligned}
& \underbrace{op_r(x1, i(x2), i(x2))} \\
= & \quad \text{by Lemma 360 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{\overbrace{rd(rd(x1, x2), i(x2))}} \\
= & \quad \text{by Lemma 39 LR with } \{x3 \leftarrow x2, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{\overbrace{rd(x2, rd(x2, x1))}} \\
= & \quad \text{by Lemma 353 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \overbrace{op_r(x1, x2, x2)}
\end{aligned}
$$

Lemma 362: $rd(x1, rd(x2, i(x1))) = op_r(i(x2), x1, x1)$

$$
\begin{aligned}
& \underbrace{rd(x1, rd(x2, i(x1)))} \\
= & \quad \text{by Lemma 348 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{\overbrace{rd(i(x1), rd(x2, x1))}} \\
= & \quad \text{by Lemma 40 LR with } \{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{\overbrace{rd(rd(x1, x2), x1)}} \\
= & \quad \text{by Lemma 354 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_r(i(x2), x1, x1)}
\end{aligned}
$$

Lemma 363: $op_r(mult(x1, i(x2)), x2, x2) = i(rd(x2, x1))$

$$
\begin{aligned}
& \underbrace{op_r(mult(x1, i(x2)), x2, x2)} \\
= & \quad \text{by Lemma 353 RL with } \{x2 \leftarrow mult(x1, i(x2)), x1 \leftarrow x2\} \\
& \overbrace{rd(x2, \underbrace{rd(x2, mult(x1, i(x2)))})} \\
= & \quad \text{by Lemma 334 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \underbrace{rd(x2, \overbrace{mult(rd(x2, x1), x2)})} \\
= & \quad \text{by Lemma 9 LR with } \{x2 \leftarrow rd(x2, x1), x1 \leftarrow x2\} \\
& \overbrace{i(rd(x2, x1))}
\end{aligned}
$$

Lemma 364: $op_r(mult(x1, i(x2)), x2, x2) = rd(x1, x2)$

$$
\begin{aligned}
& \overbrace{op_r(mult(x1, i(x2)), x2, x2)} \\
= & \quad \text{by Lemma 363 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\overbrace{i(rd(x2, x1))}} \\
= & \quad \text{by Lemma 10 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \overbrace{rd(x1, x2)}
\end{aligned}
$$

Lemma 365: $x1 = mult(op_r(x2, x1, x1), i(rd(x2, x1)))$

$$
\begin{aligned}
& \overbrace{x1} \\
= & \quad \text{by Lemma 3 RL with } \{x1 \leftarrow x1\} \\
& \overbrace{\overbrace{i(i(x1))}} \\
= & \quad \text{by Lemma 20 RL with } \{x2 \leftarrow i(x1), x1 \leftarrow op_r(x2, x1, x1)\} \\
& \overbrace{mult(op_r(x2, x1, x1), \overbrace{i(\underbrace{mult(op_r(x2, x1, x1), i(x1))})})} \\
= & \quad \text{by Lemma 352 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& mult(op_r(x2, x1, x1), \overbrace{i(\overbrace{rd(x2, x1)})})
\end{aligned}
$$

Lemma 366: $mult(op_r(x1, x2, x2), rd(x2, x1)) = x2$

$$
\begin{aligned}
& mult(op_r(x1, x2, x2), \underbrace{rd(x2, x1)}) \\
= & \quad \text{by Lemma 10 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{mult(op_r(x1, x2, x2), \overbrace{i(\overbrace{rd(x1, x2)})})} \\
= & \quad \text{by Lemma 365 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \overbrace{x2}
\end{aligned}
$$

Lemma 367: $mult(rd(x2, x1), x1) = op_t(op_r(x2, x1, x1), rd(x1, x2))$

$$
\begin{aligned}
& mult(\underbrace{rd(x2, x1)}, x1) \\
= & \quad \text{by Lemma 10 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{mult(\overbrace{i(\overbrace{rd(x1, x2)})}, x1)} \\
= & \quad \text{by Lemma 14 RL with } \{x2 \leftarrow rd(x1, x2), x1 \leftarrow x1\} \\
& \overbrace{op_t(\underbrace{rd(x1, rd(x1, x2))}, rd(x1, x2))} \\
= & \quad \text{by Lemma 353 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_t(\overbrace{op_r(x2, x1, x1)}, rd(x1, x2))
\end{aligned}
$$

Lemma 368: $op_r(op_t(x1, rd(x2, x1)), x2, x2) = x1$

$$
\begin{aligned}
& \overbrace{op_r(op_t(x1, rd(x2, x1)), x2, x2)} \\
= & \quad \text{by Axiom 14 RL with } \{x4 \leftarrow rd(x2, x1), x3 \leftarrow x2, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\overbrace{op_t(op_r(x1, x2, x2), rd(x2, x1))}} \\
= & \quad \text{by Lemma 367 RL with } \{x1 \leftarrow x2, x2 \leftarrow x1\} \\
& \overbrace{\overbrace{mult(\overbrace{rd(x1, x2)}, x2)}} \\
= & \quad \text{by Axiom 4 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{x1}
\end{aligned}
$$

Lemma 369: $rd(x1, op_r(x2, x1, x1)) = op_r(rd(x1, x2), x1, x1)$

$$
\begin{aligned}
& rd(x1, \overbrace{op_r(x2, x1, x1)}) \\
= & \quad \text{by Lemma 353 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{rd(x1, \overbrace{rd(x1, rd(x1, x2))})} \\
= & \quad \text{by Lemma 353 LR with } \{x2 \leftarrow rd(x1, x2), x1 \leftarrow x1\} \\
& \overbrace{op_r(rd(x1, x2), x1, x1)}
\end{aligned}
$$

Lemma 370: $op_r(rd(x2, x1), x1, x1) = rd(op_r(x2, x1, x1), x1)$

$$
\begin{aligned}
& op_r(\overbrace{rd(x2, x1)}, x1, x1) \\
= & \quad \text{by Lemma 10 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_r(\overbrace{i(\overbrace{rd(x1, x2)})}, x1, x1)} \\
= & \quad \text{by Lemma 354 RL with } \{x2 \leftarrow rd(x1, x2), x1 \leftarrow x1\} \\
& \overbrace{rd(\overbrace{rd(x1, \overbrace{rd(x1, x2)})}, x1)} \\
= & \quad \text{by Lemma 353 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& rd(\overbrace{op_r(x2, x1, x1)}, x1)
\end{aligned}
$$

Lemma 371: $op_r(i(mult(x1, x2)), x2, x2) = rd(i(x1), x2)$

$$
\begin{aligned}
& \overbrace{op_r(i(mult(x1, x2)), x2, x2)} \\
= & \quad \text{by Lemma 354 RL with } \{x2 \leftarrow mult(x1, x2), x1 \leftarrow x2\} \\
& \overbrace{rd(\overbrace{rd(x2, mult(x1, x2))}, x2)} \\
= & \quad \text{by Lemma 9 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& rd(\overbrace{i(x1)}, x2)
\end{aligned}
$$

Lemma 372: $rd(op_t(x1, rd(x2, x1)), x2) = mult(x1, i(x2))$

$$
\begin{aligned}
& \underbrace{rd(op_t(x1, rd(x2, x1)), x2)} \\
= \quad & \text{by Lemma 352 RL with } \{x2 \leftarrow x2, x1 \leftarrow op_t(x1, rd(x2, x1))\} \\
& \overbrace{mult(\underbrace{op_r(op_t(x1, rd(x2, x1)), x2, x2)}, i(x2))} \\
= \quad & \text{by Lemma 368 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& mult(\overbrace{x1}, i(x2))
\end{aligned}
$$

Lemma 373: $rd(op_t(x1, x2), mult(x2, x1)) = i(op_t(x2, x1))$

$$
\begin{aligned}
& rd(op_t(x1, \underbrace{x2}), mult(x2, x1)) \\
= \quad & \text{by Axiom 5 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \underbrace{rd(op_t(x1, \overbrace{rd(mult(x2, x1), x1)}), mult(x2, x1))} \\
= \quad & \text{by Lemma 372 LR with } \{x2 \leftarrow mult(x2, x1), x1 \leftarrow x1\} \\
& \underbrace{\overbrace{mult(x1, i(\underbrace{mult(x2, x1)}))}} \\
= \quad & \text{by Lemma 31 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{i(\overbrace{op_t(x2, x1)})}
\end{aligned}
$$

Lemma 374: $op_t(x1, op_t(x2, x1)) = op_t(x1, x2)$

$$
\begin{aligned}
& \underbrace{op_t(x1, op_t(x2, x1))} \\
= \quad & \text{by Lemma 133 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \overbrace{mult(\underbrace{i(op_t(x2, x1))}, mult(x2, x1))} \\
= \quad & \text{by Lemma 373 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{mult(\overbrace{rd(op_t(x1, x2), mult(x2, x1))}, mult(x2, x1))} \\
= \quad & \text{by Axiom 4 RL with } \{x2 \leftarrow mult(x2, x1), x1 \leftarrow op_t(x1, x2)\} \\
& \overbrace{op_t(x1, x2)}
\end{aligned}
$$

Lemma 375: $op_t(x1, mult(i(x1), x2)) = op_t(x1, rd(x2, x1))$

$$
\begin{aligned}
& op_t(x1, \underbrace{mult(i(x1), x2)}) \\
= \quad & \text{by Lemma 14 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \underbrace{op_t(x1, \overbrace{op_t(rd(x2, x1), x1)})} \\
= \quad & \text{by Lemma 374 LR with } \{x2 \leftarrow rd(x2, x1), x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, rd(x2, x1))}
\end{aligned}
$$

Lemma 376: $rd(mult(x1,x2),op_t(x2,x1)) = op_t(x1,x2)$

$$
\begin{aligned}
& rd(mult(x1,x2),\underbrace{op_t(x2,x1)}) \\
= \quad & \text{by Lemma 374 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& rd(\underbrace{mult(x1,x2)},\overbrace{op_t(x2,op_t(x1,x2))}) \\
= \quad & \text{by Lemma 13 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \underbrace{rd(\overbrace{mult(x2,op_t(x1,x2))},op_t(x2,op_t(x1,x2)))} \\
= \quad & \text{by Lemma 29 LR with } \{x2 \leftarrow op_t(x1,x2), x1 \leftarrow x2\} \\
& \overbrace{op_t(x1,x2)}
\end{aligned}
$$

Lemma 377: $mult(x2,x1) = mult(op_t(x2,x1),op_t(x1,x2))$

$$
\begin{aligned}
& \underbrace{mult(x2,x1)} \\
= \quad & \text{by Lemma 13 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{\overbrace{mult(x1,op_t(x2,x1))}} \\
= \quad & \text{by Lemma 13 RL with } \{x2 \leftarrow x1, x1 \leftarrow op_t(x2,x1)\} \\
& \overbrace{mult(op_t(x2,x1),\underbrace{op_t(x1,op_t(x2,x1))})} \\
= \quad & \text{by Lemma 374 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& mult(op_t(x2,x1),\overbrace{op_t(x1,x2)})
\end{aligned}
$$

Lemma 378: $op_t(x1,rd(mult(x1,x2),x1)) = op_t(x1,x2)$

$$
\begin{aligned}
& \underbrace{op_t(x1,rd(mult(x1,x2),x1))} \\
= \quad & \text{by Lemma 375 RL with } \{x2 \leftarrow mult(x1,x2), x1 \leftarrow x1\} \\
& \overbrace{op_t(x1,\underbrace{mult(i(x1),mult(x1,x2))})} \\
= \quad & \text{by Axiom 2 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& op_t(x1,\overbrace{x2}^{})
\end{aligned}
$$

Lemma 379: $op_t(x1,i(mult(x1,x2))) = op_t(x1,rd(i(x1),x2))$

$$
\begin{aligned}
& op_t(x1,\underbrace{i(mult(x1,x2))}) \\
= \quad & \text{by Lemma 33 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{op_t(x1,\overbrace{op_t(rd(i(x1),x2),x1)})} \\
= \quad & \text{by Lemma 374 LR with } \{x2 \leftarrow rd(i(x1),x2), x1 \leftarrow x1\} \\
& \overbrace{op_t(x1,rd(i(x1),x2))}
\end{aligned}
$$

Lemma 380: $rd(x1, op_t(x2, rd(x1, x2))) = mult(i(x2), x1)$

$$
\begin{aligned}
& rd(x1, \underbrace{op_t(x2, rd(x1, x2))}) \\
= & \quad \text{by Lemma 375 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \underbrace{rd(x1, \overbrace{op_t(x2, mult(i(x2), x1))})} \\
= & \quad \text{by Lemma 77 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{mult(i(x2), x1)}
\end{aligned}
$$

Lemma 381: $op_r(op_t(x1, i(x2)), x2, x2) = mult(x2, rd(x1, x2))$

$$
\begin{aligned}
& \underbrace{op_r(op_t(x1, i(x2)), x2, x2)} \\
= & \quad \text{by Axiom 14 RL with } \{x4 \leftarrow i(x2), x3 \leftarrow x2, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_t(op_r(x1, x2, x2), i(x2))}} \\
= & \quad \text{by Axiom 10 RL with } \{x2 \leftarrow op_r(x1, x2, x2), x1 \leftarrow i(x2)\} \\
& \overbrace{mult(i(i(x2)), \underbrace{mult(op_r(x1, x2, x2), i(x2))})} \\
= & \quad \text{by Lemma 352 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& mult(\underbrace{i(i(x2))}, \overbrace{rd(x1, x2)}) \\
= & \quad \text{by Lemma 3 LR with } \{x1 \leftarrow x2\} \\
& mult(\overbrace{x2}, rd(x1, x2))
\end{aligned}
$$

Lemma 382: $mult(x1, op_r(x2, x1, x1)) = op_r(mult(i(i(x1)), x2), x1, x1)$

$$
\begin{aligned}
& mult(x1, \underbrace{op_r(x2, x1, x1)}) \\
= & \quad \text{by Lemma 358 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \underbrace{mult(x1, \overbrace{rd(rd(x2, i(x1)), x1)})} \\
= & \quad \text{by Lemma 381 RL with } \{x2 \leftarrow x1, x1 \leftarrow rd(x2, i(x1))\} \\
& \overbrace{op_r(\underbrace{op_t(rd(x2, i(x1)), i(x1))}, x1, x1)} \\
= & \quad \text{by Lemma 14 LR with } \{x2 \leftarrow i(x1), x1 \leftarrow x2\} \\
& op_r(\overbrace{mult(i(i(x1)), x2)}, x1, x1)
\end{aligned}
$$

Lemma 383: $mult(x1, op_r(x2, x1, x1)) = op_r(mult(x1, x2), x1, x1)$

$$
\begin{aligned}
& \underbrace{mult(x1, op_r(x2, x1, x1))} \\
= & \quad \text{by Lemma 382 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_r(mult(\underbrace{i(i(x1))}, x2), x1, x1)} \\
= & \quad \text{by Lemma 3 LR with } \{x1 \leftarrow x1\} \\
& op_r(mult(\overbrace{x1}, x2), x1, x1)
\end{aligned}
$$

Lemma 384: $mult(i(op_t(x1,x2)),mult(x1,x2)) = op_t(x2,x1)$

$\overbrace{mult(i(op_t(x1,x2)),mult(x1,x2))}$

$=$ by Lemma 133 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_t(x2,op_t(x1,x2))}$

$=$ by Lemma 374 LR with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$\overbrace{op_t(x2,x1)}$

Lemma 385: $op_r(op_t(mult(x1,x2),x1),x1,x1) = mult(x2,x1)$

$\overbrace{op_r(op_t(mult(x1,x2),x1),x1,x1)}$

$=$ by Axiom 14 RL with $\{x4 \leftarrow x1, x3 \leftarrow x1, x2 \leftarrow x1, x1 \leftarrow mult(x1,x2)\}$

$\overbrace{op_t(\underbrace{op_r(mult(x1,x2),x1,x1)},x1)}$

$=$ by Lemma 383 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{op_t(\overbrace{mult(x1,op_r(x2,x1,x1))},x1)}$

$=$ by Axiom 5 LR with $\{x2 \leftarrow x1, x1 \leftarrow op_t(mult(x1,op_r(x2,x1,x1)),x1)\}$

$\overbrace{rd(\underbrace{mult(op_t(mult(x1,op_r(x2,x1,x1)),x1),x1)},x1)}$

$=$ by Lemma 350 LR with $\{x2 \leftarrow op_r(x2,x1,x1), x1 \leftarrow x1\}$

$rd(\overbrace{\underbrace{mult(op_r(x2,x1,x1),mult(x1,x1))},x1)}$

$=$ by Lemma 91 LR with $\{x3 \leftarrow x1, x2 \leftarrow x1, x1 \leftarrow x2\}$

$\underbrace{rd(\overbrace{mult(mult(x2,x1),x1)},x1)}$

$=$ by Axiom 5 RL with $\{x2 \leftarrow x1, x1 \leftarrow mult(x2,x1)\}$

$\overbrace{mult(x2,x1)}$

Lemma 386: $rd(x1,op_t(i(x2),rd(x2,x1))) = mult(x2,x1)$

$rd(x1,\underbrace{op_t(i(x2),rd(x2,x1))})$

$=$ by Lemma 47 RL with $\{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x2\}$

$\underbrace{rd(x1,\overbrace{i(op_t(x2,rd(x1,x2)))})}$

$=$ by Lemma 357 RL with $\{x2 \leftarrow x1, x1 \leftarrow op_t(x2,rd(x1,x2))\}$

$\overbrace{mult(\underbrace{op_r(op_t(x2,rd(x1,x2)),x1,x1)},x1)}$

$=$ by Lemma 368 LR with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$mult(\overbrace{x2},x1)$

Lemma 387: $mult(rd(op_t(x1,x2),x1),op_t(x2,x1)) = x2$

$$
\begin{aligned}
& mult(\underbrace{rd(op_t(x1,x2),x1)}, op_t(x2,x1)) \\
= & \quad \text{by Lemma 97 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& mult(\overbrace{mult(\underbrace{op_t(x1,x2)}, i(x1))}, op_t(x2,x1)) \\
= & \quad \text{by Lemma 376 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& mult(\underbrace{mult(\overbrace{rd(mult(x1,x2),op_t(x2,x1))}, i(x1))}, op_t(x2,x1)) \\
= & \quad \text{by Lemma 44 RL with } \{x3 \leftarrow x1, x2 \leftarrow mult(x1,x2), x1 \leftarrow op_t(x2,x1)\} \\
& mult(\overbrace{i(\underbrace{mult(rd(op_t(x2,x1),mult(x1,x2)),x1)})}, \underbrace{op_t(x2,x1)}) \\
= & \quad \text{by Axiom 4 LR with } \{x2 \leftarrow mult(x1,x2), x1 \leftarrow op_t(x2,x1)\} \\
& \underbrace{mult(i(mult(rd(op_t(x2,x1),mult(x1,x2)),x1)), \overbrace{mult(rd(op_t(x2,x1),mult(x1,x2)),mult(x1,x2))})} \\
= & \quad \text{by Axiom 12 LR with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow rd(op_t(x2,x1),mult(x1,x2))\} \\
& \overbrace{op_l(x2,x1,\underbrace{rd(op_t(x2,x1),mult(x1,x2))})} \\
= & \quad \text{by Lemma 373 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \underbrace{op_l(x2,x1,\overbrace{i(op_t(x1,x2))})} \\
= & \quad \text{by Lemma 289 LR with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \overbrace{x2}
\end{aligned}
$$

Lemma 388: $mult(rd(x1,op_t(x1,x2)),x2) = op_t(x2,x1)$

$$
\begin{aligned}
& mult(\underbrace{rd(x1,op_t(x1,x2))}, x2) \\
= & \quad \text{by Lemma 10 RL with } \{x2 \leftarrow x1, x1 \leftarrow op_t(x1,x2)\} \\
& mult(\overbrace{i(rd(op_t(x1,x2),x1))}, \underbrace{x2}) \\
= & \quad \text{by Lemma 387 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{mult(i(rd(op_t(x1,x2),x1)), \overbrace{mult(rd(op_t(x1,x2),x1),op_t(x2,x1))})} \\
= & \quad \text{by Axiom 2 RL with } \{x2 \leftarrow rd(op_t(x1,x2),x1), x1 \leftarrow op_t(x2,x1)\} \\
& \overbrace{op_t(x2,x1)}
\end{aligned}
$$

Lemma 389: $rd(x1,rd(op_t(x2,x1),x2)) = op_r(op_t(x1,x2),x1,x1)$

$$
\begin{aligned}
& rd(x1,\underbrace{rd(op_t(x2,x1),x2)}) \\
= & \quad \text{by Lemma 10 RL with } \{x2 \leftarrow op_t(x2,x1), x1 \leftarrow x2\} \\
& \underbrace{rd(x1,\overbrace{i(rd(x2,op_t(x2,x1)))})} \\
= & \quad \text{by Lemma 359 RL with } \{x2 \leftarrow x1, x1 \leftarrow rd(x2,op_t(x2,x1))\} \\
& \overbrace{op_r(\underbrace{mult(rd(x2,op_t(x2,x1)),x1)}, x1, x1)} \\
= & \quad \text{by Lemma 388 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& op_r(\overbrace{op_t(x1,x2)}, x1, x1)
\end{aligned}
$$

Lemma 390: $rd(x1, rd(op_t(x2, x1), x2)) = op_t(x1, x2)$

$$
\begin{aligned}
& \overbrace{rd(x1, rd(op_t(x2, x1), x2))} \\
= \quad & \text{by Lemma 389 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_r(op_t(x1, x2), x1, x1)} \\
= \quad & \text{by Lemma 28 LR with } \{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, x2)}
\end{aligned}
$$

Lemma 391: $mult(x1, rd(x2, op_t(x2, x1))) = op_t(op_t(x1, x2), i(x1))$

$$
\begin{aligned}
& mult(x1, \overbrace{rd(x2, op_t(x2, x1))}) \\
= \quad & \text{by Lemma 10 RL with } \{x2 \leftarrow x2, x1 \leftarrow op_t(x2, x1)\} \\
& \overbrace{mult(x1, \overbrace{i(rd(op_t(x2, x1), x2))})} \\
= \quad & \text{by Lemma 34 RL with } \{x2 \leftarrow rd(op_t(x2, x1), x2), x1 \leftarrow x1\} \\
& \overbrace{op_t(\overbrace{rd(x1, rd(op_t(x2, x1), x2))}, i(x1))} \\
= \quad & \text{by Lemma 390 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_t(\overbrace{op_t(x1, x2)}, i(x1))
\end{aligned}
$$

Lemma 392: $op_l(x1, rd(x2, op_t(x2, x3)), x3) = x1$

$$
\begin{aligned}
& \overbrace{op_l(x1, rd(x2, op_t(x2, x3)), x3)} \\
= \quad & \text{by Lemma 285 RL with } \{x3 \leftarrow x3, x2 \leftarrow rd(x2, op_t(x2, x3)), x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, \underbrace{mult(rd(x2, op_t(x2, x3)), x3)}, x3)} \\
= \quad & \text{by Lemma 388 LR with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& \overbrace{op_l(x1, \overbrace{op_t(x3, x2)}, x3)} \\
= \quad & \text{by Lemma 163 LR with } \{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{x1}
\end{aligned}
$$

Lemma 393: $rd(mult(x1, x2), x1) = mult(rd(op_t(x1, x2), x1), mult(i(x1), mult(x1, x2)))$

$$
\begin{aligned}
& \overbrace{rd(mult(x1, x2), x1)} \\
= \quad & \text{by Lemma 387 RL with } \{x2 \leftarrow rd(mult(x1, x2), x1), x1 \leftarrow x1\} \\
& \overbrace{mult(rd(\underbrace{op_t(x1, rd(mult(x1, x2), x1))}, x1), op_t(rd(mult(x1, x2), x1), x1))} \\
= \quad & \text{by Lemma 378 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& mult(rd(\overbrace{op_t(x1, x2)}, x1), \underbrace{op_t(rd(mult(x1, x2), x1), x1)}) \\
= \quad & \text{by Lemma 14 LR with } \{x2 \leftarrow x1, x1 \leftarrow mult(x1, x2)\} \\
& mult(rd(op_t(x1, x2), x1), \overbrace{mult(i(x1), mult(x1, x2))})
\end{aligned}
$$

Lemma 394: $mult(rd(op_t(x1,x2),x1),x2) = rd(mult(x1,x2),x1)$

$$
\begin{aligned}
& mult(rd(op_t(x1,x2),x1),\underbrace{x2}) \\
= & \quad \text{by Axiom 2 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \underbrace{mult(rd(op_t(x1,x2),x1),\overbrace{mult(i(x1),mult(x1,x2))})} \\
= & \quad \text{by Lemma 393 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{rd(mult(x1,x2),x1)}
\end{aligned}
$$

Lemma 395: $rd(op_t(x2,x1),x2) = rd(x1,op_t(x1,x2))$

$$
\begin{aligned}
& \underbrace{rd(op_t(x2,x1),x2)} \\
= & \quad \text{by Lemma 96 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \overbrace{mult(i(x2),\underbrace{op_t(x2,x1)})} \\
= & \quad \text{by Lemma 388 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{mult(i(x2),\overbrace{mult(rd(x1,op_t(x1,x2)),x2)})} \\
= & \quad \text{by Axiom 10 LR with } \{x2 \leftarrow rd(x1,op_t(x1,x2)), x1 \leftarrow x2\} \\
& \underbrace{\overbrace{op_t(rd(x1,\underbrace{op_t(x1,x2)}),x2)}} \\
= & \quad \text{by Lemma 376 RL with } \{x2 \leftarrow x2, x1 \leftarrow rd(x1,op_t(x1,x2))\} \\
& \overbrace{rd(mult(rd(x1,op_t(x1,x2)),x2),\underbrace{op_t(x2,rd(x1,op_t(x1,x2)))})} \\
= & \quad \text{by Axiom 10 RL with } \{x2 \leftarrow x2, x1 \leftarrow rd(x1,op_t(x1,x2))\} \\
& rd(mult(rd(x1,op_t(x1,x2)),x2),\overbrace{mult(i(rd(x1,op_t(x1,x2))),\underbrace{mult(x2,rd(x1,op_t(x1,x2)))})}) \\
= & \quad \text{by Lemma 391 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& rd(mult(rd(x1,op_t(x1,x2)),x2),mult(i(rd(x1,op_t(x1,x2))),\overbrace{op_t(op_t(x2,x1),i(x2))})) \\
= & \quad \text{by Axiom 13 RL with } \{x3 \leftarrow x1, x2 \leftarrow i(x2), x1 \leftarrow x2\} \\
& rd(mult(rd(x1,op_t(x1,x2)),x2),mult(i(rd(x1,op_t(x1,x2))),\overbrace{op_t(\underbrace{op_t(x2,i(x2))},x1)})) \\
= & \quad \text{by Lemma 8 RL with } \{x1 \leftarrow x2\} \\
& rd(mult(rd(x1,op_t(x1,x2)),x2),mult(\underbrace{i(rd(x1,op_t(x1,x2)))},op_t(\overbrace{x2},x1))) \\
= & \quad \text{by Lemma 10 LR with } \{x2 \leftarrow op_t(x1,x2), x1 \leftarrow x1\} \\
& rd(mult(rd(x1,op_t(x1,x2)),x2),\underbrace{mult(\overbrace{rd(op_t(x1,x2),x1)},op_t(x2,x1))}) \\
= & \quad \text{by Lemma 387 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{rd(mult(rd(x1,op_t(x1,x2)),x2),\overbrace{x2})} \\
= & \quad \text{by Axiom 5 RL with } \{x2 \leftarrow x2, x1 \leftarrow rd(x1,op_t(x1,x2))\} \\
& \overbrace{rd(x1,op_t(x1,x2))}
\end{aligned}
$$

Lemma 396: $rd(x1, op_t(x1, x2)) = rd(x2, rd(mult(x1, x2), x1))$

$$
\begin{aligned}
& \underbrace{rd(x1, op_t(x1, x2))} \\
= \quad & \text{by Lemma 10 RL with } \{x2 \leftarrow x1, x1 \leftarrow op_t(x1, x2)\} \\
& \overbrace{i(rd(op_t(x1, x2), x1))} \\
= \quad & \text{by Lemma 9 RL with } \{x2 \leftarrow rd(op_t(x1, x2), x1), x1 \leftarrow x2\} \\
& \overbrace{rd(x2, \underbrace{mult(rd(op_t(x1, x2), x1), x2)})} \\
= \quad & \text{by Lemma 394 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& rd(x2, \overbrace{rd(mult(x1, x2), x1)})
\end{aligned}
$$

Lemma 397: $rd(x1, rd(x2, op_t(x2, x1))) = op_r(rd(mult(x2, x1), x2), x1, x1)$

$$
\begin{aligned}
& rd(x1, \underbrace{rd(x2, op_t(x2, x1))}) \\
= \quad & \text{by Lemma 10 RL with } \{x2 \leftarrow x2, x1 \leftarrow op_t(x2, x1)\} \\
& \underbrace{rd(x1, \overbrace{i(rd(op_t(x2, x1), x2))})} \\
= \quad & \text{by Lemma 359 RL with } \{x2 \leftarrow x1, x1 \leftarrow rd(op_t(x2, x1), x2)\} \\
& \overbrace{op_r(\underbrace{mult(rd(op_t(x2, x1), x2), x1)}, x1, x1)} \\
= \quad & \text{by Lemma 394 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& op_r(\overbrace{rd(mult(x2, x1), x2)}, x1, x1)
\end{aligned}
$$

Lemma 398: $rd(x1, rd(x2, op_t(x2, x1))) = rd(mult(x2, x1), x2)$

$$
\begin{aligned}
& \underbrace{rd(x1, rd(x2, op_t(x2, x1)))} \\
= \quad & \text{by Lemma 397 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_r(rd(mult(x2, x1), x2), \underbrace{x1}, x1)} \\
= \quad & \text{by Axiom 2 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_r(rd(mult(x2, x1), x2), \overbrace{\underbrace{mult(i(x2), mult(x2, x1))}, x1}) \\
= \quad & \text{by Lemma 14 RL with } \{x2 \leftarrow x2, x1 \leftarrow mult(x2, x1)\} \\
& \underbrace{op_r(rd(mult(x2, x1), x2), \overbrace{op_t(rd(mult(x2, x1), x2), x2)}, x1)} \\
= \quad & \text{by Lemma 157 LR with } \{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow rd(mult(x2, x1), x2)\} \\
& \overbrace{rd(mult(x2, x1), x2)}
\end{aligned}
$$

Lemma 399: $mult(i(x1), mult(x1, x2)) = op_t(rd(mult(x1, x2), x1), op_t(x1, x2))$

$$
\begin{aligned}
& \underbrace{mult(i(x1), mult(x1, x2))} \\
= \quad & \text{by Lemma 14 RL with } \{x2 \leftarrow x1, x1 \leftarrow mult(x1, x2)\} \\
& \overbrace{\underbrace{op_t(rd(mult(x1, x2), x1), x1)}} \\
= \quad & \text{by Lemma 374 RL with } \{x2 \leftarrow x1, x1 \leftarrow rd(mult(x1, x2), x1)\} \\
& \overbrace{op_t(rd(mult(x1, x2), x1), \underbrace{op_t(x1, rd(mult(x1, x2), x1))})} \\
= \quad & \text{by Lemma 378 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_t(rd(mult(x1, x2), x1), \overbrace{op_t(x1, x2)})
\end{aligned}
$$

Lemma 400: $op_t(rd(mult(x1, x2), x1), op_t(x1, x2)) = x2$

$$
\begin{aligned}
& \underbrace{op_t(rd(mult(x1, x2), x1), op_t(x1, x2))} \\
= & \quad \text{by Lemma 399 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{mult(i(x1), mult(x1, x2))}} \\
= & \quad \text{by Axiom 2 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \overbrace{x2}
\end{aligned}
$$

Lemma 401: $op_t(rd(x1, rd(op_t(x1, x2), x1)), x2) = x1$

$$
\begin{aligned}
& op_t(rd(x1, \underbrace{rd(op_t(x1, x2), x1)}), x2) \\
= & \quad \text{by Lemma 96 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_t(\underbrace{rd(x1, \overbrace{mult(i(x1), op_t(x1, x2))})}, x2) \\
= & \quad \text{by Lemma 60 RL with } \{x2 \leftarrow op_t(x1, x2), x1 \leftarrow x1\} \\
& \underbrace{op_t(\overbrace{mult(x1, mult(i(op_t(x1, x2)), x1))}, x2)} \\
= & \quad \text{by Axiom 10 RL with } \{x2 \leftarrow mult(x1, mult(i(op_t(x1, x2)), x1)), x1 \leftarrow x2\} \\
& \overbrace{mult(i(x2), \underbrace{mult(mult(x1, mult(i(op_t(x1, x2)), x1)), x2)})} \\
= & \quad \text{by Axiom 6 RL with } \{x3 \leftarrow x2, x2 \leftarrow i(op_t(x1, x2)), x1 \leftarrow x1\} \\
& mult(i(x2), \overbrace{mult(x1, \underbrace{mult(i(op_t(x1, x2)), mult(x1, x2))})}) \\
= & \quad \text{by Lemma 384 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& mult(i(x2), \underbrace{mult(x1, \overbrace{op_t(x2, x1)})}) \\
= & \quad \text{by Lemma 13 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{mult(i(x2), \overbrace{mult(x2, x1)})} \\
= & \quad \text{by Axiom 2 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{x1}
\end{aligned}
$$

Lemma 402: $mult(x1, op_r(x2, x1, i(x1))) = rd(x1, op_t(i(x2), x1))$

$$
\begin{aligned}
& \underbrace{mult(x1, op_r(x2, x1, i(x1)))} \\
= & \quad \text{by Lemma 110 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_t(mult(x2, x1), i(x1))}} \\
= & \quad \text{by Lemma 326 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \overbrace{rd(x1, op_t(i(x2), x1))}
\end{aligned}
$$

Lemma 403: $mult(x1, rd(rd(x2, x1), x2)) = rd(x1, op_r(x1, x2, x2))$

$\underbrace{mult(x1, rd(rd(x2, x1), x2))}$

$=$ by Lemma 153 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{\underbrace{rd(i(rd(x2, rd(x2, x1))), i(x1))}}$

$=$ by Lemma 23 LR with $\{x2 \leftarrow x1, x1 \leftarrow rd(x2, rd(x2, x1))\}$

$\overbrace{rd(x1, \underbrace{rd(x2, rd(x2, x1))})}$

$=$ by Lemma 353 LR with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$rd(x1, \overbrace{op_r(x1, x2, x2)})$

Lemma 404: $rd(x1, op_t(mult(x1, x2), x1)) = op_r(i(x2), x1, i(x1))$

$rd(x1, op_t(mult(x1, x2), \underbrace{x1}))$

$=$ by Lemma 3 RL with $\{x1 \leftarrow x1\}$

$\underbrace{rd(x1, op_t(mult(x1, x2), \overbrace{i(i(x1))}))}$

$=$ by Lemma 67 RL with $\{x3 \leftarrow x1, x2 \leftarrow i(x1), x1 \leftarrow mult(x1, x2)\}$

$\overbrace{rd(op_t(\underbrace{i(mult(x1, x2))}, i(x1)), i(x1))}$

$=$ by Axiom 11 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{rd(op_t(\overbrace{mult(i(x1), i(x2))}, i(x1)), i(x1))}$

$=$ by Lemma 317 LR with $\{x2 \leftarrow i(x2), x1 \leftarrow i(x1)\}$

$\overbrace{op_r(i(x2), i(x1), \underbrace{i(i(x1))})}$

$=$ by Lemma 3 LR with $\{x1 \leftarrow x1\}$

$\underbrace{op_r(i(x2), i(x1), \overbrace{x1})}$

$=$ by Lemma 321 LR with $\{x2 \leftarrow x1, x1 \leftarrow i(x2)\}$

$\overbrace{op_r(i(x2), x1, i(x1))}$

Lemma 405: $op_r(mult(x1, x2), x1, i(x1)) = rd(x1, op_t(i(x2), x1))$

$\underbrace{op_r(mult(x1, x2), x1, i(x1))}$

$=$ by Lemma 109 RL with $\{x2 \leftarrow mult(x1, x2), x1 \leftarrow x1\}$

$\overbrace{mult(x1, \underbrace{rd(op_t(mult(x1, x2), x1), x1)})}$

$=$ by Lemma 317 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{mult(x1, \overbrace{op_r(x2, x1, i(x1))})}$

$=$ by Lemma 110 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_t(mult(x2, x1), i(x1))}}$

$=$ by Lemma 326 LR with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$\overbrace{rd(x1, op_t(i(x2), x1))}$

Lemma 406: $op_t(mult(x1, rd(x1, x2)), x1) = mult(mult(i(x2), x1), x1)$

$$
\begin{aligned}
& \underbrace{op_t(mult(x1, rd(x1, x2)), x1)} \\
= & \quad \text{by Lemma 318 RL with } \{x2 \leftarrow x1, x1 \leftarrow rd(x1, x2)\} \\
& \overbrace{mult(\underbrace{op_r(rd(x1, x2), x1, i(x1))}, x1)} \\
= & \quad \text{by Lemma 330 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& mult(\overbrace{mult(i(x2), x1)}, x1)
\end{aligned}
$$

Lemma 407: $mult(mult(x1, i(x2)), x1) = rd(x1, rd(op_t(x2, x1), x1))$

$$
\begin{aligned}
& mult(\underbrace{mult(x1, i(x2))}, x1) \\
= & \quad \text{by Lemma 230 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{mult(\overbrace{rd(x1, op_l(x2, i(x2), x1))}, x1)} \\
= & \quad \text{by Lemma 334 RL with } \{x2 \leftarrow op_l(x2, i(x2), x1), x1 \leftarrow x1\} \\
& \overbrace{rd(x1, \underbrace{mult(op_l(x2, i(x2), x1), i(x1))})} \\
= & \quad \text{by Lemma 316 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& rd(x1, \overbrace{rd(op_t(x2, x1), x1)})
\end{aligned}
$$

Lemma 408: $op_t(rd(i(x1), x2), i(x2)) = i(mult(op_t(x1, x2), x2))$

$$
\begin{aligned}
& \underbrace{op_t(rd(i(x1), x2), i(x2))} \\
= & \quad \text{by Lemma 46 RL with } \{x3 \leftarrow x2, x2 \leftarrow i(x1), x1 \leftarrow x2\} \\
& \overbrace{i(\underbrace{op_t(rd(x2, i(x1)), x2)})} \\
= & \quad \text{by Lemma 341 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& i(\overbrace{mult(op_t(x1, x2), x2)})
\end{aligned}
$$

Lemma 409: $i(op_t(mult(x1, x2), x1)) = rd(x1, op_t(mult(x1, mult(x2, x1)), x1))$

$$
\begin{aligned}
& \underbrace{i(op_t(mult(x1, x2), x1))} \\
= & \quad \text{by Lemma 15 RL with } \{x2 \leftarrow x1, x1 \leftarrow mult(x1, x2)\} \\
& \overbrace{\underbrace{op_t(i(mult(x1, x2)), i(x1))}} \\
= & \quad \text{by Lemma 344 RL with } \{x2 \leftarrow i(mult(x1, x2)), x1 \leftarrow x1\} \\
& \overbrace{rd(x1, op_t(\underbrace{rd(x1, i(mult(x1, x2)))}, x1))} \\
= & \quad \text{by Lemma 58 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& rd(x1, op_t(\overbrace{mult(x1, mult(x2, x1))}, x1))
\end{aligned}
$$

Lemma 410: $rd(x1, mult(x2, mult(x1, x1))) = i(op_t(mult(x1, x2), x1))$

$$
\begin{aligned}
& rd(x1, \underbrace{mult(x2, mult(x1, x1))}) \\
= & \quad \text{by Lemma 200 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{rd(x1, \overbrace{op_t(mult(x1, mult(x2, x1)), x1)})} \\
= & \quad \text{by Lemma 409 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{i(op_t(mult(x1, x2), x1))}
\end{aligned}
$$

Lemma 411: $op_t(i(op_r(x1, x2, x2)), x2) = mult(i(x2), i(rd(x1, x2)))$

$$
\begin{aligned}
& \underbrace{op_t(i(op_r(x1, x2, x2)), x2)} \\
= & \quad \text{by Lemma 19 RL with } \{x2 \leftarrow x2, x1 \leftarrow op_r(x1, x2, x2)\} \\
& \underbrace{\overbrace{i(op_t(op_r(x1, x2, x2), i(x2)))}} \\
= & \quad \text{by Lemma 31 RL with } \{x2 \leftarrow op_r(x1, x2, x2), x1 \leftarrow i(x2)\} \\
& \overbrace{mult(i(x2), i(\underbrace{mult(op_r(x1, x2, x2), i(x2))}))} \\
= & \quad \text{by Lemma 352 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& mult(i(x2), i(\overbrace{rd(x1, x2)}))
\end{aligned}
$$

Lemma 412: $op_r(mult(x1, x2), x1, x1) = rd(mult(x1, mult(x2, x1)), x1)$

$$
\begin{aligned}
& op_r(\underbrace{mult(x1, x2)}, x1, x1) \\
= & \quad \text{by Lemma 3 RL with } \{x1 \leftarrow mult(x1, x2)\} \\
& \underbrace{op_r(\overbrace{i(i(mult(x1, x2)))}, x1, x1)} \\
= & \quad \text{by Lemma 354 RL with } \{x2 \leftarrow i(mult(x1, x2)), x1 \leftarrow x1\} \\
& \overbrace{rd(\underbrace{rd(x1, i(mult(x1, x2)))}, x1)} \\
= & \quad \text{by Lemma 58 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& rd(\overbrace{mult(x1, mult(x2, x1))}, x1)
\end{aligned}
$$

Lemma 413: $op_r(i(op_t(x1, x2)), x2, x2) = mult(x2, rd(i(x1), x2))$

$$
\begin{aligned}
& \underbrace{op_r(i(op_t(x1, x2)), x2, x2)} \\
= & \quad \text{by Lemma 356 RL with } \{x2 \leftarrow x2, x1 \leftarrow op_t(x1, x2)\} \\
& \overbrace{i(\underbrace{op_r(op_t(x1, x2), x2, x2)})} \\
= & \quad \text{by Axiom 14 RL with } \{x4 \leftarrow x2, x3 \leftarrow x2, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{i(\overbrace{op_t(op_r(x1, x2, x2), x2)})} \\
= & \quad \text{by Lemma 31 RL with } \{x2 \leftarrow op_r(x1, x2, x2), x1 \leftarrow x2\} \\
& \overbrace{mult(x2, i(\underbrace{mult(op_r(x1, x2, x2), x2)}))} \\
= & \quad \text{by Lemma 357 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& mult(x2, \underbrace{i(\overbrace{rd(x2, i(x1))})}) \\
= & \quad \text{by Lemma 10 LR with } \{x2 \leftarrow i(x1), x1 \leftarrow x2\} \\
& mult(x2, \overbrace{rd(i(x1), x2)})
\end{aligned}
$$

Lemma 414: $op_t(i(x1), mult(x1, i(x2))) = op_t(i(x1), rd(x1, x2))$

$$
\begin{aligned}
& op_t(i(x1), \underbrace{mult(x1, i(x2))}) \\
= & \quad \text{by Lemma 34 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{op_t(i(x1), \overbrace{op_t(rd(x1, x2), i(x1))})} \\
= & \quad \text{by Lemma 374 LR with } \{x2 \leftarrow rd(x1, x2), x1 \leftarrow i(x1)\} \\
& \overbrace{op_t(i(x1), rd(x1, x2))}
\end{aligned}
$$

Lemma 415: $op_t(i(x1), rd(x2, i(x1))) = op_t(i(x1), mult(x1, x2))$

$$
\begin{aligned}
& \underbrace{op_t(i(x1), rd(x2, i(x1)))} \\
= & \quad \text{by Lemma 375 RL with } \{x2 \leftarrow x2, x1 \leftarrow i(x1)\} \\
& \overbrace{op_t(i(x1), mult(\underbrace{i(i(x1))}, x2))} \\
= & \quad \text{by Lemma 3 LR with } \{x1 \leftarrow x1\} \\
& op_t(i(x1), mult(\overbrace{x1}, x2))
\end{aligned}
$$

Lemma 416: $mult(x1, rd(rd(x2, x1), x1)) = op_r(mult(op_t(x2, i(x1)), i(x1)), x1, x1)$

$$
\begin{aligned}
& mult(x1, rd(rd(x2, \underbrace{x1}), x1)) \\
= & \quad \text{by Lemma 3 RL with } \{x1 \leftarrow x1\} \\
& \underbrace{mult(x1, rd(rd(x2, \overbrace{i(i(x1))}), x1))} \\
= & \quad \text{by Lemma 381 RL with } \{x2 \leftarrow x1, x1 \leftarrow rd(x2, i(i(x1)))\} \\
& \overbrace{op_r(\underbrace{op_t(rd(x2, i(i(x1))), i(x1))}, x1, x1)} \\
= & \quad \text{by Lemma 337 LR with } \{x2 \leftarrow i(x1), x1 \leftarrow x2\} \\
& op_r(\overbrace{mult(op_t(x2, i(x1)), i(x1))}, x1, x1)
\end{aligned}
$$

Lemma 417: $mult(x1, rd(rd(x2, x1), x1)) = rd(op_t(x2, i(x1)), x1)$

$$
\begin{aligned}
& \underbrace{mult(x1, rd(rd(x2, x1), x1))} \\
= & \quad \text{by Lemma 416 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{\overbrace{op_r(mult(op_t(x2, i(x1)), i(x1)), x1, x1)}} \\
= & \quad \text{by Lemma 364 LR with } \{x2 \leftarrow x1, x1 \leftarrow op_t(x2, i(x1))\} \\
& \overbrace{rd(op_t(x2, i(x1)), x1)}
\end{aligned}
$$

Lemma 418: $op_l(i(mult(x1,x2)),i(x2),x1) = i(mult(x2,x1))$

$$
\begin{aligned}
& op_l(\underbrace{i(mult(x1,x2))}, i(x2), x1) \\
= & \quad \text{by Axiom 11 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_l(\overbrace{mult(\underbrace{i(x1)}, i(x2))}, i(x2), x1) \\
= & \quad \text{by Lemma 21 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& op_l(\underbrace{mult(\overbrace{mult(rd(i(x2),x1),x2)}, i(x2))}, i(x2), x1) \\
= & \quad \text{by Lemma 92 LR with } \{x2 \leftarrow x2, x1 \leftarrow rd(i(x2),x1)\} \\
& \underbrace{op_l(\overbrace{op_r(rd(i(x2),x1),x2,i(x2))}, i(x2), x1)} \\
= & \quad \text{by Axiom 18 RL with } \{x5 \leftarrow i(x2), x4 \leftarrow x2, x3 \leftarrow x1, x2 \leftarrow i(x2), x1 \leftarrow rd(i(x2),x1)\} \\
& \underbrace{\overbrace{op_r(op_l(rd(i(x2),x1),i(x2),x1),x2,i(x2))}} \\
= & \quad \text{by Lemma 323 RL with } \{x2 \leftarrow x2, x1 \leftarrow op_l(rd(i(x2),x1),i(x2),x1)\} \\
& \overbrace{mult(\underbrace{mult(op_l(rd(i(x2),x1),i(x2),x1),i(x2))}, x2)} \\
= & \quad \text{by Lemma 284 LR with } \{x2 \leftarrow x1, x1 \leftarrow i(x2)\} \\
& \underbrace{mult(\overbrace{rd(i(x2),mult(i(i(x2)),x1))}, x2)} \\
= & \quad \text{by Lemma 21 LR with } \{x2 \leftarrow mult(i(i(x2)),x1), x1 \leftarrow x2\} \\
& \underbrace{\overbrace{i(mult(i(i(x2)),x1))}} \\
= & \quad \text{by Lemma 16 LR with } \{x2 \leftarrow x1, x1 \leftarrow i(x2)\} \\
& \underbrace{\overbrace{mult(i(x2),i(x1))}} \\
= & \quad \text{by Axiom 11 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \overbrace{i(mult(x2,x1))}
\end{aligned}
$$

Lemma 419: $mult(mult(x1,x2),i(x2)) = op_l(i(i(op_t(x1,x2))),i(i(mult(x1,x2))),x2)$

$$
\begin{aligned}
& \underbrace{mult(mult(x1,x2),i(x2))} \\
= & \quad \text{by Lemma 16 RL with } \{x2 \leftarrow x2, x1 \leftarrow mult(x1,x2)\} \\
& \underbrace{\overbrace{i(mult(i(mult(x1,x2)),x2))}} \\
= & \quad \text{by Lemma 418 RL with } \{x2 \leftarrow i(mult(x1,x2)), x1 \leftarrow x2\} \\
& \overbrace{op_l(i(\underbrace{mult(x2,i(mult(x1,x2)))}), i(i(mult(x1,x2))), x2)} \\
= & \quad \text{by Lemma 31 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& op_l(i(\overbrace{i(op_t(x1,x2))}), i(i(mult(x1,x2))), x2)
\end{aligned}
$$

Lemma 420: $op_r(x1, x2, i(x2)) = op_l(op_t(x1, x2), i(i(mult(x1, x2))), x2)$

$$
\begin{aligned}
& \overbrace{op_r(x1, x2, i(x2))} \\
= & \quad \text{by Lemma 92 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\overbrace{mult(mult(x1, x2), i(x2))}} \\
= & \quad \text{by Lemma 419 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(\overbrace{i(i(op_t(x1, x2)))}, i(i(mult(x1, x2))), x2)} \\
= & \quad \text{by Lemma 3 LR with } \{x1 \leftarrow op_t(x1, x2)\} \\
& op_l(\overbrace{op_t(x1, x2)}, i(i(mult(x1, x2))), x2)
\end{aligned}
$$

Lemma 421: $op_r(x1, x2, i(x2)) = op_l(op_t(x1, x2), x1, x2)$

$$
\begin{aligned}
& \overbrace{op_r(x1, x2, i(x2))} \\
= & \quad \text{by Lemma 420 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(op_t(x1, x2), \overbrace{i(i(mult(x1, x2)))}, x2)} \\
= & \quad \text{by Lemma 3 LR with } \{x1 \leftarrow mult(x1, x2)\} \\
& \overbrace{op_l(op_t(x1, x2), \overbrace{mult(x1, x2)}, x2)} \\
= & \quad \text{by Lemma 285 LR with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow op_t(x1, x2)\} \\
& \overbrace{op_l(op_t(x1, x2), x1, x2)}
\end{aligned}
$$

Lemma 422: $op_l(mult(x1, x2), x2, i(x1)) = mult(x2, x1)$

$$
\begin{aligned}
& op_l(mult(x1, x2), \underbrace{x2}, i(x1)) \\
= & \quad \text{by Lemma 3 RL with } \{x1 \leftarrow x2\} \\
& op_l(\underbrace{mult(x1, x2)}, \overbrace{i(i(x2))}, i(x1)) \\
= & \quad \text{by Lemma 3 RL with } \{x1 \leftarrow mult(x1, x2)\} \\
& op_l(\overbrace{i(\underbrace{i(mult(x1, x2))})}, i(i(x2)), i(x1)) \\
= & \quad \text{by Axiom 11 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{op_l(i(\overbrace{mult(i(x1), i(x2))}), i(i(x2)), i(x1))} \\
= & \quad \text{by Lemma 418 LR with } \{x2 \leftarrow i(x2), x1 \leftarrow i(x1)\} \\
& \overbrace{\underbrace{i(mult(i(x2), i(x1)))}} \\
= & \quad \text{by Lemma 16 LR with } \{x2 \leftarrow i(x1), x1 \leftarrow x2\} \\
& \overbrace{mult(x2, \underbrace{i(i(x1))})} \\
= & \quad \text{by Lemma 3 LR with } \{x1 \leftarrow x1\} \\
& mult(x2, \overbrace{x1}^{})
\end{aligned}
$$

Lemma 423: $op_l(mult(x1, x2), i(x2), x1) = mult(x2, x1)$

$$
\begin{aligned}
& op_l(\underbrace{mult(x1, x2)}, i(x2), x1) \\
= & \quad \text{by Lemma 422 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \underbrace{op_l(\overbrace{op_l(mult(x2, x1), x1, i(x2))}, i(x2), x1)} \\
= & \quad \text{by Lemma 138 LR with } \{x3 \leftarrow i(x2), x2 \leftarrow x1, x1 \leftarrow mult(x2, x1)\} \\
& \overbrace{mult(x2, x1)}
\end{aligned}
$$

Lemma 424: $mult(op_l(x1, x1, x2), x2) = rd(x2, op_t(i(x1), x2))$

$$
\begin{aligned}
& \underbrace{mult(op_l(x1, x1, x2), x2)} \\
= & \quad \text{by Lemma 167 RL with } \{x4 \leftarrow x2, x3 \leftarrow x1, x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \overbrace{mult(x2, \underbrace{op_l(op_t(x1, x2), x1, x2)})} \\
= & \quad \text{by Lemma 421 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{mult(x2, \overbrace{op_r(x1, x2, i(x2))})} \\
= & \quad \text{by Lemma 402 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \overbrace{rd(x2, op_t(i(x1), x2))}
\end{aligned}
$$

Lemma 425: $mult(x1, i(x2)) = op_l(i(mult(x2, i(x1))), x1, x2)$

$$
\begin{aligned}
& \underbrace{mult(x1, i(x2))} \\
= & \quad \text{by Lemma 16 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{\overbrace{i(mult(i(x1), x2))}} \\
= & \quad \text{by Lemma 418 RL with } \{x2 \leftarrow i(x1), x1 \leftarrow x2\} \\
& \overbrace{op_l(i(mult(x2, i(x1))), \underbrace{i(i(x1))}, x2)} \\
= & \quad \text{by Lemma 3 LR with } \{x1 \leftarrow x1\} \\
& op_l(i(mult(x2, i(x1))), \overbrace{x1}, x2)
\end{aligned}
$$

Lemma 426: $op_l(mult(i(x1), x2), x2, x1) = mult(x2, i(x1))$

$$
\begin{aligned}
& op_l(\underbrace{mult(i(x1), x2)}, x2, x1) \\
= & \quad \text{by Lemma 17 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{op_l(\overbrace{i(mult(x1, i(x2)))}, x2, x1)} \\
= & \quad \text{by Lemma 425 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \overbrace{mult(x2, i(x1))}
\end{aligned}
$$

Lemma 427: $op_l(x1, x1, x2) = mult(x2, rd(x1, x2))$

$$
\begin{aligned}
& \underbrace{op_l(x1, x1, x2)} \\
= \quad & \text{by Axiom 5 LR with } \{x2 \leftarrow x2, x1 \leftarrow op_l(x1, x1, x2)\} \\
& \overbrace{rd(\underbrace{mult(op_l(x1, x1, x2), x2)}, x2)} \\
= \quad & \text{by Lemma 424 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{rd(\overbrace{rd(x2, op_t(i(x1), x2))}, x2)} \\
= \quad & \text{by Lemma 354 LR with } \{x2 \leftarrow op_t(i(x1), x2), x1 \leftarrow x2\} \\
& \overbrace{\underbrace{op_r(i(op_t(i(x1), x2)), x2, x2)}} \\
= \quad & \text{by Lemma 413 LR with } \{x2 \leftarrow x2, x1 \leftarrow i(x1)\} \\
& \overbrace{mult(x2, rd(\underbrace{i(i(x1))}, x2))} \\
= \quad & \text{by Lemma 3 LR with } \{x1 \leftarrow x1\} \\
& mult(x2, rd(\overbrace{x1}, x2))
\end{aligned}
$$

Lemma 428: $op_l(x1, i(x1), x2) = mult(op_l(x1, x1, i(x2)), mult(x2, i(x2)))$

$$
\begin{aligned}
& \underbrace{op_l(x1, i(x1), x2)} \\
= \quad & \text{by Lemma 295 RL with } \{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{\overbrace{op_l(x1, rd(i(x2), x1), x2)}} \\
= \quad & \text{by Lemma 280 LR with } \{x3 \leftarrow x2, x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& \overbrace{mult(\underbrace{mult(i(x2), rd(x1, i(x2)))}, mult(x2, i(x2)))} \\
= \quad & \text{by Lemma 427 RL with } \{x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& mult(\overbrace{op_l(x1, x1, i(x2))}, mult(x2, i(x2)))
\end{aligned}
$$

Lemma 429: $op_l(x1, i(x1), x2) = op_l(x1, x1, i(x2))$

$$
\begin{aligned}
& \underbrace{op_l(x1, i(x1), x2)} \\
= \quad & \text{by Lemma 428 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{mult(op_l(x1, x1, i(x2)), \underbrace{mult(x2, i(x2))})} \\
= \quad & \text{by Axiom 7 LR with } \{x1 \leftarrow x2\} \\
& \underbrace{mult(op_l(x1, x1, i(x2)), \overbrace{unit()})} \\
= \quad & \text{by Axiom 1 RL with } \{x1 \leftarrow op_l(x1, x1, i(x2))\} \\
& \overbrace{op_l(x1, x1, i(x2))}
\end{aligned}
$$

Lemma 430: $i(op_l(x1, x1, x2)) = op_l(i(x1), x1, x2)$

$$
\begin{aligned}
& \underbrace{i(op_l(x1, x1, x2))} \\
= & \quad \text{by Lemma 276 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(i(x1), i(x1), i(x2))} \\
= & \quad \text{by Lemma 429 RL with } \{x2 \leftarrow x2, x1 \leftarrow i(x1)\} \\
& \overbrace{op_l(i(x1), \underbrace{i(i(x1))}, x2)} \\
= & \quad \text{by Lemma 3 LR with } \{x1 \leftarrow x1\} \\
& op_l(i(x1), \overbrace{x1}, x2)
\end{aligned}
$$

Lemma 431: $op_l(x1, x1, mult(i(x1), x2)) = op_l(x1, i(x2), i(i(x1)))$

$$
\begin{aligned}
& op_l(x1, x1, \underbrace{mult(i(x1), x2)}) \\
= & \quad \text{by Lemma 3 RL with } \{x1 \leftarrow mult(i(x1), x2)\} \\
& \underbrace{op_l(x1, x1, \overbrace{i(i(mult(i(x1), x2)))})} \\
= & \quad \text{by Lemma 429 RL with } \{x2 \leftarrow i(mult(i(x1), x2)), x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, i(x1), i(mult(i(x1), x2)))}} \\
= & \quad \text{by Lemma 245 LR with } \{x3 \leftarrow x2, x2 \leftarrow i(x1), x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, i(x2), i(i(x1)))}
\end{aligned}
$$

Lemma 432: $op_l(x1, x2, i(x1)) = op_l(x1, i(x2), x1)$

$$
\begin{aligned}
& \underbrace{op_l(x1, x2, i(x1))} \\
= & \quad \text{by Lemma 243 RL with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, x1, mult(i(x1), x2))}} \\
= & \quad \text{by Lemma 431 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, i(x2), \underbrace{i(i(x1))})} \\
= & \quad \text{by Lemma 3 LR with } \{x1 \leftarrow x1\} \\
& op_l(x1, i(x2), \overbrace{x1})
\end{aligned}
$$

Lemma 433: $i(op_l(x1, x2, x1)) = op_l(i(x1), x2, x1)$

$$
\begin{aligned}
& \underbrace{i(op_l(x1, x2, x1))} \\
= & \quad \text{by Lemma 307 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(i(x1), i(x2), i(x1))}} \\
= & \quad \text{by Lemma 432 RL with } \{x2 \leftarrow x2, x1 \leftarrow i(x1)\} \\
& \overbrace{op_l(i(x1), x2, \underbrace{i(i(x1))})} \\
= & \quad \text{by Lemma 3 LR with } \{x1 \leftarrow x1\} \\
& op_l(i(x1), x2, \overbrace{x1})
\end{aligned}
$$

Lemma 434: $mult(mult(i(x1), mult(x1, x2)), x1) = op_l(mult(x1, x2), x1, x2)$

$$
\begin{aligned}
& \overbrace{mult(mult(i(x1), mult(x1, x2)), x1)} \\
= \quad & \text{by Lemma 69 RL with } \{x2 \leftarrow mult(x1, x2), x1 \leftarrow x1\} \\
& \overbrace{mult(i(x1), \underbrace{rd(mult(x1, x2), i(x1))}_{})} \\
= \quad & \text{by Lemma 427 RL with } \{x2 \leftarrow i(x1), x1 \leftarrow mult(x1, x2)\} \\
& \overbrace{op_l(mult(x1, x2), mult(x1, x2), i(x1))} \\
= \quad & \text{by Lemma 242 LR with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow mult(x1, x2)\} \\
& \overbrace{op_l(mult(x1, x2), x1, x2)}
\end{aligned}
$$

Lemma 435: $op_l(mult(x1, x2), x1, x2) = mult(x2, x1)$

$$
\begin{aligned}
& \overbrace{op_l(mult(x1, x2), x1, x2)} \\
= \quad & \text{by Lemma 434 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{mult(\underbrace{mult(i(x1), mult(x1, x2))}, x1)} \\
= \quad & \text{by Axiom 2 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& mult(\overbrace{x2}, x1)
\end{aligned}
$$

Lemma 436: $i(op_l(i(x1), x1, x2)) = op_l(x1, x1, x2)$

$$
\begin{aligned}
& \overbrace{i(op_l(i(x1), x1, x2))} \\
= \quad & \text{by Lemma 304 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, i(x1), i(x2))} \\
= \quad & \text{by Lemma 429 LR with } \{x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x1, \underbrace{i(i(x2))})} \\
= \quad & \text{by Lemma 3 LR with } \{x1 \leftarrow x2\} \\
& op_l(x1, x1, \overbrace{x2})
\end{aligned}
$$

Lemma 437: $op_l(x1, i(x2), i(x1)) = mult(i(mult(x1, x2)), mult(x1, mult(x2, x1)))$

$$
\begin{aligned}
& \overbrace{op_l(x1, i(x2), i(x1))} \\
= \quad & \text{by Lemma 245 RL with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, x1, i(mult(x1, x2)))}} \\
= \quad & \text{by Lemma 427 LR with } \{x2 \leftarrow i(mult(x1, x2)), x1 \leftarrow x1\} \\
& \overbrace{mult(i(mult(x1, x2)), \underbrace{rd(x1, i(mult(x1, x2)))})} \\
= \quad & \text{by Lemma 58 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& mult(i(mult(x1, x2)), \overbrace{mult(x1, mult(x2, x1))})
\end{aligned}
$$

Lemma 438: $op_l(x1, i(x2), i(x1)) = op_l(x1, x2, x1)$

$$
\begin{aligned}
& \underbrace{op_l(x1, i(x2), i(x1))} \\
= & \quad \text{by Lemma 437 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{mult(i(mult(x1, x2)), mult(x1, mult(x2, x1)))}} \\
= & \quad \text{by Axiom 12 LR with } \{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x2, x1)}
\end{aligned}
$$

Lemma 439: $op_l(x1, x1, i(x2)) = mult(mult(i(x2), x1), x2)$

$$
\begin{aligned}
& \underbrace{op_l(x1, x1, i(x2))} \\
= & \quad \text{by Lemma 429 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, i(x1), x2)}} \\
= & \quad \text{by Axiom 12 RL with } \{x3 \leftarrow x1, x2 \leftarrow i(x1), x1 \leftarrow x2\} \\
& \overbrace{mult(i(mult(x2, i(x1))), mult(x2, \underbrace{mult(i(x1), x1)}))} \\
= & \quad \text{by Axiom 9 LR with } \{x1 \leftarrow x1\} \\
& mult(i(mult(x2, i(x1))), \underbrace{mult(x2, \overbrace{unit()})}) \\
= & \quad \text{by Axiom 1 RL with } \{x1 \leftarrow x2\} \\
& \underbrace{mult(i(mult(x2, i(x1))), \overbrace{x2})} \\
= & \quad \text{by Lemma 56 LR with } \{x2 \leftarrow i(x1), x1 \leftarrow x2\} \\
& \overbrace{\underbrace{mult(i(x2), rd(x2, i(x1)))}} \\
= & \quad \text{by Lemma 71 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \overbrace{mult(mult(i(x2), x1), x2)}
\end{aligned}
$$

Lemma 440: $mult(i(x1), op_l(x2, x2, x1)) = rd(x2, x1)$

$$
\begin{aligned}
& mult(i(x1), \underbrace{op_l(x2, x2, x1)}) \\
= & \quad \text{by Lemma 427 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \underbrace{mult(i(x1), \overbrace{mult(x1, rd(x2, x1))})} \\
= & \quad \text{by Axiom 2 RL with } \{x2 \leftarrow x1, x1 \leftarrow rd(x2, x1)\} \\
& \overbrace{rd(x2, x1)}
\end{aligned}
$$

Lemma 441: $rd(op_l(x1, x1, x2), rd(x1, x2)) = x2$

$$
\begin{aligned}
& rd(op_l(x1, x1, \underbrace{x2}), rd(x1, x2)) \\
= & \quad \text{by Lemma 3 RL with } \{x1 \leftarrow x2\} \\
& rd(\underbrace{op_l(x1, x1, \overbrace{i(i(x2))})}, rd(x1, x2)) \\
= & \quad \text{by Lemma 429 RL with } \{x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& \underbrace{rd(\overbrace{op_l(x1, i(x1), i(x2))}, rd(x1, x2))} \\
= & \quad \text{by Lemma 236 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \overbrace{x2}
\end{aligned}
$$

Lemma 442: $op_l(mult(x1, rd(x2, x1)), x1, x2) = x2$

$$
\begin{aligned}
& op_l(\underbrace{mult(x1, rd(x2, x1))}, x1, x2) \\
= & \quad \text{by Lemma 427 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \underbrace{op_l(\overbrace{op_l(x2, x2, x1)}, x1, x2)} \\
= & \quad \text{by Lemma 138 LR with } \{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x2\} \\
& \overbrace{x2}
\end{aligned}
$$

Lemma 443: $op_l(i(x1), i(x1), x2) = op_l(i(x1), x1, i(x2))$

$$
\begin{aligned}
& \underbrace{op_l(i(x1), i(x1), x2)} \\
= & \quad \text{by Lemma 270 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{\overbrace{i(op_l(x1, x1, i(x2)))}} \\
= & \quad \text{by Lemma 430 LR with } \{x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& \overbrace{op_l(i(x1), x1, i(x2))}
\end{aligned}
$$

Lemma 444: $op_l(i(x1), i(x2), x1) = op_l(i(x1), x2, i(x1))$

$$
\begin{aligned}
& \underbrace{op_l(i(x1), i(x2), x1)} \\
= & \quad \text{by Lemma 268 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{i(\underbrace{op_l(x1, x2, i(x1))})} \\
= & \quad \text{by Lemma 432 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{i(\overbrace{op_l(x1, i(x2), x1)})} \\
= & \quad \text{by Lemma 306 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(i(x1), x2, i(x1))}
\end{aligned}
$$

Lemma 445: $rd(x1, op_l(x2, x2, i(x1))) = mult(x1, i(x2))$

$$
\begin{aligned}
& rd(x1, \underbrace{op_l(x2, x2, i(x1))}) \\
= & \quad \text{by Lemma 429 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \underbrace{rd(x1, \overbrace{op_l(x2, i(x2), x1)})} \\
= & \quad \text{by Lemma 230 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{mult(x1, i(x2))}
\end{aligned}
$$

Lemma 446: $rd(op_l(x1, x1, i(x2)), x2) = mult(i(x2), x1)$

$$
\begin{aligned}
& rd(\underbrace{op_l(x1, x1, i(x2))}, x2) \\
= & \quad \text{by Lemma 429 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{rd(\overbrace{op_l(x1, i(x1), x2)}, x2)} \\
= & \quad \text{by Lemma 233 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{mult(i(x2), x1)}
\end{aligned}
$$

Lemma 447: $rd(i(x1), op_l(x2, x2, x1)) = i(mult(x1, x2))$

$$
\begin{aligned}
& rd(i(x1), \underbrace{op_l(x2, x2, x1)}) \\
= & \quad \text{by Lemma 427 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \underbrace{rd(i(x1), \overbrace{mult(x1, rd(x2, x1))})} \\
= & \quad \text{by Lemma 83 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{i(mult(x1, x2))}
\end{aligned}
$$

Lemma 448: $rd(mult(x1, rd(x2, x1)), x2) = asoc(x2, x1, x2)$

$$
\begin{aligned}
& rd(\underbrace{mult(x1, rd(x2, x1))}, x2) \\
= & \quad \text{by Lemma 427 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \underbrace{rd(\overbrace{op_l(x2, x2, x1)}, x2)} \\
= & \quad \text{by Lemma 172 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \overbrace{asoc(x2, x1, x2)}
\end{aligned}
$$

Lemma 449: $rd(rd(x1, x2), op_l(x1, x1, x2)) = i(x2)$

$$
\begin{aligned}
& rd(rd(x1, x2), \underbrace{op_l(x1, x1, x2)}) \\
= & \quad \text{by Lemma 427 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{rd(rd(x1, x2), \overbrace{mult(x2, rd(x1, x2))})} \\
= & \quad \text{by Lemma 9 LR with } \{x2 \leftarrow x2, x1 \leftarrow rd(x1, x2)\} \\
& \overbrace{i(x2)}
\end{aligned}
$$

Lemma 450: $rd(i(x1), op_t(i(x2), i(x1))) = mult(op_l(x2, x1, x2), i(x1))$

$$
\begin{aligned}
& \underbrace{rd(i(x1), op_t(i(x2), i(x1)))} \\
= & \quad \text{by Lemma 256 RL with } \{x2 \leftarrow i(x1), x1 \leftarrow i(x2)\} \\
& \overbrace{mult(op_l(\underbrace{i(i(x2))}, i(x1), i(x2)), i(x1))} \\
= & \quad \text{by Lemma 3 LR with } \{x1 \leftarrow x2\} \\
& mult(\underbrace{op_l(\overbrace{\ x2\ }, i(x1), i(x2))}, i(x1)) \\
= & \quad \text{by Lemma 438 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& mult(\overbrace{op_l(x2, x1, x2)}, i(x1))
\end{aligned}
$$

Lemma 451: $rd(op_t(x2, x1), x1) = mult(op_l(x2, x1, x2), i(x1))$

$$
\begin{aligned}
& rd(op_t(x2, \underbrace{\ x1\ }), x1) \\
= & \quad \text{by Lemma 3 RL with } \{x1 \leftarrow x1\} \\
& \underbrace{rd(op_t(x2, \overbrace{i(i(x1))}), x1)} \\
= & \quad \text{by Lemma 68 RL with } \{x3 \leftarrow i(x1), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{rd(i(x1), op_t(i(x2), i(x1)))}} \\
= & \quad \text{by Lemma 450 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{mult(op_l(x2, x1, x2), i(x1))}
\end{aligned}
$$

Lemma 452: $rd(mult(x1, x2), x1) = op_r(op_l(x2, x2, i(i(x1))), x1, x1)$

$rd(mult(\underbrace{x1}, x2), x1)$

$=$ by Lemma 3 RL with $\{x1 \leftarrow x1\}$

$\underbrace{rd(mult(\overbrace{i(i(x1))}, x2), x1)}$

$=$ by Lemma 364 RL with $\{x2 \leftarrow x1, x1 \leftarrow mult(i(i(x1)), x2)\}$

$\overbrace{op_r(\underbrace{mult(mult(i(i(x1)), x2), i(x1))}, x1, x1)}$

$=$ by Lemma 439 RL with $\{x2 \leftarrow i(x1), x1 \leftarrow x2\}$

$op_r(\overbrace{op_l(x2, x2, i(i(x1)))}, x1, x1)$

Lemma 453: $op_l(op_r(x1, x2, x2), x1, x2) = rd(mult(x2, x1), x2)$

$op_l(op_r(x1, x2, x2), x1, \underbrace{x2})$

$=$ by Lemma 3 RL with $\{x1 \leftarrow x2\}$

$\underbrace{op_l(op_r(x1, x2, x2), x1, \overbrace{i(i(x2))})}$

$=$ by Axiom 18 RL with $\{x5 \leftarrow x2, x4 \leftarrow x2, x3 \leftarrow i(i(x2)), x2 \leftarrow x1, x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_r(op_l(x1, x1, i(i(x2))), x2, x2)}}$

$=$ by Lemma 452 RL with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$\overbrace{rd(mult(x2, x1), x2)}$

Lemma 454: $mult(i(x1), rd(x1, x2)) = op_l(i(x2), x1, rd(x1, x2))$

$\underbrace{mult(i(x1), rd(x1, x2))}$

$=$ by Lemma 423 RL with $\{x2 \leftarrow i(x1), x1 \leftarrow rd(x1, x2)\}$

$\overbrace{op_l(\underbrace{mult(rd(x1, x2), i(x1))}, i(i(x1)), rd(x1, x2))}$

$=$ by Lemma 22 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$op_l(\overbrace{i(x2)}, \underbrace{i(i(x1))}, rd(x1, x2))$

$=$ by Lemma 3 LR with $\{x1 \leftarrow x1\}$

$op_l(i(x2), \overbrace{x1}, rd(x1, x2))$

Lemma 455: $op_l(op_t(x1, rd(x2, x1)), x2, x1) = op_t(x1, x2)$

$$
\begin{aligned}
& op_l(op_t(x1, \underbrace{rd(x2, x1)}), x2, x1) \\
= & \quad \text{by Lemma 10 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_l(\underbrace{op_t(x1, \overbrace{i(rd(x1, x2))})}, x2, x1) \\
= & \quad \text{by Lemma 18 RL with } \{x2 \leftarrow rd(x1, x2), x1 \leftarrow x1\} \\
& \underbrace{op_l(\overbrace{i(op_t(i(x1), rd(x1, x2)))}, x2, x1)} \\
= & \quad \text{by Lemma 291 RL with } \{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow i(op_t(i(x1), rd(x1, x2)))\} \\
& \overbrace{op_l(i(op_t(i(x1), rd(x1, x2))), x2, \underbrace{mult(x1, x2)})} \\
= & \quad \text{by Lemma 386 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \underbrace{op_l(i(op_t(i(x1), rd(x1, x2))), x2, \overbrace{rd(x2, op_t(i(x1), rd(x1, x2)))})} \\
= & \quad \text{by Lemma 454 RL with } \{x2 \leftarrow op_t(i(x1), rd(x1, x2)), x1 \leftarrow x2\} \\
& \overbrace{mult(i(x2), \underbrace{rd(x2, op_t(i(x1), rd(x1, x2)))})} \\
= & \quad \text{by Lemma 386 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \underbrace{mult(i(x2), \overbrace{mult(x1, x2)})} \\
= & \quad \text{by Axiom 10 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \overbrace{op_t(x1, x2)}
\end{aligned}
$$

Lemma 456: $op_t(op_t(mult(x1, x2), x1), x2) = mult(op_t(x1, i(x2)), x2)$

$$
\begin{aligned}
& op_t(\underbrace{op_t(mult(x1, x2), x1)}, x2) \\
= & \quad \text{by Lemma 264 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \underbrace{op_t(\overbrace{mult(x2, op_l(x1, x1, x2))}, x2)} \\
= & \quad \text{by Lemma 320 RL with } \{x2 \leftarrow x2, x1 \leftarrow op_l(x1, x1, x2)\} \\
& \overbrace{op_r(\underbrace{mult(op_l(x1, x1, x2), x2)}, x2, i(x2))} \\
= & \quad \text{by Lemma 424 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{op_r(\overbrace{rd(x2, op_t(i(x1), x2))}, x2, i(x2))} \\
= & \quad \text{by Lemma 330 LR with } \{x2 \leftarrow op_t(i(x1), x2), x1 \leftarrow x2\} \\
& \overbrace{mult(\underbrace{i(op_t(i(x1), x2))}, x2)} \\
= & \quad \text{by Lemma 18 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& mult(\overbrace{op_t(x1, i(x2))}, x2)
\end{aligned}
$$

Lemma 457: $rd(op_t(x2, i(x1)), x1) = mult(op_l(x2, x2, x1), i(x1))$

$$
\begin{aligned}
& \underbrace{rd(op_t(x2, i(x1)), x1)} \\
= \quad & \text{by Lemma 417 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{mult(x1, rd(rd(x2, x1), x1))}} \\
= \quad & \text{by Lemma 55 RL with } \{x2 \leftarrow rd(x2, x1), x1 \leftarrow x1\} \\
& \overbrace{mult(\underbrace{mult(x1, rd(x2, x1))}, i(x1))} \\
= \quad & \text{by Lemma 427 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& mult(\overbrace{op_l(x2, x2, x1)}, i(x1))
\end{aligned}
$$

Lemma 458: $asoc(x1, mult(i(x1), i(x2)), x1) = mult(i(x1), op_l(x1, x2, x1))$

$$
\begin{aligned}
& \underbrace{asoc(x1, mult(i(x1), i(x2)), x1)} \\
= \quad & \text{by Lemma 248 RL with } \{x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& \overbrace{\underbrace{rd(op_l(x1, i(x2), i(x1)), x1)}} \\
= \quad & \text{by Lemma 171 RL with } \{x3 \leftarrow i(x1), x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& \overbrace{mult(i(x1), \underbrace{op_l(x1, i(x2), i(x1))})} \\
= \quad & \text{by Lemma 438 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& mult(i(x1), \overbrace{op_l(x1, x2, x1)})
\end{aligned}
$$

Lemma 459: $rd(op_t(x1, mult(x1, x2)), x2) = mult(i(x2), op_t(x1, x2))$

$$
\begin{aligned}
& rd(\underbrace{op_t(x1, mult(x1, x2))}, x2) \\
= \quad & \text{by Lemma 251 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& rd(\overbrace{\underbrace{op_l(op_t(x1, x2), x2, x1)}}, x2) \\
= \quad & \text{by Axiom 15 RL with } \{x4 \leftarrow x2, x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{rd(\overbrace{op_t(op_l(x1, x2, x1), x2)}, x2)} \\
= \quad & \text{by Lemma 324 RL with } \{x2 \leftarrow x2, x1 \leftarrow op_l(x1, x2, x1)\} \\
& \overbrace{op_t(\underbrace{mult(op_l(x1, x2, x1), i(x2))}, x2)} \\
= \quad & \text{by Lemma 451 RL with } \{x1 \leftarrow x2, x2 \leftarrow x1\} \\
& \underbrace{op_t(\overbrace{rd(op_t(x1, x2), x2)}, x2)} \\
= \quad & \text{by Lemma 14 LR with } \{x2 \leftarrow x2, x1 \leftarrow op_t(x1, x2)\} \\
& \overbrace{mult(i(x2), op_t(x1, x2))}
\end{aligned}
$$

Lemma 460: $mult(op_l(x2, x1, x2), x1) = rd(x1, mult(i(x1), mult(x1, i(op_t(x2, x1)))))$

$\underbrace{mult(op_l(x2, x1, x2), x1)}$

$=$ by Lemma 255 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{\underbrace{mult(x1, op_t(x2, \overbrace{mult(x2, x1)}))}}$

$=$ by Lemma 57 RL with $\{x2 \leftarrow op_t(x2, mult(x2, x1)), x1 \leftarrow x1\}$

$\overbrace{rd(x1, mult(i(x1), rd(x1, \underbrace{op_t(x2, mult(x2, x1))})))}$

$=$ by Lemma 251 RL with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$rd(x1, mult(i(x1), rd(x1, \overbrace{op_l(op_t(x2, \underbrace{x1}), x1, x2)})))$

$=$ by Lemma 3 RL with $\{x1 \leftarrow x1\}$

$rd(x1, mult(i(x1), rd(x1, \underbrace{op_l(op_t(x2, \overbrace{i(\overbrace{i(x1)})}), x1, x2)})))$

$=$ by Axiom 15 RL with $\{x4 \leftarrow i(i(x1)), x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x2\}$

$rd(x1, mult(i(x1), \underbrace{rd(x1, \overbrace{op_t(op_l(x2, x1, x2), i(i(x1)))})}))$

$=$ by Lemma 67 RL with $\{x3 \leftarrow x1, x2 \leftarrow i(x1), x1 \leftarrow op_l(x2, x1, x2)\}$

$rd(x1, mult(i(x1), \overbrace{\underbrace{rd(op_t(i(op_l(x2, x1, x2)), i(x1)), i(x1))}}))$

$=$ by Lemma 329 RL with $\{x2 \leftarrow i(x1), x1 \leftarrow op_l(x2, x1, x2)\}$

$rd(x1, mult(i(x1), \overbrace{op_t(i(\underbrace{mult(op_l(x2, x1, x2), i(x1))}), i(x1))}))$

$=$ by Lemma 451 RL with $\{x1 \leftarrow x1, x2 \leftarrow x2\}$

$rd(x1, mult(i(x1), \underbrace{op_t(i(\overbrace{rd(op_t(x2, x1), x1)}), i(x1))}))$

$=$ by Lemma 15 LR with $\{x2 \leftarrow x1, x1 \leftarrow rd(op_t(x2, x1), x1)\}$

$rd(x1, mult(i(x1), \overbrace{i(\underbrace{op_t(rd(op_t(x2, x1), x1), x1)})}))$

$=$ by Lemma 46 LR with $\{x3 \leftarrow x1, x2 \leftarrow x1, x1 \leftarrow op_t(x2, x1)\}$

$rd(x1, mult(i(x1), \underbrace{op_t(rd(x1, op_t(x2, x1)), i(x1))}))$

$=$ by Lemma 34 LR with $\{x2 \leftarrow op_t(x2, x1), x1 \leftarrow x1\}$

$rd(x1, mult(i(x1), \overbrace{mult(x1, i(op_t(x2, x1)))}))$

Lemma 461: $mult(x1, op_t(x2, mult(x2, x1))) = rd(x1, i(op_t(x2, x1)))$

$\underbrace{mult(x1, op_t(x2, mult(x2, x1)))}$

$=$ by Lemma 255 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{\underbrace{mult(op_l(x2, x1, x2), x1)}}$

$=$ by Lemma 460 LR with $\{x1 \leftarrow x1, x2 \leftarrow x2\}$

$\overbrace{rd(x1, \underbrace{mult(i(x1), mult(x1, i(op_t(x2, x1))))})}$

$=$ by Axiom 2 RL with $\{x2 \leftarrow x1, x1 \leftarrow i(op_t(x2, x1))\}$

$rd(x1, \overbrace{i(op_t(x2, x1))})$

Lemma 462: $mult(x1, op_t(rd(x1, x2), i(x1))) = rd(op_t(rd(x1, x2), i(x2)), i(x1))$

$mult(\underbrace{x1}, op_t(rd(x1, x2), i(x1)))$

$=$ by Lemma 3 RL with $\{x1 \leftarrow x1\}$

$\underbrace{mult(\overbrace{i(i(x1))}, op_t(rd(x1, x2), i(x1)))}$

$=$ by Lemma 459 RL with $\{x2 \leftarrow i(x1), x1 \leftarrow rd(x1, x2)\}$

$\overbrace{rd(op_t(rd(x1, x2), \underbrace{mult(rd(x1, x2), i(x1))}), i(x1))}$

$=$ by Lemma 22 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$rd(op_t(rd(x1, x2), \overbrace{i(x2)}), i(x1))$

Lemma 463: $mult(x1, mult(x1, i(x2))) = rd(x1, op_t(i(rd(x1, x2)), x2))$

$mult(x1, \underbrace{mult(x1, i(x2))})$

$=$ by Lemma 34 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{mult(x1, \overbrace{op_t(rd(x1, x2), i(x1))})}$

$=$ by Lemma 462 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{\overbrace{rd(op_t(rd(x1, x2), i(x2)), i(x1))}}$

$=$ by Lemma 65 LR with $\{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow rd(x1, x2)\}$

$\overbrace{rd(x1, op_t(i(rd(x1, x2)), x2))}$

Lemma 464: $rd(x1, op_t(rd(x2, x1), x2)) = mult(x1, mult(x1, i(x2)))$

$rd(x1, op_t(\underbrace{rd(x2, x1)}, x2))$

$=$ by Lemma 10 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{rd(x1, op_t(\overbrace{i(rd(x1, x2))}, x2))}$

$=$ by Lemma 463 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{mult(x1, mult(x1, i(x2)))}$

Lemma 465: $mult(x1, op_t(rd(x2, x1), x2)) = rd(x1, mult(x1, i(x2)))$

$mult(x1, op_t(rd(x2, x1), \underbrace{x2}))$

$=$ by Axiom 4 LR with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$\underbrace{mult(x1, op_t(rd(x2, x1), \overbrace{mult(rd(x2, x1), x1)}))}$

$=$ by Lemma 461 LR with $\{x2 \leftarrow rd(x2, x1), x1 \leftarrow x1\}$

$\overbrace{rd(x1, \underbrace{i(op_t(rd(x2, x1), x1))})}$

$=$ by Lemma 46 LR with $\{x3 \leftarrow x1, x2 \leftarrow x1, x1 \leftarrow x2\}$

$rd(x1, \overbrace{\underbrace{op_t(rd(x1, x2), i(x1))}})$

$=$ by Lemma 34 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$rd(x1, \overbrace{mult(x1, i(x2))})$

Lemma 466: $mult(mult(x1, x2), op_t(i(x2), x1)) = op_l(x1, x2, x1)$

$$
\begin{aligned}
& mult(mult(x1, \overbrace{x2}), op_t(i(x2), x1)) \\
= \quad & \text{by Lemma 3 RL with } \{x1 \leftarrow x2\} \\
& \overbrace{mult(mult(x1, \overbrace{i(i(x2))}), op_t(i(x2), x1))} \\
= \quad & \text{by Lemma 258 LR with } \{x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& \overbrace{\overbrace{op_l(x1, i(x2), i(x1))}} \\
= \quad & \text{by Lemma 438 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x2, x1)}
\end{aligned}
$$

Lemma 467: $op_r(rd(op_t(x1, x2), x2), x2, x2) = mult(i(x2), x1)$

$$
\begin{aligned}
& \overbrace{op_r(rd(op_t(x1, x2), x2), x2, x2)} \\
= \quad & \text{by Lemma 370 LR with } \{x1 \leftarrow x2, x2 \leftarrow op_t(x1, x2)\} \\
& \overbrace{rd(\overbrace{op_r(op_t(x1, x2), x2, x2)}, x2)} \\
= \quad & \text{by Axiom 14 RL with } \{x4 \leftarrow x2, x3 \leftarrow x2, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{rd(\overbrace{op_t(op_r(x1, x2, x2), x2)}, x2)} \\
= \quad & \text{by Lemma 139 RL with } \{x2 \leftarrow op_r(x1, x2, x2), x1 \leftarrow x2\} \\
& \overbrace{mult(i(x2), \overbrace{op_r(op_r(x1, x2, x2), x2, i(x2))})} \\
= \quad & \text{by Lemma 351 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& mult(i(x2), \overbrace{x1})
\end{aligned}
$$

Lemma 468: $rd(x2, op_t(op_r(x1, x2, x2), i(i(x2)))) = op_t(i(rd(x1, x2)), i(x2))$

$$
\begin{aligned}
& \overbrace{rd(x2, op_t(op_r(x1, x2, x2), i(i(x2))))} \\
= \quad & \text{by Lemma 67 RL with } \{x3 \leftarrow x2, x2 \leftarrow i(x2), x1 \leftarrow op_r(x1, x2, x2)\} \\
& \overbrace{\overbrace{rd(op_t(i(op_r(x1, x2, x2)), i(x2)), i(x2))}} \\
= \quad & \text{by Lemma 329 RL with } \{x2 \leftarrow i(x2), x1 \leftarrow op_r(x1, x2, x2)\} \\
& \overbrace{op_t(i(\overbrace{mult(op_r(x1, x2, x2), i(x2))}), i(x2))} \\
= \quad & \text{by Lemma 352 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_t(i(\overbrace{rd(x1, x2)}), i(x2))
\end{aligned}
$$

Lemma 469: $op_r(rd(x1, op_t(x2, x1)), x1, x1) = mult(x1, i(x2))$

$$
\begin{aligned}
& \underbrace{op_r(rd(x1, op_t(x2, x1)), x1, x1)} \\
= \quad & \text{by Lemma 369 RL with } \{x2 \leftarrow op_t(x2, x1), x1 \leftarrow x1\} \\
& \overbrace{rd(x1, op_r(op_t(x2, \underbrace{x1}), x1, x1))} \\
= \quad & \text{by Lemma 3 RL with } \{x1 \leftarrow x1\} \\
& rd(x1, \underbrace{op_r(op_t(x2, \overbrace{i(i(x1))}), x1, x1)}) \\
= \quad & \text{by Axiom 14 RL with } \{x4 \leftarrow i(i(x1)), x3 \leftarrow x1, x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \underbrace{rd(x1, \overbrace{op_t(op_r(x2, x1, x1), i(i(x1)))})} \\
= \quad & \text{by Lemma 468 LR with } \{x1 \leftarrow x2, x2 \leftarrow x1\} \\
& \overbrace{\underbrace{op_t(i(rd(x2, x1)), i(x1))}} \\
= \quad & \text{by Lemma 15 LR with } \{x2 \leftarrow x1, x1 \leftarrow rd(x2, x1)\} \\
& \overbrace{\underbrace{i(op_t(rd(x2, x1), x1))}} \\
= \quad & \text{by Lemma 46 LR with } \{x3 \leftarrow x1, x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \overbrace{\underbrace{op_t(rd(x1, x2), i(x1))}} \\
= \quad & \text{by Lemma 34 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{mult(x1, i(x2))}
\end{aligned}
$$

Lemma 470: $op_t(rd(x1, x2), op_r(x2, x1, x1)) = mult(i(x2), x1)$

$$
\begin{aligned}
& op_t(rd(x1, x2), \underbrace{op_r(x2, x1, x1)}) \\
= \quad & \text{by Lemma 353 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_t(rd(x1, x2), \overbrace{rd(\underbrace{x1}, rd(x1, x2))}) \\
= \quad & \text{by Axiom 4 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{op_t(rd(x1, x2), rd(\overbrace{mult(rd(x1, x2), x2)}, rd(x1, x2)))} \\
= \quad & \text{by Lemma 378 LR with } \{x2 \leftarrow x2, x1 \leftarrow rd(x1, x2)\} \\
& \overbrace{\underbrace{op_t(rd(x1, x2), x2)}} \\
= \quad & \text{by Lemma 14 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{mult(i(x2), x1)}
\end{aligned}
$$

Lemma 471: $mult(rd(op_t(x1, i(x2)), x1), x2) = i(op_t(i(x2), x1))$

$$
\begin{aligned}
& mult(\underbrace{rd(op_t(x1, i(x2)), x1)}, x2) \\
= \quad & \text{by Lemma 10 RL with } \{x2 \leftarrow op_t(x1, i(x2)), x1 \leftarrow x1\} \\
& \underbrace{mult(\overbrace{i(rd(x1, op_t(x1, i(x2))))}, x2)} \\
= \quad & \text{by Lemma 17 RL with } \{x2 \leftarrow x2, x1 \leftarrow rd(x1, op_t(x1, i(x2)))\} \\
& \overbrace{i(\underbrace{mult(rd(x1, op_t(x1, i(x2))), i(x2))})} \\
= \quad & \text{by Lemma 388 LR with } \{x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& i(\overbrace{op_t(i(x2), x1)})
\end{aligned}
$$

Lemma 472: $i(op_t(rd(x2,x1),x1)) = rd(x1,op_l(x2,rd(x1,x2),x1))$

$$
\begin{aligned}
& \underbrace{i(op_t(\underbrace{rd(x2,x1)}, x1))} \\
= & \quad \text{by Lemma 12 RL with } \{x2 \leftarrow rd(x2,x1), x1 \leftarrow x1\} \\
& \underbrace{i(op_t(\overbrace{mult(x1, mult(i(x1), rd(x2,x1)))}, x1))} \\
= & \quad \text{by Lemma 410 RL with } \{x2 \leftarrow mult(i(x1), rd(x2,x1)), x1 \leftarrow x1\} \\
& \overbrace{rd(x1, \underbrace{mult(mult(i(x1), rd(x2,x1)), mult(x1,x1))})} \\
= & \quad \text{by Lemma 280 RL with } \{x3 \leftarrow x1, x2 \leftarrow x1, x1 \leftarrow x2\} \\
& rd(x1, \overbrace{op_l(x2, rd(x1,x2), x1)})
\end{aligned}
$$

Lemma 473: $mult(x1, i(x2)) = rd(x1, op_l(x2, rd(x1,x2), x1))$

$$
\begin{aligned}
& \underbrace{mult(x1, i(x2))} \\
= & \quad \text{by Lemma 16 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{i(\underbrace{mult(i(x1), x2)})} \\
= & \quad \text{by Lemma 14 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \underbrace{i(\overbrace{op_t(rd(x2,x1), x1)})} \\
= & \quad \text{by Lemma 472 LR with } \{x1 \leftarrow x1, x2 \leftarrow x2\} \\
& \overbrace{rd(x1, op_l(x2, rd(x1,x2), x1))}
\end{aligned}
$$

Lemma 474: $i(op_l(x1, rd(x2,x1), x2)) = mult(x2, rd(i(x1), x2))$

$$
\begin{aligned}
& \underbrace{i(op_l(x1, rd(x2,x1), x2))} \\
= & \quad \text{by Lemma 22 RL with } \{x2 \leftarrow op_l(x1, rd(x2,x1), x2), x1 \leftarrow x2\} \\
& \overbrace{mult(\underbrace{rd(x2, op_l(x1, rd(x2,x1), x2))}, i(x2))} \\
= & \quad \text{by Lemma 473 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \underbrace{mult(\overbrace{mult(x2, i(x1))}, i(x2))} \\
= & \quad \text{by Lemma 55 LR with } \{x2 \leftarrow i(x1), x1 \leftarrow x2\} \\
& \overbrace{mult(x2, rd(i(x1), x2))}
\end{aligned}
$$

Lemma 475: $op_l(op_r(x1, x2, i(x2)), x1, x2) = op_t(x1, i(x2))$

$$
\begin{aligned}
& \overbrace{op_l(op_r(x1, x2, i(x2)), x1, x2)} \\
= \quad & \text{by Axiom 18 RL with } \{x5 \leftarrow i(x2), x4 \leftarrow x2, x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x1\} \\
& \overbrace{op_r(\underbrace{op_l(x1, x1, x2)}, x2, i(x2))} \\
= \quad & \text{by Lemma 92 RL with } \{x2 \leftarrow x2, x1 \leftarrow op_l(x1, x1, x2)\} \\
& \overbrace{mult(\underbrace{mult(op_l(x1, x1, x2), x2)}, i(x2))} \\
= \quad & \text{by Lemma 424 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{mult(\overbrace{rd(x2, op_t(i(x1), x2))}, i(x2))} \\
= \quad & \text{by Lemma 22 LR with } \{x2 \leftarrow op_t(i(x1), x2), x1 \leftarrow x2\} \\
& \underbrace{\overbrace{i(op_t(i(x1), x2))}} \\
= \quad & \text{by Lemma 18 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, i(x2))}
\end{aligned}
$$

Lemma 476: $op_t(rd(x1, x2), i(x2)) = op_l(mult(x1, i(x2)), x1, x2)$

$$
\begin{aligned}
& \underbrace{op_t(rd(x1, x2), i(x2))} \\
= \quad & \text{by Lemma 475 RL with } \{x2 \leftarrow x2, x1 \leftarrow rd(x1, x2)\} \\
& \overbrace{op_l(\underbrace{op_r(rd(x1, x2), x2, i(x2))}, rd(x1, x2), x2)} \\
= \quad & \text{by Lemma 104 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{op_l(\overbrace{mult(x1, i(x2))}, rd(x1, x2), x2)} \\
= \quad & \text{by Lemma 292 LR with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow mult(x1, i(x2))\} \\
& \overbrace{op_l(mult(x1, i(x2)), x1, x2)}
\end{aligned}
$$

Lemma 477: $op_t(rd(x1, i(x2)), x2) = op_l(mult(x1, x2), x1, i(x2))$

$$
\begin{aligned}
& op_t(rd(x1, i(x2)), \underbrace{x2}) \\
= \quad & \text{by Lemma 3 RL with } \{x1 \leftarrow x2\} \\
& \underbrace{op_t(rd(x1, i(x2)), \overbrace{i(i(x2))})} \\
= \quad & \text{by Lemma 476 LR with } \{x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& \overbrace{op_l(mult(x1, \underbrace{i(i(x2))}), x1, i(x2))} \\
= \quad & \text{by Lemma 3 LR with } \{x1 \leftarrow x2\} \\
& op_l(mult(x1, \overbrace{x2}), x1, i(x2))
\end{aligned}
$$

Lemma 478: $op_t(i(x2), i(mult(x2, x1))) = op_l(i(op_t(x2, x1)), x1, mult(x2, x1))$

$$
\begin{aligned}
& op_t(\underbrace{i(x2)}, i(mult(x2, x1))) \\
= & \quad \text{by Lemma 9 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{op_t(\overbrace{rd(x1, mult(x2, x1))}, i(mult(x2, x1)))} \\
= & \quad \text{by Lemma 476 LR with } \{x2 \leftarrow mult(x2, x1), x1 \leftarrow x1\} \\
& \overbrace{op_l(\underbrace{mult(x1, i(mult(x2, x1)))}, x1, mult(x2, x1))} \\
= & \quad \text{by Lemma 31 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_l(\overbrace{i(op_t(x2, x1))}, x1, mult(x2, x1))
\end{aligned}
$$

Lemma 479: $i(op_t(x2, mult(x2, x1))) = op_l(i(op_t(x2, x1)), x1, x2)$

$$
\begin{aligned}
& \underbrace{i(op_t(x2, mult(x2, x1)))} \\
= & \quad \text{by Lemma 15 RL with } \{x2 \leftarrow mult(x2, x1), x1 \leftarrow x2\} \\
& \underbrace{\overbrace{op_t(i(x2), i(mult(x2, x1)))}} \\
= & \quad \text{by Lemma 478 LR with } \{x1 \leftarrow x1, x2 \leftarrow x2\} \\
& \underbrace{\overbrace{op_l(i(op_t(x2, x1)), x1, mult(x2, x1))}} \\
= & \quad \text{by Lemma 291 LR with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow i(op_t(x2, x1))\} \\
& \overbrace{op_l(i(op_t(x2, x1)), x1, x2)}
\end{aligned}
$$

Lemma 480: $rd(x1, op_t(rd(x1, x2), x1)) = mult(op_l(rd(x2, x1), x2, x1), x1)$

$$
\begin{aligned}
& rd(x1, op_t(\underbrace{rd(x1, x2)}, x1)) \\
= & \quad \text{by Lemma 10 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \underbrace{rd(x1, op_t(\overbrace{i(rd(x2, x1))}, x1))} \\
= & \quad \text{by Lemma 424 RL with } \{x2 \leftarrow x1, x1 \leftarrow rd(x2, x1)\} \\
& \overbrace{mult(\underbrace{op_l(rd(x2, x1), rd(x2, x1), x1)}, x1)} \\
= & \quad \text{by Lemma 292 LR with } \{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow rd(x2, x1)\} \\
& mult(\overbrace{op_l(rd(x2, x1), x2, x1)}, x1)
\end{aligned}
$$

Lemma 481: $mult(op_l(rd(x1, x2), x1, x2), x2) = op_t(x1, i(x2))$

$$
\begin{aligned}
& \underbrace{mult(op_l(rd(x1, x2), x1, x2), x2)} \\
= & \quad \text{by Lemma 480 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \underbrace{\overbrace{rd(x2, op_t(rd(x2, x1), x2))}} \\
= & \quad \text{by Lemma 344 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \overbrace{op_t(x1, i(x2))}
\end{aligned}
$$

Lemma 482: $asoc(x1, x2, x1) = mult(mult(x2, rd(x1, x2)), i(x1))$

$$
\begin{aligned}
& \underbrace{asoc(x1, x2, x1)} \\
= \quad & \text{by Lemma 172 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{rd(op_l(x1, x1, x2), x1)}} \\
= \quad & \text{by Lemma 178 RL with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x1\} \\
& \overbrace{mult(\underbrace{op_l(x1, x1, x2)}, i(x1))} \\
= \quad & \text{by Lemma 427 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& mult(\overbrace{mult(x2, rd(x1, x2))}, i(x1))
\end{aligned}
$$

Lemma 483: $op_l(x1, x2, x3) = op_l(x1, i(x3), op_l(x2, x2, x3))$

$$
\begin{aligned}
& \underbrace{op_l(x1, x2, x3)} \\
= \quad & \text{by Lemma 292 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, rd(x2, x3), x3)}} \\
= \quad & \text{by Lemma 241 RL with } \{x3 \leftarrow rd(x2, x3), x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, i(x3), \underbrace{mult(x3, rd(x2, x3))})} \\
= \quad & \text{by Lemma 427 RL with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& op_l(x1, i(x3), \overbrace{op_l(x2, x2, x3)})
\end{aligned}
$$

Lemma 484: $op_l(x1, x2, x3) = op_l(x1, op_l(x3, x3, x2), i(x2))$

$$
\begin{aligned}
& \underbrace{op_l(x1, x2, x3)} \\
= \quad & \text{by Lemma 286 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, x2, rd(x3, x2))}} \\
= \quad & \text{by Lemma 242 RL with } \{x3 \leftarrow rd(x3, x2), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, \underbrace{mult(x2, rd(x3, x2))}, i(x2))} \\
= \quad & \text{by Lemma 427 RL with } \{x2 \leftarrow x2, x1 \leftarrow x3\} \\
& op_l(x1, \overbrace{op_l(x3, x3, x2)}, i(x2))
\end{aligned}
$$

Lemma 485: $op_l(rd(mult(x1, x2), x1), x1, x2) = op_r(x2, x1, x1)$

$$
\begin{aligned}
& op_l(\underbrace{rd(mult(x1, x2), x1)}, x1, x2) \\
= \quad & \text{by Lemma 453 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \underbrace{op_l(\overbrace{op_l(op_r(x2, x1, x1), x2, x1)}, x1, x2)} \\
= \quad & \text{by Lemma 138 LR with } \{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow op_r(x2, x1, x1)\} \\
& \overbrace{op_r(x2, x1, x1)}
\end{aligned}
$$

Lemma 486: $mult(x1, asoc(i(x2), x1, i(x1))) = op_l(x1, x2, x1)$

$mult(x1, \underbrace{asoc(i(x2), x1, i(x1))})$

$=$ by Lemma 210 RL with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$mult(x1, \overbrace{mult(x2, \underbrace{op_r(i(x2), x1, i(x1))})})$

$=$ by Lemma 325 RL with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$mult(x1, mult(x2, \overbrace{mult(i(mult(x2, x1)), x1)}))$

$=$ by Lemma 13 RL with $\{x2 \leftarrow i(mult(x2, x1)), x1 \leftarrow x1\}$

$\underbrace{mult(x1, mult(x2, \overbrace{mult(x1, op_t(i(mult(x2, x1)), x1))}))}$

$=$ by Axiom 6 LR with $\{x3 \leftarrow op_t(i(mult(x2, x1)), x1), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{\underbrace{mult(mult(x1, mult(x2, x1)), op_t(i(mult(x2, x1)), x1))}}$

$=$ by Lemma 466 LR with $\{x2 \leftarrow mult(x2, x1), x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_l(x1, mult(x2, x1), x1)}}$

$=$ by Lemma 285 LR with $\{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_l(x1, x2, x1)}$

Lemma 487: $rd(x1, op_l(x1, x2, x1)) = asoc(x2, x1, i(x1))$

$\underbrace{rd(x1, op_l(x1, x2, x1))}$

$=$ by Lemma 176 LR with $\{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{mult(x1, i(\underbrace{op_l(x1, x2, x1)}))}$

$=$ by Lemma 486 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{mult(x1, i(\overbrace{mult(x1, asoc(i(x2), x1, i(x1)))}))}$

$=$ by Lemma 20 LR with $\{x2 \leftarrow asoc(i(x2), x1, i(x1)), x1 \leftarrow x1\}$

$\overbrace{\underbrace{i(asoc(i(x2), x1, i(x1)))}}$

$=$ by Lemma 145 RL with $\{x2 \leftarrow x1, x1 \leftarrow i(x2)\}$

$\overbrace{\underbrace{asoc(i(i(x2)), i(x1), x1)}}$

$=$ by Lemma 322 RL with $\{x2 \leftarrow x1, x1 \leftarrow i(i(x2))\}$

$\overbrace{asoc(\underbrace{i(i(x2))}, x1, i(x1))}$

$=$ by Lemma 3 LR with $\{x1 \leftarrow x2\}$

$asoc(\overbrace{x2}, x1, i(x1))$

Lemma 488: $asoc(mult(x1,x2),i(x1),x1) = rd(i(x1),op_l(i(x1),x1,x2))$

$$
\begin{aligned}
& asoc(mult(x1,x2),i(x1),\underbrace{x1}) \\
= & \quad \text{by Lemma 3 RL with } \{x1 \leftarrow x1\} \\
& \underbrace{asoc(mult(x1,x2),i(x1),\overbrace{i(i(x1))})} \\
= & \quad \text{by Lemma 487 RL with } \{x2 \leftarrow mult(x1,x2), x1 \leftarrow i(x1)\} \\
& \overbrace{rd(i(x1),\underbrace{op_l(i(x1),mult(x1,x2),i(x1))})} \\
= & \quad \text{by Lemma 242 LR with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow i(x1)\} \\
& rd(i(x1),\overbrace{op_l(i(x1),x1,x2)})
\end{aligned}
$$

Lemma 489: $asoc(mult(x1,x2),x1,i(x1)) = asoc(x1,x2,x1)$

$$
\begin{aligned}
& \underbrace{asoc(mult(x1,x2),x1,i(x1))} \\
= & \quad \text{by Lemma 322 LR with } \{x2 \leftarrow x1, x1 \leftarrow mult(x1,x2)\} \\
& \overbrace{\underbrace{asoc(mult(x1,x2),i(x1),x1)}} \\
= & \quad \text{by Lemma 488 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{rd(i(x1),op_l(i(x1),x1,x2))}} \\
= & \quad \text{by Lemma 176 LR with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow i(x1)\} \\
& \overbrace{mult(i(x1),\underbrace{i(op_l(i(x1),x1,x2))})} \\
= & \quad \text{by Lemma 436 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{mult(i(x1),\overbrace{op_l(x1,x1,x2)})} \\
= & \quad \text{by Lemma 171 LR with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{rd(op_l(x1,x1,x2),x1)}} \\
= & \quad \text{by Lemma 172 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{asoc(x1,x2,x1)}
\end{aligned}
$$

Lemma 490: $rd(x1,asoc(x1,x2,x1)) = op_l(x1,mult(x1,x2),x1)$

$$
\begin{aligned}
& rd(x1,\underbrace{asoc(x1,x2,x1)}) \\
= & \quad \text{by Lemma 489 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& rd(x1,\underbrace{\overbrace{asoc(mult(x1,x2),x1,i(x1))}}) \\
= & \quad \text{by Lemma 487 RL with } \{x2 \leftarrow mult(x1,x2), x1 \leftarrow x1\} \\
& \underbrace{rd(x1,\overbrace{rd(x1,op_l(x1,mult(x1,x2),x1))})} \\
= & \quad \text{by Lemma 353 LR with } \{x2 \leftarrow op_l(x1,mult(x1,x2),x1), x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_r(op_l(x1,mult(x1,x2),x1),x1,x1)}} \\
= & \quad \text{by Axiom 18 LR with } \{x5 \leftarrow x1, x4 \leftarrow x1, x3 \leftarrow x1, x2 \leftarrow mult(x1,x2), x1 \leftarrow x1\} \\
& \overbrace{op_l(\underbrace{op_r(x1,x1,x1)},mult(x1,x2),x1)} \\
= & \quad \text{by Lemma 26 LR with } \{x2 \leftarrow x1, x1 \leftarrow x1\} \\
& op_l(\overbrace{x1},mult(x1,x2),x1)
\end{aligned}
$$

Lemma 491: $mult(x1, asoc(x2, x1, i(x1))) = op_l(x1, x2, i(x1))$

$$
\begin{aligned}
& mult(x1, asoc(\underbrace{x2}, x1, i(x1))) \\
= \quad & \text{by Lemma 3 RL with } \{x1 \leftarrow x2\} \\
& \underbrace{mult(x1, asoc(\overbrace{i(i(x2))}, x1, i(x1)))} \\
= \quad & \text{by Lemma 486 LR with } \{x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, i(x2), x1)}} \\
= \quad & \text{by Lemma 432 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x2, i(x1))}
\end{aligned}
$$

Lemma 492: $asoc(mult(x1, x2), x2, i(x2)) = asoc(x1, x2, i(x2))$

$$
\begin{aligned}
& \underbrace{asoc(mult(x1, x2), x2, i(x2))} \\
= \quad & \text{by Lemma 487 RL with } \{x2 \leftarrow mult(x1, x2), x1 \leftarrow x2\} \\
& \overbrace{rd(x2, \underbrace{op_l(x2, mult(x1, x2), x2)})} \\
= \quad & \text{by Lemma 285 LR with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \underbrace{rd(x2, \overbrace{op_l(x2, x1, x2)})} \\
= \quad & \text{by Lemma 487 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \overbrace{asoc(x1, x2, i(x2))}
\end{aligned}
$$

Lemma 493: $op_l(x1, x1, op_t(x2, x1)) = op_l(x1, x2, i(x1))$

$$
\begin{aligned}
& op_l(x1, \underbrace{x1}, op_t(x2, x1)) \\
= \quad & \text{by Lemma 3 RL with } \{x1 \leftarrow x1\} \\
& op_l(x1, \overbrace{i(i(x1))}, \underbrace{op_t(x2, x1)}) \\
= \quad & \text{by Axiom 10 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{op_l(x1, i(i(x1)), \overbrace{mult(i(x1), mult(x2, x1))})} \\
= \quad & \text{by Lemma 241 LR with } \{x3 \leftarrow mult(x2, x1), x2 \leftarrow i(x1), x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, mult(x2, x1), i(x1))}} \\
= \quad & \text{by Lemma 491 RL with } \{x2 \leftarrow mult(x2, x1), x1 \leftarrow x1\} \\
& \overbrace{mult(x1, \underbrace{asoc(mult(x2, x1), x1, i(x1)))}} \\
= \quad & \text{by Lemma 492 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \underbrace{mult(x1, \overbrace{asoc(x2, x1, i(x1))})} \\
= \quad & \text{by Lemma 491 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x2, i(x1))}
\end{aligned}
$$

Lemma 494: $op_l(x1, rd(x1, x2), x1) = op_l(x1, x2, i(x1))$

$$
\begin{aligned}
& op_l(x1, \underbrace{rd(x1, x2)}, x1) \\
= & \quad \text{by Lemma 10 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \underbrace{op_l(x1, \overbrace{i(rd(x2, x1))}, x1)} \\
= & \quad \text{by Lemma 432 RL with } \{x2 \leftarrow rd(x2, x1), x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, rd(x2, x1), i(x1))}} \\
= & \quad \text{by Lemma 493 RL with } \{x2 \leftarrow rd(x2, x1), x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x1, \underbrace{op_t(rd(x2, x1), x1)})} \\
= & \quad \text{by Lemma 14 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \underbrace{op_l(x1, x1, \overbrace{mult(i(x1), x2)})} \\
= & \quad \text{by Lemma 243 LR with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x2, i(x1))}
\end{aligned}
$$

Lemma 495: $op_l(x1, x1, rd(x1, x2)) = op_l(x1, x1, mult(mult(i(x2), x1), x1))$

$$
\begin{aligned}
& \underbrace{op_l(x1, x1, rd(x1, x2))} \\
= & \quad \text{by Lemma 242 RL with } \{x3 \leftarrow rd(x1, x2), x2 \leftarrow x1, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, mult(x1, rd(x1, x2)), i(x1))}} \\
= & \quad \text{by Lemma 493 RL with } \{x2 \leftarrow mult(x1, rd(x1, x2)), x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x1, \underbrace{op_t(mult(x1, rd(x1, x2)), x1)})} \\
= & \quad \text{by Lemma 406 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_l(x1, x1, \overbrace{mult(mult(i(x2), x1), x1)})
\end{aligned}
$$

Lemma 496: $op_l(x1, x1, rd(x1, x2)) = op_l(x1, x1, i(x2))$

$$
\begin{aligned}
& \underbrace{op_l(x1, x1, rd(x1, x2))} \\
= & \quad \text{by Lemma 495 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, x1, mult(mult(i(x2), x1), x1))}} \\
= & \quad \text{by Lemma 291 LR with } \{x3 \leftarrow mult(i(x2), x1), x2 \leftarrow x1, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, x1, mult(i(x2), x1))}} \\
= & \quad \text{by Lemma 291 LR with } \{x3 \leftarrow i(x2), x2 \leftarrow x1, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x1, i(x2))}
\end{aligned}
$$

Lemma 497: $op_l(x1, op_t(x2, x1), x1) = op_l(x1, x1, i(x2))$

$$
\begin{aligned}
& \underbrace{op_l(x1, op_t(x2, x1), x1)} \\
= \quad & \text{by Lemma 246 RL with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, i(x1), mult(x2, x1))}} \\
= \quad & \text{by Lemma 429 LR with } \{x2 \leftarrow mult(x2, x1), x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, x1, i(mult(x2, x1)))}} \\
= \quad & \text{by Lemma 496 RL with } \{x2 \leftarrow mult(x2, x1), x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x1, \underbrace{rd(x1, mult(x2, x1))})} \\
= \quad & \text{by Lemma 9 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_l(x1, x1, \overbrace{i(x2)})
\end{aligned}
$$

Lemma 498: $asoc(x1, rd(x1, x2), x1) = asoc(x1, i(x2), x1)$

$$
\begin{aligned}
& \underbrace{asoc(x1, rd(x1, x2), x1)} \\
= \quad & \text{by Lemma 172 RL with } \{x2 \leftarrow rd(x1, x2), x1 \leftarrow x1\} \\
& \overbrace{\underbrace{rd(op_l(x1, x1, rd(x1, x2)), x1)}} \\
= \quad & \text{by Lemma 171 RL with } \{x3 \leftarrow rd(x1, x2), x2 \leftarrow x1, x1 \leftarrow x1\} \\
& \overbrace{mult(i(x1), \underbrace{op_l(x1, x1, rd(x1, x2))})} \\
= \quad & \text{by Lemma 496 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{mult(i(x1), \overbrace{op_l(x1, x1, i(x2))})} \\
= \quad & \text{by Lemma 171 LR with } \{x3 \leftarrow i(x2), x2 \leftarrow x1, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{rd(op_l(x1, x1, i(x2)), x1)}} \\
= \quad & \text{by Lemma 172 LR with } \{x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& \overbrace{asoc(x1, i(x2), x1)}
\end{aligned}
$$

Lemma 499: $op_l(x1, rd(x2, i(x1)), x1) = op_l(x1, x2, x1)$

$$
\begin{aligned}
& op_l(x1, \underbrace{rd(x2, i(x1))}, x1) \\
= & \quad \text{by Lemma 10 RL with } \{x2 \leftarrow x2, x1 \leftarrow i(x1)\} \\
& \underbrace{op_l(x1, \overbrace{i(rd(i(x1), x2))}, x1)} \\
= & \quad \text{by Lemma 432 RL with } \{x2 \leftarrow rd(i(x1), x2), x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, rd(i(x1), x2), i(x1))}} \\
= & \quad \text{by Lemma 493 RL with } \{x2 \leftarrow rd(i(x1), x2), x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x1, \underbrace{op_t(rd(i(x1), x2), x1)})} \\
= & \quad \text{by Lemma 33 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{op_l(x1, x1, \overbrace{i(mult(x1, x2))})} \\
= & \quad \text{by Lemma 245 LR with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, i(x2), i(x1))}} \\
= & \quad \text{by Lemma 438 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x2, x1)}
\end{aligned}
$$

Lemma 500: $op_l(x1, i(x2), i(x1)) = op_l(x1, op_r(x2, x1, x1), x1)$

$$
\begin{aligned}
& \underbrace{op_l(x1, i(x2), i(x1))} \\
= & \quad \text{by Lemma 299 RL with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, rd(x1, x2), i(x1))}} \\
= & \quad \text{by Lemma 494 RL with } \{x2 \leftarrow rd(x1, x2), x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, \underbrace{rd(x1, rd(x1, x2))}, x1)} \\
= & \quad \text{by Lemma 353 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_l(x1, \overbrace{op_r(x2, x1, x1)}, x1)
\end{aligned}
$$

Lemma 501: $op_l(x1, x1, op_r(x2, x1, x1)) = op_l(x1, x1, x2)$

$$
\begin{aligned}
& op_l(x1, x1, \underbrace{op_r(x2, x1, x1)}) \\
= & \quad \text{by Lemma 353 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{op_l(x1, x1, \overbrace{rd(x1, rd(x1, x2))})} \\
= & \quad \text{by Lemma 496 LR with } \{x2 \leftarrow rd(x1, x2), x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x1, \underbrace{i(rd(x1, x2))})} \\
= & \quad \text{by Lemma 10 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{op_l(x1, x1, \overbrace{rd(x2, x1)})} \\
= & \quad \text{by Lemma 286 LR with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x1, x2)}
\end{aligned}
$$

Lemma 502: $asoc(x1, op_r(x2, x1, x1), x1) = asoc(x1, x2, x1)$

$$
\begin{aligned}
& asoc(x1, \underbrace{op_r(x2, x1, x1)}, x1) \\
= & \quad \text{by Lemma 353 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{asoc(x1, \overbrace{rd(x1, rd(x1, x2))}, x1)} \\
= & \quad \text{by Lemma 498 LR with } \{x2 \leftarrow rd(x1, x2), x1 \leftarrow x1\} \\
& \overbrace{asoc(x1, \underbrace{i(rd(x1, x2))}, x1)} \\
= & \quad \text{by Lemma 10 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{asoc(x1, \overbrace{rd(x2, x1)}, x1)} \\
= & \quad \text{by Lemma 263 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{asoc(x1, x2, x1)}
\end{aligned}
$$

Lemma 503: $op_r(op_l(i(x1), x2, x1), x1, x1) = rd(asoc(x2, x1, i(x1)), x1)$

$$
\begin{aligned}
& op_r(\underbrace{op_l(i(x1), x2, x1)}, x1, x1) \\
= & \quad \text{by Lemma 433 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{op_r(\overbrace{i(op_l(x1, x2, x1))}, x1, x1)} \\
= & \quad \text{by Lemma 354 RL with } \{x2 \leftarrow op_l(x1, x2, x1), x1 \leftarrow x1\} \\
& \overbrace{rd(\underbrace{rd(x1, op_l(x1, x2, x1))}, x1)} \\
= & \quad \text{by Lemma 487 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& rd(\overbrace{asoc(x2, x1, i(x1))}, x1)
\end{aligned}
$$

Lemma 504: $asoc(x1, mult(i(x1), x2), x1) = asoc(x2, x1, i(x1))$

$$
\begin{aligned}
& \underbrace{asoc(x1, mult(i(x1), x2), x1)} \\
= & \quad \text{by Lemma 248 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{rd(\underbrace{op_l(x1, x2, i(x1))}, x1)} \\
= & \quad \text{by Lemma 432 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{rd(\overbrace{op_l(x1, i(x2), x1)}, x1)} \\
= & \quad \text{by Lemma 171 RL with } \{x3 \leftarrow x1, x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& \overbrace{mult(i(x1), \underbrace{op_l(x1, i(x2), x1)})} \\
= & \quad \text{by Lemma 486 RL with } \{x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& \underbrace{mult(i(x1), \overbrace{mult(x1, asoc(i(i(x2)), x1, i(x1)))})} \\
= & \quad \text{by Axiom 2 RL with } \{x2 \leftarrow x1, x1 \leftarrow asoc(i(i(x2)), x1, i(x1))\} \\
& \overbrace{asoc(\underbrace{i(i(x2))}, x1, i(x1))} \\
= & \quad \text{by Lemma 3 LR with } \{x1 \leftarrow x2\} \\
& asoc(\overbrace{x2}, x1, i(x1))
\end{aligned}
$$

Lemma 505: $asoc(rd(x1,x2),i(x1),x1) = rd(i(x1),op_l(i(x1),i(x2),i(x1)))$

$$
\begin{aligned}
& asoc(rd(x1,x2),i(x1),\underbrace{x1}) \\
= & \quad \text{by Lemma 3 RL with } \{x1 \leftarrow x1\} \\
& \underbrace{asoc(rd(x1,x2),i(x1),\overbrace{i(i(x1))})} \\
= & \quad \text{by Lemma 487 RL with } \{x2 \leftarrow rd(x1,x2), x1 \leftarrow i(x1)\} \\
& \overbrace{rd(i(x1),\underbrace{op_l(i(x1),rd(x1,x2),i(x1))})} \\
= & \quad \text{by Lemma 299 LR with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow i(x1)\} \\
& rd(i(x1),\overbrace{op_l(i(x1),i(x2),i(x1))})
\end{aligned}
$$

Lemma 506: $rd(asoc(x1,x2,x1),x1) = op_l(i(x1),mult(x1,x2),x1)$

$$
\begin{aligned}
& \underbrace{rd(asoc(x1,x2,x1),x1)} \\
= & \quad \text{by Lemma 10 RL with } \{x2 \leftarrow asoc(x1,x2,x1), x1 \leftarrow x1\} \\
& \overbrace{i(\underbrace{rd(x1,asoc(x1,x2,x1))})} \\
= & \quad \text{by Lemma 490 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{i(\overbrace{op_l(x1,mult(x1,x2),x1)})} \\
= & \quad \text{by Lemma 433 LR with } \{x2 \leftarrow mult(x1,x2), x1 \leftarrow x1\} \\
& \overbrace{op_l(i(x1),mult(x1,x2),x1)}
\end{aligned}
$$

Lemma 507: $op_l(i(x1),rd(x1,x2),x1) = op_t(i(op_l(x1,x2,i(x1))),x1)$

$$
\begin{aligned}
& \underbrace{op_l(i(x1),rd(x1,x2),x1)} \\
= & \quad \text{by Lemma 433 RL with } \{x2 \leftarrow rd(x1,x2), x1 \leftarrow x1\} \\
& \underbrace{\overbrace{i(op_l(x1,rd(x1,x2),x1))}} \\
= & \quad \text{by Lemma 193 RL with } \{x3 \leftarrow x1, x2 \leftarrow rd(x1,x2), x1 \leftarrow x1\} \\
& \overbrace{op_t(i(\underbrace{op_l(x1,rd(x1,x2),x1)}),x1)} \\
= & \quad \text{by Lemma 494 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_t(i(\overbrace{op_l(x1,x2,i(x1))}),x1)
\end{aligned}
$$

Lemma 508: $op_l(i(x1),rd(x1,x2),x1) = op_l(i(x1),x2,i(x1))$

$$
\begin{aligned}
& \underbrace{op_l(i(x1),rd(x1,x2),x1)} \\
= & \quad \text{by Lemma 507 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{\overbrace{op_t(i(op_l(x1,x2,i(x1))),x1)}} \\
= & \quad \text{by Lemma 193 LR with } \{x3 \leftarrow i(x1), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{\overbrace{i(op_l(x1,x2,i(x1)))}} \\
= & \quad \text{by Lemma 268 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{\overbrace{op_l(i(x1),i(x2),x1)}} \\
= & \quad \text{by Lemma 444 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(i(x1),x2,i(x1))}
\end{aligned}
$$

Lemma 509: $op_l(i(x1), x1, i(i(i(x2)))) = op_l(i(x1), i(op_t(x2, x1)), i(x1))$

$$
\begin{aligned}
& \overbrace{op_l(i(x1), x1, i(i(i(x2))))} \\
= & \quad \text{by Lemma 443 RL with } \{x2 \leftarrow i(i(x2)), x1 \leftarrow x1\} \\
& \overbrace{\overbrace{op_l(i(x1), i(x1), i(i(x2)))}} \\
= & \quad \text{by Lemma 497 RL with } \{x2 \leftarrow i(x2), x1 \leftarrow i(x1)\} \\
& \overbrace{op_l(i(x1), \underbrace{op_t(i(x2), i(x1))}, i(x1))} \\
= & \quad \text{by Lemma 15 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& op_l(i(x1), \overbrace{i(op_t(x2, x1))}, i(x1))
\end{aligned}
$$

Lemma 510: $rd(i(x1), op_t(x2, rd(x1, x2))) = i(mult(x2, x1))$

$$
\begin{aligned}
& rd(i(x1), op_t(x2, \underbrace{rd(x1, x2)})) \\
= & \quad \text{by Lemma 23 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& rd(i(x1), op_t(\underbrace{x2}, \overbrace{rd(i(x2), i(x1))})) \\
= & \quad \text{by Lemma 3 RL with } \{x1 \leftarrow x2\} \\
& \overbrace{rd(i(x1), op_t(\overbrace{i(i(x2))}, rd(i(x2), i(x1))))} \\
= & \quad \text{by Lemma 386 LR with } \{x2 \leftarrow i(x2), x1 \leftarrow i(x1)\} \\
& \overbrace{\overbrace{mult(i(x2), i(x1))}} \\
= & \quad \text{by Axiom 11 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \overbrace{i(mult(x2, x1))}
\end{aligned}
$$

Lemma 511: $op_l(op_t(x1, rd(x2, x1)), x1, x2) = op_t(x1, i(x2))$

$$
\begin{aligned}
& op_l(op_t(x1, rd(x2, x1)), \underbrace{x1}, x2) \\
= & \quad \text{by Lemma 368 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_l(\underbrace{op_t(x1, rd(x2, x1))}, \overbrace{op_r(op_t(x1, rd(x2, x1)), x2, x2)}, x2) \\
= & \quad \text{by Lemma 351 RL with } \{x2 \leftarrow x2, x1 \leftarrow op_t(x1, rd(x2, x1))\} \\
& \underbrace{op_l(\overbrace{op_r(op_r(op_t(x1, rd(x2, x1)), x2, x2), x2, i(x2))}, op_r(op_t(x1, rd(x2, x1)), x2, x2), x2)} \\
= & \quad \text{by Lemma 475 LR with } \{x2 \leftarrow x2, x1 \leftarrow op_r(op_t(x1, rd(x2, x1)), x2, x2)\} \\
& \overbrace{\overbrace{op_t(op_r(op_t(x1, rd(x2, x1)), x2, x2), i(x2))}} \\
= & \quad \text{by Axiom 14 LR with } \{x4 \leftarrow i(x2), x3 \leftarrow x2, x2 \leftarrow x2, x1 \leftarrow op_t(x1, rd(x2, x1))\} \\
& \overbrace{\overbrace{op_r(op_t(op_t(x1, rd(x2, x1)), i(x2)), x2, x2)}} \\
= & \quad \text{by Lemma 381 LR with } \{x2 \leftarrow x2, x1 \leftarrow op_t(x1, rd(x2, x1))\} \\
& \overbrace{mult(x2, \underbrace{rd(op_t(x1, rd(x2, x1)), x2)})} \\
= & \quad \text{by Lemma 372 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{mult(x2, \overbrace{mult(x1, i(x2))})} \\
= & \quad \text{by Lemma 30 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \overbrace{op_t(x1, i(x2))}
\end{aligned}
$$

Lemma 512: $op_l(x1, x1, mult(x2, x1)) = op_l(x1, x1, mult(x2, mult(x1, x1)))$

$$
\begin{aligned}
& \underbrace{op_l(x1, x1, mult(x2, x1))} \\
= \quad & \text{by Lemma 242 RL with } \{x3 \leftarrow mult(x2, x1), x2 \leftarrow x1, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, mult(x1, mult(x2, x1)), i(x1))}} \\
= \quad & \text{by Lemma 493 RL with } \{x2 \leftarrow mult(x1, mult(x2, x1)), x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x1, \underbrace{op_t(mult(x1, mult(x2, x1)), x1)})} \\
= \quad & \text{by Lemma 200 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_l(x1, x1, \overbrace{mult(x2, mult(x1, x1))})
\end{aligned}
$$

Lemma 513: $op_l(x1, x1, rd(x2, x1)) = op_l(x1, x1, op_r(x2, x1, i(x1)))$

$$
\begin{aligned}
& \underbrace{op_l(x1, x1, rd(x2, x1))} \\
= \quad & \text{by Lemma 242 RL with } \{x3 \leftarrow rd(x2, x1), x2 \leftarrow x1, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, mult(x1, rd(x2, x1)), i(x1))}} \\
= \quad & \text{by Lemma 493 RL with } \{x2 \leftarrow mult(x1, rd(x2, x1)), x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x1, \underbrace{op_t(mult(x1, rd(x2, x1)), x1)})} \\
= \quad & \text{by Lemma 349 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_l(x1, x1, \overbrace{op_r(x2, x1, i(x1))})
\end{aligned}
$$

Lemma 514: $op_l(x1, x1, op_t(x2, i(x1))) = op_l(x1, x1, mult(x1, x2))$

$$
\begin{aligned}
& op_l(x1, x1, \underbrace{op_t(x2, i(x1))}) \\
= \quad & \text{by Lemma 18 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \underbrace{op_l(x1, x1, \overbrace{i(op_t(i(x2), x1))})} \\
= \quad & \text{by Lemma 496 RL with } \{x2 \leftarrow op_t(i(x2), x1), x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x1, \underbrace{rd(x1, op_t(i(x2), x1))})} \\
= \quad & \text{by Lemma 405 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{op_l(x1, x1, \overbrace{op_r(mult(x1, x2), x1, i(x1))})} \\
= \quad & \text{by Lemma 513 RL with } \{x2 \leftarrow mult(x1, x2), x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, x1, rd(mult(x1, x2), x1))}} \\
= \quad & \text{by Lemma 286 LR with } \{x3 \leftarrow mult(x1, x2), x2 \leftarrow x1, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x1, mult(x1, x2))}
\end{aligned}
$$

Lemma 515: $asoc(x1, op_t(x2, i(x1)), x1) = mult(i(x1), op_l(x1, x1, mult(x1, x2)))$

$$
\begin{aligned}
& \underbrace{asoc(x1, op_t(x2, i(x1)), x1)} \\
= \quad & \text{by Lemma 172 RL with } \{x2 \leftarrow op_t(x2, i(x1)), x1 \leftarrow x1\} \\
& \overbrace{\underbrace{rd(op_l(x1, x1, op_t(x2, i(x1))), x1)}} \\
= \quad & \text{by Lemma 171 RL with } \{x3 \leftarrow op_t(x2, i(x1)), x2 \leftarrow x1, x1 \leftarrow x1\} \\
& \overbrace{mult(i(x1), \underbrace{op_l(x1, x1, op_t(x2, i(x1)))})} \\
= \quad & \text{by Lemma 514 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& mult(i(x1), \overbrace{op_l(x1, x1, mult(x1, x2))})
\end{aligned}
$$

Lemma 516: $asoc(x1, op_t(x2, i(x1)), x1) = asoc(x1, mult(x1, x2), x1)$

$$
\begin{aligned}
& \underbrace{asoc(x1, op_t(x2, i(x1)), x1)} \\
= \quad & \text{by Lemma 515 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{mult(i(x1), op_l(x1, x1, mult(x1, x2)))}} \\
= \quad & \text{by Lemma 171 LR with } \{x3 \leftarrow mult(x1, x2), x2 \leftarrow x1, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{rd(op_l(x1, x1, mult(x1, x2)), x1)}} \\
= \quad & \text{by Lemma 172 LR with } \{x2 \leftarrow mult(x1, x2), x1 \leftarrow x1\} \\
& \overbrace{asoc(x1, mult(x1, x2), x1)}
\end{aligned}
$$

Lemma 517: $i(op_l(x1, i(x2), i(x3))) = op_l(i(x1), x2, x3)$

$$
\begin{aligned}
& i(\underbrace{op_l(x1, i(x2), i(x3))}) \\
= \quad & \text{by Axiom 12 RL with } \{x3 \leftarrow x1, x2 \leftarrow i(x2), x1 \leftarrow i(x3)\} \\
& i(\overbrace{mult(i(\underbrace{mult(i(x3), i(x2))}), mult(i(x3), mult(i(x2), x1)))}) \\
= \quad & \text{by Axiom 11 LR with } \{x2 \leftarrow x2, x1 \leftarrow x3\} \\
& i(mult(i(\overbrace{i(mult(x3, x2))}), mult(i(x3), mult(i(x2), x1)))) \\
= \quad & \text{by Lemma 3 LR with } \{x1 \leftarrow mult(x3, x2)\} \\
& \underbrace{i(mult(\overbrace{mult(x3, x2)}, mult(i(x3), mult(i(x2), x1))))} \\
= \quad & \text{by Lemma 87 LR with } \{x3 \leftarrow mult(i(x2), x1), x2 \leftarrow x3, x1 \leftarrow mult(x3, x2)\} \\
& \overbrace{mult(i(mult(x3, x2)), mult(x3, \underbrace{i(mult(i(x2), x1))}))} \\
= \quad & \text{by Lemma 16 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \underbrace{mult(i(mult(x3, x2)), mult(x3, \overbrace{mult(x2, i(x1))}))} \\
= \quad & \text{by Axiom 12 LR with } \{x3 \leftarrow i(x1), x2 \leftarrow x2, x1 \leftarrow x3\} \\
& \overbrace{op_l(i(x1), x2, x3)}
\end{aligned}
$$

Lemma 518: $i(op_l(i(x1), x2, x3)) = op_l(x1, i(x2), i(x3))$

$$
\begin{aligned}
& i(\underbrace{op_l(i(x1), x2, x3)}) \\
= \quad & \text{by Lemma 517 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{i(\overbrace{i(op_l(x1, i(x2), i(x3)))})} \\
= \quad & \text{by Lemma 3 LR with } \{x1 \leftarrow op_l(x1, i(x2), i(x3))\} \\
& \overbrace{op_l(x1, i(x2), i(x3))}
\end{aligned}
$$

Lemma 519: $op_l(i(x1), i(x2), i(x3)) = i(op_l(x1, x2, x3))$

$$
\begin{aligned}
& \underbrace{op_l(i(x1), i(x2), i(x3))} \\
= \quad & \text{by Lemma 518 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow i(x1)\} \\
& \overbrace{i(op_l(\underbrace{i(i(x1))}, x2, x3))} \\
= \quad & \text{by Lemma 3 LR with } \{x1 \leftarrow x1\} \\
& i(op_l(\overbrace{x1}, x2, x3))
\end{aligned}
$$

Lemma 520: $op_r(x1, x2, x2) = rd(op_t(op_r(mult(x2, x1), x2, x2), x2), op_l(x2, x2, x1))$

$$
\begin{aligned}
& \underbrace{op_r(x1, x2, x2)} \\
= \quad & \text{by Lemma 283 RL with } \{x2 \leftarrow op_r(x1, x2, x2), x1 \leftarrow x2\} \\
& \overbrace{rd(op_t(mult(x2, op_r(x1, x2, x2)), x2), \underbrace{op_l(x2, x2, op_r(x1, x2, x2))})} \\
= \quad & \text{by Lemma 501 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& rd(op_t(\underbrace{mult(x2, op_r(x1, x2, x2))}, x2), \overbrace{op_l(x2, x2, x1)}) \\
= \quad & \text{by Lemma 383 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& rd(op_t(\overbrace{op_r(mult(x2, x1), x2, x2)}, x2), op_l(x2, x2, x1))
\end{aligned}
$$

Lemma 521: $op_l(x1, i(x1), i(x2)) = op_l(x1, op_t(x2, i(x1)), i(x1))$

$$
\begin{aligned}
& op_l(\underbrace{x1}, i(x1), i(x2)) \\
= \quad & \text{by Lemma 3 RL with } \{x1 \leftarrow x1\} \\
& op_l(\overbrace{i(i(x1))}, i(x1), \underbrace{i(x2)}) \\
= \quad & \text{by Lemma 3 RL with } \{x1 \leftarrow i(x2)\} \\
& \underbrace{op_l(i(i(x1)), i(x1), \overbrace{i(i(i(x2)))})} \\
= \quad & \text{by Lemma 509 LR with } \{x2 \leftarrow x2, x1 \leftarrow i(x1)\} \\
& \underbrace{\overbrace{op_l(i(i(x1)), i(op_t(x2, i(x1))), i(i(x1)))}} \\
= \quad & \text{by Lemma 432 RL with } \{x2 \leftarrow op_t(x2, i(x1)), x1 \leftarrow i(i(x1))\} \\
& \overbrace{op_l(i(i(x1)), op_t(x2, i(x1)), \underbrace{i(i(i(x1)))})} \\
= \quad & \text{by Lemma 3 LR with } \{x1 \leftarrow i(x1)\} \\
& op_l(\underbrace{i(i(x1))}, op_t(x2, i(x1)), \overbrace{i(x1)}) \\
= \quad & \text{by Lemma 3 LR with } \{x1 \leftarrow x1\} \\
& op_l(\overbrace{x1}, op_t(x2, i(x1)), i(x1))
\end{aligned}
$$

Lemma 522: $op_l(x1, x1, x2) = op_l(x1, op_t(x2, i(x1)), i(x1))$

$op_l(x1, x1, \underbrace{x2}) $

$=$ by Lemma 3 RL with $\{x1 \leftarrow x2\}$

$\underbrace{op_l(x1, x1, \overbrace{i(i(x2))})}$

$=$ by Lemma 429 RL with $\{x2 \leftarrow i(x2), x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_l(x1, i(x1), i(x2))}}$

$=$ by Lemma 521 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_l(x1, op_t(x2, i(x1)), i(x1))}$

Lemma 523: $op_l(x1, rd(i(x2), x1), i(x2)) = op_l(x1, x1, i(i(x2)))$

$\underbrace{op_l(x1, rd(i(x2), x1), i(x2))}$

$=$ by Lemma 226 LR with $\{x1 \leftarrow x2, x2 \leftarrow op_l(x1, rd(i(x2), x1), i(x2))\}$

$\overbrace{op_l(mult(\underbrace{mult(i(x2), op_l(x1, rd(i(x2), x1), i(x2)))}, x2), x2, i(op_l(x1, rd(i(x2), x1), i(x2))))}$

$=$ by Lemma 57 RL with $\{x2 \leftarrow op_l(x1, rd(i(x2), x1), i(x2)), x1 \leftarrow i(x2)\}$

$op_l(mult(\overbrace{rd(i(x2), mult(i(i(x2)), \underbrace{rd(i(x2), op_l(x1, rd(i(x2), x1), i(x2)))}))}, x2), x2, i(op_l(x1, rd(i(x2), x1), i(x2))))$

$=$ by Lemma 473 RL with $\{x2 \leftarrow x1, x1 \leftarrow i(x2)\}$

$op_l(mult(rd(i(x2), \underbrace{mult(i(i(x2)), \overbrace{mult(i(x2), i(x1))})}), x2), x2, i(op_l(x1, rd(i(x2), x1), i(x2))))$

$=$ by Axiom 2 RL with $\{x2 \leftarrow i(x2), x1 \leftarrow i(x1)\}$

$op_l(\underbrace{mult(rd(i(x2), \overbrace{i(x1)}), x2)}, x2, i(op_l(x1, rd(i(x2), x1), i(x2))))$

$=$ by Lemma 21 LR with $\{x2 \leftarrow i(x1), x1 \leftarrow x2\}$

$op_l(\overbrace{\underbrace{i(i(x1))}}, x2, i(op_l(x1, rd(i(x2), x1), i(x2))))$

$=$ by Lemma 3 LR with $\{x1 \leftarrow x1\}$

$op_l(\overbrace{x1}, x2, \underbrace{i(op_l(x1, rd(i(x2), x1), i(x2))))}$

$=$ by Lemma 474 LR with $\{x2 \leftarrow i(x2), x1 \leftarrow x1\}$

$op_l(x1, x2, \overbrace{mult(i(x2), rd(i(x1), i(x2)))})$

$=$ by Lemma 69 LR with $\{x2 \leftarrow i(x1), x1 \leftarrow x2\}$

$\underbrace{op_l(x1, x2, \overbrace{mult(mult(i(x2), i(x1)), x2)})}$

$=$ by Lemma 291 LR with $\{x3 \leftarrow mult(i(x2), i(x1)), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_l(x1, x2, mult(i(x2), i(x1)))}}$

$=$ by Lemma 243 LR with $\{x3 \leftarrow i(x1), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_l(x1, i(x1), i(x2))}}$

$=$ by Lemma 429 LR with $\{x2 \leftarrow i(x2), x1 \leftarrow x1\}$

$\overbrace{op_l(x1, x1, i(i(x2)))}$

Lemma 524: $op_l(x1, x1, i(x2)) = op_l(x1, rd(x2, x1), i(i(x2)))$

$$
\begin{aligned}
& op_l(x1, x1, \underbrace{i(x2)}) \\
= & \quad \text{by Lemma 3 RL with } \{x1 \leftarrow i(x2)\} \\
& \underbrace{op_l(x1, x1, \overbrace{i(i(i(x2)))})} \\
= & \quad \text{by Lemma 523 RL with } \{x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, rd(\underbrace{i(i(x2))}, x1), i(i(x2)))} \\
= & \quad \text{by Lemma 3 LR with } \{x1 \leftarrow x2\} \\
& op_l(x1, rd(\overbrace{\ x2\ }, x1), i(i(x2)))
\end{aligned}
$$

Lemma 525: $op_l(x1, rd(x2, x1), x2) = op_l(x1, x1, i(x2))$

$$
\begin{aligned}
& op_l(x1, rd(x2, x1), \underbrace{\ x2\ }) \\
= & \quad \text{by Lemma 3 RL with } \{x1 \leftarrow x2\} \\
& \underbrace{op_l(x1, rd(x2, x1), \overbrace{i(i(x2))})} \\
= & \quad \text{by Lemma 524 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x1, i(x2))}
\end{aligned}
$$

Lemma 526: $mult(op_t(rd(x1, x2), x3), x3) = op_t(rd(x3, rd(x2, x1)), x3)$

$$
\begin{aligned}
& \underbrace{mult(op_t(rd(x1, x2), x3), x3)} \\
= & \quad \text{by Lemma 337 RL with } \{x2 \leftarrow x3, x1 \leftarrow rd(x1, x2)\} \\
& \overbrace{op_t(\underbrace{rd(rd(x1, x2), i(x3))}, x3)} \\
= & \quad \text{by Lemma 39 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_t(\overbrace{rd(x3, rd(x2, x1))}, x3)
\end{aligned}
$$

Lemma 527: $rd(x1, mult(x1, mult(x2, x1))) = op_r(i(mult(x1, x2)), x1, x1)$

$$
\begin{aligned}
& rd(x1, \underbrace{mult(x1, mult(x2, x1))}) \\
= & \quad \text{by Lemma 58 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{rd(x1, \overbrace{rd(x1, i(mult(x1, x2)))})} \\
= & \quad \text{by Lemma 353 LR with } \{x2 \leftarrow i(mult(x1, x2)), x1 \leftarrow x1\} \\
& \overbrace{op_r(i(mult(x1, x2)), x1, x1)}
\end{aligned}
$$

Lemma 528: $op_t(x1, mult(x1, rd(x2, x1))) = op_t(x1, op_r(x2, x1, i(x1)))$

$$
\begin{aligned}
& \underbrace{op_t(x1, mult(x1, rd(x2, x1)))} \\
= & \quad \text{by Lemma 374 RL with } \{x2 \leftarrow mult(x1, rd(x2, x1)), x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, \underbrace{op_t(mult(x1, rd(x2, x1)), x1)})} \\
= & \quad \text{by Lemma 349 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_t(x1, \overbrace{op_r(x2, x1, i(x1))})
\end{aligned}
$$

Lemma 529: $mult(i(mult(x1, x2)), op_l(x2, x2, x1)) = i(x1)$

$$
\begin{aligned}
& mult(i(mult(x1, x2)), \underbrace{op_l(x2, x2, x1)}) \\
= & \quad \text{by Lemma 427 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& mult(i(mult(x1, x2)), \overbrace{mult(x1, rd(x2, x1))}) \\
= & \quad \text{by Lemma 55 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{mult(i(mult(x1, x2)), \overbrace{mult(mult(x1, x2), i(x1))})} \\
= & \quad \text{by Axiom 2 RL with } \{x2 \leftarrow mult(x1, x2), x1 \leftarrow i(x1)\} \\
& \overbrace{i(x1)}
\end{aligned}
$$

Lemma 530: $mult(mult(x1, x2), mult(x1, x2)) = mult(x1, mult(x2, mult(x2, x1)))$

$$
\begin{aligned}
& mult(mult(x1, x2), \underbrace{mult(x1, x2)}) \\
= & \quad \text{by Lemma 435 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \underbrace{mult(mult(x1, x2), \overbrace{op_l(mult(x2, x1), x2, x1)})} \\
= & \quad \text{by Lemma 132 LR with } \{x3 \leftarrow mult(x2, x1), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{mult(x1, mult(x2, mult(x2, x1)))}
\end{aligned}
$$

Lemma 531: $rd(x1, i(asoc(x2, x1, x2))) = op_r(x1, rd(x2, x1), i(x2))$

$$
\begin{aligned}
& \underbrace{rd(x1, i(asoc(x2, x1, x2)))} \\
= & \quad \text{by Lemma 38 RL with } \{x1 \leftarrow x1, x2 \leftarrow asoc(x2, x1, x2)\} \\
& \overbrace{rd(asoc(x2, x1, x2), \underbrace{i(x1)})} \\
= & \quad \text{by Lemma 22 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& rd(\underbrace{asoc(x2, x1, x2)}, \overbrace{mult(rd(x2, x1), i(x2))}) \\
= & \quad \text{by Lemma 482 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \underbrace{rd(\overbrace{mult(mult(x1, rd(x2, x1)), i(x2))}, mult(rd(x2, x1), i(x2)))} \\
= & \quad \text{by Axiom 16 RL with } \{x3 \leftarrow i(x2), x2 \leftarrow rd(x2, x1), x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, rd(x2, x1), i(x2))}
\end{aligned}
$$

Lemma 532: $op_l(x1, x1, asoc(x2, x1, x2)) = x1$

$op_l(x1, x1, \underbrace{asoc(x2, x1, x2)})$

$=$ by Lemma 3 RL with $\{x1 \leftarrow asoc(x2, x1, x2)\}$

$\underbrace{op_l(x1, x1, \overbrace{i(i(asoc(x2, x1, x2)))})}$

$=$ by Lemma 496 RL with $\{x2 \leftarrow i(asoc(x2, x1, x2)), x1 \leftarrow x1\}$

$\overbrace{op_l(x1, x1, \underbrace{rd(x1, i(asoc(x2, x1, x2)))})}$

$=$ by Lemma 531 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$op_l(\underbrace{x1}, x1, \overbrace{op_r(x1, rd(x2, x1), i(x2))})$

$=$ by Axiom 2 LR with $\{x2 \leftarrow mult(x1, op_r(x1, rd(x2, x1), i(x2))), x1 \leftarrow x1\}$

$op_l(\overbrace{mult(i(mult(x1, op_r(x1, rd(x2, x1), i(x2)))), mult(mult(x1, op_r(x1, rd(x2, x1), i(x2))), x1))}, x1, op_r(x1, rd(x2, x1), i(x2$

$=$ by Lemma 239 RL with $\{x3 \leftarrow i(x2), x2 \leftarrow rd(x2, x1), x1 \leftarrow x1\}$

$op_l(\overbrace{op_t(x1, mult(x1, op_r(x1, rd(x2, x1), i(x2))))}, x1, op_r(x1, rd(x2, x1), i(x2)))$

$=$ by Lemma 251 RL with $\{x2 \leftarrow op_r(x1, rd(x2, x1), i(x2)), x1 \leftarrow x1\}$

$op_l(\overbrace{op_l(\underbrace{op_t(x1, op_r(x1, rd(x2, x1), i(x2)))}, op_r(x1, rd(x2, x1), i(x2)), x1)}, x1, op_r(x1, rd(x2, x1), i(x2)))$

$=$ by Lemma 120 LR with $\{x3 \leftarrow i(x2), x2 \leftarrow rd(x2, x1), x1 \leftarrow x1\}$

$\underbrace{op_l(op_l(\overbrace{x1}, op_r(x1, rd(x2, x1), i(x2)), x1), x1, op_r(x1, rd(x2, x1), i(x2)))}$

$=$ by Lemma 138 LR with $\{x3 \leftarrow x1, x2 \leftarrow op_r(x1, rd(x2, x1), i(x2)), x1 \leftarrow x1\}$

$\overbrace{x1}$

Lemma 533: $asoc(i(i(asoc(x1, x2, x1))), x2, i(x2)) = asoc(op_r(x2, rd(x1, x2), i(x1)), x2, i(x2))$

$\underbrace{asoc(i(i(asoc(x1, x2, x1))), x2, i(x2))}$

$=$ by Lemma 322 LR with $\{x2 \leftarrow x2, x1 \leftarrow i(i(asoc(x1, x2, x1)))\}$

$\overbrace{asoc(i(i(asoc(x1, x2, x1))), i(x2), \underbrace{x2})}$

$=$ by Lemma 3 RL with $\{x1 \leftarrow x2\}$

$\underbrace{asoc(i(i(asoc(x1, x2, x1))), i(x2), \overbrace{i(i(x2))})}$

$=$ by Lemma 487 RL with $\{x2 \leftarrow i(i(asoc(x1, x2, x1))), x1 \leftarrow i(x2)\}$

$\overbrace{\underbrace{rd(i(x2), op_l(i(x2), i(i(asoc(x1, x2, x1))), i(x2)))}}$

$=$ by Lemma 505 RL with $\{x2 \leftarrow i(asoc(x1, x2, x1)), x1 \leftarrow x2\}$

$\overbrace{\underbrace{asoc(rd(x2, i(asoc(x1, x2, x1))), i(x2), x2)}}$

$=$ by Lemma 322 RL with $\{x2 \leftarrow x2, x1 \leftarrow rd(x2, i(asoc(x1, x2, x1)))\}$

$\overbrace{asoc(\underbrace{rd(x2, i(asoc(x1, x2, x1)))}, x2, i(x2))}$

$=$ by Lemma 531 LR with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$asoc(\overbrace{op_r(x2, rd(x1, x2), i(x1))}, x2, i(x2))$

Lemma 534: $mult(x1, asoc(x2, x1, x2)) = op_t(op_r(x1, rd(x2, x1), i(x2)), i(x1))$

$$
\begin{aligned}
& mult(x1, \underbrace{asoc(x2, x1, x2)}) \\
= & \quad \text{by Lemma 3 RL with } \{x1 \leftarrow asoc(x2, x1, x2)\} \\
& \underbrace{mult(x1, \overbrace{i(i(asoc(x2, x1, x2)))})} \\
= & \quad \text{by Lemma 34 RL with } \{x2 \leftarrow i(asoc(x2, x1, x2)), x1 \leftarrow x1\} \\
& \overbrace{op_t(\underbrace{rd(x1, i(asoc(x2, x1, x2)))}, i(x1))} \\
= & \quad \text{by Lemma 531 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_t(\overbrace{op_r(x1, rd(x2, x1), i(x2))}, i(x1))
\end{aligned}
$$

Lemma 535: $op_t(x1, asoc(x2, x1, x2)) = x1$

$$
\begin{aligned}
& \underbrace{op_t(x1, asoc(x2, x1, x2))} \\
= & \quad \text{by Lemma 378 RL with } \{x2 \leftarrow asoc(x2, x1, x2), x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, rd(\underbrace{mult(x1, asoc(x2, x1, x2))}, x1))} \\
= & \quad \text{by Lemma 534 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_t(x1, rd(\overbrace{\underbrace{op_t(op_r(x1, rd(x2, x1), i(x2)), i(x1))}}, x1)) \\
= & \quad \text{by Axiom 14 LR with } \{x4 \leftarrow i(x1), x3 \leftarrow i(x2), x2 \leftarrow rd(x2, x1), x1 \leftarrow x1\} \\
& op_t(x1, rd(\overbrace{op_r(\underbrace{op_t(x1, i(x1))}, rd(x2, x1), i(x2))}, x1)) \\
= & \quad \text{by Lemma 8 RL with } \{x1 \leftarrow x1\} \\
& \underbrace{op_t(x1, rd(op_r(\overbrace{x1}, rd(x2, x1), i(x2)), x1))} \\
= & \quad \text{by Axiom 10 RL with } \{x2 \leftarrow x1, x1 \leftarrow rd(op_r(x1, rd(x2, x1), i(x2)), x1)\} \\
& \overbrace{mult(i(rd(op_r(x1, rd(x2, x1), i(x2)), x1)), \underbrace{mult(x1, rd(op_r(x1, rd(x2, x1), i(x2)), x1))})} \\
= & \quad \text{by Lemma 123 LR with } \{x3 \leftarrow i(x2), x2 \leftarrow rd(x2, x1), x1 \leftarrow x1\} \\
& mult(\underbrace{i(rd(op_r(x1, rd(x2, x1), i(x2)), x1))}, \overbrace{op_r(x1, rd(x2, x1), i(x2))}) \\
= & \quad \text{by Lemma 10 LR with } \{x2 \leftarrow x1, x1 \leftarrow op_r(x1, rd(x2, x1), i(x2))\} \\
& \underbrace{mult(\overbrace{rd(x1, op_r(x1, rd(x2, x1), i(x2)))}, op_r(x1, rd(x2, x1), i(x2)))} \\
= & \quad \text{by Axiom 4 RL with } \{x2 \leftarrow op_r(x1, rd(x2, x1), i(x2)), x1 \leftarrow x1\} \\
& \overbrace{x1}
\end{aligned}
$$

Lemma 536: $op_t(x1, rd(asoc(x2, x1, x2), x1)) = op_l(x1, asoc(x2, x1, x2), i(x1))$

$$
\begin{aligned}
& \underbrace{op_t(x1, rd(asoc(x2, x1, x2), x1))} \\
= & \quad \text{by Lemma 375 RL with } \{x2 \leftarrow asoc(x2, x1, x2), x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_t(x1, mult(i(x1), asoc(x2, x1, x2)))}} \\
= & \quad \text{by Lemma 257 LR with } \{x2 \leftarrow asoc(x2, x1, x2), x1 \leftarrow x1\} \\
& \overbrace{op_l(\underbrace{op_t(x1, asoc(x2, x1, x2))}, asoc(x2, x1, x2), i(x1))} \\
= & \quad \text{by Lemma 535 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_l(\overbrace{x1}, asoc(x2, x1, x2), i(x1))
\end{aligned}
$$

Lemma 537: $op_t(x1, i(asoc(x2, x1, x2))) = x1$

$$
\begin{aligned}
& \overbrace{op_t(x1, i(asoc(x2, x1, x2)))} \\
= \quad & \text{by Lemma 511 RL with } \{x2 \leftarrow asoc(x2, x1, x2), x1 \leftarrow x1\} \\
& \overbrace{op_l(\underbrace{op_t(x1, rd(asoc(x2, x1, x2), x1))}, x1, asoc(x2, x1, x2))} \\
= \quad & \text{by Lemma 536 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_l(\overbrace{\underbrace{op_l(x1, asoc(x2, x1, x2), i(x1))}}, x1, asoc(x2, x1, x2)) \\
= \quad & \text{by Lemma 491 RL with } \{x2 \leftarrow asoc(x2, x1, x2), x1 \leftarrow x1\} \\
& op_l(\overbrace{mult(x1, asoc(\underbrace{asoc(x2, x1, x2)}, x1, i(x1)))}, x1, asoc(x2, x1, x2)) \\
= \quad & \text{by Lemma 3 RL with } \{x1 \leftarrow asoc(x2, x1, x2)\} \\
& op_l(mult(x1, \underbrace{asoc(\overbrace{i(i(asoc(x2, x1, x2)))}, x1, i(x1)))}, x1, asoc(x2, x1, x2)) \\
= \quad & \text{by Lemma 533 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& op_l(mult(x1, \overbrace{\underbrace{asoc(op_r(x1, rd(x2, x1), i(x2)), x1, i(x1))}}, x1, asoc(x2, x1, x2)) \\
= \quad & \text{by Lemma 141 RL with } \{x2 \leftarrow x1, x1 \leftarrow op_r(x1, rd(x2, x1), i(x2))\} \\
& op_l(mult(x1, \overbrace{rd(\underbrace{op_r(op_r(x1, rd(x2, x1), i(x2)), x1, i(x1))}, op_r(x1, rd(x2, x1), i(x2))))}, x1, asoc(x2, x1, x2)) \\
= \quad & \text{by Axiom 17 RL with } \{x5 \leftarrow i(x2), x4 \leftarrow rd(x2, x1), x3 \leftarrow i(x1), x2 \leftarrow x1, x1 \leftarrow x1\} \\
& op_l(mult(x1, rd(\overbrace{op_r(\underbrace{op_r(x1, x1, i(x1))}, rd(x2, x1), i(x2))}, op_r(x1, rd(x2, x1), i(x2))))), x1, asoc(x2, x1, x2)) \\
= \quad & \text{by Lemma 26 LR with } \{x2 \leftarrow i(x1), x1 \leftarrow x1\} \\
& op_l(mult(x1, \underbrace{rd(op_r(\overbrace{x1}, rd(x2, x1), i(x2)), op_r(x1, rd(x2, x1), i(x2))))}, x1, asoc(x2, x1, x2)) \\
= \quad & \text{by Lemma 2 LR with } \{x1 \leftarrow op_r(x1, rd(x2, x1), i(x2))\} \\
& op_l(\underbrace{mult(x1, \overbrace{unit()})}, x1, asoc(x2, x1, x2)) \\
= \quad & \text{by Axiom 1 RL with } \{x1 \leftarrow x1\} \\
& \underbrace{op_l(\overbrace{x1}, x1, asoc(x2, x1, x2))} \\
= \quad & \text{by Lemma 532 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{x1}
\end{aligned}
$$

Lemma 538: $op_t(x1, asoc(x2, i(x1), x2)) = i(i(x1))$

$$
\begin{aligned}
& op_t(x1, \underbrace{asoc(x2, i(x1), x2)}) \\
= \quad & \text{by Lemma 3 RL with } \{x1 \leftarrow asoc(x2, i(x1), x2)\} \\
& \underbrace{op_t(x1, \overbrace{i(i(asoc(x2, i(x1), x2)))})} \\
= \quad & \text{by Lemma 18 RL with } \{x2 \leftarrow i(asoc(x2, i(x1), x2)), x1 \leftarrow x1\} \\
& \overbrace{i(\underbrace{op_t(i(x1), i(asoc(x2, i(x1), x2)))})} \\
= \quad & \text{by Lemma 537 LR with } \{x2 \leftarrow x2, x1 \leftarrow i(x1)\} \\
& i(\overbrace{i(x1)})
\end{aligned}
$$

Lemma 539: $op_t(x1, asoc(x2, i(x1), x2)) = x1$

$$
\begin{aligned}
& \overbrace{op_t(x1, asoc(x2, i(x1), x2))} \\
= & \quad \text{by Lemma 538 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{i(i(x1))}} \\
= & \quad \text{by Lemma 3 LR with } \{x1 \leftarrow x1\} \\
& \overbrace{x1}
\end{aligned}
$$

Lemma 540: $mult(asoc(x1, x2, x1), x2) = mult(x2, asoc(x1, x2, x1))$

$$
\begin{aligned}
& mult(asoc(x1, x2, x1), \underbrace{x2}) \\
= & \quad \text{by Lemma 535 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \underbrace{mult(asoc(x1, x2, x1), \overbrace{op_t(x2, asoc(x1, x2, x1))})} \\
= & \quad \text{by Lemma 13 LR with } \{x2 \leftarrow x2, x1 \leftarrow asoc(x1, x2, x1)\} \\
& \overbrace{mult(x2, asoc(x1, x2, x1))}
\end{aligned}
$$

Lemma 541: $op_t(rd(x1, x2), asoc(x1, x2, x1)) = rd(x1, x2)$

$$
\begin{aligned}
& op_t(rd(x1, x2), \underbrace{asoc(x1, x2, x1)}) \\
= & \quad \text{by Lemma 3 RL with } \{x1 \leftarrow asoc(x1, x2, x1)\} \\
& op_t(rd(x1, x2), \overbrace{i(\underbrace{i(asoc(x1, x2, x1))})}) \\
= & \quad \text{by Lemma 174 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_t(rd(x1, x2), i(\overbrace{rd(x1, op_l(x1, x1, x2))})) \\
= & \quad \text{by Lemma 176 LR with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x1\} \\
& op_t(rd(x1, x2), i(\overbrace{mult(x1, i(op_l(x1, x1, x2)))})) \\
= & \quad \text{by Lemma 282 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{op_t(rd(x1, x2), i(\overbrace{asoc(i(x1), rd(x1, x2), i(x1))}))} \\
= & \quad \text{by Lemma 537 LR with } \{x2 \leftarrow i(x1), x1 \leftarrow rd(x1, x2)\} \\
& \overbrace{rd(x1, x2)}
\end{aligned}
$$

Lemma 542: $mult(asoc(x1,x2,x1),i(x2)) = rd(asoc(x1,x2,x1),x2)$

$\underbrace{mult(asoc(x1,x2,x1),i(x2))}$

$=$ by Lemma 445 RL with $\{x2 \leftarrow x2, x1 \leftarrow asoc(x1,x2,x1)\}$

$\overbrace{rd(asoc(x1,x2,x1),\underbrace{op_l(x2,x2,i(asoc(x1,x2,x1)))}}$

$=$ by Lemma 525 RL with $\{x2 \leftarrow asoc(x1,x2,x1), x1 \leftarrow x2\}$

$rd(asoc(x1,x2,x1),\overbrace{op_l(\underbrace{x2},rd(asoc(x1,x2,x1),x2),asoc(x1,x2,x1))})$

$=$ by Lemma 532 RL with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$rd(asoc(x1,x2,x1),op_l(\overbrace{op_l(x2,x2,asoc(x1,x2,x1))},\underbrace{rd(asoc(x1,x2,x1),x2)},asoc(x1,x2,x1)))$

$=$ by Lemma 10 RL with $\{x2 \leftarrow asoc(x1,x2,x1), x1 \leftarrow x2\}$

$rd(asoc(x1,x2,x1),op_l(\underbrace{op_l(x2,x2,asoc(x1,x2,x1))},\overbrace{i(rd(x2,asoc(x1,x2,x1)))},asoc(x1,x2,x1)))$

$=$ by Lemma 427 LR with $\{x2 \leftarrow asoc(x1,x2,x1), x1 \leftarrow x2\}$

$rd(asoc(x1,x2,x1),\underbrace{op_l(\overbrace{mult(asoc(x1,x2,x1),rd(x2,asoc(x1,x2,x1)))},i(rd(x2,asoc(x1,x2,x1))),asoc(x1,x2,x1))})$

$=$ by Lemma 423 LR with $\{x2 \leftarrow rd(x2,asoc(x1,x2,x1)), x1 \leftarrow asoc(x1,x2,x1)\}$

$rd(asoc(x1,x2,x1),\overbrace{\underbrace{mult(rd(x2,asoc(x1,x2,x1)),asoc(x1,x2,x1))}})$

$=$ by Axiom 4 RL with $\{x2 \leftarrow asoc(x1,x2,x1), x1 \leftarrow x2\}$

$rd(asoc(x1,x2,x1),\overbrace{x2})$

Lemma 543: $i(mult(x2,asoc(x1,x2,x1))) = rd(i(asoc(x1,x2,x1)),x2)$

$i(\underbrace{mult(x2,asoc(x1,x2,x1))})$

$=$ by Lemma 540 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{i(\overbrace{mult(asoc(x1,x2,x1),x2)})}$

$=$ by Lemma 447 RL with $\{x2 \leftarrow x2, x1 \leftarrow asoc(x1,x2,x1)\}$

$\overbrace{rd(i(asoc(x1,x2,x1)),\underbrace{op_l(x2,x2,asoc(x1,x2,x1))})}$

$=$ by Lemma 532 LR with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$rd(i(asoc(x1,x2,x1)),\overbrace{x2})$

Lemma 544: $mult(asoc(x1,i(x2),x1),x2) = mult(x2,asoc(x1,i(x2),x1))$

$mult(asoc(x1,i(x2),x1),\underbrace{x2})$

$=$ by Lemma 539 RL with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$\underbrace{mult(asoc(x1,i(x2),x1),\overbrace{op_t(x2,asoc(x1,i(x2),x1))})}$

$=$ by Lemma 13 LR with $\{x2 \leftarrow x2, x1 \leftarrow asoc(x1,i(x2),x1)\}$

$\overbrace{mult(x2,asoc(x1,i(x2),x1))}$

Lemma 545: $op_r(i(i(x1)), rd(i(x1), x2), x2) = rd(x1, asoc(x2, i(x1), x2))$

$$
\begin{aligned}
& op_r(i(i(x1)), rd(i(x1), x2), \underbrace{x2}) \\
= & \quad \text{by Lemma 3 RL with } \{x1 \leftarrow x2\} \\
& op_r(i(i(x1)), \underbrace{rd(i(x1), x2)}, \overbrace{i(i(x2))}) \\
= & \quad \text{by Lemma 23 RL with } \{x2 \leftarrow i(x1), x1 \leftarrow x2\} \\
& op_r(\underbrace{i(i(x1))}, \overbrace{rd(i(x2), i(i(x1)))}, i(i(x2))) \\
= & \quad \text{by Lemma 8 LR with } \{x1 \leftarrow i(i(x1))\} \\
& \underbrace{op_r(\overbrace{op_t(i(i(x1)), i(i(i(x1))))}, rd(i(x2), i(i(x1))), i(i(x2)))} \\
= & \quad \text{by Axiom 14 RL with } \{x4 \leftarrow i(i(i(x1))), x3 \leftarrow i(i(x2)), x2 \leftarrow rd(i(x2), i(i(x1))), x1 \leftarrow i(i(x1))\} \\
& \overbrace{\underbrace{op_t(op_r(i(i(x1)), rd(i(x2), i(i(x1))), i(i(x2))), i(i(i(x1))))}} \\
= & \quad \text{by Lemma 534 RL with } \{x2 \leftarrow i(x2), x1 \leftarrow i(i(x1))\} \\
& \overbrace{mult(i(i(x1)), \underbrace{asoc(i(x2), i(i(x1)), i(x2))})} \\
= & \quad \text{by Lemma 271 RL with } \{x2 \leftarrow i(i(x1)), x1 \leftarrow x2\} \\
& mult(i(i(x1)), \overbrace{i(asoc(x2, \underbrace{i(i(i(x1)))}, x2))}) \\
= & \quad \text{by Lemma 3 LR with } \{x1 \leftarrow i(x1)\} \\
& \underbrace{mult(i(i(x1)), i(asoc(x2, \overbrace{i(x1)}, x2)))} \\
= & \quad \text{by Axiom 11 LR with } \{x2 \leftarrow asoc(x2, i(x1), x2), x1 \leftarrow i(x1)\} \\
& \overbrace{\underbrace{i(mult(i(x1), asoc(x2, i(x1), x2)))}} \\
= & \quad \text{by Lemma 543 LR with } \{x1 \leftarrow x2, x2 \leftarrow i(x1)\} \\
& \overbrace{\underbrace{rd(i(asoc(x2, i(x1), x2)), i(x1))}} \\
= & \quad \text{by Lemma 37 LR with } \{x2 \leftarrow i(x1), x1 \leftarrow asoc(x2, i(x1), x2)\} \\
& \overbrace{rd(\underbrace{i(i(x1))}, asoc(x2, i(x1), x2))} \\
= & \quad \text{by Lemma 3 LR with } \{x1 \leftarrow x1\} \\
& rd(\overbrace{x1}, asoc(x2, i(x1), x2))
\end{aligned}
$$

Lemma 546: $rd(x1, asoc(x2, i(x1), x2)) = op_r(x1, rd(i(x1), x2), x2)$

$$
\begin{aligned}
& \underbrace{rd(x1, asoc(x2, i(x1), x2))} \\
= & \quad \text{by Lemma 545 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_r(\underbrace{i(i(x1))}, rd(i(x1), x2), x2)} \\
= & \quad \text{by Lemma 3 LR with } \{x1 \leftarrow x1\} \\
& op_r(\overbrace{x1}, rd(i(x1), x2), x2)
\end{aligned}
$$

Lemma 547: $op_t(x1, rd(x2, x1)) = op_r(x1, x2, i(x2))$

$$
\begin{aligned}
& \underbrace{op_t(x1, rd(x2, x1))} \\
= \quad & \text{by Lemma 351 RL with } \{x2 \leftarrow x2, x1 \leftarrow op_t(x1, rd(x2, x1))\} \\
& \overbrace{op_r(\underbrace{op_r(op_t(x1, rd(x2, x1)), x2, x2)}, x2, i(x2))} \\
= \quad & \text{by Lemma 368 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_r(\overbrace{x1}, x2, i(x2))
\end{aligned}
$$

Lemma 548: $op_t(x1, op_r(x2, x1, i(x1))) = op_l(op_r(x1, x2, i(x2)), rd(x2, x1), x1)$

$$
\begin{aligned}
& \underbrace{op_t(x1, op_r(x2, x1, i(x1)))} \\
= \quad & \text{by Lemma 528 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_t(x1, mult(x1, rd(x2, x1)))}} \\
= \quad & \text{by Lemma 251 RL with } \{x2 \leftarrow rd(x2, x1), x1 \leftarrow x1\} \\
& \overbrace{op_l(\underbrace{op_t(x1, rd(x2, x1))}, rd(x2, x1), x1)} \\
= \quad & \text{by Lemma 547 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_l(\overbrace{op_r(x1, x2, i(x2))}, rd(x2, x1), x1)
\end{aligned}
$$

Lemma 549: $op_t(x1, op_r(x2, x1, i(x1))) = op_t(x1, x2)$

$$
\begin{aligned}
& \underbrace{op_t(x1, op_r(x2, x1, i(x1)))} \\
= \quad & \text{by Lemma 548 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(op_r(x1, x2, i(x2)), rd(x2, x1), x1)}} \\
= \quad & \text{by Lemma 292 LR with } \{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow op_r(x1, x2, i(x2))\} \\
& \overbrace{op_l(\underbrace{op_r(x1, x2, i(x2))}, x2, x1)} \\
= \quad & \text{by Lemma 421 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{op_l(\overbrace{op_l(op_t(x1, x2), x1, x2)}, x2, x1)} \\
= \quad & \text{by Lemma 138 LR with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow op_t(x1, x2)\} \\
& \overbrace{op_t(x1, x2)}
\end{aligned}
$$

Lemma 550: $op_t(x1, op_r(x2, x1, x1)) = op_t(x1, x2)$

$$
\begin{aligned}
& \underbrace{op_t(x1, op_r(x2, x1, x1))} \\
= \quad & \text{by Lemma 549 RL with } \{x2 \leftarrow op_r(x2, x1, x1), x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, \underbrace{op_r(op_r(x2, x1, x1), x1, i(x1))})} \\
= \quad & \text{by Lemma 351 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& op_t(x1, \overbrace{x2})
\end{aligned}
$$

Lemma 551: $op_t(x1, mult(x2, i(x1))) = op_t(x1, rd(x2, x1))$

$$
\begin{aligned}
& op_t(x1, \underbrace{mult(x2, i(x1))}) \\
= \quad & \text{by Lemma 104 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \underbrace{op_t(x1, \overbrace{op_r(rd(x2, x1), x1, i(x1))})} \\
= \quad & \text{by Lemma 549 LR with } \{x2 \leftarrow rd(x2, x1), x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, rd(x2, x1))}
\end{aligned}
$$

Lemma 552: $op_t(x1, rd(mult(x1, x2), x1)) = op_t(x1, op_l(x2, x2, i(i(x1))))$

$$
\begin{aligned}
& op_t(x1, rd(mult(\underbrace{x1}, x2), x1)) \\
= \quad & \text{by Lemma 3 RL with } \{x1 \leftarrow x1\} \\
& \underbrace{op_t(x1, rd(mult(\overbrace{i(i(x1))}, x2), x1))} \\
= \quad & \text{by Lemma 551 RL with } \{x2 \leftarrow mult(i(i(x1)), x2), x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, \underbrace{mult(mult(i(i(x1)), x2), i(x1))})} \\
= \quad & \text{by Lemma 439 RL with } \{x2 \leftarrow i(x1), x1 \leftarrow x2\} \\
& op_t(x1, \overbrace{op_l(x2, x2, i(i(x1)))})
\end{aligned}
$$

Lemma 553: $op_t(x1, x2) = op_t(x1, op_l(x2, x2, x1))$

$$
\begin{aligned}
& \underbrace{op_t(x1, x2)} \\
= \quad & \text{by Lemma 378 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{\overbrace{op_t(x1, rd(mult(x1, x2), x1))}} \\
= \quad & \text{by Lemma 552 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, op_l(x2, x2, \underbrace{i(i(x1))}))} \\
= \quad & \text{by Lemma 3 LR with } \{x1 \leftarrow x1\} \\
& op_t(x1, op_l(x2, x2, \overbrace{x1}))
\end{aligned}
$$

Lemma 554: $op_t(x1, mult(i(x2), x1)) = op_t(x1, rd(x1, x2))$

$$
\begin{aligned}
& op_t(x1, \underbrace{mult(i(x2), x1)}) \\
= \quad & \text{by Lemma 330 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{op_t(x1, \overbrace{op_r(rd(x1, x2), x1, i(x1))})} \\
= \quad & \text{by Lemma 549 LR with } \{x2 \leftarrow rd(x1, x2), x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, rd(x1, x2))}
\end{aligned}
$$

Lemma 555: $op_t(x1, rd(x2, i(x1))) = op_t(x1, mult(x2, x1))$

$$
\begin{aligned}
& \underbrace{op_t(x1, rd(x2, i(x1)))} \\
= \quad & \text{by Lemma 549 RL with } \{x2 \leftarrow rd(x2, i(x1)), x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, \underbrace{op_r(rd(x2, i(x1)), x1, i(x1))})} \\
= \quad & \text{by Lemma 321 RL with } \{x2 \leftarrow x1, x1 \leftarrow rd(x2, i(x1))\} \\
& op_t(x1, \overbrace{\underbrace{op_r(rd(x2, i(x1)), i(x1), x1)}}) \\
= \quad & \text{by Lemma 93 RL with } \{x2 \leftarrow x1, x1 \leftarrow rd(x2, i(x1))\} \\
& op_t(x1, \overbrace{mult(\underbrace{mult(rd(x2, i(x1)), i(x1))}, x1)}) \\
= \quad & \text{by Axiom 4 RL with } \{x2 \leftarrow i(x1), x1 \leftarrow x2\} \\
& op_t(x1, mult(\overbrace{x2}, x1))
\end{aligned}
$$

Lemma 556: $op_t(x1, rd(x1, i(x2))) = op_t(x1, mult(x2, x1))$

$$
\begin{aligned}
& op_t(x1, \underbrace{rd(x1, i(x2))}) \\
= \quad & \text{by Lemma 359 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \underbrace{op_t(x1, \overbrace{op_r(mult(x2, x1), x1, x1)})} \\
= \quad & \text{by Lemma 550 LR with } \{x2 \leftarrow mult(x2, x1), x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, mult(x2, x1))}
\end{aligned}
$$

Lemma 557: $op_t(x1, mult(x1, rd(x2, x1))) = op_t(x1, x2)$

$$
\begin{aligned}
& \underbrace{op_t(x1, mult(x1, rd(x2, x1)))} \\
= \quad & \text{by Lemma 528 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{\overbrace{op_t(x1, op_r(x2, x1, i(x1)))}} \\
= \quad & \text{by Lemma 549 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, x2)}
\end{aligned}
$$

Lemma 558: $op_t(x1, x2) = op_t(x1, mult(x1, mult(x2, i(x1))))$

$$
\begin{aligned}
& \underbrace{op_t(x1, x2)} \\
= \quad & \text{by Lemma 549 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{\overbrace{op_t(x1, op_r(x2, x1, i(x1)))}} \\
= \quad & \text{by Lemma 557 RL with } \{x2 \leftarrow op_r(x2, x1, i(x1)), x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, mult(x1, \underbrace{rd(op_r(x2, x1, i(x1)), x1)}))} \\
= \quad & \text{by Lemma 342 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& op_t(x1, mult(x1, \overbrace{mult(x2, i(x1))}))
\end{aligned}
$$

Lemma 559: $op_t(x1, op_t(x2, i(x1))) = op_t(x1, x2)$

$$
\begin{aligned}
& op_t(x1, \underbrace{op_t(x2, i(x1))}) \\
= & \quad \text{by Lemma 30 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{op_t(x1, \overbrace{mult(x1, mult(x2, i(x1)))})} \\
= & \quad \text{by Lemma 558 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, x2)}
\end{aligned}
$$

Lemma 560: $op_t(x1, op_l(x2, x1, x2)) = op_t(x1, x2)$

$$
\begin{aligned}
& op_t(x1, \underbrace{op_l(x2, x1, x2)}) \\
= & \quad \text{by Axiom 5 LR with } \{x2 \leftarrow x1, x1 \leftarrow op_l(x2, x1, x2)\} \\
& op_t(x1, \overbrace{rd(\underbrace{mult(op_l(x2, x1, x2), x1)}, x1)}) \\
= & \quad \text{by Lemma 255 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{op_t(x1, rd(\overbrace{mult(x1, op_t(x2, mult(x2, x1)))}, x1))} \\
= & \quad \text{by Lemma 378 LR with } \{x2 \leftarrow op_t(x2, mult(x2, x1)), x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_t(x1, op_t(x2, mult(x2, x1)))}} \\
= & \quad \text{by Lemma 557 RL with } \{x2 \leftarrow op_t(x2, mult(x2, x1)), x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, mult(x1, \underbrace{rd(op_t(x2, mult(x2, x1)), x1)}))} \\
= & \quad \text{by Lemma 459 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& op_t(x1, \underbrace{mult(x1, \overbrace{mult(i(x1), op_t(x2, x1))})}) \\
= & \quad \text{by Lemma 12 LR with } \{x2 \leftarrow op_t(x2, x1), x1 \leftarrow x1\} \\
& \underbrace{op_t(x1, \overbrace{op_t(x2, x1)})} \\
= & \quad \text{by Lemma 374 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, x2)}
\end{aligned}
$$

Lemma 561: $op_t(i(x1), op_t(x2, x1)) = op_t(i(x1), x2)$

$$
\begin{aligned}
& op_t(i(x1), op_t(x2, \underbrace{x1})) \\
= & \quad \text{by Lemma 3 RL with } \{x1 \leftarrow x1\} \\
& \underbrace{op_t(i(x1), op_t(x2, \overbrace{i(\underbrace{i(x1)})}))} \\
= & \quad \text{by Lemma 559 LR with } \{x2 \leftarrow x2, x1 \leftarrow i(x1)\} \\
& \overbrace{op_t(i(x1), x2)}
\end{aligned}
$$

Lemma 562: $op_t(x1, i(op_t(x2, x1))) = op_t(x1, i(x2))$

$$
\begin{aligned}
& op_t(x1, \underbrace{i(op_t(x2, x1))}) \\
= & \quad \text{by Lemma 15 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \underbrace{op_t(x1, \overbrace{op_t(i(x2), i(x1))})} \\
= & \quad \text{by Lemma 559 LR with } \{x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, i(x2))}
\end{aligned}
$$

Lemma 563: $op_t(x1, rd(x1, x2)) = op_t(x1, op_r(mult(x1, i(x2)), x1, x1))$

$$
\begin{aligned}
& \underbrace{op_t(x1, rd(x1, x2))} \\
= \quad & \text{by Lemma 557 RL with } \{x2 \leftarrow rd(x1, x2), x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, mult(x1, \underbrace{rd(rd(x1, x2), x1)}))} \\
= \quad & \text{by Lemma 354 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_t(x1, \underbrace{mult(x1, \overbrace{op_r(i(x2), x1, x1)})}) \\
= \quad & \text{by Lemma 383 LR with } \{x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& op_t(x1, \overbrace{op_r(mult(x1, i(x2)), x1, x1)})
\end{aligned}
$$

Lemma 564: $op_t(x1, mult(x1, i(x2))) = op_t(x1, rd(x1, x2))$

$$
\begin{aligned}
& \underbrace{op_t(x1, mult(x1, i(x2)))} \\
= \quad & \text{by Lemma 550 RL with } \{x2 \leftarrow mult(x1, i(x2)), x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_t(x1, op_r(mult(x1, i(x2)), x1, x1))}} \\
= \quad & \text{by Lemma 563 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, rd(x1, x2))}
\end{aligned}
$$

Lemma 565: $op_t(x1, op_t(x2, mult(x2, x1))) = op_t(x1, x2)$

$$
\begin{aligned}
& \underbrace{op_t(x1, op_t(x2, mult(x2, x1)))} \\
= \quad & \text{by Lemma 378 RL with } \{x2 \leftarrow op_t(x2, mult(x2, x1)), x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, rd(\underbrace{mult(x1, op_t(x2, mult(x2, x1)))}, x1))} \\
= \quad & \text{by Lemma 255 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_t(x1, \underbrace{rd(\overbrace{mult(op_l(x2, x1, x2), x1)}, x1)}) \\
= \quad & \text{by Axiom 5 RL with } \{x2 \leftarrow x1, x1 \leftarrow op_l(x2, x1, x2)\} \\
& \underbrace{op_t(x1, \overbrace{op_l(x2, x1, x2)})} \\
= \quad & \text{by Lemma 560 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, x2)}
\end{aligned}
$$

Lemma 566: $op_t(x1, op_l(x2, x2, i(x1))) = op_t(x1, x2)$

$$
\begin{aligned}
& \underbrace{op_t(x1, op_l(x2, x2, i(x1)))} \\
= \quad & \text{by Lemma 557 RL with } \{x2 \leftarrow op_l(x2, x2, i(x1)), x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, mult(x1, \underbrace{rd(op_l(x2, x2, i(x1)), x1)}))} \\
= \quad & \text{by Lemma 446 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& op_t(x1, \underbrace{mult(x1, \overbrace{mult(i(x1), x2)})}) \\
= \quad & \text{by Lemma 12 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_t(x1, \overbrace{x2})
\end{aligned}
$$

Lemma 567: $op_t(x1, x2) = op_t(x1, op_t(op_r(x2, x1, x1), rd(x1, i(x2))))$

$$
\begin{aligned}
& \overbrace{op_t(x1, x2)} \\
= \quad & \text{by Lemma 550 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_t(x1, op_r(x2, x1, x1))}} \\
= \quad & \text{by Lemma 565 RL with } \{x2 \leftarrow op_r(x2, x1, x1), x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, op_t(op_r(x2, x1, x1), \underbrace{mult(op_r(x2, x1, x1), x1)}))} \\
= \quad & \text{by Lemma 357 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& op_t(x1, op_t(op_r(x2, x1, x1), \overbrace{rd(x1, i(x2))}))
\end{aligned}
$$

Lemma 568: $op_t(x1, op_t(x2, mult(x1, x2))) = op_t(x1, x2)$

$$
\begin{aligned}
& op_t(x1, \underbrace{op_t(x2, mult(x1, x2))}) \\
= \quad & \text{by Lemma 555 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \underbrace{op_t(x1, \overbrace{op_t(x2, rd(x1, i(x2)))})} \\
= \quad & \text{by Lemma 550 RL with } \{x2 \leftarrow op_t(x2, rd(x1, i(x2))), x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, \underbrace{op_r(op_t(x2, rd(x1, i(x2))), x1, x1)})} \\
= \quad & \text{by Axiom 14 RL with } \{x4 \leftarrow rd(x1, i(x2)), x3 \leftarrow x1, x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \underbrace{op_t(x1, \overbrace{op_t(op_r(x2, x1, x1), rd(x1, i(x2)))})} \\
= \quad & \text{by Lemma 567 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, x2)}
\end{aligned}
$$

Lemma 569: $op_t(i(x1), op_r(x2, x1, x1)) = op_t(i(x1), x2)$

$$
\begin{aligned}
& op_t(i(x1), \underbrace{op_r(x2, x1, x1)}) \\
= \quad & \text{by Lemma 361 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \underbrace{op_t(i(x1), \overbrace{op_r(x2, i(x1), i(x1))})} \\
= \quad & \text{by Lemma 550 LR with } \{x2 \leftarrow x2, x1 \leftarrow i(x1)\} \\
& \overbrace{op_t(i(x1), x2)}
\end{aligned}
$$

Lemma 570: $op_t(x1, rd(x1, mult(x1, x2))) = op_t(x1, rd(rd(x1, x2), x1))$

$$
\begin{aligned}
& \underbrace{op_t(x1, rd(x1, mult(x1, x2)))} \\
= \quad & \text{by Lemma 554 RL with } \{x2 \leftarrow mult(x1, x2), x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, \underbrace{mult(i(mult(x1, x2)), x1)})} \\
= \quad & \text{by Lemma 56 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{op_t(x1, \overbrace{mult(i(x1), rd(x1, x2))})} \\
= \quad & \text{by Lemma 375 LR with } \{x2 \leftarrow rd(x1, x2), x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, rd(rd(x1, x2), x1))}
\end{aligned}
$$

Lemma 571: $op_t(x1, rd(x1, mult(x1, x2))) = op_t(x1, i(x2))$

$$
\begin{aligned}
& \underbrace{op_t(x1, rd(x1, mult(x1, x2)))} \\
= & \quad \text{by Lemma 570 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, \underbrace{rd(rd(x1, x2), x1)})} \\
= & \quad \text{by Lemma 354 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{op_t(x1, \overbrace{op_r(i(x2), x1, x1)})} \\
= & \quad \text{by Lemma 550 LR with } \{x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, i(x2))}
\end{aligned}
$$

Lemma 572: $op_t(x1, op_l(i(x2), x2, x1)) = op_t(x1, i(x2))$

$$
\begin{aligned}
& \underbrace{op_t(x1, op_l(i(x2), x2, x1))} \\
= & \quad \text{by Lemma 557 RL with } \{x2 \leftarrow op_l(i(x2), x2, x1), x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, mult(x1, \underbrace{rd(op_l(i(x2), x2, x1), x1)}))} \\
= & \quad \text{by Lemma 10 RL with } \{x2 \leftarrow op_l(i(x2), x2, x1), x1 \leftarrow x1\} \\
& op_t(x1, mult(x1, \overbrace{i(\underbrace{rd(x1, op_l(i(x2), x2, x1))})})) \\
= & \quad \text{by Lemma 231 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_t(x1, \underbrace{mult(x1, \overbrace{i(\overbrace{mult(x1, x2)})})}) \\
= & \quad \text{by Lemma 20 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_t(x1, \overbrace{i(x2)})
\end{aligned}
$$

Lemma 573: $mult(i(x2), x1) = op_t(rd(x1, x2), op_t(x2, op_l(x1, x1, x2)))$

$$
\begin{aligned}
& \underbrace{mult(i(x2), x1)} \\
= & \quad \text{by Lemma 14 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{\overbrace{op_t(rd(x1, x2), x2)}} \\
= & \quad \text{by Lemma 565 RL with } \{x2 \leftarrow x2, x1 \leftarrow rd(x1, x2)\} \\
& \overbrace{op_t(rd(x1, x2), op_t(x2, \underbrace{mult(x2, rd(x1, x2))}))} \\
= & \quad \text{by Lemma 427 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_t(rd(x1, x2), op_t(x2, \overbrace{op_l(x1, x1, x2)}))
\end{aligned}
$$

Lemma 574: $op_t(x1, rd(op_t(x2, x1), x1)) = op_t(x1, rd(x2, x1))$

$$
\begin{aligned}
& op_t(x1, \underbrace{rd(op_t(x2, x1), x1)}) \\
= & \quad \text{by Lemma 324 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \underbrace{op_t(x1, \overbrace{op_t(mult(x2, i(x1)), x1)})} \\
= & \quad \text{by Lemma 374 LR with } \{x2 \leftarrow mult(x2, i(x1)), x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_t(x1, mult(x2, i(x1)))}} \\
= & \quad \text{by Lemma 551 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, rd(x2, x1))}
\end{aligned}
$$

Lemma 575: $op_t(mult(x1, x2), x2) = op_t(mult(x1, x2), op_t(x2, x1))$

$$
\begin{aligned}
& op_t(mult(x1, x2), \underbrace{x2}) \\
= \quad & \text{by Lemma 29 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{op_t(mult(x1, x2), \overbrace{rd(mult(x1, x2), op_t(x1, x2))})} \\
= \quad & \text{by Lemma 554 RL with } \{x2 \leftarrow op_t(x1, x2), x1 \leftarrow mult(x1, x2)\} \\
& \overbrace{op_t(mult(x1, x2), \underbrace{mult(i(op_t(x1, x2)), mult(x1, x2))})} \\
= \quad & \text{by Lemma 384 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_t(mult(x1, x2), \overbrace{op_t(x2, x1)})
\end{aligned}
$$

Lemma 576: $op_t(x1, mult(i(x2), x1)) = op_t(x1, rd(op_t(i(x2), i(x1)), i(x1)))$

$$
\begin{aligned}
& op_t(x1, \underbrace{mult(i(x2), x1)}) \\
= \quad & \text{by Lemma 17 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \underbrace{op_t(x1, \overbrace{i(mult(x2, i(x1)))})} \\
= \quad & \text{by Lemma 559 RL with } \{x2 \leftarrow i(mult(x2, i(x1))), x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, \underbrace{op_t(i(mult(x2, i(x1))), i(x1))})} \\
= \quad & \text{by Lemma 329 LR with } \{x2 \leftarrow i(x1), x1 \leftarrow x2\} \\
& op_t(x1, \overbrace{rd(op_t(i(x2), i(x1)), i(x1))})
\end{aligned}
$$

Lemma 577: $op_t(x1, rd(x1, x2)) = op_t(x1, mult(op_t(i(x2), i(x1)), x1))$

$$
\begin{aligned}
& \underbrace{op_t(x1, rd(x1, x2))} \\
= \quad & \text{by Lemma 554 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{\overbrace{op_t(x1, mult(i(x2), x1))}} \\
= \quad & \text{by Lemma 576 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{\overbrace{op_t(x1, rd(op_t(i(x2), i(x1)), i(x1)))}} \\
= \quad & \text{by Lemma 555 LR with } \{x2 \leftarrow op_t(i(x2), i(x1)), x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, mult(op_t(i(x2), i(x1)), x1))}
\end{aligned}
$$

Lemma 578: $op_t(x1, rd(x1, x2)) = op_t(x1, rd(x1, op_t(x2, x1)))$

$$
\begin{aligned}
& \underbrace{op_t(x1, rd(x1, x2))} \\
= \quad & \text{by Lemma 577 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, mult(\underbrace{op_t(i(x2), i(x1))}, x1))} \\
= \quad & \text{by Lemma 15 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \underbrace{op_t(x1, mult(\overbrace{i(op_t(x2, x1))}, x1))} \\
= \quad & \text{by Lemma 554 LR with } \{x2 \leftarrow op_t(x2, x1), x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, rd(x1, op_t(x2, x1)))}
\end{aligned}
$$

Lemma 579: $op_t(x1, rd(x1, x2)) = op_l(op_t(x1, i(x2)), op_l(i(x2), x2, x1), x1)$

$$
\begin{aligned}
& op_t(x1, \underbrace{rd(x1, x2)}) \\
= & \quad \text{by Lemma 232 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{op_t(x1, \overbrace{mult(x1, op_l(i(x2), x2, x1))})} \\
= & \quad \text{by Lemma 251 RL with } \{x2 \leftarrow op_l(i(x2), x2, x1), x1 \leftarrow x1\} \\
& \overbrace{op_l(\underbrace{op_t(x1, op_l(i(x2), x2, x1))}, op_l(i(x2), x2, x1), x1)} \\
= & \quad \text{by Lemma 572 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_l(\overbrace{op_t(x1, i(x2))}, op_l(i(x2), x2, x1), x1)
\end{aligned}
$$

Lemma 580: $op_l(op_t(x1, i(x2)), x1, x2) = op_t(x1, rd(x1, x2))$

$$
\begin{aligned}
& op_l(op_t(x1, i(x2)), x1, \underbrace{x2}) \\
= & \quad \text{by Lemma 3 RL with } \{x1 \leftarrow x2\} \\
& \underbrace{op_l(op_t(x1, i(x2)), x1, \overbrace{i(i(x2))})} \\
= & \quad \text{by Axiom 15 RL with } \{x4 \leftarrow i(x2), x3 \leftarrow i(i(x2)), x2 \leftarrow x1, x1 \leftarrow x1\} \\
& \overbrace{op_t(\underbrace{op_l(x1, x1, i(i(x2)))}, i(x2))} \\
= & \quad \text{by Lemma 429 RL with } \{x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& \underbrace{op_t(\overbrace{op_l(x1, i(x1), i(x2))}, i(x2))} \\
= & \quad \text{by Axiom 15 LR with } \{x4 \leftarrow i(x2), x3 \leftarrow i(x2), x2 \leftarrow i(x1), x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(op_t(x1, i(x2)), i(x1), i(x2))}} \\
= & \quad \text{by Lemma 247 RL with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow op_t(x1, i(x2))\} \\
& \overbrace{op_l(op_t(x1, i(x2)), \underbrace{i(mult(x1, x2))}, x1)} \\
= & \quad \text{by Lemma 231 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_l(op_t(x1, i(x2)), \underbrace{i(\overbrace{rd(x1, op_l(i(x2), x2, x1))})}, x1) \\
= & \quad \text{by Lemma 10 LR with } \{x2 \leftarrow op_l(i(x2), x2, x1), x1 \leftarrow x1\} \\
& \underbrace{op_l(op_t(x1, i(x2)), \overbrace{rd(op_l(i(x2), x2, x1), x1)}, x1)} \\
= & \quad \text{by Lemma 292 LR with } \{x3 \leftarrow x1, x2 \leftarrow op_l(i(x2), x2, x1), x1 \leftarrow op_t(x1, i(x2))\} \\
& \overbrace{\underbrace{op_l(op_t(x1, i(x2)), op_l(i(x2), x2, x1), x1)}} \\
= & \quad \text{by Lemma 579 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, rd(x1, x2))}
\end{aligned}
$$

Lemma 581: $op_t(x1, mult(x2, x1)) = op_l(op_t(x1, x2), x1, i(x2))$

$$
\begin{aligned}
& \underbrace{op_t(x1, mult(x2, x1))} \\
= \quad & \text{by Lemma 556 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_t(x1, rd(x1, i(x2)))}} \\
= \quad & \text{by Lemma 580 RL with } \{x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& \overbrace{op_l(op_t(x1, \underbrace{i(i(x2))}), x1, i(x2))} \\
= \quad & \text{by Lemma 3 LR with } \{x1 \leftarrow x2\} \\
& op_l(op_t(x1, \overbrace{x2}), x1, i(x2))
\end{aligned}
$$

Lemma 582: $op_t(x1, x2) = op_l(op_t(x1, mult(x2, x1)), x1, i(i(x2)))$

$$
\begin{aligned}
& \underbrace{op_t(x1, x2)} \\
= \quad & \text{by Lemma 254 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(op_t(x1, mult(x1, x2)), x1, \underbrace{x2})} \\
= \quad & \text{by Lemma 3 RL with } \{x1 \leftarrow x2\} \\
& op_l(\underbrace{op_t(x1, mult(x1, x2))}, x1, \overbrace{i(i(x2))}) \\
= \quad & \text{by Lemma 374 RL with } \{x2 \leftarrow mult(x1, x2), x1 \leftarrow x1\} \\
& op_l(\overbrace{op_t(x1, \underbrace{op_t(mult(x1, x2), x1)})}, x1, i(i(x2))) \\
= \quad & \text{by Lemma 264 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& op_l(op_t(x1, \overbrace{mult(x2, op_l(x1, x1, \underbrace{x2}))}), x1, i(i(x2))) \\
= \quad & \text{by Lemma 3 RL with } \{x1 \leftarrow x2\} \\
& \underbrace{op_l(op_t(x1, mult(x2, op_l(x1, x1, \overbrace{i(i(x2))}))), x1, i(i(x2)))} \\
= \quad & \text{by Axiom 15 RL with } \{x4 \leftarrow mult(x2, op_l(x1, x1, i(i(x2)))), x3 \leftarrow i(i(x2)), x2 \leftarrow x1, x1 \leftarrow x1\} \\
& \underbrace{\overbrace{op_t(op_l(x1, x1, i(i(x2))), mult(x2, op_l(x1, x1, i(i(x2)))))}} \\
= \quad & \text{by Lemma 556 RL with } \{x2 \leftarrow x2, x1 \leftarrow op_l(x1, x1, i(i(x2)))\} \\
& \overbrace{op_t(op_l(x1, x1, i(i(x2))), \underbrace{rd(op_l(x1, x1, i(i(x2))), i(x2))})} \\
= \quad & \text{by Lemma 446 LR with } \{x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& \underbrace{op_t(op_l(x1, x1, i(i(x2))), \overbrace{mult(i(i(x2)), x1)})} \\
= \quad & \text{by Axiom 15 LR with } \{x4 \leftarrow mult(i(i(x2)), x1), x3 \leftarrow i(i(x2)), x2 \leftarrow x1, x1 \leftarrow x1\} \\
& \overbrace{op_l(\underbrace{op_t(x1, mult(i(i(x2)), x1))}, x1, i(i(x2)))} \\
= \quad & \text{by Lemma 554 LR with } \{x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& op_l(\overbrace{\underbrace{op_t(x1, rd(x1, i(x2)))}}, x1, i(i(x2))) \\
= \quad & \text{by Lemma 556 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_l(\overbrace{op_t(x1, mult(x2, x1))}, x1, i(i(x2)))
\end{aligned}
$$

Lemma 583: $op_l(op_t(x1, mult(x2, x1)), x1, x2) = op_t(x1, x2)$

$$
\begin{aligned}
& op_l(op_t(x1, mult(x2, x1)), x1, \underbrace{x2}) \\
= & \quad \text{by Lemma 3 RL with } \{x1 \leftarrow x2\} \\
& \underbrace{op_l(op_t(x1, mult(x2, x1)), x1, \overbrace{i(i(x2))})} \\
= & \quad \text{by Lemma 582 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, x2)}
\end{aligned}
$$

Lemma 584: $op_t(x1, mult(x1, op_l(i(x1), x2, x3))) = op_l(x1, x1, i(op_l(i(x1), x2, x3)))$

$$
\begin{aligned}
& op_t(x1, \underbrace{mult(x1, op_l(i(x1), x2, x3))}) \\
= & \quad \text{by Lemma 188 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{op_t(x1, \overbrace{mult(op_l(i(x1), x2, x3), x1)})} \\
= & \quad \text{by Lemma 581 LR with } \{x2 \leftarrow op_l(i(x1), x2, x3), x1 \leftarrow x1\} \\
& \overbrace{op_l(\underbrace{op_t(x1, op_l(i(x1), x2, x3))}, x1, i(op_l(i(x1), x2, x3)))} \\
= & \quad \text{by Lemma 189 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_l(\overbrace{x1}, x1, i(op_l(i(x1), x2, x3)))
\end{aligned}
$$

Lemma 585: $op_t(x1, mult(x1, op_l(i(x1), x2, x3))) = x1$

$$
\begin{aligned}
& \underbrace{op_t(x1, mult(x1, op_l(i(x1), x2, x3)))} \\
= & \quad \text{by Lemma 584 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x1, \underbrace{i(op_l(i(x1), x2, x3))})} \\
= & \quad \text{by Lemma 518 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{op_l(x1, x1, \overbrace{op_l(x1, i(x2), i(x3))})} \\
= & \quad \text{by Lemma 252 LR with } \{x3 \leftarrow i(x3), x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& \overbrace{x1}
\end{aligned}
$$

Lemma 586: $op_r(x1, rd(x2, x1), rd(x1, x2)) = op_t(x1, i(x2))$

$$
\begin{aligned}
& op_r(x1, rd(x2, x1), \underbrace{rd(x1, x2)}) \\
= & \quad \text{by Lemma 10 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \underbrace{op_r(x1, rd(x2, x1), \overbrace{i(rd(x2, x1))})} \\
= & \quad \text{by Lemma 421 LR with } \{x2 \leftarrow rd(x2, x1), x1 \leftarrow x1\} \\
& \overbrace{op_l(\underbrace{op_t(x1, rd(x2, x1))}, x1, rd(x2, x1))} \\
= & \quad \text{by Lemma 547 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{op_l(\overbrace{op_r(x1, x2, i(x2))}, x1, rd(x2, x1))} \\
= & \quad \text{by Lemma 286 LR with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow op_r(x1, x2, i(x2))\} \\
& \overbrace{\underbrace{op_l(op_r(x1, x2, i(x2)), x1, x2)}} \\
= & \quad \text{by Lemma 475 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, i(x2))}
\end{aligned}
$$

Lemma 587: $mult(i(x2), x1) = op_t(rd(x1, x2), op_l(x2, x1, x2))$

$\overbrace{mult(i(x2), x1)}$

$=$ by Lemma 14 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_t(rd(x1, x2), x2)}$

$=$ by Lemma 560 RL with $\{x2 \leftarrow x2, x1 \leftarrow rd(x1, x2)\}$

$\overbrace{op_t(rd(x1, x2), \underbrace{op_l(x2, rd(x1, x2), x2)})}$

$=$ by Lemma 292 LR with $\{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x2\}$

$op_t(rd(x1, x2), \overbrace{op_l(x2, x1, x2)})$

Lemma 588: $op_t(i(x1), op_t(x2, mult(x2, i(x1)))) = i(op_t(x1, i(x2)))$

$op_t(i(x1), op_t(x2, \underbrace{mult(x2, i(x1))}))$

$=$ by Lemma 16 RL with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$\underbrace{op_t(i(x1), op_t(x2, \overbrace{i(mult(i(x2), x1))}))}$

$=$ by Lemma 90 RL with $\{x3 \leftarrow mult(i(x2), x1), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{i(\underbrace{op_t(x1, op_t(i(x2), mult(i(x2), x1)))})}$

$=$ by Lemma 565 LR with $\{x2 \leftarrow i(x2), x1 \leftarrow x1\}$

$i(\overbrace{op_t(x1, i(x2))})$

Lemma 589: $op_t(i(x1), op_t(x2, rd(x2, x1))) = op_t(i(x1), x2)$

$op_t(i(x1), \underbrace{op_t(x2, rd(x2, x1))})$

$=$ by Lemma 564 RL with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$\underbrace{op_t(i(x1), \overbrace{op_t(x2, mult(x2, i(x1)))})}$

$=$ by Lemma 588 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{\overbrace{i(op_t(x1, i(x2)))}}$

$=$ by Lemma 19 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_t(i(x1), x2)}$

Lemma 590: $mult(x1, i(x2)) = op_t(rd(x1, x2), op_t(i(x1), i(x2)))$

$\underbrace{mult(x1, i(x2))}$

$=$ by Lemma 34 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{\overbrace{op_t(rd(x1, x2), i(x1))}}$

$=$ by Lemma 568 RL with $\{x2 \leftarrow i(x1), x1 \leftarrow rd(x1, x2)\}$

$\overbrace{op_t(rd(x1, x2), op_t(i(x1), \underbrace{mult(rd(x1, x2), i(x1))}))}$

$=$ by Lemma 22 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$op_t(rd(x1, x2), op_t(i(x1), \overbrace{i(x2)}))$

Lemma 591: $op_t(rd(x1,x2), i(op_t(x1,x2))) = mult(x1, i(x2))$

$$
\begin{aligned}
& op_t(rd(x1,x2), \underbrace{i(op_t(x1,x2))}) \\
= & \quad \text{by Lemma 15 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{op_t(rd(x1,x2), \overbrace{op_t(i(x1), i(x2))})} \\
= & \quad \text{by Lemma 590 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{mult(x1, i(x2))}
\end{aligned}
$$

Lemma 592: $op_t(i(x1), op_t(x2, rd(x1,x2))) = op_t(i(x1), x2)$

$$
\begin{aligned}
& \underbrace{op_t(i(x1), op_t(x2, rd(x1,x2)))} \\
= & \quad \text{by Lemma 569 RL with } \{x2 \leftarrow op_t(x2, rd(x1,x2)), x1 \leftarrow x1\} \\
& \overbrace{op_t(i(x1), \underbrace{op_r(op_t(x2, rd(x1,x2)), x1, x1)})} \\
= & \quad \text{by Lemma 368 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& op_t(i(x1), \overbrace{x2}^{})
\end{aligned}
$$

Lemma 593: $op_t(i(x1), x2) = op_t(i(x1), mult(x1, rd(x2,x1)))$

$$
\begin{aligned}
& \underbrace{op_t(i(x1), x2)} \\
= & \quad \text{by Lemma 374 RL with } \{x2 \leftarrow x2, x1 \leftarrow i(x1)\} \\
& \overbrace{\underbrace{op_t(i(x1), op_t(x2, i(x1)))}} \\
= & \quad \text{by Lemma 569 RL with } \{x2 \leftarrow op_t(x2, i(x1)), x1 \leftarrow x1\} \\
& \overbrace{op_t(i(x1), \underbrace{op_r(op_t(x2, i(x1)), x1, x1)})} \\
= & \quad \text{by Lemma 381 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& op_t(i(x1), \overbrace{mult(x1, rd(x2,x1))})
\end{aligned}
$$

Lemma 594: $rd(rd(x1,x2), op_t(i(x2), x1)) = op_l(x1, x1, x2)$

$$
\begin{aligned}
& rd(\underbrace{rd(x1,x2)}, op_t(i(x2), x1)) \\
= & \quad \text{by Lemma 440 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& rd(\overbrace{mult(i(x2), op_l(x1,x1,x2))}, op_t(i(x2), \underbrace{x1})) \\
= & \quad \text{by Lemma 138 RL with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x1\} \\
& rd(mult(i(x2), op_l(x1,x1,x2)), op_t(i(x2), \overbrace{\underbrace{op_l(op_l(x1,x1,x2), x2, x1)}})) \\
= & \quad \text{by Lemma 484 LR with } \{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow op_l(x1,x1,x2)\} \\
& rd(mult(i(x2), op_l(x1,x1,x2)), \underbrace{op_t(i(x2), \overbrace{op_l(op_l(x1,x1,x2), op_l(x1,x1,x2), i(x2))})}) \\
= & \quad \text{by Lemma 553 RL with } \{x2 \leftarrow op_l(x1,x1,x2), x1 \leftarrow i(x2)\} \\
& \underbrace{rd(mult(i(x2), op_l(x1,x1,x2)), \overbrace{op_t(i(x2), op_l(x1,x1,x2))})} \\
= & \quad \text{by Lemma 29 LR with } \{x2 \leftarrow op_l(x1,x1,x2), x1 \leftarrow i(x2)\} \\
& \overbrace{op_l(x1,x1,x2)}
\end{aligned}
$$

Lemma 595: $rd(x1, rd(op_t(x1, x2), x2)) = mult(mult(i(x1), x2), x1)$

$rd(x1, rd(op_t(\underbrace{x1}, x2), x2))$

$=$ by Lemma 3 RL with $\{x1 \leftarrow x1\}$

$rd(x1, \underbrace{rd(op_t(\overbrace{i(i(x1))}, x2), x2)})$

$=$ by Lemma 329 RL with $\{x2 \leftarrow x2, x1 \leftarrow i(x1)\}$

$rd(x1, \overbrace{op_t(i(mult(i(x1), x2)), \underbrace{x2})})$

$=$ by Lemma 12 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$rd(\underbrace{x1}, op_t(i(mult(i(x1), x2)), \overbrace{mult(x1, mult(i(x1), x2))}))$

$=$ by Axiom 5 LR with $\{x2 \leftarrow mult(i(x1), x2), x1 \leftarrow x1\}$

$\underbrace{rd(\overbrace{rd(mult(x1, mult(i(x1), x2)), mult(i(x1), x2)}), op_t(i(mult(i(x1), x2)), mult(x1, mult(i(x1), x2))))}$

$=$ by Lemma 594 LR with $\{x2 \leftarrow mult(i(x1), x2), x1 \leftarrow mult(x1, mult(i(x1), x2))\}$

$\underbrace{\overbrace{op_l(mult(x1, mult(i(x1), x2)), mult(x1, mult(i(x1), x2)), mult(i(x1), x2))}}$

$=$ by Lemma 285 LR with $\{x3 \leftarrow mult(i(x1), x2), x2 \leftarrow x1, x1 \leftarrow mult(x1, mult(i(x1), x2))\}$

$\underbrace{\overbrace{op_l(mult(x1, mult(i(x1), x2)), x1, mult(i(x1), x2))}}$

$=$ by Lemma 435 LR with $\{x2 \leftarrow mult(i(x1), x2), x1 \leftarrow x1\}$

$\overbrace{mult(mult(i(x1), x2), x1)}$

Lemma 596: $mult(rd(x2, op_t(x1, x2)), x1) = op_r(mult(mult(i(x1), x2), x1), x1, i(x1))$

$mult(\underbrace{rd(x2, op_t(x1, x2))}, x1)$

$=$ by Lemma 10 RL with $\{x2 \leftarrow x2, x1 \leftarrow op_t(x1, x2)\}$

$\underbrace{mult(\overbrace{i(rd(op_t(x1, x2), x2))}, x1)}$

$=$ by Lemma 330 RL with $\{x2 \leftarrow rd(op_t(x1, x2), x2), x1 \leftarrow x1\}$

$\overbrace{op_r(\underbrace{rd(x1, rd(op_t(x1, x2), x2))}, x1, i(x1))}$

$=$ by Lemma 595 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$op_r(\overbrace{mult(mult(i(x1), x2), x1)}, x1, i(x1))$

Lemma 597: $mult(rd(x1, op_t(x2, x1)), x2) = op_t(x1, x2)$

$\underbrace{mult(rd(x1, op_t(x2, x1)), x2)}$

$=$ by Lemma 596 LR with $\{x1 \leftarrow x2, x2 \leftarrow x1\}$

$\underbrace{\overbrace{op_r(mult(mult(i(x2), x1), x2), x2, i(x2))}}$

$=$ by Lemma 320 LR with $\{x2 \leftarrow x2, x1 \leftarrow mult(i(x2), x1)\}$

$\overbrace{op_t(\underbrace{mult(x2, mult(i(x2), x1))}, x2)}$

$=$ by Lemma 12 LR with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$op_t(\overbrace{x1}, x2)$

Lemma 598: $mult(mult(mult(x1,x2),x2),i(mult(x1,x2))) = rd(i(mult(x1,x2)),rd(rd(op_t(i(x1),x2),x2),x2))$

$$
\begin{aligned}
& mult(mult(\underbrace{mult(x1,x2)}_{},x2),i(mult(x1,x2))) \\
= & \quad \text{by Lemma 3 RL with } \{x1 \leftarrow mult(x1,x2)\} \\
& \underbrace{mult(mult(\overbrace{i(i(mult(x1,x2)))}^{},x2),i(mult(x1,x2)))}_{} \\
= & \quad \text{by Lemma 595 RL with } \{x2 \leftarrow x2, x1 \leftarrow i(mult(x1,x2))\} \\
& \overbrace{rd(i(mult(x1,x2)),rd(\underbrace{op_t(i(mult(x1,x2)),x2)}_{},x2))}^{} \\
= & \quad \text{by Lemma 329 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& rd(i(mult(x1,x2)),rd(\overbrace{rd(op_t(i(x1),x2),x2)}^{},x2))
\end{aligned}
$$

Lemma 599: $mult(mult(x1,x2),i(x1)) = rd(i(mult(x1,x2)),rd(rd(op_t(i(x1),x2),x2),x2))$

$$
\begin{aligned}
& mult(mult(x1,x2),\underbrace{i(x1)}_{}) \\
= & \quad \text{by Lemma 9 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \underbrace{mult(mult(x1,x2),\overbrace{rd(x2,mult(x1,x2))}^{})}_{} \\
= & \quad \text{by Lemma 55 RL with } \{x2 \leftarrow x2, x1 \leftarrow mult(x1,x2)\} \\
& \overbrace{\underbrace{mult(mult(mult(x1,x2),x2),i(mult(x1,x2)))}_{}}^{} \\
= & \quad \text{by Lemma 598 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{rd(i(mult(x1,x2)),rd(rd(op_t(i(x1),x2),x2),x2))}^{}
\end{aligned}
$$

Lemma 600: $mult(x1,rd(x2,x1)) = rd(rd(x2,rd(op_t(i(x1),x2),x2)),mult(x1,x2))$

$$
\begin{aligned}
& \underbrace{mult(x1,rd(x2,x1))}_{} \\
= & \quad \text{by Lemma 55 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{mult(mult(x1,x2),i(x1))}_{}}^{} \\
= & \quad \text{by Lemma 599 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{rd(i(mult(x1,x2)),rd(rd(op_t(i(x1),x2),x2),x2))}_{}}^{} \\
= & \quad \text{by Lemma 40 LR with } \{x3 \leftarrow x2, x2 \leftarrow rd(op_t(i(x1),x2),x2), x1 \leftarrow mult(x1,x2)\} \\
& \overbrace{rd(rd(x2,rd(op_t(i(x1),x2),x2)),mult(x1,x2))}^{}
\end{aligned}
$$

Lemma 601: $op_r(x1, x2, x1) = op_l(x1, x1, x2)$

$$
\begin{aligned}
& \overbrace{op_r(x1, x2, x1)} \\
= \quad & \text{by Lemma 123 RL with } \{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{mult(x1, rd(\underbrace{op_r(x1, x2, x1)}, x1))} \\
= \quad & \text{by Axiom 16 LR with } \{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& mult(x1, rd(\overbrace{rd(mult(mult(x1, \underbrace{x2}), x1), mult(x2, x1))}, x1)) \\
= \quad & \text{by Lemma 3 RL with } \{x1 \leftarrow x2\} \\
& mult(x1, rd(rd(\underbrace{mult(mult(x1, \overbrace{i(i(x2))}), x1)}, mult(x2, x1)), x1)) \\
= \quad & \text{by Lemma 407 LR with } \{x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& mult(x1, rd(\underbrace{rd(\overbrace{rd(x1, rd(op_t(i(x2), x1), x1))}, mult(x2, x1))}, x1)) \\
= \quad & \text{by Lemma 600 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& mult(x1, \underbrace{rd(\overbrace{mult(x2, rd(x1, x2))}, x1)}) \\
= \quad & \text{by Lemma 448 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \underbrace{mult(x1, \overbrace{asoc(x1, x2, x1)})} \\
= \quad & \text{by Lemma 158 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x1, x2)}
\end{aligned}
$$

Lemma 602: $mult(x1, rd(x2, op_t(x1, x2))) = op_t(mult(mult(i(x1), x2), x1), i(x1))$

$$
\begin{aligned}
& mult(x1, \underbrace{rd(x2, op_t(x1, x2))}) \\
= \quad & \text{by Lemma 10 RL with } \{x2 \leftarrow x2, x1 \leftarrow op_t(x1, x2)\} \\
& \underbrace{mult(x1, \overbrace{i(\overbrace{rd(op_t(x1, x2), x2)})})} \\
= \quad & \text{by Lemma 34 RL with } \{x2 \leftarrow rd(op_t(x1, x2), x2), x1 \leftarrow x1\} \\
& \overbrace{op_t(\underbrace{rd(x1, rd(op_t(x1, x2), x2))}, i(x1))} \\
= \quad & \text{by Lemma 595 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_t(\overbrace{mult(mult(i(x1), x2), x1)}, i(x1))
\end{aligned}
$$

Lemma 603: $mult(x1, rd(x2, op_t(x1, x2))) = rd(x1, op_t(mult(x1, i(x2)), x1))$

$$
\begin{aligned}
& \underbrace{mult(x1, rd(x2, op_t(x1, x2)))} \\
= \quad & \text{by Lemma 602 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_t(mult(mult(i(x1), x2), x1), i(x1))}} \\
= \quad & \text{by Lemma 326 LR with } \{x2 \leftarrow x1, x1 \leftarrow mult(i(x1), x2)\} \\
& \overbrace{rd(x1, op_t(\underbrace{i(mult(i(x1), x2))}, x1))} \\
= \quad & \text{by Lemma 16 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& rd(x1, op_t(\overbrace{mult(x1, i(x2))}, x1))
\end{aligned}
$$

Lemma 604: $mult(x1, rd(x2, op_t(x1, x2))) = op_r(x2, x1, i(x1))$

$$
\begin{aligned}
& \underbrace{mult(x1, rd(x2, op_t(x1, x2)))} \\
= \quad & \text{by Lemma 603 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{rd(x1, op_t(mult(x1, i(x2)), x1))}} \\
= \quad & \text{by Lemma 404 LR with } \{x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& \overbrace{op_r(\underbrace{i(i(x2))}, x1, i(x1))} \\
= \quad & \text{by Lemma 3 LR with } \{x1 \leftarrow x2\} \\
& op_r(\overbrace{x2}, x1, i(x1))
\end{aligned}
$$

Lemma 605: $mult(rd(op_t(x1, x2), x2), op_t(x2, x1)) = x1$

$$
\begin{aligned}
& mult(\underbrace{rd(op_t(x1, x2), x2)}, op_t(x2, x1)) \\
= \quad & \text{by Lemma 10 RL with } \{x2 \leftarrow op_t(x1, x2), x1 \leftarrow x2\} \\
& mult(\overbrace{i(rd(x2, op_t(x1, x2)))}, \underbrace{op_t(x2, x1)}) \\
= \quad & \text{by Lemma 597 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \underbrace{mult(i(rd(x2, op_t(x1, x2))), \overbrace{mult(rd(x2, op_t(x1, x2)), x1)})} \\
= \quad & \text{by Axiom 2 RL with } \{x2 \leftarrow rd(x2, op_t(x1, x2)), x1 \leftarrow x1\} \\
& \overbrace{x1}
\end{aligned}
$$

Lemma 606: $op_l(x1, x2, i(x1)) = op_l(x1, x1, x2)$

$$
\begin{aligned}
& \underbrace{op_l(x1, x2, i(x1))} \\
= \quad & \text{by Lemma 258 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{mult(\underbrace{mult(x1, i(x2))}, op_t(x2, x1))} \\
= \quad & \text{by Lemma 342 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& mult(\overbrace{rd(\underbrace{op_r(x1, x2, i(x2))}, x2)}, op_t(x2, x1)) \\
= \quad & \text{by Lemma 421 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& mult(rd(\overbrace{\underbrace{op_l(op_t(x1, x2), x1, x2)}}, x2), op_t(x2, x1)) \\
= \quad & \text{by Axiom 15 RL with } \{x4 \leftarrow x2, x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x1\} \\
& mult(\overbrace{rd(\overbrace{op_t(op_l(x1, x1, x2), x2)}, x2)}, \underbrace{op_t(x2, x1)}) \\
= \quad & \text{by Lemma 553 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \underbrace{mult(rd(op_t(op_l(x1, x1, x2), x2), x2), \overbrace{op_t(x2, op_l(x1, x1, x2))})} \\
= \quad & \text{by Lemma 605 LR with } \{x2 \leftarrow x2, x1 \leftarrow op_l(x1, x1, x2)\} \\
& \overbrace{op_l(x1, x1, x2)}
\end{aligned}
$$

Lemma 607: $op_l(x1, i(x2), x1) = op_l(x1, x1, x2)$

$$
\begin{aligned}
& \underbrace{op_l(x1, i(x2), x1)} \\
= & \quad \text{by Lemma 432 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, x2, i(x1))}} \\
= & \quad \text{by Lemma 606 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x1, x2)}
\end{aligned}
$$

Lemma 608: $op_l(x1, x1, i(x2)) = op_l(x1, x2, x1)$

$$
\begin{aligned}
& \underbrace{op_l(x1, x1, i(x2))} \\
= & \quad \text{by Lemma 606 RL with } \{x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, i(x2), i(x1))}} \\
= & \quad \text{by Lemma 438 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x2, x1)}
\end{aligned}
$$

Lemma 609: $op_l(x1, i(x1), x2) = op_l(x1, x2, x1)$

$$
\begin{aligned}
& \underbrace{op_l(x1, i(x1), x2)} \\
= & \quad \text{by Lemma 429 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, x1, i(x2))}} \\
= & \quad \text{by Lemma 608 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x2, x1)}
\end{aligned}
$$

Lemma 610: $rd(op_l(x1, x1, x2), x1) = asoc(x2, x1, i(x1))$

$$
\begin{aligned}
& rd(\underbrace{op_l(x1, x1, x2)}, x1) \\
= & \quad \text{by Lemma 606 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{rd(\overbrace{op_l(x1, x2, i(x1))}, x1)} \\
= & \quad \text{by Lemma 248 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{asoc(x1, mult(i(x1), x2), x1)}} \\
= & \quad \text{by Lemma 504 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{asoc(x2, x1, i(x1))}
\end{aligned}
$$

Lemma 611: $asoc(x1, x2, i(x2)) = asoc(x2, x1, x2)$

$$
\begin{aligned}
& \underbrace{asoc(x1, x2, i(x2))} \\
= & \quad \text{by Lemma 610 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \overbrace{\underbrace{rd(op_l(x2, x2, x1), x2)}} \\
= & \quad \text{by Lemma 172 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \overbrace{asoc(x2, x1, x2)}
\end{aligned}
$$

Lemma 612: $i(asoc(x1, x2, x1)) = asoc(x1, i(x2), x1)$

$$
\begin{aligned}
& i(\underbrace{asoc(x1, x2, x1)}) \\
= & \quad \text{by Lemma 611 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \underbrace{i(\overbrace{asoc(x2, x1, i(x1))})} \\
= & \quad \text{by Lemma 145 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \underbrace{\overbrace{asoc(i(x2), i(x1), x1)}} \\
= & \quad \text{by Lemma 322 RL with } \{x2 \leftarrow x1, x1 \leftarrow i(x2)\} \\
& \underbrace{\overbrace{asoc(i(x2), x1, i(x1))}} \\
= & \quad \text{by Lemma 611 LR with } \{x2 \leftarrow x1, x1 \leftarrow i(x2)\} \\
& \overbrace{asoc(x1, i(x2), x1)}
\end{aligned}
$$

Lemma 613: $asoc(x1, x2, x1) = asoc(i(x1), x2, i(x1))$

$$
\begin{aligned}
& asoc(x1, \underbrace{x2}, x1) \\
= & \quad \text{by Lemma 3 RL with } \{x1 \leftarrow x2\} \\
& \underbrace{asoc(x1, \overbrace{i(i(x2))}, x1)} \\
= & \quad \text{by Lemma 612 RL with } \{x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& \underbrace{\overbrace{i(asoc(x1, i(x2), x1))}} \\
= & \quad \text{by Lemma 271 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{asoc(i(x1), x2, i(x1))}
\end{aligned}
$$

Lemma 614: $rd(x1, op_l(x2, x1, x2)) = mult(x1, i(x2))$

$$
\begin{aligned}
& rd(x1, \underbrace{op_l(x2, x1, x2)}) \\
= & \quad \text{by Lemma 608 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \underbrace{rd(x1, \overbrace{op_l(x2, x2, i(x1))})} \\
= & \quad \text{by Lemma 445 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{mult(x1, i(x2))}
\end{aligned}
$$

Lemma 615: $rd(op_l(x1, x2, x1), x2) = mult(i(x2), x1)$

$$
\begin{aligned}
& rd(\underbrace{op_l(x1, x2, x1)}, x2) \\
= & \quad \text{by Lemma 608 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{rd(\overbrace{op_l(x1, x1, i(x2))}, x2)} \\
= & \quad \text{by Lemma 446 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{mult(i(x2), x1)}
\end{aligned}
$$

Lemma 616: $mult(x1, op_l(x2, x1, x2)) = rd(x1, i(x2))$

$$
\begin{aligned}
& mult(x1, \underbrace{op_l(x2, x1, x2)}) \\
= & \quad \text{by Lemma 608 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& mult(x1, \overbrace{\underbrace{op_l(x2, x2, i(x1))}}) \\
= & \quad \text{by Lemma 429 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& mult(x1, \overbrace{\underbrace{op_l(x2, i(x2), x1)}}) \\
= & \quad \text{by Lemma 229 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \underbrace{mult(x1, \overbrace{op_l(x2, mult(i(x2), x1), x2)})} \\
= & \quad \text{by Lemma 57 RL with } \{x2 \leftarrow op_l(x2, mult(i(x2), x1), x2), x1 \leftarrow x1\} \\
& \overbrace{rd(x1, mult(i(x1), \underbrace{rd(x1, op_l(x2, mult(i(x2), x1), x2))}))} \\
= & \quad \text{by Lemma 218 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& rd(x1, \underbrace{mult(i(x1), \overbrace{mult(x1, i(x2))})}) \\
= & \quad \text{by Axiom 2 RL with } \{x2 \leftarrow x1, x1 \leftarrow i(x2)\} \\
& rd(x1, \overbrace{i(x2)})
\end{aligned}
$$

Lemma 617: $rd(x1, op_l(x1, x2, x1)) = asoc(x1, x2, x1)$

$$
\begin{aligned}
& \underbrace{rd(x1, op_l(x1, x2, x1))} \\
= & \quad \text{by Lemma 487 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{\overbrace{asoc(x2, x1, i(x1))}} \\
= & \quad \text{by Lemma 611 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \overbrace{asoc(x1, x2, x1)}
\end{aligned}
$$

Lemma 618: $asoc(x1, mult(x1, x2), x1) = asoc(x1, x2, x1)$

$$
\begin{aligned}
& \underbrace{asoc(x1, mult(x1, x2), x1)} \\
= & \quad \text{by Lemma 611 RL with } \{x2 \leftarrow x1, x1 \leftarrow mult(x1, x2)\} \\
& \underbrace{\overbrace{asoc(mult(x1, x2), x1, i(x1))}} \\
= & \quad \text{by Lemma 489 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{asoc(x1, x2, x1)}
\end{aligned}
$$

Lemma 619: $op_l(x1, mult(x1, x2), x1) = op_l(x1, x2, x1)$

$$
\begin{aligned}
& \underbrace{op_l(x1, mult(x1, x2), x1)} \\
= & \quad \text{by Lemma 490 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{rd(x1, \underbrace{asoc(x1, x2, x1)})} \\
= & \quad \text{by Lemma 611 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& rd(x1, \overbrace{\underbrace{asoc(x2, x1, i(x1))}}) \\
= & \quad \text{by Lemma 487 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{rd(x1, \overbrace{rd(x1, op_l(x1, x2, x1))})} \\
= & \quad \text{by Lemma 353 LR with } \{x2 \leftarrow op_l(x1, x2, x1), x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_r(op_l(x1, x2, x1), x1, x1)}} \\
= & \quad \text{by Axiom 18 LR with } \{x5 \leftarrow x1, x4 \leftarrow x1, x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(\underbrace{op_r(x1, x1, x1)}, x2, x1)} \\
= & \quad \text{by Lemma 26 LR with } \{x2 \leftarrow x1, x1 \leftarrow x1\} \\
& op_l(\overbrace{x1}, x2, x1)
\end{aligned}
$$

Lemma 620: $rd(x1, asoc(x1, x2, x1)) = op_l(x1, x2, x1)$

$$
\begin{aligned}
& \underbrace{rd(x1, asoc(x1, x2, x1))} \\
= & \quad \text{by Lemma 490 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, mult(x1, x2), x1)}} \\
= & \quad \text{by Lemma 619 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x2, x1)}
\end{aligned}
$$

Lemma 621: $op_l(x1, x1, op_t(x2, x1)) = op_l(x1, x1, x2)$

$$
\begin{aligned}
& \underbrace{op_l(x1, x1, op_t(x2, x1))} \\
= & \quad \text{by Lemma 493 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, x2, i(x1))}} \\
= & \quad \text{by Lemma 606 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x1, x2)}
\end{aligned}
$$

Lemma 622: $op_l(x1, rd(x1, x2), x1) = op_l(x1, x1, x2)$

$$
\begin{aligned}
& \underbrace{op_l(x1, rd(x1, x2), x1)} \\
= & \quad \text{by Lemma 494 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, x2, i(x1))}} \\
= & \quad \text{by Lemma 606 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x1, x2)}
\end{aligned}
$$

Lemma 623: $op_l(x1, op_t(x2, x1), x1) = op_l(x1, x2, x1)$

$$
\begin{aligned}
& \underbrace{op_l(x1, op_t(x2, x1), x1)} \\
= & \quad \text{by Lemma 497 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, x1, i(x2))}} \\
= & \quad \text{by Lemma 608 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x2, x1)}
\end{aligned}
$$

Lemma 624: $op_l(x1, x1, mult(x1, x2)) = op_l(x1, x1, x2)$

$$
\begin{aligned}
& \underbrace{op_l(x1, x1, mult(x1, x2))} \\
= & \quad \text{by Lemma 514 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, x1, op_t(x2, i(x1)))}} \\
= & \quad \text{by Lemma 606 RL with } \{x2 \leftarrow op_t(x2, i(x1)), x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, op_t(x2, i(x1)), i(x1))}} \\
= & \quad \text{by Lemma 522 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x1, x2)}
\end{aligned}
$$

Lemma 625: $op_l(x1, rd(x2, x1), x2) = op_l(x1, x2, x1)$

$$
\begin{aligned}
& \underbrace{op_l(x1, rd(x2, x1), x2)} \\
= & \quad \text{by Lemma 525 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, x1, i(x2))}} \\
= & \quad \text{by Lemma 608 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x2, x1)}
\end{aligned}
$$

Lemma 626: $mult(x1, asoc(x2, x1, x2)) = op_r(x1, x2, i(x2))$

$$
\begin{aligned}
& mult(x1, \underbrace{asoc(x2, x1, x2)}) \\
= & \quad \text{by Lemma 611 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{mult(x1, \overbrace{asoc(x1, x2, i(x2))})} \\
= & \quad \text{by Lemma 107 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, x2, i(x2))}
\end{aligned}
$$

Lemma 627: $rd(asoc(x1, x2, x1), x1) = op_l(i(x1), x2, x1)$

$$
\begin{aligned}
& rd(\underbrace{asoc(x1, x2, x1)}, x1) \\
= & \quad \text{by Lemma 611 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \underbrace{rd(\overbrace{asoc(x2, x1, i(x1))}, x1)} \\
= & \quad \text{by Lemma 503 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{\overbrace{op_r(op_l(i(x1), x2, x1), x1, x1)}} \\
= & \quad \text{by Axiom 18 LR with } \{x5 \leftarrow x1, x4 \leftarrow x1, x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow i(x1)\} \\
& \overbrace{op_l(op_r(\underbrace{i(x1)}, x1, x1), x2, x1)} \\
= & \quad \text{by Lemma 9 RL with } \{x2 \leftarrow x1, x1 \leftarrow x1\} \\
& op_l(op_r(\overbrace{rd(x1, mult(x1, x1))}, \underbrace{x1}, x1), x2, x1) \\
= & \quad \text{by Axiom 5 LR with } \{x2 \leftarrow x1, x1 \leftarrow x1\} \\
& op_l(\underbrace{op_r(rd(x1, mult(x1, x1)), \overbrace{rd(mult(x1, x1), x1)}, x1)}, x2, x1) \\
= & \quad \text{by Lemma 51 LR with } \{x3 \leftarrow x1, x2 \leftarrow mult(x1, x1), x1 \leftarrow x1\} \\
& op_l(\overbrace{rd(x1, mult(x1, x1))}, x2, x1) \\
= & \quad \text{by Lemma 9 LR with } \{x2 \leftarrow x1, x1 \leftarrow x1\} \\
& op_l(\overbrace{i(x1)}, x2, x1)
\end{aligned}
$$

Lemma 628: $mult(asoc(x1, x2, x1), x2) = op_r(x2, x1, i(x1))$

$$
\begin{aligned}
& \underbrace{mult(asoc(x1, x2, x1), x2)} \\
= & \quad \text{by Lemma 540 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{\overbrace{mult(x2, asoc(x1, x2, x1))}} \\
= & \quad \text{by Lemma 626 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \overbrace{op_r(x2, x1, i(x1))}
\end{aligned}
$$

Lemma 629: $op_l(i(x1), x1, i(x2)) = op_l(i(x1), x2, x1)$

$$
\begin{aligned}
& \underbrace{op_l(i(x1), x1, i(x2))} \\
= & \quad \text{by Lemma 443 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{\overbrace{op_l(i(x1), i(x1), x2)}} \\
= & \quad \text{by Lemma 606 RL with } \{x2 \leftarrow x2, x1 \leftarrow i(x1)\} \\
& \overbrace{op_l(i(x1), x2, \underbrace{i(i(x1))})} \\
= & \quad \text{by Lemma 3 LR with } \{x1 \leftarrow x1\} \\
& op_l(i(x1), x2, \overbrace{x1}^{})
\end{aligned}
$$

Lemma 630: $op_l(i(x1), i(x1), x2) = op_l(i(x1), x2, x1)$

$$
\begin{aligned}
& \underbrace{op_l(i(x1), i(x1), x2)} \\
= \quad & \text{by Lemma 443 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(i(x1), x1, i(x2))}} \\
= \quad & \text{by Lemma 629 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(i(x1), x2, x1)}
\end{aligned}
$$

Lemma 631: $op_l(i(x1), i(x2), x1) = op_t(i(op_l(x1, x1, x2)), x1)$

$$
\begin{aligned}
& \underbrace{op_l(i(x1), i(x2), x1)} \\
= \quad & \text{by Lemma 517 RL with } \{x3 \leftarrow x1, x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& \overbrace{i(op_l(x1, \underbrace{i(i(x2))}, i(x1)))} \\
= \quad & \text{by Lemma 3 LR with } \{x1 \leftarrow x2\} \\
& \underbrace{i(op_l(x1, \overbrace{x2}, i(x1)))} \\
= \quad & \text{by Lemma 193 RL with } \{x3 \leftarrow i(x1), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_t(i(\underbrace{op_l(x1, x2, i(x1))}), x1)} \\
= \quad & \text{by Lemma 606 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_t(i(\overbrace{op_l(x1, x1, x2)}), x1)
\end{aligned}
$$

Lemma 632: $op_l(i(x1), x2, i(x1)) = i(op_l(x1, x1, x2))$

$$
\begin{aligned}
& \underbrace{op_l(i(x1), x2, i(x1))} \\
= \quad & \text{by Lemma 444 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(i(x1), i(x2), x1)}} \\
= \quad & \text{by Lemma 631 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_t(i(op_l(x1, x1, x2)), x1)}} \\
= \quad & \text{by Lemma 193 LR with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x1\} \\
& \overbrace{i(op_l(x1, x1, x2))}
\end{aligned}
$$

Lemma 633: $op_l(i(x1), x2, i(x1)) = op_l(i(x1), x1, x2)$

$$
\begin{aligned}
& \underbrace{op_l(i(x1), x2, i(x1))} \\
= \quad & \text{by Lemma 632 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{i(op_l(x1, x1, x2))}} \\
= \quad & \text{by Lemma 430 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(i(x1), x1, x2)}
\end{aligned}
$$

Lemma 634: $op_l(i(x1), i(x2), x1) = op_l(i(x1), x1, x2)$

$$
\begin{aligned}
& \underbrace{op_l(i(x1), i(x2), x1)} \\
= & \quad \text{by Lemma 444 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(i(x1), x2, i(x1))}} \\
= & \quad \text{by Lemma 633 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(i(x1), x1, x2)}
\end{aligned}
$$

Lemma 635: $op_l(mult(x1, x2), x2, x1) = op_l(mult(x1, x2), i(x1), x2)$

$$
\begin{aligned}
& \underbrace{op_l(mult(x1, x2), x2, x1)} \\
= & \quad \text{by Lemma 291 RL with } \{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow mult(x1, x2)\} \\
& \overbrace{\underbrace{op_l(mult(x1, x2), x2, mult(x1, x2))}} \\
= & \quad \text{by Lemma 625 RL with } \{x2 \leftarrow x2, x1 \leftarrow mult(x1, x2)\} \\
& \overbrace{op_l(mult(x1, x2), \underbrace{rd(x2, mult(x1, x2))}, x2)} \\
= & \quad \text{by Lemma 9 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& op_l(mult(x1, x2), \overbrace{i(x1)}, x2)
\end{aligned}
$$

Lemma 636: $op_l(mult(x1, x2), x2, x1) = mult(op_t(x1, x2), x2)$

$$
\begin{aligned}
& \underbrace{op_l(mult(x1, x2), x2, x1)} \\
= & \quad \text{by Lemma 635 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(mult(x1, x2), i(x1), x2)}} \\
= & \quad \text{by Axiom 12 RL with } \{x3 \leftarrow mult(x1, x2), x2 \leftarrow i(x1), x1 \leftarrow x2\} \\
& \overbrace{mult(i(mult(x2, i(x1))), mult(x2, \underbrace{mult(i(x1), mult(x1, x2))}))} \\
= & \quad \text{by Axiom 2 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& mult(\underbrace{i(mult(x2, i(x1)))}, mult(x2, \overbrace{x2})) \\
= & \quad \text{by Lemma 17 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \underbrace{mult(\overbrace{mult(i(x2), x1)}, mult(x2, x2))} \\
= & \quad \text{by Lemma 213 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \overbrace{\underbrace{op_t(rd(x1, i(x2)), x2)}} \\
= & \quad \text{by Lemma 337 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{mult(op_t(x1, x2), x2)}
\end{aligned}
$$

Lemma 637: $op_l(x1, x2, asoc(x3, i(x2), x3)) = x1$

$$
\begin{aligned}
& op_l(x1, x2, \underbrace{asoc(x3, i(x2), x3)}) \\
= & \quad \text{by Lemma 611 RL with } \{x2 \leftarrow x3, x1 \leftarrow i(x2)\} \\
& op_l(x1, \underbrace{x2}, \overbrace{asoc(i(x2), x3, i(x3))}) \\
= & \quad \text{by Lemma 3 RL with } \{x1 \leftarrow x2\} \\
& \underbrace{op_l(x1, \overbrace{i(i(x2))}, asoc(i(x2), x3, i(x3)))} \\
= & \quad \text{by Lemma 297 RL with } \{x3 \leftarrow x3, x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& \overbrace{x1}
\end{aligned}
$$

Lemma 638: $op_l(x1, asoc(x3, i(x2), x3), x2) = x1$

$$
\begin{aligned}
& op_l(x1, \underbrace{asoc(x3, i(x2), x3)}, x2) \\
= & \quad \text{by Lemma 611 RL with } \{x2 \leftarrow x3, x1 \leftarrow i(x2)\} \\
& op_l(x1, \overbrace{asoc(i(x2), x3, i(x3))}, \underbrace{x2}) \\
= & \quad \text{by Lemma 3 RL with } \{x1 \leftarrow x2\} \\
& \underbrace{op_l(x1, asoc(i(x2), x3, i(x3)), \overbrace{i(i(x2))})} \\
= & \quad \text{by Lemma 298 RL with } \{x3 \leftarrow x3, x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& \overbrace{x1}
\end{aligned}
$$

Lemma 639: $rd(op_r(x1, x2, i(x2)), x1) = asoc(x2, x1, x2)$

$$
\begin{aligned}
& \underbrace{rd(op_r(x1, x2, i(x2)), x1)} \\
= & \quad \text{by Lemma 141 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{asoc(x1, x2, i(x2))}} \\
= & \quad \text{by Lemma 611 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{asoc(x2, x1, x2)}
\end{aligned}
$$

Lemma 640: $asoc(x1, x2, x1) = rd(x2, op_r(x2, x1, x1))$

$$
\begin{aligned}
& \underbrace{asoc(x1, x2, x1)} \\
= & \quad \text{by Lemma 502 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{asoc(x1, op_r(x2, x1, x1), x1)}} \\
= & \quad \text{by Lemma 639 RL with } \{x2 \leftarrow x1, x1 \leftarrow op_r(x2, x1, x1)\} \\
& \overbrace{rd(\underbrace{op_r(op_r(x2, x1, x1), x1, i(x1))}, op_r(x2, x1, x1))} \\
= & \quad \text{by Lemma 351 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& rd(\overbrace{x2}, op_r(x2, x1, x1))
\end{aligned}
$$

Lemma 641: $rd(x1, asoc(x2, x1, x2)) = op_r(x1, x2, x2)$

$$
\begin{aligned}
& rd(x1, \underbrace{asoc(x2, x1, x2)}) \\
= & \quad \text{by Lemma 165 RL with } \{x3 \leftarrow x2, x2 \leftarrow op_r(x1, x2, x2), x1 \leftarrow asoc(x2, x1, x2)\} \\
& rd(x1, \overbrace{op_l(asoc(x2, x1, x2), op_r(x1, x2, x2), \underbrace{op_r(op_r(x1, x2, x2), x2, i(x2))})}) \\
= & \quad \text{by Lemma 351 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& rd(x1, op_l(\underbrace{asoc(x2, x1, x2)}, op_r(x1, x2, x2), \overbrace{x1})) \\
= & \quad \text{by Lemma 640 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& rd(x1, op_l(\overbrace{rd(x1, op_r(x1, x2, x2))}, op_r(x1, x2, x2), x1)) \\
= & \quad \text{by Lemma 10 RL with } \{x2 \leftarrow x1, x1 \leftarrow op_r(x1, x2, x2)\} \\
& rd(x1, \underbrace{op_l(\overbrace{i(rd(op_r(x1, x2, x2), x1))}, op_r(x1, x2, x2), x1)}) \\
= & \quad \text{by Lemma 292 RL with } \{x3 \leftarrow x1, x2 \leftarrow op_r(x1, x2, x2), x1 \leftarrow i(rd(op_r(x1, x2, x2), x1))\} \\
& \underbrace{rd(x1, \overbrace{op_l(i(rd(op_r(x1, x2, x2), x1)), rd(op_r(x1, x2, x2), x1), x1)})} \\
= & \quad \text{by Lemma 231 RL with } \{x2 \leftarrow rd(op_r(x1, x2, x2), x1), x1 \leftarrow x1\} \\
& \underbrace{\overbrace{mult(x1, rd(op_r(x1, x2, x2), x1))}} \\
= & \quad \text{by Lemma 123 LR with } \{x3 \leftarrow x2, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, x2, x2)}
\end{aligned}
$$

Lemma 642: $rd(asoc(x1, x2, x1), x2) = op_r(i(x2), x1, x1)$

$$
\begin{aligned}
& \underbrace{rd(asoc(x1, x2, x1), x2)} \\
= & \quad \text{by Lemma 542 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{mult(\underbrace{asoc(x1, x2, x1)}, i(x2))} \\
= & \quad \text{by Lemma 640 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{mult(\overbrace{rd(x2, op_r(x2, x1, x1))}, i(x2))} \\
= & \quad \text{by Lemma 22 LR with } \{x2 \leftarrow op_r(x2, x1, x1), x1 \leftarrow x2\} \\
& \underbrace{\overbrace{i(op_r(x2, x1, x1))}} \\
= & \quad \text{by Lemma 356 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \overbrace{op_r(i(x2), x1, x1)}
\end{aligned}
$$

Lemma 643: $mult(x1, op_r(i(x1), x2, x2)) = asoc(x2, x1, x2)$

$$
\begin{aligned}
& mult(x1, \underbrace{op_r(i(x1), x2, x2)}) \\
= & \quad \text{by Lemma 354 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \underbrace{mult(x1, \overbrace{rd(rd(x2, x1), x2)})} \\
= & \quad \text{by Lemma 403 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{rd(x1, op_r(x1, x2, x2))}} \\
= & \quad \text{by Lemma 640 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \overbrace{asoc(x2, x1, x2)}
\end{aligned}
$$

Lemma 644: $mult(x1, asoc(x2, i(x1), x2)) = op_r(x1, x2, x2)$

$\overbrace{mult(x1, asoc(x2, i(x1), x2))}$

$=$ by Lemma 544 RL with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$\overbrace{mult(\underbrace{asoc(x2, i(x1), x2)}, x1)}$

$=$ by Lemma 640 LR with $\{x2 \leftarrow i(x1), x1 \leftarrow x2\}$

$\underbrace{mult(\overbrace{rd(i(x1), op_r(i(x1), x2, x2))}, x1)}$

$=$ by Lemma 21 LR with $\{x2 \leftarrow op_r(i(x1), x2, x2), x1 \leftarrow x1\}$

$\overbrace{\underbrace{i(op_r(i(x1), x2, x2))}}$

$=$ by Lemma 356 LR with $\{x2 \leftarrow x2, x1 \leftarrow i(x1)\}$

$\overbrace{op_r(\underbrace{i(i(x1))}, x2, x2)}$

$=$ by Lemma 3 LR with $\{x1 \leftarrow x1\}$

$op_r(\overbrace{x1}, x2, x2)$

Lemma 645: $rd(x1, op_r(x1, x2, i(x2))) = asoc(x2, i(x1), x2)$

$rd(x1, \underbrace{op_r(x1, x2, i(x2))})$

$=$ by Lemma 107 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$rd(\underbrace{x1}, \overbrace{mult(x1, asoc(x1, x2, i(x2)))})$

$=$ by Axiom 4 LR with $\{x2 \leftarrow op_r(x1, x2, i(x2)), x1 \leftarrow x1\}$

$rd(\overbrace{mult(\underbrace{rd(x1, op_r(x1, x2, i(x2)))}, op_r(x1, x2, i(x2)))}, mult(x1, asoc(x1, x2, i(x2))))$

$=$ by Lemma 10 RL with $\{x2 \leftarrow x1, x1 \leftarrow op_r(x1, x2, i(x2))\}$

$rd(mult(\overbrace{i(\overbrace{rd(op_r(x1, x2, i(x2)), x1)})}, \underbrace{op_r(x1, x2, i(x2))}), mult(x1, asoc(x1, x2, i(x2))))$

$=$ by Lemma 123 RL with $\{x3 \leftarrow i(x2), x2 \leftarrow x2, x1 \leftarrow x1\}$

$rd(\underbrace{mult(i(rd(op_r(x1, x2, i(x2)), x1)), \overbrace{mult(x1, rd(op_r(x1, x2, i(x2)), x1))})}, mult(x1, asoc(x1, x2, i(x2))))$

$=$ by Axiom 10 LR with $\{x2 \leftarrow x1, x1 \leftarrow rd(op_r(x1, x2, i(x2)), x1)\}$

$rd(\overbrace{op_t(x1, \underbrace{rd(op_r(x1, x2, i(x2)), x1)})}, mult(x1, asoc(x1, x2, i(x2))))$

$=$ by Lemma 141 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{rd(op_t(x1, \overbrace{asoc(x1, x2, i(x2))}), mult(x1, asoc(x1, x2, i(x2))))}$

$=$ by Lemma 36 LR with $\{x2 \leftarrow asoc(x1, x2, i(x2)), x1 \leftarrow x1\}$

$\overbrace{\underbrace{i(asoc(x1, x2, i(x2)))}}$

$=$ by Lemma 145 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{\underbrace{asoc(i(x1), i(x2), x2)}}$

$=$ by Lemma 322 RL with $\{x2 \leftarrow x2, x1 \leftarrow i(x1)\}$

$\overbrace{\underbrace{asoc(i(x1), x2, i(x2))}}$

$=$ by Lemma 611 LR with $\{x2 \leftarrow x2, x1 \leftarrow i(x1)\}$

$\overbrace{asoc(x2, i(x1), x2)}$

Lemma 646: $asoc(x1, i(mult(x1, x2)), x1) = asoc(x1, i(x2), x1)$

$$
\begin{aligned}
& asoc(x1, \underbrace{i(mult(x1, x2))}, x1) \\
= & \quad \text{by Axiom 11 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{asoc(x1, \overbrace{mult(i(x1), i(x2))}, x1)} \\
= & \quad \text{by Lemma 458 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{mult(i(x1), \underbrace{op_l(x1, x2, x1)})} \\
= & \quad \text{by Lemma 486 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{mult(i(x1), \overbrace{mult(x1, asoc(i(x2), x1, i(x1)))})} \\
= & \quad \text{by Axiom 2 RL with } \{x2 \leftarrow x1, x1 \leftarrow asoc(i(x2), x1, i(x1))\} \\
& \underbrace{\overbrace{asoc(i(x2), x1, i(x1))}} \\
= & \quad \text{by Lemma 611 LR with } \{x2 \leftarrow x1, x1 \leftarrow i(x2)\} \\
& \overbrace{asoc(x1, i(x2), x1)}
\end{aligned}
$$

Lemma 647: $op_l(i(x1), mult(x1, x2), x1) = op_l(i(x1), x2, x1)$

$$
\begin{aligned}
& \underbrace{op_l(i(x1), mult(x1, x2), x1)} \\
= & \quad \text{by Lemma 506 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{\overbrace{rd(asoc(x1, x2, x1), x1)}} \\
= & \quad \text{by Lemma 627 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(i(x1), x2, x1)}
\end{aligned}
$$

Lemma 648: $asoc(x1, i(op_t(x2, x1)), x1) = asoc(x1, i(x2), x1)$

$$
\begin{aligned}
& \underbrace{asoc(x1, i(op_t(x2, x1)), x1)} \\
= & \quad \text{by Lemma 498 RL with } \{x2 \leftarrow op_t(x2, x1), x1 \leftarrow x1\} \\
& \overbrace{asoc(x1, \underbrace{rd(x1, op_t(x2, x1))}, x1)} \\
= & \quad \text{by Lemma 10 RL with } \{x2 \leftarrow x1, x1 \leftarrow op_t(x2, x1)\} \\
& \underbrace{asoc(x1, \overbrace{i(rd(op_t(x2, x1), x1))}, x1)} \\
= & \quad \text{by Lemma 498 RL with } \{x2 \leftarrow rd(op_t(x2, x1), x1), x1 \leftarrow x1\} \\
& \overbrace{asoc(x1, \underbrace{rd(x1, rd(op_t(x2, x1), x1))}, x1)} \\
= & \quad \text{by Lemma 407 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{asoc(x1, \overbrace{mult(mult(x1, i(x2)), x1)}, x1)} \\
= & \quad \text{by Lemma 262 LR with } \{x2 \leftarrow mult(x1, i(x2)), x1 \leftarrow x1\} \\
& \overbrace{\underbrace{asoc(x1, mult(x1, i(x2)), x1)}} \\
= & \quad \text{by Lemma 618 LR with } \{x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& \overbrace{asoc(x1, i(x2), x1)}
\end{aligned}
$$

Lemma 649: $rd(mult(x1,x2),x1) = mult(rd(mult(i(x1),mult(x1,x2)),x1),op_t(x1,x2))$

$$
\begin{aligned}
& \overbrace{rd(mult(x1,x2),x1)} \\
= \quad & \text{by Lemma 605 RL with } \{x2 \leftarrow x1, x1 \leftarrow rd(mult(x1,x2),x1)\} \\
& \overbrace{mult(rd(op_t(rd(mult(x1,x2),x1),x1),x1),\underbrace{op_t(x1,rd(mult(x1,x2),x1))}_{})} \\
= \quad & \text{by Lemma 378 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& mult(rd(\underbrace{op_t(rd(mult(x1,x2),x1),x1)}_{},x1),\overbrace{op_t(x1,x2)}) \\
= \quad & \text{by Lemma 14 LR with } \{x2 \leftarrow x1, x1 \leftarrow mult(x1,x2)\} \\
& mult(rd(\overbrace{mult(i(x1),mult(x1,x2))},x1),op_t(x1,x2))
\end{aligned}
$$

Lemma 650: $mult(rd(x1,x2),op_t(x2,x1)) = rd(mult(x2,x1),x2)$

$$
\begin{aligned}
& mult(rd(\underbrace{x1}_{},x2),op_t(x2,x1)) \\
= \quad & \text{by Axiom 2 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{mult(rd(\overbrace{mult(i(x2),mult(x2,x1))},x2),op_t(x2,x1))}_{} \\
= \quad & \text{by Lemma 649 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \overbrace{rd(mult(x2,x1),x2)}
\end{aligned}
$$

Lemma 651: $op_l(x1,x2,i(x3)) = op_l(x1,x3,op_l(x2,x3,x2))$

$$
\begin{aligned}
& \overbrace{op_l(x1,x2,i(x3))} \\
= \quad & \text{by Lemma 483 LR with } \{x3 \leftarrow i(x3), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1,i(i(x3)),\underbrace{op_l(x2,x2,i(x3))}_{})} \\
= \quad & \text{by Lemma 608 LR with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& op_l(x1,\underbrace{i(i(x3))}_{},\overbrace{op_l(x2,x3,x2)}) \\
= \quad & \text{by Lemma 3 LR with } \{x1 \leftarrow x3\} \\
& op_l(x1,\overbrace{x3}^{},op_l(x2,x3,x2))
\end{aligned}
$$

Lemma 652: $op_l(i(x1),x1,mult(x1,x2)) = op_l(i(x1),x1,x2)$

$$
\begin{aligned}
& \overbrace{op_l(i(x1),x1,mult(x1,x2))} \\
= \quad & \text{by Lemma 430 RL with } \{x2 \leftarrow mult(x1,x2), x1 \leftarrow x1\} \\
& \overbrace{\underbrace{i(op_l(x1,x1,mult(x1,x2)))}_{}} \\
= \quad & \text{by Lemma 182 RL with } \{x2 \leftarrow mult(x1,x2), x1 \leftarrow x1\} \\
& \overbrace{rd(i(x1),\underbrace{asoc(x1,mult(x1,x2),x1)}_{})} \\
= \quad & \text{by Lemma 618 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{rd(i(x1),\overbrace{asoc(x1,x2,x1)})}_{} \\
= \quad & \text{by Lemma 182 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{i(op_l(x1,x1,x2))}_{}} \\
= \quad & \text{by Lemma 430 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(i(x1),x1,x2)}
\end{aligned}
$$

Lemma 653: $op_r(x1, x2, i(x2)) = op_r(x1, rd(i(x1), x2), x2)$

$\underbrace{op_r(x1, x2, i(x2))}$

$=$ by Lemma 626 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{\underbrace{mult(x1, asoc(x2, x1, x2))}}$

$=$ by Lemma 29 RL with $\{x2 \leftarrow mult(x1, asoc(x2, x1, x2)), x1 \leftarrow i(x1)\}$

$\overbrace{rd(\underbrace{mult(i(x1), mult(x1, asoc(x2, x1, x2)))}, op_t(i(x1), mult(x1, asoc(x2, x1, x2))))}$

$=$ by Axiom 2 RL with $\{x2 \leftarrow x1, x1 \leftarrow asoc(x2, x1, x2)\}$

$rd(\overbrace{asoc(x2, x1, x2)}, op_t(i(x1), \underbrace{mult(x1, asoc(x2, x1, x2))}))$

$=$ by Lemma 626 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$rd(asoc(x2, x1, x2), op_t(i(x1), \overbrace{op_r(\underbrace{x1}, x2, i(x2))}))$

$=$ by Lemma 3 RL with $\{x1 \leftarrow x1\}$

$rd(asoc(x2, x1, x2), \underbrace{op_t(i(x1), op_r(\overbrace{i(i(x1))}, x2, i(x2)))})$

$=$ by Axiom 10 RL with $\{x2 \leftarrow i(x1), x1 \leftarrow op_r(i(i(x1)), x2, i(x2))\}$

$rd(asoc(x2, x1, x2), \overbrace{mult(i(op_r(i(i(x1)), x2, i(x2))), \underbrace{mult(i(x1), op_r(i(i(x1)), x2, i(x2)))})})$

$=$ by Lemma 159 RL with $\{x3 \leftarrow i(x2), x2 \leftarrow x2, x1 \leftarrow i(x1)\}$

$rd(asoc(x2, x1, x2), \underbrace{mult(i(op_r(i(i(x1)), x2, i(x2))), \overbrace{mult(op_r(i(i(x1)), x2, i(x2)), i(x1))})})$

$=$ by Axiom 2 RL with $\{x2 \leftarrow op_r(i(i(x1)), x2, i(x2)), x1 \leftarrow i(x1)\}$

$\underbrace{rd(asoc(x2, x1, x2), \overbrace{i(x1)})}$

$=$ by Lemma 38 LR with $\{x1 \leftarrow x1, x2 \leftarrow asoc(x2, x1, x2)\}$

$\overbrace{rd(x1, \underbrace{i(asoc(x2, x1, x2))})}$

$=$ by Lemma 612 LR with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$\underbrace{rd(x1, \overbrace{asoc(x2, i(x1), x2)})}$

$=$ by Lemma 546 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_r(x1, rd(i(x1), x2), x2)}$

Lemma 654: $op_r(i(x1), mult(x1, x2), x3) = i(op_r(x1, x2, x3))$

$\overbrace{op_r(i(x1), mult(x1, x2), x3)}$

$=$ by Axiom 5 LR with $\{x2 \leftarrow i(x1), x1 \leftarrow op_r(i(x1), mult(x1, x2), x3)\}$

$\overbrace{rd(\underbrace{mult(op_r(i(x1), mult(x1, x2), x3), i(x1))}, i(x1))}$

$=$ by Lemma 119 LR with $\{x3 \leftarrow x3, x2 \leftarrow mult(x1, x2), x1 \leftarrow i(x1)\}$

$rd(\overbrace{mult(i(x1), \underbrace{op_r(i(x1), mult(x1, x2), x3)})}, i(x1))$

$=$ by Axiom 16 LR with $\{x3 \leftarrow x3, x2 \leftarrow mult(x1, x2), x1 \leftarrow i(x1)\}$

$rd(mult(i(x1), \overbrace{rd(mult(\underbrace{mult(i(x1), mult(x1, x2))}, x3), mult(mult(x1, x2), x3))}), i(x1))$

$=$ by Axiom 2 RL with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$\underbrace{rd(mult(i(x1), rd(mult(\overbrace{x2}, x3), mult(mult(x1, x2), x3))), i(x1))}$

$=$ by Lemma 63 LR with $\{x3 \leftarrow x1, x2 \leftarrow rd(mult(x2, x3), mult(mult(x1, x2), x3)), x1 \leftarrow x1\}$

$\overbrace{rd(x1, mult(x1, \underbrace{i(rd(mult(x2, x3), mult(mult(x1, x2), x3)))}))}$

$=$ by Lemma 10 LR with $\{x2 \leftarrow mult(mult(x1, x2), x3), x1 \leftarrow mult(x2, x3)\}$

$rd(x1, mult(x1, \overbrace{rd(mult(mult(x1, x2), x3), mult(x2, x3))}))$

$=$ by Axiom 16 RL with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{rd(x1, mult(x1, \overbrace{op_r(x1, x2, x3)}))}$

$=$ by Lemma 146 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{i(op_r(x1, x2, x3))}$

Lemma 655: $i(op_r(x1, x2, i(x1))) = op_l(i(x1), mult(x1, x2), x1)$

$\underbrace{i(op_r(x1, x2, i(x1)))}$

$=$ by Lemma 654 RL with $\{x3 \leftarrow i(x1), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{\overbrace{op_r(i(x1), mult(x1, x2), i(x1))}}$

$=$ by Lemma 601 LR with $\{x2 \leftarrow mult(x1, x2), x1 \leftarrow i(x1)\}$

$\underbrace{\overbrace{op_l(i(x1), i(x1), mult(x1, x2))}}$

$=$ by Lemma 630 LR with $\{x2 \leftarrow mult(x1, x2), x1 \leftarrow x1\}$

$\overbrace{op_l(i(x1), mult(x1, x2), x1)}$

Lemma 656: $op_r(x1, x2, i(x1)) = i(op_l(i(x1), x2, x1))$

$\overbrace{op_r(x1, x2, i(x1))}$

$=$ by Lemma 3 RL with $\{x1 \leftarrow op_r(x1, x2, i(x1))\}$

$\overbrace{i(\underbrace{i(op_r(x1, x2, i(x1)))})}$

$=$ by Lemma 655 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$i(\overbrace{\underbrace{op_l(i(x1), mult(x1, x2), x1)}})$

$=$ by Lemma 647 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$i(\overbrace{op_l(i(x1), x2, x1)})$

Lemma 657: $op_r(x1, x2, i(x1)) = op_l(x1, x1, i(x2))$

$$
\begin{aligned}
& \underbrace{op_r(x1, x2, i(x1))} \\
= & \quad \text{by Lemma 656 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{i(op_l(i(x1), x2, x1))}} \\
= & \quad \text{by Lemma 518 LR with } \{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, i(x2), i(x1))}} \\
= & \quad \text{by Lemma 606 LR with } \{x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x1, i(x2))}
\end{aligned}
$$

Lemma 658: $op_r(x1, x2, i(x1)) = op_l(x1, x2, x1)$

$$
\begin{aligned}
& \underbrace{op_r(x1, x2, i(x1))} \\
= & \quad \text{by Lemma 657 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, x1, i(x2))}} \\
= & \quad \text{by Lemma 608 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x2, x1)}
\end{aligned}
$$

Lemma 659: $x1 = op_r(x1, asoc(x2, x1, x2), x3)$

$\overbrace{x1}$

$=$ by Lemma 3 RL with $\{x1 \leftarrow x1\}$

$\overbrace{i(\underbrace{i(x1)}_{x1})}$

$=$ by Axiom 5 LR with $\{x2 \leftarrow i(rd(x2, x1)), x1 \leftarrow i(x1)\}$

$i(\overbrace{rd(\underbrace{mult(i(x1), i(rd(x2, x1)))}_{x1}, i(rd(x2, x1))))}$

$=$ by Lemma 166 RL with $\{x3 \leftarrow x3, x2 \leftarrow i(rd(x2, x1)), x1 \leftarrow i(x1)\}$

$i(\underbrace{rd(\overbrace{mult(op_r(i(x1), i(op_t(i(x1), \overbrace{i(rd(x2, x1))}))), x3)}, i(rd(x2, x1)))}, i(rd(x2, x1))))$

$=$ by Axiom 5 RL with $\{x2 \leftarrow i(rd(x2, x1)), x1 \leftarrow op_r(i(x1), i(op_t(i(x1), i(rd(x2, x1)))), x3)\}$

$i(\overbrace{op_r(i(x1), \underbrace{i(op_t(i(x1), i(rd(x2, x1))))}, x3)})$

$=$ by Lemma 15 LR with $\{x2 \leftarrow rd(x2, x1), x1 \leftarrow x1\}$

$i(op_r(i(x1), \underbrace{i(\overbrace{i(op_t(x1, rd(x2, x1)))})}, x3))$

$=$ by Lemma 3 LR with $\{x1 \leftarrow op_t(x1, rd(x2, x1))\}$

$\underbrace{i(op_r(i(x1), \overbrace{op_t(x1, rd(x2, x1))}, x3))}$

$=$ by Lemma 654 RL with $\{x3 \leftarrow x3, x2 \leftarrow op_t(x1, rd(x2, x1)), x1 \leftarrow i(x1)\}$

$\overbrace{op_r(i(i(x1)), \underbrace{mult(i(x1), op_t(x1, rd(x2, x1)))}, x3)}$

$=$ by Lemma 96 LR with $\{x2 \leftarrow rd(x2, x1), x1 \leftarrow x1\}$

$op_r(\underbrace{i(i(x1))}, \overbrace{rd(op_t(x1, rd(x2, x1)), x1)}, x3)$

$=$ by Lemma 3 LR with $\{x1 \leftarrow x1\}$

$op_r(\overbrace{x1}, rd(\underbrace{op_t(x1, rd(x2, x1))}, x1), x3)$

$=$ by Lemma 547 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$op_r(x1, \underbrace{rd(\overbrace{op_r(x1, x2, i(x2))}, x1)}, x3)$

$=$ by Lemma 639 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$op_r(x1, \overbrace{asoc(x2, x1, x2)}, x3)$

Lemma 660: $op_l(mult(x1, x2), x1, x2) = op_l(mult(x1, x2), op_t(x1, x2), x2)$

$\underbrace{op_l(mult(x1, x2), x1, x2)}$

$=$ by Lemma 285 RL with $\{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow mult(x1, x2)\}$

$\underbrace{\overbrace{op_l(mult(x1, x2), mult(x1, x2), x2)}}$

$=$ by Lemma 607 RL with $\{x2 \leftarrow x2, x1 \leftarrow mult(x1, x2)\}$

$\underbrace{\overbrace{op_l(mult(x1, x2), i(x2), mult(x1, x2))}}$

$=$ by Lemma 246 LR with $\{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow mult(x1, x2)\}$

$\overbrace{op_l(mult(x1, x2), op_t(x1, x2), x2)}$

Lemma 661: $op_l(mult(x1,x2),x1,mult(x1,x2)) = mult(x2,x1)$

$$
\begin{aligned}
& \underbrace{op_l(mult(x1,x2),x1,mult(x1,x2))} \\
= & \quad \text{by Lemma 608 RL with } \{x2 \leftarrow x1, x1 \leftarrow mult(x1,x2)\} \\
& \overbrace{\underbrace{op_l(mult(x1,x2),mult(x1,x2),i(x1))}} \\
= & \quad \text{by Lemma 242 LR with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow mult(x1,x2)\} \\
& \overbrace{\underbrace{op_l(mult(x1,x2),x1,x2)}} \\
= & \quad \text{by Lemma 435 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{mult(x2,x1)}
\end{aligned}
$$

Lemma 662: $x1 = mult(i(mult(i(x1),x2)),op_l(x2,x1,x2))$

$$
\begin{aligned}
& \underbrace{x1} \\
= & \quad \text{by Lemma 3 RL with } \{x1 \leftarrow x1\} \\
& \underbrace{\overbrace{i(i(x1))}} \\
= & \quad \text{by Lemma 529 RL with } \{x2 \leftarrow x2, x1 \leftarrow i(x1)\} \\
& \overbrace{mult(i(mult(i(x1),x2)),\underbrace{op_l(x2,x2,i(x1))})} \\
= & \quad \text{by Lemma 608 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& mult(i(mult(i(x1),x2)),\overbrace{op_l(x2,x1,x2)})
\end{aligned}
$$

Lemma 663: $mult(mult(x1,i(x2)),op_l(x2,x1,x2)) = x1$

$$
\begin{aligned}
& mult(\underbrace{mult(x1,i(x2))},op_l(x2,x1,x2)) \\
= & \quad \text{by Lemma 16 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{mult(\overbrace{i(mult(i(x1),x2))},op_l(x2,x1,x2))} \\
= & \quad \text{by Lemma 662 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{x1}
\end{aligned}
$$

Lemma 664: $x1 = op_l(x1,rd(x2,x3),asoc(x3,x2,x3))$

$$
\begin{aligned}
& \underbrace{x1} \\
= & \quad \text{by Lemma 293 LR with } \{x3 \leftarrow x3, x2 \leftarrow rd(x2,x3), x1 \leftarrow x1\} \\
& \overbrace{op_l(x1,rd(x2,x3),\underbrace{asoc(rd(x2,x3),x3,i(x3))})} \\
= & \quad \text{by Lemma 611 LR with } \{x2 \leftarrow x3, x1 \leftarrow rd(x2,x3)\} \\
& op_l(x1,rd(x2,x3),\overbrace{\underbrace{asoc(x3,rd(x2,x3),x3)}}) \\
= & \quad \text{by Lemma 263 LR with } \{x2 \leftarrow x2, x1 \leftarrow x3\} \\
& op_l(x1,rd(x2,x3),\overbrace{asoc(x3,x2,x3)})
\end{aligned}
$$

Lemma 665: $rd(mult(mult(x1, i(x2)), mult(x2, x3)), x3) = op_r(x1, i(x2), mult(x2, x3))$

$$
\begin{aligned}
& \underbrace{rd(mult(mult(x1, i(x2)), mult(x2, x3)), x3)} \\
= & \quad \text{by Axiom 2 LR with } \{x2 \leftarrow x2, x1 \leftarrow x3\} \\
& \overbrace{rd(mult(mult(x1, i(x2)), mult(x2, x3)), \overbrace{mult(i(x2), mult(x2, x3))})} \\
= & \quad \text{by Axiom 16 RL with } \{x3 \leftarrow mult(x2, x3), x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, i(x2), mult(x2, x3))}
\end{aligned}
$$

Lemma 666: $op_t(rd(x2, op_t(x1, x2)), x1) = op_t(i(rd(op_t(x1, x2), x2)), op_t(x1, mult(mult(i(x1), x2), x1)))$

$$
\begin{aligned}
& op_t(\underbrace{rd(x2, op_t(x1, x2))}, x1) \\
= & \quad \text{by Lemma 10 RL with } \{x2 \leftarrow x2, x1 \leftarrow op_t(x1, x2)\} \\
& \underbrace{op_t(\overbrace{i(rd(op_t(x1, x2), x2))}, x1)} \\
= & \quad \text{by Lemma 589 RL with } \{x2 \leftarrow x1, x1 \leftarrow rd(op_t(x1, x2), x2)\} \\
& \overbrace{op_t(i(rd(op_t(x1, x2), x2)), op_t(x1, \underbrace{rd(x1, rd(op_t(x1, x2), x2))}))} \\
= & \quad \text{by Lemma 595 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_t(i(rd(op_t(x1, x2), x2)), op_t(x1, \overbrace{mult(mult(i(x1), x2), x1)}))
\end{aligned}
$$

Lemma 667: $op_t(rd(x2, op_t(x1, x2)), x1) = op_t(rd(x2, op_t(x1, x2)), op_t(x1, x2))$

$$
\begin{aligned}
& \underbrace{op_t(rd(x2, op_t(x1, x2)), x1)} \\
= & \quad \text{by Lemma 666 LR with } \{x1 \leftarrow x1, x2 \leftarrow x2\} \\
& \overbrace{op_t(\underbrace{i(rd(op_t(x1, x2), x2))}, op_t(x1, mult(mult(i(x1), x2), x1)))} \\
= & \quad \text{by Lemma 10 LR with } \{x2 \leftarrow x2, x1 \leftarrow op_t(x1, x2)\} \\
& op_t(\overbrace{rd(x2, op_t(x1, x2))}, op_t(x1, \underbrace{mult(mult(i(x1), x2), x1)})) \\
= & \quad \text{by Lemma 439 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& op_t(rd(x2, op_t(x1, x2)), \underbrace{op_t(x1, \overbrace{op_l(x2, x2, i(x1))})}) \\
= & \quad \text{by Lemma 566 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_t(rd(x2, op_t(x1, x2)), \overbrace{op_t(x1, x2)})
\end{aligned}
$$

Lemma 668: $op_t(rd(x1, op_t(x2, x1)), x2) = mult(i(op_t(x2, x1)), x1)$

$$
\begin{aligned}
& \underbrace{op_t(rd(x1, op_t(x2, x1)), x2)} \\
= & \quad \text{by Lemma 667 LR with } \{x1 \leftarrow x2, x2 \leftarrow x1\} \\
& \overbrace{\underbrace{op_t(rd(x1, op_t(x2, x1)), op_t(x2, x1))}} \\
= & \quad \text{by Lemma 14 LR with } \{x2 \leftarrow op_t(x2, x1), x1 \leftarrow x1\} \\
& \overbrace{mult(i(op_t(x2, x1)), x1)}
\end{aligned}
$$

Lemma 669: $mult(i(op_t(x1, x2)), x2) = mult(i(x1), op_t(x2, x1))$

$$
\begin{aligned}
& \underbrace{mult(i(op_t(x1, x2)), x2)} \\
= \quad & \text{by Lemma 668 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \overbrace{op_t(rd(x2, op_t(x1, x2)), x1)} \\
= \quad & \text{by Axiom 10 RL with } \{x2 \leftarrow rd(x2, op_t(x1, x2)), x1 \leftarrow x1\} \\
& \overbrace{mult(i(x1), \underbrace{mult(rd(x2, op_t(x1, x2)), x1)})} \\
= \quad & \text{by Lemma 597 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& mult(i(x1), \overbrace{op_t(x2, x1)})
\end{aligned}
$$

Lemma 670: $op_t(x1, x2) = rd(x2, mult(i(x1), op_t(x2, x1)))$

$$
\begin{aligned}
& \underbrace{op_t(x1, x2)} \\
= \quad & \text{by Lemma 3 RL with } \{x1 \leftarrow op_t(x1, x2)\} \\
& \overbrace{\underbrace{i(i(op_t(x1, x2)))}} \\
= \quad & \text{by Lemma 9 RL with } \{x2 \leftarrow i(op_t(x1, x2)), x1 \leftarrow x2\} \\
& \overbrace{rd(x2, \underbrace{mult(i(op_t(x1, x2)), x2)})} \\
= \quad & \text{by Lemma 669 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& rd(x2, \overbrace{mult(i(x1), op_t(x2, x1))})
\end{aligned}
$$

Lemma 671: $op_l(op_t(x1, x2), i(x3), x1) = op_l(op_t(x1, x2), x1, x3)$

$$
\begin{aligned}
& \underbrace{op_l(op_t(x1, x2), i(x3), x1)} \\
= \quad & \text{by Axiom 15 RL with } \{x4 \leftarrow x2, x3 \leftarrow x1, x2 \leftarrow i(x3), x1 \leftarrow x1\} \\
& \overbrace{op_t(\underbrace{op_l(x1, i(x3), x1)}, x2)} \\
= \quad & \text{by Lemma 432 RL with } \{x2 \leftarrow x3, x1 \leftarrow x1\} \\
& op_t(\overbrace{\underbrace{op_l(x1, x3, i(x1))}}, x2) \\
= \quad & \text{by Lemma 606 LR with } \{x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \underbrace{op_t(\overbrace{op_l(x1, x1, x3)}, x2)} \\
= \quad & \text{by Axiom 15 LR with } \{x4 \leftarrow x2, x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow x1\} \\
& \overbrace{op_l(op_t(x1, x2), x1, x3)}
\end{aligned}
$$

Lemma 672: $op_l(op_t(x1, x2), x1, i(x3)) = op_l(op_t(x1, x2), x3, x1)$

$$
\begin{aligned}
& \underbrace{op_l(op_t(x1, x2), x1, i(x3))} \\
= \quad & \text{by Axiom 15 RL with } \{x4 \leftarrow x2, x3 \leftarrow i(x3), x2 \leftarrow x1, x1 \leftarrow x1\} \\
& \overbrace{op_t(\underbrace{op_l(x1, x1, i(x3))}, x2)} \\
= \quad & \text{by Lemma 608 LR with } \{x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \underbrace{op_t(\overbrace{op_l(x1, x3, x1)}, x2)} \\
= \quad & \text{by Axiom 15 LR with } \{x4 \leftarrow x2, x3 \leftarrow x1, x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{op_l(op_t(x1, x2), x3, x1)}
\end{aligned}
$$

Lemma 673: $op_t(mult(i(x1),x2),x1) = op_t(mult(x2,i(x1)),x2)$

$op_t(mult(i(x1),x2),\underbrace{x1}) =$ by Axiom 1 LR with $\{x1 \leftarrow x1\}$

$op_t(mult(i(x1),x2),\overbrace{mult(x1,\underbrace{unit()})}) =$ by Axiom 9 RL with $\{x1 \leftarrow x2\}$

$op_t(\underbrace{mult(i(x1),x2)},mult(x1,\overbrace{mult(i(x2),x2)})) =$ by Lemma 17 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{op_t(\overbrace{i(mult(x1,i(x2)))},mult(x1,mult(i(x2),x2)))} =$ by Axiom 10 RL with $\{x2 \leftarrow i(mult(x1,i(x2))), x1 \leftarrow mult(x1,mult(i(x2),x2))\}$

$\overbrace{mult(i(mult(x1,mult(i(x2),x2))),\underbrace{mult(i(mult(x1,i(x2))),mult(x1,mult(i(x2),x2)))})} =$ by Axiom 12 LR with $\{x3 \leftarrow x2, x2 \leftarrow i(x2), x1 \leftarrow x1\}$

$mult(i(mult(x1,mult(i(x2),x2))),\overbrace{\underbrace{op_l(x2,i(x2),x1)}}) =$ by Lemma 609 LR with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$mult(\underbrace{i(mult(x1,mult(i(x2),x2)))},\overbrace{op_l(x2,x1,x2)}) =$ by Lemma 87 LR with $\{x3 \leftarrow x2, x2 \leftarrow x2, x1 \leftarrow x1\}$

$mult(\overbrace{mult(i(x1),\underbrace{mult(x2,i(x2))})},op_l(x2,x1,x2)) =$ by Axiom 7 LR with $\{x1 \leftarrow x2\}$

$mult(\underbrace{mult(i(x1),\overbrace{unit()})},op_l(x2,x1,x2)) =$ by Axiom 1 RL with $\{x1 \leftarrow i(x1)\}$

$mult(\overbrace{i(x1)},\underbrace{op_l(x2,x1,x2)}) =$ by Lemma 608 RL with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$\underbrace{mult(i(x1),\overbrace{op_l(x2,x2,i(x1))})} =$ by Lemma 264 LR with $\{x2 \leftarrow x2, x1 \leftarrow i(x1)\}$

$\overbrace{op_t(mult(x2,i(x1)),x2)}$

Lemma 674: $op_r(x1,mult(i(x1),x2),x3) = i(op_r(i(x1),x2,x3))$

$op_r(\underbrace{x1},mult(i(x1),x2),x3) =$ by Lemma 3 RL with $\{x1 \leftarrow x1\}$

$\underbrace{op_r(\overbrace{i(i(x1))},mult(i(x1),x2),x3)} =$ by Lemma 654 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow i(x1)\}$

$\overbrace{i(op_r(i(x1),x2,x3))}$

Lemma 675: $mult(x1, op_t(mult(x2, i(x1)), x2)) = op_l(x2, x1, x2)$

$$
\begin{aligned}
& mult(x1, \underbrace{op_t(mult(x2, i(x1)), x2)}) \\
= & \quad \text{by Lemma 264 RL with } \{x2 \leftarrow x2, x1 \leftarrow i(x1)\} \\
& \underbrace{mult(x1, \overbrace{mult(i(x1), op_l(x2, x2, i(x1)))})} \\
= & \quad \text{by Lemma 12 LR with } \{x2 \leftarrow op_l(x2, x2, i(x1)), x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x2, x2, i(x1))}} \\
= & \quad \text{by Lemma 608 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \overbrace{op_l(x2, x1, x2)}
\end{aligned}
$$

Lemma 676: $op_r(rd(x1, x2), x2, x3) = rd(mult(x1, x3), mult(x2, x3))$

$$
\begin{aligned}
& \underbrace{op_r(rd(x1, x2), x2, x3)} \\
= & \quad \text{by Axiom 16 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow rd(x1, x2)\} \\
& \overbrace{rd(mult(\underbrace{mult(rd(x1, x2), x2)}, x3), mult(x2, x3))} \\
= & \quad \text{by Axiom 4 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& rd(mult(\overbrace{x1}, x3), mult(x2, x3))
\end{aligned}
$$

Lemma 677: $op_l(x1, i(x1), i(x2)) = rd(op_t(x2, i(x1)), mult(x2, op_l(i(x1), x1, x2)))$

$$
\begin{aligned}
& \underbrace{op_l(x1, i(x1), i(x2))} \\
= & \quad \text{by Lemma 518 RL with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{i(op_l(i(x1), x1, x2))}} \\
= & \quad \text{by Lemma 36 RL with } \{x2 \leftarrow op_l(i(x1), x1, x2), x1 \leftarrow x2\} \\
& \overbrace{rd(\underbrace{op_t(x2, op_l(i(x1), x1, x2))}, mult(x2, op_l(i(x1), x1, x2)))} \\
= & \quad \text{by Lemma 572 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& rd(\overbrace{op_t(x2, i(x1))}, mult(x2, op_l(i(x1), x1, x2)))
\end{aligned}
$$

Lemma 678: $op_l(x1, x1, x2) = rd(op_t(x2, i(x1)), mult(x2, op_l(i(x1), x1, x2)))$

$$
\begin{aligned}
& op_l(x1, x1, \underbrace{x2}) \\
= & \quad \text{by Lemma 3 RL with } \{x1 \leftarrow x2\} \\
& \underbrace{op_l(x1, x1, \overbrace{i(i(x2))})} \\
= & \quad \text{by Lemma 429 RL with } \{x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, i(x1), i(x2))}} \\
= & \quad \text{by Lemma 677 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{rd(op_t(x2, i(x1)), mult(x2, op_l(i(x1), x1, x2)))}
\end{aligned}
$$

Lemma 679: $rd(op_t(x1, i(x2)), rd(x1, x2)) = op_l(x2, x2, x1)$

$$
\begin{aligned}
& rd(op_t(x1, i(x2)), \underbrace{rd(x1, x2)}) \\
= & \quad \text{by Lemma 232 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{rd(op_t(x1, i(x2)), \overbrace{mult(x1, op_l(i(x2), x2, x1))})} \\
= & \quad \text{by Lemma 678 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \overbrace{op_l(x2, x2, x1)}
\end{aligned}
$$

Lemma 680: $op_t(x1, i(x2)) = mult(rd(x1, x2), op_t(x2, i(x1)))$

$$
\begin{aligned}
& \underbrace{op_t(x1, i(x2))} \\
= & \quad \text{by Lemma 511 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(op_t(x1, rd(x2, x1)), x1, \underbrace{x2})} \\
= & \quad \text{by Lemma 3 RL with } \{x1 \leftarrow x2\} \\
& op_l(op_t(x1, \underbrace{rd(x2, x1)}), x1, \overbrace{i(i(x2))}) \\
= & \quad \text{by Lemma 232 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \underbrace{op_l(op_t(x1, \overbrace{mult(x2, op_l(i(x1), x1, x2))}), x1, i(i(x2)))} \\
= & \quad \text{by Axiom 15 RL with } \{x4 \leftarrow mult(x2, op_l(i(x1), x1, x2)), x3 \leftarrow i(i(x2)), x2 \leftarrow x1, x1 \leftarrow x1\} \\
& \overbrace{op_t(\underbrace{op_l(x1, x1, i(i(x2)))}, mult(x2, op_l(i(x1), x1, x2)))} \\
= & \quad \text{by Lemma 429 RL with } \{x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& op_t(\overbrace{\underbrace{op_l(x1, i(x1), i(x2))}}, mult(x2, op_l(i(x1), x1, x2))) \\
= & \quad \text{by Lemma 518 RL with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x1\} \\
& \underbrace{op_t(\overbrace{i(op_l(i(x1), x1, x2))}, mult(x2, op_l(i(x1), x1, x2)))} \\
= & \quad \text{by Axiom 10 RL with } \{x2 \leftarrow i(op_l(i(x1), x1, x2)), x1 \leftarrow mult(x2, op_l(i(x1), x1, x2))\} \\
& \overbrace{mult(i(mult(x2, op_l(i(x1), x1, x2))), \underbrace{mult(i(op_l(i(x1), x1, x2)), mult(x2, op_l(i(x1), x1, x2)))})} \\
= & \quad \text{by Axiom 10 LR with } \{x2 \leftarrow x2, x1 \leftarrow op_l(i(x1), x1, x2)\} \\
& mult(i(mult(x2, op_l(i(x1), x1, x2))), \overbrace{op_t(x2, op_l(i(x1), x1, x2))}) \\
= & \quad \text{by Lemma 572 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& mult(i(\underbrace{mult(x2, op_l(i(x1), x1, x2))}), \overbrace{op_t(x2, i(x1))}) \\
= & \quad \text{by Lemma 232 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& mult(\underbrace{i(\overbrace{rd(x2, x1)})}, op_t(x2, i(x1))) \\
= & \quad \text{by Lemma 10 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& mult(\overbrace{rd(x1, x2)}, op_t(x2, i(x1)))
\end{aligned}
$$

Lemma 681: $op_l(op_t(x1, rd(x1, x2)), x2, x1) = op_t(x1, i(x2))$

$$
\begin{aligned}
& op_l(\underbrace{op_t(x1, rd(x1, x2))}, x2, x1) \\
= & \quad \text{by Lemma 580 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{op_l(\overbrace{op_l(op_t(x1, i(x2)), x1, x2)}, x2, x1)} \\
= & \quad \text{by Lemma 138 LR with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow op_t(x1, i(x2))\} \\
& \overbrace{op_t(x1, i(x2))}
\end{aligned}
$$

Lemma 682: $i(mult(x1, x2)) = op_t(rd(i(x1), x2), op_t(x1, x2))$

$$
\begin{aligned}
& \underbrace{i(mult(x1, x2))} \\
= & \quad \text{by Lemma 33 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_t(rd(i(x1), x2), \underbrace{x1})} \\
= & \quad \text{by Axiom 5 LR with } \{x2 \leftarrow rd(i(x1), x2), x1 \leftarrow x1\} \\
& op_t(rd(i(x1), x2), \overbrace{rd(\underbrace{mult(x1, rd(i(x1), x2))}, rd(i(x1), x2))}) \\
= & \quad \text{by Lemma 29 RL with } \{x2 \leftarrow mult(x1, rd(i(x1), x2)), x1 \leftarrow i(x1)\} \\
& op_t(rd(i(x1), x2), rd(\overbrace{rd(\underbrace{mult(i(x1), mult(x1, rd(i(x1), x2)))}, op_t(i(x1), mult(x1, rd(i(x1), x2))))}, rd(i(x1), x2))) \\
= & \quad \text{by Axiom 2 RL with } \{x2 \leftarrow x1, x1 \leftarrow rd(i(x1), x2)\} \\
& op_t(rd(i(x1), x2), \underbrace{rd(rd(\overbrace{rd(i(x1), x2)}, op_t(i(x1), mult(x1, rd(i(x1), x2)))), rd(i(x1), x2))}) \\
= & \quad \text{by Lemma 354 LR with } \{x2 \leftarrow op_t(i(x1), mult(x1, rd(i(x1), x2))), x1 \leftarrow rd(i(x1), x2)\} \\
& op_t(rd(i(x1), x2), \overbrace{op_r(\underbrace{i(op_t(i(x1), mult(x1, rd(i(x1), x2))))}, rd(i(x1), x2), rd(i(x1), x2))}) \\
= & \quad \text{by Lemma 18 LR with } \{x2 \leftarrow mult(x1, rd(i(x1), x2)), x1 \leftarrow x1\} \\
& op_t(rd(i(x1), x2), op_r(\overbrace{op_t(x1, i(mult(x1, rd(i(x1), x2))))}, rd(i(x1), x2), rd(i(x1), x2))) \\
= & \quad \text{by Lemma 379 LR with } \{x2 \leftarrow rd(i(x1), x2), x1 \leftarrow x1\} \\
& \underbrace{op_t(rd(i(x1), x2), op_r(\overbrace{op_t(x1, rd(i(x1), \underbrace{rd(i(x1), x2)}))}, rd(i(x1), x2), rd(i(x1), x2)))} \\
= & \quad \text{by Lemma 550 LR with } \{x2 \leftarrow op_t(x1, rd(i(x1), rd(i(x1), x2))), x1 \leftarrow rd(i(x1), x2)\} \\
& \overbrace{op_t(rd(i(x1), x2), op_t(x1, \underbrace{rd(i(x1), rd(i(x1), x2))}))} \\
= & \quad \text{by Lemma 353 LR with } \{x2 \leftarrow x2, x1 \leftarrow i(x1)\} \\
& op_t(rd(i(x1), x2), op_t(x1, \overbrace{op_r(x2, i(x1), i(x1))})) \\
= & \quad \text{by Lemma 361 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& op_t(rd(i(x1), x2), \underbrace{op_t(x1, \overbrace{op_r(x2, x1, x1)})}) \\
= & \quad \text{by Lemma 550 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_t(rd(i(x1), x2), \overbrace{op_t(x1, x2)})
\end{aligned}
$$

Lemma 683: $op_l(x1, x1, x2) = rd(i(x2), op_t(i(mult(x2, x1)), x1))$

$\overbrace{op_l(x1, x1, x2)}$

$=$ by Lemma 291 RL with $\{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x1\}$

$\overbrace{op_l(x1, x1, mult(x2, x1))}$

$=$ by Lemma 594 RL with $\{x2 \leftarrow mult(x2, x1), x1 \leftarrow x1\}$

$\overbrace{rd(\underbrace{rd(x1, mult(x2, x1))}, op_t(i(mult(x2, x1)), x1))}$

$=$ by Lemma 9 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$rd(\overbrace{i(x2)}, op_t(i(mult(x2, x1)), x1))$

Lemma 684: $op_l(x1, x1, x2) = rd(rd(x1, op_t(i(x2), x1)), x2)$

$\overbrace{op_l(x1, x1, x2)}$

$=$ by Lemma 683 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{rd(i(x2), op_t(i(mult(x2, x1)), x1))}$

$=$ by Lemma 68 LR with $\{x3 \leftarrow x1, x2 \leftarrow mult(x2, x1), x1 \leftarrow x2\}$

$\overbrace{rd(\underbrace{op_t(mult(x2, x1), i(x1))}, x2)}$

$=$ by Lemma 326 LR with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$rd(\overbrace{rd(x1, op_t(i(x2), x1))}, x2)$

Lemma 685: $unit() = asoc(mult(rd(x1, op_t(i(i(x2)), x1)), x2), op_l(x1, x1, i(x2)), x3)$

$\overbrace{unit()}$

= by Axiom 9 RL with $\{x1 \leftarrow mult(rd(x2, i(rd(x1, op_t(i(i(x2)), x1)))), mult(op_t(rd(x2, i(rd(x1, op_t(i(i(x2)), x1)))), i(rd(x1, op$

$\overbrace{mult(i(mult(rd(x2, i(rd(x1, op_t(i(i(x2)), x1)))), mult(op_t(rd(x2, i(rd(x1, op_t(i(i(x2)), x1)))), i(rd(x1, op_t(i(i(x2)), x1)))), x$

= by Lemma 162 RL with $\{x3 \leftarrow x3, x2 \leftarrow i(rd(x1, op_t(i(i(x2)), x1))), x1 \leftarrow rd(x2, i(rd(x1, op_t(i(i(x2)), x1))))\}$

$mult(i(mult(rd(x2, i(rd(x1, op_t(i(i(x2)), x1)))), mult(op_t(rd(x2, i(rd(x1, op_t(i(i(x2)), x1)))), i(rd(x1, op_t(i(i(x2)), x1)))), x$

= by Lemma 27 RL with $\{x2 \leftarrow i(rd(x1, op_t(i(i(x2)), x1))), x1 \leftarrow rd(x2, i(rd(x1, op_t(i(i(x2)), x1))))\}$

$mult(i(\underbrace{mult(rd(x2, i(rd(x1, op_t(i(i(x2)), x1)))), mult(op_t(rd(x2, i(rd(x1, op_t(i(i(x2)), x1)))), i(rd(x1, op_t(i(i(x2)), x1)))), x}$

= by Lemma 161 RL with $\{x3 \leftarrow x3, x2 \leftarrow i(rd(x1, op_t(i(i(x2)), x1))), x1 \leftarrow rd(x2, i(rd(x1, op_t(i(i(x2)), x1))))\}$

$\underbrace{mult(i(\overbrace{mult(op_t(rd(x2, i(rd(x1, op_t(i(i(x2)), x1)))), i(rd(x1, op_t(i(i(x2)), x1)))), mult(rd(x2, i(rd(x1, op_t(i(i(x2)), x1)))), x}$

= by Lemma 108 LR with $\{x3 \leftarrow x3, x2 \leftarrow rd(x2, i(rd(x1, op_t(i(i(x2)), x1)))), x1 \leftarrow op_t(rd(x2, i(rd(x1, op_t(i(i(x2)), x1)))), i(rd$

$\overbrace{asoc(\underbrace{op_t(rd(x2, i(rd(x1, op_t(i(i(x2)), x1)))), i(rd(x1, op_t(i(i(x2)), x1)))}, rd(x2, i(rd(x1, op_t(i(i(x2)), x1)))), x3)}$

= by Lemma 14 LR with $\{x2 \leftarrow i(rd(x1, op_t(i(i(x2)), x1))), x1 \leftarrow x2\}$

$asoc(\overbrace{mult(i(i(rd(x1, op_t(i(i(x2)), x1))))), x2)}, \underbrace{rd(x2, i(rd(x1, op_t(i(i(x2)), x1))))}, x3)$

= by Lemma 38 LR with $\{x1 \leftarrow rd(x1, op_t(i(i(x2)), x1)), x2 \leftarrow x2\}$

$asoc(\underbrace{mult(i(i(rd(x1, op_t(i(i(x2)), x1))))), x2)}, \overbrace{rd(rd(x1, op_t(i(i(x2)), x1)), i(x2))}, x3)$

= by Lemma 3 LR with $\{x1 \leftarrow rd(x1, op_t(i(i(x2)), x1))\}$

$asoc(\overbrace{mult(rd(x1, op_t(i(i(x2)), x1)), x2)}, \underbrace{rd(rd(x1, op_t(i(i(x2)), x1)), i(x2))}, x3)$

= by Lemma 684 RL with $\{x2 \leftarrow i(x2), x1 \leftarrow x1\}$

$asoc(mult(rd(x1, op_t(i(i(x2)), x1)), x2), \overbrace{op_l(x1, x1, i(x2))}, x3)$

Lemma 686: $unit() = asoc(op_t(x1, x2), op_l(x1, x1, i(x2)), x3)$

$\overbrace{unit()}$

= by Lemma 685 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{asoc(mult(rd(x1, op_t(\underbrace{i(i(x2))}, x1)), x2), op_l(x1, x1, i(x2)), x3)}$

= by Lemma 3 LR with $\{x1 \leftarrow x2\}$

$asoc(\underbrace{mult(rd(x1, op_t(\overbrace{x2}, x1)), x2)}, op_l(x1, x1, i(x2)), x3)$

= by Lemma 597 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$asoc(\overbrace{op_t(x1, x2)}, op_l(x1, x1, i(x2)), x3)$

Lemma 687: $op_t(x1, i(x2)) = mult(op_l(x2, x2, x1), i(rd(x2, x1)))$

$$
\begin{aligned}
& \underbrace{op_t(x1, i(x2))} \\
= \quad & \text{by Lemma 18 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{i(op_t(i(x1), x2))}} \\
= \quad & \text{by Lemma 22 RL with } \{x2 \leftarrow op_t(i(x1), x2), x1 \leftarrow rd(x2, x1)\} \\
& \overbrace{mult(\underbrace{rd(rd(x2, x1), op_t(i(x1), x2))}, i(rd(x2, x1)))} \\
= \quad & \text{by Lemma 594 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& mult(\overbrace{op_l(x2, x2, x1)}, i(rd(x2, x1)))
\end{aligned}
$$

Lemma 688: $mult(op_l(x1, x1, x2), rd(x2, x1)) = op_t(x2, i(x1))$

$$
\begin{aligned}
& mult(op_l(x1, x1, x2), \underbrace{rd(x2, x1)}) \\
= \quad & \text{by Lemma 10 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{mult(op_l(x1, x1, x2), \overbrace{i(rd(x1, x2))})} \\
= \quad & \text{by Lemma 687 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \overbrace{op_t(x2, i(x1))}
\end{aligned}
$$

Lemma 689: $op_r(rd(op_t(x1, x2), x2), x1, x1) = i(mult(i(x1), x2))$

$$
\begin{aligned}
& \underbrace{op_r(rd(op_t(x1, x2), x2), x1, x1)} \\
= \quad & \text{by Lemma 353 RL with } \{x2 \leftarrow rd(op_t(x1, x2), x2), x1 \leftarrow x1\} \\
& \overbrace{rd(x1, \underbrace{rd(x1, rd(op_t(x1, x2), x2))})} \\
= \quad & \text{by Lemma 595 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{rd(x1, \overbrace{mult(mult(i(x1), x2), x1)})} \\
= \quad & \text{by Lemma 9 LR with } \{x2 \leftarrow mult(i(x1), x2), x1 \leftarrow x1\} \\
& \overbrace{i(mult(i(x1), x2))}
\end{aligned}
$$

Lemma 690: $op_r(rd(op_t(x1, x2), x2), x1, x1) = mult(x1, i(x2))$

$$
\begin{aligned}
& \underbrace{op_r(rd(op_t(x1, x2), x2), x1, x1)} \\
= \quad & \text{by Lemma 689 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{i(mult(i(x1), x2))}} \\
= \quad & \text{by Lemma 16 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{mult(x1, i(x2))}
\end{aligned}
$$

Lemma 691: $op_r(rd(x2, op_t(x1, x2)), x1, x1) = rd(mult(mult(i(x1), x2), x1), x1)$

$$
\begin{aligned}
& op_r(\underbrace{rd(x2, op_t(x1, x2))}, x1, x1) \\
= & \quad \text{by Lemma 10 RL with } \{x2 \leftarrow x2, x1 \leftarrow op_t(x1, x2)\} \\
& \underbrace{op_r(\overbrace{i(rd(op_t(x1, x2), x2))}, x1, x1)} \\
= & \quad \text{by Lemma 354 RL with } \{x2 \leftarrow rd(op_t(x1, x2), x2), x1 \leftarrow x1\} \\
& \overbrace{rd(\underbrace{rd(x1, rd(op_t(x1, x2), x2))}, x1)} \\
= & \quad \text{by Lemma 595 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& rd(\overbrace{mult(mult(i(x1), x2), x1)}, x1)
\end{aligned}
$$

Lemma 692: $mult(rd(op_t(i(x2), x1), x1), x2) = i(op_t(x1, i(x2)))$

$$
\begin{aligned}
& mult(\underbrace{rd(op_t(i(x2), x1), x1)}, x2) \\
= & \quad \text{by Lemma 10 RL with } \{x2 \leftarrow op_t(i(x2), x1), x1 \leftarrow x1\} \\
& \underbrace{mult(\overbrace{i(rd(x1, op_t(i(x2), x1)))}, x2)} \\
= & \quad \text{by Lemma 17 RL with } \{x2 \leftarrow x2, x1 \leftarrow rd(x1, op_t(i(x2), x1))\} \\
& \overbrace{i(\underbrace{mult(rd(x1, op_t(i(x2), x1)), i(x2))})} \\
= & \quad \text{by Lemma 597 LR with } \{x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& i(\overbrace{op_t(x1, i(x2))})
\end{aligned}
$$

Lemma 693: $rd(mult(mult(x1, rd(x2, x3)), x3), x2) = op_r(x1, rd(x2, x3), x3)$

$$
\begin{aligned}
& rd(mult(mult(x1, rd(x2, x3)), x3), \underbrace{x2}) \\
= & \quad \text{by Axiom 4 LR with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& \underbrace{rd(mult(mult(x1, rd(x2, x3)), x3), \overbrace{mult(rd(x2, x3), x3)})} \\
= & \quad \text{by Axiom 16 RL with } \{x3 \leftarrow x3, x2 \leftarrow rd(x2, x3), x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, rd(x2, x3), x3)}
\end{aligned}
$$

Lemma 694: $op_r(x1, x2, x2) = mult(rd(x2, op_t(i(op_r(x1, x2, x2)), x2)), op_t(i(x2), x1))$

$$
\begin{aligned}
& \underbrace{op_r(x1, x2, x2)} \\
= & \quad \text{by Lemma 605 RL with } \{x2 \leftarrow i(x2), x1 \leftarrow op_r(x1, x2, x2)\} \\
& \overbrace{mult(rd(op_t(op_r(x1, x2, x2), i(x2)), i(x2)), \underbrace{op_t(i(x2), op_r(x1, x2, x2))})} \\
= & \quad \text{by Lemma 569 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& mult(\underbrace{rd(op_t(op_r(x1, x2, x2), i(x2)), i(x2))}, \overbrace{op_t(i(x2), x1)}) \\
= & \quad \text{by Lemma 65 LR with } \{x3 \leftarrow x2, x2 \leftarrow x2, x1 \leftarrow op_r(x1, x2, x2)\} \\
& mult(\overbrace{rd(x2, op_t(i(op_r(x1, x2, x2)), x2))}, op_t(i(x2), x1))
\end{aligned}
$$

Lemma 695: $mult(mult(x1, x2), op_t(i(x1), x2)) = op_r(x2, x1, x1)$

$$
\begin{aligned}
& mult(\underbrace{mult(x1, x2)}, op_t(i(x1), x2)) \\
= \quad & \text{by Lemma 57 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& mult(\overbrace{rd(x1, \underbrace{mult(i(x1), rd(x1, x2))})}, op_t(i(x1), x2)) \\
= \quad & \text{by Lemma 45 RL with } \{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& mult(rd(x1, \overbrace{\underbrace{i(mult(x1, rd(x2, x1)))}}), op_t(i(x1), x2)) \\
= \quad & \text{by Axiom 11 RL with } \{x2 \leftarrow rd(x2, x1), x1 \leftarrow x1\} \\
& mult(rd(x1, \overbrace{mult(i(x1), \underbrace{i(rd(x2, x1))})}), op_t(i(x1), x2)) \\
= \quad & \text{by Lemma 411 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \underbrace{mult(rd(x1, \overbrace{op_t(i(op_r(x2, x1, x1)), x1)}), op_t(i(x1), x2))} \\
= \quad & \text{by Lemma 694 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \overbrace{op_r(x2, x1, x1)}
\end{aligned}
$$

Lemma 696: $op_l(x1, x2, x1) = rd(rd(x2, i(x1)), op_t(x2, x1))$

$$
\begin{aligned}
& \underbrace{op_l(x1, x2, x1)} \\
= \quad & \text{by Lemma 29 RL with } \{x2 \leftarrow op_l(x1, x2, x1), x1 \leftarrow x2\} \\
& \overbrace{rd(\underbrace{mult(x2, op_l(x1, x2, x1))}, op_t(x2, op_l(x1, x2, x1)))} \\
= \quad & \text{by Lemma 616 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& rd(\overbrace{rd(x2, i(x1))}, \underbrace{op_t(x2, op_l(x1, x2, x1))}) \\
= \quad & \text{by Lemma 560 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& rd(rd(x2, i(x1)), \overbrace{op_t(x2, x1)})
\end{aligned}
$$

Lemma 697: $rd(x2, rd(x2, x1)) = mult(rd(x1, x2), op_l(x2, x1, x2))$

$$
\begin{aligned}
& \underbrace{rd(x2, rd(x2, x1))} \\
= \quad & \text{by Lemma 39 RL with } \{x3 \leftarrow x2, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{\overbrace{rd(rd(x1, x2), i(x2))}} \\
= \quad & \text{by Lemma 616 RL with } \{x2 \leftarrow x2, x1 \leftarrow rd(x1, x2)\} \\
& \overbrace{mult(rd(x1, x2), \underbrace{op_l(x2, rd(x1, x2), x2)})} \\
= \quad & \text{by Lemma 292 LR with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x2\} \\
& mult(rd(x1, x2), \overbrace{op_l(x2, x1, x2)})
\end{aligned}
$$

Lemma 698: $mult(rd(x1, x2), op_l(x2, x1, x2)) = op_r(x1, x2, x2)$

$$
\begin{aligned}
& \underbrace{mult(rd(x1, x2), op_l(x2, x1, x2))} \\
= \quad & \text{by Lemma 697 RL with } \{x1 \leftarrow x1, x2 \leftarrow x2\} \\
& \underbrace{\overbrace{rd(x2, rd(x2, x1))}} \\
= \quad & \text{by Lemma 353 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \overbrace{op_r(x1, x2, x2)}
\end{aligned}
$$

Lemma 699: $mult(x1, i(x2)) = mult(rd(x1, x2), asoc(x2, x1, x2))$

$$
\begin{aligned}
& \underbrace{mult(x1, i(x2))} \\
= \quad & \text{by Lemma 104 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_r(rd(x1, x2), x2, i(x2))}} \\
= \quad & \text{by Lemma 626 RL with } \{x2 \leftarrow x2, x1 \leftarrow rd(x1, x2)\} \\
& \overbrace{mult(rd(x1, x2), \underbrace{asoc(x2, rd(x1, x2), x2)})} \\
= \quad & \text{by Lemma 263 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& mult(rd(x1, x2), \overbrace{asoc(x2, x1, x2)})
\end{aligned}
$$

Lemma 700: $mult(x1, i(x2)) = mult(asoc(x2, x1, x2), rd(x1, x2))$

$$
\begin{aligned}
& \underbrace{mult(x1, i(x2))} \\
= \quad & \text{by Lemma 104 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_r(rd(x1, x2), x2, i(x2))}} \\
= \quad & \text{by Lemma 628 RL with } \{x2 \leftarrow rd(x1, x2), x1 \leftarrow x2\} \\
& \overbrace{mult(\underbrace{asoc(x2, rd(x1, x2), x2)}, rd(x1, x2))} \\
= \quad & \text{by Lemma 263 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& mult(\overbrace{asoc(x2, x1, x2)}, rd(x1, x2))
\end{aligned}
$$

Lemma 701: $asoc(x1, x2, x1) = rd(mult(x2, i(x1)), rd(x2, x1))$

$$
\begin{aligned}
& \underbrace{asoc(x1, x2, x1)} \\
= \quad & \text{by Lemma 263 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{asoc(x1, rd(x2, x1), x1)}} \\
= \quad & \text{by Lemma 639 RL with } \{x2 \leftarrow x1, x1 \leftarrow rd(x2, x1)\} \\
& \overbrace{rd(\underbrace{op_r(rd(x2, x1), x1, i(x1))}, rd(x2, x1))} \\
= \quad & \text{by Lemma 104 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& rd(\overbrace{mult(x2, i(x1))}, rd(x2, x1))
\end{aligned}
$$

Lemma 702: $rd(mult(x1, x2), rd(x1, i(x2))) = asoc(x2, x1, x2)$

$rd(mult(\underbrace{x1}, x2), rd(x1, i(x2)))$

$=$ by Axiom 4 LR with $\{x2 \leftarrow i(x2), x1 \leftarrow x1\}$

$rd(\underbrace{mult(\overbrace{mult(rd(x1, i(x2)), i(x2))}, x2)}, rd(x1, i(x2)))$

$=$ by Lemma 93 LR with $\{x2 \leftarrow x2, x1 \leftarrow rd(x1, i(x2))\}$

$rd(\overbrace{op_r(rd(x1, i(x2)), i(x2), x2)}, rd(x1, i(x2)))$

$=$ by Lemma 321 LR with $\{x2 \leftarrow x2, x1 \leftarrow rd(x1, i(x2))\}$

$\underbrace{rd(\overbrace{op_r(rd(x1, i(x2)), x2, i(x2))}, rd(x1, i(x2)))}$

$=$ by Lemma 639 LR with $\{x2 \leftarrow x2, x1 \leftarrow rd(x1, i(x2))\}$

$\overbrace{asoc(x2, \underbrace{rd(x1, i(x2))}, x2)}$

$=$ by Lemma 38 RL with $\{x1 \leftarrow x1, x2 \leftarrow x2\}$

$\underbrace{asoc(x2, \overbrace{rd(x2, i(x1))}, x2)}$

$=$ by Lemma 498 LR with $\{x2 \leftarrow i(x1), x1 \leftarrow x2\}$

$\overbrace{asoc(x2, \underbrace{i(i(x1))}, x2)}$

$=$ by Lemma 3 LR with $\{x1 \leftarrow x1\}$

$asoc(x2, \overbrace{x1}, x2)$

Lemma 703: $asoc(x1, rd(x2, x1), x1) = rd(rd(x1, x2), mult(i(x2), x1))$

$asoc(x1, \underbrace{rd(x2, x1)}, x1)$

$=$ by Lemma 10 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{asoc(x1, \overbrace{i(rd(x1, x2))}, x1)}$

$=$ by Lemma 645 RL with $\{x2 \leftarrow x1, x1 \leftarrow rd(x1, x2)\}$

$\overbrace{rd(rd(x1, x2), \underbrace{op_r(rd(x1, x2), x1, i(x1))})}$

$=$ by Lemma 330 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$rd(rd(x1, x2), \overbrace{mult(i(x2), x1)})$

Lemma 704: $rd(rd(x1, x2), mult(i(x2), x1)) = asoc(x1, x2, x1)$

$\underbrace{rd(rd(x1, x2), mult(i(x2), x1))}$

$=$ by Lemma 703 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{\underbrace{asoc(x1, rd(x2, x1), x1)}}$

$=$ by Lemma 263 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{asoc(x1, x2, x1)}$

Lemma 705: $i(op_t(x1, x2)) = op_t(i(x1), rd(mult(x1, x2), i(x1)))$

$$
\begin{aligned}
& \underbrace{i(op_t(x1, x2))} \\
= & \quad \text{by Lemma 79 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{i(\overbrace{op_r(x1, x2, i(mult(x1, x2)))})} \\
= & \quad \text{by Lemma 654 RL with } \{x3 \leftarrow i(mult(x1, x2)), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{\overbrace{op_r(i(x1), mult(x1, x2), i(mult(x1, x2)))}} \\
= & \quad \text{by Lemma 547 RL with } \{x2 \leftarrow mult(x1, x2), x1 \leftarrow i(x1)\} \\
& \overbrace{op_t(i(x1), rd(\overbrace{mult(x1, x2)}, i(x1)))}
\end{aligned}
$$

Lemma 706: $op_t(i(x1), mult(x1, mult(x1, x2))) = i(op_t(x1, x2))$

$$
\begin{aligned}
& \underbrace{op_t(i(x1), mult(x1, mult(x1, x2)))} \\
= & \quad \text{by Lemma 415 RL with } \{x2 \leftarrow mult(x1, x2), x1 \leftarrow x1\} \\
& \underbrace{\overbrace{op_t(i(x1), rd(\overbrace{mult(x1, x2)}, i(x1)))}} \\
= & \quad \text{by Lemma 705 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{i(\overbrace{op_t(x1, x2)})}
\end{aligned}
$$

Lemma 707: $op_t(x1, rd(i(x1), x2)) = op_t(x1, rd(x2, x1))$

$$
\begin{aligned}
& \underbrace{op_t(x1, rd(\overbrace{i(x1)}, x2))} \\
= & \quad \text{by Lemma 379 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{\overbrace{op_t(x1, i(mult(x1, x2)))}} \\
= & \quad \text{by Lemma 18 RL with } \{x2 \leftarrow mult(x1, x2), x1 \leftarrow x1\} \\
& \underbrace{\overbrace{i(op_t(i(x1), mult(x1, x2)))}} \\
= & \quad \text{by Lemma 706 RL with } \{x2 \leftarrow mult(x1, x2), x1 \leftarrow i(x1)\} \\
& \overbrace{op_t(i(i(x1)), mult(i(x1), \underbrace{mult(i(x1), mult(x1, x2))}))} \\
= & \quad \text{by Axiom 2 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& op_t(\underbrace{i(i(x1))}, mult(i(x1), \overbrace{x2})) \\
= & \quad \text{by Lemma 3 LR with } \{x1 \leftarrow x1\} \\
& \underbrace{op_t(\overbrace{x1}, mult(i(x1), x2))} \\
= & \quad \text{by Lemma 375 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, rd(x2, x1))}
\end{aligned}
$$

Lemma 708: $op_t(x1, i(mult(x1, x2))) = op_t(x1, rd(x2, x1))$

$$
\begin{aligned}
& \underbrace{op_t(x1, i(mult(x1, x2)))} \\
= & \quad \text{by Lemma 379 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{\overbrace{op_t(x1, rd(\overbrace{i(x1)}, x2))}} \\
= & \quad \text{by Lemma 707 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, rd(x2, x1))}
\end{aligned}
$$

Lemma 709: $op_t(x1, i(mult(x2, x1))) = op_t(x1, rd(x2, x1))$

$op_t(x1, \underbrace{i(mult(x2, x1))})$

$=$ by Axiom 11 RL with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$op_t(x1, \overbrace{mult(\underbrace{i(x2)}, i(x1))})$

$=$ by Lemma 21 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$op_t(x1, \underbrace{mult(\overbrace{mult(rd(i(x1), x2), x1)}, i(x1))})$

$=$ by Lemma 92 LR with $\{x2 \leftarrow x1, x1 \leftarrow rd(i(x1), x2)\}$

$\underbrace{op_t(x1, \overbrace{op_r(rd(i(x1), x2), x1, i(x1))})}$

$=$ by Lemma 549 LR with $\{x2 \leftarrow rd(i(x1), x2), x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_t(x1, rd(i(x1), x2))}}$

$=$ by Lemma 707 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_t(x1, rd(x2, x1))}$

Lemma 710: $op_t(x1, i(rd(x1, i(x2)))) = op_t(i(i(x1)), mult(i(x1), mult(mult(i(x1), x2), x1)))$

$\underbrace{op_t(x1, i(rd(x1, i(x2))))}$

$=$ by Lemma 18 RL with $\{x2 \leftarrow rd(x1, i(x2)), x1 \leftarrow x1\}$

$\overbrace{\underbrace{i(op_t(i(x1), rd(x1, i(x2))))}}$

$=$ by Lemma 706 RL with $\{x2 \leftarrow rd(x1, i(x2)), x1 \leftarrow i(x1)\}$

$\overbrace{op_t(i(i(x1)), mult(i(x1), \underbrace{mult(i(x1), rd(x1, i(x2)))}))}$

$=$ by Lemma 71 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$op_t(i(i(x1)), mult(i(x1), \overbrace{mult(mult(i(x1), x2), x1)}))$

Lemma 711: $op_t(x1, rd(i(x2), x1)) = op_t(x1, mult(i(x1), mult(mult(i(x1), x2), x1)))$

$op_t(x1, \underbrace{rd(i(x2), x1)})$

$=$ by Lemma 10 RL with $\{x2 \leftarrow i(x2), x1 \leftarrow x1\}$

$\underbrace{op_t(x1, \overbrace{i(rd(x1, i(x2)))})}$

$=$ by Lemma 710 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_t(\underbrace{i(i(x1))}, mult(i(x1), mult(mult(i(x1), x2), x1)))}$

$=$ by Lemma 3 LR with $\{x1 \leftarrow x1\}$

$op_t(\overbrace{x1}, mult(i(x1), mult(mult(i(x1), x2), x1)))$

Lemma 712: $op_t(x1, rd(i(x2), x1)) = op_t(x1, rd(x2, x1))$

$$
\begin{aligned}
& \underbrace{op_t(x1, rd(i(x2), x1))} \\
= & \quad \text{by Lemma 711 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_t(x1, mult(i(x1), mult(mult(i(x1), x2), x1)))}} \\
= & \quad \text{by Lemma 375 LR with } \{x2 \leftarrow mult(mult(i(x1), x2), x1), x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, \underbrace{rd(mult(mult(i(x1), x2), x1), x1)})} \\
= & \quad \text{by Axiom 5 RL with } \{x2 \leftarrow x1, x1 \leftarrow mult(i(x1), x2)\} \\
& \underbrace{op_t(x1, \overbrace{mult(i(x1), x2)})} \\
= & \quad \text{by Lemma 375 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, rd(x2, x1))}
\end{aligned}
$$

Lemma 713: $op_t(i(x1), mult(x1, i(x2))) = op_t(i(x1), mult(x1, x2))$

$$
\begin{aligned}
& op_t(i(x1), \underbrace{mult(x1, i(x2))}) \\
= & \quad \text{by Lemma 16 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{op_t(i(x1), \overbrace{i(mult(i(x1), x2))})} \\
= & \quad \text{by Lemma 15 LR with } \{x2 \leftarrow mult(i(x1), x2), x1 \leftarrow x1\} \\
& \overbrace{\underbrace{i(op_t(x1, mult(i(x1), x2)))}} \\
= & \quad \text{by Lemma 706 RL with } \{x2 \leftarrow mult(i(x1), x2), x1 \leftarrow x1\} \\
& \overbrace{op_t(i(x1), mult(x1, \underbrace{mult(x1, mult(i(x1), x2))}))} \\
= & \quad \text{by Lemma 12 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_t(i(x1), mult(x1, \overbrace{x2}))
\end{aligned}
$$

Lemma 714: $op_t(i(x1), mult(x1, x2)) = op_t(i(x1), rd(x1, x2))$

$$
\begin{aligned}
& \underbrace{op_t(i(x1), mult(x1, x2))} \\
= & \quad \text{by Lemma 713 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_t(i(x1), mult(x1, i(x2)))}} \\
= & \quad \text{by Lemma 414 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_t(i(x1), rd(x1, x2))}
\end{aligned}
$$

Lemma 715: $op_t(i(x1), mult(x2, x1)) = op_t(i(x1), rd(x1, x2))$

$$
\begin{aligned}
& \underbrace{op_t(i(x1), mult(x2, x1))} \\
= \quad & \text{by Lemma 19 RL with } \{x2 \leftarrow mult(x2, x1), x1 \leftarrow x1\} \\
& \underbrace{\overbrace{i(op_t(x1, i(mult(x2, x1))))}} \\
= \quad & \text{by Lemma 15 RL with } \{x2 \leftarrow i(mult(x2, x1)), x1 \leftarrow x1\} \\
& \overbrace{op_t(i(x1), \underbrace{i(\overbrace{i(mult(x2, x1))})})} \\
= \quad & \text{by Axiom 11 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \underbrace{op_t(i(x1), i(\overbrace{mult(i(x2), i(x1))}))} \\
= \quad & \text{by Lemma 709 LR with } \{x2 \leftarrow i(x2), x1 \leftarrow i(x1)\} \\
& \underbrace{\overbrace{op_t(i(x1), rd(i(x2), i(x1)))}} \\
= \quad & \text{by Lemma 415 LR with } \{x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& \underbrace{\overbrace{op_t(i(x1), mult(x1, i(x2)))}} \\
= \quad & \text{by Lemma 414 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_t(i(x1), rd(x1, x2))}
\end{aligned}
$$

Lemma 716: $op_t(x1, rd(mult(x2, x1), x1)) = op_t(x1, mult(rd(i(x1), x2), i(x1)))$

$$
\begin{aligned}
& op_t(x1, rd(mult(x2, \underbrace{x1}), x1)) \\
= \quad & \text{by Lemma 3 RL with } \{x1 \leftarrow x1\} \\
& \underbrace{op_t(x1, rd(mult(x2, \overbrace{i(i(x1))}), x1))} \\
= \quad & \text{by Lemma 707 RL with } \{x2 \leftarrow mult(x2, i(i(x1))), x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, \underbrace{rd(i(x1), mult(x2, i(i(x1))))})} \\
= \quad & \text{by Lemma 334 LR with } \{x2 \leftarrow x2, x1 \leftarrow i(x1)\} \\
& op_t(x1, \overbrace{mult(rd(i(x1), x2), i(x1))})
\end{aligned}
$$

Lemma 717: $op_t(i(x1), rd(x2, i(x1))) = op_t(i(x1), rd(x1, x2))$

$$
\begin{aligned}
& \underbrace{op_t(i(x1), rd(x2, i(x1)))} \\
= \quad & \text{by Lemma 415 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{\overbrace{op_t(i(x1), mult(x1, x2))}} \\
= \quad & \text{by Lemma 714 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_t(i(x1), rd(x1, x2))}
\end{aligned}
$$

Lemma 718: $op_t(i(x1), rd(x1, rd(x2, x1))) = op_t(i(x1), x2)$

$$
\begin{aligned}
& \underbrace{op_t(i(x1), rd(x1, rd(x2, x1)))} \\
= \quad & \text{by Lemma 714 RL with } \{x2 \leftarrow rd(x2, x1), x1 \leftarrow x1\} \\
& \underbrace{\overbrace{op_t(i(x1), mult(x1, rd(x2, x1)))}} \\
= \quad & \text{by Lemma 593 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_t(i(x1), x2)}
\end{aligned}
$$

Lemma 719: $mult(rd(x1,x2),op_r(x2,x1,x1)) = op_l(x1,op_t(op_r(x2,x1,x1),rd(x1,x2)),rd(x1,x2))$

$$
\begin{aligned}
& \overbrace{mult(rd(x1,x2),op_r(x2,x1,x1))} \\
= & \quad \text{by Lemma 435 RL with } \{x2 \leftarrow rd(x1,x2), x1 \leftarrow op_r(x2,x1,x1)\} \\
& \overbrace{op_l(\underbrace{mult(op_r(x2,x1,x1),rd(x1,x2))}, op_r(x2,x1,x1), rd(x1,x2))} \\
= & \quad \text{by Lemma 660 LR with } \{x2 \leftarrow rd(x1,x2), x1 \leftarrow op_r(x2,x1,x1)\} \\
& \overbrace{op_l(\underbrace{mult(op_r(x2,x1,x1),rd(x1,x2))}, op_t(op_r(x2,x1,x1),rd(x1,x2)),rd(x1,x2))} \\
= & \quad \text{by Lemma 366 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& op_l(\overbrace{x1}, op_t(op_r(x2,x1,x1),rd(x1,x2)),rd(x1,x2))
\end{aligned}
$$

Lemma 720: $mult(rd(x1,x2),op_r(x2,x1,x1)) = op_l(x1,x2,x1)$

$$
\begin{aligned}
& \overbrace{mult(rd(x1,x2),op_r(x2,x1,x1))} \\
= & \quad \text{by Lemma 719 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1,\underbrace{op_t(op_r(x2,x1,x1),rd(x1,x2))},rd(x1,x2))} \\
= & \quad \text{by Axiom 14 LR with } \{x4 \leftarrow rd(x1,x2), x3 \leftarrow x1, x2 \leftarrow x1, x1 \leftarrow x2\} \\
& op_l(x1,\overbrace{op_r(op_t(x2,rd(x1,x2)),x1,x1)},rd(x1,x2)) \\
= & \quad \text{by Lemma 368 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \underbrace{op_l(x1,\overbrace{x2},rd(x1,x2))} \\
= & \quad \text{by Lemma 286 LR with } \{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1,x2,x1)}
\end{aligned}
$$

Lemma 721: $i(mult(x1,i(x2))) = rd(op_t(x2,x1),op_l(x1,x1,x2))$

$$
\begin{aligned}
& \overbrace{i(mult(x1,i(x2)))} \\
= & \quad \text{by Lemma 9 RL with } \{x2 \leftarrow mult(x1,i(x2)), x1 \leftarrow op_t(x2,x1)\} \\
& \overbrace{rd(op_t(x2,x1),\underbrace{mult(mult(x1,i(x2)),op_t(x2,x1))})} \\
= & \quad \text{by Lemma 258 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& rd(op_t(x2,x1),\overbrace{op_l(x1,x2,i(x1))}) \\
= & \quad \text{by Lemma 606 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& rd(op_t(x2,x1),\overbrace{op_l(x1,x1,x2)})
\end{aligned}
$$

Lemma 722: $rd(mult(x1, x2), op_r(x2, x1, x1)) = op_l(x1, x1, x2)$

$$
\begin{aligned}
& rd(mult(x1, x2), \underbrace{op_r(x2, x1, x1)}) \\
= & \quad \text{by Lemma 358 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& rd(mult(\underbrace{x1}, x2), \overbrace{rd(rd(x2, i(x1)), x1)}) \\
= & \quad \text{by Lemma 3 RL with } \{x1 \leftarrow x1\} \\
& rd(\underbrace{mult(\overbrace{i(i(x1))}, x2)}, rd(rd(x2, i(x1)), x1)) \\
= & \quad \text{by Lemma 14 RL with } \{x2 \leftarrow i(x1), x1 \leftarrow x2\} \\
& \underbrace{rd(\overbrace{op_t(rd(x2, i(x1)), i(x1))}, rd(rd(x2, i(x1)), x1))} \\
= & \quad \text{by Lemma 679 LR with } \{x2 \leftarrow x1, x1 \leftarrow rd(x2, i(x1))\} \\
& \overbrace{op_l(x1, x1, \underbrace{rd(x2, i(x1))})} \\
= & \quad \text{by Lemma 38 RL with } \{x1 \leftarrow x2, x2 \leftarrow x1\} \\
& \underbrace{op_l(x1, x1, \overbrace{rd(x1, i(x2))})} \\
= & \quad \text{by Lemma 496 LR with } \{x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x1, \underbrace{i(i(x2))})} \\
= & \quad \text{by Lemma 3 LR with } \{x1 \leftarrow x2\} \\
& op_l(x1, x1, \overbrace{x2}) \\
\end{aligned}
$$

Lemma 723: $op_l(x1, x2, x1) = rd(i(op_t(x2, x1)), rd(i(x2), x1))$

$$
\begin{aligned}
& \underbrace{op_l(x1, x2, x1)} \\
= & \quad \text{by Lemma 608 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, x1, i(x2))}} \\
= & \quad \text{by Lemma 679 RL with } \{x2 \leftarrow x1, x1 \leftarrow i(x2)\} \\
& \overbrace{rd(\underbrace{op_t(i(x2), i(x1))}, rd(i(x2), x1))} \\
= & \quad \text{by Lemma 15 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& rd(\overbrace{i(op_t(x2, x1))}, rd(i(x2), x1)) \\
\end{aligned}
$$

Lemma 724: $mult(rd(i(x1), x2), op_t(x2, x1)) = i(op_t(x1, x2))$

$$
\begin{aligned}
& mult(rd(i(x1), x2), op_t(x2, \underbrace{x1})) \\
= & \quad \text{by Lemma 3 RL with } \{x1 \leftarrow x1\} \\
& \underbrace{mult(rd(i(x1), x2), op_t(x2, \overbrace{i(i(x1))}))} \\
= & \quad \text{by Lemma 680 RL with } \{x2 \leftarrow x2, x1 \leftarrow i(x1)\} \\
& \overbrace{\underbrace{op_t(i(x1), i(x2))}} \\
= & \quad \text{by Lemma 15 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{i(op_t(x1, x2))} \\
\end{aligned}
$$

Lemma 725: $op_l(x1, rd(x2, i(x1)), x1) = rd(mult(x1, rd(rd(x2, i(x1)), x1)), x2)$

$$
\begin{aligned}
& \overbrace{op_l(x1, rd(x2, i(x1)), x1)} \\
= & \quad \text{by Lemma 658 RL with } \{x2 \leftarrow rd(x2, i(x1)), x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_r(x1, rd(x2, i(x1)), i(x1))}} \\
= & \quad \text{by Lemma 693 RL with } \{x3 \leftarrow i(x1), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{rd(\underbrace{mult(mult(x1, rd(x2, i(x1))), i(x1))}, x2)} \\
= & \quad \text{by Lemma 55 LR with } \{x2 \leftarrow rd(x2, i(x1)), x1 \leftarrow x1\} \\
& rd(\overbrace{mult(x1, rd(rd(x2, i(x1)), x1))}, x2)
\end{aligned}
$$

Lemma 726: $op_l(x1, x2, x1) = rd(mult(x1, op_r(x2, x1, x1)), x2)$

$$
\begin{aligned}
& \overbrace{op_l(x1, x2, x1)} \\
= & \quad \text{by Lemma 499 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, rd(x2, i(x1)), x1)}} \\
= & \quad \text{by Lemma 725 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{rd(mult(x1, \underbrace{rd(rd(x2, i(x1)), x1)}), x2)} \\
= & \quad \text{by Lemma 358 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& rd(mult(x1, \overbrace{op_r(x2, x1, x1)}), x2)
\end{aligned}
$$

Lemma 727: $rd(op_r(mult(x1, x2), x1, x1), x2) = op_l(x1, x2, x1)$

$$
\begin{aligned}
& rd(\underbrace{op_r(mult(x1, x2), x1, x1)}, x2) \\
= & \quad \text{by Lemma 383 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{rd(\overbrace{mult(x1, op_r(x2, x1, x1))}, x2)} \\
= & \quad \text{by Lemma 726 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x2, x1)}
\end{aligned}
$$

Lemma 728: $asoc(x1, x2, x1) = mult(mult(x2, i(x1)), rd(x1, x2))$

$$
\begin{aligned}
& \underbrace{asoc(x1, x2, x1)} \\
= & \quad \text{by Lemma 664 LR with } \{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow asoc(x1, x2, x1)\} \\
& \overbrace{\underbrace{op_l(asoc(x1, x2, x1), rd(x2, x1), asoc(x1, x2, x1))}} \\
= & \quad \text{by Lemma 466 RL with } \{x2 \leftarrow rd(x2, x1), x1 \leftarrow asoc(x1, x2, x1)\} \\
& \overbrace{mult(\underbrace{mult(asoc(x1, x2, x1), rd(x2, x1))}, op_t(i(rd(x2, x1)), asoc(x1, x2, x1)))} \\
= & \quad \text{by Lemma 700 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& mult(\overbrace{mult(x2, i(x1))}, op_t(\underbrace{i(rd(x2, x1))}, asoc(x1, x2, x1))) \\
= & \quad \text{by Lemma 10 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& mult(mult(x2, i(x1)), \underbrace{op_t(\overbrace{rd(x1, x2)}, asoc(x1, x2, x1))}) \\
= & \quad \text{by Lemma 541 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& mult(mult(x2, i(x1)), \overbrace{rd(x1, x2)})
\end{aligned}
$$

Lemma 729: $asoc(x1, x2, x1) = rd(mult(rd(x2, op_t(i(i(x1)), x2)), x1), op_l(x2, x2, i(x1)))$

$\underbrace{asoc(x1, x2, x1)}$

$=$ by Lemma 262 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{asoc(x1, mult(x2, \underbrace{x1}), x1)}$

$=$ by Lemma 3 RL with $\{x1 \leftarrow x1\}$

$asoc(x1, \underbrace{mult(x2, \overbrace{i(i(x1))})}, x1)$

$=$ by Lemma 342 RL with $\{x2 \leftarrow i(x1), x1 \leftarrow x2\}$

$asoc(x1, \overbrace{\underbrace{rd(op_r(x2, i(x1)), i(i(x1)))}, i(x1)}, x1)$

$=$ by Lemma 38 RL with $\{x1 \leftarrow op_r(x2, i(x1)), i(i(x1))), x2 \leftarrow x1\}$

$\underbrace{asoc(x1, \overbrace{rd(x1, i(\underbrace{op_r(x2, i(x1)), i(i(x1))}))}, x1)}$

$=$ by Lemma 498 LR with $\{x2 \leftarrow i(op_r(x2, i(x1)), i(i(x1)))), x1 \leftarrow x1\}$

$\overbrace{asoc(x1, \underbrace{i(i(op_r(x2, i(x1)), i(i(x1))))}, x1)}$

$=$ by Lemma 3 LR with $\{x1 \leftarrow op_r(x2, i(x1)), i(i(x1)))\}$

$asoc(x1, \overbrace{op_r(x2, i(x1)), \underbrace{i(i(x1))}}, x1)$

$=$ by Lemma 3 LR with $\{x1 \leftarrow x1\}$

$asoc(x1, \underbrace{op_r(x2, i(x1), \overbrace{x1})}, x1)$

$=$ by Lemma 321 LR with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$asoc(x1, \overbrace{op_r(x2, x1, i(x1))}, x1)$

$=$ by Lemma 604 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{asoc(x1, \overbrace{mult(x1, rd(x2, op_t(x1, x2)))}, x1)}$

$=$ by Lemma 618 LR with $\{x2 \leftarrow rd(x2, op_t(x1, x2)), x1 \leftarrow x1\}$

$\overbrace{asoc(x1, rd(x2, op_t(\underbrace{x1}, x2)), x1)}$

$=$ by Lemma 3 RL with $\{x1 \leftarrow x1\}$

$\underbrace{asoc(x1, rd(x2, op_t(\overbrace{i(i(x1))}, x2)), x1)}$

$=$ by Lemma 702 RL with $\{x2 \leftarrow x1, x1 \leftarrow rd(x2, op_t(i(i(x1)), x2))\}$

$\overbrace{rd(mult(rd(x2, op_t(i(i(x1)), x2)), x1), \underbrace{rd(rd(x2, op_t(i(i(x1)), x2)), i(x1))})}$

$=$ by Lemma 684 RL with $\{x2 \leftarrow i(x1), x1 \leftarrow x2\}$

$rd(mult(rd(x2, op_t(i(i(x1)), x2)), x1), \overbrace{op_l(x2, x2, i(x1))})$

Lemma 730: $asoc(x1, x2, x1) = rd(op_t(x2, x1), op_l(x2, x2, i(x1)))$

$$
\begin{aligned}
& \underbrace{asoc(x1, x2, x1)} \\
= \quad & \text{by Lemma 729 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{rd(mult(rd(x2, op_t(\underbrace{i(i(x1))}, x2)), x1), op_l(x2, x2, i(x1)))} \\
= \quad & \text{by Lemma 3 LR with } \{x1 \leftarrow x1\} \\
& rd(\underbrace{mult(rd(x2, op_t(\overbrace{x1}, x2)), x1)}, op_l(x2, x2, i(x1))) \\
= \quad & \text{by Lemma 597 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& rd(\overbrace{op_t(x2, x1)}, op_l(x2, x2, i(x1)))
\end{aligned}
$$

Lemma 731: $rd(op_t(x1, x2), op_l(x1, x2, x1)) = asoc(x2, x1, x2)$

$$
\begin{aligned}
& rd(op_t(x1, x2), \underbrace{op_l(x1, x2, x1)}) \\
= \quad & \text{by Lemma 608 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{rd(op_t(x1, x2), \overbrace{op_l(x1, x1, i(x2))})} \\
= \quad & \text{by Lemma 730 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \overbrace{asoc(x2, x1, x2)}
\end{aligned}
$$

Lemma 732: $rd(x2, i(x1)) = rd(op_t(x2, x1), i(op_t(x1, x2)))$

$$
\begin{aligned}
& \underbrace{rd(x2, i(x1))} \\
= \quad & \text{by Lemma 10 RL with } \{x2 \leftarrow x2, x1 \leftarrow i(x1)\} \\
& \underbrace{\overbrace{i(rd(i(x1), x2))}} \\
= \quad & \text{by Lemma 9 RL with } \{x2 \leftarrow rd(i(x1), x2), x1 \leftarrow op_t(x2, x1)\} \\
& \overbrace{rd(op_t(x2, x1), \underbrace{mult(rd(i(x1), x2), op_t(x2, x1))})} \\
= \quad & \text{by Lemma 724 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& rd(op_t(x2, x1), \overbrace{i(op_t(x1, x2))})
\end{aligned}
$$

Lemma 733: $rd(rd(x2, i(x1)), op_t(x1, x2)) = rd(op_t(x2, x1), rd(rd(i(x1), x2), i(mult(x2, x1))))$

$$
\begin{aligned}
& \underbrace{rd(rd(x2, i(x1)), op_t(x1, x2))} \\
= \quad & \text{by Lemma 40 RL with } \{x3 \leftarrow x2, x2 \leftarrow i(x1), x1 \leftarrow op_t(x1, x2)\} \\
& \overbrace{rd(\underbrace{i(op_t(x1, x2))}, rd(i(x1), x2))} \\
= \quad & \text{by Lemma 724 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{rd(\overbrace{mult(rd(i(x1), x2), op_t(x2, x1))}, rd(i(x1), x2))} \\
= \quad & \text{by Lemma 398 RL with } \{x2 \leftarrow rd(i(x1), x2), x1 \leftarrow op_t(x2, x1)\} \\
& \overbrace{rd(op_t(x2, x1), rd(rd(i(x1), x2), op_t(\underbrace{rd(i(x1), x2)}, op_t(x2, x1))))} \\
= \quad & \text{by Lemma 37 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& rd(op_t(x2, x1), rd(rd(i(x1), x2), \underbrace{op_t(\overbrace{rd(i(x2), x1)}, op_t(x2, x1))})) \\
= \quad & \text{by Lemma 682 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& rd(op_t(x2, x1), rd(rd(i(x1), x2), \overbrace{i(mult(x2, x1))}))
\end{aligned}
$$

Lemma 734: $op_l(x2, x1, x2) = rd(op_t(x2, x1), rd(rd(i(x1), x2), i(mult(x2, x1))))$

$$
\begin{aligned}
& \overbrace{op_l(x2, x1, x2)} \\
= \quad & \text{by Lemma 723 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \overbrace{rd(\overbrace{i(op_t(x1, x2))}, rd(i(x1), x2))} \\
= \quad & \text{by Lemma 40 LR with } \{x3 \leftarrow x2, x2 \leftarrow i(x1), x1 \leftarrow op_t(x1, x2)\} \\
& \overbrace{rd(rd(x2, i(x1)), op_t(x1, x2))} \\
= \quad & \text{by Lemma 733 LR with } \{x1 \leftarrow x1, x2 \leftarrow x2\} \\
& \overbrace{rd(op_t(x2, x1), rd(rd(i(x1), x2), i(mult(x2, x1))))}
\end{aligned}
$$

Lemma 735: $op_l(x2, x1, x2) = rd(op_t(x2, x1), asoc(x1, x2, x1))$

$$
\begin{aligned}
& \overbrace{op_l(x2, x1, x2)} \\
= \quad & \text{by Lemma 734 LR with } \{x1 \leftarrow x1, x2 \leftarrow x2\} \\
& \overbrace{rd(op_t(x2, x1), \underbrace{rd(rd(i(x1), x2), i(mult(x2, x1)))})} \\
= \quad & \text{by Lemma 39 LR with } \{x3 \leftarrow mult(x2, x1), x2 \leftarrow x2, x1 \leftarrow i(x1)\} \\
& rd(op_t(x2, x1), \overbrace{rd(mult(x2, x1), rd(x2, i(x1)))}) \\
= \quad & \text{by Lemma 702 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& rd(op_t(x2, x1), \overbrace{asoc(x1, x2, x1)})
\end{aligned}
$$

Lemma 736: $asoc(op_t(x1, x2), x2, op_t(x1, x2)) = rd(op_t(x2, x1), op_l(x2, x1, x2))$

$$
\begin{aligned}
& \overbrace{asoc(op_t(x1, x2), x2, op_t(x1, x2))} \\
= \quad & \text{by Lemma 731 RL with } \{x2 \leftarrow op_t(x1, x2), x1 \leftarrow x2\} \\
& \overbrace{rd(\underbrace{op_t(x2, op_t(x1, x2))}, op_l(x2, op_t(x1, x2), x2))} \\
= \quad & \text{by Lemma 374 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& rd(\overbrace{op_t(x2, x1)}, \underbrace{op_l(x2, op_t(x1, x2), x2)}) \\
= \quad & \text{by Lemma 623 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& rd(op_t(x2, x1), \overbrace{op_l(x2, x1, x2)})
\end{aligned}
$$

Lemma 737: $op_t(x1, mult(rd(x2, x3), x1)) = op_t(x1, rd(x1, rd(x3, x2)))$

$$
\begin{aligned}
& op_t(x1, mult(\underbrace{rd(x2, x3)}, x1)) \\
= \quad & \text{by Lemma 10 RL with } \{x2 \leftarrow x2, x1 \leftarrow x3\} \\
& \underbrace{op_t(x1, mult(\overbrace{i(\overbrace{rd(x3, x2)})}, x1))} \\
= \quad & \text{by Lemma 554 LR with } \{x2 \leftarrow rd(x3, x2), x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, rd(x1, rd(x3, x2)))}
\end{aligned}
$$

Lemma 738: $op_t(x1, mult(x1, rd(x2, x3))) = op_t(x1, rd(x1, rd(x3, x2)))$

$$
\begin{aligned}
& op_t(x1, mult(x1, \underbrace{rd(x2, x3)})) \\
= & \quad \text{by Lemma 10 RL with } \{x2 \leftarrow x2, x1 \leftarrow x3\} \\
& \underbrace{op_t(x1, mult(x1, \overbrace{i(rd(x3, x2))}))} \\
= & \quad \text{by Lemma 564 LR with } \{x2 \leftarrow rd(x3, x2), x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, rd(x1, rd(x3, x2)))}
\end{aligned}
$$

Lemma 739: $op_r(op_r(x1, x2, x3), x4, x1) = op_l(op_r(x1, x2, x3), x1, x4)$

$$
\begin{aligned}
& \underbrace{op_r(op_r(x1, x2, x3), x4, x1)} \\
= & \quad \text{by Axiom 17 RL with } \{x5 \leftarrow x3, x4 \leftarrow x2, x3 \leftarrow x1, x2 \leftarrow x4, x1 \leftarrow x1\} \\
& \overbrace{op_r(\underbrace{op_r(x1, x4, x1)}, x2, x3)} \\
= & \quad \text{by Lemma 601 LR with } \{x2 \leftarrow x4, x1 \leftarrow x1\} \\
& \underbrace{op_r(\overbrace{op_l(x1, x1, x4)}, x2, x3)} \\
= & \quad \text{by Axiom 18 LR with } \{x5 \leftarrow x3, x4 \leftarrow x2, x3 \leftarrow x4, x2 \leftarrow x1, x1 \leftarrow x1\} \\
& \overbrace{op_l(op_r(x1, x2, x3), x1, x4)}
\end{aligned}
$$

Lemma 740: $i(x1) = op_l(op_l(i(x1), x1, x2), i(i(x1)), i(x2))$

$$
\begin{aligned}
& \underbrace{i(x1)} \\
= & \quad \text{by Lemma 138 RL with } \{x3 \leftarrow i(i(x1)), x2 \leftarrow i(x2), x1 \leftarrow i(x1)\} \\
& \overbrace{op_l(\underbrace{op_l(i(x1), i(x2), i(i(x1)))}, i(i(x1)), i(x2))} \\
= & \quad \text{by Lemma 438 LR with } \{x2 \leftarrow x2, x1 \leftarrow i(x1)\} \\
& op_l(\overbrace{\underbrace{op_l(i(x1), x2, i(x1))}}, i(i(x1)), i(x2)) \\
= & \quad \text{by Lemma 633 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_l(\overbrace{op_l(i(x1), x1, x2)}, i(i(x1)), i(x2))
\end{aligned}
$$

Lemma 741: $op_l(op_l(i(x1), x1, x2), x1, i(x2)) = i(x1)$

$$
\begin{aligned}
& op_l(op_l(i(x1), x1, x2), \underbrace{x1}, i(x2)) \\
= & \quad \text{by Lemma 3 RL with } \{x1 \leftarrow x1\} \\
& \underbrace{op_l(op_l(i(x1), x1, x2), \overbrace{i(i(x1))}, i(x2))} \\
= & \quad \text{by Lemma 740 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{i(x1)}
\end{aligned}
$$

Lemma 742: $i(op_r(x1, op_t(x2, x1), x3)) = op_r(i(x1), mult(x2, x1), x3)$

$$
\begin{aligned}
& \overbrace{i(op_r(x1, op_t(x2, x1), x3))} \\
= \quad & \text{by Lemma 654 RL with } \{x3 \leftarrow x3, x2 \leftarrow op_t(x2, x1), x1 \leftarrow x1\} \\
& \overbrace{op_r(i(x1), \underbrace{mult(x1, op_t(x2, x1))}, x3)} \\
= \quad & \text{by Lemma 13 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_r(i(x1), \overbrace{mult(x2, x1)}, x3)
\end{aligned}
$$

Lemma 743: $op_l(x1, op_r(x1, x2, x2), x3) = op_r(x1, x3, op_r(x1, x2, x2))$

$$
\begin{aligned}
& op_l(\underbrace{x1}, op_r(x1, x2, x2), x3) \\
= \quad & \text{by Lemma 351 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{op_l(\overbrace{op_r(op_r(x1, x2, x2), x2, i(x2))}, op_r(x1, x2, x2), x3)} \\
= \quad & \text{by Lemma 739 RL with } \{x4 \leftarrow x3, x3 \leftarrow i(x2), x2 \leftarrow x2, x1 \leftarrow op_r(x1, x2, x2)\} \\
& \overbrace{op_r(\underbrace{op_r(op_r(x1, x2, x2), x2, i(x2))}, x3, op_r(x1, x2, x2))} \\
= \quad & \text{by Lemma 351 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_r(\overbrace{x1}, x3, op_r(x1, x2, x2))
\end{aligned}
$$

Lemma 744: $op_l(x1, x2, x1) = mult(mult(i(x2), x1), x2)$

$$
\begin{aligned}
& \underbrace{op_l(x1, x2, x1)} \\
= \quad & \text{by Lemma 608 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{\overbrace{op_l(x1, x1, i(x2))}} \\
= \quad & \text{by Lemma 439 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{mult(mult(i(x2), x1), x2)}
\end{aligned}
$$

Lemma 745: $op_l(x1, x2, x1) = op_r(op_t(x1, x2), x2, x2)$

$$
\begin{aligned}
& \underbrace{op_l(x1, x2, x1)} \\
= \quad & \text{by Axiom 5 LR with } \{x2 \leftarrow i(x2), x1 \leftarrow op_l(x1, x2, x1)\} \\
& \overbrace{rd(\underbrace{mult(op_l(x1, x2, x1), i(x2))}, \overbrace{i(x2)})} \\
= \quad & \text{by Lemma 451 RL with } \{x1 \leftarrow x2, x2 \leftarrow x1\} \\
& \underbrace{rd(\overbrace{rd(op_t(x1, x2), x2)}, i(x2))} \\
= \quad & \text{by Lemma 39 LR with } \{x3 \leftarrow x2, x2 \leftarrow x2, x1 \leftarrow op_t(x1, x2)\} \\
& \underbrace{\overbrace{rd(x2, rd(x2, op_t(x1, x2)))}} \\
= \quad & \text{by Lemma 353 LR with } \{x2 \leftarrow op_t(x1, x2), x1 \leftarrow x2\} \\
& \overbrace{op_r(op_t(x1, x2), x2, x2)}
\end{aligned}
$$

Lemma 746: $rd(mult(x2,x1),x1) = op_l(op_l(x2,x1,x2),op_t(x2,x1),x1)$

$$
\begin{aligned}
& rd(\underbrace{mult(x2,x1)}, x1) \\
= & \quad \text{by Lemma 13 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{rd(\overbrace{mult(x1,op_t(x2,x1))}, x1)} \\
= & \quad \text{by Lemma 453 RL with } \{x2 \leftarrow x1, x1 \leftarrow op_t(x2,x1)\} \\
& \overbrace{op_l(\underbrace{op_r(op_t(x2,x1),x1,x1)}, op_t(x2,x1), x1)} \\
= & \quad \text{by Lemma 745 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& op_l(\overbrace{op_l(x2,x1,x2)}, op_t(x2,x1), x1)
\end{aligned}
$$

Lemma 747: $rd(x1,op_r(op_t(x1,x2),x2,x2)) = asoc(x1,x2,x1)$

$$
\begin{aligned}
& rd(x1, \underbrace{op_r(op_t(x1,x2),x2,x2)}) \\
= & \quad \text{by Lemma 745 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{rd(x1, \overbrace{op_l(x1,x2,x1)})} \\
= & \quad \text{by Lemma 617 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{asoc(x1,x2,x1)}
\end{aligned}
$$

Lemma 748: $rd(x1,op_t(x2,x1)) = rd(op_t(x1,x2),x2)$

$$
\begin{aligned}
& \underbrace{rd(x1,op_t(x2,x1))} \\
= & \quad \text{by Axiom 5 LR with } \{x2 \leftarrow x2, x1 \leftarrow rd(x1,op_t(x2,x1))\} \\
& \overbrace{rd(\underbrace{mult(rd(x1,op_t(x2,x1)),x2)}, x2)} \\
= & \quad \text{by Lemma 597 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& rd(\overbrace{op_t(x1,x2)}, x2)
\end{aligned}
$$

Lemma 749: $mult(mult(i(x1),x2),mult(x1,x1)) = mult(op_t(x2,x1),x1)$

$$
\begin{aligned}
& \underbrace{mult(mult(i(x1),x2),mult(x1,x1))} \\
= & \quad \text{by Lemma 213 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{\overbrace{op_t(rd(x2,i(x1)),x1)}} \\
= & \quad \text{by Lemma 337 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \overbrace{mult(op_t(x2,x1),x1)}
\end{aligned}
$$

Lemma 750: $rd(x1, op_t(i(op_t(x2, x1)), x1)) = op_t(mult(x1, x2), x1)$

$$
\begin{aligned}
& \underbrace{rd(x1, op_t(i(op_t(x2, x1)), x1))} \\
= & \quad \text{by Lemma 65 RL with } \{x3 \leftarrow x1, x2 \leftarrow x1, x1 \leftarrow op_t(x2, x1)\} \\
& \overbrace{rd(op_t(\underbrace{op_t(x2, x1)}, i(x1)), i(x1))} \\
= & \quad \text{by Axiom 2 LR with } \{x2 \leftarrow x1, x1 \leftarrow op_t(x2, x1)\} \\
& \underbrace{rd(op_t(\overbrace{mult(i(x1), mult(x1, op_t(x2, x1)))}, i(x1)), i(x1))} \\
= & \quad \text{by Lemma 317 LR with } \{x2 \leftarrow mult(x1, op_t(x2, x1)), x1 \leftarrow i(x1)\} \\
& \overbrace{op_r(mult(x1, op_t(x2, x1)), i(x1), \underbrace{i(i(x1))})} \\
= & \quad \text{by Lemma 3 LR with } \{x1 \leftarrow x1\} \\
& \underbrace{op_r(mult(x1, op_t(x2, x1)), i(x1), \overbrace{x1})} \\
= & \quad \text{by Lemma 125 LR with } \{x2 \leftarrow op_t(x2, x1), x1 \leftarrow x1\} \\
& \overbrace{mult(mult(x1, rd(op_t(x2, x1), x1)), \underbrace{i(i(x1))})} \\
= & \quad \text{by Lemma 3 LR with } \{x1 \leftarrow x1\} \\
& mult(\underbrace{mult(x1, rd(op_t(x2, x1), x1))}, \overbrace{x1}) \\
= & \quad \text{by Lemma 109 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{mult(\overbrace{op_r(x2, x1, i(x1))}, x1)} \\
= & \quad \text{by Lemma 318 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \overbrace{op_t(mult(x1, x2), x1)}
\end{aligned}
$$

Lemma 751: $op_t(mult(x1, x2), x2) = op_t(mult(op_t(x1, x2), x2), i(x1))$

$$
\begin{aligned}
& op_t(mult(x1, \underbrace{x2}), x2) \\
= & \quad \text{by Lemma 3 RL with } \{x1 \leftarrow x2\} \\
& \underbrace{op_t(mult(x1, \overbrace{i(i(x2))}), x2)} \\
= & \quad \text{by Lemma 194 RL with } \{x3 \leftarrow x2, x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& \overbrace{op_t(\underbrace{op_t(rd(x1, i(x2)), x2)}, i(x1))} \\
= & \quad \text{by Lemma 337 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_t(\overbrace{mult(op_t(x1, x2), x2)}, i(x1))
\end{aligned}
$$

Lemma 752: $rd(x1, i(mult(op_r(x2, i(x1), x1), x1))) = mult(mult(x2, i(i(x1))), x1)$

$$
\begin{aligned}
& rd(x1, i(mult(op_r(x2, i(x1), \underbrace{x1}), x1))) \\
= & \quad \text{by Lemma 3 RL with } \{x1 \leftarrow x1\} \\
& \underbrace{rd(x1, i(mult(op_r(x2, i(x1), \overbrace{i(i(x1))}), x1)))} \\
= & \quad \text{by Lemma 336 RL with } \{x2 \leftarrow x1, x1 \leftarrow op_r(x2, i(x1), i(i(x1)))\} \\
& \overbrace{mult(\underbrace{rd(op_r(x2, i(x1), i(i(x1))), i(x1))}, x1)} \\
= & \quad \text{by Lemma 342 LR with } \{x2 \leftarrow i(x1), x1 \leftarrow x2\} \\
& mult(\overbrace{mult(x2, i(i(x1)))}, x1)
\end{aligned}
$$

Lemma 753: $rd(x1, i(op_t(mult(x1, x2), x1))) = mult(mult(x2, i(i(x1))), x1)$

$$
\begin{aligned}
& rd(x1, i(\underbrace{op_t(mult(x1, x2), x1)})) \\
= & \quad \text{by Lemma 318 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& rd(x1, i(\overbrace{mult(\underbrace{op_r(x2, x1, i(x1))}, x1)})) \\
= & \quad \text{by Lemma 321 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \underbrace{rd(x1, i(mult(\overbrace{op_r(x2, i(x1), x1)}, x1)))} \\
= & \quad \text{by Lemma 752 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{mult(mult(x2, i(i(x1))), x1)}
\end{aligned}
$$

Lemma 754: $asoc(x1, i(op_t(mult(x1, x2), x1)), x1) = mult(mult(i(op_t(mult(x1, x2), x1)), i(x1)), mult(mult(x2, x1), x1))$

$$
\begin{aligned}
& \underbrace{asoc(x1, i(op_t(mult(x1, x2), x1)), x1)} \\
= & \quad \text{by Lemma 728 LR with } \{x2 \leftarrow i(op_t(mult(x1, x2), x1)), x1 \leftarrow x1\} \\
& \overbrace{mult(mult(i(op_t(mult(x1, x2), x1)), i(x1)), \underbrace{rd(x1, i(op_t(mult(x1, x2), x1)))})} \\
= & \quad \text{by Lemma 753 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& mult(mult(i(op_t(mult(x1, x2), x1)), i(x1)), \overbrace{mult(mult(x2, \underbrace{i(i(x1))}), x1)}) \\
= & \quad \text{by Lemma 3 LR with } \{x1 \leftarrow x1\} \\
& mult(mult(i(op_t(mult(x1, x2), x1)), i(x1)), mult(mult(x2, \overbrace{x1}), x1))
\end{aligned}
$$

Lemma 755: $asoc(x1, i(x2), x1) = mult(mult(i(op_t(mult(x1, x2), x1)), i(x1)), mult(mult(x2, x1), x1))$

$$
\begin{aligned}
& \underbrace{asoc(x1, i(x2), x1)} \\
= & \quad \text{by Lemma 646 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{asoc(x1, i(mult(x1, x2)), x1)}} \\
= & \quad \text{by Lemma 648 RL with } \{x2 \leftarrow mult(x1, x2), x1 \leftarrow x1\} \\
& \overbrace{\underbrace{asoc(x1, i(op_t(mult(x1, x2), x1)), x1)}} \\
= & \quad \text{by Lemma 754 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{mult(mult(i(op_t(mult(x1, x2), x1)), i(x1)), mult(mult(x2, x1), x1))}
\end{aligned}
$$

Lemma 756: $asoc(x1, i(x2), x1) = asoc(x2, x1, x1)$

$$
\begin{aligned}
& \underbrace{asoc(x1, i(x2), x1)} \\
= & \quad \text{by Lemma 755 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{mult(\underbrace{mult(i(op_t(mult(x1, x2), x1)), i(x1))}, mult(mult(x2, x1), x1))} \\
= & \quad \text{by Axiom 11 LR with } \{x2 \leftarrow x1, x1 \leftarrow op_t(mult(x1, x2), x1)\} \\
& mult(\overbrace{i(\underbrace{mult(op_t(mult(x1, x2), x1), x1)})}, mult(mult(x2, x1), x1)) \\
= & \quad \text{by Lemma 350 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{mult(i(\overbrace{mult(x2, mult(x1, x1))}), mult(mult(x2, x1), x1))} \\
= & \quad \text{by Lemma 108 LR with } \{x3 \leftarrow x1, x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \overbrace{asoc(x2, x1, x1)}
\end{aligned}
$$

Lemma 757: $i(asoc(x1, x2, x1)) = asoc(x2, x1, x1)$

$$
\begin{aligned}
& \underbrace{i(asoc(x1, x2, x1))} \\
= \quad & \text{by Lemma 612 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{asoc(x1, i(x2), x1)}} \\
= \quad & \text{by Lemma 756 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{asoc(x2, x1, x1)}
\end{aligned}
$$

Lemma 758: $asoc(i(x1), x2, x2) = asoc(x2, x1, x2)$

$$
\begin{aligned}
& \underbrace{asoc(i(x1), x2, x2)} \\
= \quad & \text{by Lemma 756 RL with } \{x2 \leftarrow i(x1), x1 \leftarrow x2\} \\
& \overbrace{asoc(x2, \underbrace{i(i(x1))}, x2)} \\
= \quad & \text{by Lemma 3 LR with } \{x1 \leftarrow x1\} \\
& asoc(x2, \overbrace{x1}, x2)
\end{aligned}
$$

Lemma 759: $i(asoc(x1, x2, x2)) = asoc(x2, x1, x2)$

$$
\begin{aligned}
& i(\underbrace{asoc(x1, x2, x2)}) \\
= \quad & \text{by Lemma 757 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \underbrace{i(\overbrace{i(asoc(x2, x1, x2))})} \\
= \quad & \text{by Lemma 3 LR with } \{x1 \leftarrow asoc(x2, x1, x2)\} \\
& \overbrace{asoc(x2, x1, x2)}
\end{aligned}
$$

Lemma 760: $op_l(x1, x2, asoc(x2, x3, x3)) = x1$

$$
\begin{aligned}
& op_l(x1, x2, \underbrace{asoc(x2, x3, x3)}) \\
= \quad & \text{by Lemma 756 RL with } \{x2 \leftarrow x2, x1 \leftarrow x3\} \\
& \underbrace{op_l(x1, x2, \overbrace{asoc(x3, i(x2), x3)})} \\
= \quad & \text{by Lemma 637 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{x1}
\end{aligned}
$$

Lemma 761: $x1 = op_l(x1, x1, asoc(x2, x1, x1))$

$$
\begin{aligned}
& \underbrace{x1} \\
= \quad & \text{by Lemma 138 RL with } \{x3 \leftarrow x1, x2 \leftarrow asoc(x1, i(x2), x1), x1 \leftarrow x1\} \\
& \overbrace{op_l(\underbrace{op_l(x1, asoc(x1, i(x2), x1), x1)}, x1, asoc(x1, i(x2), x1))} \\
= \quad & \text{by Lemma 228 LR with } \{x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& op_l(\overbrace{x1}, x1, \underbrace{asoc(x1, i(x2), x1)}) \\
= \quad & \text{by Lemma 756 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_l(x1, x1, \overbrace{asoc(x2, x1, x1)})
\end{aligned}
$$

Lemma 762: $asoc(x1, rd(x1, x2), x1) = asoc(x2, x1, x1)$

$$
\begin{aligned}
& \overbrace{asoc(x1, rd(x1, x2), x1)} \\
= & \quad \text{by Lemma 498 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\overbrace{asoc(x1, i(x2), x1)}} \\
= & \quad \text{by Lemma 756 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{asoc(x2, x1, x1)}
\end{aligned}
$$

Lemma 763: $asoc(mult(x1, x2), x2, x2) = asoc(x1, x2, x2)$

$$
\begin{aligned}
& \overbrace{asoc(mult(x1, x2), x2, x2)} \\
= & \quad \text{by Lemma 756 RL with } \{x2 \leftarrow mult(x1, x2), x1 \leftarrow x2\} \\
& \overbrace{\overbrace{asoc(x2, i(mult(x1, x2)), x2)}} \\
= & \quad \text{by Lemma 498 RL with } \{x2 \leftarrow mult(x1, x2), x1 \leftarrow x2\} \\
& \overbrace{asoc(x2, \underbrace{rd(x2, mult(x1, x2))}, x2)} \\
= & \quad \text{by Lemma 9 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \overbrace{asoc(x2, \overbrace{i(x1)}, x2)} \\
= & \quad \text{by Lemma 756 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \overbrace{asoc(x1, x2, x2)}
\end{aligned}
$$

Lemma 764: $rd(x1, op_l(x1, x1, x2)) = asoc(x2, x1, x1)$

$$
\begin{aligned}
& \overbrace{rd(x1, op_l(x1, x1, x2))} \\
= & \quad \text{by Lemma 174 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\overbrace{i(asoc(x1, x2, x1))}} \\
= & \quad \text{by Lemma 612 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\overbrace{asoc(x1, i(x2), x1)}} \\
= & \quad \text{by Lemma 756 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{asoc(x2, x1, x1)}
\end{aligned}
$$

Lemma 765: $rd(op_l(x1, x2, x1), x1) = asoc(x2, x1, x1)$

$$
\begin{aligned}
& \overbrace{rd(op_l(x1, x2, x1), x1)} \\
= & \quad \text{by Lemma 171 RL with } \{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{mult(i(x1), \overbrace{op_l(x1, x2, x1)})} \\
= & \quad \text{by Lemma 486 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{mult(i(x1), \overbrace{mult(x1, asoc(i(x2), x1, i(x1)))})} \\
= & \quad \text{by Axiom 2 RL with } \{x2 \leftarrow x1, x1 \leftarrow asoc(i(x2), x1, i(x1))\} \\
& \overbrace{\overbrace{asoc(i(x2), x1, i(x1))}} \\
= & \quad \text{by Lemma 611 LR with } \{x2 \leftarrow x1, x1 \leftarrow i(x2)\} \\
& \overbrace{\overbrace{asoc(x1, i(x2), x1)}} \\
= & \quad \text{by Lemma 756 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{asoc(x2, x1, x1)}
\end{aligned}
$$

Lemma 766: $rd(op_r(x1, x2, x2), x1) = asoc(x1, x2, x2)$

$$
\begin{aligned}
& \overbrace{rd(op_r(x1, x2, x2), x1)} \\
= & \quad \text{by Lemma 10 RL with } \{x2 \leftarrow op_r(x1, x2, x2), x1 \leftarrow x1\} \\
& \overbrace{i(\overbrace{rd(x1, op_r(x1, x2, x2))})} \\
= & \quad \text{by Lemma 640 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \overbrace{\overbrace{i(\overbrace{asoc(x2, x1, x2)})}} \\
= & \quad \text{by Lemma 612 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \overbrace{\overbrace{asoc(x2, i(x1), x2)}} \\
= & \quad \text{by Lemma 756 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \overbrace{asoc(x1, x2, x2)}
\end{aligned}
$$

Lemma 767: $mult(asoc(x2, x1, x1), x2) = op_r(x2, x1, x1)$

$$
\begin{aligned}
& mult(\overbrace{asoc(x2, x1, x1)}, x2) \\
= & \quad \text{by Lemma 756 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{mult(\overbrace{asoc(x1, i(x2), x1)}, x2)} \\
= & \quad \text{by Lemma 544 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\overbrace{mult(x2, asoc(x1, i(x2), x1))}} \\
= & \quad \text{by Lemma 644 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \overbrace{op_r(x2, x1, x1)}
\end{aligned}
$$

Lemma 768: $asoc(mult(x1, x2), x1, x1) = asoc(x2, x1, x1)$

$$
\begin{aligned}
& \overbrace{asoc(mult(x1, x2), x1, x1)} \\
= \quad & \text{by Lemma 756 RL with } \{x2 \leftarrow mult(x1, x2), x1 \leftarrow x1\} \\
& \overbrace{asoc(x1, i(mult(x1, x2)), x1)} \\
= \quad & \text{by Lemma 646 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{asoc(x1, i(x2), x1)} \\
= \quad & \text{by Lemma 756 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{asoc(x2, x1, x1)}
\end{aligned}
$$

Lemma 769: $asoc(op_t(x2, x1), x1, x1) = asoc(x2, x1, x1)$

$$
\begin{aligned}
& \overbrace{asoc(op_t(x2, x1), x1, x1)} \\
= \quad & \text{by Lemma 756 RL with } \{x2 \leftarrow op_t(x2, x1), x1 \leftarrow x1\} \\
& \overbrace{asoc(x1, i(op_t(x2, x1)), x1)} \\
= \quad & \text{by Lemma 648 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{asoc(x1, i(x2), x1)} \\
= \quad & \text{by Lemma 756 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{asoc(x2, x1, x1)}
\end{aligned}
$$

Lemma 770: $op_l(x1, x2, x1) = mult(x1, asoc(x2, x1, x1))$

$$
\begin{aligned}
& \overbrace{op_l(x1, x2, x1)} \\
= \quad & \text{by Lemma 608 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x1, i(x2))} \\
= \quad & \text{by Lemma 158 RL with } \{x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& \overbrace{mult(x1, \underbrace{asoc(x1, i(x2), x1)})} \\
= \quad & \text{by Lemma 756 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& mult(x1, \overbrace{asoc(x2, x1, x1)})
\end{aligned}
$$

Lemma 771: $op_l(x1, x2, x1) = mult(asoc(x2, x1, x1), x1)$

$$
\begin{aligned}
& \overbrace{op_l(x1, x2, x1)} \\
= \quad & \text{by Lemma 608 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x1, i(x2))} \\
= \quad & \text{by Lemma 173 RL with } \{x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& \overbrace{mult(\underbrace{asoc(x1, i(x2), x1)}, x1)} \\
= \quad & \text{by Lemma 756 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& mult(\overbrace{asoc(x2, x1, x1)}, x1)
\end{aligned}
$$

Lemma 772: $asoc(x1, i(x2), i(x2)) = asoc(x1, x2, x2)$

$$
\begin{aligned}
& \underbrace{asoc(x1, i(x2), i(x2))} \\
= \quad & \text{by Lemma 756 RL with } \{x2 \leftarrow x1, x1 \leftarrow i(x2)\} \\
& \overbrace{\underbrace{asoc(i(x2), i(x1), i(x2))}} \\
= \quad & \text{by Lemma 613 RL with } \{x2 \leftarrow i(x1), x1 \leftarrow x2\} \\
& \overbrace{\underbrace{asoc(x2, i(x1), x2)}} \\
= \quad & \text{by Lemma 756 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \overbrace{asoc(x1, x2, x2)}
\end{aligned}
$$

Lemma 773: $op_l(x1, x1, x2) = rd(x1, asoc(x2, x1, x1))$

$$
\begin{aligned}
& \underbrace{op_l(x1, x1, x2)} \\
= \quad & \text{by Lemma 607 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, i(x2), x1)}} \\
= \quad & \text{by Lemma 620 RL with } \{x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& \overbrace{rd(x1, \underbrace{asoc(x1, i(x2), x1)})} \\
= \quad & \text{by Lemma 756 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& rd(x1, \overbrace{asoc(x2, x1, x1)})
\end{aligned}
$$

Lemma 774: $asoc(rd(x1, x2), x2, x2) = asoc(x1, x2, x2)$

$$
\begin{aligned}
& \underbrace{asoc(rd(x1, x2), x2, x2)} \\
= \quad & \text{by Lemma 757 RL with } \{x2 \leftarrow rd(x1, x2), x1 \leftarrow x2\} \\
& \overbrace{i(\underbrace{asoc(x2, rd(x1, x2), x2)})} \\
= \quad & \text{by Lemma 263 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \underbrace{i(\overbrace{asoc(x2, x1, x2)})} \\
= \quad & \text{by Lemma 757 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \overbrace{asoc(x1, x2, x2)}
\end{aligned}
$$

Lemma 775: $rd(x1, asoc(x1, x2, x2)) = op_r(x1, x2, i(x2))$

$$
\begin{aligned}
& rd(x1, \underbrace{asoc(x1, x2, x2)}) \\
= \quad & \text{by Lemma 756 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \underbrace{rd(x1, \overbrace{asoc(x2, i(x1), x2)})} \\
= \quad & \text{by Lemma 546 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_r(x1, rd(i(x1), x2), x2)}} \\
= \quad & \text{by Lemma 653 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, x2, i(x2))}
\end{aligned}
$$

Lemma 776: $op_l(i(x1), x1, x2) = rd(asoc(x2, x1, x1), x1)$

$$
\begin{aligned}
& \underbrace{op_l(i(x1), x1, x2)} \\
= & \quad \text{by Lemma 634 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(i(x1), i(x2), x1)}} \\
= & \quad \text{by Lemma 627 RL with } \{x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& \overbrace{rd(\underbrace{asoc(x1, i(x2), x1)}, x1)} \\
= & \quad \text{by Lemma 756 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& rd(\overbrace{asoc(x2, x1, x1)}, x1)
\end{aligned}
$$

Lemma 777: $rd(x1, op_r(x1, x2, i(x2))) = asoc(x1, x2, x2)$

$$
\begin{aligned}
& \underbrace{rd(x1, op_r(x1, x2, i(x2)))} \\
= & \quad \text{by Lemma 645 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{asoc(x2, i(x1), x2)}} \\
= & \quad \text{by Lemma 756 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \overbrace{asoc(x1, x2, x2)}
\end{aligned}
$$

Lemma 778: $mult(x1, op_l(i(x1), x1, x2)) = asoc(x2, x1, x1)$

$$
\begin{aligned}
& mult(x1, \underbrace{op_l(i(x1), x1, x2)}) \\
= & \quad \text{by Lemma 430 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{mult(x1, \overbrace{i(op_l(x1, x1, x2))})} \\
= & \quad \text{by Lemma 176 RL with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{rd(x1, op_l(x1, x1, x2))}} \\
= & \quad \text{by Lemma 174 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{i(asoc(x1, x2, x1))}} \\
= & \quad \text{by Lemma 612 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{asoc(x1, i(x2), x1)}} \\
= & \quad \text{by Lemma 756 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{asoc(x2, x1, x1)}
\end{aligned}
$$

Lemma 779: $asoc(mult(x1, i(x2)), x2, x2) = asoc(x1, x2, x2)$

$$
\begin{aligned}
& \underbrace{asoc(mult(x1, i(x2)), x2, x2)} \\
= \quad & \text{by Lemma 757 RL with } \{x2 \leftarrow mult(x1, i(x2)), x1 \leftarrow x2\} \\
& \overbrace{i(\underbrace{asoc(x2, \underbrace{mult(x1, i(x2))}, x2)})} \\
= \quad & \text{by Lemma 3 RL with } \{x1 \leftarrow mult(x1, i(x2))\} \\
& i(\underbrace{asoc(x2, \overbrace{i(i(mult(x1, i(x2))))}, x2)}) \\
= \quad & \text{by Lemma 498 RL with } \{x2 \leftarrow i(mult(x1, i(x2))), x1 \leftarrow x2\} \\
& i(\overbrace{asoc(x2, \underbrace{rd(x2, i(mult(x1, i(x2))))}, x2)}) \\
= \quad & \text{by Lemma 38 LR with } \{x1 \leftarrow mult(x1, i(x2)), x2 \leftarrow x2\} \\
& i(asoc(x2, \overbrace{\underbrace{rd(mult(x1, i(x2)), i(x2))}}, x2)) \\
= \quad & \text{by Axiom 5 RL with } \{x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& \underbrace{i(asoc(x2, \overbrace{x1}, x2))} \\
= \quad & \text{by Lemma 757 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \overbrace{asoc(x1, x2, x2)}
\end{aligned}
$$

Lemma 780: $asoc(mult(i(x1), x2), x1, x1) = i(asoc(x1, x2, x1))$

$$
\begin{aligned}
& \underbrace{asoc(mult(i(x1), x2), x1, x1)} \\
= \quad & \text{by Lemma 757 RL with } \{x2 \leftarrow mult(i(x1), x2), x1 \leftarrow x1\} \\
& \overbrace{i(\underbrace{asoc(x1, mult(i(x1), x2), x1)})} \\
= \quad & \text{by Lemma 504 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& i(\overbrace{\underbrace{asoc(x2, x1, i(x1))}}) \\
= \quad & \text{by Lemma 611 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& i(\overbrace{asoc(x1, x2, x1)})
\end{aligned}
$$

Lemma 781: $asoc(op_t(x1, i(x2)), x2, x2) = i(asoc(x2, x1, x2))$

$$
\begin{aligned}
& \underbrace{asoc(op_t(x1, i(x2)), x2, x2)} \\
= \quad & \text{by Lemma 757 RL with } \{x2 \leftarrow op_t(x1, i(x2)), x1 \leftarrow x2\} \\
& \overbrace{i(\underbrace{asoc(x2, op_t(x1, i(x2)), x2)})} \\
= \quad & \text{by Lemma 516 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& i(\overbrace{\underbrace{asoc(x2, mult(x2, x1), x2)}}) \\
= \quad & \text{by Lemma 618 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& i(\overbrace{asoc(x2, x1, x2)})
\end{aligned}
$$

Lemma 782: $asoc(rd(i(x1),x2),x1,x1) = asoc(x1,x2,x1)$

$$
\begin{aligned}
& \underbrace{asoc(rd(i(x1),x2),x1,x1)} \\
= \quad & \text{by Lemma 763 RL with } \{x2 \leftarrow x1, x1 \leftarrow rd(i(x1),x2)\} \\
& \overbrace{asoc(\underbrace{mult(rd(i(x1),x2),x1)},x1,x1)} \\
= \quad & \text{by Lemma 21 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{asoc(\overbrace{i(x2)},x1,x1)} \\
= \quad & \text{by Lemma 758 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \overbrace{asoc(x1,x2,x1)}
\end{aligned}
$$

Lemma 783: $op_l(i(x1),i(x2),x1) = mult(i(x1),asoc(x2,x1,x1))$

$$
\begin{aligned}
& \underbrace{op_l(i(x1),i(x2),x1)} \\
= \quad & \text{by Lemma 627 RL with } \{x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& \underbrace{\overbrace{rd(asoc(x1,i(x2),x1),x1)}} \\
= \quad & \text{by Lemma 185 RL with } \{x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& \overbrace{mult(i(x1),\underbrace{asoc(x1,i(x2),x1)})} \\
= \quad & \text{by Lemma 756 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& mult(i(x1),\overbrace{asoc(x2,x1,x1)})
\end{aligned}
$$

Lemma 784: $rd(i(x1),asoc(x2,x3,x3)) = rd(asoc(x3,x2,x3),x1)$

$$
\begin{aligned}
& rd(i(x1),\underbrace{asoc(x2,x3,x3)}) \\
= \quad & \text{by Lemma 757 RL with } \{x2 \leftarrow x2, x1 \leftarrow x3\} \\
& \underbrace{rd(i(x1),\overbrace{i(asoc(x3,x2,x3))})} \\
= \quad & \text{by Lemma 23 LR with } \{x2 \leftarrow asoc(x3,x2,x3), x1 \leftarrow x1\} \\
& \overbrace{rd(asoc(x3,x2,x3),x1)}
\end{aligned}
$$

Lemma 785: $rd(i(x1),asoc(x2,x3,x2)) = rd(asoc(x3,x2,x2),x1)$

$$
\begin{aligned}
& \underbrace{rd(i(x1),asoc(x2,x3,x2))} \\
= \quad & \text{by Lemma 37 RL with } \{x2 \leftarrow x1, x1 \leftarrow asoc(x2,x3,x2)\} \\
& \overbrace{rd(\underbrace{i(asoc(x2,x3,x2))},x1)} \\
= \quad & \text{by Lemma 757 LR with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& rd(\overbrace{asoc(x3,x2,x2)},x1)
\end{aligned}
$$

Lemma 786: $mult(x1, op_r(i(x1), x2, i(x2))) = asoc(x1, x2, x2)$

$$
\begin{aligned}
& \underbrace{mult(x1, op_r(i(x1), x2, i(x2)))} \\
= \quad & \text{by Lemma 210 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{asoc(i(x1), x2, i(x2))}} \\
= \quad & \text{by Lemma 611 LR with } \{x2 \leftarrow x2, x1 \leftarrow i(x1)\} \\
& \overbrace{\underbrace{asoc(x2, i(x1), x2)}} \\
= \quad & \text{by Lemma 756 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \overbrace{asoc(x1, x2, x2)}
\end{aligned}
$$

Lemma 787: $mult(i(x2), x1) = mult(rd(x1, x2), asoc(x2, x1, x1))$

$$
\begin{aligned}
& \underbrace{mult(i(x2), x1)} \\
= \quad & \text{by Lemma 330 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_r(rd(x1, x2), x1, i(x1))}} \\
= \quad & \text{by Lemma 626 RL with } \{x2 \leftarrow x1, x1 \leftarrow rd(x1, x2)\} \\
& \overbrace{mult(rd(x1, x2), \underbrace{asoc(x1, rd(x1, x2), x1)})} \\
= \quad & \text{by Lemma 762 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& mult(rd(x1, x2), \overbrace{asoc(x2, x1, x1)})
\end{aligned}
$$

Lemma 788: $mult(i(x2), x1) = mult(asoc(x2, x1, x1), rd(x1, x2))$

$$
\begin{aligned}
& \underbrace{mult(i(x2), x1)} \\
= \quad & \text{by Lemma 330 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_r(rd(x1, x2), x1, i(x1))}} \\
= \quad & \text{by Lemma 628 RL with } \{x2 \leftarrow rd(x1, x2), x1 \leftarrow x1\} \\
& \overbrace{mult(\underbrace{asoc(x1, rd(x1, x2), x1)}, rd(x1, x2))} \\
= \quad & \text{by Lemma 762 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& mult(\overbrace{asoc(x2, x1, x1)}, rd(x1, x2))
\end{aligned}
$$

Lemma 789: $asoc(x1, x2, x2) = rd(rd(x2, i(x1)), mult(x1, x2))$

$$
\begin{aligned}
& \underbrace{asoc(x1, x2, x2)} \\
= \quad & \text{by Lemma 763 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{asoc(mult(x1, x2), x2, x2)}} \\
= \quad & \text{by Lemma 766 RL with } \{x2 \leftarrow x2, x1 \leftarrow mult(x1, x2)\} \\
& \overbrace{rd(\underbrace{op_r(mult(x1, x2), x2, x2)}, mult(x1, x2))} \\
= \quad & \text{by Lemma 359 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& rd(\overbrace{rd(x2, i(x1))}, mult(x1, x2))
\end{aligned}
$$

Lemma 790: $mult(i(x1), x2) = rd(asoc(x1, x2, x2), rd(x1, x2))$

$\underbrace{mult(i(x1), x2)}$

$=$ by Lemma 330 RL with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$\overbrace{op_r(\underbrace{rd(x2, x1)}, x2, i(x2))}$

$=$ by Lemma 10 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{op_r(\overbrace{i(rd(x1, x2))}, x2, i(x2))}$

$=$ by Lemma 211 LR with $\{x2 \leftarrow x2, x1 \leftarrow rd(x1, x2)\}$

$\overbrace{rd(asoc(i(rd(x1, x2)), x2, i(x2)), op_t(rd(x1, x2), \underbrace{op_r(i(rd(x1, x2)), x2, i(x2))})}$

$=$ by Lemma 92 RL with $\{x2 \leftarrow x2, x1 \leftarrow i(rd(x1, x2))\}$

$rd(asoc(i(rd(x1, x2)), x2, i(x2)), op_t(rd(x1, x2), \overbrace{mult(\underbrace{mult(i(rd(x1, x2)), x2)}, i(x2))}))$

$=$ by Lemma 17 RL with $\{x2 \leftarrow x2, x1 \leftarrow rd(x1, x2)\}$

$rd(asoc(i(rd(x1, x2)), x2, i(x2)), op_t(\underbrace{rd(x1, x2)}, mult(\overbrace{i(mult(rd(x1, x2), i(x2)))}, i(x2))))$

$=$ by Axiom 5 LR with $\{x2 \leftarrow i(x2), x1 \leftarrow rd(x1, x2)\}$

$rd(asoc(i(rd(x1, x2)), x2, i(x2)), \underbrace{op_t(\overbrace{rd(mult(rd(x1, x2), i(x2)), i(x2))}, mult(i(mult(rd(x1, x2), i(x2))), i(x2)))})$

$=$ by Lemma 53 RL with $\{x2 \leftarrow i(x2), x1 \leftarrow mult(rd(x1, x2), i(x2))\}$

$rd(asoc(i(rd(x1, x2)), x2, i(x2)), \underbrace{\overbrace{rd(mult(rd(x1, x2), i(x2)), i(x2))})$

$=$ by Axiom 5 RL with $\{x2 \leftarrow i(x2), x1 \leftarrow rd(x1, x2)\}$

$rd(\underbrace{asoc(i(rd(x1, x2)), x2, i(x2))}, \overbrace{rd(x1, x2)})$

$=$ by Lemma 611 LR with $\{x2 \leftarrow x2, x1 \leftarrow i(rd(x1, x2))\}$

$rd(\overbrace{\underbrace{asoc(x2, i(rd(x1, x2)), x2)}}, rd(x1, x2))$

$=$ by Lemma 756 LR with $\{x2 \leftarrow rd(x1, x2), x1 \leftarrow x2\}$

$rd(\overbrace{\underbrace{asoc(rd(x1, x2), x2, x2)}}, rd(x1, x2))$

$=$ by Lemma 774 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$rd(\overbrace{asoc(x1, x2, x2)}, rd(x1, x2))$

Lemma 791: $op_r(x1, i(i(x2)), mult(i(x2), rd(x2, mult(x1, i(i(x2))))))) = rd(asoc(x2, mult(x1, i(i(x2))), mult(x1, i(i(x2)))), rd(x2$

$\overbrace{op_r(x1, i(i(x2)), mult(i(x2), rd(x2, mult(x1, i(i(x2))))))}$

$=$ by Lemma 665 RL with $\{x3 \leftarrow rd(x2, mult(x1, i(i(x2)))), x2 \leftarrow i(x2), x1 \leftarrow x1\}$

$\overbrace{rd(mult(mult(x1, i(i(x2))), \underbrace{mult(i(x2), rd(x2, mult(x1, i(i(x2)))))}), rd(x2, mult(x1, i(i(x2)))))}$

$=$ by Lemma 56 RL with $\{x2 \leftarrow mult(x1, i(i(x2))), x1 \leftarrow x2\}$

$rd(mult(mult(x1, i(i(x2))), \overbrace{mult(\underbrace{i(mult(x2, mult(x1, i(i(x2)))))}, x2)}), rd(x2, mult(x1, i(i(x2)))))$

$=$ by Axiom 11 RL with $\{x2 \leftarrow mult(x1, i(i(x2))), x1 \leftarrow x2\}$

$rd(mult(mult(x1, i(i(x2))), mult(\overbrace{mult(\underbrace{i(x2)}, i(mult(x1, i(i(x2)))))}, x2)), rd(x2, mult(x1, i(i(x2)))))$

$=$ by Lemma 5 RL with $\{x1 \leftarrow x2\}$

$rd(mult(mult(x1, i(i(x2))), mult(mult(\overbrace{rd(\underbrace{unit()}, x2)}, i(mult(x1, i(i(x2)))))), x2)), rd(x2, mult(x1, i(i(x2)))))$

$=$ by Axiom 7 RL with $\{x1 \leftarrow mult(x1, i(i(x2)))\}$

$rd(mult(mult(x1, i(i(x2))), mult(\underbrace{mult(rd(\overbrace{mult(mult(x1, i(i(x2))), i(mult(x1, i(i(x2)))))}, x2), i(mult(x1, i(i(x2)))))}, x2)),$

$=$ by Lemma 44 RL with $\{x3 \leftarrow mult(x1, i(i(x2))), x2 \leftarrow mult(mult(x1, i(i(x2))), i(mult(x1, i(i(x2))))), x1 \leftarrow x2\}$

$rd(mult(mult(x1, i(i(x2))), mult(\overbrace{i(mult(rd(x2, mult(mult(x1, i(i(x2))), i(mult(x1, i(i(x2)))))), mult(x1, i(i(x2)))))}, \underbrace{x2}))$

$=$ by Axiom 4 LR with $\{x2 \leftarrow mult(mult(x1, i(i(x2))), i(mult(x1, i(i(x2))))), x1 \leftarrow x2\}$

$rd(mult(mult(x1, i(i(x2))), \underbrace{mult(i(mult(rd(x2, mult(mult(x1, i(i(x2))), i(mult(x1, i(i(x2)))))), mult(x1, i(i(x2)))))}, \overbrace{mult($

$=$ by Axiom 12 LR with $\{x3 \leftarrow i(mult(x1, i(i(x2)))), x2 \leftarrow mult(x1, i(i(x2))), x1 \leftarrow rd(x2, mult(mult(x1, i(i(x2))), i(mult(x1,$

$rd(\underbrace{mult(mult(x1, i(i(x2))), \overbrace{op_l(i(mult(x1, i(i(x2)))), mult(x1, i(i(x2))), rd(x2, mult(mult(x1, i(i(x2))), i(mult(x1, i(i(x2)))}$

$=$ by Lemma 778 LR with $\{x2 \leftarrow rd(x2, mult(mult(x1, i(i(x2))), i(mult(x1, i(i(x2)))))), x1 \leftarrow mult(x1, i(i(x2)))\}$

$rd(\overbrace{asoc(rd(x2, \underbrace{mult(mult(x1, i(i(x2))), i(mult(x1, i(i(x2)))))}), mult(x1, i(i(x2))), mult(x1, i(i(x2))))}, rd(x2, mult(x1, i(i(x$

$=$ by Axiom 7 LR with $\{x1 \leftarrow mult(x1, i(i(x2)))\}$

$rd(asoc(\underbrace{rd(x2, \overbrace{unit()})}, mult(x1, i(i(x2))), mult(x1, i(i(x2)))), rd(x2, mult(x1, i(i(x2)))))$

$=$ by Lemma 1 LR with $\{x1 \leftarrow x2\}$

$rd(asoc(\overbrace{x2}, mult(x1, i(i(x2))), mult(x1, i(i(x2)))), rd(x2, mult(x1, i(i(x2)))))$

Lemma 792: $op_r(x1, x2, mult(i(x2), rd(x2, mult(x1, x2)))) = rd(asoc(x2, mult(x1, i(i(x2))), mult(x1, i(i(x2)))), rd(x2, mult(x1,$

$op_r(x1, x2, mult(i(x2), rd(x2, mult(x1, \underbrace{x2}))))$

$=$ by Lemma 3 RL with $\{x1 \leftarrow x2\}$

$op_r(x1, \underbrace{x2}, mult(i(x2), rd(x2, mult(x1, \overbrace{i(i(x2))}))))$

$=$ by Lemma 3 RL with $\{x1 \leftarrow x2\}$

$\underbrace{op_r(x1, \overbrace{i(i(x2))}, mult(i(x2), rd(x2, mult(x1, i(i(x2))))))}$

$=$ by Lemma 791 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{rd(asoc(x2, mult(x1, i(i(x2))), mult(x1, i(i(x2)))), rd(x2, mult(x1, i(i(x2)))))}$

Lemma 793: $rd(op_t(x1,x2),i(asoc(x1,x2,x2))) = mult(op_t(x1,x2),asoc(x1,x2,x2))$

$\overbrace{rd(op_t(x1,x2),i(asoc(x1,x2,x2)))}$

$=$ by Lemma 616 RL with $\{x2 \leftarrow asoc(x1,x2,x2), x1 \leftarrow op_t(x1,x2)\}$

$\overbrace{mult(op_t(x1,x2),op_l(asoc(x1,x2,x2),op_t(x1,x2),\underbrace{asoc(x1,x2,x2)}))}$

$=$ by Lemma 769 RL with $\{x1 \leftarrow x2, x2 \leftarrow x1\}$

$mult(op_t(x1,x2),\underbrace{op_l(asoc(x1,x2,x2),op_t(x1,x2),\overbrace{asoc(op_t(x1,x2),x2,x2)})})$

$=$ by Lemma 760 LR with $\{x3 \leftarrow x2, x2 \leftarrow op_t(x1,x2), x1 \leftarrow asoc(x1,x2,x2)\}$

$mult(op_t(x1,x2),\overbrace{asoc(x1,x2,x2)})$

Lemma 794: $asoc(x1,x2,x2) = rd(mult(i(x1),x2),rd(x2,x1))$

$\underbrace{asoc(x1,x2,x2)}$

$=$ by Lemma 29 RL with $\{x2 \leftarrow asoc(x1,x2,x2), x1 \leftarrow rd(x2,x1)\}$

$\overbrace{rd(\underbrace{mult(rd(x2,x1),asoc(x1,x2,x2))},op_t(rd(x2,x1),asoc(x1,x2,x2)))}$

$=$ by Lemma 787 RL with $\{x1 \leftarrow x2, x2 \leftarrow x1\}$

$rd(\overbrace{mult(i(x1),x2)},op_t(rd(x2,x1),\underbrace{asoc(x1,x2,x2)}))$

$=$ by Lemma 762 RL with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$rd(mult(i(x1),x2),\underbrace{op_t(rd(x2,x1),\overbrace{asoc(x2,rd(x2,x1),x2)})})$

$=$ by Lemma 535 LR with $\{x2 \leftarrow x2, x1 \leftarrow rd(x2,x1)\}$

$rd(mult(i(x1),x2),\overbrace{rd(x2,x1)})$

Lemma 795: $rd(op_l(x1,x2,x1),op_t(x1,x2)) = asoc(x1,x2,x2)$

$rd(op_l(x1,x2,x1),op_t(\underbrace{x1},x2))$

$=$ by Lemma 539 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$rd(op_l(x1,x2,x1),op_t(\overbrace{op_t(x1,\underbrace{asoc(x2,i(x1),x2)})},x2))$

$=$ by Lemma 756 LR with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$rd(op_l(x1,x2,x1),\underbrace{op_t(op_t(x1,\overbrace{asoc(x1,x2,x2)}),x2)})$

$=$ by Axiom 13 LR with $\{x3 \leftarrow x2, x2 \leftarrow asoc(x1,x2,x2), x1 \leftarrow x1\}$

$rd(\underbrace{op_l(x1,x2,x1)},\overbrace{op_t(op_t(x1,x2),asoc(x1,x2,x2))})$

$=$ by Lemma 735 LR with $\{x1 \leftarrow x2, x2 \leftarrow x1\}$

$rd(\overbrace{rd(op_t(x1,x2),\underbrace{asoc(x2,x1,x2)})},op_t(op_t(x1,x2),asoc(x1,x2,x2)))$

$=$ by Lemma 759 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$rd(\underbrace{rd(op_t(x1,x2),\overbrace{i(asoc(x1,x2,x2))})},op_t(op_t(x1,x2),asoc(x1,x2,x2)))$

$=$ by Lemma 793 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{rd(\overbrace{mult(op_t(x1,x2),asoc(x1,x2,x2))},op_t(op_t(x1,x2),asoc(x1,x2,x2)))}$

$=$ by Lemma 29 LR with $\{x2 \leftarrow asoc(x1,x2,x2), x1 \leftarrow op_t(x1,x2)\}$

$\overbrace{asoc(x1,x2,x2)}$

Lemma 796: $i(x1) = rd(asoc(x2, x1, x1), op_l(x1, x1, i(x2)))$

$$
\begin{aligned}
& \underbrace{i(x1)} \\
= \quad & \text{by Lemma 9 RL with } \{x2 \leftarrow x1, x1 \leftarrow asoc(x1, i(x2), x1)\} \\
& \overbrace{rd(asoc(x1, i(x2), x1), \underbrace{mult(x1, asoc(x1, i(x2), x1))})} \\
= \quad & \text{by Lemma 158 LR with } \{x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& rd(\underbrace{asoc(x1, i(x2), x1)}, \overbrace{op_l(x1, x1, i(x2))}) \\
= \quad & \text{by Lemma 756 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& rd(\overbrace{asoc(x2, x1, x1)}, op_l(x1, x1, i(x2)))
\end{aligned}
$$

Lemma 797: $rd(asoc(x1, x2, x2), op_l(x2, x1, x2)) = i(x2)$

$$
\begin{aligned}
& rd(asoc(x1, x2, x2), \underbrace{op_l(x2, x1, x2)}) \\
= \quad & \text{by Lemma 608 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \underbrace{rd(asoc(x1, x2, x2), \overbrace{op_l(x2, x2, i(x1))})} \\
= \quad & \text{by Lemma 796 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \overbrace{i(x2)}
\end{aligned}
$$

Lemma 798: $op_l(x1, asoc(x2, x3, x3), x3) = op_l(x1, x3, asoc(x3, x2, x3))$

$$
\begin{aligned}
& op_l(x1, asoc(x2, x3, x3), \underbrace{x3}) \\
= \quad & \text{by Lemma 228 RL with } \{x2 \leftarrow i(x2), x1 \leftarrow x3\} \\
& op_l(x1, asoc(x2, x3, x3), \overbrace{op_l(x3, \underbrace{asoc(x3, i(x2), x3)}, x3)}) \\
= \quad & \text{by Lemma 756 LR with } \{x2 \leftarrow x2, x1 \leftarrow x3\} \\
& \underbrace{op_l(x1, asoc(x2, x3, x3), op_l(x3, \overbrace{asoc(x2, x3, x3)}, x3))} \\
= \quad & \text{by Lemma 651 RL with } \{x3 \leftarrow asoc(x2, x3, x3), x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x3, \underbrace{i(asoc(x2, x3, x3))})} \\
= \quad & \text{by Lemma 759 LR with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& op_l(x1, x3, \overbrace{asoc(x3, x2, x3)})
\end{aligned}
$$

Lemma 799: $op_l(x1, x2, asoc(x3, x2, x2)) = op_l(x1, asoc(x2, x3, x2), x2)$

$$
\begin{aligned}
& \underbrace{op_l(x1, x2, asoc(x3, x2, x2))} \\
= \quad & \text{by Lemma 483 LR with } \{x3 \leftarrow asoc(x3, x2, x2), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, i(asoc(x3, x2, x2)), \underbrace{op_l(x2, x2, asoc(x3, x2, x2))})} \\
= \quad & \text{by Lemma 761 RL with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& op_l(x1, \underbrace{i(asoc(x3, x2, x2))}, \overbrace{x2}) \\
= \quad & \text{by Lemma 759 LR with } \{x2 \leftarrow x2, x1 \leftarrow x3\} \\
& op_l(x1, \overbrace{asoc(x2, x3, x2)}, x2)
\end{aligned}
$$

Lemma 800: $op_r(x1, mult(x2, x1), mult(x2, x1)) = rd(mult(x2, x1), x2)$

$$
\begin{aligned}
& \underbrace{op_r(x1, mult(x2, x1), mult(x2, x1))} \\
= \quad & \text{by Lemma 353 RL with } \{x2 \leftarrow x1, x1 \leftarrow mult(x2, x1)\} \\
& \overbrace{rd(mult(x2, x1), \underbrace{rd(mult(x2, x1), x1)})} \\
= \quad & \text{by Axiom 5 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& rd(mult(x2, x1), \overbrace{x2}^{})
\end{aligned}
$$

Lemma 801: $op_r(mult(x1, i(rd(x1, x2))), x1, x1) = rd(mult(x1, x2), x1)$

$$
\begin{aligned}
& op_r(\underbrace{mult(x1, i(rd(x1, x2)))}, x1, x1) \\
= \quad & \text{by Lemma 16 RL with } \{x2 \leftarrow rd(x1, x2), x1 \leftarrow x1\} \\
& \underbrace{op_r(\overbrace{i(mult(i(x1), rd(x1, x2)))}, x1, x1)} \\
= \quad & \text{by Lemma 354 RL with } \{x2 \leftarrow mult(i(x1), rd(x1, x2)), x1 \leftarrow x1\} \\
& \overbrace{rd(\underbrace{rd(x1, mult(i(x1), rd(x1, x2)))}, x1)} \\
= \quad & \text{by Lemma 57 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& rd(\overbrace{mult(x1, x2)}, x1)
\end{aligned}
$$

Lemma 802: $op_r(mult(x1, rd(x2, x1)), x1, x1) = rd(mult(x1, x2), x1)$

$$
\begin{aligned}
& op_r(mult(x1, \underbrace{rd(x2, x1)}), x1, x1) \\
= \quad & \text{by Lemma 10 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{op_r(mult(x1, \overbrace{i(rd(x1, x2))}), x1, x1)} \\
= \quad & \text{by Lemma 801 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{rd(mult(x1, x2), x1)}
\end{aligned}
$$

Lemma 803: $op_t(op_t(x1, x2), x2) = mult(i(op_t(x2, x1)), mult(x1, x2))$

$$
\begin{aligned}
& \underbrace{op_t(op_t(x1, x2), x2)} \\
= \quad & \text{by Lemma 384 RL with } \{x2 \leftarrow op_t(x1, x2), x1 \leftarrow x2\} \\
& \overbrace{mult(i(op_t(x2, op_t(x1, x2))), \underbrace{mult(x2, op_t(x1, x2))})} \\
= \quad & \text{by Lemma 13 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& mult(i(\underbrace{op_t(x2, op_t(x1, x2))}), \overbrace{mult(x1, x2)}) \\
= \quad & \text{by Lemma 374 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& mult(i(\overbrace{op_t(x2, x1)}), mult(x1, x2))
\end{aligned}
$$

Lemma 804: $rd(x2, rd(x1, x2)) = mult(x2, i(rd(op_t(x1, i(x2)), x2)))$

$$
\begin{aligned}
& \underbrace{rd(x2, rd(x1, x2))} \\
= & \quad \text{by Lemma 10 RL with } \{x2 \leftarrow x2, x1 \leftarrow rd(x1, x2)\} \\
& \underbrace{\overbrace{i(rd(rd(x1, x2), x2))}} \\
= & \quad \text{by Lemma 20 RL with } \{x2 \leftarrow rd(rd(x1, x2), x2), x1 \leftarrow x2\} \\
& \overbrace{mult(x2, i(\underbrace{mult(x2, rd(rd(x1, x2), x2))}))} \\
= & \quad \text{by Lemma 417 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& mult(x2, i(\overbrace{rd(op_t(x1, i(x2)), x2)}))
\end{aligned}
$$

Lemma 805: $mult(x1, rd(x1, op_t(x2, i(x1)))) = rd(x1, rd(x2, x1))$

$$
\begin{aligned}
& mult(x1, \underbrace{rd(x1, op_t(x2, i(x1)))}) \\
= & \quad \text{by Lemma 10 RL with } \{x2 \leftarrow x1, x1 \leftarrow op_t(x2, i(x1))\} \\
& \underbrace{mult(x1, \overbrace{i(rd(op_t(x2, i(x1)), x1))})} \\
= & \quad \text{by Lemma 804 RL with } \{x1 \leftarrow x2, x2 \leftarrow x1\} \\
& \overbrace{rd(x1, rd(x2, x1))}
\end{aligned}
$$

Lemma 806: $rd(x1, op_r(x2, x1, x1)) = mult(x1, rd(x1, mult(i(i(x1)), x2)))$

$$
\begin{aligned}
& rd(x1, \underbrace{op_r(x2, x1, x1)}) \\
= & \quad \text{by Lemma 358 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \underbrace{rd(x1, \overbrace{rd(rd(x2, i(x1)), x1)})} \\
= & \quad \text{by Lemma 805 RL with } \{x2 \leftarrow rd(x2, i(x1)), x1 \leftarrow x1\} \\
& \overbrace{mult(x1, rd(x1, \underbrace{op_t(rd(x2, i(x1)), i(x1))}))} \\
= & \quad \text{by Lemma 14 LR with } \{x2 \leftarrow i(x1), x1 \leftarrow x2\} \\
& mult(x1, rd(x1, \overbrace{mult(i(i(x1)), x2)}))
\end{aligned}
$$

Lemma 807: $op_r(rd(x1, x2), x1, x1) = mult(x1, rd(x1, mult(x1, x2)))$

$$
\begin{aligned}
& \underbrace{op_r(rd(x1, x2), x1, x1)} \\
= & \quad \text{by Lemma 369 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{\overbrace{rd(x1, op_r(x2, x1, x1))}} \\
= & \quad \text{by Lemma 806 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{mult(x1, rd(x1, mult(\underbrace{i(i(x1))}, x2)))} \\
= & \quad \text{by Lemma 3 LR with } \{x1 \leftarrow x1\} \\
& mult(x1, rd(x1, mult(\overbrace{x1}, x2)))
\end{aligned}
$$

Lemma 808: $mult(op_r(rd(x1,x2),x1,x1),x2) = op_l(x1,x1,x2)$

$$
\begin{aligned}
& mult(op_r(rd(x1,x2),x1,x1),\underbrace{x2}) \\
= & \quad \text{by Axiom 2 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& mult(op_r(rd(x1,x2),x1,x1),\overbrace{mult(i(x1),mult(x1,x2))}) \\
= & \quad \text{by Lemma 14 RL with } \{x2 \leftarrow x1, x1 \leftarrow mult(x1,x2)\} \\
& mult(op_r(rd(x1,x2),x1,x1),\overbrace{op_t(\underbrace{rd(mult(x1,x2),x1)},x1)}) \\
= & \quad \text{by Lemma 10 RL with } \{x2 \leftarrow mult(x1,x2), x1 \leftarrow x1\} \\
& mult(\underbrace{op_r(rd(x1,x2),x1,x1)},op_t(\overbrace{i(rd(x1,mult(x1,x2)))},x1)) \\
= & \quad \text{by Lemma 807 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{mult(\overbrace{mult(x1,rd(x1,mult(x1,x2)))},op_t(i(rd(x1,mult(x1,x2))),x1))} \\
= & \quad \text{by Lemma 466 LR with } \{x2 \leftarrow rd(x1,mult(x1,x2)), x1 \leftarrow x1\} \\
& \underbrace{\overbrace{op_l(x1,rd(x1,mult(x1,x2)),x1)}} \\
= & \quad \text{by Lemma 622 LR with } \{x2 \leftarrow mult(x1,x2), x1 \leftarrow x1\} \\
& \underbrace{\overbrace{op_l(x1,x1,mult(x1,x2))}} \\
= & \quad \text{by Lemma 624 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1,x1,x2)}
\end{aligned}
$$

Lemma 809: $op_r(x1,i(x2),mult(x2,x1)) = op_t(x1,x2)$

$$
\begin{aligned}
& \underbrace{op_r(x1,i(x2),mult(x2,x1))} \\
= & \quad \text{by Axiom 16 LR with } \{x3 \leftarrow mult(x2,x1), x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& \overbrace{rd(\underbrace{mult(mult(x1,i(x2)),mult(x2,x1))},mult(i(x2),mult(x2,x1)))} \\
= & \quad \text{by Lemma 137 RL with } \{x3 \leftarrow mult(x2,x1), x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& rd(\overbrace{mult(x1,mult(i(x2),\underbrace{op_l(mult(x2,x1),x1,i(x2))}))},mult(i(x2),mult(x2,x1))) \\
= & \quad \text{by Lemma 422 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& rd(mult(x1,\underbrace{mult(i(x2),\overbrace{mult(x1,x2)})}),mult(i(x2),mult(x2,x1))) \\
= & \quad \text{by Axiom 10 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& rd(mult(x1,\overbrace{op_t(x1,x2)}),\underbrace{mult(i(x2),mult(x2,x1))}) \\
= & \quad \text{by Axiom 2 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& rd(\underbrace{mult(x1,op_t(x1,x2))},\overbrace{x1}) \\
= & \quad \text{by Lemma 27 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{rd(\overbrace{mult(op_t(x1,x2),x1)},x1)} \\
= & \quad \text{by Axiom 5 RL with } \{x2 \leftarrow x1, x1 \leftarrow op_t(x1,x2)\} \\
& \overbrace{op_t(x1,x2)}
\end{aligned}
$$

Lemma 810: $mult(mult(x1, x2), x1) = mult(x1, op_t(mult(x2, x1), i(x2)))$

$\underbrace{mult(mult(x1, x2), x1)}$

$=$ by Lemma 137 RL with $\{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{mult(x1, mult(x2, \underbrace{op_l(x1, x1, x2)}))}$

$=$ by Lemma 427 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$mult(x1, \underbrace{mult(x2, \overbrace{mult(x2, rd(x1, x2))})})$

$=$ by Lemma 70 LR with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$mult(x1, \overbrace{op_t(mult(x2, x1), i(x2))})$

Lemma 811: $rd(x1, op_t(rd(x2, i(x1)), i(x1))) = mult(x1, op_t(rd(i(x1), x2), i(x2)))$

$rd(x1, \underbrace{op_t(rd(x2, i(x1)), i(x1))})$

$=$ by Lemma 46 RL with $\{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow i(x1)\}$

$\underbrace{rd(x1, \overbrace{i(op_t(rd(i(x1), x2), x1))})}$

$=$ by Lemma 461 RL with $\{x2 \leftarrow rd(i(x1), x2), x1 \leftarrow x1\}$

$\overbrace{mult(x1, op_t(rd(i(x1), x2), \underbrace{mult(rd(i(x1), x2), x1)}))}$

$=$ by Lemma 21 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$mult(x1, op_t(rd(i(x1), x2), \overbrace{i(x2)}))$

Lemma 812: $rd(x1, mult(x1, x2)) = mult(x1, op_t(rd(i(x1), x2), i(x2)))$

$rd(x1, mult(\underbrace{x1}, x2))$

$=$ by Lemma 3 RL with $\{x1 \leftarrow x1\}$

$rd(x1, \underbrace{mult(\overbrace{i(i(x1))}, x2)})$

$=$ by Lemma 14 RL with $\{x2 \leftarrow i(x1), x1 \leftarrow x2\}$

$\underbrace{rd(x1, \overbrace{op_t(rd(x2, i(x1)), i(x1))})}$

$=$ by Lemma 811 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{mult(x1, op_t(rd(i(x1), x2), i(x2)))}$

Lemma 813: $i(op_r(i(x1), x2, x3)) = op_r(x1, i(x2), i(x3))$

$$
\begin{aligned}
& \underbrace{i(op_r(i(x1), x2, x3))} \\
= \quad & \text{by Lemma 674 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_r(\underbrace{x1}, mult(i(x1), x2), x3)} \\
= \quad & \text{by Lemma 3 RL with } \{x1 \leftarrow x1\} \\
& op_r(\overbrace{i(i(x1))}, mult(i(x1), x2), x3) \\
= \quad & \text{by Lemma 9 RL with } \{x2 \leftarrow i(x1), x1 \leftarrow x2\} \\
& \underbrace{op_r(\overbrace{rd(x2, mult(i(x1), x2))}, mult(i(x1), x2), x3)} \\
= \quad & \text{by Lemma 676 LR with } \{x3 \leftarrow x3, x2 \leftarrow mult(i(x1), x2), x1 \leftarrow x2\} \\
& \overbrace{rd(mult(x2, x3), mult(\underbrace{mult(i(x1), x2)}, x3))} \\
= \quad & \text{by Lemma 17 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{rd(mult(x2, x3), mult(\overbrace{i(mult(x1, i(x2)))}, x3))} \\
= \quad & \text{by Lemma 61 RL with } \{x3 \leftarrow mult(x2, x3), x2 \leftarrow x3, x1 \leftarrow mult(x1, i(x2))\} \\
& \overbrace{rd(mult(mult(x1, i(x2)), i(x3)), \underbrace{i(mult(x2, x3))})} \\
= \quad & \text{by Axiom 11 RL with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& \underbrace{rd(mult(mult(x1, i(x2)), i(x3)), \overbrace{mult(i(x2), i(x3))})} \\
= \quad & \text{by Axiom 16 RL with } \{x3 \leftarrow i(x3), x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, i(x2), i(x3))}
\end{aligned}
$$

Lemma 814: $i(op_r(x1, x2, i(x3))) = op_r(i(x1), i(x2), x3)$

$$
\begin{aligned}
& i(op_r(x1, \underbrace{x2}, i(x3))) \\
= \quad & \text{by Lemma 3 RL with } \{x1 \leftarrow x2\} \\
& i(\underbrace{op_r(x1, \overbrace{i(i(x2))}, i(x3))}) \\
= \quad & \text{by Axiom 16 LR with } \{x3 \leftarrow i(x3), x2 \leftarrow i(i(x2)), x1 \leftarrow x1\} \\
& i(\overbrace{rd(mult(mult(x1, i(i(x2))), i(x3)), \underbrace{mult(i(i(x2)), i(x3))})}) \\
= \quad & \text{by Axiom 11 LR with } \{x2 \leftarrow x3, x1 \leftarrow i(x2)\} \\
& i(\underbrace{rd(mult(mult(x1, i(i(x2))), i(x3)), \overbrace{i(mult(i(x2), x3))})}) \\
= \quad & \text{by Lemma 61 LR with } \{x3 \leftarrow mult(i(x2), x3), x2 \leftarrow x3, x1 \leftarrow mult(x1, i(i(x2)))\} \\
& i(\overbrace{rd(mult(i(x2), x3), mult(\underbrace{i(mult(x1, i(i(x2))))}, x3))}) \\
= \quad & \text{by Lemma 17 LR with } \{x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& \underbrace{i(rd(mult(i(x2), x3), mult(\overbrace{mult(i(x1), i(x2))}, x3)))} \\
= \quad & \text{by Lemma 10 LR with } \{x2 \leftarrow mult(mult(i(x1), i(x2)), x3), x1 \leftarrow mult(i(x2), x3)\} \\
& \overbrace{rd(mult(mult(i(x1), i(x2)), x3), mult(i(x2), x3))} \\
= \quad & \text{by Axiom 16 RL with } \{x3 \leftarrow x3, x2 \leftarrow i(x2), x1 \leftarrow i(x1)\} \\
& \overbrace{op_r(i(x1), i(x2), x3)}
\end{aligned}
$$

Lemma 815: $op_r(i(x1), i(x2), i(x3)) = i(op_r(x1, x2, x3))$

$$
\begin{aligned}
& \underbrace{op_r(i(x1), i(x2), i(x3))} \\
= \quad & \text{by Lemma 813 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow i(x1)\} \\
& \overbrace{i(op_r(\underbrace{i(i(x1))}, x2, x3))} \\
= \quad & \text{by Lemma 3 LR with } \{x1 \leftarrow x1\} \\
& i(op_r(\overbrace{x1}, x2, x3))
\end{aligned}
$$

Lemma 816: $i(op_r(x1, i(x2), x3)) = op_r(i(x1), x2, i(x3))$

$$
\begin{aligned}
& \underbrace{i(op_r(x1, i(x2), x3))} \\
= \quad & \text{by Lemma 815 RL with } \{x3 \leftarrow x3, x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& \overbrace{op_r(i(x1), \underbrace{i(i(x2))}, i(x3))} \\
= \quad & \text{by Lemma 3 LR with } \{x1 \leftarrow x2\} \\
& op_r(i(x1), \overbrace{x2}, i(x3))
\end{aligned}
$$

Lemma 817: $op_r(x1, i(mult(x1, x2)), x3) = i(i(op_r(x1, x2, i(x3))))$

$$
\begin{aligned}
& op_r(\underbrace{x1}, i(mult(x1, x2)), x3) \\
= \quad & \text{by Lemma 3 RL with } \{x1 \leftarrow x1\} \\
& \underbrace{op_r(\overbrace{i(i(x1))}, i(mult(x1, x2)), x3)} \\
= \quad & \text{by Lemma 814 RL with } \{x3 \leftarrow x3, x2 \leftarrow mult(x1, x2), x1 \leftarrow i(x1)\} \\
& \overbrace{i(\underbrace{op_r(i(x1), mult(x1, x2), i(x3))})} \\
= \quad & \text{by Lemma 654 LR with } \{x3 \leftarrow i(x3), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& i(\overbrace{i(op_r(x1, x2, i(x3)))})
\end{aligned}
$$

Lemma 818: $op_r(x1, i(mult(x1, x2)), x3) = op_r(x1, x2, i(x3))$

$$
\begin{aligned}
& \underbrace{op_r(x1, i(mult(x1, x2)), x3)} \\
= \quad & \text{by Lemma 817 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{i(i(op_r(x1, x2, i(x3))))}} \\
= \quad & \text{by Lemma 3 LR with } \{x1 \leftarrow op_r(x1, x2, i(x3))\} \\
& \overbrace{op_r(x1, x2, i(x3))}
\end{aligned}
$$

Lemma 819: $op_r(x1, mult(i(x1), x2), x3) = op_r(x1, i(x2), i(x3))$

$$
\begin{aligned}
& \underbrace{op_r(x1, mult(i(x1), x2), x3)} \\
= \quad & \text{by Lemma 674 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{i(op_r(i(x1), x2, x3))}} \\
= \quad & \text{by Lemma 813 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, i(x2), i(x3))}
\end{aligned}
$$

Lemma 820: $op_r(x1, i(mult(x2, x1)), x3) = op_r(x1, op_t(x2, x1), i(x3))$

$op_r(x1, i(mult(x2, x1)), x3)$

= by Lemma 13 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$op_r(x1, i(mult(x1, op_t(x2, x1))), x3)$

= by Lemma 818 LR with $\{x3 \leftarrow x3, x2 \leftarrow op_t(x2, x1), x1 \leftarrow x1\}$

$op_r(x1, op_t(x2, x1), i(x3))$

Lemma 821: $i(op_r(x1, x2, mult(x1, x2))) = rd(mult(i(x1), mult(x1, x2)), op_l(mult(x1, x2), x1, x2))$

$i(op_r(x1, x2, mult(x1, x2)))$

= by Lemma 654 RL with $\{x3 \leftarrow mult(x1, x2), x2 \leftarrow x2, x1 \leftarrow x1\}$

$op_r(i(x1), mult(x1, x2), mult(x1, x2))$

= by Lemma 356 RL with $\{x2 \leftarrow mult(x1, x2), x1 \leftarrow x1\}$

$i(op_r(x1, mult(x1, x2), mult(x1, x2)))$

= by Lemma 815 RL with $\{x3 \leftarrow mult(x1, x2), x2 \leftarrow mult(x1, x2), x1 \leftarrow x1\}$

$op_r(i(x1), i(mult(x1, x2)), i(mult(x1, x2)))$

= by Lemma 356 RL with $\{x2 \leftarrow i(mult(x1, x2)), x1 \leftarrow x1\}$

$i(op_r(x1, i(mult(x1, x2)), i(mult(x1, x2))))$

= by Lemma 520 LR with $\{x2 \leftarrow i(mult(x1, x2)), x1 \leftarrow x1\}$

$i(rd(op_t(op_r(mult(i(mult(x1, x2)), x1), i(mult(x1, x2)), i(mult(x1, x2))), i(mult(x1, x2))), op_l(i(mult(x1, x2)), i(mult(x1,$

= by Axiom 14 LR with $\{x4 \leftarrow i(mult(x1, x2)), x3 \leftarrow i(mult(x1, x2)), x2 \leftarrow i(mult(x1, x2)), x1 \leftarrow mult(i(mult(x1, x2)), x1)\}$

$i(rd(op_r(op_t(mult(i(mult(x1, x2)), x1), i(mult(x1, x2))), i(mult(x1, x2)), i(mult(x1, x2))), op_l(i(mult(x1, x2)), i(mult(x1,$

= by Lemma 385 LR with $\{x2 \leftarrow x1, x1 \leftarrow i(mult(x1, x2))\}$

$i(rd(mult(x1, i(mult(x1, x2))), op_l(i(mult(x1, x2)), i(mult(x1, x2)), x1)))$

= by Lemma 10 LR with $\{x2 \leftarrow op_l(i(mult(x1, x2)), i(mult(x1, x2)), x1), x1 \leftarrow mult(x1, i(mult(x1, x2)))\}$

$rd(op_l(i(mult(x1, x2)), i(mult(x1, x2)), x1), mult(x1, i(mult(x1, x2))))$

= by Lemma 247 LR with $\{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow i(mult(x1, x2))\}$

$rd(op_l(i(mult(x1, x2)), i(x1), i(x2)), mult(x1, i(mult(x1, x2))))$

= by Lemma 519 LR with $\{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow mult(x1, x2)\}$

$rd(i(op_l(mult(x1, x2), x1, x2)), mult(x1, i(mult(x1, x2))))$

= by Lemma 62 LR with $\{x3 \leftarrow mult(x1, x2), x2 \leftarrow x1, x1 \leftarrow op_l(mult(x1, x2), x1, x2)\}$

$rd(mult(i(x1), mult(x1, x2)), op_l(mult(x1, x2), x1, x2))$

Lemma 822: $op_r(x1, x2, mult(x1, x2)) = rd(mult(x2, x1), x2)$

$op_r(x1, x2, \underbrace{mult(x1, x2)})$

$=$ by Lemma 3 RL with $\{x1 \leftarrow mult(x1, x2)\}$

$op_r(x1, \underbrace{x2}, \overbrace{i(i(mult(x1, x2)))})$

$=$ by Lemma 3 RL with $\{x1 \leftarrow x2\}$

$\underbrace{op_r(x1, \overbrace{i(i(x2))}, i(i(mult(x1, x2))))}$

$=$ by Lemma 813 RL with $\{x3 \leftarrow i(mult(x1, x2)), x2 \leftarrow i(x2), x1 \leftarrow x1\}$

$\overbrace{i(op_r(i(x1), i(x2), \underbrace{i(mult(x1, x2))}))}$

$=$ by Axiom 11 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{i(op_r(i(x1), i(x2), \overbrace{mult(i(x1), i(x2))}))}$

$=$ by Lemma 821 LR with $\{x2 \leftarrow i(x2), x1 \leftarrow i(x1)\}$

$\overbrace{rd(\underbrace{mult(i(i(x1)), mult(i(x1), i(x2)))}, op_l(mult(i(x1), i(x2)), i(x1), i(x2)))}$

$=$ by Axiom 2 RL with $\{x2 \leftarrow i(x1), x1 \leftarrow i(x2)\}$

$rd(\overbrace{i(x2)}, \underbrace{op_l(mult(i(x1), i(x2)), i(x1), i(x2))})$

$=$ by Lemma 435 LR with $\{x2 \leftarrow i(x2), x1 \leftarrow i(x1)\}$

$\underbrace{rd(i(x2), \overbrace{mult(i(x2), i(x1))})}$

$=$ by Lemma 64 LR with $\{x3 \leftarrow i(x1), x2 \leftarrow x2, x1 \leftarrow x2\}$

$\overbrace{rd(mult(x2, \underbrace{i(i(x1))}), x2)}$

$=$ by Lemma 3 LR with $\{x1 \leftarrow x1\}$

$rd(mult(x2, \overbrace{x1}), x2)$

Lemma 823: $op_r(x1, mult(x1, x2), mult(x1, x2)) = rd(mult(x2, x1), x2)$

$\overbrace{op_r(x1, mult(x1, x2), mult(x1, x2))}$

$=$ by Lemma 361 RL with $\{x2 \leftarrow mult(x1, x2), x1 \leftarrow x1\}$

$\overbrace{op_r(x1, \overbrace{i(mult(x1, x2))}, \overbrace{i(mult(x1, x2))})}$

$=$ by Lemma 520 LR with $\{x2 \leftarrow i(mult(x1, x2)), x1 \leftarrow x1\}$

$\overbrace{rd(\underbrace{op_t(op_r(mult(\overbrace{i(mult(x1, x2))}, x1), \overbrace{i(mult(x1, x2))}, \overbrace{i(mult(x1, x2))}), \overbrace{i(mult(x1, x2))})}, op_l(i(mult(x1, x2)), i(mult(x1, x2$

$=$ by Axiom 14 LR with $\{x4 \leftarrow i(mult(x1, x2)), x3 \leftarrow i(mult(x1, x2)), x2 \leftarrow i(mult(x1, x2)), x1 \leftarrow mult(i(mult(x1, x2)), x1)\}$

$rd(\overbrace{\underbrace{op_r(op_t(mult(\overbrace{i(mult(x1, x2))}, x1), \overbrace{i(mult(x1, x2))}), \overbrace{i(mult(x1, x2))}, \overbrace{i(mult(x1, x2))})}}, op_l(i(mult(x1, x2)), i(mult(x1, x2$

$=$ by Lemma 385 LR with $\{x2 \leftarrow x1, x1 \leftarrow i(mult(x1, x2))\}$

$rd(\overbrace{\underbrace{mult(x1, \overbrace{i(mult(x1, x2))})}}, op_l(i(mult(x1, x2)), i(mult(x1, x2)), x1))$

$=$ by Lemma 20 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$rd(\overbrace{i(x2)}, \underbrace{op_l(i(mult(x1, x2)), i(mult(x1, x2)), x1)})$

$=$ by Lemma 443 LR with $\{x2 \leftarrow x1, x1 \leftarrow mult(x1, x2)\}$

$rd(i(x2), \overbrace{\underbrace{op_l(i(mult(x1, x2)), mult(x1, x2), i(x1))}})$

$=$ by Lemma 242 LR with $\{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow i(mult(x1, x2))\}$

$rd(i(x2), \overbrace{\underbrace{op_l(i(mult(x1, x2)), x1, x2)}})$

$=$ by Lemma 196 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{rd(i(x2), \overbrace{i(mult(x2, x1))})}$

$=$ by Lemma 23 LR with $\{x2 \leftarrow mult(x2, x1), x1 \leftarrow x2\}$

$\overbrace{rd(mult(x2, x1), x2)}$

Lemma 824: $op_t(rd(x1, i(x2)), x2) = op_t(rd(x1, i(x2)), op_t(x2, op_r(x1, x2, x2)))$

$\underbrace{op_t(rd(x1, i(x2)), x2)}$

$=$ by Lemma 549 RL with $\{x2 \leftarrow x2, x1 \leftarrow rd(x1, i(x2))\}$

$\overbrace{op_t(rd(x1, i(x2)), \underbrace{op_r(x2, rd(x1, i(x2)), i(rd(x1, i(x2))))})}$

$=$ by Lemma 547 RL with $\{x2 \leftarrow rd(x1, i(x2)), x1 \leftarrow x2\}$

$op_t(rd(x1, i(x2)), \overbrace{op_t(x2, \underbrace{rd(rd(x1, i(x2)), x2)})})$

$=$ by Lemma 358 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$op_t(rd(x1, i(x2)), op_t(x2, \overbrace{op_r(x1, x2, x2)}))$

Lemma 825: $mult(op_t(x1, x2), x2) = op_t(rd(x1, i(x2)), op_t(x2, x1))$

$$
\begin{aligned}
& \underbrace{mult(op_t(x1, x2), x2)} \\
= \quad & \text{by Lemma 337 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_t(rd(x1, i(x2)), x2)}} \\
= \quad & \text{by Lemma 824 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_t(rd(x1, i(x2)), \underbrace{op_t(x2, op_r(x1, x2, x2))})} \\
= \quad & \text{by Lemma 550 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& op_t(rd(x1, i(x2)), \overbrace{op_t(x2, x1)})
\end{aligned}
$$

Lemma 826: $op_t(x1, rd(x2, x1)) = op_t(x1, op_t(mult(i(x1), x2), x2))$

$$
\begin{aligned}
& \underbrace{op_t(x1, rd(x2, x1))} \\
= \quad & \text{by Lemma 375 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_t(x1, mult(i(x1), x2))}} \\
= \quad & \text{by Lemma 568 RL with } \{x2 \leftarrow mult(i(x1), x2), x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, op_t(mult(i(x1), x2), \underbrace{mult(x1, mult(i(x1), x2))}))} \\
= \quad & \text{by Lemma 12 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_t(x1, op_t(mult(i(x1), x2), \overbrace{x2}))
\end{aligned}
$$

Lemma 827: $op_l(i(x1), x1, x2) = rd(rd(x2, x1), op_t(x2, i(x1)))$

$$
\begin{aligned}
& \underbrace{op_l(i(x1), x1, x2)} \\
= \quad & \text{by Lemma 29 RL with } \{x2 \leftarrow op_l(i(x1), x1, x2), x1 \leftarrow x2\} \\
& \overbrace{rd(mult(x2, op_l(i(x1), x1, x2)), \underbrace{op_t(x2, op_l(i(x1), x1, x2))})} \\
= \quad & \text{by Lemma 572 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& rd(\underbrace{mult(x2, op_l(i(x1), x1, x2))}, \overbrace{op_t(x2, i(x1))}) \\
= \quad & \text{by Lemma 232 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& rd(\overbrace{rd(x2, x1)}, op_t(x2, i(x1)))
\end{aligned}
$$

Lemma 828: $op_t(mult(x1, x2), i(op_t(x1, x2))) = op_r(mult(x1, x2), i(x2), x2)$

$$
\begin{aligned}
& \underbrace{op_t(mult(x1, x2), i(op_t(x1, x2)))} \\
= \quad & \text{by Lemma 586 RL with } \{x2 \leftarrow op_t(x1, x2), x1 \leftarrow mult(x1, x2)\} \\
& \overbrace{op_r(mult(x1, x2), rd(op_t(x1, x2), mult(x1, x2)), \underbrace{rd(mult(x1, x2), op_t(x1, x2))})} \\
= \quad & \text{by Lemma 29 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_r(mult(x1, x2), \underbrace{rd(op_t(x1, x2), mult(x1, x2))}, \overbrace{x2}) \\
= \quad & \text{by Lemma 36 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_r(mult(x1, x2), \overbrace{i(x2)}, x2)
\end{aligned}
$$

Lemma 829: $op_t(x1, i(op_t(mult(x1, x2), x2))) = op_r(x1, x2, i(x2))$

$$
\begin{aligned}
& op_t(\underbrace{x1}, i(op_t(mult(x1, x2), x2))) \\
= & \quad \text{by Axiom 5 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{op_t(\overbrace{rd(mult(x1, x2), x2)}, i(op_t(mult(x1, x2), x2)))} \\
= & \quad \text{by Lemma 591 LR with } \{x2 \leftarrow x2, x1 \leftarrow mult(x1, x2)\} \\
& \underbrace{\overbrace{mult(mult(x1, x2), i(x2))}} \\
= & \quad \text{by Lemma 92 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, x2, i(x2))}
\end{aligned}
$$

Lemma 830: $mult(op_t(x1, i(x2)), rd(x2, x1)) = op_t(op_l(x2, x2, x1), op_t(i(x1), x2))$

$$
\begin{aligned}
& mult(\underbrace{op_t(x1, i(x2))}, rd(x2, x1)) \\
= & \quad \text{by Lemma 18 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{mult(\overbrace{i(op_t(i(x1), x2))}, rd(x2, x1))} \\
= & \quad \text{by Lemma 14 RL with } \{x2 \leftarrow op_t(i(x1), x2), x1 \leftarrow rd(x2, x1)\} \\
& \overbrace{op_t(\underbrace{rd(rd(x2, x1), op_t(i(x1), x2))}, op_t(i(x1), x2))} \\
= & \quad \text{by Lemma 594 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& op_t(\overbrace{op_l(x2, x2, x1)}, op_t(i(x1), x2))
\end{aligned}
$$

Lemma 831: $rd(x1, rd(op_t(i(x1), x2), x2)) = mult(x1, mult(op_r(x2, x1, i(x1)), x1))$

$$
\begin{aligned}
& rd(x1, \underbrace{rd(op_t(i(x1), x2), x2)}) \\
= & \quad \text{by Lemma 10 RL with } \{x2 \leftarrow op_t(i(x1), x2), x1 \leftarrow x2\} \\
& \underbrace{rd(x1, \overbrace{i(rd(x2, op_t(i(x1), x2)))})} \\
= & \quad \text{by Lemma 86 RL with } \{x2 \leftarrow rd(x2, op_t(i(x1), x2)), x1 \leftarrow x1\} \\
& \overbrace{mult(x1, mult(\underbrace{mult(i(x1), rd(x2, op_t(i(x1), x2)))}, x1))} \\
= & \quad \text{by Lemma 604 LR with } \{x2 \leftarrow x2, x1 \leftarrow i(x1)\} \\
& mult(x1, mult(\overbrace{op_r(x2, i(x1), \underbrace{i(i(x1))})}, x1)) \\
= & \quad \text{by Lemma 3 LR with } \{x1 \leftarrow x1\} \\
& mult(x1, mult(\underbrace{op_r(x2, i(x1), \overbrace{x1})}, x1)) \\
= & \quad \text{by Lemma 321 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& mult(x1, mult(\overbrace{op_r(x2, x1, i(x1))}, x1))
\end{aligned}
$$

Lemma 832: $rd(x1, rd(op_t(i(x1), x2), x2)) = mult(mult(x1, x2), x1)$

$$
\begin{aligned}
& \underbrace{rd(x1, rd(op_t(i(x1), x2), x2))} \\
= & \quad \text{by Lemma 831 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{mult(x1, \underbrace{mult(op_r(x2, x1, i(x1)), x1)})} \\
= & \quad \text{by Lemma 318 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \underbrace{mult(x1, \overbrace{op_t(mult(x1, x2), x1)})} \\
= & \quad \text{by Lemma 13 LR with } \{x2 \leftarrow mult(x1, x2), x1 \leftarrow x1\} \\
& \overbrace{mult(mult(x1, x2), x1)}
\end{aligned}
$$

Lemma 833: $mult(rd(x1, x2), i(op_t(x1, x2))) = rd(x1, mult(x1, x2))$

$$
\begin{aligned}
& mult(rd(x1, \underbrace{x2}), i(op_t(x1, x2))) \\
= & \quad \text{by Axiom 2 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& mult(rd(x1, \overbrace{mult(i(x1), mult(x1, x2))}), i(op_t(x1, x2))) \\
= & \quad \text{by Lemma 14 RL with } \{x2 \leftarrow x1, x1 \leftarrow mult(x1, x2)\} \\
& mult(rd(x1, \overbrace{op_t(\underbrace{rd(mult(x1, x2), x1)}, x1)}), i(op_t(x1, x2))) \\
= & \quad \text{by Lemma 10 RL with } \{x2 \leftarrow mult(x1, x2), x1 \leftarrow x1\} \\
& mult(\underbrace{rd(x1, op_t(\overbrace{i(rd(x1, mult(x1, x2)))}, x1))}, i(op_t(x1, x2))) \\
= & \quad \text{by Lemma 65 RL with } \{x3 \leftarrow x1, x2 \leftarrow x1, x1 \leftarrow rd(x1, mult(x1, x2))\} \\
& \overbrace{mult(\overbrace{rd(op_t(rd(x1, mult(x1, x2)), i(x1)), i(x1))}, \underbrace{i(op_t(x1, x2))})} \\
= & \quad \text{by Lemma 15 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& mult(rd(op_t(rd(x1, mult(x1, x2)), i(x1)), i(x1)), \underbrace{\overbrace{op_t(i(x1), i(x2))}}) \\
= & \quad \text{by Lemma 378 RL with } \{x2 \leftarrow i(x2), x1 \leftarrow i(x1)\} \\
& mult(rd(op_t(rd(x1, mult(x1, x2)), i(x1)), i(x1)), \overbrace{op_t(i(x1), rd(\underbrace{mult(i(x1), i(x2))}, i(x1)))}) \\
= & \quad \text{by Axiom 11 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& mult(rd(op_t(rd(x1, mult(x1, x2)), i(x1)), i(x1)), op_t(i(x1), \underbrace{rd(\overbrace{i(mult(x1, x2))}, i(x1))})) \\
= & \quad \text{by Lemma 23 LR with } \{x2 \leftarrow x1, x1 \leftarrow mult(x1, x2)\} \\
& \underbrace{mult(rd(op_t(rd(x1, mult(x1, x2)), i(x1)), i(x1)), op_t(i(x1), \overbrace{rd(x1, mult(x1, x2))}))} \\
= & \quad \text{by Lemma 605 LR with } \{x2 \leftarrow i(x1), x1 \leftarrow rd(x1, mult(x1, x2))\} \\
& \overbrace{rd(x1, mult(x1, x2))}
\end{aligned}
$$

Lemma 834: $op_t(mult(x2,x1),x1) = mult(op_t(x2,x1),op_l(x1,x1,x2))$

$$
\begin{aligned}
& op_t(\underbrace{mult(x2,x1)}, x1) \\
= & \quad \text{by Lemma 13 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{op_t(\overbrace{mult(x1,op_t(x2,x1))}, x1)} \\
= & \quad \text{by Lemma 264 RL with } \{x2 \leftarrow x1, x1 \leftarrow op_t(x2,x1)\} \\
& \overbrace{mult(op_t(x2,x1), \underbrace{op_l(x1,x1,op_t(x2,x1))})} \\
= & \quad \text{by Lemma 621 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& mult(op_t(x2,x1), \overbrace{op_l(x1,x1,x2)})
\end{aligned}
$$

Lemma 835: $i(op_l(x1,x2,x1)) = op_l(i(x1),rd(x1,x2),i(x2))$

$$
\begin{aligned}
& \underbrace{i(op_l(x1,x2,x1))} \\
= & \quad \text{by Lemma 519 RL with } \{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(i(x1),i(x2),i(x1))}} \\
= & \quad \text{by Lemma 625 RL with } \{x2 \leftarrow i(x2), x1 \leftarrow i(x1)\} \\
& \overbrace{op_l(i(x1), \underbrace{rd(i(x2),i(x1))}, i(x2))} \\
= & \quad \text{by Lemma 23 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& op_l(i(x1), \overbrace{rd(x1,x2)}, i(x2))
\end{aligned}
$$

Lemma 836: $asoc(mult(x1,x2),x2,mult(x1,x2)) = mult(mult(x1,x2),op_l(i(mult(x1,x2)),x2,x1))$

$$
\begin{aligned}
& \underbrace{asoc(mult(x1,x2),x2,mult(x1,x2))} \\
= & \quad \text{by Lemma 187 LR with } \{x2 \leftarrow x2, x1 \leftarrow mult(x1,x2)\} \\
& \overbrace{mult(mult(x1,x2), \underbrace{rd(asoc(mult(x1,x2),x2,mult(x1,x2)),mult(x1,x2))})} \\
= & \quad \text{by Lemma 627 LR with } \{x2 \leftarrow x2, x1 \leftarrow mult(x1,x2)\} \\
& mult(mult(x1,x2), \underbrace{\overbrace{op_l(i(mult(x1,x2)),x2,mult(x1,x2))}}) \\
= & \quad \text{by Lemma 291 LR with } \{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow i(mult(x1,x2))\} \\
& mult(mult(x1,x2), \overbrace{op_l(i(mult(x1,x2)),x2,x1)})
\end{aligned}
$$

Lemma 837: $asoc(mult(x1,x2),x2,mult(x1,x2)) = mult(x1,i(op_t(x1,x2)))$

$$
\begin{aligned}
& \underbrace{asoc(mult(x1,x2),x2,mult(x1,x2))} \\
= & \quad \text{by Lemma 836 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{\overbrace{mult(mult(x1,x2),op_l(i(mult(x1,x2)),x2,x1))}} \\
= & \quad \text{by Lemma 132 LR with } \{x3 \leftarrow i(mult(x1,x2)), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{mult(x1, \underbrace{mult(x2,i(mult(x1,x2)))})} \\
= & \quad \text{by Lemma 31 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& mult(x1, \overbrace{i(op_t(x1,x2))})
\end{aligned}
$$

Lemma 838: $asoc(mult(x1, x2), x2, mult(x1, x2)) = rd(x1, op_t(x1, x2))$

$$
\begin{aligned}
& \underbrace{asoc(mult(x1, x2), x2, mult(x1, x2))} \\
= \quad & \text{by Lemma 837 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{mult(x1, i(op_t(x1, x2)))}} \\
= \quad & \text{by Lemma 98 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{rd(x1, op_t(x1, x2))}
\end{aligned}
$$

Lemma 839: $op_r(rd(rd(x1, x2), x3), x3, x2) = rd(x1, mult(x3, x2))$

$$
\begin{aligned}
& \underbrace{op_r(rd(rd(x1, x2), x3), x3, x2)} \\
= \quad & \text{by Lemma 676 LR with } \{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow rd(x1, x2)\} \\
& \overbrace{rd(\underbrace{mult(rd(x1, x2), x2)}, mult(x3, x2))} \\
= \quad & \text{by Axiom 4 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& rd(\overbrace{x1}, mult(x3, x2))
\end{aligned}
$$

Lemma 840: $rd(mult(x2,x2),mult(x1,x2)) = op_r(rd(i(x1),i(x2)),i(x2),i(x2))$

$rd(mult(x2,\underbrace{x2}),mult(x1,x2))$

$=$ by Lemma 3 RL with $\{x1 \leftarrow x2\}$

$\underbrace{rd(mult(x2,\overbrace{i(i(x2))}),mult(x1,x2))}$

$=$ by Lemma 64 RL with $\{x3 \leftarrow i(x2), x2 \leftarrow x2, x1 \leftarrow mult(x1,x2)\}$

$\overbrace{rd(\underbrace{i(mult(x1,x2))},mult(i(x2),i(x2)))}$

$=$ by Lemma 9 RL with $\{x2 \leftarrow mult(x1,x2), x1 \leftarrow x2\}$

$rd(\overbrace{rd(x2,mult(\overbrace{mult(x1,x2)},\underbrace{x2}))},mult(i(x2),i(x2)))$

$=$ by Lemma 3 RL with $\{x1 \leftarrow x2\}$

$rd(\underbrace{rd(x2,mult(mult(x1,x2),\overbrace{i(i(x2))}))},mult(i(x2),i(x2)))$

$=$ by Lemma 63 RL with $\{x3 \leftarrow x2, x2 \leftarrow i(x2), x1 \leftarrow mult(x1,x2)\}$

$\underbrace{rd(\overbrace{rd(mult(\overbrace{i(mult(x1,x2))},i(x2)),i(x2))},mult(i(x2),i(x2)))}$

$=$ by Lemma 839 RL with $\{x3 \leftarrow i(x2), x2 \leftarrow i(x2), x1 \leftarrow rd(mult(i(mult(x1,x2)),i(x2)),i(x2))\}$

$\underbrace{\overbrace{op_r(rd(rd(rd(mult(\overbrace{i(mult(x1,x2))},i(x2)),i(x2)),i(x2)),i(x2)),i(x2),i(x2))}}$

$=$ by Lemma 370 LR with $\{x1 \leftarrow i(x2), x2 \leftarrow rd(rd(mult(i(mult(x1,x2)),i(x2)),i(x2)),i(x2))\}$

$\overbrace{rd(\underbrace{op_r(rd(rd(mult(\overbrace{i(mult(x1,x2))},i(x2)),i(x2)),i(x2)),i(x2),i(x2))},i(x2))}$

$=$ by Lemma 839 LR with $\{x3 \leftarrow i(x2), x2 \leftarrow i(x2), x1 \leftarrow mult(i(mult(x1,x2)),i(x2))\}$

$rd(\overbrace{rd(mult(\underbrace{i(mult(x1,x2))},i(x2)),mult(i(x2),i(x2)))},i(x2))$

$=$ by Axiom 11 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$rd(\underbrace{rd(mult(\overbrace{mult(i(x1),i(x2))},i(x2)),mult(i(x2),i(x2)))},i(x2))$

$=$ by Axiom 16 RL with $\{x3 \leftarrow i(x2), x2 \leftarrow i(x2), x1 \leftarrow i(x1)\}$

$\underbrace{rd(\overbrace{op_r(i(x1),i(x2),i(x2))},i(x2))}$

$=$ by Lemma 370 RL with $\{x1 \leftarrow i(x2), x2 \leftarrow i(x1)\}$

$\overbrace{op_r(rd(i(x1),i(x2)),i(x2),i(x2))}$

Lemma 841: $rd(mult(x1,rd(x2,x1)),x1) = mult(x1,rd(x2,mult(x1,x1)))$

$\underbrace{rd(mult(x1,rd(x2,x1)),x1)}$

$=$ by Lemma 802 RL with $\{x2 \leftarrow rd(x2,x1), x1 \leftarrow x1\}$

$\underbrace{\overbrace{op_r(mult(x1,rd(rd(x2,x1),x1)),x1,x1)}}$

$=$ by Lemma 383 RL with $\{x2 \leftarrow rd(rd(x2,x1),x1), x1 \leftarrow x1\}$

$\overbrace{mult(x1,\underbrace{op_r(rd(rd(x2,x1),x1),x1,x1)})}$

$=$ by Lemma 839 LR with $\{x3 \leftarrow x1, x2 \leftarrow x1, x1 \leftarrow x2\}$

$mult(x1,\overbrace{rd(x2,mult(x1,x1))})$

Lemma 842: $op_r(mult(x1, rd(i(x2), x1)), x1, x2) = i(x2)$

$$
\begin{aligned}
& op_r(\underbrace{mult(x1, rd(i(x2), x1))}, x1, x2) \\
= & \quad \text{by Lemma 55 RL with } \{x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& op_r(\overbrace{mult(\underbrace{mult(x1, i(x2))}, i(x1))}, x1, x2) \\
= & \quad \text{by Lemma 16 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_r(\underbrace{mult(\overbrace{i(mult(i(x1), x2))}, i(x1))}, x1, x2) \\
= & \quad \text{by Axiom 11 LR with } \{x2 \leftarrow x1, x1 \leftarrow mult(i(x1), x2)\} \\
& op_r(\overbrace{i(\underbrace{mult(mult(i(x1), x2), x1)})}, x1, x2) \\
= & \quad \text{by Lemma 595 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_r(\underbrace{i(\overbrace{rd(x1, rd(op_t(x1, x2), x2))})}, x1, x2) \\
= & \quad \text{by Lemma 10 LR with } \{x2 \leftarrow rd(op_t(x1, x2), x2), x1 \leftarrow x1\} \\
& \underbrace{op_r(\overbrace{rd(rd(op_t(x1, x2), x2), x1)}, x1, x2)} \\
= & \quad \text{by Lemma 839 LR with } \{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow op_t(x1, x2)\} \\
& \underbrace{\overbrace{rd(op_t(x1, x2), mult(x1, x2))}} \\
= & \quad \text{by Lemma 36 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{i(x2)}
\end{aligned}
$$

Lemma 843: $x1 = op_r(mult(x2, rd(x1, x2)), x2, i(x1))$

$$
\begin{aligned}
& \underbrace{x1} \\
= & \quad \text{by Lemma 3 RL with } \{x1 \leftarrow x1\} \\
& \underbrace{\overbrace{i(i(x1))}} \\
= & \quad \text{by Lemma 842 RL with } \{x2 \leftarrow i(x1), x1 \leftarrow x2\} \\
& \overbrace{op_r(mult(x2, rd(\underbrace{i(i(x1))}, x2)), x2, i(x1))} \\
= & \quad \text{by Lemma 3 LR with } \{x1 \leftarrow x1\} \\
& op_r(mult(x2, rd(\overbrace{x1}, x2)), x2, i(x1))
\end{aligned}
$$

Lemma 844: $op_r(x1, mult(x2, rd(i(x1), x2)), x3) = x1$

$op_r(\underbrace{x1}, mult(x2, rd(i(x1), x2)), x3)$

$=$ by Lemma 3 RL with $\{x1 \leftarrow x1\}$

$op_r(\overbrace{i(\underbrace{i(x1)})}, mult(x2, rd(i(x1), x2)), x3)$

$=$ by Lemma 842 RL with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$op_r(i(\overbrace{op_r(mult(x2, rd(i(x1), x2)), x2, x1)}), \underbrace{mult(x2, rd(i(x1), x2))}, x3)$

$=$ by Lemma 3 RL with $\{x1 \leftarrow mult(x2, rd(i(x1), x2))\}$

$op_r(\underbrace{i(op_r(mult(x2, rd(i(x1), x2)), x2, x1))}, \overbrace{i(i(mult(x2, rd(i(x1), x2))))}, x3)$

$=$ by Lemma 654 RL with $\{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow mult(x2, rd(i(x1), x2))\}$

$\underbrace{op_r(\overbrace{op_r(i(mult(x2, rd(i(x1), x2))), mult(mult(x2, rd(i(x1), x2)), x2), x1)}, i(i(mult(x2, rd(i(x1), x2)))), x3)}$

$=$ by Axiom 17 RL with $\{x5 \leftarrow x1, x4 \leftarrow mult(mult(x2, rd(i(x1), x2)), x2), x3 \leftarrow x3, x2 \leftarrow i(i(mult(x2, rd(i(x1), x2)))), x1 \leftarrow i$

$\overbrace{op_r(\underbrace{op_r(i(mult(x2, rd(i(x1), x2))), i(i(mult(x2, rd(i(x1), x2)))), x3)}, mult(mult(x2, rd(i(x1), x2)), x2), x1)}$

$=$ by Lemma 52 LR with $\{x2 \leftarrow x3, x1 \leftarrow i(mult(x2, rd(i(x1), x2)))\}$

$\underbrace{op_r(\overbrace{i(mult(x2, rd(i(x1), x2)))}, mult(mult(x2, rd(i(x1), x2)), x2), x1)}$

$=$ by Lemma 654 LR with $\{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow mult(x2, rd(i(x1), x2))\}$

$\overbrace{i(\underbrace{op_r(mult(x2, rd(i(x1), x2)), x2, x1)})}$

$=$ by Lemma 842 LR with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$\underbrace{i(\overbrace{i(x1)})}$

$=$ by Lemma 3 LR with $\{x1 \leftarrow x1\}$

$\overbrace{x1}$

Lemma 845: $x1 = op_r(x1, rd(op_r(x2, x1, x1), x2), i(x3))$

$$
\begin{aligned}
& \overbrace{x1} \\
= \quad & \text{by Lemma 844 RL with } \{x3 \leftarrow x3, x2 \leftarrow mult(i(x1), mult(x2, i(x1))), x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, mult(mult(i(x1), mult(x2, i(x1))), \underbrace{rd(i(x1), mult(i(x1), mult(x2, i(x1))))}), x3)} \\
= \quad & \text{by Lemma 527 LR with } \{x2 \leftarrow x2, x1 \leftarrow i(x1)\} \\
& op_r(x1, \underbrace{mult(mult(i(x1), mult(x2, i(x1))), \overbrace{op_r(i(mult(i(x1), x2)), i(x1), i(x1))})}, x3) \\
= \quad & \text{by Axiom 6 RL with } \{x3 \leftarrow op_r(i(mult(i(x1), x2)), i(x1), i(x1)), x2 \leftarrow x2, x1 \leftarrow i(x1)\} \\
& \underbrace{op_r(x1, \overbrace{mult(i(x1), mult(x2, mult(i(x1), op_r(i(mult(i(x1), x2)), i(x1), i(x1)))))}, x3)} \\
= \quad & \text{by Lemma 819 LR with } \{x3 \leftarrow x3, x2 \leftarrow mult(x2, mult(i(x1), op_r(i(mult(i(x1), x2)), i(x1), i(x1)))), x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, \underbrace{i(mult(x2, mult(i(x1), op_r(i(mult(i(x1), x2)), i(x1), i(x1)))))}, i(x3))} \\
= \quad & \text{by Lemma 87 LR with } \{x3 \leftarrow op_r(i(mult(i(x1), x2)), i(x1), i(x1)), x2 \leftarrow x1, x1 \leftarrow x2\} \\
& op_r(x1, \overbrace{mult(i(x2), mult(x1, \underbrace{i(op_r(i(mult(i(x1), x2)), i(x1), i(x1)))))}), i(x3))} \\
= \quad & \text{by Lemma 813 LR with } \{x3 \leftarrow i(x1), x2 \leftarrow i(x1), x1 \leftarrow mult(i(x1), x2)\} \\
& op_r(x1, mult(i(x2), mult(x1, \underbrace{op_r(mult(i(x1), x2), i(i(x1)), i(i(x1)))})), i(x3)) \\
= \quad & \text{by Lemma 361 LR with } \{x2 \leftarrow i(x1), x1 \leftarrow mult(i(x1), x2)\} \\
& op_r(x1, mult(i(x2), mult(x1, \overbrace{op_r(mult(i(x1), x2), i(x1), i(x1))})), i(x3)) \\
= \quad & \text{by Lemma 361 LR with } \{x2 \leftarrow x1, x1 \leftarrow mult(i(x1), x2)\} \\
& op_r(x1, mult(i(x2), \underbrace{mult(x1, \overbrace{op_r(mult(i(x1), x2), x1, x1)})}), i(x3)) \\
= \quad & \text{by Lemma 383 LR with } \{x2 \leftarrow mult(i(x1), x2), x1 \leftarrow x1\} \\
& op_r(x1, mult(i(x2), \overbrace{op_r(\underbrace{mult(x1, mult(i(x1), x2))}, x1, x1)}), i(x3)) \\
= \quad & \text{by Lemma 12 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_r(x1, \underbrace{mult(i(x2), op_r(\overbrace{\ x2\ }, x1, x1))}, i(x3)) \\
= \quad & \text{by Lemma 117 LR with } \{x3 \leftarrow x1, x2 \leftarrow x1, x1 \leftarrow x2\} \\
& op_r(x1, \overbrace{rd(op_r(x2, x1, x1), x2)}, i(x3))
\end{aligned}
$$

Lemma 846: $x1 = op_r(x1, asoc(x2, x1, x1), x3)$

$$
\begin{aligned}
& \overbrace{x1} \\
= \quad & \text{by Lemma 845 LR with } \{x3 \leftarrow i(x3), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, \underbrace{rd(op_r(x2, x1, x1), x2)}, i(i(x3)))} \\
= \quad & \text{by Lemma 766 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& op_r(x1, \overbrace{asoc(x2, x1, x1)}, \underbrace{i(i(x3))}) \\
= \quad & \text{by Lemma 3 LR with } \{x1 \leftarrow x3\} \\
& op_r(x1, asoc(x2, x1, x1), \overbrace{\ x3\ })
\end{aligned}
$$

Lemma 847: $op_l(x1, x2, x1) = op_r(x1, x2, op_l(i(x1), x2, x1))$

$$
\begin{aligned}
& \overbrace{op_l(x1, x2, x1)} \\
= \quad & \text{by Lemma 843 LR with } \{x2 \leftarrow x2, x1 \leftarrow op_l(x1, x2, x1)\} \\
& \overbrace{op_r(mult(x2, \underbrace{rd(op_l(x1, x2, x1), x2)}), x2, i(op_l(x1, x2, x1)))} \\
= \quad & \text{by Lemma 615 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_r(\underbrace{mult(x2, \overbrace{mult(i(x2), x1)})}, x2, i(op_l(x1, x2, x1))) \\
= \quad & \text{by Lemma 12 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& op_r(\overbrace{x1}, x2, \underbrace{i(op_l(x1, x2, x1))}) \\
= \quad & \text{by Lemma 433 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_r(x1, x2, \overbrace{op_l(i(x1), x2, x1)})
\end{aligned}
$$

Lemma 848: $op_l(x1, i(x1), i(x2)) = op_r(x1, i(mult(x1, x2)), op_l(i(x1), i(x1), i(x2)))$

$$
\begin{aligned}
& \overbrace{op_l(x1, i(x1), i(x2))} \\
= \quad & \text{by Lemma 247 RL with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, i(\overbrace{mult(x1, x2)}), x1)}} \\
= \quad & \text{by Lemma 847 LR with } \{x2 \leftarrow i(mult(x1, x2)), x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, i(mult(x1, x2)), \underbrace{op_l(i(x1), i(mult(x1, x2)), x1)})} \\
= \quad & \text{by Lemma 247 LR with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow i(x1)\} \\
& op_r(x1, i(mult(x1, x2)), \overbrace{op_l(i(x1), i(x1), i(x2))})
\end{aligned}
$$

Lemma 849: $op_l(x1, x1, x2) = op_r(x1, i(mult(x1, x2)), op_l(i(x1), i(x1), i(x2)))$

$$
\begin{aligned}
& \overbrace{op_l(x1, x1, x2)} \\
= \quad & \text{by Lemma 607 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, i(x2), x1)}} \\
= \quad & \text{by Lemma 609 RL with } \{x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, i(x1), i(x2))}} \\
= \quad & \text{by Lemma 848 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, i(mult(x1, x2)), op_l(i(x1), i(x1), i(x2)))}
\end{aligned}
$$

Lemma 850: $op_l(x1, x1, x2) = op_r(x1, x2, op_l(x1, i(i(x1)), i(i(x2))))$

$$
\begin{aligned}
& \overbrace{op_l(x1, x1, x2)} \\
= \quad & \text{by Lemma 849 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_r(x1, i(mult(x1, x2)), op_l(i(x1), i(x1), i(x2)))}} \\
= \quad & \text{by Lemma 818 LR with } \{x3 \leftarrow op_l(i(x1), i(x1), i(x2)), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, x2, \underbrace{i(op_l(i(x1), i(x1), i(x2)))})} \\
= \quad & \text{by Lemma 518 LR with } \{x3 \leftarrow i(x2), x2 \leftarrow i(x1), x1 \leftarrow x1\} \\
& op_r(x1, x2, \overbrace{op_l(x1, i(i(x1)), i(i(x2)))})
\end{aligned}
$$

Lemma 851: $op_l(x1, x1, x2) = op_r(x1, x2, op_l(x1, i(x2), x1))$

$$
\begin{aligned}
& \underbrace{op_l(x1, x1, x2)} \\
= \quad & \text{by Lemma 850 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, x2, op_l(x1, \underbrace{i(i(x1))}, i(i(x2))))} \\
= \quad & \text{by Lemma 3 LR with } \{x1 \leftarrow x1\} \\
& op_r(x1, x2, \underbrace{op_l(x1, \overbrace{x1}, i(i(x2)))}) \\
= \quad & \text{by Lemma 608 LR with } \{x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& op_r(x1, x2, \overbrace{op_l(x1, i(x2), x1)})
\end{aligned}
$$

Lemma 852: $op_l(mult(x1, x2), mult(x1, x2), x2) = op_r(mult(x1, x2), x2, op_l(mult(x1, x2), x1, x2))$

$$
\begin{aligned}
& \underbrace{op_l(mult(x1, x2), mult(x1, x2), x2)} \\
= \quad & \text{by Lemma 851 LR with } \{x2 \leftarrow x2, x1 \leftarrow mult(x1, x2)\} \\
& \overbrace{op_r(mult(x1, x2), x2, \underbrace{op_l(mult(x1, x2), i(x2), mult(x1, x2))})} \\
= \quad & \text{by Lemma 607 LR with } \{x2 \leftarrow x2, x1 \leftarrow mult(x1, x2)\} \\
& op_r(mult(x1, x2), x2, \overbrace{\underbrace{op_l(mult(x1, x2), mult(x1, x2), x2)}}) \\
= \quad & \text{by Lemma 285 LR with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow mult(x1, x2)\} \\
& op_r(mult(x1, x2), x2, \overbrace{op_l(mult(x1, x2), x1, x2)})
\end{aligned}
$$

Lemma 853: $mult(x2, x1) = op_r(mult(x1, x2), x2, op_l(mult(x1, x2), x1, x2))$

$$
\begin{aligned}
& \underbrace{mult(x2, x1)} \\
= \quad & \text{by Lemma 435 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(mult(x1, x2), x1, x2)}} \\
= \quad & \text{by Lemma 285 RL with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow mult(x1, x2)\} \\
& \overbrace{\underbrace{op_l(mult(x1, x2), mult(x1, x2), x2)}} \\
= \quad & \text{by Lemma 852 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_r(mult(x1, x2), x2, op_l(mult(x1, x2), x1, x2))}
\end{aligned}
$$

Lemma 854: $asoc(mult(x1, x2), x1, mult(x1, x2)) = rd(rd(mult(x1, x2), x1), x2)$

$$
\begin{aligned}
& \underbrace{asoc(mult(x1, x2), x1, mult(x1, x2))} \\
= \quad & \text{by Lemma 701 LR with } \{x2 \leftarrow x1, x1 \leftarrow mult(x1, x2)\} \\
& \overbrace{rd(\underbrace{mult(x1, i(mult(x1, x2)))}, rd(x1, mult(x1, x2)))} \\
= \quad & \text{by Lemma 20 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{rd(\overbrace{i(x2)}, rd(x1, mult(x1, x2)))} \\
= \quad & \text{by Lemma 40 LR with } \{x3 \leftarrow mult(x1, x2), x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \overbrace{rd(rd(mult(x1, x2), x1), x2)}
\end{aligned}
$$

Lemma 855: $mult(x1, rd(x1, x2)) = rd(rd(x1, x2), op_t(i(x1), x2))$

$$
\begin{aligned}
& mult(x1, \underbrace{rd(x1, x2)}) \\
= & \quad \text{by Lemma 10 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \underbrace{mult(x1, \overbrace{i(rd(x2, x1))})} \\
= & \quad \text{by Lemma 34 RL with } \{x2 \leftarrow rd(x2, x1), x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_t(rd(x1, rd(x2, x1)), i(x1))}} \\
= & \quad \text{by Lemma 376 RL with } \{x2 \leftarrow i(x1), x1 \leftarrow rd(x1, rd(x2, x1))\} \\
& \overbrace{rd(\underbrace{mult(rd(x1, rd(x2, x1)), i(x1))}, \underbrace{op_t(i(x1), rd(x1, rd(x2, x1)))})} \\
= & \quad \text{by Lemma 718 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& rd(\underbrace{mult(rd(x1, rd(x2, x1)), i(x1))}, \overbrace{op_t(i(x1), x2)}) \\
= & \quad \text{by Lemma 22 LR with } \{x2 \leftarrow rd(x2, x1), x1 \leftarrow x1\} \\
& rd(\overbrace{\underbrace{i(rd(x2, x1))}}, op_t(i(x1), x2)) \\
= & \quad \text{by Lemma 10 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& rd(\overbrace{rd(x1, x2)}, op_t(i(x1), x2))
\end{aligned}
$$

Lemma 856: $op_t(x1, i(x2)) = mult(mult(x1, rd(x1, x2)), i(rd(x1, x2)))$

$$
\begin{aligned}
& \underbrace{op_t(x1, i(x2))} \\
= & \quad \text{by Lemma 18 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{i(op_t(i(x1), x2))}} \\
= & \quad \text{by Lemma 22 RL with } \{x2 \leftarrow op_t(i(x1), x2), x1 \leftarrow rd(x1, x2)\} \\
& \overbrace{mult(\underbrace{rd(rd(x1, x2), op_t(i(x1), x2))}, i(rd(x1, x2)))} \\
= & \quad \text{by Lemma 855 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& mult(\overbrace{mult(x1, rd(x1, x2))}, i(rd(x1, x2)))
\end{aligned}
$$

Lemma 857: $op_t(x1, i(x2)) = op_r(x1, rd(x1, x2), rd(x2, x1))$

$$
\begin{aligned}
& \underbrace{op_t(x1, i(x2))} \\
= & \quad \text{by Lemma 856 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{mult(mult(x1, rd(x1, x2)), i(rd(x1, x2)))}} \\
= & \quad \text{by Lemma 92 LR with } \{x2 \leftarrow rd(x1, x2), x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, rd(x1, x2), \underbrace{i(rd(x1, x2))})} \\
= & \quad \text{by Lemma 10 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_r(x1, rd(x1, x2), \overbrace{rd(x2, x1)})
\end{aligned}
$$

Lemma 858: $asoc(x1, mult(x2, x1), mult(x2, x1)) = rd(op_t(x2, x1), x2)$

$\underbrace{asoc(x1, mult(x2, x1), mult(x2, x1))}$

$=$ by Lemma 756 RL with $\{x2 \leftarrow x1, x1 \leftarrow mult(x2, x1)\}$

$\overbrace{\underbrace{asoc(mult(x2, x1), i(x1), mult(x2, x1))}}$

$=$ by Axiom 1 LR with $\{x1 \leftarrow asoc(mult(x2, x1), i(x1), mult(x2, x1))\}$

$\overbrace{mult(asoc(mult(x2, x1), i(x1), mult(x2, x1)), \underbrace{unit()})}$

$=$ by Axiom 7 RL with $\{x1 \leftarrow mult(x2, x1)\}$

$mult(asoc(mult(x2, x1), i(x1), mult(x2, x1)), \overbrace{mult(\underbrace{mult(x2, x1)}, i(mult(x2, x1)))})$

$=$ by Axiom 4 LR with $\{x2 \leftarrow x1, x1 \leftarrow mult(x2, x1)\}$

$mult(\underbrace{asoc(mult(x2, x1), i(x1), mult(x2, x1))}, mult(\overbrace{mult(rd(mult(x2, x1), x1), x1)}, i(mult(x2, x1))))$

$=$ by Lemma 612 RL with $\{x2 \leftarrow x1, x1 \leftarrow mult(x2, x1)\}$

$mult(\overbrace{i(\underbrace{asoc(mult(x2, x1), x1, mult(x2, x1))})}, mult(mult(rd(mult(x2, x1), x1), x1), i(mult(x2, x1))))$

$=$ by Axiom 2 LR with $\{x2 \leftarrow rd(x1, mult(x2, x1)), x1 \leftarrow asoc(mult(x2, x1), x1, mult(x2, x1))\}$

$mult(i(\overbrace{mult(i(rd(x1, mult(x2, x1))), \underbrace{mult(rd(x1, mult(x2, x1)), asoc(mult(x2, x1), x1, mult(x2, x1)))})}), mult(mult(rd(mu$

$=$ by Lemma 699 RL with $\{x2 \leftarrow mult(x2, x1), x1 \leftarrow x1\}$

$mult(i(mult(\underbrace{i(rd(x1, mult(x2, x1)))}, \overbrace{mult(x1, i(mult(x2, x1)))})), mult(mult(rd(mult(x2, x1), x1), x1), i(mult(x2, x1))))$

$=$ by Lemma 10 LR with $\{x2 \leftarrow mult(x2, x1), x1 \leftarrow x1\}$

$\underbrace{mult(i(mult(\overbrace{rd(mult(x2, x1), x1)}, mult(x1, i(mult(x2, x1))))), mult(mult(rd(mult(x2, x1), x1), x1), i(mult(x2, x1))))}$

$=$ by Lemma 108 LR with $\{x3 \leftarrow i(mult(x2, x1)), x2 \leftarrow x1, x1 \leftarrow rd(mult(x2, x1), x1)\}$

$\overbrace{asoc(\underbrace{rd(mult(x2, x1), x1)}, x1, i(mult(x2, x1)))}$

$=$ by Axiom 5 RL with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$\underbrace{asoc(\overbrace{x2}, x1, i(mult(x2, x1)))}$

$=$ by Lemma 108 RL with $\{x3 \leftarrow i(mult(x2, x1)), x2 \leftarrow x1, x1 \leftarrow x2\}$

$\overbrace{mult(i(mult(x2, mult(x1, i(mult(x2, x1))))), \underbrace{mult(mult(x2, x1), i(mult(x2, x1)))})$

$=$ by Axiom 7 LR with $\{x1 \leftarrow mult(x2, x1)\}$

$\underbrace{mult(i(mult(x2, mult(x1, i(mult(x2, x1))))), \overbrace{unit()})}$

$=$ by Axiom 1 RL with $\{x1 \leftarrow i(mult(x2, mult(x1, i(mult(x2, x1)))))\}$

$\overbrace{\underbrace{i(mult(x2, mult(x1, i(mult(x2, x1)))))}}$

$=$ by Axiom 11 RL with $\{x2 \leftarrow mult(x1, i(mult(x2, x1))), x1 \leftarrow x2\}$

$\overbrace{mult(i(x2), \underbrace{i(mult(x1, i(mult(x2, x1))))})}$

$=$ by Lemma 17 LR with $\{x2 \leftarrow mult(x2, x1), x1 \leftarrow x1\}$

$mult(i(x2), \overbrace{\underbrace{mult(i(x1), mult(x2, x1))}})$

$=$ by Axiom 10 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{mult(i(x2), \overbrace{op_t(x2, x1)})}$

$=$ by Lemma 96 LR with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$\overbrace{rd(op_t(x2, x1), x2)}$

Lemma 859: $asoc(x1, mult(x1, x2), mult(x1, x2)) = rd(x1, op_r(x1, x2, i(mult(x1, x2))))$

$$
\begin{aligned}
& \overbrace{asoc(x1, mult(x1, x2), mult(x1, x2))} \\
= \quad & \text{by Lemma 786 RL with } \{x2 \leftarrow mult(x1, x2), x1 \leftarrow x1\} \\
& \overbrace{mult(x1, \underbrace{op_r(i(x1), mult(x1, x2), i(mult(x1, x2)))})} \\
= \quad & \text{by Lemma 654 LR with } \{x3 \leftarrow i(mult(x1, x2)), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{mult(x1, \overbrace{i(op_r(x1, x2, i(mult(x1, x2))))})} \\
= \quad & \text{by Lemma 115 LR with } \{x3 \leftarrow i(mult(x1, x2)), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{rd(x1, \underbrace{op_r(x1, x2, i(mult(x1, x2)))})}
\end{aligned}
$$

Lemma 860: $asoc(x1, mult(x1, x2), mult(x1, x2)) = rd(x1, op_t(x1, x2))$

$$
\begin{aligned}
& \overbrace{asoc(x1, mult(x1, x2), mult(x1, x2))} \\
= \quad & \text{by Lemma 859 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{rd(x1, \underbrace{op_r(x1, x2, i(mult(x1, x2)))})} \\
= \quad & \text{by Lemma 79 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& rd(x1, \overbrace{op_t(x1, x2)})
\end{aligned}
$$

Lemma 861: $mult(mult(x1, x2), i(mult(x2, x1))) = rd(op_t(x1, x2), x1)$

$$
\begin{aligned}
& \underbrace{mult(mult(x1, x2), i(mult(x2, x1)))} \\
= \quad & \text{by Lemma 91 RL with } \{x3 \leftarrow i(mult(x2, x1)), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{mult(op_r(x1, x2, \underbrace{i(mult(x2, x1))}), mult(x2, i(mult(x2, x1))))} \\
= \quad & \text{by Axiom 11 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& mult(op_r(x1, x2, \overbrace{mult(i(x2), \underbrace{i(x1)})}), mult(x2, i(mult(x2, x1)))) \\
= \quad & \text{by Lemma 9 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& mult(\underbrace{op_r(x1, x2, mult(i(x2), \overbrace{rd(x2, mult(x1, x2))}))}, mult(x2, i(mult(x2, x1)))) \\
= \quad & \text{by Lemma 792 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& mult(\overbrace{rd(asoc(x2, mult(x1, i(i(x2))), mult(x1, i(i(x2)))), rd(x2, mult(x1, i(i(x2)))))}, mult(x2, i(mult(x2, x1)))) \\
= \quad & \text{by Lemma 790 RL with } \{x2 \leftarrow mult(x1, i(i(x2))), x1 \leftarrow x2\} \\
& mult(\overbrace{mult(i(x2), mult(x1, i(i(x2))))}, mult(x2, i(mult(x2, x1)))) \\
= \quad & \text{by Lemma 30 LR with } \{x2 \leftarrow x1, x1 \leftarrow i(x2)\} \\
& mult(\overbrace{op_t(x1, \underbrace{i(i(x2))})}, mult(x2, i(mult(x2, x1)))) \\
= \quad & \text{by Lemma 3 LR with } \{x1 \leftarrow x2\} \\
& mult(op_t(x1, \overbrace{x2}), \underbrace{mult(x2, i(mult(x2, x1)))}) \\
= \quad & \text{by Lemma 20 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \underbrace{mult(op_t(x1, x2), \overbrace{i(x1)})} \\
= \quad & \text{by Lemma 97 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{rd(op_t(x1, x2), x1)}
\end{aligned}
$$

Lemma 862: $rd(mult(x2,x1),mult(x1,x2)) = rd(rd(op_t(x2,x1),x2),unit())$

$\underbrace{rd(mult(x2,x1),mult(x1,x2))}$

$=$ by Lemma 23 RL with $\{x2 \leftarrow mult(x2,x1), x1 \leftarrow mult(x1,x2)\}$

$\overbrace{rd(i(mult(x1,x2)),i(mult(x2,x1)))}$

$=$ by Lemma 359 RL with $\{x2 \leftarrow i(mult(x1,x2)), x1 \leftarrow mult(x2,x1)\}$

$\overbrace{op_r(mult(mult(x2,x1),i(mult(x1,x2))),i(mult(x1,x2)),i(mult(x1,x2)))}$

$=$ by Lemma 641 RL with $\{x2 \leftarrow i(mult(x1,x2)), x1 \leftarrow mult(mult(x2,x1),i(mult(x1,x2)))\}$

$\overbrace{rd(mult(mult(x2,x1),i(mult(x1,x2))),\underbrace{asoc(i(mult(x1,x2)),mult(mult(x2,x1),i(mult(x1,x2))),i(mult(x1,x2)))}}$

$=$ by Lemma 262 LR with $\{x2 \leftarrow mult(x2,x1), x1 \leftarrow i(mult(x1,x2))\}$

$rd(\underbrace{mult(mult(x2,x1),i(mult(x1,x2)))},\overbrace{asoc(i(mult(x1,x2)),mult(x2,x1),i(mult(x1,x2))))}$

$=$ by Lemma 861 LR with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$rd(\overbrace{rd(op_t(x2,x1),x2)},\underbrace{asoc(i(mult(x1,x2)),mult(x2,x1),i(mult(x1,x2))))}$

$=$ by Lemma 613 RL with $\{x2 \leftarrow mult(x2,x1), x1 \leftarrow mult(x1,x2)\}$

$rd(rd(op_t(x2,x1),x2),\overbrace{asoc(mult(x1,x2),mult(\underbrace{x2},x1),mult(x1,x2)))}$

$=$ by Axiom 2 LR with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$rd(rd(op_t(x2,x1),x2),asoc(mult(x1,x2),\underbrace{mult(\overbrace{mult(i(x1),mult(x1,x2))},x1)},mult(x1,x2)))$

$=$ by Lemma 195 RL with $\{x2 \leftarrow x1, x1 \leftarrow mult(x1,x2)\}$

$rd(rd(op_t(x2,x1),x2),asoc(mult(x1,x2),\overbrace{op_l(mult(x1,x2),i(x1),\underbrace{mult(x1,i(mult(x1,x2)))})},mult(x1,x2)))$

$=$ by Lemma 20 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$rd(rd(op_t(x2,x1),x2),\underbrace{asoc(mult(x1,x2),op_l(mult(x1,x2),i(x1),\overbrace{i(x2)}),mult(x1,x2)))}$

$=$ by Lemma 253 LR with $\{x3 \leftarrow i(x2), x2 \leftarrow i(x1), x1 \leftarrow mult(x1,x2)\}$

$rd(rd(op_t(x2,x1),x2),\overbrace{\underbrace{mult(i(mult(x1,x2)),mult(x1,x2))})$

$=$ by Axiom 9 LR with $\{x1 \leftarrow mult(x1,x2)\}$

$rd(rd(op_t(x2,x1),x2),\overbrace{unit()})$

Lemma 863: $op_l(mult(x1,x2),x2,mult(x1,x2)) = op_l(mult(x1,x2),rd(mult(x1,x2),x1),mult(x1,x2))$

$\underbrace{op_l(mult(x1,x2),x2,mult(x1,x2))}$

$=$ by Lemma 438 RL with $\{x2 \leftarrow x2, x1 \leftarrow mult(x1,x2)\}$

$\overbrace{\underbrace{op_l(mult(x1,x2),i(x2),i(mult(x1,x2)))}}$

$=$ by Lemma 500 LR with $\{x2 \leftarrow x2, x1 \leftarrow mult(x1,x2)\}$

$\overbrace{op_l(mult(x1,x2),\underbrace{op_r(x2,mult(x1,x2),mult(x1,x2))},mult(x1,x2))}$

$=$ by Lemma 800 LR with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$op_l(mult(x1,x2),\overbrace{rd(mult(x1,x2),x1)},mult(x1,x2))$

Lemma 864: $mult(op_t(x1, x2), x2) = op_l(mult(x1, x2), rd(mult(x1, x2), x1), mult(x1, x2))$

$$
\begin{aligned}
& \underbrace{mult(op_t(x1, x2), x2)} \\
= \quad & \text{by Lemma 636 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(mult(x1, x2), x2, x1)}} \\
= \quad & \text{by Lemma 291 RL with } \{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow mult(x1, x2)\} \\
& \overbrace{\underbrace{op_l(mult(x1, x2), x2, mult(x1, x2))}} \\
= \quad & \text{by Lemma 863 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(mult(x1, x2), rd(mult(x1, x2), x1), mult(x1, x2))}
\end{aligned}
$$

Lemma 865: $mult(op_t(x1, x2), x2) = mult(x1, rd(mult(x1, x2), x1))$

$$
\begin{aligned}
& \underbrace{mult(op_t(x1, x2), x2)} \\
= \quad & \text{by Lemma 864 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(mult(x1, x2), rd(mult(x1, x2), x1), mult(x1, x2))}} \\
= \quad & \text{by Lemma 622 LR with } \{x2 \leftarrow x1, x1 \leftarrow mult(x1, x2)\} \\
& \overbrace{\underbrace{op_l(mult(x1, x2), mult(x1, x2), x1)}} \\
= \quad & \text{by Lemma 427 LR with } \{x2 \leftarrow x1, x1 \leftarrow mult(x1, x2)\} \\
& \overbrace{mult(x1, rd(mult(x1, x2), x1))}
\end{aligned}
$$

Lemma 866: $rd(x1, op_r(i(rd(x2, x1)), x2, x2)) = op_r(op_l(x2, x2, x1), x1, x1)$

$$
\begin{aligned}
& rd(x1, \underbrace{op_r(i(rd(x2, x1)), x2, x2)}) \\
= \quad & \text{by Lemma 356 RL with } \{x2 \leftarrow x2, x1 \leftarrow rd(x2, x1)\} \\
& \underbrace{rd(x1, \overbrace{i(op_r(rd(x2, x1), x2, x2))})} \\
= \quad & \text{by Lemma 359 RL with } \{x2 \leftarrow x1, x1 \leftarrow op_r(rd(x2, x1), x2, x2)\} \\
& \overbrace{op_r(\underbrace{mult(op_r(rd(x2, x1), x2, x2), x1)}, x1, x1)} \\
= \quad & \text{by Lemma 808 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& op_r(\overbrace{op_l(x2, x2, x1)}, x1, x1)
\end{aligned}
$$

Lemma 867: $op_r(rd(x2, rd(x1, x2)), x2, x2) = i(rd(x1, mult(x2, x2)))$

$$
\begin{aligned}
& op_r(\underbrace{rd(x2, rd(x1, x2))}, x2, x2) \\
= \quad & \text{by Lemma 10 RL with } \{x2 \leftarrow x2, x1 \leftarrow rd(x1, x2)\} \\
& \underbrace{op_r(\overbrace{i(rd(rd(x1, x2), x2))}, x2, x2)} \\
= \quad & \text{by Lemma 356 RL with } \{x2 \leftarrow x2, x1 \leftarrow rd(rd(x1, x2), x2)\} \\
& \overbrace{i(\underbrace{op_r(rd(rd(x1, x2), x2), x2, x2)})} \\
= \quad & \text{by Lemma 839 LR with } \{x3 \leftarrow x2, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& i(\overbrace{rd(x1, mult(x2, x2))})
\end{aligned}
$$

Lemma 868: $op_r(rd(x1, rd(x2, x1)), x1, x1) = rd(mult(x1, x1), x2)$

$$
\begin{aligned}
& \underbrace{op_r(rd(x1, rd(x2, x1)), x1, x1)} \\
= & \quad \text{by Lemma 867 LR with } \{x1 \leftarrow x2, x2 \leftarrow x1\} \\
& \underbrace{\overbrace{i(rd(x2, mult(x1, x1)))}} \\
= & \quad \text{by Lemma 10 LR with } \{x2 \leftarrow mult(x1, x1), x1 \leftarrow x2\} \\
& \overbrace{rd(mult(x1, x1), x2)}
\end{aligned}
$$

Lemma 869: $rd(x1, rd(rd(x2, x1), x1)) = mult(rd(mult(x1, x1), x2), x1)$

$$
\begin{aligned}
& rd(x1, \underbrace{rd(rd(x2, x1), x1)}) \\
= & \quad \text{by Lemma 10 RL with } \{x2 \leftarrow rd(x2, x1), x1 \leftarrow x1\} \\
& \underbrace{rd(x1, \overbrace{i(rd(x1, rd(x2, x1)))})} \\
= & \quad \text{by Lemma 357 RL with } \{x2 \leftarrow x1, x1 \leftarrow rd(x1, rd(x2, x1))\} \\
& \overbrace{mult(\underbrace{op_r(rd(x1, rd(x2, x1)), x1, x1)}, x1)} \\
= & \quad \text{by Lemma 868 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& mult(\overbrace{rd(mult(x1, x1), x2)}, x1)
\end{aligned}
$$

Lemma 870: $mult(mult(x1,x2),op_t(x2,x1)) = op_l(mult(x2,mult(x2,x1)),rd(op_t(x2,x1),i(mult(x1,x2))),mult(x1,x2))$

$mult(\overbrace{mult(x1,x2)},op_t(x2,x1))$

$=$ by Lemma 3 RL with $\{x1 \leftarrow mult(x1,x2)\}$

$\overbrace{mult(\overbrace{i(i(mult(x1,x2)))},op_t(x2,x1))}$

$=$ by Lemma 14 RL with $\{x2 \leftarrow i(mult(x1,x2)), x1 \leftarrow op_t(x2,x1)\}$

$\overbrace{op_t(\overbrace{rd(op_t(x2,x1),i(mult(x1,x2)))},i(mult(x1,x2)))}$

$=$ by Lemma 475 RL with $\{x2 \leftarrow mult(x1,x2), x1 \leftarrow rd(op_t(x2,x1),i(mult(x1,x2)))\}$

$\overbrace{op_l(\overbrace{op_r(rd(op_t(x2,x1),i(mult(x1,x2))),mult(x1,x2),i(mult(x1,x2)))},rd(op_t(x2,x1),i(mult(x1,x2))),mult(x1,x2))}$

$=$ by Lemma 321 RL with $\{x2 \leftarrow mult(x1,x2), x1 \leftarrow rd(op_t(x2,x1),i(mult(x1,x2)))\}$

$op_l(\overbrace{op_r(rd(op_t(x2,x1),i(mult(x1,x2))),i(mult(x1,x2)),mult(x1,x2))},rd(op_t(x2,x1),i(mult(x1,x2))),mult(x1,x2))$

$=$ by Lemma 93 RL with $\{x2 \leftarrow mult(x1,x2), x1 \leftarrow rd(op_t(x2,x1),i(mult(x1,x2)))\}$

$op_l(\overbrace{mult(\overbrace{mult(rd(op_t(x2,x1),i(mult(x1,x2))),i(mult(x1,x2)))},mult(x1,x2))},rd(op_t(x2,x1),i(mult(x1,x2))),mult(x1,x$

$=$ by Axiom 4 RL with $\{x2 \leftarrow i(mult(x1,x2)), x1 \leftarrow op_t(x2,x1)\}$

$op_l(mult(\overbrace{op_t(x2,x1)},mult(x1,x2)),rd(op_t(x2,x1),i(mult(x1,x2))),mult(x1,x2))$

$=$ by Lemma 583 RL with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$op_l(mult(\overbrace{op_l(op_t(x2,mult(x1,x2)),x2,x1)},mult(x1,x2)),rd(op_t(x2,x1),i(mult(x1,x2))),mult(x1,x2))$

$=$ by Axiom 15 RL with $\{x4 \leftarrow mult(x1,x2), x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x2\}$

$op_l(\overbrace{mult(\overbrace{op_t(op_l(x2,x2,x1),mult(x1,x2))},mult(x1,x2))},rd(op_t(x2,x1),i(mult(x1,x2))),mult(x1,x2))$

$=$ by Lemma 749 RL with $\{x2 \leftarrow op_l(x2,x2,x1), x1 \leftarrow mult(x1,x2)\}$

$op_l(\overbrace{mult(\overbrace{mult(i(mult(x1,x2)),op_l(x2,x2,x1))},mult(mult(x1,x2),mult(x1,x2)))},rd(op_t(x2,x1),i(mult(x1,x2))),mult(x$

$=$ by Lemma 529 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$op_l(mult(\overbrace{i(x1)},\overbrace{mult(mult(x1,x2),mult(x1,x2))}),rd(op_t(x2,x1),i(mult(x1,x2))),mult(x1,x2))$

$=$ by Lemma 530 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$op_l(\overbrace{mult(i(x1),\overbrace{mult(x1,mult(x2,mult(x2,x1)))})},rd(op_t(x2,x1),i(mult(x1,x2))),mult(x1,x2))$

$=$ by Axiom 2 RL with $\{x2 \leftarrow x1, x1 \leftarrow mult(x2,mult(x2,x1))\}$

$op_l(\overbrace{mult(x2,mult(x2,x1))},rd(op_t(x2,x1),i(mult(x1,x2))),mult(x1,x2))$

Lemma 871: $mult(mult(x1, x2), op_t(x2, x1)) = mult(mult(x2, x1), x2)$

$mult(mult(x1, x2), op_t(x2, x1))$

$=$ by Lemma 870 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$op_l(mult(x2, mult(x2, x1)), rd(op_t(x2, x1), i(mult(x1, x2))), mult(x1, x2))$

$=$ by Lemma 38 RL with $\{x1 \leftarrow op_t(x2, x1), x2 \leftarrow mult(x1, x2)\}$

$op_l(mult(x2, mult(x2, x1)), rd(mult(x1, x2), i(op_t(x2, x1))), mult(x1, x2))$

$=$ by Lemma 15 RL with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$op_l(mult(x2, mult(x2, x1)), rd(mult(x1, x2), op_t(i(x2), i(x1))), mult(x1, x2))$

$=$ by Lemma 9 RL with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$op_l(mult(x2, mult(x2, x1)), rd(mult(x1, x2), op_t(i(x2), rd(x2, mult(x1, x2)))), mult(x1, x2))$

$=$ by Lemma 386 LR with $\{x2 \leftarrow x2, x1 \leftarrow mult(x1, x2)\}$

$op_l(mult(x2, mult(x2, x1)), mult(x2, mult(x1, x2)), mult(x1, x2))$

$=$ by Lemma 285 LR with $\{x3 \leftarrow mult(x1, x2), x2 \leftarrow x2, x1 \leftarrow mult(x2, mult(x2, x1))\}$

$op_l(mult(x2, mult(x2, x1)), x2, mult(x1, x2))$

$=$ by Lemma 242 RL with $\{x3 \leftarrow mult(x1, x2), x2 \leftarrow x2, x1 \leftarrow mult(x2, mult(x2, x1))\}$

$op_l(mult(x2, mult(x2, x1)), mult(x2, mult(x1, x2)), i(x2))$

$=$ by Lemma 134 LR with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$op_l(op_t(mult(x2, mult(x1, x2)), i(x2)), mult(x2, mult(x1, x2)), i(x2))$

$=$ by Lemma 421 RL with $\{x2 \leftarrow i(x2), x1 \leftarrow mult(x2, mult(x1, x2))\}$

$op_r(mult(x2, mult(x1, x2)), i(x2), i(i(x2)))$

$=$ by Lemma 3 LR with $\{x1 \leftarrow x2\}$

$op_r(mult(x2, mult(x1, x2)), i(x2), x2)$

$=$ by Lemma 321 LR with $\{x2 \leftarrow x2, x1 \leftarrow mult(x2, mult(x1, x2))\}$

$op_r(mult(x2, mult(x1, x2)), x2, i(x2))$

$=$ by Lemma 405 LR with $\{x2 \leftarrow mult(x1, x2), x1 \leftarrow x2\}$

$rd(x2, op_t(i(mult(x1, x2)), x2))$

$=$ by Lemma 329 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$rd(x2, rd(op_t(i(x1), x2), x2))$

$=$ by Lemma 407 RL with $\{x2 \leftarrow i(x1), x1 \leftarrow x2\}$

$mult(mult(x2, i(i(x1))), x2)$

$=$ by Lemma 3 LR with $\{x1 \leftarrow x1\}$

$mult(mult(x2, x1), x2)$

Lemma 872: $mult(x1, mult(x2, x1)) = mult(rd(x1, i(x2)), op_t(x1, op_r(x2, x1, x1)))$

$$
\begin{aligned}
& \underbrace{mult(x1, mult(x2, x1))} \\
= \quad & \text{by Lemma 58 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{\overbrace{rd(x1, i(mult(x1, x2)))}} \\
= \quad & \text{by Lemma 357 RL with } \{x2 \leftarrow x1, x1 \leftarrow mult(x1, x2)\} \\
& \overbrace{mult(\underbrace{op_r(mult(x1, x2), x1, x1)}, x1)} \\
= \quad & \text{by Lemma 383 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{mult(\overbrace{mult(x1, op_r(x2, x1, x1))}, x1)} \\
= \quad & \text{by Lemma 871 RL with } \{x2 \leftarrow x1, x1 \leftarrow op_r(x2, x1, x1)\} \\
& \overbrace{mult(\underbrace{mult(op_r(x2, x1, x1), x1)}, op_t(x1, op_r(x2, x1, x1)))} \\
= \quad & \text{by Lemma 357 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& mult(\overbrace{rd(x1, i(x2))}, op_t(x1, op_r(x2, x1, x1)))
\end{aligned}
$$

Lemma 873: $op_t(op_r(mult(x1, x2), x1, x1), x1) = mult(op_r(x2, x1, x1), op_l(x1, x1, x2))$

$$
\begin{aligned}
& op_t(\underbrace{op_r(mult(x1, x2), x1, x1)}, x1) \\
= \quad & \text{by Lemma 383 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{op_t(\overbrace{mult(x1, op_r(x2, x1, x1))}, x1)} \\
= \quad & \text{by Lemma 264 RL with } \{x2 \leftarrow x1, x1 \leftarrow op_r(x2, x1, x1)\} \\
& \overbrace{mult(op_r(x2, x1, x1), \underbrace{op_l(x1, x1, op_r(x2, x1, x1))})} \\
= \quad & \text{by Lemma 501 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& mult(op_r(x2, x1, x1), \overbrace{op_l(x1, x1, x2)})
\end{aligned}
$$

Lemma 874: $op_r(x2, x1, i(x1)) = mult(asoc(x2, op_t(x1, x2), x2), op_t(x2, x1))$

$\overbrace{op_r(x2, x1, i(x1))}$

$=$ by Lemma 604 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{mult(x1, rd(x2, op_t(x1, x2)))}$

$=$ by Lemma 287 RL with $\{x3 \leftarrow x1, x2 \leftarrow i(rd(x2, op_t(x1, x2))), x1 \leftarrow mult(x1, rd(x2, op_t(x1, x2)))\}$

$\overbrace{op_l(mult(x1, rd(x2, op_t(x1, x2))), i(rd(x2, op_t(x1, x2))), op_t(\underbrace{i(i(rd(x2, op_t(x1, x2))))}, x1))}$

$=$ by Lemma 3 LR with $\{x1 \leftarrow rd(x2, op_t(x1, x2))\}$

$\underbrace{op_l(mult(x1, rd(x2, op_t(x1, x2))), i(rd(x2, op_t(x1, x2))), op_t(\overbrace{rd(x2, op_t(x1, x2))}, x1))}$

$=$ by Axiom 12 RL with $\{x3 \leftarrow mult(x1, rd(x2, op_t(x1, x2))), x2 \leftarrow i(rd(x2, op_t(x1, x2))), x1 \leftarrow op_t(rd(x2, op_t(x1, x2)), x1)\}$

$\overbrace{mult(i(mult(op_t(rd(x2, op_t(x1, x2)), x1), i(rd(x2, op_t(x1, x2))))), mult(op_t(rd(x2, op_t(x1, x2)), x1), \underbrace{mult(i(rd(x2, op_t(x1, x}$

$=$ by Axiom 10 LR with $\{x2 \leftarrow x1, x1 \leftarrow rd(x2, op_t(x1, x2))\}$

$mult(\underbrace{i(mult(op_t(rd(x2, op_t(x1, x2)), x1), i(rd(x2, op_t(x1, x2)))))}, mult(op_t(rd(x2, op_t(x1, x2)), x1), \overbrace{op_t(x1, rd(x2, op_t(x1, x}$

$=$ by Lemma 17 LR with $\{x2 \leftarrow rd(x2, op_t(x1, x2)), x1 \leftarrow op_t(rd(x2, op_t(x1, x2)), x1)\}$

$mult(\overbrace{mult(i(op_t(rd(x2, op_t(x1, x2)), x1)), rd(x2, op_t(x1, x2)))}, \underbrace{mult(op_t(rd(x2, op_t(x1, x2)), x1), op_t(x1, rd(x2, op_t(x1, x2}$

$=$ by Lemma 377 RL with $\{x1 \leftarrow x1, x2 \leftarrow rd(x2, op_t(x1, x2))\}$

$mult(\underbrace{mult(i(op_t(rd(x2, op_t(x1, x2)), x1)), rd(x2, op_t(x1, x2)))}, \overbrace{mult(rd(x2, op_t(x1, x2)), x1)})$

$=$ by Lemma 99 LR with $\{x2 \leftarrow x1, x1 \leftarrow rd(x2, op_t(x1, x2))\}$

$mult(\overbrace{rd(rd(x2, op_t(x1, x2)), op_t(rd(x2, op_t(x1, x2)), x1))}, \underbrace{mult(rd(x2, op_t(x1, x2)), x1)})$

$=$ by Lemma 597 LR with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$mult(rd(rd(x2, op_t(x1, x2)), \underbrace{op_t(rd(x2, op_t(x1, x2)), x1)}), \overbrace{op_t(x2, x1)})$

$=$ by Lemma 668 LR with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$mult(\underbrace{rd(rd(x2, op_t(x1, x2)), \overbrace{mult(i(op_t(x1, x2)), x2)})}, op_t(x2, x1))$

$=$ by Lemma 704 LR with $\{x2 \leftarrow op_t(x1, x2), x1 \leftarrow x2\}$

$mult(\overbrace{asoc(x2, op_t(x1, x2), x2)}, op_t(x2, x1))$

Lemma 875: $rd(op_t(x1,x2),asoc(x2,x1,x1)) = op_t(op_r(x1,x2,i(x2)),i(op_t(x1,x2)))$

$rd(op_t(x1,x2),\underbrace{asoc(x2,x1,x1)})$

$=$ by Lemma 756 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$rd(op_t(x1,x2),\overbrace{asoc(x1,i(x2),x1)})$

$=$ by Axiom 2 LR with $\{x2 \leftarrow op_t(x1,x2), x1 \leftarrow asoc(x1,i(x2),x1)\}$

$rd(op_t(x1,x2),\overbrace{mult(i(op_t(x1,x2)),\underbrace{mult(op_t(x1,x2),asoc(x1,i(x2),x1))})})$

$=$ by Lemma 179 RL with $\{x3 \leftarrow x2, x2 \leftarrow i(x2), x1 \leftarrow x1\}$

$rd(op_t(x1,x2),\overbrace{op_t(\underbrace{asoc(x1,i(x2),x1)},op_t(x1,x2))})$

$=$ by Lemma 756 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$rd(op_t(x1,x2),op_t(\overbrace{asoc(x2,x1,x1)},op_t(x1,x2)))$

$=$ by Lemma 757 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{rd(op_t(x1,x2),op_t(\overbrace{i(asoc(x1,x2,x1))},op_t(x1,x2)))}$

$=$ by Lemma 326 RL with $\{x2 \leftarrow op_t(x1,x2), x1 \leftarrow asoc(x1,x2,x1)\}$

$\overbrace{op_t(mult(\underbrace{asoc(x1,x2,x1)},op_t(x1,x2)),i(op_t(x1,x2)))}$

$=$ by Lemma 611 RL with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$op_t(mult(\overbrace{asoc(x2,x1,i(x1))},op_t(x1,x2)),i(op_t(x1,x2)))$

$=$ by Lemma 492 RL with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$op_t(mult(\overbrace{asoc(mult(x2,x1),x1,i(x1))},op_t(x1,x2)),i(op_t(x1,x2)))$

$=$ by Lemma 504 RL with $\{x2 \leftarrow mult(x2,x1), x1 \leftarrow x1\}$

$op_t(mult(\overbrace{asoc(x1,\underbrace{mult(i(x1),mult(x2,x1))},x1)},op_t(x1,x2)),i(op_t(x1,x2)))$

$=$ by Axiom 10 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$op_t(\underbrace{mult(asoc(x1,\overbrace{op_t(x2,x1)},x1),op_t(x1,x2))},i(op_t(x1,x2)))$

$=$ by Lemma 874 RL with $\{x1 \leftarrow x2, x2 \leftarrow x1\}$

$op_t(\overbrace{op_r(x1,x2,i(x2))},i(op_t(x1,x2)))$

Lemma 876: $op_t(x1, rd(x2, i(x1))) = mult(asoc(x1, rd(rd(i(x1), x2), x1), x1), op_t(x1, x2))$

$op_t(x1, \underbrace{rd(x2, i(x1))})$

$=$ by Lemma 10 RL with $\{x2 \leftarrow x2, x1 \leftarrow i(x1)\}$

$\underbrace{op_t(x1, \overbrace{i(rd(i(x1), x2))})}$

$=$ by Lemma 586 RL with $\{x2 \leftarrow rd(i(x1), x2), x1 \leftarrow x1\}$

$\overbrace{op_r(x1, rd(rd(i(x1), x2), x1), \underbrace{rd(x1, rd(i(x1), x2))})}$

$=$ by Lemma 10 RL with $\{x2 \leftarrow x1, x1 \leftarrow rd(i(x1), x2)\}$

$\underbrace{op_r(x1, rd(rd(i(x1), x2), x1), \overbrace{i(rd(rd(i(x1), x2), x1))})}$

$=$ by Lemma 874 LR with $\{x1 \leftarrow rd(rd(i(x1), x2), x1), x2 \leftarrow x1\}$

$\overbrace{mult(asoc(x1, \underbrace{op_t(rd(rd(i(x1), x2), x1), x1)}, x1), op_t(x1, rd(rd(i(x1), x2), x1)))}$

$=$ by Axiom 10 RL with $\{x2 \leftarrow rd(rd(i(x1), x2), x1), x1 \leftarrow x1\}$

$mult(\underbrace{asoc(x1, \overbrace{mult(i(x1), mult(rd(rd(i(x1), x2), x1), x1))}, x1)}, op_t(x1, rd(rd(i(x1), x2), x1)))$

$=$ by Lemma 504 LR with $\{x2 \leftarrow mult(rd(rd(i(x1), x2), x1), x1), x1 \leftarrow x1\}$

$mult(\overbrace{asoc(mult(rd(rd(i(x1), x2), x1), x1), x1, i(x1))}, op_t(x1, rd(rd(i(x1), x2), x1)))$

$=$ by Lemma 492 LR with $\{x2 \leftarrow x1, x1 \leftarrow rd(rd(i(x1), x2), x1)\}$

$mult(\overbrace{asoc(rd(rd(i(x1), x2), x1), x1, i(x1))}, op_t(x1, rd(rd(i(x1), x2), x1)))$

$=$ by Lemma 611 LR with $\{x2 \leftarrow x1, x1 \leftarrow rd(rd(i(x1), x2), x1)\}$

$mult(\overbrace{asoc(x1, rd(rd(i(x1), x2), x1), x1)}, \underbrace{op_t(x1, rd(rd(i(x1), x2), x1))})$

$=$ by Lemma 551 RL with $\{x2 \leftarrow rd(i(x1), x2), x1 \leftarrow x1\}$

$mult(asoc(x1, rd(rd(i(x1), x2), x1), x1), \overbrace{op_t(x1, mult(rd(i(x1), x2), i(x1)))})$

$=$ by Lemma 716 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$mult(asoc(x1, rd(rd(i(x1), x2), x1), x1), \overbrace{op_t(x1, \underbrace{rd(mult(x2, x1), x1)})})$

$=$ by Axiom 5 RL with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$mult(asoc(x1, rd(rd(i(x1), x2), x1), x1), op_t(x1, \overbrace{x2}))$

Lemma 877: $op_t(x1, mult(x2, x1)) = mult(asoc(x1, rd(i(x1), x2), x1), op_t(x1, x2))$

$\underbrace{op_t(x1, mult(x2, x1))}$

$=$ by Lemma 555 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_t(x1, rd(x2, i(x1)))}$

$=$ by Lemma 876 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{mult(\underbrace{asoc(x1, rd(rd(i(x1), x2), x1), x1)}, op_t(x1, x2))}$

$=$ by Lemma 263 LR with $\{x2 \leftarrow rd(i(x1), x2), x1 \leftarrow x1\}$

$mult(\overbrace{asoc(x1, rd(i(x1), x2), x1)}, op_t(x1, x2))$

Lemma 878: $op_t(x1, mult(x2, x1)) = rd(op_t(x1, x2), asoc(rd(i(x1), x2), x1, x1))$

$\overbrace{op_t(x1, mult(x2, x1))}$

$=$ by Lemma 555 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_t(x1, \underbrace{rd(x2, i(x1))})}$

$=$ by Lemma 10 RL with $\{x2 \leftarrow x2, x1 \leftarrow i(x1)\}$

$\underbrace{op_t(x1, \overbrace{i(rd(i(x1), x2))})}$

$=$ by Lemma 586 RL with $\{x2 \leftarrow rd(i(x1), x2), x1 \leftarrow x1\}$

$\overbrace{op_r(x1, \underbrace{rd(rd(i(x1), x2), x1)}, \underbrace{rd(x1, rd(i(x1), x2))})}$

$=$ by Lemma 10 RL with $\{x2 \leftarrow x1, x1 \leftarrow rd(i(x1), x2)\}$

$op_r(\underbrace{x1}, rd(rd(i(x1), x2), x1), \overbrace{i(rd(rd(i(x1), x2), x1))})$

$=$ by Lemma 48 RL with $\{x2 \leftarrow rd(rd(i(x1), x2), x1), x1 \leftarrow x1\}$

$\underbrace{op_r(\overbrace{op_t(x1, i(op_t(x1, rd(rd(i(x1), x2), x1))))}, rd(rd(i(x1), x2), x1), i(rd(rd(i(x1), x2), x1)))}$

$=$ by Axiom 14 RL with $\{x4 \leftarrow i(op_t(x1, rd(rd(i(x1), x2), x1))), x3 \leftarrow i(rd(rd(i(x1), x2), x1)), x2 \leftarrow rd(rd(i(x1), x2), x1), x1 \leftarrow$

$\overbrace{\underbrace{op_t(op_r(x1, rd(rd(i(x1), x2), x1), i(rd(rd(i(x1), x2), x1))), i(op_t(x1, rd(rd(i(x1), x2), x1))))}}$

$=$ by Lemma 875 RL with $\{x2 \leftarrow rd(rd(i(x1), x2), x1), x1 \leftarrow x1\}$

$\overbrace{rd(\underbrace{op_t(x1, rd(rd(i(x1), x2), x1))}, asoc(rd(rd(i(x1), x2), x1), x1, x1))}$

$=$ by Lemma 551 RL with $\{x2 \leftarrow rd(i(x1), x2), x1 \leftarrow x1\}$

$rd(\overbrace{op_t(x1, mult(rd(i(x1), x2), i(x1)))}, asoc(rd(rd(i(x1), x2), x1), x1, x1))$

$=$ by Lemma 716 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$rd(\overbrace{op_t(x1, \underbrace{rd(mult(x2, x1), x1)})}, asoc(rd(rd(i(x1), x2), x1), x1, x1))$

$=$ by Axiom 5 RL with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$rd(op_t(x1, \overbrace{x2}), \underbrace{asoc(rd(rd(i(x1), x2), x1), x1, x1)})$

$=$ by Lemma 774 LR with $\{x2 \leftarrow x1, x1 \leftarrow rd(i(x1), x2)\}$

$rd(op_t(x1, x2), \overbrace{asoc(rd(i(x1), x2), x1, x1)})$

Lemma 879: $rd(op_t(x1, x2), asoc(x1, x2, x1)) = op_t(x1, mult(x2, x1))$

$rd(op_t(x1, x2), \underbrace{asoc(x1, x2, x1)})$

$=$ by Lemma 782 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{rd(op_t(x1, x2), \overbrace{asoc(rd(i(x1), x2), x1, x1)})}$

$=$ by Lemma 878 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_t(x1, mult(x2, x1))}$

Lemma 880: $mult(x1, x2) = op_t(mult(x1, x2), mult(x1, i(op_t(x1, x2))))$

$\overbrace{mult(x1, x2)}$

$=$ by Lemma 585 RL with $\{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow mult(x1, x2)\}$

$\overbrace{op_t(mult(x1, x2), \underbrace{mult(mult(x1, x2), op_l(i(mult(x1, x2)), x2, x1))})}$

$=$ by Lemma 132 LR with $\{x3 \leftarrow i(mult(x1, x2)), x2 \leftarrow x2, x1 \leftarrow x1\}$

$op_t(mult(x1, x2), \overbrace{mult(x1, \underbrace{mult(x2, i(mult(x1, x2)))})})$

$=$ by Lemma 31 LR with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$op_t(mult(x1, x2), mult(x1, \overbrace{i(op_t(x1, x2))}))$

Lemma 881: $mult(mult(i(x1), x2), op_l(x1, x1, x2)) = op_t(x2, x1)$

$mult(\underbrace{mult(i(x1), x2)}, op_l(x1, x1, x2))$

$=$ by Lemma 14 RL with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$mult(\overbrace{op_t(rd(x2, x1), x1)}, op_l(x1, x1, x2))$

$=$ by Lemma 553 LR with $\{x2 \leftarrow x1, x1 \leftarrow rd(x2, x1)\}$

$mult(\overbrace{op_t(rd(x2, x1), \underbrace{op_l(x1, x1, rd(x2, x1))})}, op_l(x1, x1, x2))$

$=$ by Lemma 286 LR with $\{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x1\}$

$\underbrace{mult(op_t(rd(x2, x1), \overbrace{op_l(x1, x1, x2)}), op_l(x1, x1, x2))}$

$=$ by Lemma 526 LR with $\{x3 \leftarrow op_l(x1, x1, x2), x2 \leftarrow x1, x1 \leftarrow x2\}$

$\overbrace{op_t(\underbrace{rd(op_l(x1, x1, x2), rd(x1, x2))}, op_l(x1, x1, x2))}$

$=$ by Lemma 441 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{op_t(\overbrace{x2}, op_l(x1, x1, x2))}$

$=$ by Lemma 553 RL with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$\overbrace{op_t(x2, x1)}$

Lemma 882: $op_t(mult(x1,x2),x1) = rd(op_l(op_t(x1,rd(x2,x1)),x1,x2),i(op_t(x2,x1)))$

$\overbrace{op_t(mult(x1,x2),x1)}$

$=$ by Lemma 264 RL with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$\overbrace{\underbrace{mult(x2,op_l(x1,x1,x2))}}$

$=$ by Lemma 13 RL with $\{x2 \leftarrow x2, x1 \leftarrow op_l(x1,x1,x2)\}$

$\overbrace{mult(op_l(x1,x1,x2),\underbrace{op_t(x2,op_l(x1,x1,x2))})}$

$=$ by Lemma 553 RL with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$mult(op_l(x1,x1,x2),\overbrace{op_t(x2,x1)})$

$=$ by Lemma 881 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{mult(op_l(x1,x1,x2),\overbrace{mult(mult(i(x1),x2),op_l(x1,x1,x2))})}$

$=$ by Lemma 386 RL with $\{x2 \leftarrow op_l(x1,x1,x2), x1 \leftarrow mult(mult(i(x1),x2),op_l(x1,x1,x2))\}$

$\overbrace{rd(mult(mult(i(x1),x2),op_l(x1,x1,x2)),op_t(i(op_l(x1,x1,x2)),\underbrace{rd(op_l(x1,x1,x2),mult(mult(i(x1),x2),op_l(x1,x1,x2)))}))}$

$=$ by Lemma 9 LR with $\{x2 \leftarrow mult(i(x1),x2), x1 \leftarrow op_l(x1,x1,x2)\}$

$rd(mult(mult(i(x1),x2),op_l(x1,x1,x2)),\underbrace{op_t(i(op_l(x1,x1,x2)),\overbrace{i(mult(i(x1),x2))})})$

$=$ by Lemma 15 LR with $\{x2 \leftarrow mult(i(x1),x2), x1 \leftarrow op_l(x1,x1,x2)\}$

$\underbrace{rd(mult(mult(i(x1),x2),op_l(x1,x1,x2)),\overbrace{i(op_t(op_l(x1,x1,x2),mult(i(x1),x2)))})}$

$=$ by Lemma 38 LR with $\{x1 \leftarrow op_t(op_l(x1,x1,x2),mult(i(x1),x2)), x2 \leftarrow mult(mult(i(x1),x2),op_l(x1,x1,x2))\}$

$\overbrace{rd(op_t(op_l(x1,x1,x2),mult(i(x1),x2)),i(\underbrace{mult(mult(i(x1),x2),op_l(x1,x1,x2))}))}$

$=$ by Lemma 881 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$rd(\underbrace{op_t(op_l(x1,x1,x2),mult(i(x1),x2))},i(\overbrace{op_t(x2,x1)}))$

$=$ by Axiom 15 LR with $\{x4 \leftarrow mult(i(x1),x2), x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x1\}$

$rd(\overbrace{op_l(\underbrace{op_t(x1,mult(i(x1),x2))},x1,x2)},i(op_t(x2,x1)))$

$=$ by Lemma 375 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$rd(op_l(\overbrace{op_t(x1,rd(x2,x1))},x1,x2),i(op_t(x2,x1)))$

Lemma 883: $op_t(mult(x1,x2),x1) = rd(op_t(x2,x1),op_t(i(x1),x2))$

$\overbrace{op_t(mult(x1,x2),x1)}$

$=$ by Lemma 882 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{rd(\underbrace{op_l(op_t(x1,rd(x2,x1)),x1,x2)},i(op_t(x2,x1)))}$

$=$ by Lemma 511 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{rd(\overbrace{op_t(x1,i(x2))},i(op_t(x2,x1)))}$

$=$ by Lemma 65 LR with $\{x3 \leftarrow op_t(x2,x1), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{rd(op_t(x2,x1),op_t(i(x1),x2))}$

Lemma 884: $mult(x1, x1) = mult(op_l(x1, x1, x2), op_t(op_l(x1, x2, x1), x1))$

$$
\begin{aligned}
& \overbrace{mult(x1, x1)} \\
= \quad & \text{by Lemma 204 RL with } \{x2 \leftarrow asoc(x1, x2, x1), x1 \leftarrow x1\} \\
& \overbrace{mult(mult(x1, asoc(x1, x2, x1)), op_t(\underbrace{rd(x1, asoc(x1, x2, x1))}, x1))} \\
= \quad & \text{by Lemma 620 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& mult(\underbrace{mult(x1, asoc(x1, x2, x1))}, op_t(\overbrace{op_l(x1, x2, x1)}, x1)) \\
= \quad & \text{by Lemma 158 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& mult(\overbrace{op_l(x1, x1, x2)}, op_t(op_l(x1, x2, x1), x1))
\end{aligned}
$$

Lemma 885: $rd(x2, x1) = rd(op_t(x2, x1), rd(mult(x2, x1), x2))$

$$
\begin{aligned}
& \underbrace{rd(x2, x1)} \\
= \quad & \text{by Lemma 10 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{i(rd(x1, x2))}} \\
= \quad & \text{by Lemma 9 RL with } \{x2 \leftarrow rd(x1, x2), x1 \leftarrow op_t(x2, x1)\} \\
& \overbrace{rd(op_t(x2, x1), \underbrace{mult(rd(x1, x2), op_t(x2, x1))})} \\
= \quad & \text{by Lemma 650 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& rd(op_t(x2, x1), \overbrace{rd(mult(x2, x1), x2)})
\end{aligned}
$$

Lemma 886: $op_t(mult(x1, rd(x2, x1)), mult(x1, x2)) = mult(i(op_r(x1, x2, i(mult(x1, x2)))), mult(x1, x2))$

$$
\begin{aligned}
& op_t(\underbrace{mult(x1, rd(x2, x1))}, mult(x1, x2)) \\
= \quad & \text{by Lemma 55 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{op_t(\overbrace{mult(mult(x1, x2), i(x1))}, mult(x1, x2))} \\
= \quad & \text{by Lemma 318 RL with } \{x2 \leftarrow mult(x1, x2), x1 \leftarrow i(x1)\} \\
& \overbrace{mult(\underbrace{op_r(i(x1), mult(x1, x2), i(mult(x1, x2)))}, mult(x1, x2))} \\
= \quad & \text{by Lemma 654 LR with } \{x3 \leftarrow i(mult(x1, x2)), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& mult(\overbrace{i(op_r(x1, x2, i(mult(x1, x2))))}, mult(x1, x2))
\end{aligned}
$$

Lemma 887: $op_t(mult(x1, rd(x2, x1)), mult(x1, x2)) = op_t(x2, x1)$

$$
\begin{aligned}
& \underbrace{op_t(mult(x1, rd(x2, x1)), mult(x1, x2))} \\
= \quad & \text{by Lemma 886 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{mult(i(\underbrace{op_r(x1, x2, i(mult(x1, x2)))}), mult(x1, x2))} \\
= \quad & \text{by Lemma 79 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{mult(i(\overbrace{op_t(x1, x2)}), mult(x1, x2))} \\
= \quad & \text{by Lemma 384 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_t(x2, x1)}
\end{aligned}
$$

Lemma 888: $rd(mult(x1, mult(x2, x3)), op_l(x3, x2, x1)) = mult(x1, x2)$

$$
\begin{aligned}
& rd(mult(x1, mult(x2, x3)), \underbrace{op_l(x3, x2, x1)}) \\
= & \quad \text{by Axiom 12 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{rd(mult(x1, mult(x2, x3)), \overbrace{mult(i(mult(x1, x2)), mult(x1, mult(x2, x3)))})} \\
= & \quad \text{by Lemma 9 LR with } \{x2 \leftarrow i(mult(x1, x2)), x1 \leftarrow mult(x1, mult(x2, x3))\} \\
& \overbrace{\underbrace{i(i(mult(x1, x2)))}} \\
= & \quad \text{by Lemma 3 LR with } \{x1 \leftarrow mult(x1, x2)\} \\
& \overbrace{mult(x1, x2)}
\end{aligned}
$$

Lemma 889: $op_r(op_t(x1, x2), mult(x1, x2), mult(x1, x2)) = x1$

$$
\begin{aligned}
& \underbrace{op_r(op_t(x1, x2), mult(x1, x2), mult(x1, x2))} \\
= & \quad \text{by Lemma 353 RL with } \{x2 \leftarrow op_t(x1, x2), x1 \leftarrow mult(x1, x2)\} \\
& \overbrace{rd(mult(x1, x2), \underbrace{rd(mult(x1, x2), op_t(x1, x2))})} \\
= & \quad \text{by Lemma 29 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{rd(mult(x1, x2), \overbrace{x2})} \\
= & \quad \text{by Axiom 5 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{x1}
\end{aligned}
$$

Lemma 890: $op_l(x1, op_t(x1, x2), x3) = op_r(x1, x3, op_t(x1, x2))$

$$
\begin{aligned}
& op_l(\underbrace{x1}, op_t(x1, x2), x3) \\
= & \quad \text{by Lemma 889 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{op_l(\overbrace{op_r(op_t(x1, x2), mult(x1, x2), mult(x1, x2))}, op_t(x1, x2), x3)} \\
= & \quad \text{by Lemma 739 RL with } \{x4 \leftarrow x3, x3 \leftarrow mult(x1, x2), x2 \leftarrow mult(x1, x2), x1 \leftarrow op_t(x1, x2)\} \\
& \overbrace{op_r(\underbrace{op_r(op_t(x1, x2), mult(x1, x2), mult(x1, x2))}, x3, op_t(x1, x2))} \\
= & \quad \text{by Lemma 889 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_r(\overbrace{x1}, x3, op_t(x1, x2))
\end{aligned}
$$

Lemma 891: $rd(mult(x1, x2), mult(mult(x3, x1), x2)) = i(op_r(x3, x1, x2))$

$$
\begin{aligned}
& \underbrace{rd(mult(x1, x2), mult(mult(x3, x1), x2))} \\
= & \quad \text{by Lemma 10 RL with } \{x2 \leftarrow mult(x1, x2), x1 \leftarrow mult(mult(x3, x1), x2)\} \\
& \overbrace{i(\underbrace{rd(mult(mult(x3, x1), x2), mult(x1, x2))})} \\
= & \quad \text{by Axiom 16 RL with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x3\} \\
& i(\overbrace{op_r(x3, x1, x2)})
\end{aligned}
$$

Lemma 892: $mult(rd(x2,x1),x1) = op_r(op_r(x2,x1,x1),rd(x1,x2),i(x1))$

$$
\begin{aligned}
& mult(\underbrace{rd(x2,x1)}, x1) \\
= & \quad \text{by Lemma 10 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{mult(\overbrace{i(rd(x1,x2))}, x1)} \\
= & \quad \text{by Lemma 82 LR with } \{x2 \leftarrow x1, x1 \leftarrow rd(x1,x2)\} \\
& \overbrace{op_r(\underbrace{rd(x1,rd(x1,x2))}, rd(x1,x2), i(x1))} \\
= & \quad \text{by Lemma 353 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_r(\overbrace{op_r(x2,x1,x1)}, rd(x1,x2), i(x1))
\end{aligned}
$$

Lemma 893: $x1 = mult(op_l(x2,x1,x2),rd(x1,op_t(x2,x1)))$

$$
\begin{aligned}
& \underbrace{x1} \\
= & \quad \text{by Lemma 3 RL with } \{x1 \leftarrow x1\} \\
& \underbrace{\overbrace{i(i(x1))}} \\
= & \quad \text{by Lemma 20 RL with } \{x2 \leftarrow i(x1), x1 \leftarrow op_l(x2,x1,x2)\} \\
& \overbrace{mult(op_l(x2,x1,x2), i(\underbrace{mult(op_l(x2,x1,x2), i(x1))}))} \\
= & \quad \text{by Lemma 451 RL with } \{x1 \leftarrow x1, x2 \leftarrow x2\} \\
& mult(op_l(x2,x1,x2), \underbrace{i(\overbrace{rd(op_t(x2,x1),x1)})}) \\
= & \quad \text{by Lemma 10 LR with } \{x2 \leftarrow x1, x1 \leftarrow op_t(x2,x1)\} \\
& mult(op_l(x2,x1,x2), \overbrace{rd(x1,op_t(x2,x1))})
\end{aligned}
$$

Lemma 894: $rd(mult(x1,x2),x1) = mult(op_l(x1,mult(x1,x2),x1),rd(rd(mult(x1,x2),x1),op_t(x1,x2)))$

$$
\begin{aligned}
& \underbrace{rd(mult(x1,x2),x1)} \\
= & \quad \text{by Lemma 893 LR with } \{x2 \leftarrow x1, x1 \leftarrow rd(mult(x1,x2),x1)\} \\
& \overbrace{mult(op_l(x1,rd(mult(x1,x2),x1),x1), rd(rd(mult(x1,x2),x1), \underbrace{op_t(x1,rd(mult(x1,x2),x1))}))} \\
= & \quad \text{by Lemma 378 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& mult(\underbrace{op_l(x1,rd(mult(x1,x2),x1),x1)}, rd(rd(mult(x1,x2),x1), \overbrace{op_t(x1,x2)})) \\
= & \quad \text{by Lemma 292 LR with } \{x3 \leftarrow x1, x2 \leftarrow mult(x1,x2), x1 \leftarrow x1\} \\
& mult(\overbrace{op_l(x1,mult(x1,x2),x1)}, rd(rd(mult(x1,x2),x1),op_t(x1,x2)))
\end{aligned}
$$

Lemma 895: $mult(op_t(x1,x2),rd(x2,x1)) = rd(op_t(mult(x2,x1),x2),x1)$

$mult(op_t(x1,x2),\underbrace{rd(x2,x1)})$

$=$ by Lemma 440 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$mult(op_t(x1,x2),\overbrace{mult(i(x1),\underbrace{op_l(x2,x2,x1)})})$

$=$ by Lemma 281 LR with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$mult(op_t(x1,x2),mult(i(x1),\overbrace{mult(i(x1),op_t(\underbrace{mult(x2,x1),x2)}))}))$

$=$ by Lemma 14 RL with $\{x2 \leftarrow x1, x1 \leftarrow op_t(mult(x2,x1),x2)\}$

$mult(\underbrace{op_t(x1,x2)},mult(i(x1),\overbrace{op_t(rd(op_t(mult(x2,x1),x2),x1),x1)}))$

$=$ by Lemma 559 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$mult(\overbrace{op_t(x1,\underbrace{op_t(x2,i(x1))})},mult(i(x1),op_t(rd(op_t(mult(x2,x1),x2),x1),x1)))$

$=$ by Axiom 5 LR with $\{x2 \leftarrow x1, x1 \leftarrow op_t(x2,i(x1))\}$

$mult(op_t(x1,\overbrace{rd(\underbrace{mult(op_t(x2,i(x1)),x1)},x1)}),mult(i(x1),op_t(rd(op_t(mult(x2,x1),x2),x1),x1)))$

$=$ by Lemma 456 RL with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$mult(\underbrace{op_t(x1,rd(\overbrace{op_t(op_t(mult(x2,x1),x2),x1)},x1))},mult(i(x1),op_t(rd(op_t(mult(x2,x1),x2),x1),x1)))$

$=$ by Lemma 574 LR with $\{x2 \leftarrow op_t(mult(x2,x1),x2), x1 \leftarrow x1\}$

$mult(\overbrace{\underbrace{op_t(x1,rd(op_t(mult(x2,x1),x2),x1))}},mult(i(x1),op_t(rd(op_t(mult(x2,x1),x2),x1),x1)))$

$=$ by Lemma 3 RL with $\{x1 \leftarrow op_t(x1,rd(op_t(mult(x2,x1),x2),x1))\}$

$mult(\overbrace{i(i(op_t(x1,rd(op_t(mult(x2,x1),x2),x1))))},\underbrace{mult(i(x1),op_t(rd(op_t(mult(x2,x1),x2),x1),x1))})$

$=$ by Lemma 669 RL with $\{x2 \leftarrow rd(op_t(mult(x2,x1),x2),x1), x1 \leftarrow x1\}$

$\underbrace{mult(i(i(op_t(x1,rd(op_t(mult(x2,x1),x2),x1)))),\overbrace{mult(i(op_t(x1,rd(op_t(mult(x2,x1),x2),x1))),rd(op_t(mult(x2,x1),x2),x}}$

$=$ by Axiom 2 RL with $\{x2 \leftarrow i(op_t(x1,rd(op_t(mult(x2,x1),x2),x1))), x1 \leftarrow rd(op_t(mult(x2,x1),x2),x1)\}$

$\overbrace{rd(op_t(mult(x2,x1),x2),x1)}$

Lemma 896: $x1 = op_t(op_t(x1, op_l(op_t(x1, x2), x3, x1)), i(op_l(op_t(x1, x2), x3, x1)))$

$\overbrace{x1}$

$=$ by Lemma 843 LR with $\{x2 \leftarrow i(mult(x1, x3)), x1 \leftarrow x1\}$

$\overbrace{op_r(\underbrace{mult(i(mult(x1, x3)), rd(x1, i(mult(x1, x3))))}, i(mult(x1, x3)), i(x1))}$

$=$ by Lemma 157 RL with $\{x3 \leftarrow rd(i(op_l(op_t(x1, x2), x3, x1)), op_l(op_t(x1, x2), x3, x1)), x2 \leftarrow x2, x1 \leftarrow mult(i(mult(x1, x3)), r$

$\underbrace{op_r(\overbrace{op_r(mult(i(mult(x1, x3)), rd(x1, i(mult(x1, x3)))), op_t(mult(i(mult(x1, x3)), rd(x1, i(mult(x1, x3)))), x2)}, rd(i(op_l(op$

$=$ by Axiom 17 LR with $\{x5 \leftarrow i(x1), x4 \leftarrow i(mult(x1, x3)), x3 \leftarrow rd(i(op_l(op_t(x1, x2), x3, x1)), op_l(op_t(x1, x2), x3, x1)), x2 \leftarrow$

$\overbrace{op_r(\underbrace{op_r(mult(i(mult(x1, x3)), rd(x1, i(mult(x1, x3)))), i(mult(x1, x3)), i(x1))}, op_t(mult(i(mult(x1, x3)), rd(x1, i(mult(x1,$

$=$ by Lemma 843 RL with $\{x2 \leftarrow i(mult(x1, x3)), x1 \leftarrow x1\}$

$op_r(\overbrace{x1}, op_t(mult(i(mult(x1, x3)), \underbrace{rd(x1, i(mult(x1, x3)))}), x2), rd(i(op_l(op_t(x1, x2), x3, x1)), op_l(op_t(x1, x2), x3, x1)))$

$=$ by Lemma 58 LR with $\{x2 \leftarrow x3, x1 \leftarrow x1\}$

$op_r(x1, op_t(\underbrace{mult(i(mult(x1, x3)), \overbrace{mult(x1, mult(x3, x1))})}, x2), rd(i(op_l(op_t(x1, x2), x3, x1)), op_l(op_t(x1, x2), x3, x1)))$

$=$ by Axiom 12 LR with $\{x3 \leftarrow x1, x2 \leftarrow x3, x1 \leftarrow x1\}$

$op_r(x1, \underbrace{op_t(\overbrace{op_l(x1, x3, x1)}, x2)}, rd(i(op_l(op_t(x1, x2), x3, x1)), op_l(op_t(x1, x2), x3, x1)))$

$=$ by Axiom 15 LR with $\{x4 \leftarrow x2, x3 \leftarrow x1, x2 \leftarrow x3, x1 \leftarrow x1\}$

$op_r(x1, \overbrace{op_l(op_t(x1, x2), x3, x1)}, \underbrace{rd(i(op_l(op_t(x1, x2), x3, x1)), op_l(op_t(x1, x2), x3, x1))})$

$=$ by Lemma 43 RL with $\{x1 \leftarrow op_l(op_t(x1, x2), x3, x1)\}$

$op_r(x1, \underbrace{op_l(op_t(x1, x2), x3, x1)}, \overbrace{i(mult(op_l(op_t(x1, x2), x3, x1), op_l(op_t(x1, x2), x3, x1))))}$

$=$ by Lemma 3 RL with $\{x1 \leftarrow op_l(op_t(x1, x2), x3, x1)\}$

$op_r(x1, \overbrace{i(i(op_l(op_t(x1, x2), x3, x1)))}, \underbrace{i(mult(op_l(op_t(x1, x2), x3, x1), op_l(op_t(x1, x2), x3, x1))))}$

$=$ by Axiom 11 RL with $\{x2 \leftarrow op_l(op_t(x1, x2), x3, x1), x1 \leftarrow op_l(op_t(x1, x2), x3, x1)\}$

$\underbrace{op_r(x1, i(i(op_l(op_t(x1, x2), x3, x1))), \overbrace{mult(i(op_l(op_t(x1, x2), x3, x1)), i(op_l(op_t(x1, x2), x3, x1))))}}$

$=$ by Lemma 665 RL with $\{x3 \leftarrow i(op_l(op_t(x1, x2), x3, x1)), x2 \leftarrow i(op_l(op_t(x1, x2), x3, x1)), x1 \leftarrow x1\}$

$\overbrace{rd(\underbrace{mult(mult(x1, i(i(op_l(op_t(x1, x2), x3, x1))))), mult(i(op_l(op_t(x1, x2), x3, x1)), i(op_l(op_t(x1, x2), x3, x1))))}, i(op_l(op_t(x1,$

$=$ by Lemma 350 RL with $\{x2 \leftarrow mult(x1, i(i(op_l(op_t(x1, x2), x3, x1)))), x1 \leftarrow i(op_l(op_t(x1, x2), x3, x1))\}$

$\underbrace{rd(\overbrace{mult(op_t(mult(i(op_l(op_t(x1, x2), x3, x1)), mult(x1, i(i(op_l(op_t(x1, x2), x3, x1))))), i(op_l(op_t(x1, x2), x3, x1))), i(op_l(op_t($

$=$ by Axiom 5 RL with $\{x2 \leftarrow i(op_l(op_t(x1, x2), x3, x1)), x1 \leftarrow op_t(mult(i(op_l(op_t(x1, x2), x3, x1)), mult(x1, i(i(op_l(op_t(x1, x2)$

$\overbrace{op_t(\underbrace{mult(i(op_l(op_t(x1, x2), x3, x1)), mult(x1, i(i(op_l(op_t(x1, x2), x3, x1)))))}, i(op_l(op_t(x1, x2), x3, x1)))}$

$=$ by Lemma 30 LR with $\{x2 \leftarrow x1, x1 \leftarrow i(op_l(op_t(x1, x2), x3, x1))\}$

$\underbrace{op_t(\overbrace{op_t(x1, i(i(op_l(op_t(x1, x2), x3, x1))))}, i(op_l(op_t(x1, x2), x3, x1)))}$

$=$ by Axiom 13 LR with $\{x3 \leftarrow i(op_l(op_t(x1, x2), x3, x1)), x2 \leftarrow i(i(op_l(op_t(x1, x2), x3, x1))), x1 \leftarrow x1\}$

$\overbrace{op_t(op_t(x1, i(op_l(op_t(x1, x2), x3, x1))), \underbrace{i(i(op_l(op_t(x1, x2), x3, x1))))}}$

$=$ by Lemma 3 LR with $\{x1 \leftarrow op_l(op_t(x1, x2), x3, x1)\}$

$\underbrace{op_t(op_t(x1, i(op_l(op_t(x1, x2), x3, x1))), \overbrace{op_l(op_t(x1, x2), x3, x1)})}$

$=$ by Axiom 13 LR with $\{x3 \leftarrow op_l(op_t(x1, x2), x3, x1), x2 \leftarrow i(op_l(op_t(x1, x2), x3, x1)), x1 \leftarrow x1\}$

$\overbrace{op_t(op_t(x1, op_l(op_t(x1, x2), x3, x1)), i(op_l(op_t(x1, x2), x3, x1)))}$

Lemma 897: $x1 = op_t(x1, i(op_l(op_t(x1, x2), x3, x1)))$

$\overbrace{x1}$

$=$ by Lemma 896 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_t(op_t(x1, \underbrace{op_l(op_t(x1, x2), x3, x1)}), i(op_l(op_t(x1, x2), x3, x1)))}$

$=$ by Axiom 15 RL with $\{x4 \leftarrow x2, x3 \leftarrow x1, x2 \leftarrow x3, x1 \leftarrow x1\}$

$op_t(op_t(x1, \overbrace{op_t(\underbrace{op_l(x1, x3, x1)}, x2)}), i(op_l(op_t(x1, x2), x3, x1)))$

$=$ by Lemma 720 RL with $\{x2 \leftarrow x3, x1 \leftarrow x1\}$

$op_t(op_t(\underbrace{x1}, op_t(\overbrace{mult(rd(x1, x3), op_r(x3, x1, x1))}, x2)), i(op_l(op_t(x1, x2), x3, x1)))$

$=$ by Lemma 366 RL with $\{x2 \leftarrow x1, x1 \leftarrow x3\}$

$op_t(op_t(\overbrace{mult(op_r(x3, x1, x1), rd(x1, x3))}, op_t(mult(rd(x1, x3), op_r(x3, x1, x1)), x2)), i(op_l(op_t(x1, x2), x3, x1)))$

$=$ by Lemma 435 RL with $\{x2 \leftarrow op_r(x3, x1, x1), x1 \leftarrow rd(x1, x3)\}$

$op_t(\underbrace{op_t(\overbrace{op_l(mult(rd(x1, x3), op_r(x3, x1, x1)), rd(x1, x3), op_r(x3, x1, x1))}, op_t(mult(rd(x1, x3), op_r(x3, x1, x1)), x2))}, i(op_l($

$=$ by Axiom 15 LR with $\{x4 \leftarrow op_t(mult(rd(x1, x3), op_r(x3, x1, x1)), x2), x3 \leftarrow op_r(x3, x1, x1), x2 \leftarrow rd(x1, x3), x1 \leftarrow mult(rd$

$op_t(\overbrace{op_l(\underbrace{op_t(mult(rd(x1, x3), op_r(x3, x1, x1)), op_t(mult(rd(x1, x3), op_r(x3, x1, x1)), x2))}, rd(x1, x3), op_r(x3, x1, x1))}, i(op_l($

$=$ by Axiom 10 RL with $\{x2 \leftarrow mult(rd(x1, x3), op_r(x3, x1, x1)), x1 \leftarrow op_t(mult(rd(x1, x3), op_r(x3, x1, x1)), x2)\}$

$op_t(op_l(\overbrace{mult(i(op_t(mult(rd(x1, x3), op_r(x3, x1, x1)), x2)), \underbrace{mult(mult(rd(x1, x3), op_r(x3, x1, x1)), op_t(mult(rd(x1, x3), op_r$

$=$ by Lemma 27 RL with $\{x2 \leftarrow x2, x1 \leftarrow mult(rd(x1, x3), op_r(x3, x1, x1))\}$

$op_t(op_l(\underbrace{mult(i(op_t(mult(rd(x1, x3), op_r(x3, x1, x1)), x2)), \overbrace{mult(op_t(mult(rd(x1, x3), op_r(x3, x1, x1)), x2), mult(rd(x1, x3)}$

$=$ by Axiom 2 RL with $\{x2 \leftarrow op_t(mult(rd(x1, x3), op_r(x3, x1, x1)), x2), x1 \leftarrow mult(rd(x1, x3), op_r(x3, x1, x1))\}$

$op_t(\underbrace{op_l(\overbrace{mult(rd(x1, x3), op_r(x3, x1, x1))}, rd(x1, x3), op_r(x3, x1, x1))}, i(op_l(op_t(x1, x2), x3, x1)))$

$=$ by Lemma 435 LR with $\{x2 \leftarrow op_r(x3, x1, x1), x1 \leftarrow rd(x1, x3)\}$

$op_t(\underbrace{\overbrace{mult(op_r(x3, x1, x1), rd(x1, x3))}}, i(op_l(op_t(x1, x2), x3, x1)))$

$=$ by Lemma 366 LR with $\{x2 \leftarrow x1, x1 \leftarrow x3\}$

$op_t(\overbrace{\underbrace{x1}}, i(op_l(op_t(x1, x2), x3, x1)))$

Lemma 898: $i(x1) = op_t(i(x1), op_l(op_t(x1, x2), x1, x3))$

$$
\begin{aligned}
& \underbrace{i(x1)} \\
= \quad & \text{by Lemma 897 LR with } \{x3 \leftarrow x3, x2 \leftarrow i(x2), x1 \leftarrow i(x1)\} \\
& \overbrace{op_t(i(x1), i(\underbrace{op_l(op_t(i(x1), i(x2)), x3, i(x1))}))} \\
= \quad & \text{by Axiom 15 RL with } \{x4 \leftarrow i(x2), x3 \leftarrow i(x1), x2 \leftarrow x3, x1 \leftarrow i(x1)\} \\
& op_t(i(x1), \underbrace{i(\overbrace{op_t(op_l(i(x1), x3, i(x1)), i(x2))}))} \\
= \quad & \text{by Lemma 15 RL with } \{x2 \leftarrow i(x2), x1 \leftarrow op_l(i(x1), x3, i(x1))\} \\
& op_t(i(x1), \overbrace{op_t(\underbrace{i(op_l(i(x1), x3, i(x1)))}, i(i(x2)))}) \\
= \quad & \text{by Lemma 433 LR with } \{x2 \leftarrow x3, x1 \leftarrow i(x1)\} \\
& op_t(i(x1), \underbrace{op_t(\overbrace{op_l(i(i(x1)), x3, i(x1))}, i(i(x2)))}) \\
= \quad & \text{by Axiom 15 LR with } \{x4 \leftarrow i(i(x2)), x3 \leftarrow i(x1), x2 \leftarrow x3, x1 \leftarrow i(i(x1))\} \\
& op_t(i(x1), \overbrace{op_l(\underbrace{op_t(i(i(x1)), i(i(x2)))}, x3, i(x1))}) \\
= \quad & \text{by Lemma 15 LR with } \{x2 \leftarrow i(x2), x1 \leftarrow i(x1)\} \\
& op_t(i(x1), op_l(\overbrace{i(\underbrace{op_t(i(x1), i(x2))})}, x3, i(x1))) \\
= \quad & \text{by Lemma 15 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_t(i(x1), op_l(\underbrace{i(\overbrace{i(op_t(x1, x2))})}, x3, i(x1))) \\
= \quad & \text{by Lemma 3 LR with } \{x1 \leftarrow op_t(x1, x2)\} \\
& op_t(i(x1), \underbrace{op_l(\overbrace{op_t(x1, x2)}, x3, i(x1)))} \\
= \quad & \text{by Axiom 15 RL with } \{x4 \leftarrow x2, x3 \leftarrow i(x1), x2 \leftarrow x3, x1 \leftarrow x1\} \\
& op_t(i(x1), \overbrace{op_t(\underbrace{op_l(x1, x3, i(x1))}, x2)}) \\
= \quad & \text{by Lemma 606 LR with } \{x2 \leftarrow x3, x1 \leftarrow x1\} \\
& op_t(i(x1), \underbrace{op_t(\overbrace{op_l(x1, x1, x3)}, x2)}) \\
= \quad & \text{by Axiom 15 LR with } \{x4 \leftarrow x2, x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow x1\} \\
& op_t(i(x1), \overbrace{op_l(op_t(x1, x2), x1, x3)})
\end{aligned}
$$

Lemma 899: $op_t(x1, i(mult(i(x1), op_l(op_t(x1, x2), x1, x3)))) = op_l(i(i(x1)), op_l(op_t(x1, x2), x1, x3), i(x1))$

$$
\begin{aligned}
& \underbrace{op_t(x1, i(mult(i(x1), op_l(op_t(x1, x2), x1, x3))))} \\
= \quad & \text{by Lemma 18 RL with } \{x2 \leftarrow mult(i(x1), op_l(op_t(x1, x2), x1, x3)), x1 \leftarrow x1\} \\
& \overbrace{\underbrace{i(op_t(i(x1), mult(i(x1), op_l(op_t(x1, x2), x1, x3))))}} \\
= \quad & \text{by Lemma 479 LR with } \{x1 \leftarrow op_l(op_t(x1, x2), x1, x3), x2 \leftarrow i(x1)\} \\
& \overbrace{op_l(i(\underbrace{op_t(i(x1), op_l(op_t(x1, x2), x1, x3))}), op_l(op_t(x1, x2), x1, x3), i(x1))} \\
= \quad & \text{by Lemma 898 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_l(i(\overbrace{i(x1)}), op_l(op_t(x1, x2), x1, x3), i(x1))
\end{aligned}
$$

Lemma 900: $op_t(x1, rd(x1, op_l(op_t(x1, x2), x1, x3))) = op_l(i(i(x1)), op_l(op_t(x1, x2), x1, x3), i(x1))$

$\underbrace{op_t(x1, rd(x1, op_l(op_t(x1, x2), x1, x3)))}$

$=$ by Lemma 564 RL with $\{x2 \leftarrow op_l(op_t(x1, x2), x1, x3), x1 \leftarrow x1\}$

$\overbrace{op_t(x1, \underbrace{mult(x1, i(op_l(op_t(x1, x2), x1, x3)))})}$

$=$ by Lemma 16 RL with $\{x2 \leftarrow op_l(op_t(x1, x2), x1, x3), x1 \leftarrow x1\}$

$\underbrace{op_t(x1, \overbrace{i(mult(i(x1), op_l(op_t(x1, x2), x1, x3)))})}$

$=$ by Lemma 899 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_l(i(i(x1)), op_l(op_t(x1, x2), x1, x3), i(x1))}$

Lemma 901: $op_t(x1, rd(x1, op_l(op_t(x1, x2), x1, x3))) = op_l(x1, x1, op_l(op_t(x1, x2), x1, x3))$

$\underbrace{op_t(x1, rd(x1, op_l(op_t(x1, x2), x1, x3)))}$

$=$ by Lemma 900 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_l(\underbrace{i(i(x1))}, op_l(op_t(x1, x2), x1, x3), i(x1))}$

$=$ by Lemma 3 LR with $\{x1 \leftarrow x1\}$

$\underbrace{op_l(\overbrace{x1}, op_l(op_t(x1, x2), x1, x3), i(x1))}$

$=$ by Lemma 606 LR with $\{x2 \leftarrow op_l(op_t(x1, x2), x1, x3), x1 \leftarrow x1\}$

$\overbrace{op_l(x1, x1, op_l(op_t(x1, x2), x1, x3))}$

Lemma 902: $op_r(i(x1), x2, i(x2)) = op_t(i(x1), mult(x1, x2))$

$\underbrace{op_r(i(x1), x2, i(x2))}$

$=$ by Lemma 325 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{\underbrace{mult(i(mult(x1, x2)), x2)}}$

$=$ by Lemma 14 RL with $\{x2 \leftarrow mult(x1, x2), x1 \leftarrow x2\}$

$\overbrace{op_t(\underbrace{rd(x2, mult(x1, x2))}, mult(x1, x2))}$

$=$ by Lemma 9 LR with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$op_t(\overbrace{i(x1)}, mult(x1, x2))$

Lemma 903: $rd(x1, op_t(op_t(x2, i(x1)), x1)) = op_t(op_t(rd(x1, x2), x1), i(x1))$

$rd(x1, op_t(\underbrace{op_t(x2, i(x1))}, x1))$

$=$ by Lemma 18 RL with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$\underbrace{rd(x1, op_t(\overbrace{i(op_t(i(x2), x1))}, x1))}$

$=$ by Lemma 326 RL with $\{x2 \leftarrow x1, x1 \leftarrow op_t(i(x2), x1)\}$

$\overbrace{op_t(\underbrace{mult(op_t(i(x2), x1), x1)}, i(x1))}$

$=$ by Lemma 340 LR with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$op_t(\overbrace{op_t(rd(x1, x2), x1)}, i(x1))$

Lemma 904: $mult(x1, rd(i(x2), x1)) = op_l(i(x2), x1, x2)$

$\overbrace{mult(x1, rd(i(x2), x1))}$

$=$ by Lemma 55 RL with $\{x2 \leftarrow i(x2), x1 \leftarrow x1\}$

$\overbrace{mult(\underbrace{mult(x1, i(x2))}, i(x1))}$

$=$ by Lemma 34 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{mult(\overbrace{op_t(rd(x1, x2), i(x1))}, i(x1))}$

$=$ by Lemma 337 RL with $\{x2 \leftarrow i(x1), x1 \leftarrow rd(x1, x2)\}$

$\underbrace{\overbrace{op_t(rd(rd(x1, x2), i(i(x1))), i(x1))}}$

$=$ by Lemma 213 RL with $\{x2 \leftarrow rd(x1, x2), x1 \leftarrow i(x1)\}$

$\overbrace{mult(\underbrace{mult(i(i(x1)), rd(x1, x2))}, mult(i(x1), i(x1)))}$

$=$ by Lemma 17 RL with $\{x2 \leftarrow rd(x1, x2), x1 \leftarrow i(x1)\}$

$mult(\overbrace{i(mult(i(x1), i(rd(x1, x2))))}, mult(i(x1), \underbrace{i(x1)}))$

$=$ by Axiom 2 LR with $\{x2 \leftarrow rd(x1, x2), x1 \leftarrow i(x1)\}$

$\underbrace{mult(i(mult(i(x1), i(rd(x1, x2)))), mult(i(x1), \overbrace{mult(i(rd(x1, x2)), mult(rd(x1, x2), i(x1)))}))}$

$=$ by Axiom 12 LR with $\{x3 \leftarrow mult(rd(x1, x2), i(x1)), x2 \leftarrow i(rd(x1, x2)), x1 \leftarrow i(x1)\}$

$\overbrace{op_l(\underbrace{mult(rd(x1, x2), i(x1))}, i(rd(x1, x2)), i(x1))}$

$=$ by Lemma 22 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$op_l(\overbrace{i(x2)}, \underbrace{i(rd(x1, x2))}, i(x1))$

$=$ by Lemma 10 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{op_l(i(x2), \overbrace{rd(x2, x1)}, i(x1))}$

$=$ by Lemma 835 RL with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$\underbrace{\overbrace{i(op_l(x2, x1, x2))}}$

$=$ by Lemma 433 LR with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$\overbrace{op_l(i(x2), x1, x2)}$

Lemma 905: $op_l(x1, x2, i(op_r(x3, x2, x2))) = op_l(x1, i(i(x2)), i(rd(x3, x2)))$

$op_l(x1, \underbrace{x2}, i(op_r(x3, x2, x2)))$

$=$ by Lemma 3 RL with $\{x1 \leftarrow x2\}$

$\underbrace{op_l(x1, \overbrace{i(i(x2))}, i(op_r(x3, x2, x2)))}$

$=$ by Lemma 312 LR with $\{x2 \leftarrow op_r(x3, x2, x2), x3 \leftarrow i(x2), x1 \leftarrow x1\}$

$\overbrace{op_l(x1, i(i(x2)), i(\underbrace{mult(op_r(x3, x2, x2), i(x2))}))}$

$=$ by Lemma 352 LR with $\{x2 \leftarrow x2, x1 \leftarrow x3\}$

$op_l(x1, i(i(x2)), i(\overbrace{rd(x3, x2)}))$

Lemma 906: $op_l(x1, x2, i(op_r(x3, x2, x2))) = op_l(x1, x2, rd(x2, x3))$

$$
\begin{aligned}
& \underbrace{op_l(x1, x2, i(op_r(x3, x2, x2)))} \\
= & \quad \text{by Lemma 905 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, \underbrace{i(i(x2))}, i(rd(x3, x2)))} \\
= & \quad \text{by Lemma 3 LR with } \{x1 \leftarrow x2\} \\
& op_l(x1, \overbrace{x2}, \underbrace{i(rd(x3, x2))}) \\
= & \quad \text{by Lemma 10 LR with } \{x2 \leftarrow x2, x1 \leftarrow x3\} \\
& op_l(x1, x2, \overbrace{rd(x2, x3)})
\end{aligned}
$$

Lemma 907: $op_l(x1, x2, op_r(i(x3), x2, x2)) = op_l(x1, x2, rd(x2, x3))$

$$
\begin{aligned}
& op_l(x1, x2, \underbrace{op_r(i(x3), x2, x2)}) \\
= & \quad \text{by Lemma 356 RL with } \{x2 \leftarrow x2, x1 \leftarrow x3\} \\
& \underbrace{op_l(x1, x2, \overbrace{i(op_r(x3, x2, x2))})} \\
= & \quad \text{by Lemma 906 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x2, rd(x2, x3))}
\end{aligned}
$$

Lemma 908: $mult(i(x1), rd(x2, i(x1))) = op_t(op_r(x2, i(x1), i(x1)), x1)$

$$
\begin{aligned}
& \underbrace{mult(i(x1), rd(x2, i(x1)))} \\
= & \quad \text{by Lemma 45 RL with } \{x3 \leftarrow x2, x2 \leftarrow i(x1), x1 \leftarrow x1\} \\
& \overbrace{\underbrace{i(mult(x1, rd(i(x1), x2)))}} \\
= & \quad \text{by Lemma 33 RL with } \{x2 \leftarrow rd(i(x1), x2), x1 \leftarrow x1\} \\
& \overbrace{op_t(\underbrace{rd(i(x1), rd(i(x1), x2))}, x1)} \\
= & \quad \text{by Lemma 353 LR with } \{x2 \leftarrow x2, x1 \leftarrow i(x1)\} \\
& op_t(\overbrace{op_r(x2, i(x1), i(x1))}, x1)
\end{aligned}
$$

Lemma 909: $op_t(rd(x1, i(op_t(x2, x1))), x2) = mult(i(x2), mult(x2, mult(x1, x2)))$

$\underbrace{op_t(rd(x1, i(op_t(x2, x1))), x2)}$

$=$ by Axiom 10 RL with $\{x2 \leftarrow rd(x1, i(op_t(x2, x1))), x1 \leftarrow x2\}$

$\overbrace{mult(i(x2), mult(\underbrace{rd(x1, i(op_t(x2, x1)))}, x2))}$

$=$ by Lemma 357 RL with $\{x2 \leftarrow x1, x1 \leftarrow op_t(x2, x1)\}$

$mult(i(x2), \underbrace{mult(\overbrace{mult(op_r(op_t(x2, x1), x1, x1), x1)}, x2)})$

$=$ by Lemma 91 RL with $\{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow op_r(op_t(x2, x1), x1, x1)\}$

$mult(i(x2), \overbrace{mult(op_r(op_r(op_t(x2, x1), x1, x1), x1, \underbrace{x2}), mult(x1, x2))})$

$=$ by Lemma 12 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$mult(i(x2), mult(op_r(\underbrace{op_r(op_t(x2, x1), x1, x1)}, x1, \overbrace{mult(x1, mult(i(x1), x2))}), mult(x1, x2)))$

$=$ by Lemma 361 RL with $\{x2 \leftarrow x1, x1 \leftarrow op_t(x2, x1)\}$

$mult(i(x2), mult(op_r(\overbrace{op_r(op_t(x2, x1), i(x1), i(x1))}, x1, mult(x1, mult(i(x1), x2))), mult(x1, x2)))$

$=$ by Axiom 14 RL with $\{x4 \leftarrow x1, x3 \leftarrow i(x1), x2 \leftarrow i(x1), x1 \leftarrow x2\}$

$mult(i(x2), mult(op_r(\overbrace{op_t(op_r(x2, i(x1), i(x1)), x1)}, x1, mult(x1, mult(i(x1), x2))), mult(x1, x2)))$

$=$ by Lemma 908 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$mult(i(x2), mult(op_r(\overbrace{mult(i(x1), rd(x2, i(x1)))}, x1, mult(x1, mult(i(x1), x2))), mult(x1, x2)))$

$=$ by Lemma 69 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$mult(i(x2), mult(op_r(\overbrace{mult(mult(i(x1), x2), x1)}, x1, \underbrace{mult(x1, mult(i(x1), x2))}), mult(x1, x2)))$

$=$ by Lemma 435 RL with $\{x2 \leftarrow x1, x1 \leftarrow mult(i(x1), x2)\}$

$mult(i(x2), mult(\underbrace{op_r(mult(mult(i(x1), x2), x1), x1, \overbrace{op_l(mult(\underbrace{mult(i(x1), x2)}, x1), mult(i(x1), x2), x1)})}, mult(x1, x2)))$

$=$ by Lemma 853 RL with $\{x1 \leftarrow mult(i(x1), x2), x2 \leftarrow x1\}$

$mult(i(x2), mult(\underbrace{mult(x1, mult(i(x1), x2))}, mult(x1, x2)))$

$=$ by Lemma 12 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$mult(i(x2), mult(\overbrace{x2}, mult(x1, x2)))$

Lemma 910: $op_t(rd(op_t(x1, i(x2)), x2), x1) = mult(i(x2), x1)$

$$
\begin{aligned}
& op_t(rd(\underbrace{op_t(x1, i(x2))}, x2), x1) \\
= & \quad \text{by Lemma 18 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_t(\underbrace{rd(\overbrace{i(op_t(i(x1), x2))}, x2)}, x1) \\
= & \quad \text{by Lemma 10 RL with } \{x2 \leftarrow i(op_t(i(x1), x2)), x1 \leftarrow x2\} \\
& \underbrace{op_t(\overbrace{i(rd(x2, i(op_t(i(x1), x2))))}, x1)} \\
= & \quad \text{by Lemma 19 RL with } \{x2 \leftarrow x1, x1 \leftarrow rd(x2, i(op_t(i(x1), x2)))\} \\
& \overbrace{i(\underbrace{op_t(rd(x2, i(op_t(i(x1), x2))), i(x1))})} \\
= & \quad \text{by Lemma 909 LR with } \{x2 \leftarrow i(x1), x1 \leftarrow x2\} \\
& i(\overbrace{mult(i(i(x1)), mult(i(x1), mult(x2, i(x1))))}) \\
= & \quad \text{by Axiom 2 RL with } \{x2 \leftarrow i(x1), x1 \leftarrow mult(x2, i(x1))\} \\
& \underbrace{i(\overbrace{mult(x2, i(x1))})} \\
= & \quad \text{by Lemma 17 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \overbrace{mult(i(x2), x1)}
\end{aligned}
$$

Lemma 911: $mult(x1, rd(x2, op_t(x1, i(x2)))) = op_r(rd(x1, mult(i(x2), x1)), x1, x1)$

$$
\begin{aligned}
& mult(x1, \underbrace{rd(x2, op_t(x1, i(x2)))}) \\
= & \quad \text{by Lemma 10 RL with } \{x2 \leftarrow x2, x1 \leftarrow op_t(x1, i(x2))\} \\
& \underbrace{mult(x1, \overbrace{i(rd(op_t(x1, i(x2)), x2))})} \\
= & \quad \text{by Lemma 469 RL with } \{x2 \leftarrow rd(op_t(x1, i(x2)), x2), x1 \leftarrow x1\} \\
& \overbrace{op_r(rd(x1, \underbrace{op_t(rd(op_t(x1, i(x2)), x2), x1)}), x1, x1)} \\
= & \quad \text{by Lemma 910 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_r(rd(x1, \overbrace{mult(i(x2), x1)}), x1, x1)
\end{aligned}
$$

Lemma 912: $mult(x1, rd(x2, op_t(x1, i(x2)))) = op_r(x2, x1, x1)$

$$
\begin{aligned}
& \underbrace{mult(x1, rd(x2, op_t(x1, i(x2))))} \\
= & \quad \text{by Lemma 911 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_r(\underbrace{rd(x1, mult(i(x2), x1))}, x1, x1)} \\
= & \quad \text{by Lemma 9 LR with } \{x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& op_r(\underbrace{i(\overbrace{i(x2)})}, x1, x1) \\
= & \quad \text{by Lemma 3 LR with } \{x1 \leftarrow x2\} \\
& op_r(\overbrace{x2}, x1, x1)
\end{aligned}
$$

Lemma 913: $mult(x1, rd(i(x2), op_t(x1, x2))) = op_r(i(x2), x1, x1)$

$mult(x1, \underbrace{rd(i(x2), op_t(x1, x2))})$

$=$ by Lemma 37 RL with $\{x2 \leftarrow x2, x1 \leftarrow op_t(x1, x2)\}$

$mult(x1, \overbrace{\underbrace{rd(i(op_t(x1, x2)), x2)}})$

$=$ by Lemma 10 RL with $\{x2 \leftarrow i(op_t(x1, x2)), x1 \leftarrow x2\}$

$\underbrace{mult(x1, \overbrace{i(rd(x2, i(op_t(x1, x2))))})}$

$=$ by Lemma 469 RL with $\{x2 \leftarrow rd(x2, i(op_t(x1, x2))), x1 \leftarrow x1\}$

$\overbrace{op_r(rd(x1, \underbrace{op_t(rd(x2, i(op_t(x1, x2))), x1)}), x1, x1)}$

$=$ by Lemma 909 LR with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$op_r(rd(x1, \overbrace{\underbrace{mult(i(x1), mult(x1, mult(x2, x1)))}}), x1, x1)$

$=$ by Axiom 2 RL with $\{x2 \leftarrow x1, x1 \leftarrow mult(x2, x1)\}$

$op_r(\underbrace{rd(x1, \overbrace{mult(x2, x1)})}, x1, x1)$

$=$ by Lemma 9 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$op_r(\overbrace{i(x2)}, x1, x1)$

Lemma 914: $rd(x1, rd(op_t(i(x1), x2), i(x2))) = mult(x1, mult(op_r(i(x2), i(x1), i(x1)), x1))$

$rd(x1, \underbrace{rd(op_t(i(x1), x2), i(x2))})$

$=$ by Lemma 10 RL with $\{x2 \leftarrow op_t(i(x1), x2), x1 \leftarrow i(x2)\}$

$\underbrace{rd(x1, \overbrace{i(rd(i(x2), op_t(i(x1), x2)))})}$

$=$ by Lemma 86 RL with $\{x2 \leftarrow rd(i(x2), op_t(i(x1), x2)), x1 \leftarrow x1\}$

$\overbrace{mult(x1, mult(\underbrace{mult(i(x1), rd(i(x2), op_t(i(x1), x2)))}, x1))}$

$=$ by Lemma 913 LR with $\{x2 \leftarrow x2, x1 \leftarrow i(x1)\}$

$mult(x1, mult(\overbrace{op_r(i(x2), i(x1), i(x1))}, x1))$

Lemma 915: $rd(x1, rd(x2, op_t(x1, i(x2)))) = mult(x1, mult(i(op_r(x2, x1, x1)), x1))$

$rd(x1, \underbrace{rd(x2, op_t(x1, i(x2)))})$

$=$ by Lemma 67 RL with $\{x3 \leftarrow x2, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{rd(x1, \overbrace{rd(op_t(i(x1), x2), i(x2))})}$

$=$ by Lemma 914 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{mult(x1, mult(\underbrace{op_r(i(x2), i(x1), i(x1))}, x1))}$

$=$ by Lemma 815 LR with $\{x3 \leftarrow x1, x2 \leftarrow x1, x1 \leftarrow x2\}$

$mult(x1, mult(\overbrace{i(op_r(x2, x1, x1))}, x1))$

Lemma 916: $rd(x1, rd(x2, op_t(x1, i(x2)))) = mult(x1, rd(x1, i(i(x2))))$

$$
\begin{aligned}
& \underbrace{rd(x1, rd(x2, op_t(x1, i(x2))))} \\
= \quad & \text{by Lemma 915 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{mult(x1, mult(\underbrace{i(op_r(x2, x1, x1))}, x1))} \\
= \quad & \text{by Lemma 356 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& mult(x1, \underbrace{mult(\overbrace{op_r(i(x2), x1, x1)}, x1)}) \\
= \quad & \text{by Lemma 357 LR with } \{x2 \leftarrow x1, x1 \leftarrow i(x2)\} \\
& mult(x1, \overbrace{rd(x1, i(i(x2)))})
\end{aligned}
$$

Lemma 917: $rd(x1, op_r(x3, x2, x2)) = rd(op_r(i(x3), x2, x2), i(x1))$

$$
\begin{aligned}
& rd(x1, \underbrace{op_r(x3, x2, x2)}) \\
= \quad & \text{by Lemma 353 RL with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& \underbrace{rd(x1, \overbrace{rd(x2, rd(x2, x3))})} \\
= \quad & \text{by Lemma 39 RL with } \{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow rd(x2, x3)\} \\
& \overbrace{rd(\underbrace{rd(rd(x2, x3), x2)}, i(x1))} \\
= \quad & \text{by Lemma 354 LR with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& rd(\overbrace{op_r(i(x3), x2, x2)}, i(x1))
\end{aligned}
$$

Lemma 918: $rd(op_r(x2, x1, x1), x3) = rd(i(x3), op_r(i(x2), x1, x1))$

$$
\begin{aligned}
& rd(\underbrace{op_r(x2, x1, x1)}, x3) \\
= \quad & \text{by Lemma 353 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{rd(\overbrace{rd(x1, rd(x1, x2))}, x3)} \\
= \quad & \text{by Lemma 40 RL with } \{x3 \leftarrow x1, x2 \leftarrow rd(x1, x2), x1 \leftarrow x3\} \\
& \overbrace{rd(i(x3), \underbrace{rd(rd(x1, x2), x1)})} \\
= \quad & \text{by Lemma 354 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& rd(i(x3), \overbrace{op_r(i(x2), x1, x1)})
\end{aligned}
$$

Lemma 919: $op_t(mult(x1,x2),i(x1)) = op_t(mult(x2,x1),x2)$

$\underbrace{op_t(mult(x1,x2),i(x1))}$

$=$ by Lemma 384 RL with $\{x2 \leftarrow mult(x1,x2), x1 \leftarrow i(x1)\}$

$\overbrace{mult(i(op_t(i(x1),mult(x1,x2))),\underbrace{mult(i(x1),mult(x1,x2))})}$

$=$ by Axiom 2 RL with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$mult(\underbrace{i(op_t(i(x1),mult(x1,x2)))},\overbrace{x2})$

$=$ by Lemma 18 LR with $\{x2 \leftarrow mult(x1,x2), x1 \leftarrow x1\}$

$mult(\overbrace{\underbrace{op_t(x1,i(mult(x1,x2)))}},x2)$

$=$ by Lemma 379 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$mult(\overbrace{\underbrace{op_t(x1,rd(i(x1),x2))}},x2)$

$=$ by Lemma 707 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$mult(\overbrace{op_t(x1,\underbrace{rd(x2,x1)})},x2)$

$=$ by Lemma 10 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$mult(\underbrace{op_t(x1,\overbrace{i(rd(x1,x2))})},x2)$

$=$ by Lemma 18 RL with $\{x2 \leftarrow rd(x1,x2), x1 \leftarrow x1\}$

$\underbrace{mult(\overbrace{i(op_t(i(x1),rd(x1,x2)))},x2)}$

$=$ by Lemma 330 RL with $\{x2 \leftarrow op_t(i(x1),rd(x1,x2)), x1 \leftarrow x2\}$

$\overbrace{op_r(\underbrace{rd(x2,op_t(i(x1),rd(x1,x2)))},x2,i(x2))}$

$=$ by Lemma 386 LR with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$\underbrace{op_r(\overbrace{mult(x1,x2)},x2,i(x2))}$

$=$ by Lemma 320 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_t(mult(x2,x1),x2)}$

Lemma 920: $mult(mult(mult(i(x1),x2),x1),x1) = op_r(mult(op_t(x2,x1),x1),x1,x1)$

$\underbrace{mult(mult(mult(i(x1),x2),x1),x1)}$

$=$ by Lemma 385 RL with $\{x2 \leftarrow mult(mult(i(x1),x2),x1), x1 \leftarrow x1\}$

$\overbrace{op_r(op_t(\underbrace{mult(x1,mult(mult(i(x1),x2),x1))},x1),x1,x1)}$

$=$ by Lemma 86 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$op_r(\underbrace{op_t(\overbrace{rd(x1,i(x2))},x1)},x1,x1)$

$=$ by Lemma 341 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$op_r(\overbrace{mult(op_t(x2,x1),x1)},x1,x1)$

Lemma 921: $mult(mult(mult(i(x1),x2),x1),x1) = rd(x1,i(op_t(x2,x1)))$

$$
\begin{aligned}
& \overbrace{mult(mult(mult(i(x1),x2),x1),x1)} \\
= \quad & \text{by Lemma 920 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_r(mult(op_t(x2,x1),x1),x1,x1)} \\
= \quad & \text{by Lemma 359 LR with } \{x2 \leftarrow x1, x1 \leftarrow op_t(x2,x1)\} \\
& \overbrace{rd(x1,i(op_t(x2,x1)))}
\end{aligned}
$$

Lemma 922: $i(mult(op_t(op_r(x1,i(x2),i(x2)),x2),x2)) = op_t(op_r(rd(i(x2),x1),i(x2),i(x2)),i(x2))$

$$
\begin{aligned}
& i(\overbrace{mult(op_t(op_r(x1,i(x2),i(x2)),x2),x2)}) \\
= \quad & \text{by Lemma 337 RL with } \{x2 \leftarrow x2, x1 \leftarrow op_r(x1,i(x2),i(x2))\} \\
& \overbrace{i(op_t(rd(op_r(x1,i(x2),i(x2)),i(x2)),x2))} \\
= \quad & \text{by Lemma 46 LR with } \{x3 \leftarrow x2, x2 \leftarrow i(x2), x1 \leftarrow op_r(x1,i(x2),i(x2))\} \\
& \overbrace{op_t(\underbrace{rd(i(x2),op_r(x1,i(x2),i(x2)))}, i(x2))} \\
= \quad & \text{by Lemma 369 LR with } \{x2 \leftarrow x1, x1 \leftarrow i(x2)\} \\
& op_t(\overbrace{op_r(rd(i(x2),x1),i(x2),i(x2))}, i(x2))
\end{aligned}
$$

Lemma 923: $mult(i(x1),rd(x1,rd(x2,x1))) = rd(x1,op_t(x2,i(x1)))$

$$
\begin{aligned}
& \overbrace{mult(i(x1),rd(x1,rd(x2,x1)))} \\
= \quad & \text{by Lemma 45 RL with } \{x3 \leftarrow x1, x2 \leftarrow rd(x2,x1), x1 \leftarrow x1\} \\
& \overbrace{i(\underbrace{mult(x1,rd(rd(x2,x1),x1))})} \\
= \quad & \text{by Lemma 417 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{i(\overbrace{rd(op_t(x2,i(x1)),x1)})} \\
= \quad & \text{by Lemma 10 LR with } \{x2 \leftarrow x1, x1 \leftarrow op_t(x2,i(x1))\} \\
& \overbrace{rd(x1,op_t(x2,i(x1)))}
\end{aligned}
$$

Lemma 924: $mult(mult(x1,x2),mult(i(x2),x1)) = mult(x1,op_t(x1,i(x2)))$

$$
\begin{aligned}
& \overbrace{mult(mult(x1,x2),mult(i(x2),x1))} \\
= \quad & \text{by Lemma 137 RL with } \{x3 \leftarrow mult(i(x2),x1), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{mult(x1,mult(x2,\underbrace{op_l(mult(i(x2),x1),x1,x2)}))} \\
= \quad & \text{by Lemma 426 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& mult(x1,\underbrace{mult(x2,\overbrace{mult(x1,i(x2))})}) \\
= \quad & \text{by Lemma 30 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& mult(x1,\overbrace{op_t(x1,i(x2))})
\end{aligned}
$$

Lemma 925: $mult(i(x1), i(mult(x1, x2))) = rd(op_t(rd(i(x1), x2), i(x2)), x1)$

$mult(i(x1), \underbrace{i(mult(x1, x2))})$

$=$ by Lemma 33 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{mult(i(x1), \overbrace{op_t(rd(i(x1), x2), x1)})}$

$=$ by Lemma 459 RL with $\{x2 \leftarrow x1, x1 \leftarrow rd(i(x1), x2)\}$

$\overbrace{rd(op_t(rd(i(x1), x2), \underbrace{mult(rd(i(x1), x2), x1)}), x1)}$

$=$ by Lemma 21 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$rd(op_t(rd(i(x1), x2), \overbrace{i(x2)}), x1)$

Lemma 926: $op_l(rd(x1, x2), i(x1), i(x2)) = op_l(rd(x1, x2), x1, x2)$

$\underbrace{op_l(rd(x1, x2), i(x1), i(x2))}$

$=$ by Lemma 301 RL with $\{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow rd(x1, x2)\}$

$\underbrace{\overbrace{op_l(rd(x1, x2), i(x1), rd(x1, x2))}}$

$=$ by Lemma 466 RL with $\{x2 \leftarrow i(x1), x1 \leftarrow rd(x1, x2)\}$

$\overbrace{mult(\underbrace{mult(rd(x1, x2), i(x1))}, op_t(i(i(x1)), rd(x1, x2)))}$

$=$ by Lemma 22 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$mult(\overbrace{i(x2)}, op_t(\underbrace{i(i(x1))}, rd(x1, x2)))$

$=$ by Lemma 3 LR with $\{x1 \leftarrow x1\}$

$\underbrace{mult(i(x2), op_t(\overbrace{x1}, rd(x1, x2)))}$

$=$ by Lemma 314 LR with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$\overbrace{op_l(rd(x1, x2), x1, x2)}$

Lemma 927: $op_l(i(mult(i(i(x1)), x2)), x1, i(x2)) = i(op_t(rd(x2, i(x1)), x2))$

$op_l(i(mult(i(i(x1)), x2)), \underbrace{x1}, i(x2))$

$=$ by Lemma 3 RL with $\{x1 \leftarrow x1\}$

$\underbrace{op_l(i(mult(i(i(x1)), x2)), \overbrace{i(i(x1))}, i(x2))}$

$=$ by Lemma 519 LR with $\{x3 \leftarrow x2, x2 \leftarrow i(x1), x1 \leftarrow mult(i(i(x1)), x2)\}$

$\overbrace{i(\underbrace{op_l(mult(i(i(x1)), x2), i(x1), x2)})}$

$=$ by Lemma 249 LR with $\{x2 \leftarrow x2, x1 \leftarrow i(x1)\}$

$i(\overbrace{op_t(rd(x2, i(x1)), x2)})$

Lemma 928: $op_l(rd(x1,x2),x1,x2) = rd(op_t(x1,i(x2)),x2)$

$\overbrace{op_l(rd(x1,x2),x1,x2)}$

$=$ by Lemma 926 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_l(\underbrace{rd(x1,x2)}, i(x1), i(x2))}$

$=$ by Lemma 12 RL with $\{x2 \leftarrow rd(x1,x2), x1 \leftarrow x1\}$

$\underbrace{op_l(\overbrace{mult(x1, mult(i(x1), rd(x1,x2)))}, i(x1), i(x2))}$

$=$ by Lemma 245 RL with $\{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow mult(x1, mult(i(x1), rd(x1,x2)))\}$

$\overbrace{\underbrace{op_l(mult(x1, mult(i(x1), rd(x1,x2))), x2, i(mult(x2,x1)))}}$

$=$ by Axiom 12 RL with $\{x3 \leftarrow mult(x1, mult(i(x1), rd(x1,x2))), x2 \leftarrow x2, x1 \leftarrow i(mult(x2,x1))\}$

$\overbrace{mult(i(mult(i(mult(x2,x1)),x2)), \underbrace{mult(i(mult(x2,x1)), mult(x2, mult(x1, mult(i(x1), rd(x1,x2)))))})$

$=$ by Axiom 12 LR with $\{x3 \leftarrow mult(i(x1), rd(x1,x2)), x2 \leftarrow x1, x1 \leftarrow x2\}$

$mult(\underbrace{i(mult(i(mult(x2,x1)),x2))}, \overbrace{op_l(mult(i(x1), rd(x1,x2)), x1, x2)})$

$=$ by Lemma 16 LR with $\{x2 \leftarrow x2, x1 \leftarrow mult(x2,x1)\}$

$mult(\overbrace{\underbrace{mult(mult(x2,x1), i(x2))}}, op_l(mult(i(x1), rd(x1,x2)), x1, x2))$

$=$ by Lemma 55 LR with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$mult(\overbrace{mult(x2, rd(x1,x2))}, \underbrace{op_l(mult(i(x1), rd(x1,x2)), x1, x2)})$

$=$ by Lemma 222 RL with $\{x1 \leftarrow x1, x2 \leftarrow x2\}$

$\underbrace{mult(mult(x2, rd(x1,x2)), \overbrace{i(x2)})}$

$=$ by Lemma 55 LR with $\{x2 \leftarrow rd(x1,x2), x1 \leftarrow x2\}$

$\overbrace{\underbrace{mult(x2, rd(rd(x1,x2),x2))}}$

$=$ by Lemma 417 LR with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$\overbrace{rd(op_t(x1, i(x2)), x2)}$

Lemma 929: $op_l(rd(i(x1),x2),x1,i(x2)) = rd(op_t(i(x1),i(x2)),x2)$

$op_l(rd(i(x1),x2),\underbrace{x1},i(x2))$

$=$ by Lemma 3 RL with $\{x1 \leftarrow x1\}$

$op_l(\underbrace{rd(i(x1),x2)},\overbrace{i(i(x1))},i(x2))$

$=$ by Axiom 2 LR with $\{x2 \leftarrow x1, x1 \leftarrow rd(i(x1),x2)\}$

$\underbrace{op_l(\overbrace{mult(i(x1),mult(x1,rd(i(x1),x2)))},i(i(x1)),i(x2))}$

$=$ by Lemma 245 RL with $\{x3 \leftarrow i(x1), x2 \leftarrow x2, x1 \leftarrow mult(i(x1),mult(x1,rd(i(x1),x2)))\}$

$\overbrace{\underbrace{op_l(mult(i(x1),mult(x1,rd(i(x1),x2)))},x2,i(mult(x2,i(x1))))}$

$=$ by Axiom 12 RL with $\{x3 \leftarrow mult(i(x1),mult(x1,rd(i(x1),x2))), x2 \leftarrow x2, x1 \leftarrow i(mult(x2,i(x1)))\}$

$\overbrace{mult(i(mult(i(mult(x2,i(x1))),x2)),\underbrace{mult(i(mult(x2,i(x1))),mult(x2,mult(i(x1),mult(x1,rd(i(x1),x2)))))})}$

$=$ by Axiom 12 LR with $\{x3 \leftarrow mult(x1,rd(i(x1),x2)), x2 \leftarrow i(x1), x1 \leftarrow x2\}$

$mult(\underbrace{i(mult(i(mult(x2,i(x1))),x2))},\overbrace{op_l(mult(x1,rd(i(x1),x2)),i(x1),x2)})$

$=$ by Lemma 16 LR with $\{x2 \leftarrow x2, x1 \leftarrow mult(x2,i(x1))\}$

$mult(\overbrace{\underbrace{mult(mult(x2,i(x1)),i(x2))},op_l(mult(x1,rd(i(x1),x2)),i(x1),x2))}$

$=$ by Lemma 55 LR with $\{x2 \leftarrow i(x1), x1 \leftarrow x2\}$

$mult(\overbrace{mult(x2,rd(i(x1),x2))},op_l(mult(\underbrace{x1},rd(i(x1),x2)),i(x1),x2))$

$=$ by Lemma 3 RL with $\{x1 \leftarrow x1\}$

$mult(mult(x2,rd(i(x1),x2)),\underbrace{op_l(mult(\overbrace{i(i(x1))},rd(i(x1),x2)),i(x1),x2)})$

$=$ by Lemma 222 RL with $\{x1 \leftarrow i(x1), x2 \leftarrow x2\}$

$\underbrace{mult(mult(x2,rd(i(x1),x2)),\overbrace{i(x2)})}$

$=$ by Lemma 55 LR with $\{x2 \leftarrow rd(i(x1),x2), x1 \leftarrow x2\}$

$\overbrace{\underbrace{mult(x2,rd(rd(i(x1),x2),x2))}}$

$=$ by Lemma 417 LR with $\{x2 \leftarrow i(x1), x1 \leftarrow x2\}$

$\overbrace{rd(op_t(i(x1),i(x2)),x2)}$

Lemma 930: $op_l(rd(i(x1),x2),x1,i(x2)) = rd(i(x2),op_t(x1,x2))$

$\underbrace{op_l(rd(i(x1),x2),x1,i(x2))}$

$=$ by Lemma 929 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{rd(\underbrace{op_t(i(x1),i(x2))},x2)}$

$=$ by Lemma 15 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{rd(\overbrace{i(op_t(x1,x2))},x2)}$

$=$ by Lemma 37 LR with $\{x2 \leftarrow x2, x1 \leftarrow op_t(x1,x2)\}$

$\overbrace{rd(i(x2),op_t(x1,x2))}$

Lemma 931: $op_t(rd(op_t(x2,x1),x1),i(x2)) = mult(x2,i(op_t(x1,x2)))$

$$
\begin{aligned}
& \underbrace{op_t(rd(op_t(x2,x1),x1),i(x2))} \\
= & \quad \text{by Lemma 46 RL with } \{x3 \leftarrow x2, x2 \leftarrow op_t(x2,x1), x1 \leftarrow x1\} \\
& \underbrace{\overbrace{i(op_t(rd(x1,op_t(x2,x1)),x2))}} \\
= & \quad \text{by Lemma 31 RL with } \{x2 \leftarrow rd(x1,op_t(x2,x1)), x1 \leftarrow x2\} \\
& \overbrace{mult(x2,i(\underbrace{mult(rd(x1,op_t(x2,x1)),x2)}))} \\
= & \quad \text{by Lemma 597 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& mult(x2,i(\overbrace{op_t(x1,x2)}))
\end{aligned}
$$

Lemma 932: $op_r(op_l(i(x1),x2,x1),x2,x2) = rd(mult(x2,i(x1)),x2)$

$$
\begin{aligned}
& op_r(\underbrace{op_l(i(x1),x2,x1)},x2,x2) \\
= & \quad \text{by Lemma 433 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{op_r(\overbrace{i(op_l(x1,x2,x1))},x2,x2)} \\
= & \quad \text{by Lemma 354 RL with } \{x2 \leftarrow op_l(x1,x2,x1), x1 \leftarrow x2\} \\
& \overbrace{rd(\underbrace{rd(x2,op_l(x1,x2,x1))},x2)} \\
= & \quad \text{by Lemma 614 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& rd(\overbrace{mult(x2,i(x1))},x2)
\end{aligned}
$$

Lemma 933: $mult(op_t(x1,x2),i(x2)) = mult(x1,i(op_t(x2,x1)))$

$$
\begin{aligned}
& mult(op_t(x1,\underbrace{x2}),i(x2)) \\
= & \quad \text{by Lemma 3 RL with } \{x1 \leftarrow x2\} \\
& mult(\underbrace{op_t(x1,\overbrace{i(i(x2))})},i(x2)) \\
= & \quad \text{by Lemma 18 RL with } \{x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& \underbrace{mult(\overbrace{i(op_t(i(x1),i(x2)))},i(x2))} \\
= & \quad \text{by Lemma 668 RL with } \{x2 \leftarrow i(x1), x1 \leftarrow i(x2)\} \\
& \overbrace{op_t(rd(i(x2),\underbrace{op_t(i(x1),i(x2))}),i(x1))} \\
= & \quad \text{by Lemma 15 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_t(\underbrace{rd(i(x2),\overbrace{i(op_t(x1,x2))})},i(x1)) \\
= & \quad \text{by Lemma 23 LR with } \{x2 \leftarrow op_t(x1,x2), x1 \leftarrow x2\} \\
& \underbrace{op_t(\overbrace{rd(op_t(x1,x2),x2)},i(x1))} \\
= & \quad \text{by Lemma 931 LR with } \{x1 \leftarrow x2, x2 \leftarrow x1\} \\
& \overbrace{mult(x1,i(op_t(x2,x1)))}
\end{aligned}
$$

Lemma 934: $mult(x1, op_t(op_t(x2, x1), i(x1))) = mult(i(op_t(i(x1), x2)), op_t(x2, x1))$

$mult(\underbrace{x1}, op_t(op_t(x2, x1), i(x1)))$

$=$ by Lemma 3 RL with $\{x1 \leftarrow x1\}$

$\underbrace{mult(\overbrace{i(i(x1))}, op_t(op_t(x2, x1), i(x1)))}$

$=$ by Lemma 669 RL with $\{x2 \leftarrow op_t(x2, x1), x1 \leftarrow i(x1)\}$

$\overbrace{mult(i(\underbrace{op_t(i(x1), op_t(x2, x1))}), op_t(x2, x1))}$

$=$ by Lemma 561 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$mult(i(\overbrace{op_t(i(x1), x2)}), op_t(x2, x1))$

Lemma 935: $mult(op_t(x2, i(x1)), x1) = mult(op_t(x1, i(x2)), op_t(x2, x1))$

$\underbrace{mult(op_t(x2, i(x1)), x1)}$

$=$ by Lemma 50 RL with $\{x3 \leftarrow i(x1), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{\overbrace{mult(x1, op_t(op_t(x2, x1), i(x1)))}}$

$=$ by Lemma 934 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{mult(\underbrace{i(op_t(i(x1), x2))}, op_t(x2, x1))}$

$=$ by Lemma 18 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$mult(\overbrace{op_t(x1, i(x2))}, op_t(x2, x1))$

Lemma 936: $mult(op_t(x2, i(x1)), x1) = mult(x2, op_t(x1, i(x2)))$

$\underbrace{mult(op_t(x2, i(x1)), x1)}$

$=$ by Lemma 935 LR with $\{x1 \leftarrow x1, x2 \leftarrow x2\}$

$\overbrace{mult(op_t(x1, i(x2)), \underbrace{op_t(x2, x1)})}$

$=$ by Lemma 559 RL with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$\underbrace{mult(op_t(x1, i(x2)), \overbrace{op_t(x2, op_t(x1, i(x2)))})}$

$=$ by Lemma 13 LR with $\{x2 \leftarrow x2, x1 \leftarrow op_t(x1, i(x2))\}$

$\overbrace{mult(x2, op_t(x1, i(x2)))}$

Lemma 937: $i(op_t(mult(x1, x2), x1)) = op_t(i(mult(x2, x1)), x2)$

$$
\begin{aligned}
& \overbrace{i(op_t(mult(x1, x2), x1))} \\
= & \quad \text{by Lemma 264 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \overbrace{i(\overbrace{mult(x2, op_l(x1, x1, x2))})} \\
= & \quad \text{by Lemma 682 LR with } \{x2 \leftarrow op_l(x1, x1, x2), x1 \leftarrow x2\} \\
& \overbrace{op_t(\underbrace{rd(i(x2), op_l(x1, x1, x2))}, op_t(x2, op_l(x1, x1, x2)))} \\
= & \quad \text{by Lemma 447 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& op_t(\overbrace{i(mult(x2, x1))}, \underbrace{op_t(x2, op_l(x1, x1, x2))}) \\
= & \quad \text{by Lemma 553 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& op_t(i(mult(x2, x1)), \overbrace{op_t(x2, x1)}) \\
= & \quad \text{by Lemma 378 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \underbrace{op_t(i(mult(x2, x1)), \overbrace{op_t(x2, rd(mult(x2, x1), x2))})} \\
= & \quad \text{by Lemma 592 LR with } \{x2 \leftarrow x2, x1 \leftarrow mult(x2, x1)\} \\
& \overbrace{op_t(i(mult(x2, x1)), x2)}
\end{aligned}
$$

Lemma 938: $asoc(rd(x1, x2), x3, x3) = asoc(x3, rd(x2, x1), x3)$

$$
\begin{aligned}
& \underbrace{asoc(rd(x1, x2), x3, x3)} \\
= & \quad \text{by Lemma 772 RL with } \{x2 \leftarrow x3, x1 \leftarrow rd(x1, x2)\} \\
& \overbrace{\underbrace{asoc(rd(x1, x2), i(x3), i(x3))}} \\
= & \quad \text{by Lemma 762 RL with } \{x2 \leftarrow rd(x1, x2), x1 \leftarrow i(x3)\} \\
& \overbrace{asoc(i(x3), \underbrace{rd(i(x3), rd(x1, x2))}, i(x3))} \\
= & \quad \text{by Lemma 40 LR with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x3\} \\
& \underbrace{asoc(i(x3), \overbrace{rd(rd(x2, x1), x3)}, i(x3))} \\
= & \quad \text{by Lemma 613 RL with } \{x2 \leftarrow rd(rd(x2, x1), x3), x1 \leftarrow x3\} \\
& \overbrace{\underbrace{asoc(x3, rd(rd(x2, x1), x3), x3)}} \\
= & \quad \text{by Lemma 263 LR with } \{x2 \leftarrow rd(x2, x1), x1 \leftarrow x3\} \\
& \overbrace{asoc(x3, rd(x2, x1), x3)}
\end{aligned}
$$

Lemma 939: $mult(x1, mult(asoc(x1, x2, x2), x3)) = mult(op_r(x1, x2, x2), op_l(x3, asoc(x1, x2, x2), x1))$

$$
\begin{aligned}
& \underbrace{mult(x1, mult(asoc(x1, x2, x2), x3))} \\
= & \quad \text{by Lemma 132 RL with } \{x3 \leftarrow x3, x2 \leftarrow asoc(x1, x2, x2), x1 \leftarrow x1\} \\
& \overbrace{mult(mult(x1, \underbrace{asoc(x1, x2, x2)}), op_l(x3, asoc(x1, x2, x2), x1))} \\
= & \quad \text{by Lemma 756 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& mult(\underbrace{mult(x1, \overbrace{asoc(x2, i(x1), x2)})}, op_l(x3, asoc(x1, x2, x2), x1)) \\
= & \quad \text{by Lemma 644 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& mult(\overbrace{op_r(x1, x2, x2)}, op_l(x3, asoc(x1, x2, x2), x1))
\end{aligned}
$$

Lemma 940: $mult(x1, mult(asoc(x1, x2, x2), x3)) = mult(op_r(x1, x2, x2), x3)$

$\underbrace{mult(x1, mult(asoc(x1, x2, x2), x3))}$

$=$ by Lemma 939 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{mult(op_r(x1, x2, x2), op_l(x3, \underbrace{asoc(x1, x2, x2)}, x1))}$

$=$ by Lemma 756 RL with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$mult(op_r(x1, x2, x2), \underbrace{op_l(x3, \overbrace{asoc(x2, i(x1), x2)}, x1)})$

$=$ by Lemma 638 LR with $\{x2 \leftarrow x1, x3 \leftarrow x2, x1 \leftarrow x3\}$

$mult(op_r(x1, x2, x2), \overbrace{x3})$

Lemma 941: $mult(asoc(x1, x2, x2), mult(x1, x3)) = mult(op_r(x1, x2, x2), x3)$

$\underbrace{mult(asoc(x1, x2, x2), mult(x1, x3))}$

$=$ by Lemma 132 RL with $\{x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow asoc(x1, x2, x2)\}$

$\overbrace{mult(\underbrace{mult(asoc(x1, x2, x2), x1)}, op_l(x3, x1, asoc(x1, x2, x2)))}$

$=$ by Lemma 767 LR with $\{x1 \leftarrow x2, x2 \leftarrow x1\}$

$mult(\overbrace{op_r(x1, x2, x2)}, \underbrace{op_l(x3, x1, asoc(x1, x2, x2))})$

$=$ by Lemma 760 LR with $\{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x3\}$

$mult(op_r(x1, x2, x2), \overbrace{x3})$

Lemma 942: $asoc(rd(x1, op_t(x1, x2)), op_t(x1, x2), x3) = unit()$

$asoc(\underbrace{rd(x1, op_t(x1, x2))}, op_t(x1, x2), x3)$

$=$ by Lemma 396 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$asoc(\overbrace{rd(\underbrace{x2}, rd(mult(x1, x2), x1))}, op_t(x1, x2), x3)$

$=$ by Lemma 400 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$asoc(\underbrace{rd(\overbrace{op_t(rd(mult(x1, x2), x1), op_t(x1, x2))}, rd(mult(x1, x2), x1))}, op_t(x1, x2), x3)$

$=$ by Lemma 858 RL with $\{x2 \leftarrow rd(mult(x1, x2), x1), x1 \leftarrow op_t(x1, x2)\}$

$\underbrace{asoc(\overbrace{asoc(op_t(x1, x2), mult(rd(mult(x1, x2), x1), op_t(x1, x2)), mult(rd(mult(x1, x2), x1), op_t(x1, x2)))}, op_t(x1, x2), x3)}$

$=$ by Lemma 108 RL with $\{x3 \leftarrow x3, x2 \leftarrow op_t(x1, x2), x1 \leftarrow asoc(op_t(x1, x2), mult(rd(mult(x1, x2), x1), op_t(x1, x2)), mult(rd$

$\overbrace{mult(i(\underbrace{mult(asoc(op_t(x1, x2), mult(rd(mult(x1, x2), x1), op_t(x1, x2)), mult(rd(mult(x1, x2), x1), op_t(x1, x2)))}, mult(op_t(x$

$=$ by Lemma 941 LR with $\{x3 \leftarrow x3, x2 \leftarrow mult(rd(mult(x1, x2), x1), op_t(x1, x2)), x1 \leftarrow op_t(x1, x2)\}$

$mult(i(\overbrace{mult(op_r(op_t(x1, x2), mult(rd(mult(x1, x2), x1), op_t(x1, x2)), mult(rd(mult(x1, x2), x1), op_t(x1, x2))), x3)}), mult($

$=$ by Lemma 767 LR with $\{x1 \leftarrow mult(rd(mult(x1, x2), x1), op_t(x1, x2)), x2 \leftarrow op_t(x1, x2)\}$

$\underbrace{mult(i(mult(op_r(op_t(x1, x2), mult(rd(mult(x1, x2), x1), op_t(x1, x2)), mult(rd(mult(x1, x2), x1), op_t(x1, x2))), x3)), mult(}$

$=$ by Axiom 9 LR with $\{x1 \leftarrow mult(op_r(op_t(x1, x2), mult(rd(mult(x1, x2), x1), op_t(x1, x2)), mult(rd(mult(x1, x2), x1), op_t(x1$

$\overbrace{unit()}$

Lemma 943: $mult(x1, mult(asoc(x2, x1, x1), x3)) = mult(op_l(x1, x2, x1), x3)$

$$
\begin{aligned}
& mult(\underbrace{x1}, mult(asoc(x2, x1, x1), x3)) \\
= \quad & \text{by Lemma 846 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{mult(\overbrace{op_r(x1, asoc(x2, x1, x1), x3)}, mult(asoc(x2, x1, x1), x3))} \\
= \quad & \text{by Lemma 91 LR with } \{x3 \leftarrow x3, x2 \leftarrow asoc(x2, x1, x1), x1 \leftarrow x1\} \\
& \overbrace{mult(\underbrace{mult(x1, asoc(x2, x1, x1))}, x3)} \\
= \quad & \text{by Lemma 770 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& mult(\overbrace{op_l(x1, x2, x1)}, x3)
\end{aligned}
$$

Lemma 944: $op_l(x1, x2, asoc(x2, x3, x2)) = x1$

$$
\begin{aligned}
& \underbrace{op_l(x1, x2, asoc(x2, x3, x2))} \\
= \quad & \text{by Lemma 3 RL with } \{x1 \leftarrow op_l(x1, x2, asoc(x2, x3, x2))\} \\
& \overbrace{i(i(\underbrace{op_l(x1, x2, asoc(x2, x3, x2))}))} \\
= \quad & \text{by Lemma 798 RL with } \{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x1\} \\
& i(i(\overbrace{op_l(x1, asoc(x3, x2, x2), x2)})) \\
= \quad & \text{by Axiom 12 RL with } \{x3 \leftarrow x1, x2 \leftarrow asoc(x3, x2, x2), x1 \leftarrow x2\} \\
& i(\underbrace{i(\overbrace{mult(i(mult(x2, asoc(x3, x2, x2))), mult(x2, mult(asoc(x3, x2, x2), x1)))})}) \\
= \quad & \text{by Lemma 16 LR with } \{x2 \leftarrow mult(x2, mult(asoc(x3, x2, x2), x1)), x1 \leftarrow mult(x2, asoc(x3, x2, x2))\} \\
& i(\overbrace{mult(mult(x2, asoc(x3, x2, x2)), \underbrace{i(\underbrace{mult(x2, mult(asoc(x3, x2, x2), x1))})})}) \\
= \quad & \text{by Lemma 943 LR with } \{x3 \leftarrow x1, x2 \leftarrow x3, x1 \leftarrow x2\} \\
& i(mult(\underbrace{mult(x2, asoc(x3, x2, x2))}, i(\overbrace{mult(op_l(x2, x3, x2), x1)}))) \\
= \quad & \text{by Lemma 770 RL with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& i(\underbrace{mult(\overbrace{op_l(x2, x3, x2)}, i(mult(op_l(x2, x3, x2), x1)))}) \\
= \quad & \text{by Lemma 20 LR with } \{x2 \leftarrow x1, x1 \leftarrow op_l(x2, x3, x2)\} \\
& \underbrace{i(\overbrace{i(x1)})} \\
= \quad & \text{by Lemma 3 LR with } \{x1 \leftarrow x1\} \\
& \overbrace{x1}
\end{aligned}
$$

Lemma 945: $op_l(x1, x2, op_l(x2, x2, x3)) = x1$

$$
\begin{aligned}
& \underbrace{op_l(x1, x2, op_l(x2, x2, x3))} \\
= \quad & \text{by Lemma 286 RL with } \{x3 \leftarrow op_l(x2, x2, x3), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x2, \underbrace{rd(op_l(x2, x2, x3), x2)})} \\
= \quad & \text{by Lemma 172 LR with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& \underbrace{op_l(x1, x2, \overbrace{asoc(x2, x3, x2)})} \\
= \quad & \text{by Lemma 944 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{x1}
\end{aligned}
$$

Lemma 946: $op_l(x1, asoc(x2, x3, x3), x3) = x1$

$$
\begin{aligned}
& \underbrace{op_l(x1, asoc(x2, x3, x3), x3)} \\
= & \quad \text{by Lemma 798 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, x3, asoc(x3, x2, x3))}} \\
= & \quad \text{by Lemma 944 LR with } \{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{x1}
\end{aligned}
$$

Lemma 947: $op_l(x1, x2, asoc(x3, x2, x2)) = x1$

$$
\begin{aligned}
& \underbrace{op_l(x1, x2, asoc(x3, x2, x2))} \\
= & \quad \text{by Lemma 799 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(\underbrace{x1}, asoc(x2, x3, x2), x2)} \\
= & \quad \text{by Lemma 944 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{op_l(\overbrace{op_l(x1, x2, asoc(x2, x3, x2))}, asoc(x2, x3, x2), x2)} \\
= & \quad \text{by Lemma 138 LR with } \{x3 \leftarrow asoc(x2, x3, x2), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{x1}
\end{aligned}
$$

Lemma 948: $op_l(x1, op_l(x2, x2, x3), x2) = x1$

$$
\begin{aligned}
& op_l(x1, \underbrace{op_l(x2, x2, x3)}, x2) \\
= & \quad \text{by Lemma 173 RL with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& \underbrace{op_l(x1, \overbrace{mult(asoc(x2, x3, x2), x2)}, x2)} \\
= & \quad \text{by Lemma 285 LR with } \{x3 \leftarrow x2, x2 \leftarrow asoc(x2, x3, x2), x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, asoc(x2, x3, x2), x2)}} \\
= & \quad \text{by Lemma 799 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, x2, asoc(x3, x2, x2))}} \\
= & \quad \text{by Lemma 947 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{x1}
\end{aligned}
$$

Lemma 949: $x1 = op_l(x1, mult(x2, x3), mult(x3, x2))$

$$
\begin{aligned}
& \underbrace{x1} \\
= & \quad \text{by Lemma 945 RL with } \{x3 \leftarrow i(x2), x2 \leftarrow mult(x2, x3), x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, mult(x2, x3), \underbrace{op_l(mult(x2, x3), mult(x2, x3), i(x2))})} \\
= & \quad \text{by Lemma 242 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow mult(x2, x3)\} \\
& op_l(x1, mult(x2, x3), \overbrace{\underbrace{op_l(mult(x2, x3), x2, x3)}}) \\
= & \quad \text{by Lemma 435 LR with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& op_l(x1, mult(x2, x3), \overbrace{mult(x3, x2)})
\end{aligned}
$$

Lemma 950: $x1 = op_l(x1, mult(x2, mult(x2, mult(i(x2), x3))), mult(op_t(x3, x2), x2))$

$$
\begin{aligned}
& \overbrace{x1} \\
= \quad & \text{by Lemma 949 LR with } \{x3 \leftarrow mult(i(x2), x3), x2 \leftarrow mult(x2, x2), x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, \underbrace{mult(mult(x2, x2), mult(i(x2), x3))}, \underbrace{mult(mult(i(x2), x3), mult(x2, x2))})} \\
= \quad & \text{by Lemma 749 LR with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& op_l(x1, \underbrace{mult(mult(x2, x2), mult(i(x2), x3))}, \overbrace{mult(op_t(x3, x2), x2)}) \\
= \quad & \text{by Lemma 24 LR with } \{x2 \leftarrow mult(i(x2), x3), x1 \leftarrow x2\} \\
& op_l(x1, \overbrace{mult(x2, mult(x2, mult(i(x2), x3)))}, mult(op_t(x3, x2), x2))
\end{aligned}
$$

Lemma 951: $mult(mult(x1, asoc(x1, x2, x1)), x3) = mult(x1, mult(asoc(x1, x2, x1), x3))$

$$
\begin{aligned}
& \underbrace{mult(mult(x1, asoc(x1, x2, x1)), x3)} \\
= \quad & \text{by Lemma 91 RL with } \{x3 \leftarrow x3, x2 \leftarrow asoc(x1, x2, x1), x1 \leftarrow x1\} \\
& \overbrace{mult(op_r(x1, \underbrace{asoc(x1, x2, x1)}, x3), mult(asoc(x1, x2, x1), x3))} \\
= \quad & \text{by Lemma 758 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& mult(\underbrace{op_r(x1, \overbrace{asoc(i(x2), x1, x1)}, x3)}, mult(asoc(x1, x2, x1), x3)) \\
= \quad & \text{by Lemma 846 RL with } \{x3 \leftarrow x3, x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& mult(\overbrace{x1}, mult(asoc(x1, x2, x1), x3))
\end{aligned}
$$

Lemma 952: $mult(op_l(x1, x2, x1), rd(x1, x2)) = rd(x1, mult(i(x1), x2))$

$$
\begin{aligned}
& mult(\underbrace{op_l(x1, x2, x1)}, rd(x1, x2)) \\
= \quad & \text{by Lemma 720 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{mult(\overbrace{mult(rd(x1, x2), \overbrace{op_r(x2, x1, x1)}), rd(x1, x2))}} \\
= \quad & \text{by Lemma 871 RL with } \{x2 \leftarrow rd(x1, x2), x1 \leftarrow op_r(x2, x1, x1)\} \\
& \overbrace{mult(\underbrace{mult(op_r(x2, x1, x1), rd(x1, x2))}, op_t(rd(x1, x2), op_r(x2, x1, x1)))} \\
= \quad & \text{by Lemma 366 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& mult(\overbrace{x1}, \underbrace{op_t(rd(x1, x2), op_r(x2, x1, x1))}) \\
= \quad & \text{by Lemma 470 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{mult(x1, \overbrace{mult(i(x2), x1)})} \\
= \quad & \text{by Lemma 60 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{rd(x1, mult(i(x1), x2))}
\end{aligned}
$$

Lemma 953: $rd(asoc(x2,x1,x2),op_t(x2,x1)) = i(op_t(x2,mult(x1,x2)))$

$rd(\underbrace{asoc(x2,x1,x2)}, op_t(x2,x1))$

$=$ by Axiom 2 LR with $\{x2 \leftarrow op_t(x2,x1), x1 \leftarrow asoc(x2,x1,x2)\}$

$rd(\overbrace{mult(i(op_t(x2,x1)), mult(op_t(x2,x1), asoc(x2,x1,x2)))}, op_t(x2,x1))$

$=$ by Lemma 179 RL with $\{x3 \leftarrow x1, x2 \leftarrow x1, x1 \leftarrow x2\}$

$rd(\overbrace{op_t(\underbrace{asoc(x2,x1,x2)}, op_t(x2,x1))}, op_t(x2,x1))$

$=$ by Lemma 759 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{rd(op_t(\overbrace{i(\underbrace{asoc(x1,x2,x2)}))}, op_t(x2,x1)), op_t(x2,x1))}$

$=$ by Lemma 329 RL with $\{x2 \leftarrow op_t(x2,x1), x1 \leftarrow asoc(x1,x2,x2)\}$

$\overbrace{op_t(i(mult(\underbrace{asoc(x1,x2,x2)}, op_t(x2,x1))), op_t(x2,x1))}$

$=$ by Lemma 756 RL with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$op_t(i(mult(\overbrace{asoc(x2,\underbrace{i(x1)}, x2)}, op_t(x2,x1))), op_t(x2,x1))$

$=$ by Lemma 21 RL with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$op_t(i(mult(\underbrace{asoc(x2,\overbrace{mult(rd(i(x2),x1),x2)}, x2)}, op_t(x2,x1))), op_t(x2,x1))$

$=$ by Lemma 262 LR with $\{x2 \leftarrow rd(i(x2),x1), x1 \leftarrow x2\}$

$op_t(i(\underbrace{mult(\overbrace{asoc(x2,rd(i(x2),x1),x2)}, op_t(x2,x1))}), op_t(x2,x1))$

$=$ by Lemma 877 RL with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$\underbrace{op_t(i(\overbrace{op_t(x2,mult(x1,x2))}), op_t(x2,x1))}$

$=$ by Lemma 19 RL with $\{x2 \leftarrow op_t(x2,x1), x1 \leftarrow op_t(x2,mult(x1,x2))\}$

$\overbrace{i(\underbrace{op_t(op_t(x2,mult(x1,x2)), i(op_t(x2,x1)))})}$

$=$ by Axiom 13 RL with $\{x3 \leftarrow mult(x1,x2), x2 \leftarrow i(op_t(x2,x1)), x1 \leftarrow x2\}$

$i(\overbrace{op_t(\underbrace{op_t(x2,i(op_t(x2,x1)))}, mult(x1,x2))})$

$=$ by Lemma 48 LR with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$i(op_t(\overbrace{x2}, mult(x1,x2)))$

Lemma 954: $mult(rd(op_t(i(x2),x1),i(x1)),x2) = mult(i(x2),rd(x2,op_r(i(x1),i(x2),i(x2))))$

$mult(\underbrace{rd(op_t(i(x2),x1),i(x1))}, x2)$

$=$ by Lemma 10 RL with $\{x2 \leftarrow op_t(i(x2),x1), x1 \leftarrow i(x1)\}$

$\underbrace{mult(\overbrace{i(\underbrace{rd(i(x1),op_t(i(x2),x1))})}, x2)}$

$=$ by Lemma 217 RL with $\{x2 \leftarrow rd(i(x1),op_t(i(x2),x1)), x1 \leftarrow x2\}$

$\overbrace{mult(i(x2),rd(x2,\underbrace{mult(i(x2),rd(i(x1),op_t(i(x2),x1)))}))}$

$=$ by Lemma 913 LR with $\{x2 \leftarrow x1, x1 \leftarrow i(x2)\}$

$mult(i(x2),rd(x2,\overbrace{op_r(i(x1),i(x2),i(x2))}))$

Lemma 955: $mult(rd(x1, op_t(x2, i(x1))), x2) = mult(i(x2), rd(x2, i(op_r(x1, x2, x2))))$

$$
\begin{aligned}
& mult(\underbrace{rd(x1, op_t(x2, i(x1)))}, x2) \\
= & \quad \text{by Lemma 67 RL with } \{x3 \leftarrow x1, x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \underbrace{mult(\overbrace{rd(op_t(i(x2), x1), i(x1))}, x2)} \\
= & \quad \text{by Lemma 954 LR with } \{x1 \leftarrow x1, x2 \leftarrow x2\} \\
& \overbrace{mult(i(x2), rd(x2, \underbrace{op_r(i(x1), i(x2), i(x2))}))} \\
= & \quad \text{by Lemma 815 LR with } \{x3 \leftarrow x2, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& mult(i(x2), rd(x2, \overbrace{i(op_r(x1, x2, x2))}))
\end{aligned}
$$

Lemma 956: $mult(rd(x1, op_t(x2, i(x1))), x2) = mult(i(x2), op_r(rd(x2, i(x1)), x2, x2))$

$$
\begin{aligned}
& \underbrace{mult(rd(x1, op_t(x2, i(x1))), x2)} \\
= & \quad \text{by Lemma 955 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{mult(i(x2), rd(x2, \underbrace{i(op_r(x1, x2, x2))}))} \\
= & \quad \text{by Lemma 356 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& mult(i(x2), \underbrace{rd(x2, \overbrace{op_r(i(x1), x2, x2)})}) \\
= & \quad \text{by Lemma 369 LR with } \{x2 \leftarrow i(x1), x1 \leftarrow x2\} \\
& mult(i(x2), \overbrace{op_r(rd(x2, i(x1)), x2, x2)})
\end{aligned}
$$

Lemma 957: $mult(asoc(x1, x2, x1), mult(x1, x3)) = mult(op_l(x1, x1, x2), x3)$

$$
\begin{aligned}
& \underbrace{mult(asoc(x1, x2, x1), mult(x1, x3))} \\
= & \quad \text{by Lemma 132 RL with } \{x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow asoc(x1, x2, x1)\} \\
& \overbrace{mult(mult(asoc(x1, x2, x1), x1), \underbrace{op_l(x3, x1, asoc(x1, x2, x1))})} \\
= & \quad \text{by Lemma 944 LR with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x3\} \\
& mult(\underbrace{mult(asoc(x1, x2, x1), x1)}, \overbrace{x3}) \\
= & \quad \text{by Lemma 173 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& mult(\overbrace{op_l(x1, x1, x2)}, x3)
\end{aligned}
$$

Lemma 958: $asoc(asoc(x1, x2, x2), x2, x3) = unit()$

$$
\begin{aligned}
& asoc(\underbrace{asoc(x1, x2, x2)}, x2, x3) \\
= \quad & \text{by Lemma 756 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \underbrace{asoc(\overbrace{asoc(x2, i(x1), x2)}, x2, x3)} \\
= \quad & \text{by Lemma 108 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow asoc(x2, i(x1), x2)\} \\
& \overbrace{mult(i(\underbrace{mult(asoc(x2, i(x1), x2), mult(x2, x3))}), mult(mult(asoc(x2, i(x1), x2), x2), x3))} \\
= \quad & \text{by Lemma 957 LR with } \{x3 \leftarrow x3, x2 \leftarrow i(x1), x1 \leftarrow x2\} \\
& mult(i(\overbrace{mult(op_l(x2, x2, i(x1)), x3)}), mult(\underbrace{mult(asoc(x2, i(x1), x2), x2)}, x3)) \\
= \quad & \text{by Lemma 173 LR with } \{x2 \leftarrow i(x1), x1 \leftarrow x2\} \\
& \underbrace{mult(i(mult(op_l(x2, x2, i(x1)), x3)), mult(\overbrace{op_l(x2, x2, i(x1))}, x3))} \\
= \quad & \text{by Axiom 9 LR with } \{x1 \leftarrow mult(op_l(x2, x2, i(x1)), x3)\} \\
& \overbrace{unit()}
\end{aligned}
$$

Lemma 959: $mult(asoc(x1, x2, x2), mult(x2, x3)) = mult(op_l(x2, x1, x2), x3)$

$$
\begin{aligned}
& mult(asoc(x1, x2, x2), mult(x2, \underbrace{x3})) \\
= \quad & \text{by Lemma 946 RL with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x3\} \\
& \underbrace{mult(asoc(x1, x2, x2), mult(x2, \overbrace{op_l(x3, asoc(x1, x2, x2), x2)}))} \\
= \quad & \text{by Lemma 137 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow asoc(x1, x2, x2)\} \\
& \overbrace{mult(\underbrace{mult(asoc(x1, x2, x2), x2)}, x3)} \\
= \quad & \text{by Lemma 771 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& mult(\overbrace{op_l(x2, x1, x2)}, x3)
\end{aligned}
$$

Lemma 960: $op_l(x1, x2, x1) = mult(op_l(i(x2), x2, x1), mult(x1, x2))$

$$
\begin{aligned}
& \underbrace{op_l(x1, x2, x1)} \\
= \quad & \text{by Lemma 292 RL with } \{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, rd(x2, x1), x1)}} \\
= \quad & \text{by Lemma 280 LR with } \{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{mult(\underbrace{mult(i(x1), rd(x1, x2))}, mult(x1, x2))} \\
= \quad & \text{by Lemma 310 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& mult(\overbrace{op_l(i(x2), x2, x1)}, mult(x1, x2))
\end{aligned}
$$

Lemma 961: $i(mult(x1,x2)) = op_t(rd(i(x1),x2), rd(i(x2), rd(i(x1),x2)))$

$$
\begin{aligned}
& \overbrace{i(mult(x1,x2))} \\
= & \quad \text{by Lemma 33 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_t(rd(i(x1),x2),x1)} \\
= & \quad \text{by Lemma 378 RL with } \{x2 \leftarrow x1, x1 \leftarrow rd(i(x1),x2)\} \\
& \overbrace{op_t(rd(i(x1),x2), rd(\underbrace{mult(rd(i(x1),x2),x1)}, rd(i(x1),x2)))} \\
= & \quad \text{by Lemma 21 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_t(rd(i(x1),x2), rd(\overbrace{i(x2)}, rd(i(x1),x2)))
\end{aligned}
$$

Lemma 962: $i(mult(x1,x2)) = op_t(rd(i(x1),x2), op_r(i(i(x1)),x2,x2))$

$$
\begin{aligned}
& \overbrace{i(mult(x1,x2))} \\
= & \quad \text{by Lemma 961 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_t(rd(i(x1),x2), \underbrace{rd(i(x2), rd(i(x1),x2))})} \\
= & \quad \text{by Lemma 40 LR with } \{x3 \leftarrow x2, x2 \leftarrow i(x1), x1 \leftarrow x2\} \\
& op_t(rd(i(x1),x2), \overbrace{\underbrace{rd(rd(x2,i(x1)),x2)}}) \\
= & \quad \text{by Lemma 354 LR with } \{x2 \leftarrow i(x1), x1 \leftarrow x2\} \\
& op_t(rd(i(x1),x2), \overbrace{op_r(i(i(x1)),x2,x2)})
\end{aligned}
$$

Lemma 963: $mult(i(rd(i(x1),x2)), op_t(op_r(x1,x2,x2), rd(i(x1),x2))) = mult(i(i(mult(x1,x2))), op_r(x1,x2,x2))$

$$
\begin{aligned}
& \underbrace{mult(i(rd(i(x1),x2)), op_t(op_r(x1,x2,x2), rd(i(x1),x2)))} \\
= & \quad \text{by Lemma 669 RL with } \{x2 \leftarrow op_r(x1,x2,x2), x1 \leftarrow rd(i(x1),x2)\} \\
& \overbrace{mult(i(op_t(rd(i(x1),x2), op_r(\underbrace{x1}, x2, x2))), op_r(x1,x2,x2))} \\
= & \quad \text{by Lemma 3 RL with } \{x1 \leftarrow x1\} \\
& mult(i(\underbrace{op_t(rd(i(x1),x2), op_r(\overbrace{i(i(x1))}, x2, x2))}), op_r(x1,x2,x2)) \\
= & \quad \text{by Lemma 962 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& mult(i(\overbrace{i(mult(x1,x2))}), op_r(x1,x2,x2))
\end{aligned}
$$

Lemma 964: $mult(mult(x1, mult(x2,x3)), i(mult(mult(x1,x2),x3))) = i(asoc(x1,x2,x3))$

$$
\begin{aligned}
& mult(mult(x1, mult(x2,x3)), i(\underbrace{mult(mult(x1,x2),x3)})) \\
= & \quad \text{by Axiom 8 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{mult(mult(x1, mult(x2,x3)), i(\overbrace{mult(mult(x1, mult(x2,x3)), asoc(x1,x2,x3))}))} \\
= & \quad \text{by Lemma 20 LR with } \{x2 \leftarrow asoc(x1,x2,x3), x1 \leftarrow mult(x1, mult(x2,x3))\} \\
& \overbrace{i(asoc(x1,x2,x3))}
\end{aligned}
$$

Lemma 965: $op_l(x1, x2, x3) = op_l(x1, rd(x2, x3), op_l(x2, x2, x3))$

$$
\begin{aligned}
& \overbrace{op_l(x1, x2, x3)} \\
= \quad & \text{by Lemma 292 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, rd(x2, x3), x3)}} \\
= \quad & \text{by Lemma 291 RL with } \{x3 \leftarrow x3, x2 \leftarrow rd(x2, x3), x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, rd(x2, x3), \underbrace{mult(x3, rd(x2, x3))})} \\
= \quad & \text{by Lemma 427 RL with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& op_l(x1, rd(x2, x3), \overbrace{op_l(x2, x2, x3)})
\end{aligned}
$$

Lemma 966: $op_l(op_t(i(x1), rd(x1, x2)), x2, x1) = i(op_t(x1, x2))$

$$
\begin{aligned}
& op_l(\underbrace{op_t(i(x1), rd(x1, x2))}, x2, x1) \\
= \quad & \text{by Lemma 47 RL with } \{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_l(\overbrace{\underbrace{i(op_t(x1, rd(x2, x1)))}}, x2, x1) \\
= \quad & \text{by Lemma 15 RL with } \{x2 \leftarrow rd(x2, x1), x1 \leftarrow x1\} \\
& \underbrace{op_l(\overbrace{op_t(i(x1), i(rd(x2, x1)))}, x2, x1)} \\
= \quad & \text{by Axiom 15 RL with } \{x4 \leftarrow i(rd(x2, x1)), x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow i(x1)\} \\
& \overbrace{op_t(\underbrace{op_l(i(x1), x2, x1)}, i(rd(x2, x1)))} \\
= \quad & \text{by Lemma 433 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{op_t(\overbrace{i(op_l(x1, x2, x1))}, i(rd(x2, x1)))} \\
= \quad & \text{by Lemma 15 LR with } \{x2 \leftarrow rd(x2, x1), x1 \leftarrow op_l(x1, x2, x1)\} \\
& \overbrace{i(\underbrace{op_t(op_l(x1, x2, x1), rd(x2, x1))})} \\
= \quad & \text{by Axiom 15 LR with } \{x4 \leftarrow rd(x2, x1), x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& i(\overbrace{\underbrace{op_l(op_t(x1, rd(x2, x1)), x2, x1)}}) \\
= \quad & \text{by Lemma 455 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& i(\overbrace{op_t(x1, x2)})
\end{aligned}
$$

Lemma 967: $op_t(mult(x1, i(x2)), asoc(x1, x2, x1)) = mult(x1, i(x2))$

$$
\begin{aligned}
& \underbrace{op_t(mult(x1, i(x2)), asoc(x1, x2, x1))} \\
= \quad & \text{by Lemma 194 RL with } \{x3 \leftarrow asoc(x1, x2, x1), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_t(\underbrace{op_t(rd(x1, x2), asoc(x1, x2, x1))}, i(x1))} \\
= \quad & \text{by Lemma 541 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{op_t(\overbrace{rd(x1, x2)}, i(x1))} \\
= \quad & \text{by Lemma 34 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{mult(x1, i(x2))}
\end{aligned}
$$

Lemma 968: $mult(x1, i(x2)) = op_t(rd(x1, x2), op_l(i(x1), i(x2), i(x1)))$

$$
\begin{aligned}
& \underbrace{mult(x1, i(x2))} \\
= \quad & \text{by Lemma 34 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_t(rd(x1, x2), i(x1))}} \\
= \quad & \text{by Lemma 560 RL with } \{x2 \leftarrow i(x1), x1 \leftarrow rd(x1, x2)\} \\
& \overbrace{op_t(rd(x1, x2), \underbrace{op_l(i(x1), rd(x1, x2), i(x1))})} \\
= \quad & \text{by Lemma 299 LR with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow i(x1)\} \\
& op_t(rd(x1, x2), \overbrace{op_l(i(x1), i(x2), i(x1))})
\end{aligned}
$$

Lemma 969: $mult(x1, i(x2)) = op_t(rd(x1, x2), op_l(i(x1), x2, x1))$

$$
\begin{aligned}
& \underbrace{mult(x1, i(x2))} \\
= \quad & \text{by Lemma 968 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_t(rd(x1, x2), \underbrace{op_l(i(x1), i(x2), i(x1))})} \\
= \quad & \text{by Lemma 519 LR with } \{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_t(rd(x1, x2), \overbrace{\underbrace{i(op_l(x1, x2, x1))}}) \\
= \quad & \text{by Lemma 433 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_t(rd(x1, x2), \overbrace{op_l(i(x1), x2, x1)})
\end{aligned}
$$

Lemma 970: $op_t(i(x1), rd(rd(x2, i(x1)), x1)) = op_t(i(x1), mult(mult(i(x1), x2), x1))$

$$
\begin{aligned}
& \underbrace{op_t(i(x1), rd(rd(x2, i(x1)), x1))} \\
= \quad & \text{by Lemma 561 RL with } \{x2 \leftarrow rd(rd(x2, i(x1)), x1), x1 \leftarrow x1\} \\
& \overbrace{op_t(i(x1), \underbrace{op_t(rd(rd(x2, i(x1)), x1), x1)})} \\
= \quad & \text{by Lemma 14 LR with } \{x2 \leftarrow x1, x1 \leftarrow rd(x2, i(x1))\} \\
& op_t(i(x1), \overbrace{\underbrace{mult(i(x1), rd(x2, i(x1)))}}) \\
= \quad & \text{by Lemma 69 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_t(i(x1), \overbrace{mult(mult(i(x1), x2), x1)})
\end{aligned}
$$

Lemma 971: $op_t(i(x1), x2) = op_t(i(x1), mult(mult(i(x1), x2), x1))$

$$
\begin{aligned}
& \underbrace{op_t(i(x1), x2)} \\
= \quad & \text{by Lemma 569 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_t(i(x1), \underbrace{op_r(x2, x1, x1)})} \\
= \quad & \text{by Lemma 358 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \underbrace{op_t(i(x1), \overbrace{rd(rd(x2, i(x1)), x1)})} \\
= \quad & \text{by Lemma 970 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_t(i(x1), mult(mult(i(x1), x2), x1))}
\end{aligned}
$$

Lemma 972: $op_t(rd(mult(x1,x2),x1),mult(x1,x2)) = op_l(x2,x1,x2)$

$$
\begin{aligned}
& \overbrace{op_t(rd(mult(x1,x2),x1),mult(x1,x2))} \\
= \quad & \text{by Lemma 340 RL with } \{x2 \leftarrow mult(x1,x2), x1 \leftarrow x1\} \\
& \overbrace{mult(\underbrace{op_t(i(x1),mult(x1,x2))}, mult(x1,x2))} \\
= \quad & \text{by Lemma 636 RL with } \{x2 \leftarrow mult(x1,x2), x1 \leftarrow i(x1)\} \\
& \overbrace{op_l(\underbrace{mult(i(x1),mult(x1,x2))}, mult(x1,x2), i(x1))} \\
= \quad & \text{by Axiom 2 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \underbrace{op_l(\overbrace{x2}, mult(x1,x2), i(x1))} \\
= \quad & \text{by Lemma 242 LR with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \overbrace{op_l(x2,x1,x2)}
\end{aligned}
$$

Lemma 973: $i(x1) = op_t(rd(rd(mult(x1,x2),x1),mult(x1,x2)),i(op_l(x2,x1,x2)))$

$$
\begin{aligned}
& \underbrace{i(x1)} \\
= \quad & \text{by Lemma 22 RL with } \{x2 \leftarrow x1, x1 \leftarrow mult(x1,x2)\} \\
& \overbrace{mult(rd(mult(x1,x2),x1),i(mult(x1,x2)))} \\
= \quad & \text{by Lemma 591 RL with } \{x2 \leftarrow mult(x1,x2), x1 \leftarrow rd(mult(x1,x2),x1)\} \\
& \overbrace{op_t(rd(rd(mult(x1,x2),x1),mult(x1,x2)),i(\underbrace{op_t(rd(mult(x1,x2),x1),mult(x1,x2))}))} \\
= \quad & \text{by Lemma 972 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_t(rd(rd(mult(x1,x2),x1),mult(x1,x2)),i(\overbrace{op_l(x2,x1,x2)}))
\end{aligned}
$$

Lemma 974: $i(x1) = op_t(i(op_r(x1,x2,mult(x1,x2))),i(op_l(x2,x1,x2)))$

$$
\begin{aligned}
& \underbrace{i(x1)} \\
= \quad & \text{by Lemma 973 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_t(\underbrace{rd(rd(mult(x1,x2),x1),mult(x1,x2))}, i(op_l(x2,x1,x2)))} \\
= \quad & \text{by Lemma 354 LR with } \{x2 \leftarrow x1, x1 \leftarrow mult(x1,x2)\} \\
& op_t(\overbrace{op_r(i(x1),mult(x1,x2),mult(x1,x2))}, i(op_l(x2,x1,x2))) \\
= \quad & \text{by Lemma 654 LR with } \{x3 \leftarrow mult(x1,x2), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_t(\overbrace{i(op_r(x1,x2,mult(x1,x2)))}, i(op_l(x2,x1,x2)))
\end{aligned}
$$

Lemma 975: $op_r(x1, x2, x2) = op_r(op_t(i(op_r(i(x1), x2, x2)), rd(x2, x1)), i(rd(x1, x2)), x2)$

$\overbrace{op_r(x1, x2, x2)}$

$=$ by Lemma 361 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_r(x1, i(x2), i(x2))}$

$=$ by Lemma 813 RL with $\{x3 \leftarrow x2, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{i(op_r(i(x1), x2, x2))}$

$=$ by Lemma 974 LR with $\{x2 \leftarrow rd(x1, x2), x1 \leftarrow op_r(i(x1), x2, x2)\}$

$\overbrace{op_t(i(op_r(op_r(i(x1), x2, x2), rd(x1, x2), mult(op_r(i(x1), x2, x2), rd(x1, x2)))), i(op_l(rd(x1, x2), op_r(i(x1), x2, x2), rd(x1, x2}$

$=$ by Lemma 15 LR with $\{x2 \leftarrow op_l(rd(x1, x2), op_r(i(x1), x2, x2), rd(x1, x2)), x1 \leftarrow op_r(op_r(i(x1), x2, x2), rd(x1, x2), mult(op_r$

$\overbrace{i(op_t(op_r(op_r(i(x1), x2, x2), rd(x1, x2), mult(op_r(i(x1), x2, x2), rd(x1, x2))), op_l(rd(x1, x2), op_r(i(x1), x2, x2), rd(x1, x2))))}$

$=$ by Axiom 14 LR with $\{x4 \leftarrow op_l(rd(x1, x2), op_r(i(x1), x2, x2), rd(x1, x2)), x3 \leftarrow mult(op_r(i(x1), x2, x2), rd(x1, x2)), x2 \leftarrow rd$

$i(\overbrace{op_r(\underbrace{op_t(op_r(i(x1), x2, x2), op_l(rd(x1, x2), op_r(i(x1), x2, x2), rd(x1, x2)))}, rd(x1, x2), mult(op_r(i(x1), x2, x2), rd(x1, x2)))}$

$=$ by Lemma 560 LR with $\{x2 \leftarrow rd(x1, x2), x1 \leftarrow op_r(i(x1), x2, x2)\}$

$i(op_r(\overbrace{op_t(op_r(i(x1), x2, x2), rd(x1, x2))}, rd(x1, x2), mult(\underbrace{op_r(i(x1), x2, x2)}, rd(x1, x2))))$

$=$ by Lemma 356 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$i(op_r(op_t(op_r(i(x1), x2, x2), rd(x1, x2)), rd(x1, x2), mult(\overbrace{i(op_r(x1, x2, x2))}, \underbrace{rd(x1, x2)})))$

$=$ by Lemma 352 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$i(op_r(op_t(op_r(i(x1), x2, x2), rd(x1, x2)), rd(x1, x2), \underbrace{mult(i(op_r(x1, x2, x2)), \overbrace{mult(op_r(x1, x2, x2), i(x2))})})$

$=$ by Axiom 2 RL with $\{x2 \leftarrow op_r(x1, x2, x2), x1 \leftarrow i(x2)\}$

$\underbrace{i(op_r(op_t(op_r(i(x1), x2, x2), rd(x1, x2)), rd(x1, x2), \overbrace{i(x2)}))}$

$=$ by Lemma 814 LR with $\{x3 \leftarrow x2, x2 \leftarrow rd(x1, x2), x1 \leftarrow op_t(op_r(i(x1), x2, x2), rd(x1, x2))\}$

$\overbrace{op_r(\underbrace{i(op_t(op_r(i(x1), x2, x2), rd(x1, x2)))}, i(rd(x1, x2)), x2)}$

$=$ by Lemma 47 LR with $\{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow op_r(i(x1), x2, x2)\}$

$op_r(\overbrace{op_t(i(op_r(i(x1), x2, x2)), rd(x2, x1))}, i(rd(x1, x2)), x2)$

Lemma 976: $rd(x2, x1) = op_r(op_t(rd(x2, x1), x1), x1, i(i(x2)))$

$\overbrace{rd(x2, x1)}$

$=$ by Lemma 10 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{i(\overbrace{rd(x1, x2)})}$

$=$ by Lemma 974 LR with $\{x2 \leftarrow i(x1), x1 \leftarrow rd(x1, x2)\}$

$\overbrace{op_t(i(op_r(rd(x1, x2), i(x1), \overbrace{mult(rd(x1, x2), i(x1))})), i(op_l(i(x1), rd(x1, x2), i(x1))))}$

$=$ by Lemma 15 LR with $\{x2 \leftarrow op_l(i(x1), rd(x1, x2), i(x1)), x1 \leftarrow op_r(rd(x1, x2), i(x1), mult(rd(x1, x2), i(x1)))\}$

$\overbrace{i(\underbrace{op_t(op_r(rd(x1, x2), i(x1), \overbrace{mult(rd(x1, x2), i(x1))}), op_l(i(x1), rd(x1, x2), i(x1)))})}$

$=$ by Axiom 14 LR with $\{x4 \leftarrow op_l(i(x1), rd(x1, x2), i(x1)), x3 \leftarrow mult(rd(x1, x2), i(x1)), x2 \leftarrow i(x1), x1 \leftarrow rd(x1, x2)\}$

$i(\overbrace{op_r(\underbrace{op_t(rd(x1, x2), op_l(i(x1), rd(x1, x2), i(x1)))}, i(x1), mult(rd(x1, x2), i(x1))))}$

$=$ by Lemma 560 LR with $\{x2 \leftarrow i(x1), x1 \leftarrow rd(x1, x2)\}$

$i(op_r(\overbrace{op_t(rd(x1, x2), i(x1))}, i(x1), \underbrace{mult(rd(x1, x2), i(x1))}))$

$=$ by Lemma 22 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$i(\underbrace{op_r(op_t(rd(x1, x2), i(x1)), i(x1), \overbrace{i(x2)})})$

$=$ by Axiom 14 RL with $\{x4 \leftarrow i(x1), x3 \leftarrow i(x2), x2 \leftarrow i(x1), x1 \leftarrow rd(x1, x2)\}$

$\underbrace{i(\overbrace{op_t(op_r(rd(x1, x2), i(x1), i(x2)), i(x1))})}$

$=$ by Lemma 19 LR with $\{x2 \leftarrow x1, x1 \leftarrow op_r(rd(x1, x2), i(x1), i(x2))\}$

$\overbrace{op_t(\underbrace{i(op_r(rd(x1, x2), i(x1), i(x2)))}, x1)}$

$=$ by Lemma 816 LR with $\{x3 \leftarrow i(x2), x2 \leftarrow x1, x1 \leftarrow rd(x1, x2)\}$

$\underbrace{op_t(\overbrace{op_r(i(rd(x1, x2)), x1, i(i(x2)))}, x1)}$

$=$ by Axiom 14 LR with $\{x4 \leftarrow x1, x3 \leftarrow i(i(x2)), x2 \leftarrow x1, x1 \leftarrow i(rd(x1, x2))\}$

$\overbrace{op_r(op_t(\underbrace{i(rd(x1, x2))}, x1), x1, i(i(x2)))}$

$=$ by Lemma 10 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$op_r(op_t(\overbrace{rd(x2, x1)}, x1), x1, i(i(x2)))$

Lemma 977: $rd(x2, x1) = op_r(mult(i(x1), x2), x1, x2)$

$\overbrace{rd(x2, x1)}$

$=$ by Lemma 976 LR with $\{x1 \leftarrow x1, x2 \leftarrow x2\}$

$\overbrace{op_r(\underbrace{op_t(rd(x2, x1), x1)}, x1, i(i(x2)))}$

$=$ by Lemma 14 LR with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$op_r(\overbrace{mult(i(x1), x2)}, x1, \underbrace{i(i(x2))})$

$=$ by Lemma 3 LR with $\{x1 \leftarrow x2\}$

$op_r(mult(i(x1), x2), x1, \overbrace{x2})$

Lemma 978: $mult(op_l(x1, x2, x1), mult(x2, i(x1))) = op_l(x2, mult(x2, i(x1)), x2)$

$$
\begin{aligned}
& \underbrace{mult(op_l(x1, x2, x1), mult(x2, i(x1)))} \\
= \quad & \text{by Lemma 661 RL with } \{x2 \leftarrow op_l(x1, x2, x1), x1 \leftarrow mult(x2, i(x1))\} \\
& \overbrace{op_l(\underbrace{mult(mult(x2, i(x1)), op_l(x1, x2, x1))}, mult(x2, i(x1)), mult(mult(x2, i(x1)), op_l(x1, x2, x1)))} \\
= \quad & \text{by Lemma 663 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& op_l(\overbrace{x2}, mult(x2, i(x1)), \underbrace{mult(mult(x2, i(x1)), op_l(x1, x2, x1))}) \\
= \quad & \text{by Lemma 663 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& op_l(x2, mult(x2, i(x1)), \overbrace{x2})
\end{aligned}
$$

Lemma 979: $mult(op_l(x1, x2, x1), mult(x2, i(x1))) = op_l(x2, x2, x1)$

$$
\begin{aligned}
& \underbrace{mult(op_l(x1, x2, x1), mult(x2, i(x1)))} \\
= \quad & \text{by Lemma 978 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x2, mult(x2, i(x1)), x2)}} \\
= \quad & \text{by Lemma 619 LR with } \{x2 \leftarrow i(x1), x1 \leftarrow x2\} \\
& \overbrace{\underbrace{op_l(x2, i(x1), x2)}} \\
= \quad & \text{by Lemma 607 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \overbrace{op_l(x2, x2, x1)}
\end{aligned}
$$

Lemma 980: $op_t(x1, i(x2)) = mult(mult(x1, x2), i(op_r(x2, x1, x1)))$

$$
\begin{aligned}
& \underbrace{op_t(x1, i(x2))} \\
= \quad & \text{by Lemma 18 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{i(op_t(i(x1), x2))}} \\
= \quad & \text{by Lemma 20 RL with } \{x2 \leftarrow op_t(i(x1), x2), x1 \leftarrow mult(x1, x2)\} \\
& \overbrace{mult(mult(x1, x2), i(\underbrace{mult(mult(x1, x2), op_t(i(x1), x2))}))} \\
= \quad & \text{by Lemma 695 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& mult(mult(x1, x2), i(\overbrace{op_r(x2, x1, x1)}))
\end{aligned}
$$

Lemma 981: $asoc(x2, x1, x2) = op_t(asoc(x2, x1, x2), rd(i(x1), x2))$

$\overbrace{asoc(x2, x1, x2)}$

$=$ by Lemma 262 RL with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$\overbrace{asoc(x2, mult(x1, x2), x2)}$

$=$ by Lemma 643 RL with $\{x2 \leftarrow x2, x1 \leftarrow mult(x1, x2)\}$

$\overbrace{mult(mult(x1, x2), \underbrace{op_r(i(mult(x1, x2)), x2, x2)})}$

$=$ by Lemma 371 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$mult(\underbrace{mult(x1, x2)}, \overbrace{rd(i(x1), x2)})$

$=$ by Lemma 75 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$mult(\overbrace{op_t(mult(x1, x2), \underbrace{rd(x2, i(x1))})}, rd(i(x1), x2))$

$=$ by Lemma 10 RL with $\{x2 \leftarrow x2, x1 \leftarrow i(x1)\}$

$\underbrace{mult(op_t(mult(x1, x2), \overbrace{i(\overbrace{rd(i(x1), x2)}))}, rd(i(x1), x2))}$

$=$ by Lemma 456 RL with $\{x2 \leftarrow rd(i(x1), x2), x1 \leftarrow mult(x1, x2)\}$

$\overbrace{op_t(op_t(mult(mult(x1, x2), \underbrace{rd(i(x1), x2)}), mult(x1, x2)), rd(i(x1), x2))}$

$=$ by Lemma 371 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$op_t(op_t(\underbrace{mult(mult(x1, x2), \overbrace{op_r(i(\overbrace{mult(x1, x2)}), x2, x2)})}, mult(x1, x2)), rd(i(x1), x2))$

$=$ by Lemma 643 LR with $\{x2 \leftarrow x2, x1 \leftarrow mult(x1, x2)\}$

$op_t(\underbrace{op_t(\overbrace{asoc(x2, mult(x1, x2), x2)}, mult(x1, x2))}, rd(i(x1), x2))$

$=$ by Lemma 376 RL with $\{x2 \leftarrow mult(x1, x2), x1 \leftarrow asoc(x2, mult(x1, x2), x2)\}$

$op_t(\overbrace{rd(mult(asoc(x2, mult(x1, x2), x2), mult(x1, x2)), \underbrace{op_t(mult(x1, x2), asoc(x2, mult(x1, x2), x2))})}, rd(i(x1), x2))$

$=$ by Lemma 535 LR with $\{x2 \leftarrow x2, x1 \leftarrow mult(x1, x2)\}$

$op_t(\underbrace{rd(mult(asoc(x2, mult(x1, x2), x2), mult(x1, x2)), \overbrace{mult(x1, x2)})}, rd(i(x1), x2))$

$=$ by Axiom 5 RL with $\{x2 \leftarrow mult(x1, x2), x1 \leftarrow asoc(x2, mult(x1, x2), x2)\}$

$op_t(\underbrace{\overbrace{asoc(x2, mult(x1, x2), x2)}}, rd(i(x1), x2))$

$=$ by Lemma 262 LR with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$op_t(\overbrace{asoc(x2, x1, x2)}, rd(i(x1), x2))$

Lemma 982: $mult(x1, mult(rd(i(x1), x2), mult(x1, x3))) = mult(mult(x1, i(x2)), x3)$

$\overbrace{mult(x1, mult(rd(i(x1), x2), mult(x1, x3)))}$

$=$ by Axiom 6 LR with $\{x3 \leftarrow x3, x2 \leftarrow rd(i(x1), x2), x1 \leftarrow x1\}$

$\overbrace{mult(mult(x1, \underbrace{mult(rd(i(x1), x2), x1)}), x3)}$

$=$ by Lemma 21 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$mult(mult(x1, \overbrace{i(x2)}), x3)$

Lemma 983: $op_r(x1, x2, x2) = rd(op_l(x2, x1, x2), mult(i(x1), x2))$

$$
\begin{aligned}
& \underbrace{op_r(x1, x2, x2)} \\
= \quad & \text{by Lemma 29 RL with } \{x2 \leftarrow op_r(x1, x2, x2), x1 \leftarrow rd(x2, x1)\} \\
& \overbrace{rd(\underbrace{mult(rd(x2, x1), op_r(x1, x2, x2))}, op_t(rd(x2, x1), op_r(x1, x2, x2)))} \\
= \quad & \text{by Lemma 720 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& rd(\overbrace{op_l(x2, x1, x2)}, \underbrace{op_t(rd(x2, x1), op_r(x1, x2, x2))}) \\
= \quad & \text{by Lemma 470 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& rd(op_l(x2, x1, x2), \overbrace{mult(i(x1), x2)})
\end{aligned}
$$

Lemma 984: $op_l(x1, x2, x1) = rd(op_r(op_l(x2, x2, x1), x1, x1), rd(x2, x1))$

$$
\begin{aligned}
& \underbrace{op_l(x1, x2, x1)} \\
= \quad & \text{by Lemma 292 RL with } \{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, rd(x2, x1), x1)}} \\
= \quad & \text{by Lemma 727 RL with } \{x2 \leftarrow rd(x2, x1), x1 \leftarrow x1\} \\
& \overbrace{rd(op_r(\underbrace{mult(x1, rd(x2, x1))}, x1, x1), rd(x2, x1))} \\
= \quad & \text{by Lemma 427 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& rd(op_r(\overbrace{op_l(x2, x2, x1)}, x1, x1), rd(x2, x1))
\end{aligned}
$$

Lemma 985: $rd(mult(x1, x2), op_t(x1, rd(x2, x1))) = op_r(x2, x1, x1)$

$$
\begin{aligned}
& rd(mult(x1, x2), \underbrace{op_t(x1, rd(x2, x1))}) \\
= \quad & \text{by Lemma 707 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& rd(\underbrace{mult(x1, x2)}, \overbrace{op_t(x1, rd(i(x1), x2))}) \\
= \quad & \text{by Lemma 3 RL with } \{x1 \leftarrow mult(x1, x2)\} \\
& rd(\overbrace{i(\underbrace{i(mult(x1, x2))})}, op_t(x1, rd(i(x1), x2))) \\
= \quad & \text{by Lemma 33 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& rd(\underbrace{i(\overbrace{op_t(rd(i(x1), x2), x1)})}, op_t(x1, rd(i(x1), x2))) \\
= \quad & \text{by Lemma 724 RL with } \{x2 \leftarrow x1, x1 \leftarrow rd(i(x1), x2)\} \\
& \underbrace{rd(\overbrace{mult(rd(i(rd(i(x1), x2)), x1), op_t(x1, rd(i(x1), x2)))}, op_t(x1, rd(i(x1), x2)))} \\
= \quad & \text{by Axiom 5 RL with } \{x2 \leftarrow op_t(x1, rd(i(x1), x2)), x1 \leftarrow rd(i(rd(i(x1), x2)), x1)\} \\
& \overbrace{rd(\underbrace{i(rd(i(x1), x2))}, x1)} \\
= \quad & \text{by Lemma 10 LR with } \{x2 \leftarrow x2, x1 \leftarrow i(x1)\} \\
& \underbrace{rd(\overbrace{rd(x2, i(x1))}, x1)} \\
= \quad & \text{by Lemma 358 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \overbrace{op_r(x2, x1, x1)}
\end{aligned}
$$

Lemma 986: $asoc(x1, x2, x1) = rd(mult(x2, rd(x1, op_t(x2, x1))), op_t(x1, x2))$

$$
\begin{aligned}
& \underbrace{asoc(x1, x2, x1)} \\
= \quad & \text{by Lemma 731 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \overbrace{\underbrace{rd(op_t(x2, x1), op_l(x2, x1, x2))}} \\
= \quad & \text{by Lemma 736 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{asoc(op_t(x1, x2), x2, op_t(x1, x2))}} \\
= \quad & \text{by Lemma 448 RL with } \{x2 \leftarrow op_t(x1, x2), x1 \leftarrow x2\} \\
& \overbrace{rd(mult(x2, \underbrace{rd(op_t(x1, x2), x2)}), op_t(x1, x2))} \\
= \quad & \text{by Lemma 748 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& rd(mult(x2, \overbrace{rd(x1, op_t(x2, x1))}), op_t(x1, x2))
\end{aligned}
$$

Lemma 987: $mult(x1, mult(rd(i(x2), x1), mult(x2, x3))) = op_l(x3, x2, i(x1))$

$$
\begin{aligned}
& mult(\underbrace{x1}, mult(rd(i(x2), x1), mult(x2, x3))) \\
= \quad & \text{by Lemma 3 RL with } \{x1 \leftarrow x1\} \\
& mult(\overbrace{i(\underbrace{i(x1)})}, mult(rd(i(x2), x1), mult(x2, x3))) \\
= \quad & \text{by Lemma 21 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \underbrace{mult(i(\overbrace{mult(rd(i(x2), x1), x2)}), mult(rd(i(x2), x1), mult(x2, x3)))} \\
= \quad & \text{by Axiom 12 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow rd(i(x2), x1)\} \\
& \overbrace{\underbrace{op_l(x3, x2, rd(i(x2), x1))}} \\
= \quad & \text{by Lemma 296 LR with } \{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x3\} \\
& \overbrace{op_l(x3, x2, i(x1))}
\end{aligned}
$$

Lemma 988: $rd(op_l(x1,x1,x2),asoc(x1,x2,x2)) = op_t(x1,i(x2))$

$$
\begin{aligned}
& rd(op_l(x1,x1,x2),\underbrace{asoc(x1,x2,x2)}) \\
= & \quad \text{by Lemma 757 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& rd(op_l(x1,x1,x2),\overbrace{i(\underbrace{asoc(x2,x1,x2)})}) \\
= & \quad \text{by Lemma 618 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& rd(op_l(x1,x1,x2),i(\overbrace{asoc(x2,\underbrace{mult(x2,x1)},x2)})) \\
= & \quad \text{by Lemma 231 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& rd(op_l(x1,x1,x2),i(\underbrace{asoc(x2,\overbrace{rd(x2,op_l(i(x1),x1,x2))},x2)})) \\
= & \quad \text{by Lemma 498 LR with } \{x2 \leftarrow op_l(i(x1),x1,x2), x1 \leftarrow x2\} \\
& rd(op_l(x1,x1,x2),i(\overbrace{asoc(x2,\underbrace{i(op_l(i(x1),x1,x2))},x2)})) \\
= & \quad \text{by Lemma 436 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& rd(op_l(x1,x1,x2),\underbrace{i(asoc(x2,\overbrace{op_l(x1,x1,x2)},x2))}) \\
= & \quad \text{by Lemma 757 LR with } \{x2 \leftarrow op_l(x1,x1,x2), x1 \leftarrow x2\} \\
& \underbrace{rd(op_l(x1,x1,x2),\overbrace{asoc(op_l(x1,x1,x2),x2,x2)})} \\
= & \quad \text{by Lemma 775 LR with } \{x2 \leftarrow x2, x1 \leftarrow op_l(x1,x1,x2)\} \\
& \underbrace{\overbrace{op_r(op_l(x1,x1,x2),x2,i(x2))}} \\
= & \quad \text{by Axiom 18 LR with } \{x5 \leftarrow i(x2), x4 \leftarrow x2, x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x1\} \\
& \underbrace{\overbrace{op_l(op_r(x1,x2,i(x2)),x1,x2)}} \\
= & \quad \text{by Lemma 475 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_t(x1,i(x2))}
\end{aligned}
$$

Lemma 989: $mult(i(op_t(x1,x2)),op_l(x1,x2,x1)) = rd(unit(),asoc(x2,x1,x2))$

$$
\begin{aligned}
& \underbrace{mult(i(op_t(x1,x2)),op_l(x1,x2,x1))} \\
= & \quad \text{by Lemma 790 LR with } \{x2 \leftarrow op_l(x1,x2,x1), x1 \leftarrow op_t(x1,x2)\} \\
& \overbrace{rd(asoc(op_t(x1,x2),op_l(x1,x2,x1),op_l(x1,x2,x1)),\underbrace{rd(op_t(x1,x2),op_l(x1,x2,x1))})} \\
= & \quad \text{by Lemma 731 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& rd(asoc(op_t(x1,x2),\underbrace{op_l(x1,x2,x1)},op_l(x1,x2,x1)),\overbrace{asoc(x2,x1,x2)}) \\
= & \quad \text{by Lemma 608 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& rd(\underbrace{asoc(op_t(x1,x2),\overbrace{op_l(x1,x1,i(x2))},op_l(x1,x2,x1))},asoc(x2,x1,x2)) \\
= & \quad \text{by Lemma 686 RL with } \{x3 \leftarrow op_l(x1,x2,x1), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& rd(\overbrace{unit()},asoc(x2,x1,x2))
\end{aligned}
$$

Lemma 990: $mult(i(op_t(x1,x2)),op_l(x1,x2,x1)) = asoc(x1,x2,x2)$

$\overbrace{mult(i(op_t(x1,x2)),op_l(x1,x2,x1))}$

$=$ by Lemma 989 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{rd(unit(),asoc(x2,x1,x2))}$

$=$ by Lemma 5 LR with $\{x1 \leftarrow asoc(x2,x1,x2)\}$

$\overbrace{i(asoc(x2,x1,x2))}$

$=$ by Lemma 757 LR with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$\overbrace{asoc(x1,x2,x2)}$

Lemma 991: $op_t(mult(x2,x1),x2) = op_r(mult(x1,x2),x1,x2)$

$\overbrace{op_t(mult(x2,x1),x2)}$

$=$ by Lemma 320 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_r(mult(x1,x2),x2,i(x2))}$

$=$ by Lemma 321 RL with $\{x2 \leftarrow x2, x1 \leftarrow mult(x1,x2)\}$

$\overbrace{op_r(mult(x1,x2),i(x2),x2)}$

$=$ by Lemma 828 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_t(mult(x1,x2),i(op_t(x1,x2)))}$

$=$ by Lemma 550 RL with $\{x2 \leftarrow i(op_t(x1,x2)), x1 \leftarrow mult(x1,x2)\}$

$\overbrace{op_t(mult(x1,x2),\underbrace{op_r(i(op_t(x1,x2)),mult(x1,x2),mult(x1,x2))})}$

$=$ by Lemma 354 RL with $\{x2 \leftarrow op_t(x1,x2), x1 \leftarrow mult(x1,x2)\}$

$\underbrace{op_t(mult(x1,x2),\overbrace{rd(rd(mult(x1,x2),op_t(x1,x2)),mult(x1,x2))})}$

$=$ by Lemma 809 RL with $\{x2 \leftarrow rd(rd(mult(x1,x2),op_t(x1,x2)),mult(x1,x2)), x1 \leftarrow mult(x1,x2)\}$

$\overbrace{op_r(mult(x1,x2),i(rd(rd(mult(x1,x2),op_t(x1,x2)),mult(x1,x2))),\underbrace{mult(rd(rd(mult(x1,x2),op_t(x1,x2)),mult(x1,x2)),$

$=$ by Axiom 4 RL with $\{x2 \leftarrow mult(x1,x2), x1 \leftarrow rd(mult(x1,x2),op_t(x1,x2))\}$

$op_r(mult(x1,x2),\underbrace{i(rd(rd(mult(x1,x2),op_t(x1,x2)),mult(x1,x2)))},\overbrace{rd(mult(x1,x2),op_t(x1,x2))})$

$=$ by Lemma 10 LR with $\{x2 \leftarrow mult(x1,x2), x1 \leftarrow rd(mult(x1,x2),op_t(x1,x2))\}$

$op_r(mult(x1,x2),\overbrace{rd(mult(x1,x2),rd(mult(x1,x2),op_t(x1,x2)))},rd(mult(x1,x2),op_t(x1,x2)))$

$=$ by Lemma 353 LR with $\{x2 \leftarrow op_t(x1,x2), x1 \leftarrow mult(x1,x2)\}$

$op_r(mult(x1,x2),\overbrace{op_r(op_t(x1,x2),mult(x1,x2),mult(x1,x2))},\underbrace{rd(mult(x1,x2),op_t(x1,x2))})$

$=$ by Lemma 29 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$op_r(mult(x1,x2),\underbrace{op_r(op_t(x1,x2),mult(x1,x2),mult(x1,x2))},\overbrace{x2})$

$=$ by Lemma 889 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$op_r(mult(x1,x2),\overbrace{x1},x2)$

Lemma 992: $op_l(x1, x1, x2) = mult(op_r(rd(x1, x2), x2, x1), x2)$

$\underbrace{op_l(x1, x1, x2)}$

$=$ by Lemma 291 RL with $\{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x1\}$

$\overbrace{op_l(x1, x1, \underbrace{mult(x2, x1)})}$

$=$ by Axiom 4 LR with $\{x2 \leftarrow mult(x1, x1), x1 \leftarrow mult(x2, x1)\}$

$\underbrace{op_l(x1, x1, \overbrace{mult(rd(mult(x2, x1), mult(x1, x1)), mult(x1, x1))})}$

$=$ by Lemma 512 RL with $\{x2 \leftarrow rd(mult(x2, x1), mult(x1, x1)), x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_l(x1, x1, mult(rd(mult(x2, x1), mult(x1, x1)), x1))}}$

$=$ by Lemma 291 LR with $\{x3 \leftarrow rd(mult(x2, x1), mult(x1, x1)), x2 \leftarrow x1, x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_l(x1, x1, rd(mult(x2, x1), mult(x1, x1)))}}$

$=$ by Lemma 281 LR with $\{x2 \leftarrow rd(mult(x2, x1), mult(x1, x1)), x1 \leftarrow x1\}$

$\overbrace{mult(i(rd(mult(x2, x1), mult(x1, x1))), op_t(\underbrace{mult(x1, rd(mult(x2, x1), mult(x1, x1)))}, x1))}$

$=$ by Lemma 841 RL with $\{x2 \leftarrow mult(x2, x1), x1 \leftarrow x1\}$

$mult(\underbrace{i(rd(mult(x2, x1), mult(x1, x1)))}, op_t(\overbrace{rd(mult(x1, rd(mult(x2, x1), x1)), x1)}, x1))$

$=$ by Lemma 10 LR with $\{x2 \leftarrow mult(x1, x1), x1 \leftarrow mult(x2, x1)\}$

$mult(\overbrace{rd(mult(x1, x1), mult(x2, x1))}, \underbrace{op_t(rd(mult(x1, rd(mult(x2, x1), x1)), x1), x1)})$

$=$ by Lemma 14 LR with $\{x2 \leftarrow x1, x1 \leftarrow mult(x1, rd(mult(x2, x1), x1))\}$

$mult(rd(mult(x1, x1), mult(x2, x1)), \overbrace{\underbrace{mult(i(x1), mult(x1, rd(mult(x2, x1), x1)))}})$

$=$ by Axiom 2 RL with $\{x2 \leftarrow x1, x1 \leftarrow rd(mult(x2, x1), x1)\}$

$mult(\underbrace{rd(mult(x1, x1), mult(x2, x1))}, \overbrace{rd(mult(x2, x1), x1)})$

$=$ by Lemma 676 RL with $\{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x1\}$

$mult(\overbrace{op_r(rd(x1, x2), x2, x1)}, \underbrace{rd(mult(x2, x1), x1)})$

$=$ by Axiom 5 RL with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$mult(op_r(rd(x1, x2), x2, x1), \overbrace{x2})$

Lemma 993: $rd(op_t(x1, i(x2)), x2) = op_r(mult(x1, i(x2)), x2, x1)$

$\overbrace{rd(op_t(x1, i(x2)), x2)}$

$=$ by Lemma 457 LR with $\{x1 \leftarrow x2, x2 \leftarrow x1\}$

$\overbrace{mult(op_l(x1, x1, x2), i(x2))}$

$=$ by Lemma 34 RL with $\{x2 \leftarrow x2, x1 \leftarrow op_l(x1, x1, x2)\}$

$\overbrace{op_t(rd(\underbrace{op_l(x1, x1, x2)}, x2), i(op_l(x1, x1, x2)))}$

$=$ by Lemma 992 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$op_t(\underbrace{rd(\overbrace{mult(op_r(rd(x1, x2), x2, x1), x2)}, x2)}, i(op_l(x1, x1, x2)))$

$=$ by Axiom 5 RL with $\{x2 \leftarrow x2, x1 \leftarrow op_r(rd(x1, x2), x2, x1)\}$

$\underbrace{op_t(\overbrace{op_r(rd(x1, x2), x2, x1)}, i(op_l(x1, x1, x2)))}$

$=$ by Axiom 14 LR with $\{x4 \leftarrow i(op_l(x1, x1, x2)), x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow rd(x1, x2)\}$

$\overbrace{op_r(op_t(rd(x1, x2), \underbrace{i(op_l(x1, x1, x2))}), x2, x1)}$

$=$ by Lemma 519 RL with $\{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x1\}$

$op_r(op_t(rd(x1, x2), \underbrace{\overbrace{op_l(i(x1), i(x1), i(x2))}}), x2, x1)$

$=$ by Lemma 301 RL with $\{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow i(x1)\}$

$op_r(\underbrace{op_t(rd(x1, x2), \overbrace{op_l(i(x1), i(x1), rd(x1, x2))})}, x2, x1)$

$=$ by Lemma 553 RL with $\{x2 \leftarrow i(x1), x1 \leftarrow rd(x1, x2)\}$

$op_r(\overbrace{op_t(rd(x1, x2), i(x1))}, x2, x1)$

$=$ by Lemma 34 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$op_r(\overbrace{mult(x1, i(x2))}, x2, x1)$

Lemma 994: $op_r(x2, x1, x1) = op_r(x2, op_t(x1, x2), i(rd(x2, i(i(mult(x1, x2))))))$

$\underbrace{op_r(x2, x1, x1)}$

$=$ by Lemma 485 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_l(rd(mult(x1, x2), x1), x1, x2)}}$

$=$ by Lemma 285 RL with $\{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow rd(mult(x1, x2), x1)\}$

$\overbrace{op_l(\underbrace{rd(mult(x1, x2), x1)}, mult(x1, x2), x2)}$

$=$ by Lemma 800 RL with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$\underbrace{op_l(\overbrace{op_r(x2, mult(x1, x2), mult(x1, x2))}, mult(x1, x2), x2)}$

$=$ by Axiom 18 RL with $\{x5 \leftarrow mult(x1, x2), x4 \leftarrow mult(x1, x2), x3 \leftarrow x2, x2 \leftarrow mult(x1, x2), x1 \leftarrow x2\}$

$\overbrace{\underbrace{op_r(op_l(x2, mult(x1, x2), x2), mult(x1, x2), mult(x1, x2))}}$

$=$ by Lemma 353 RL with $\{x2 \leftarrow op_l(x2, mult(x1, x2), x2), x1 \leftarrow mult(x1, x2)\}$

$\overbrace{rd(mult(x1, x2), \underbrace{rd(mult(x1, x2), op_l(x2, mult(x1, x2), x2))})}$

$=$ by Lemma 614 LR with $\{x2 \leftarrow x2, x1 \leftarrow mult(x1, x2)\}$

$\underbrace{rd(mult(x1, x2), \overbrace{mult(mult(x1, x2), i(x2))})}$

$=$ by Lemma 63 RL with $\{x3 \leftarrow mult(x1, x2), x2 \leftarrow x2, x1 \leftarrow mult(x1, x2)\}$

$\overbrace{rd(mult(\underbrace{i(mult(x1, x2))}, x2), i(mult(x1, x2)))}$

$=$ by Lemma 3 RL with $\{x1 \leftarrow i(mult(x1, x2))\}$

$rd(\underbrace{mult(\overbrace{i(i(i(mult(x1, x2))))}, x2)}, i(mult(x1, x2)))$

$=$ by Lemma 14 RL with $\{x2 \leftarrow i(i(mult(x1, x2))), x1 \leftarrow x2\}$

$\underbrace{rd(\overbrace{op_t(rd(x2, i(i(mult(x1, x2))))), i(i(mult(x1, x2))))}, i(mult(x1, x2)))}$

$=$ by Lemma 993 LR with $\{x2 \leftarrow i(mult(x1, x2)), x1 \leftarrow rd(x2, i(i(mult(x1, x2))))\}$

$\overbrace{op_r(\underbrace{mult(rd(x2, i(i(mult(x1, x2))))), i(i(mult(x1, x2))))}, i(mult(x1, x2)), rd(x2, i(i(mult(x1, x2)))))}$

$=$ by Axiom 4 RL with $\{x2 \leftarrow i(i(mult(x1, x2))), x1 \leftarrow x2\}$

$\underbrace{op_r(\overbrace{x2}, i(mult(x1, x2)), rd(x2, i(i(mult(x1, x2)))))}$

$=$ by Lemma 820 LR with $\{x3 \leftarrow rd(x2, i(i(mult(x1, x2)))), x2 \leftarrow x1, x1 \leftarrow x2\}$

$\overbrace{op_r(x2, op_t(x1, x2), i(rd(x2, i(i(mult(x1, x2))))))}$

Lemma 995: $op_r(x2, x1, x1) = op_r(x2, op_t(x1, x2), rd(mult(x1, x2), x2))$

$\underbrace{op_r(x2, x1, x1)}$

$=$ by Lemma 994 LR with $\{x1 \leftarrow x1, x2 \leftarrow x2\}$

$\overbrace{op_r(x2, op_t(x1, x2), \underbrace{i(rd(x2, i(i(mult(x1, x2)))))})}$

$=$ by Lemma 10 LR with $\{x2 \leftarrow i(i(mult(x1, x2))), x1 \leftarrow x2\}$

$op_r(x2, op_t(x1, x2), \overbrace{rd(\underbrace{i(i(mult(x1, x2)))}, x2)})$

$=$ by Lemma 3 LR with $\{x1 \leftarrow mult(x1, x2)\}$

$op_r(x2, op_t(x1, x2), rd(\overbrace{mult(x1, x2)}, x2))$

Lemma 996: $op_r(i(x1), mult(x2, x1), x2) = op_r(i(x1), x2, x2)$

$\overbrace{op_r(i(x1), mult(x2, x1), x2)}$

$=$ by Lemma 742 RL with $\{x3 \leftarrow x2, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{\underbrace{i(op_r(x1, op_t(x2, x1), x2))}}$

$=$ by Lemma 815 RL with $\{x3 \leftarrow x2, x2 \leftarrow op_t(x2, x1), x1 \leftarrow x1\}$

$\overbrace{op_r(i(x1), \underbrace{i(op_t(x2, x1))}, i(x2))}$

$=$ by Lemma 15 RL with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$op_r(i(x1), \overbrace{op_t(i(x2), i(x1))}, \underbrace{i(x2)})$

$=$ by Axiom 5 LR with $\{x2 \leftarrow i(x1), x1 \leftarrow i(x2)\}$

$\underbrace{op_r(i(x1), op_t(i(x2), i(x1)), \overbrace{rd(mult(i(x2), i(x1)), i(x1))})}$

$=$ by Lemma 995 RL with $\{x1 \leftarrow i(x2), x2 \leftarrow i(x1)\}$

$\overbrace{\underbrace{op_r(i(x1), i(x2), i(x2))}}$

$=$ by Lemma 815 LR with $\{x3 \leftarrow x2, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{\underbrace{i(op_r(x1, x2, x2))}}$

$=$ by Lemma 356 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_r(i(x1), x2, x2)}$

Lemma 997: $op_r(x1, x2, x2) = op_r(x1, mult(x2, i(x1)), x2)$

$op_r(\underbrace{x1}, x2, x2)$

$=$ by Lemma 3 RL with $\{x1 \leftarrow x1\}$

$\underbrace{op_r(\overbrace{i(i(x1))}, x2, x2)}$

$=$ by Lemma 996 RL with $\{x2 \leftarrow x2, x1 \leftarrow i(x1)\}$

$\overbrace{op_r(\underbrace{i(i(x1))}, mult(x2, i(x1)), x2)}$

$=$ by Lemma 3 LR with $\{x1 \leftarrow x1\}$

$op_r(\overbrace{x1}, mult(x2, i(x1)), x2)$

Lemma 998: $op_l(x1, x2, x1) = rd(rd(x1, x2), op_r(op_l(i(x2), x2, x1), i(x1), i(x1)))$

$\overbrace{op_l(x1, x2, x1)}$

$=$ by Lemma 608 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_l(x1, x1, i(x2))}$

$=$ by Lemma 606 RL with $\{x2 \leftarrow i(x2), x1 \leftarrow x1\}$

$\overbrace{op_l(x1, i(x2), i(x1))}$

$=$ by Lemma 518 RL with $\{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{i(\overbrace{op_l(i(x1), x2, x1)})}$

$=$ by Lemma 519 RL with $\{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow i(x1)\}$

$\overbrace{op_l(\overbrace{i(i(x1))}, i(x2), i(x1))}$

$=$ by Lemma 299 RL with $\{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow i(i(x1))\}$

$\overbrace{op_l(i(i(x1)), \overbrace{rd(x1, x2)}, i(x1))}$

$=$ by Lemma 647 RL with $\{x2 \leftarrow rd(x1, x2), x1 \leftarrow i(x1)\}$

$\overbrace{op_l(i(i(x1)), \overbrace{mult(i(x1), rd(x1, x2))}, i(x1))}$

$=$ by Lemma 630 RL with $\{x2 \leftarrow mult(i(x1), rd(x1, x2)), x1 \leftarrow i(x1)\}$

$\overbrace{op_l(i(i(x1)), \overbrace{i(i(x1))}, \overbrace{mult(i(x1), rd(x1, x2))})$

$=$ by Lemma 722 RL with $\{x2 \leftarrow mult(i(x1), rd(x1, x2)), x1 \leftarrow i(i(x1))\}$

$\overbrace{rd(\underbrace{mult(i(i(x1)), mult(i(x1), rd(x1, x2)))}, \overbrace{op_r(mult(i(x1), rd(x1, x2)), i(i(x1)), i(i(x1)))})$

$=$ by Axiom 2 RL with $\{x2 \leftarrow i(x1), x1 \leftarrow rd(x1, x2)\}$

$rd(\overbrace{rd(x1, x2)}, \underbrace{op_r(mult(i(x1), rd(x1, x2)), i(i(x1)), i(i(x1)))})$

$=$ by Lemma 361 LR with $\{x2 \leftarrow i(x1), x1 \leftarrow mult(i(x1), rd(x1, x2))\}$

$rd(rd(x1, x2), \overbrace{op_r(\underbrace{mult(i(x1), rd(x1, x2))}, i(x1), i(x1)))})$

$=$ by Lemma 310 RL with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$rd(rd(x1, x2), op_r(\overbrace{op_l(i(x2), x2, x1)}, i(x1), i(x1)))$

Lemma 999: $op_r(op_r(x1,x2,x2),rd(x1,x2),x2) = rd(mult(rd(mult(x1,x1),x2),x2),x1)$

$\underbrace{op_r(op_r(x1,x2,x2),rd(x1,x2),x2)}$

$=$ by Lemma 693 RL with $\{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow op_r(x1,x2,x2)\}$

$\overbrace{rd(mult(mult(op_r(x1,x2,x2),\underbrace{rd(x1,x2)}),x2),x1)}$

$=$ by Lemma 10 RL with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$rd(mult(mult(\underbrace{op_r(x1,x2,x2)},\overbrace{i(rd(x2,x1))}),x2),x1)$

$=$ by Lemma 353 RL with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$rd(mult(mult(\overbrace{rd(\underbrace{x2},rd(x2,x1))},i(rd(x2,x1))),x2),x1)$

$=$ by Axiom 4 LR with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$rd(mult(mult(\underbrace{rd(\overbrace{mult(rd(x2,x1),x1)},rd(x2,x1))},i(rd(x2,x1))),x2),x1)$

$=$ by Lemma 453 RL with $\{x2 \leftarrow rd(x2,x1), x1 \leftarrow x1\}$

$rd(mult(mult(\overbrace{\underbrace{op_l(op_r(x1,rd(x2,x1),rd(x2,x1)),x1,rd(x2,x1))}},i(rd(x2,x1))),x2),x1)$

$=$ by Axiom 18 RL with $\{x5 \leftarrow rd(x2,x1), x4 \leftarrow rd(x2,x1), x3 \leftarrow rd(x2,x1), x2 \leftarrow x1, x1 \leftarrow x1\}$

$rd(mult(mult(\overbrace{\underbrace{op_r(op_l(x1,x1,rd(x2,x1)),rd(x2,x1),rd(x2,x1))}},i(rd(x2,x1))),x2),x1)$

$=$ by Lemma 866 RL with $\{x2 \leftarrow x1, x1 \leftarrow rd(x2,x1)\}$

$rd(mult(mult(\overbrace{rd(rd(x2,x1),op_r(\underbrace{i(rd(x1,rd(x2,x1)))},x1,x1))},i(rd(x2,x1))),x2),x1)$

$=$ by Lemma 10 LR with $\{x2 \leftarrow rd(x2,x1), x1 \leftarrow x1\}$

$rd(mult(mult(rd(rd(x2,x1),\underbrace{op_r(\overbrace{rd(rd(x2,x1),x1)},x1,x1)}),i(rd(x2,x1))),x2),x1)$

$=$ by Lemma 839 LR with $\{x3 \leftarrow x1, x2 \leftarrow x1, x1 \leftarrow x2\}$

$rd(mult(\underbrace{mult(rd(rd(x2,x1),\overbrace{rd(x2,mult(x1,x1))}),i(rd(x2,x1)))},x2),x1)$

$=$ by Lemma 22 LR with $\{x2 \leftarrow rd(x2,mult(x1,x1)), x1 \leftarrow rd(x2,x1)\}$

$rd(mult(\underbrace{\overbrace{i(rd(x2,mult(x1,x1)))}},x2),x1)$

$=$ by Lemma 10 LR with $\{x2 \leftarrow mult(x1,x1), x1 \leftarrow x2\}$

$rd(mult(\overbrace{rd(mult(x1,x1),x2)},x2),x1)$

Lemma 1000: $mult(op_r(mult(x1, i(x2)), x1, x1), x2) = op_l(op_t(x1, op_t(mult(i(x1), x2), x2)), x1, x2)$

$mult(\underbrace{op_r(mult(x1, i(x2)), x1, x1)}, x2)$

$=$ by Lemma 3 RL with $\{x1 \leftarrow op_r(mult(x1, i(x2)), x1, x1)\}$

$\underbrace{mult(\overbrace{i(i(op_r(mult(x1, i(x2)), x1, x1)))}, x2)}$

$=$ by Lemma 573 LR with $\{x1 \leftarrow x2, x2 \leftarrow i(op_r(mult(x1, i(x2)), x1, x1))\}$

$\overbrace{op_t(rd(x2, i(op_r(mult(x1, i(x2)), x1, x1))), \underbrace{op_t(i(op_r(mult(x1, i(x2)), x1, x1)), op_l(x2, x2, i(op_r(mult(x1, i(x2)), x1, x1))))}}$

$=$ by Lemma 553 RL with $\{x2 \leftarrow x2, x1 \leftarrow i(op_r(mult(x1, i(x2)), x1, x1))\}$

$op_t(\underbrace{rd(x2, i(op_r(mult(x1, i(x2)), x1, x1)))}, \overbrace{op_t(i(op_r(mult(x1, i(x2)), x1, x1)), x2))}$

$=$ by Lemma 38 LR with $\{x1 \leftarrow op_r(mult(x1, i(x2)), x1, x1), x2 \leftarrow x2\}$

$op_t(\overbrace{rd(op_r(mult(x1, i(x2)), x1, x1), i(x2))}, op_t(i(op_r(mult(x1, i(x2)), x1, x1)), x2))$

$=$ by Lemma 727 LR with $\{x2 \leftarrow i(x2), x1 \leftarrow x1\}$

$\underbrace{op_t(\overbrace{op_l(x1, i(x2), x1)}, op_t(i(op_r(mult(x1, i(x2)), x1, x1)), x2))}$

$=$ by Axiom 15 LR with $\{x4 \leftarrow op_t(i(op_r(mult(x1, i(x2)), x1, x1)), x2), x3 \leftarrow x1, x2 \leftarrow i(x2), x1 \leftarrow x1\}$

$\underbrace{\overbrace{op_l(op_t(x1, op_t(i(op_r(mult(x1, i(x2)), x1, x1)), x2)), i(x2), x1)}}$

$=$ by Lemma 671 LR with $\{x3 \leftarrow x2, x2 \leftarrow op_t(i(op_r(mult(x1, i(x2)), x1, x1)), x2), x1 \leftarrow x1\}$

$\overbrace{op_l(op_t(x1, op_t(\underbrace{i(op_r(mult(x1, i(x2)), x1, x1))}, x2)), x1, x2)}$

$=$ by Lemma 356 LR with $\{x2 \leftarrow x1, x1 \leftarrow mult(x1, i(x2))\}$

$op_l(op_t(x1, \underbrace{op_t(\overbrace{op_r(i(mult(x1, i(x2))), x1, x1)}, x2)}), x1, x2)$

$=$ by Axiom 14 LR with $\{x4 \leftarrow x2, x3 \leftarrow x1, x2 \leftarrow x1, x1 \leftarrow i(mult(x1, i(x2)))\}$

$op_l(\underbrace{op_t(x1, \overbrace{op_r(op_t(i(mult(x1, i(x2))), x2), x1, x1)})}, x1, x2)$

$=$ by Lemma 550 LR with $\{x2 \leftarrow op_t(i(mult(x1, i(x2))), x2), x1 \leftarrow x1\}$

$op_l(\overbrace{op_t(x1, op_t(\underbrace{i(mult(x1, i(x2)))}, x2))}, x1, x2)$

$=$ by Lemma 17 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$op_l(op_t(x1, op_t(\overbrace{mult(i(x1), x2)}, x2)), x1, x2)$

Lemma 1001: $mult(op_r(mult(x1, i(x2)), x1, x1), x2) = op_t(x1, i(x2))$

$\underbrace{mult(op_r(mult(x1, i(x2)), x1, x1), x2)}$

$=$ by Lemma 1000 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_l(\underbrace{op_t(x1, op_t(mult(i(x1), x2), x2))}, x1, x2)}$

$=$ by Lemma 826 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{op_l(\overbrace{op_t(x1, rd(x2, x1))}, x1, x2)}$

$=$ by Lemma 511 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_t(x1, i(x2))}$

Lemma 1002: $i(op_t(x1, x2)) = mult(op_r(i(mult(x1, x2)), x1, x1), i(i(x2)))$

$$
\begin{aligned}
& \overbrace{i(op_t(x1, x2))} \\
= \quad & \text{by Lemma 15 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\overbrace{op_t(i(x1), i(x2))}} \\
= \quad & \text{by Lemma 455 RL with } \{x2 \leftarrow i(x2), x1 \leftarrow i(x1)\} \\
& \overbrace{op_l(\overbrace{op_t(i(x1), rd(i(x2), i(x1)))}, i(x2), i(x1))} \\
= \quad & \text{by Lemma 708 RL with } \{x2 \leftarrow i(x2), x1 \leftarrow i(x1)\} \\
& op_l(\overbrace{\overbrace{op_t(i(x1), i(mult(i(x1), i(x2))))}}, i(x2), i(x1)) \\
= \quad & \text{by Lemma 550 RL with } \{x2 \leftarrow i(mult(i(x1), i(x2))), x1 \leftarrow i(x1)\} \\
& op_l(\overbrace{op_t(i(x1), \overbrace{op_r(i(mult(i(x1), i(x2))), i(x1), i(x1))})}, i(x2), i(x1)) \\
= \quad & \text{by Lemma 356 RL with } \{x2 \leftarrow i(x1), x1 \leftarrow mult(i(x1), i(x2))\} \\
& \underbrace{op_l(op_t(i(x1), \overbrace{i(op_r(mult(i(x1), i(x2)), i(x1), i(x1)))}), i(x2), i(x1))} \\
= \quad & \text{by Axiom 15 RL with } \{x4 \leftarrow i(op_r(mult(i(x1), i(x2)), i(x1), i(x1))), x3 \leftarrow i(x1), x2 \leftarrow i(x2), x1 \leftarrow i(x1)\} \\
& \overbrace{op_t(\underbrace{op_l(i(x1), i(x2), i(x1))}, i(op_r(mult(i(x1), i(x2)), i(x1), i(x1))))} \\
= \quad & \text{by Lemma 727 RL with } \{x2 \leftarrow i(x2), x1 \leftarrow i(x1)\} \\
& \underbrace{op_t(\overbrace{rd(op_r(mult(i(x1), i(x2)), i(x1), i(x1)), i(x2))}, i(op_r(mult(i(x1), i(x2)), i(x1), i(x1))))} \\
= \quad & \text{by Lemma 34 LR with } \{x2 \leftarrow i(x2), x1 \leftarrow op_r(mult(i(x1), i(x2)), i(x1), i(x1))\} \\
& \overbrace{mult(op_r(\underbrace{mult(i(x1), i(x2))}, i(x1), i(x1)), i(i(x2)))} \\
= \quad & \text{by Axiom 11 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& mult(\underbrace{op_r(\overbrace{i(mult(x1, x2))}, i(x1), i(x1))}, i(i(x2))) \\
= \quad & \text{by Lemma 815 LR with } \{x3 \leftarrow x1, x2 \leftarrow x1, x1 \leftarrow mult(x1, x2)\} \\
& mult(\overbrace{i(op_r(mult(x1, x2), x1, x1))}, i(i(x2))) \\
= \quad & \text{by Lemma 356 LR with } \{x2 \leftarrow x1, x1 \leftarrow mult(x1, x2)\} \\
& mult(\overbrace{op_r(i(mult(x1, x2)), x1, x1)}, i(i(x2)))
\end{aligned}
$$

Lemma 1003: $mult(rd(mult(i(x1), x2), x3), mult(x1, i(x2))) = i(x3)$

$$
\begin{aligned}
& mult(rd(\underbrace{mult(i(x1), x2)}, x3), mult(x1, i(x2))) \\
= \quad & \text{by Lemma 17 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{mult(rd(\overbrace{i(mult(x1, i(x2)))}, x3), mult(x1, i(x2)))} \\
= \quad & \text{by Lemma 21 LR with } \{x2 \leftarrow x3, x1 \leftarrow mult(x1, i(x2))\} \\
& \overbrace{i(x3)}
\end{aligned}
$$

Lemma 1004: $op_r(asoc(x1,x2,x2),mult(mult(x2,x1),x3),unit()) = rd(mult(mult(asoc(x1,x2,x2),mult(x2,x1)),x3),mult(m$

$op_r(asoc(x1,x2,x2),mult(mult(x2,x1),x3),\underbrace{unit()})$

$=$ by Axiom 9 RL with $\{x1 \leftarrow mult(op_r(mult(x2,x1),x2,x2),x3)\}$

$op_r(asoc(x1,x2,x2),mult(mult(x2,x1),x3),\overbrace{mult(i(mult(op_r(mult(x2,x1),x2,x2),x3)),mult(\underbrace{op_r(mult(x2,x1),x2,x2)}$

$=$ by Lemma 767 RL with $\{x1 \leftarrow x2, x2 \leftarrow mult(x2,x1)\}$

$op_r(asoc(x1,x2,x2),mult(mult(x2,x1),x3),mult(i(\underbrace{mult(op_r(mult(x2,x1),x2,x2),x3)}),mult(\overbrace{mult(asoc(mult(x2,x1),$

$=$ by Lemma 941 RL with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow mult(x2,x1)\}$

$op_r(asoc(x1,x2,x2),mult(mult(x2,x1),x3),\underbrace{mult(i(\overbrace{mult(asoc(mult(x2,x1),x2,x2),mult(mult(x2,x1),x3))}),mult(mult($

$=$ by Lemma 108 LR with $\{x3 \leftarrow x3, x2 \leftarrow mult(x2,x1), x1 \leftarrow asoc(mult(x2,x1),x2,x2)\}$

$op_r(asoc(x1,x2,x2),mult(mult(x2,x1),x3),\overbrace{asoc(\underbrace{asoc(mult(x2,x1),x2,x2)},mult(x2,x1),x3))}$

$=$ by Lemma 768 LR with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$\underbrace{op_r(asoc(x1,x2,x2),mult(mult(x2,x1),x3),asoc(\overbrace{asoc(x1,x2,x2)},mult(x2,x1),x3))}$

$=$ by Axiom 16 LR with $\{x3 \leftarrow asoc(asoc(x1,x2,x2),mult(x2,x1),x3), x2 \leftarrow mult(mult(x2,x1),x3), x1 \leftarrow asoc(x1,x2,x2)\}$

$\overbrace{rd(\underbrace{mult(mult(asoc(x1,x2,x2),mult(mult(x2,x1),x3)),asoc(asoc(x1,x2,x2),mult(x2,x1),x3))},mult(mult(mult(x2,x1),$

$=$ by Axiom 8 RL with $\{x3 \leftarrow x3, x2 \leftarrow mult(x2,x1), x1 \leftarrow asoc(x1,x2,x2)\}$

$rd(\overbrace{mult(mult(asoc(x1,x2,x2),mult(x2,x1)),x3)},mult(mult(mult(x2,x1),x3),asoc(\underbrace{asoc(x1,x2,x2)},mult(x2,x1),x3)))$

$=$ by Lemma 768 RL with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$rd(mult(mult(asoc(x1,x2,x2),mult(x2,x1)),x3),mult(mult(mult(x2,x1),x3),\underbrace{asoc(\overbrace{asoc(mult(x2,x1),x2,x2)},mult(x2,$

$=$ by Lemma 108 RL with $\{x3 \leftarrow x3, x2 \leftarrow mult(x2,x1), x1 \leftarrow asoc(mult(x2,x1),x2,x2)\}$

$rd(mult(mult(asoc(x1,x2,x2),mult(x2,x1)),x3),mult(mult(mult(x2,x1),x3),\overbrace{mult(i(\underbrace{mult(asoc(mult(x2,x1),x2,x2),m}$

$=$ by Lemma 941 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow mult(x2,x1)\}$

$rd(mult(mult(asoc(x1,x2,x2),mult(x2,x1)),x3),mult(mult(mult(x2,x1),x3),mult(i(\overbrace{mult(op_r(mult(x2,x1),x2,x2),x3$

$=$ by Lemma 767 LR with $\{x1 \leftarrow x2, x2 \leftarrow mult(x2,x1)\}$

$rd(mult(mult(asoc(x1,x2,x2),mult(x2,x1)),x3),mult(mult(mult(x2,x1),x3),\underbrace{mult(i(mult(op_r(mult(x2,x1),x2,x2),x3}$

$=$ by Axiom 9 LR with $\{x1 \leftarrow mult(op_r(mult(x2,x1),x2,x2),x3)\}$

$rd(mult(mult(asoc(x1,x2,x2),mult(x2,x1)),x3),mult(mult(mult(x2,x1),x3),\overbrace{unit()}))$

Lemma 1005: $rd(x1, op_t(x1, x2)) = op_r(rd(x1, op_t(x1, x2)), x1, x3)$

$\overbrace{rd(x1, op_t(x1, x2))}$

$=$ by Lemma 860 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{asoc(x1, mult(x1, x2), mult(x1, x2))}$

$=$ by Lemma 768 RL with $\{x2 \leftarrow x1, x1 \leftarrow mult(x1, x2)\}$

$\overbrace{asoc(mult(mult(x1, x2), x1), mult(x1, x2), mult(x1, x2))}$

$=$ by Lemma 772 RL with $\{x2 \leftarrow mult(x1, x2), x1 \leftarrow mult(mult(x1, x2), x1)\}$

$\overbrace{asoc(mult(mult(x1, x2), x1), i(mult(x1, x2)), i(mult(x1, x2)))}$

$=$ by Lemma 6 RL with $\{x2 \leftarrow mult(mult(i(mult(x1, x2)), mult(mult(x1, x2), x1)), x3), x1 \leftarrow asoc(mult(mult(x1, x2), x1), i(m$

$\overbrace{op_r(asoc(mult(mult(x1, x2), x1), i(mult(x1, x2)), i(mult(x1, x2))), mult(mult(i(mult(x1, x2)), mult(mult(x1, x2), x1)), x3}$

$=$ by Lemma 1004 LR with $\{x3 \leftarrow x3, x2 \leftarrow i(mult(x1, x2)), x1 \leftarrow mult(mult(x1, x2), x1)\}$

$\overbrace{rd(mult(mult(asoc(mult(mult(x1, x2), x1), i(mult(x1, x2)), i(mult(x1, x2))), mult(i(mult(x1, x2)), mult(mult(x1, x2), x1)}$

$=$ by Axiom 1 RL with $\{x1 \leftarrow mult(mult(i(mult(x1, x2)), mult(mult(x1, x2), x1)), x3)\}$

$\overbrace{rd(mult(mult(asoc(mult(mult(x1, x2), x1), i(mult(x1, x2)), i(mult(x1, x2))), mult(i(mult(x1, x2)), mult(mult(x1, x2), x1)}$

$=$ by Axiom 16 RL with $\{x3 \leftarrow x3, x2 \leftarrow mult(i(mult(x1, x2)), mult(mult(x1, x2), x1)), x1 \leftarrow asoc(mult(mult(x1, x2), x1), i(m$

$\overbrace{op_r(asoc(mult(mult(x1, x2), x1), i(mult(x1, x2)), i(mult(x1, x2))), \underbrace{mult(i(mult(x1, x2)), mult(mult(x1, x2), x1))}, x3)}$

$=$ by Axiom 2 RL with $\{x2 \leftarrow mult(x1, x2), x1 \leftarrow x1\}$

$op_r(\underbrace{asoc(mult(mult(x1, x2), x1), i(mult(x1, x2)), i(mult(x1, x2)))}, \overbrace{x1}, x3)$

$=$ by Lemma 772 LR with $\{x2 \leftarrow mult(x1, x2), x1 \leftarrow mult(mult(x1, x2), x1)\}$

$op_r(\overbrace{asoc(mult(mult(x1, x2), x1), mult(x1, x2), mult(x1, x2))}, x1, x3)$

$=$ by Lemma 768 LR with $\{x2 \leftarrow x1, x1 \leftarrow mult(x1, x2)\}$

$op_r(\overbrace{asoc(x1, mult(x1, x2), mult(x1, x2))}, x1, x3)$

$=$ by Lemma 860 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$op_r(\overbrace{rd(x1, op_t(x1, x2))}, x1, x3)$

Lemma 1006: $op_t(mult(x1, rd(x2, x1)), rd(x1, x2)) = op_t(x2, i(x1))$

$\overbrace{op_t(mult(x1, rd(x2, x1)), rd(x1, x2))}$

$=$ by Axiom 10 RL with $\{x2 \leftarrow mult(x1, rd(x2, x1)), x1 \leftarrow rd(x1, x2)\}$

$\overbrace{mult(i(rd(x1, x2)), \underbrace{mult(mult(x1, rd(x2, x1)), rd(x1, x2))})}$

$=$ by Lemma 91 RL with $\{x3 \leftarrow rd(x1, x2), x2 \leftarrow rd(x2, x1), x1 \leftarrow x1\}$

$mult(i(rd(x1, x2)), \overbrace{mult(\underbrace{op_r(x1, rd(x2, x1), rd(x1, x2))}, mult(rd(x2, x1), rd(x1, x2)))})$

$=$ by Lemma 586 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$mult(i(rd(x1, x2)), mult(\overbrace{op_t(x1, i(x2))}, mult(rd(x2, x1), \underbrace{rd(x1, x2)})))$

$=$ by Lemma 10 RL with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$mult(i(rd(x1, x2)), mult(op_t(x1, i(x2)), \underbrace{mult(rd(x2, x1), \overbrace{i(rd(x2, x1))})}))$

$=$ by Axiom 7 LR with $\{x1 \leftarrow rd(x2, x1)\}$

$mult(i(rd(x1, x2)), \underbrace{mult(op_t(x1, i(x2)), \overbrace{unit()})})$

$=$ by Axiom 1 RL with $\{x1 \leftarrow op_t(x1, i(x2))\}$

$mult(\underbrace{i(rd(x1, x2))}, \overbrace{op_t(x1, i(x2))})$

$=$ by Lemma 10 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{mult(\overbrace{rd(x2, x1)}, op_t(x1, i(x2)))}$

$=$ by Lemma 680 RL with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$\overbrace{op_t(x2, i(x1))}$

Lemma 1007: $rd(x1, rd(x2, op_t(x2, rd(x1, x2)))) = rd(x1, rd(op_l(x1, x2, x1), x1))$

$\underbrace{rd(x1, rd(x2, op_t(x2, rd(x1, x2))))}$

$=$ by Lemma 885 LR with $\{x1 \leftarrow rd(x2, op_t(x2, rd(x1, x2))), x2 \leftarrow x1\}$

$\overbrace{rd(op_t(x1, rd(x2, op_t(x2, rd(x1, x2)))), rd(mult(x1, rd(x2, op_t(x2, \underbrace{rd(x1, x2)}))), x1))}$

$=$ by Lemma 10 RL with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$rd(op_t(x1, rd(x2, op_t(x2, rd(x1, x2)))), rd(mult(x1, \underbrace{rd(x2, op_t(x2, \overbrace{i(rd(x2, x1))}))}), x1))$

$=$ by Lemma 102 LR with $\{x2 \leftarrow rd(x2, x1), x1 \leftarrow x2\}$

$rd(op_t(x1, rd(x2, op_t(x2, rd(x1, x2)))), rd(mult(x1, \overbrace{mult(x2, \underbrace{op_t(i(x2), rd(x2, x1))})}), x1))$

$=$ by Lemma 714 RL with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$rd(op_t(x1, rd(x2, op_t(x2, rd(x1, x2)))), rd(mult(x1, mult(x2, \overbrace{op_t(i(x2), mult(x2, x1))})), x1))$

$=$ by Lemma 254 RL with $\{x2 \leftarrow mult(x2, x1), x1 \leftarrow i(x2)\}$

$rd(op_t(x1, rd(x2, op_t(x2, rd(x1, x2)))), rd(mult(x1, mult(x2, \overbrace{op_l(op_t(i(x2), \underbrace{mult(i(x2), mult(x2, x1))}), i(x2), mult(x2, x1))}))$

$=$ by Axiom 2 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$rd(op_t(x1, rd(x2, op_t(x2, rd(x1, x2)))), rd(mult(x1, mult(x2, \underbrace{op_l(op_t(i(x2), \overbrace{x1}), i(x2), mult(x2, x1))})), x1))$

$=$ by Lemma 241 LR with $\{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow op_t(i(x2), x1)\}$

$rd(op_t(x1, rd(x2, op_t(x2, rd(x1, x2)))), rd(\underbrace{mult(x1, mult(x2, \overbrace{op_l(op_t(i(x2), x1), x1, x2)}))}, x1))$

$=$ by Lemma 137 LR with $\{x3 \leftarrow op_t(i(x2), x1), x2 \leftarrow x2, x1 \leftarrow x1\}$

$rd(op_t(x1, rd(x2, op_t(x2, rd(x1, x2)))), rd(\overbrace{\underbrace{mult(mult(x1, x2), op_t(i(x2), x1))}}, x1))$

$=$ by Lemma 466 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$rd(op_t(x1, rd(x2, \underbrace{op_t(x2, rd(x1, x2))})), rd(\overbrace{op_l(x1, x2, x1)}, x1))$

$=$ by Lemma 375 RL with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$rd(op_t(\underbrace{x1}, rd(x2, \overbrace{op_t(x2, mult(i(x2), x1))})), rd(op_l(x1, x2, x1), x1))$

$=$ by Lemma 12 RL with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$rd(op_t(\overbrace{mult(x2, mult(i(x2), x1))}), \underbrace{rd(x2, op_t(x2, mult(i(x2), x1)))}), rd(op_l(x1, x2, x1), x1))$

$=$ by Lemma 98 LR with $\{x2 \leftarrow mult(i(x2), x1), x1 \leftarrow x2\}$

$rd(\underbrace{op_t(mult(x2, mult(i(x2), x1))}, \overbrace{mult(x2, i(op_t(x2, mult(i(x2), x1))))}), rd(op_l(x1, x2, x1), x1))$

$=$ by Lemma 880 RL with $\{x2 \leftarrow mult(i(x2), x1), x1 \leftarrow x2\}$

$rd(\overbrace{\underbrace{mult(x2, mult(i(x2), x1))}}, rd(op_l(x1, x2, x1), x1))$

$=$ by Lemma 12 LR with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$rd(\overbrace{x1}, rd(op_l(x1, x2, x1), x1))$

Lemma 1008: $asoc(x1, x2, x1) = rd(op_r(x1, rd(mult(x1, x2), x1), rd(x1, mult(x1, x2))), op_t(x1, x2))$

$\underbrace{asoc(x1, x2, x1)}$

$=$ by Lemma 618 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{\underbrace{asoc(x1, mult(x1, x2), x1)}}$

$=$ by Lemma 263 RL with $\{x2 \leftarrow mult(x1, x2), x1 \leftarrow x1\}$

$\overbrace{\underbrace{asoc(x1, rd(mult(x1, x2), x1), x1)}}$

$=$ by Lemma 986 LR with $\{x2 \leftarrow rd(mult(x1, x2), x1), x1 \leftarrow x1\}$

$\overbrace{rd(\underbrace{mult(rd(mult(x1, x2), x1), rd(x1, op_t(rd(mult(x1, x2), x1), x1)))}, op_t(x1, rd(mult(x1, x2), x1)))}$

$=$ by Lemma 604 LR with $\{x2 \leftarrow x1, x1 \leftarrow rd(mult(x1, x2), x1)\}$

$rd(\overbrace{op_r(x1, rd(mult(x1, x2), x1), i(rd(mult(x1, x2), x1)))}, \underbrace{op_t(x1, rd(mult(x1, x2), x1))})$

$=$ by Lemma 378 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$rd(op_r(x1, rd(mult(x1, x2), x1), \underbrace{i(rd(mult(x1, x2), x1))}, \overbrace{op_t(x1, x2)})$

$=$ by Lemma 10 LR with $\{x2 \leftarrow x1, x1 \leftarrow mult(x1, x2)\}$

$rd(op_r(x1, rd(mult(x1, x2), x1), \overbrace{rd(x1, mult(x1, x2))}), op_t(x1, x2))$

Lemma 1009: $asoc(x1, x2, x1) = rd(op_t(x1, rd(x2, x1)), op_t(x1, x2))$

$\underbrace{asoc(x1, x2, x1)}$

$=$ by Lemma 1008 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{rd(\underbrace{op_r(x1, rd(mult(x1, x2), x1), rd(x1, mult(x1, x2)))}, op_t(x1, x2))}$

$=$ by Lemma 586 LR with $\{x2 \leftarrow mult(x1, x2), x1 \leftarrow x1\}$

$rd(\overbrace{\underbrace{op_t(x1, i(mult(x1, x2)))}}, op_t(x1, x2))$

$=$ by Lemma 708 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$rd(\overbrace{op_t(x1, rd(x2, x1))}, op_t(x1, x2))$

Lemma 1010: $i(x1) = op_t(rd(rd(i(x1), i(op_t(x1, x2))), x1), i(x2))$

$\underbrace{i(x1)}$

$=$ by Lemma 401 RL with $\{x2 \leftarrow i(x2), x1 \leftarrow i(x1)\}$

$\overbrace{op_t(rd(i(x1), rd(\underbrace{op_t(i(x1), i(x2))}, i(x1))), i(x2))}$

$=$ by Lemma 15 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$op_t(\underbrace{rd(i(x1), rd(\overbrace{i(op_t(x1, x2))}, i(x1)))}, i(x2))$

$=$ by Lemma 40 LR with $\{x3 \leftarrow i(x1), x2 \leftarrow i(op_t(x1, x2)), x1 \leftarrow x1\}$

$op_t(\overbrace{rd(rd(i(x1), i(op_t(x1, x2))), x1)}, i(x2))$

Lemma 1011: $i(x1) = op_t(rd(op_t(x1,x2), mult(x1,x1)), i(x2))$

$$
\begin{aligned}
& \underbrace{i(x1)} \\
= \quad & \text{by Lemma 1010 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_t(rd(\underbrace{rd(i(x1), i(op_t(x1,x2)))}, x1), i(x2))} \\
= \quad & \text{by Lemma 23 LR with } \{x2 \leftarrow op_t(x1,x2), x1 \leftarrow x1\} \\
& op_t(\underbrace{rd(\overbrace{rd(op_t(x1,x2), x1)}, x1)}, i(x2)) \\
= \quad & \text{by Lemma 206 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_t(\overbrace{rd(op_t(x1,x2), mult(x1,x1))}, i(x2))
\end{aligned}
$$

Lemma 1012: $mult(x1, i(mult(x1,x2))) = op_t(rd(x1, mult(x1,x2)), i(op_t(x1, i(x2))))$

$$
\begin{aligned}
& \underbrace{mult(x1, i(mult(x1,x2)))} \\
= \quad & \text{by Lemma 34 RL with } \{x2 \leftarrow mult(x1,x2), x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_t(rd(x1, mult(x1,x2)), i(x1))}} \\
= \quad & \text{by Lemma 562 RL with } \{x2 \leftarrow x1, x1 \leftarrow rd(x1, mult(x1,x2))\} \\
& \overbrace{op_t(rd(x1, mult(x1,x2)), \underbrace{i(op_t(x1, rd(x1, mult(x1,x2))))})} \\
= \quad & \text{by Lemma 571 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_t(rd(x1, mult(x1,x2)), \overbrace{i(op_t(x1, i(x2)))})
\end{aligned}
$$

Lemma 1013: $i(x2) = op_t(rd(x1, mult(x1,x2)), op_t(i(x1), x2))$

$$
\begin{aligned}
& \underbrace{i(x2)} \\
= \quad & \text{by Lemma 20 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{mult(x1, i(mult(x1,x2)))}} \\
= \quad & \text{by Lemma 1012 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_t(rd(x1, mult(x1,x2)), \underbrace{i(op_t(x1, i(x2)))})} \\
= \quad & \text{by Lemma 19 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_t(rd(x1, mult(x1,x2)), \overbrace{op_t(i(x1), x2)})
\end{aligned}
$$

Lemma 1014: $mult(x1, op_t(mult(i(x1), x2), x3)) = mult(op_t(rd(x2,x1), x3), x1)$

$$
\begin{aligned}
& mult(x1, op_t(\underbrace{mult(i(x1), x2)}, x3)) \\
= \quad & \text{by Lemma 14 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \underbrace{mult(x1, op_t(\overbrace{op_t(rd(x2,x1), x1)}, x3))} \\
= \quad & \text{by Lemma 50 LR with } \{x3 \leftarrow x3, x2 \leftarrow rd(x2,x1), x1 \leftarrow x1\} \\
& \overbrace{mult(op_t(rd(x2,x1), x3), x1)}
\end{aligned}
$$

Lemma 1015: $mult(op_t(op_r(x1, x2, x2), x3), x2) = mult(x2, op_t(op_l(x1, x1, i(x2)), x3))$

$$
\begin{aligned}
& mult(op_t(\underbrace{op_r(x1, x2, x2)}, x3), x2) \\
= & \quad \text{by Lemma 358 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{mult(op_t(\overbrace{rd(rd(x1, i(x2)), x2)}, x3), x2)} \\
= & \quad \text{by Lemma 1014 RL with } \{x3 \leftarrow x3, x2 \leftarrow rd(x1, i(x2)), x1 \leftarrow x2\} \\
& \overbrace{mult(x2, op_t(\underbrace{mult(i(x2), rd(x1, i(x2)))}, x3))} \\
= & \quad \text{by Lemma 427 RL with } \{x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& mult(x2, op_t(\overbrace{op_l(x1, x1, i(x2))}, x3))
\end{aligned}
$$

Lemma 1016: $mult(x1, op_l(op_t(x2, x3), x1, x2)) = rd(x1, i(op_t(x2, x3)))$

$$
\begin{aligned}
& mult(x1, \underbrace{op_l(op_t(x2, x3), x1, x2)}) \\
= & \quad \text{by Lemma 672 RL with } \{x3 \leftarrow x1, x2 \leftarrow x3, x1 \leftarrow x2\} \\
& mult(x1, \overbrace{op_l(op_t(x2, x3), x2, i(x1))}) \\
= & \quad \text{by Axiom 15 RL with } \{x4 \leftarrow x3, x3 \leftarrow i(x1), x2 \leftarrow x2, x1 \leftarrow x2\} \\
& \underbrace{mult(x1, \overbrace{op_t(op_l(x2, x2, i(x1)), x3)})} \\
= & \quad \text{by Lemma 1015 RL with } \{x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \overbrace{mult(\underbrace{op_t(op_r(x2, x1, x1), x3)}, x1)} \\
= & \quad \text{by Axiom 14 LR with } \{x4 \leftarrow x3, x3 \leftarrow x1, x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \underbrace{mult(\overbrace{op_r(op_t(x2, x3), x1, x1)}, x1)} \\
= & \quad \text{by Lemma 357 LR with } \{x2 \leftarrow x1, x1 \leftarrow op_t(x2, x3)\} \\
& \overbrace{rd(x1, i(op_t(x2, x3)))}
\end{aligned}
$$

Lemma 1017: $op_t(x1, op_l(op_t(x2, x3), x1, x2)) = op_t(x1, op_r(i(i(op_t(x2, x3))), x1, x1))$

$$
\begin{aligned}
& \underbrace{op_t(x1, op_l(op_t(x2, x3), x1, x2))} \\
= & \quad \text{by Lemma 378 RL with } \{x2 \leftarrow op_l(op_t(x2, x3), x1, x2), x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, rd(\underbrace{mult(x1, op_l(op_t(x2, x3), x1, x2))}, x1))} \\
= & \quad \text{by Lemma 1016 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_t(x1, \underbrace{rd(\overbrace{rd(x1, i(op_t(x2, x3)))}, x1)}) \\
= & \quad \text{by Lemma 354 LR with } \{x2 \leftarrow i(op_t(x2, x3)), x1 \leftarrow x1\} \\
& op_t(x1, \overbrace{op_r(i(i(op_t(x2, x3))), x1, x1)})
\end{aligned}
$$

Lemma 1018: $op_t(x1, op_l(op_t(x2, x3), x1, x2)) = op_t(x1, op_t(x2, x3))$

$$
\begin{aligned}
& \underbrace{op_t(x1, op_l(op_t(x2, x3), x1, x2))} \\
= \quad & \text{by Lemma 1017 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_t(x1, op_r(i(i(op_t(x2, x3))), x1, x1))}} \\
= \quad & \text{by Lemma 550 LR with } \{x2 \leftarrow i(i(op_t(x2, x3))), x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, \underbrace{i(i(op_t(x2, x3)))})} \\
= \quad & \text{by Lemma 3 LR with } \{x1 \leftarrow op_t(x2, x3)\} \\
& op_t(x1, \overbrace{op_t(x2, x3)})
\end{aligned}
$$

Lemma 1019: $op_l(op_t(x1, x2), x3, x1) = rd(rd(x3, i(op_t(x1, x2))), op_t(x3, op_t(x1, x2)))$

$$
\begin{aligned}
& \underbrace{op_l(op_t(x1, x2), x3, x1)} \\
= \quad & \text{by Lemma 29 RL with } \{x2 \leftarrow op_l(op_t(x1, x2), x3, x1), x1 \leftarrow x3\} \\
& \overbrace{rd(mult(x3, op_l(op_t(x1, x2), x3, x1)), \underbrace{op_t(x3, op_l(op_t(x1, x2), x3, x1))})} \\
= \quad & \text{by Lemma 1018 LR with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x3\} \\
& rd(\underbrace{mult(x3, op_l(op_t(x1, x2), x3, x1))}, \overbrace{op_t(x3, op_t(x1, x2))}) \\
= \quad & \text{by Lemma 1016 LR with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x3\} \\
& rd(\overbrace{rd(x3, i(op_t(x1, x2)))}, op_t(x3, op_t(x1, x2)))
\end{aligned}
$$

Lemma 1020: $op_l(op_t(x1, x2), x3, op_t(x1, x2)) = op_l(op_t(x1, x2), x3, x1)$

$$
\begin{aligned}
& \underbrace{op_l(op_t(x1, x2), x3, op_t(x1, x2))} \\
= \quad & \text{by Lemma 696 LR with } \{x2 \leftarrow x3, x1 \leftarrow op_t(x1, x2)\} \\
& \overbrace{\underbrace{rd(rd(x3, i(op_t(x1, x2))), op_t(x3, op_t(x1, x2)))}} \\
= \quad & \text{by Lemma 1019 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(op_t(x1, x2), x3, x1)}
\end{aligned}
$$

Lemma 1021: $op_l(x2, x3, rd(mult(x1, x2), x1)) = op_l(x2, x3, op_t(rd(mult(x1, x2), x1), op_t(x1, x2)))$

$$
\begin{aligned}
& op_l(\underbrace{x2}, x3, rd(mult(x1, x2), x1)) \\
= \quad & \text{by Lemma 400 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{op_l(\overbrace{op_t(rd(mult(x1, x2), x1), op_t(x1, x2))}, x3, rd(mult(x1, x2), x1))} \\
= \quad & \text{by Lemma 1020 RL with } \{x3 \leftarrow x3, x2 \leftarrow op_t(x1, x2), x1 \leftarrow rd(mult(x1, x2), x1)\} \\
& \overbrace{op_l(\underbrace{op_t(rd(mult(x1, x2), x1), op_t(x1, x2))}, x3, op_t(rd(mult(x1, x2), x1), op_t(x1, x2)))} \\
= \quad & \text{by Lemma 400 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_l(\overbrace{x2}, x3, op_t(rd(mult(x1, x2), x1), op_t(x1, x2)))
\end{aligned}
$$

Lemma 1022: $op_l(x1, x2, rd(mult(x3, x1), x3)) = op_l(x1, x2, x1)$

$$
\begin{aligned}
& \overbrace{op_l(x1, x2, rd(mult(x3, x1), x3))} \\
= \quad & \text{by Lemma 1021 LR with } \{x1 \leftarrow x3, x3 \leftarrow x2, x2 \leftarrow x1\} \\
& \overbrace{op_l(x1, x2, \overbrace{op_t(rd(mult(x3, x1), x3), op_t(x3, x1))})} \\
= \quad & \text{by Lemma 400 LR with } \{x2 \leftarrow x1, x1 \leftarrow x3\} \\
& op_l(x1, x2, \overbrace{x1})
\end{aligned}
$$

Lemma 1023: $mult(op_l(x1, x2, op_t(x1, x3)), x3) = mult(op_l(x1, x2, x1), x3)$

$$
\begin{aligned}
& \overbrace{mult(op_l(x1, x2, op_t(x1, x3)), x3)} \\
= \quad & \text{by Lemma 167 RL with } \{x4 \leftarrow op_t(x1, x3), x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x3\} \\
& \overbrace{mult(x3, \overbrace{op_l(op_t(x1, x3), x2, op_t(x1, x3))})} \\
= \quad & \text{by Lemma 1020 LR with } \{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{mult(x3, \overbrace{op_l(op_t(x1, x3), x2, x1)})} \\
= \quad & \text{by Lemma 167 LR with } \{x4 \leftarrow x1, x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x3\} \\
& \overbrace{mult(op_l(x1, x2, x1), x3)}
\end{aligned}
$$

Lemma 1024: $op_l(x1, x2, op_t(x1, x3)) = op_l(x1, x2, x1)$

$$
\begin{aligned}
& \overbrace{op_l(x1, x2, op_t(x1, x3))} \\
= \quad & \text{by Axiom 5 LR with } \{x2 \leftarrow x3, x1 \leftarrow op_l(x1, x2, op_t(x1, x3))\} \\
& \overbrace{rd(\overbrace{mult(op_l(x1, x2, op_t(x1, x3)), x3)}, x3)} \\
= \quad & \text{by Lemma 1023 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{rd(\overbrace{mult(op_l(x1, x2, x1), x3)}, x3)} \\
= \quad & \text{by Axiom 5 RL with } \{x2 \leftarrow x3, x1 \leftarrow op_l(x1, x2, x1)\} \\
& \overbrace{op_l(x1, x2, x1)}
\end{aligned}
$$

Lemma 1025: $op_l(x1, mult(x2, op_t(x1, x3)), x1) = op_l(x1, x2, x1)$

$$
\begin{aligned}
& \overbrace{op_l(x1, mult(x2, op_t(x1, x3)), x1)} \\
= \quad & \text{by Lemma 1024 RL with } \{x3 \leftarrow x3, x2 \leftarrow mult(x2, op_t(x1, x3)), x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, mult(x2, op_t(x1, x3)), op_t(x1, x3))} \\
= \quad & \text{by Lemma 285 LR with } \{x3 \leftarrow op_t(x1, x3), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x2, op_t(x1, x3))} \\
= \quad & \text{by Lemma 1024 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x2, x1)}
\end{aligned}
$$

Lemma 1026: $op_l(x1, rd(x2, op_t(x1, x3)), x1) = op_l(x1, x2, x1)$

$$
\begin{aligned}
& \overbrace{op_l(x1, rd(x2, op_t(x1, x3)), x1)} \\
= \quad & \text{by Lemma 1024 RL with } \{x3 \leftarrow x3, x2 \leftarrow rd(x2, op_t(x1, x3)), x1 \leftarrow x1\} \\
& \overbrace{\overbrace{op_l(x1, rd(x2, op_t(x1, x3)), op_t(x1, x3))}} \\
= \quad & \text{by Lemma 292 LR with } \{x3 \leftarrow op_t(x1, x3), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\overbrace{op_l(x1, x2, op_t(x1, x3))}} \\
= \quad & \text{by Lemma 1024 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x2, x1)}
\end{aligned}
$$

Lemma 1027: $asoc(mult(x2, op_t(x1, x3)), x1, x1) = rd(op_l(x1, x2, x1), x1)$

$$
\begin{aligned}
& \overbrace{asoc(mult(x2, op_t(x1, x3)), x1, x1)} \\
= \quad & \text{by Lemma 765 RL with } \{x2 \leftarrow mult(x2, op_t(x1, x3)), x1 \leftarrow x1\} \\
& \overbrace{\overbrace{rd(op_l(x1, mult(x2, op_t(x1, x3)), x1), x1)}} \\
= \quad & \text{by Lemma 171 RL with } \{x3 \leftarrow x1, x2 \leftarrow mult(x2, op_t(x1, x3)), x1 \leftarrow x1\} \\
& \overbrace{mult(i(x1), \underbrace{op_l(x1, mult(x2, op_t(x1, x3)), x1)})} \\
= \quad & \text{by Lemma 1025 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{mult(i(x1), \overbrace{op_l(x1, x2, x1)})} \\
= \quad & \text{by Lemma 171 LR with } \{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{rd(op_l(x1, x2, x1), x1)}
\end{aligned}
$$

Lemma 1028: $asoc(mult(x1, op_t(x2, x3)), x2, x2) = asoc(x1, x2, x2)$

$$
\begin{aligned}
& \overbrace{asoc(mult(x1, op_t(x2, x3)), x2, x2)} \\
= \quad & \text{by Lemma 1027 LR with } \{x3 \leftarrow x3, x1 \leftarrow x2, x2 \leftarrow x1\} \\
& \overbrace{\underbrace{rd(op_l(x2, x1, x2), x2)}} \\
= \quad & \text{by Lemma 765 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \overbrace{asoc(x1, x2, x2)}
\end{aligned}
$$

Lemma 1029: $asoc(rd(x2, op_t(x1, x3)), x1, x1) = mult(i(x1), op_l(x1, x2, x1))$

$$
\begin{aligned}
& \overbrace{asoc(rd(x2, op_t(x1, x3)), x1, x1)} \\
= \quad & \text{by Lemma 765 RL with } \{x2 \leftarrow rd(x2, op_t(x1, x3)), x1 \leftarrow x1\} \\
& \overbrace{\underbrace{rd(op_l(x1, rd(x2, op_t(x1, x3)), x1), x1)}} \\
= \quad & \text{by Lemma 171 RL with } \{x3 \leftarrow x1, x2 \leftarrow rd(x2, op_t(x1, x3)), x1 \leftarrow x1\} \\
& \overbrace{mult(i(x1), \underbrace{op_l(x1, rd(x2, op_t(x1, x3)), x1)})} \\
= \quad & \text{by Lemma 1026 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& mult(i(x1), \overbrace{op_l(x1, x2, x1)})
\end{aligned}
$$

Lemma 1030: $asoc(rd(x2, op_t(x1, x3)), x1, x1) = asoc(x2, x1, x1)$

$$
\begin{aligned}
& \underbrace{asoc(rd(x2, op_t(x1, x3)), x1, x1)} \\
= \quad & \text{by Lemma 1029 LR with } \{x3 \leftarrow x3, x1 \leftarrow x1, x2 \leftarrow x2\} \\
& \overbrace{\underbrace{mult(i(x1), op_l(x1, x2, x1))}} \\
= \quad & \text{by Lemma 171 LR with } \{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{rd(op_l(x1, x2, x1), x1)}} \\
= \quad & \text{by Lemma 765 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{asoc(x2, x1, x1)}
\end{aligned}
$$

Lemma 1031: $op_l(x1, rd(op_t(x1, x2), x3), x1) = rd(x1, asoc(x3, x1, x1))$

$$
\begin{aligned}
& \underbrace{op_l(x1, rd(op_t(x1, x2), x3), x1)} \\
= \quad & \text{by Lemma 620 RL with } \{x2 \leftarrow rd(op_t(x1, x2), x3), x1 \leftarrow x1\} \\
& \overbrace{rd(x1, \underbrace{asoc(x1, rd(op_t(x1, x2), x3), x1)})} \\
= \quad & \text{by Lemma 938 RL with } \{x3 \leftarrow x1, x2 \leftarrow op_t(x1, x2), x1 \leftarrow x3\} \\
& rd(x1, \overbrace{\underbrace{asoc(rd(x3, op_t(x1, x2)), x1, x1)}}) \\
= \quad & \text{by Lemma 1030 LR with } \{x3 \leftarrow x2, x1 \leftarrow x1, x2 \leftarrow x3\} \\
& rd(x1, \overbrace{asoc(x3, x1, x1)})
\end{aligned}
$$

Lemma 1032: $op_l(i(x1), x1, x2) = op_l(i(x1), x2, i(op_t(x1, x3)))$

$$
\begin{aligned}
& \underbrace{op_l(i(x1), x1, x2)} \\
= \quad & \text{by Lemma 633 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(i(x1), x2, i(x1))}} \\
= \quad & \text{by Lemma 1024 RL with } \{x3 \leftarrow i(x3), x2 \leftarrow x2, x1 \leftarrow i(x1)\} \\
& \overbrace{op_l(i(x1), x2, \underbrace{op_t(i(x1), i(x3))})} \\
= \quad & \text{by Lemma 15 LR with } \{x2 \leftarrow x3, x1 \leftarrow x1\} \\
& op_l(i(x1), x2, \overbrace{i(op_t(x1, x3))})
\end{aligned}
$$

Lemma 1033: $op_l(op_t(x1, x2), x1, x3) = op_l(op_t(x1, x2), op_t(x1, x2), x3)$

$$
\begin{aligned}
& \underbrace{op_l(op_t(x1, x2), x1, x3)} \\
= \quad & \text{by Lemma 671 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(op_t(x1, x2), i(x3), x1)}} \\
= \quad & \text{by Lemma 1020 RL with } \{x3 \leftarrow i(x3), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(op_t(x1, x2), i(x3), op_t(x1, x2))}} \\
= \quad & \text{by Lemma 607 LR with } \{x2 \leftarrow x3, x1 \leftarrow op_t(x1, x2)\} \\
& \overbrace{op_l(op_t(x1, x2), op_t(x1, x2), x3)}
\end{aligned}
$$

Lemma 1034: $op_l(x2, rd(mult(x1, x2), x1), x3) = op_l(x2, op_t(rd(mult(x1, x2), x1), op_t(x1, x2)), x3)$

$$
\begin{aligned}
& op_l(\underbrace{x2}, rd(mult(x1, x2), x1), x3) \\
= \quad & \text{by Lemma 400 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{op_l(\overbrace{op_t(rd(mult(x1, x2), x1), op_t(x1, x2))}, rd(mult(x1, x2), x1), x3)} \\
= \quad & \text{by Lemma 1033 LR with } \{x3 \leftarrow x3, x2 \leftarrow op_t(x1, x2), x1 \leftarrow rd(mult(x1, x2), x1)\} \\
& \overbrace{op_l(\underbrace{op_t(rd(mult(x1, x2), x1), op_t(x1, x2))}, op_t(rd(mult(x1, x2), x1), op_t(x1, x2)), x3)} \\
= \quad & \text{by Lemma 400 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_l(\overbrace{x2}, op_t(rd(mult(x1, x2), x1), op_t(x1, x2)), x3)
\end{aligned}
$$

Lemma 1035: $mult(op_l(x1, op_t(x1, x2), x3), x2) = mult(op_l(x1, x1, x3), x2)$

$$
\begin{aligned}
& \underbrace{mult(op_l(x1, op_t(x1, x2), x3), x2)} \\
= \quad & \text{by Lemma 167 RL with } \{x4 \leftarrow x3, x3 \leftarrow op_t(x1, x2), x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \overbrace{mult(x2, \underbrace{op_l(op_t(x1, x2), op_t(x1, x2), x3)})} \\
= \quad & \text{by Lemma 1033 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{mult(x2, \overbrace{op_l(op_t(x1, x2), x1, x3)})} \\
= \quad & \text{by Lemma 167 LR with } \{x4 \leftarrow x3, x3 \leftarrow x1, x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \overbrace{mult(op_l(x1, x1, x3), x2)}
\end{aligned}
$$

Lemma 1036: $op_l(x1, op_t(x1, x2), x3) = op_l(x1, x1, x3)$

$$
\begin{aligned}
& \underbrace{op_l(x1, op_t(x1, x2), x3)} \\
= \quad & \text{by Axiom 5 LR with } \{x2 \leftarrow x2, x1 \leftarrow op_l(x1, op_t(x1, x2), x3)\} \\
& \overbrace{rd(\underbrace{mult(op_l(x1, op_t(x1, x2), x3), x2)}, x2)} \\
= \quad & \text{by Lemma 1035 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{rd(\overbrace{mult(op_l(x1, x1, x3), x2)}, x2)} \\
= \quad & \text{by Axiom 5 RL with } \{x2 \leftarrow x2, x1 \leftarrow op_l(x1, x1, x3)\} \\
& \overbrace{op_l(x1, x1, x3)}
\end{aligned}
$$

Lemma 1037: $op_l(x1, x1, i(mult(x1, x2))) = op_r(x1, x2, i(op_t(x1, x3)))$

$$
\begin{aligned}
& \underbrace{op_l(x1, x1, i(mult(x1, x2)))} \\
= \quad & \text{by Lemma 1036 RL with } \{x3 \leftarrow i(mult(x1, x2)), x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, op_t(x1, x3), i(mult(x1, x2)))}} \\
= \quad & \text{by Lemma 890 LR with } \{x3 \leftarrow i(mult(x1, x2)), x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \underbrace{\overbrace{op_r(x1, i(mult(x1, x2)), op_t(x1, x3))}} \\
= \quad & \text{by Lemma 818 LR with } \{x3 \leftarrow op_t(x1, x3), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, x2, i(op_t(x1, x3)))}
\end{aligned}
$$

Lemma 1038: $op_l(x1, x1, i(x2)) = op_r(x1, x2, i(op_t(x1, x3)))$

$$
\begin{aligned}
& \overbrace{op_l(x1, x1, i(x2))} \\
= \quad & \text{by Lemma 606 RL with } \{x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, \overbrace{i(x2), i(x1)})} \\
= \quad & \text{by Lemma 245 RL with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x1, \overbrace{i(mult(x1, x2))})} \\
= \quad & \text{by Lemma 1037 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, x2, \overbrace{i(op_t(x1, x3))})}
\end{aligned}
$$

Lemma 1039: $op_l(x1, rd(x2, op_r(x1, x3, x3)), x1) = op_l(x1, x2, x1)$

$$
\begin{aligned}
& \overbrace{op_l(x1, rd(x2, op_r(x1, x3, x3)), x1)} \\
= \quad & \text{by Lemma 1022 RL with } \{x3 \leftarrow rd(x3, x1), x2 \leftarrow rd(x2, op_r(x1, x3, x3)), x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, rd(x2, op_r(x1, x3, x3)), \underbrace{rd(\underbrace{mult(rd(x3, x1), x1)}, rd(x3, x1)))}} \\
= \quad & \text{by Axiom 4 RL with } \{x2 \leftarrow x1, x1 \leftarrow x3\} \\
& op_l(x1, rd(x2, op_r(x1, x3, x3)), \underbrace{rd(\overbrace{x3}, rd(x3, x1)))} \\
= \quad & \text{by Lemma 353 LR with } \{x2 \leftarrow x1, x1 \leftarrow x3\} \\
& \underbrace{op_l(x1, rd(x2, op_r(x1, x3, x3)), \overbrace{op_r(x1, x3, x3)})} \\
= \quad & \text{by Lemma 292 LR with } \{x3 \leftarrow op_r(x1, x3, x3), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x2, \underbrace{op_r(x1, x3, x3)})} \\
= \quad & \text{by Lemma 353 RL with } \{x2 \leftarrow x1, x1 \leftarrow x3\} \\
& op_l(x1, x2, \overbrace{rd(\underbrace{x3}, rd(x3, x1))}) \\
= \quad & \text{by Axiom 4 LR with } \{x2 \leftarrow x1, x1 \leftarrow x3\} \\
& \underbrace{op_l(x1, x2, rd(\overbrace{mult(rd(x3, x1), x1)}, rd(x3, x1)))} \\
= \quad & \text{by Lemma 1022 LR with } \{x3 \leftarrow rd(x3, x1), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x2, x1)}
\end{aligned}
$$

Lemma 1040: $asoc(mult(x2, op_t(i(x1), x3)), x1, x1) = mult(op_l(i(x1), x2, i(x1)), x1)$

$$
\begin{aligned}
& \underbrace{asoc(mult(x2, op_t(i(x1), x3)), x1, x1)} \\
= \quad & \text{by Lemma 778 RL with } \{x2 \leftarrow mult(x2, op_t(i(x1), x3)), x1 \leftarrow x1\} \\
& \overbrace{mult(x1, \underbrace{op_l(i(x1), x1, mult(x2, op_t(i(x1), x3)))})} \\
= \quad & \text{by Lemma 633 RL with } \{x2 \leftarrow mult(x2, op_t(i(x1), x3)), x1 \leftarrow x1\} \\
& \underbrace{mult(x1, \overbrace{op_l(i(x1), mult(x2, op_t(i(x1), x3)), i(x1))})} \\
= \quad & \text{by Lemma 188 RL with } \{x3 \leftarrow i(x1), x2 \leftarrow mult(x2, op_t(i(x1), x3)), x1 \leftarrow x1\} \\
& \overbrace{mult(\underbrace{op_l(i(x1), mult(x2, op_t(i(x1), x3)), i(x1))}, x1)} \\
= \quad & \text{by Lemma 1025 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow i(x1)\} \\
& mult(\overbrace{op_l(i(x1), x2, i(x1))}, x1)
\end{aligned}
$$

Lemma 1041: $asoc(mult(x2, op_t(i(x1), x3)), x1, x1) = mult(x1, op_l(i(x1), x1, x2))$

$$
\begin{aligned}
& \underbrace{asoc(mult(x2, op_t(i(x1), x3)), x1, x1)} \\
= \quad & \text{by Lemma 1040 LR with } \{x3 \leftarrow x3, x1 \leftarrow x1, x2 \leftarrow x2\} \\
& \overbrace{\underbrace{mult(op_l(i(x1), x2, i(x1)), x1)}} \\
= \quad & \text{by Lemma 188 LR with } \{x3 \leftarrow i(x1), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{mult(x1, \underbrace{op_l(i(x1), x2, i(x1))})} \\
= \quad & \text{by Lemma 633 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& mult(x1, \overbrace{op_l(i(x1), x1, x2)})
\end{aligned}
$$

Lemma 1042: $asoc(rd(x2, op_r(x1, x3, x3)), x1, x1) = mult(i(x1), op_l(x1, x2, x1))$

$$
\begin{aligned}
& \underbrace{asoc(rd(x2, op_r(x1, x3, x3)), x1, x1)} \\
= \quad & \text{by Lemma 765 RL with } \{x2 \leftarrow rd(x2, op_r(x1, x3, x3)), x1 \leftarrow x1\} \\
& \overbrace{\underbrace{rd(op_l(x1, rd(x2, op_r(x1, x3, x3)), x1), x1)}} \\
= \quad & \text{by Lemma 171 RL with } \{x3 \leftarrow x1, x2 \leftarrow rd(x2, op_r(x1, x3, x3)), x1 \leftarrow x1\} \\
& \overbrace{mult(i(x1), \underbrace{op_l(x1, rd(x2, op_r(x1, x3, x3)), x1)})} \\
= \quad & \text{by Lemma 1039 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& mult(i(x1), \overbrace{op_l(x1, x2, x1)})
\end{aligned}
$$

Lemma 1043: $asoc(rd(x2, op_r(x1, x3, x3)), x1, x1) = asoc(x2, x1, x1)$

$$
\begin{aligned}
& \underbrace{asoc(rd(x2, op_r(x1, x3, x3)), x1, x1)} \\
= \quad & \text{by Lemma 1042 LR with } \{x3 \leftarrow x3, x1 \leftarrow x1, x2 \leftarrow x2\} \\
& \overbrace{\underbrace{mult(i(x1), op_l(x1, x2, x1))}} \\
= \quad & \text{by Lemma 171 LR with } \{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{rd(op_l(x1, x2, x1), x1)}} \\
= \quad & \text{by Lemma 765 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{asoc(x2, x1, x1)}
\end{aligned}
$$

Lemma 1044: $op_l(x1, x1, rd(op_r(x1, x2, x2), x3)) = mult(x1, asoc(x3, x1, x1))$

$$
\begin{aligned}
& \underbrace{op_l(x1, x1, rd(op_r(x1, x2, x2), x3))} \\
= \quad & \text{by Lemma 158 RL with } \{x2 \leftarrow rd(op_r(x1, x2, x2), x3), x1 \leftarrow x1\} \\
& \overbrace{mult(x1, \underbrace{asoc(x1, rd(op_r(x1, x2, x2), x3), x1)})} \\
= \quad & \text{by Lemma 938 RL with } \{x3 \leftarrow x1, x2 \leftarrow op_r(x1, x2, x2), x1 \leftarrow x3\} \\
& mult(x1, \overbrace{\underbrace{asoc(rd(x3, op_r(x1, x2, x2)), x1, x1)}}) \\
= \quad & \text{by Lemma 1043 LR with } \{x3 \leftarrow x2, x1 \leftarrow x1, x2 \leftarrow x3\} \\
& mult(x1, \overbrace{asoc(x3, x1, x1)})
\end{aligned}
$$

Lemma 1045: $mult(i(x1), mult(mult(x1, x2), x3)) = mult(rd(x2, x1), mult(x1, x3))$

$$
\begin{aligned}
& mult(i(x1), \underbrace{mult(mult(x1, x2), x3)}) \\
= & \quad \text{by Lemma 54 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{mult(i(x1), \overbrace{mult(x1, mult(rd(x2, x1), mult(x1, x3)))})} \\
= & \quad \text{by Axiom 2 RL with } \{x2 \leftarrow x1, x1 \leftarrow mult(rd(x2, x1), mult(x1, x3))\} \\
& \overbrace{mult(rd(x2, x1), mult(x1, x3))}
\end{aligned}
$$

Lemma 1046: $mult(unit(), mult(mult(x1, x2), x3)) = mult(i(mult(x1, x2)), mult(mult(x1, mult(x2, mult(x2, x1))), x3))$

$$
\begin{aligned}
& mult(\underbrace{unit()}, mult(mult(x1, x2), x3)) \\
= & \quad \text{by Lemma 2 RL with } \{x1 \leftarrow mult(x1, x2)\} \\
& \underbrace{mult(\overbrace{rd(mult(x1, x2), mult(x1, x2))}, mult(mult(x1, x2), x3))} \\
= & \quad \text{by Lemma 1045 RL with } \{x3 \leftarrow x3, x2 \leftarrow mult(x1, x2), x1 \leftarrow mult(x1, x2)\} \\
& \overbrace{mult(i(mult(x1, x2)), mult(\underbrace{mult(mult(x1, x2), mult(x1, x2))}, x3))} \\
= & \quad \text{by Lemma 530 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& mult(i(mult(x1, x2)), mult(\overbrace{mult(x1, mult(x2, mult(x2, x1)))}, x3))
\end{aligned}
$$

Lemma 1047: $op_l(mult(x1, mult(x2, x3)), x1, x2) = mult(mult(x2, x1), x3)$

$$
\begin{aligned}
& \underbrace{op_l(mult(x1, mult(x2, x3)), x1, x2)} \\
= & \quad \text{by Axiom 12 RL with } \{x3 \leftarrow mult(x1, mult(x2, x3)), x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \overbrace{mult(i(mult(x2, x1)), mult(x2, \underbrace{mult(x1, mult(x1, mult(x2, x3)))}))} \\
= & \quad \text{by Lemma 24 RL with } \{x2 \leftarrow mult(x2, x3), x1 \leftarrow x1\} \\
& mult(i(mult(x2, x1)), \underbrace{mult(x2, \overbrace{mult(mult(x1, x1), mult(x2, x3))})}) \\
= & \quad \text{by Axiom 6 LR with } \{x3 \leftarrow x3, x2 \leftarrow mult(x1, x1), x1 \leftarrow x2\} \\
& mult(i(mult(x2, x1)), \overbrace{mult(mult(x2, \underbrace{mult(mult(x1, x1), x2)}), x3)}) \\
= & \quad \text{by Lemma 24 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{mult(i(mult(x2, x1)), mult(mult(x2, \overbrace{mult(x1, mult(x1, x2))}), x3))} \\
= & \quad \text{by Lemma 1046 RL with } \{x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \overbrace{\underbrace{mult(unit(), mult(mult(x2, x1), x3))}} \\
= & \quad \text{by Axiom 3 RL with } \{x1 \leftarrow mult(mult(x2, x1), x3)\} \\
& \overbrace{mult(mult(x2, x1), x3)}
\end{aligned}
$$

Lemma 1048: $op_l(x1, i(x2), i(x3)) = op_l(x1, x2, x3)$

$op_l(\underbrace{x1}, i(x2), i(x3))$

$=$ by Axiom 2 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$op_l(\overbrace{mult(\underbrace{i(x2)}, mult(x2, x1))}, i(x2), i(x3))$

$=$ by Lemma 20 RL with $\{x2 \leftarrow x2, x1 \leftarrow x3\}$

$op_l(\underbrace{mult(\overbrace{mult(x3, i(mult(x3, x2)))}, mult(x2, x1))}, i(x2), i(x3))$

$=$ by Lemma 1047 RL with $\{x3 \leftarrow mult(x2, x1), x2 \leftarrow x3, x1 \leftarrow i(mult(x3, x2))\}$

$op_l(\overbrace{op_l(\underbrace{mult(i(mult(x3, x2)), mult(x3, mult(x2, x1)))}, i(mult(x3, x2)), x3)}, i(x2), i(x3))$

$=$ by Axiom 12 LR with $\{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x3\}$

$op_l(\underbrace{op_l(\overbrace{op_l(x1, x2, x3)}, i(mult(x3, x2)), x3)}, i(x2), i(x3))$

$=$ by Lemma 247 LR with $\{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow op_l(x1, x2, x3)\}$

$\underbrace{op_l(\overbrace{op_l(op_l(x1, x2, x3), i(x3), i(x2))}, i(x2), i(x3))}$

$=$ by Lemma 138 LR with $\{x3 \leftarrow i(x2), x2 \leftarrow i(x3), x1 \leftarrow op_l(x1, x2, x3)\}$

$\overbrace{op_l(x1, x2, x3)}$

Lemma 1049: $i(op_l(x1, x2, x3)) = op_l(i(x1), x2, x3)$

$\underbrace{i(op_l(x1, x2, x3))}$

$=$ by Lemma 519 RL with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{\overbrace{op_l(i(x1), i(x2), i(x3))}}$

$=$ by Lemma 1048 RL with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow i(x1)\}$

$\overbrace{op_l(i(x1), x2, x3)}$

Lemma 1050: $op_l(x1, i(x2), x3) = op_l(x1, x2, i(x3))$

$\underbrace{op_l(x1, i(x2), x3)}$

$=$ by Lemma 1048 RL with $\{x3 \leftarrow x3, x2 \leftarrow i(x2), x1 \leftarrow x1\}$

$\overbrace{op_l(x1, \underbrace{i(i(x2))}, i(x3))}$

$=$ by Lemma 3 LR with $\{x1 \leftarrow x2\}$

$op_l(x1, \overbrace{x2}, i(x3))$

Lemma 1051: $op_l(x1, x2, i(mult(x2, x3))) = op_l(x1, x3, x2)$

$\underbrace{op_l(x1, x2, i(mult(x2, x3)))}$

$=$ by Lemma 1050 RL with $\{x3 \leftarrow mult(x2, x3), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{\overbrace{op_l(x1, i(x2), mult(x2, x3))}}$

$=$ by Lemma 241 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_l(x1, x3, x2)}$

Lemma 1052: $op_l(x1, x2, i(op_t(x1, x3))) = i(op_l(i(x1), x1, x2))$

$$
\begin{aligned}
& op_l(\underbrace{x1}, x2, i(op_t(x1, x3))) \\
= & \quad \text{by Lemma 3 RL with } \{x1 \leftarrow x1\} \\
& \underbrace{op_l(\overbrace{i(i(x1))}, x2, i(op_t(x1, x3)))} \\
= & \quad \text{by Lemma 1049 RL with } \{x3 \leftarrow i(op_t(x1, x3)), x2 \leftarrow x2, x1 \leftarrow i(x1)\} \\
& \overbrace{i(\underbrace{op_l(i(x1), x2, i(op_t(x1, x3)))})} \\
= & \quad \text{by Lemma 1032 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& i(\overbrace{op_l(i(x1), x1, x2)})
\end{aligned}
$$

Lemma 1053: $op_l(x1, x2, i(op_t(x1, x3))) = op_l(x1, x1, x2)$

$$
\begin{aligned}
& \underbrace{op_l(x1, x2, i(op_t(x1, x3)))} \\
= & \quad \text{by Lemma 1052 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{i(op_l(i(x1), x1, x2))}} \\
= & \quad \text{by Lemma 1049 LR with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow i(x1)\} \\
& \overbrace{op_l(\underbrace{i(i(x1))}, x1, x2)} \\
= & \quad \text{by Lemma 3 LR with } \{x1 \leftarrow x1\} \\
& op_l(\overbrace{x1}, x1, x2)
\end{aligned}
$$

Lemma 1054: $op_l(x1, mult(i(x2), x3), x2) = op_l(x1, x2, i(x3))$

$$
\begin{aligned}
& \underbrace{op_l(x1, mult(i(x2), x3), x2)} \\
= & \quad \text{by Lemma 244 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, i(x2), x3)}} \\
= & \quad \text{by Lemma 1050 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x2, i(x3))}
\end{aligned}
$$

Lemma 1055: $op_l(x1, rd(i(x2), x3), x2) = op_l(x1, x3, i(x2))$

$$
\begin{aligned}
& \underbrace{op_l(x1, rd(i(x2), x3), x2)} \\
= & \quad \text{by Lemma 295 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, i(x3), x2)}} \\
= & \quad \text{by Lemma 1050 LR with } \{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x3, i(x2))}
\end{aligned}
$$

Lemma 1056: $mult(x1, op_l(i(x1), x2, x3)) = rd(x1, op_l(x1, x2, x3))$

$$
\begin{aligned}
& mult(x1, \underbrace{op_l(i(x1), x2, x3)}) \\
= & \quad \text{by Lemma 1049 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{mult(x1, \overbrace{i(op_l(x1, x2, x3))})} \\
= & \quad \text{by Lemma 176 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{rd(x1, op_l(x1, x2, x3))}
\end{aligned}
$$

Lemma 1057: $rd(x1, op_l(i(x1), x2, x3)) = mult(x1, op_l(x1, x2, x3))$

$$
\begin{aligned}
& rd(x1, \underbrace{op_l(i(x1), x2, x3)}) \\
= & \quad \text{by Lemma 1049 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& rd(x1, \overbrace{i(\underbrace{op_l(x1, x2, x3)})}) \\
= & \quad \text{by Lemma 170 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{rd(x1, i(\overbrace{mult(x1, rd(op_l(x1, x2, x3), x1))}))} \\
= & \quad \text{by Lemma 58 LR with } \{x2 \leftarrow rd(op_l(x1, x2, x3), x1), x1 \leftarrow x1\} \\
& \overbrace{mult(x1, \underbrace{mult(rd(op_l(x1, x2, x3), x1), x1)})} \\
= & \quad \text{by Axiom 4 RL with } \{x2 \leftarrow x1, x1 \leftarrow op_l(x1, x2, x3)\} \\
& mult(x1, \overbrace{op_l(x1, x2, x3)})
\end{aligned}
$$

Lemma 1058: $mult(op_l(i(x1), x2, x3), x1) = rd(x1, op_l(x1, x2, x3))$

$$
\begin{aligned}
& \underbrace{mult(op_l(i(x1), x2, x3), x1)} \\
= & \quad \text{by Lemma 188 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{mult(x1, op_l(i(x1), x2, x3))}} \\
= & \quad \text{by Lemma 1056 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{rd(x1, op_l(x1, x2, x3))}
\end{aligned}
$$

Lemma 1059: $op_l(x1, x2, i(mult(x3, x2))) = op_l(x1, op_t(x3, x2), x2)$

$$
\begin{aligned}
& \underbrace{op_l(x1, x2, i(mult(x3, x2)))} \\
= & \quad \text{by Lemma 1050 RL with } \{x3 \leftarrow mult(x3, x2), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, i(x2), mult(x3, x2))}} \\
= & \quad \text{by Lemma 246 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, op_t(x3, x2), x2)}
\end{aligned}
$$

Lemma 1060: $op_l(x1, rd(x2, x3), i(x4)) = op_l(x1, rd(x3, x2), x4)$

$$
\begin{aligned}
& \underbrace{op_l(x1, rd(x2, x3), i(x4))} \\
= & \quad \text{by Lemma 1050 RL with } \{x3 \leftarrow x4, x2 \leftarrow rd(x2, x3), x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, \underbrace{i(rd(x2, x3))}, x4)} \\
= & \quad \text{by Lemma 10 LR with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& op_l(x1, \overbrace{rd(x3, x2)}, x4)
\end{aligned}
$$

Lemma 1061: $op_l(x1, x2, op_l(i(x3), x3, x2)) = op_l(x1, x3, x2)$

$\underbrace{op_l(x1, x2, op_l(i(x3), x3, x2))}$

$=$ by Lemma 286 RL with $\{x3 \leftarrow op_l(i(x3), x3, x2), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_l(x1, x2, \underbrace{rd(op_l(i(x3), x3, x2), x2)})}$

$=$ by Lemma 10 RL with $\{x2 \leftarrow op_l(i(x3), x3, x2), x1 \leftarrow x2\}$

$op_l(x1, x2, \overbrace{i(\underbrace{rd(x2, op_l(i(x3), x3, x2))})})$

$=$ by Lemma 231 RL with $\{x2 \leftarrow x3, x1 \leftarrow x2\}$

$\underbrace{op_l(x1, x2, \overbrace{i(mult(x2, x3))})}$

$=$ by Lemma 245 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_l(x1, i(x3), i(x2))}}$

$=$ by Lemma 1048 LR with $\{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x1\}$

$\overbrace{op_l(x1, x3, x2)}$

Lemma 1062: $op_l(x1, x2, x1) = op_l(x1, x1, i(mult(op_t(x1, x3), x2)))$

$\underbrace{op_l(x1, x2, x1)}$

$=$ by Lemma 1024 RL with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_l(x1, x2, op_t(x1, x3))}}$

$=$ by Lemma 1051 RL with $\{x3 \leftarrow x2, x2 \leftarrow op_t(x1, x3), x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_l(x1, op_t(x1, x3), i(mult(op_t(x1, x3), x2)))}}$

$=$ by Lemma 1036 LR with $\{x3 \leftarrow i(mult(op_t(x1, x3), x2)), x2 \leftarrow x3, x1 \leftarrow x1\}$

$\overbrace{op_l(x1, x1, i(mult(op_t(x1, x3), x2)))}$

Lemma 1063: $op_l(x1, x1, mult(op_t(x1, x2), x3)) = op_l(x1, x1, x3)$

$\underbrace{op_l(x1, x1, mult(op_t(x1, x2), x3))}$

$=$ by Lemma 1053 RL with $\{x3 \leftarrow x2, x2 \leftarrow mult(op_t(x1, x2), x3), x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_l(x1, mult(op_t(x1, x2), x3), i(op_t(x1, x2)))}}$

$=$ by Lemma 242 LR with $\{x3 \leftarrow x3, x2 \leftarrow op_t(x1, x2), x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_l(x1, op_t(x1, x2), x3)}}$

$=$ by Lemma 1036 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_l(x1, x1, x3)}$

Lemma 1064: $rd(x1, op_l(x1, mult(op_t(x1, x2), x3), x1)) = op_t(rd(x1, op_l(x1, x3, x1)), x1)$

$$
\begin{aligned}
& \underbrace{rd(x1, op_l(x1, mult(op_t(x1, x2), x3), x1))} \\
= & \quad \text{by Lemma 311 RL with } \{x3 \leftarrow x1, x2 \leftarrow mult(op_t(x1, x2), x3), x1 \leftarrow x1\} \\
& \overbrace{op_t(rd(x1, \underbrace{op_l(x1, mult(op_t(x1, x2), x3), x1)}), x1)} \\
= & \quad \text{by Lemma 608 RL with } \{x2 \leftarrow mult(op_t(x1, x2), x3), x1 \leftarrow x1\} \\
& op_t(rd(x1, \overbrace{op_l(x1, x1, i(mult(op_t(x1, x2), x3)))}), x1) \\
= & \quad \text{by Lemma 1062 RL with } \{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x1\} \\
& op_t(rd(x1, \overbrace{op_l(x1, x3, x1)}), x1)
\end{aligned}
$$

Lemma 1065: $asoc(x1, mult(op_t(x1, x2), x3), x1) = rd(x1, op_l(x1, x3, x1))$

$$
\begin{aligned}
& \underbrace{asoc(x1, mult(op_t(x1, x2), x3), x1)} \\
= & \quad \text{by Lemma 617 RL with } \{x2 \leftarrow mult(op_t(x1, x2), x3), x1 \leftarrow x1\} \\
& \overbrace{\underbrace{rd(x1, op_l(x1, mult(op_t(x1, x2), x3), x1))}} \\
= & \quad \text{by Lemma 1064 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_t(rd(x1, op_l(x1, x3, x1)), x1)}} \\
= & \quad \text{by Lemma 311 LR with } \{x3 \leftarrow x1, x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{rd(x1, op_l(x1, x3, x1))}
\end{aligned}
$$

Lemma 1066: $op_l(x1, rd(i(x2), x2), x3) = op_l(x1, mult(x2, x2), i(x3))$

$$
\begin{aligned}
& op_l(x1, \underbrace{rd(i(x2), x2)}, x3) \\
= & \quad \text{by Lemma 43 RL with } \{x1 \leftarrow x2\} \\
& \underbrace{op_l(x1, \overbrace{i(mult(x2, x2))}, x3)} \\
= & \quad \text{by Lemma 1050 LR with } \{x3 \leftarrow x3, x2 \leftarrow mult(x2, x2), x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, mult(x2, x2), i(x3))}
\end{aligned}
$$

Lemma 1067: $op_l(x1, mult(x2, i(x3)), x3) = op_l(x1, x3, op_t(i(x2), x3))$

$$
\begin{aligned}
& \underbrace{op_l(x1, mult(x2, i(x3)), x3)} \\
= & \quad \text{by Lemma 1051 RL with } \{x3 \leftarrow mult(x2, i(x3)), x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x3, \underbrace{i(mult(x3, mult(x2, i(x3))))})} \\
= & \quad \text{by Lemma 30 LR with } \{x2 \leftarrow x2, x1 \leftarrow x3\} \\
& op_l(x1, x3, \underbrace{i(\overbrace{op_t(x2, i(x3))})}) \\
= & \quad \text{by Lemma 19 LR with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& op_l(x1, x3, \overbrace{op_t(i(x2), x3)})
\end{aligned}
$$

Lemma 1068: $op_l(mult(mult(x1,x2),x3),x1,x2) = mult(x2,mult(x1,x3))$

$\underbrace{op_l(mult(mult(x1,x2),x3),x1,x2)}$

$=$ by Lemma 286 RL with $\{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow mult(mult(x1,x2),x3)\}$

$\overbrace{op_l(\underbrace{mult(mult(x1,x2),x3)},x1,rd(x2,x1))}$

$=$ by Lemma 54 RL with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{op_l(\overbrace{mult(x1,mult(rd(x2,x1),mult(x1,x3)))},x1,rd(x2,x1))}$

$=$ by Lemma 1047 LR with $\{x3 \leftarrow mult(x1,x3), x2 \leftarrow rd(x2,x1), x1 \leftarrow x1\}$

$\overbrace{mult(\underbrace{mult(rd(x2,x1),x1)},mult(x1,x3))}$

$=$ by Axiom 4 RL with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$mult(\overbrace{x2},mult(x1,x3))$

Lemma 1069: $x1 = op_l(x1, op_l(x2, x2, x3), i(mult(x2, x2)))$

$\overbrace{x1}$

$=$ by Lemma 950 LR with $\{x3 \leftarrow rd(x3, x2), x2 \leftarrow mult(rd(x2, x3), x2), x1 \leftarrow x1\}$

$\overbrace{op_l(x1, mult(mult(rd(x2, x3), x2), \underbrace{mult(mult(rd(x2, x3), x2), mult(i(\overbrace{mult(rd(x2, x3), x2)}), rd(x3, x2)))}), mult(op_t(rd(x3, $

$=$ by Lemma 12 LR with $\{x2 \leftarrow rd(x3, x2), x1 \leftarrow mult(rd(x2, x3), x2)\}$

$op_l(x1, mult(mult(rd(x2, x3), x2), \overbrace{rd(x3, x2)}), mult(op_t(rd(x3, x2), mult(rd(x2, x3), x2)), mult(rd(x2, x3), x2)))$

$=$ by Lemma 364 RL with $\{x2 \leftarrow x2, x1 \leftarrow x3\}$

$op_l(x1, mult(\underbrace{mult(rd(x2, x3), x2)}, \overbrace{op_r(mult(x3, \overbrace{i(x2)}), x2, x2)}), mult(op_t(rd(x3, x2), mult(rd(x2, x3), x2)), mult(rd(x2, x3$

$=$ by Lemma 334 RL with $\{x2 \leftarrow x3, x1 \leftarrow x2\}$

$op_l(x1, \underbrace{mult(\overbrace{rd(x2, mult(x3, \overbrace{i(x2)}))}, op_r(mult(x3, \overbrace{i(x2)}), x2, x2))}, mult(op_t(rd(x3, x2), mult(rd(x2, x3), x2)), mult(rd(x2, $

$=$ by Lemma 720 LR with $\{x2 \leftarrow mult(x3, i(x2)), x1 \leftarrow x2\}$

$op_l(x1, \overbrace{op_l(x2, mult(x3, \overbrace{i(x2)}), x2)}, mult(op_t(rd(x3, x2), mult(rd(x2, x3), x2)), mult(rd(x2, x3), x2)))$

$=$ by Lemma 499 RL with $\{x2 \leftarrow mult(x3, i(x2)), x1 \leftarrow x2\}$

$op_l(x1, \overbrace{op_l(x2, \underbrace{rd(mult(x3, \overbrace{i(x2)}), i(x2))}, x2)}, mult(op_t(rd(x3, x2), mult(rd(x2, x3), x2)), mult(rd(x2, x3), x2)))$

$=$ by Axiom 5 RL with $\{x2 \leftarrow i(x2), x1 \leftarrow x3\}$

$op_l(x1, op_l(x2, \overbrace{x3}, x2), \underbrace{mult(op_t(rd(x3, x2), mult(rd(x2, x3), x2)), mult(rd(x2, x3), x2))})$

$=$ by Lemma 526 LR with $\{x3 \leftarrow mult(rd(x2, x3), x2), x2 \leftarrow x2, x1 \leftarrow x3\}$

$op_l(x1, op_l(x2, x3, x2), \overbrace{\underbrace{op_t(rd(mult(rd(x2, x3), x2), rd(x2, x3)), mult(rd(x2, x3), x2))}})$

$=$ by Lemma 972 LR with $\{x2 \leftarrow x2, x1 \leftarrow rd(x2, x3)\}$

$op_l(x1, op_l(x2, x3, x2), \overbrace{\underbrace{op_l(x2, rd(x2, x3), x2)}})$

$=$ by Lemma 622 LR with $\{x2 \leftarrow x3, x1 \leftarrow x2\}$

$\underbrace{op_l(x1, op_l(x2, x3, x2), \overbrace{op_l(x2, x2, x3)})}$

$=$ by Lemma 1051 RL with $\{x3 \leftarrow op_l(x2, x3, x2), x2 \leftarrow op_l(x2, x2, x3), x1 \leftarrow x1\}$

$\overbrace{op_l(x1, op_l(x2, x2, x3), i(mult(op_l(x2, x2, x3), op_l(\underbrace{x2}, x3, x2))))}$

$=$ by Lemma 4 RL with $\{x1 \leftarrow x2\}$

$op_l(x1, op_l(x2, x2, x3), i(mult(op_l(x2, x2, x3), \underbrace{op_l(\overbrace{op_t(x2, x2)}, x3, x2)})))$

$=$ by Axiom 15 RL with $\{x4 \leftarrow x2, x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x2\}$

$op_l(x1, op_l(x2, x2, x3), i(\underbrace{mult(op_l(x2, x2, x3), \overbrace{op_t(op_l(x2, x3, x2), x2)})}))$

$=$ by Lemma 884 RL with $\{x2 \leftarrow x3, x1 \leftarrow x2\}$

$op_l(x1, op_l(x2, x2, x3), i(\overbrace{mult(x2, x2)}))$

Lemma 1070: $op_l(x1, op_l(x2, x2, x3), rd(i(x2), x2)) = x1$

$$
\begin{aligned}
& op_l(x1, op_l(x2, x2, x3), \underbrace{rd(i(x2), x2)}) \\
= & \quad \text{by Lemma 43 RL with } \{x1 \leftarrow x2\} \\
& \underbrace{op_l(x1, op_l(x2, x2, x3), \overbrace{i(mult(x2, x2))})} \\
= & \quad \text{by Lemma 1069 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{x1}
\end{aligned}
$$

Lemma 1071: $op_l(x1, mult(x2, x2), op_l(x2, x2, x3)) = op_l(x1, op_l(x2, x2, x3), rd(i(x2), x2))$

$$
\begin{aligned}
& op_l(x1, mult(x2, x2), \underbrace{op_l(x2, x2, x3)}) \\
= & \quad \text{by Lemma 3 RL with } \{x1 \leftarrow op_l(x2, x2, x3)\} \\
& \underbrace{op_l(x1, mult(x2, x2), \overbrace{i(i(op_l(x2, x2, x3)))})} \\
= & \quad \text{by Lemma 1066 RL with } \{x3 \leftarrow i(op_l(x2, x2, x3)), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, rd(i(x2), x2), i(op_l(x2, x2, x3)))}} \\
= & \quad \text{by Lemma 651 LR with } \{x3 \leftarrow op_l(x2, x2, x3), x2 \leftarrow rd(i(x2), x2), x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, op_l(x2, x2, x3), \underbrace{op_l(rd(i(x2), x2), op_l(x2, x2, x3), rd(i(x2), x2))})} \\
= & \quad \text{by Lemma 1070 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow rd(i(x2), x2)\} \\
& op_l(x1, op_l(x2, x2, x3), \overbrace{rd(i(x2), x2)})
\end{aligned}
$$

Lemma 1072: $x1 = op_l(x1, mult(x2, x2), mult(x3, rd(x2, x3)))$

$$
\begin{aligned}
& \underbrace{x1} \\
= & \quad \text{by Lemma 1070 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, op_l(x2, x2, x3), rd(i(x2), x2))}} \\
= & \quad \text{by Lemma 1071 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, mult(x2, x2), \underbrace{op_l(x2, x2, x3)})} \\
= & \quad \text{by Lemma 427 LR with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& op_l(x1, mult(x2, x2), \overbrace{mult(x3, rd(x2, x3))})
\end{aligned}
$$

Lemma 1073: $rd(op_l(i(x1), x2, x3), i(x4)) = rd(x4, op_l(x1, x2, x3))$

$$
\begin{aligned}
& rd(\underbrace{op_l(i(x1), x2, x3)}, i(x4)) \\
= & \quad \text{by Lemma 1049 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{rd(\overbrace{i(op_l(x1, x2, x3))}, i(x4))} \\
= & \quad \text{by Lemma 23 LR with } \{x2 \leftarrow x4, x1 \leftarrow op_l(x1, x2, x3)\} \\
& \overbrace{rd(x4, op_l(x1, x2, x3))}
\end{aligned}
$$

Lemma 1074: $rd(i(x1), op_l(i(x2), x3, x4)) = rd(op_l(x2, x3, x4), x1)$

$$
\begin{aligned}
& rd(i(x1), \underbrace{op_l(i(x2), x3, x4)}) \\
= & \quad \text{by Lemma 1049 RL with } \{x3 \leftarrow x4, x2 \leftarrow x3, x1 \leftarrow x2\} \\
& \underbrace{rd(i(x1), \overbrace{i(op_l(x2, x3, x4))})} \\
= & \quad \text{by Lemma 23 LR with } \{x2 \leftarrow op_l(x2, x3, x4), x1 \leftarrow x1\} \\
& \overbrace{rd(op_l(x2, x3, x4), x1)}
\end{aligned}
$$

Lemma 1075: $op_l(x1, op_l(rd(x2, x3), x3, x2), x2) = op_l(x1, x2, rd(rd(x3, x2), x2))$

$$
\begin{aligned}
& \underbrace{op_l(x1, op_l(rd(x2, x3), x3, x2), x2)} \\
= & \quad \text{by Lemma 1051 RL with } \{x3 \leftarrow op_l(rd(x2, x3), x3, x2), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x2, i(\underbrace{mult(x2, op_l(rd(x2, x3), x3, x2))}))} \\
= & \quad \text{by Lemma 308 RL with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& op_l(x1, x2, \underbrace{i(\overbrace{rd(x2, rd(x3, x2))})}) \\
= & \quad \text{by Lemma 10 LR with } \{x2 \leftarrow rd(x3, x2), x1 \leftarrow x2\} \\
& op_l(x1, x2, \overbrace{rd(rd(x3, x2), x2)})
\end{aligned}
$$

Lemma 1076: $op_l(x1, op_l(rd(x2, x3), x3, x2), x2) = op_l(x1, x2, x3)$

$$
\begin{aligned}
& \underbrace{op_l(x1, op_l(rd(x2, x3), x3, x2), x2)} \\
= & \quad \text{by Lemma 1075 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{\overbrace{op_l(x1, x2, rd(rd(x3, x2), x2))}} \\
= & \quad \text{by Lemma 286 LR with } \{x3 \leftarrow rd(x3, x2), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{\overbrace{op_l(x1, x2, rd(x3, x2))}} \\
= & \quad \text{by Lemma 286 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x2, x3)}
\end{aligned}
$$

Lemma 1077: $op_l(x1, x2, x3) = op_l(x1, op_l(x2, x3, x2), mult(x2, x3))$

$$
\begin{aligned}
& \underbrace{op_l(x1, x2, x3)} \\
= & \quad \text{by Lemma 242 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{\overbrace{op_l(x1, mult(x2, x3), i(x2))}} \\
= & \quad \text{by Lemma 1050 RL with } \{x3 \leftarrow x2, x2 \leftarrow mult(x2, x3), x1 \leftarrow x1\} \\
& \underbrace{\overbrace{op_l(x1, i(mult(x2, x3)), x2)}} \\
= & \quad \text{by Lemma 484 LR with } \{x3 \leftarrow x2, x2 \leftarrow i(mult(x2, x3)), x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, \underbrace{op_l(x2, x2, i(mult(x2, x3)))}, i(i(mult(x2, x3))))} \\
= & \quad \text{by Lemma 1051 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x2\} \\
& op_l(x1, \overbrace{op_l(x2, x3, x2)}, \underbrace{i(i(mult(x2, x3)))}) \\
= & \quad \text{by Lemma 3 LR with } \{x1 \leftarrow mult(x2, x3)\} \\
& op_l(x1, op_l(x2, x3, x2), \overbrace{mult(x2, x3)})
\end{aligned}
$$

Lemma 1078: $op_l(x1, rd(x2, op_t(x3, i(x2))), x2) = op_l(x1, x2, rd(rd(x3, x2), x2))$

$$
\begin{aligned}
& \underbrace{op_l(x1, rd(x2, op_t(x3, i(x2))), x2)} \\
= \quad & \text{by Lemma 1051 RL with } \{x3 \leftarrow rd(x2, op_t(x3, i(x2))), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x2, i(\underbrace{mult(x2, rd(x2, op_t(x3, i(x2))))}))} \\
= \quad & \text{by Lemma 805 LR with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& op_l(x1, x2, \overbrace{i(\overbrace{rd(x2, rd(x3, x2))})}) \\
= \quad & \text{by Lemma 10 LR with } \{x2 \leftarrow rd(x3, x2), x1 \leftarrow x2\} \\
& op_l(x1, x2, \overbrace{rd(rd(x3, x2), x2)})
\end{aligned}
$$

Lemma 1079: $op_l(x1, rd(x2, op_t(x3, i(x2))), x2) = op_l(x1, x2, x3)$

$$
\begin{aligned}
& \underbrace{op_l(x1, rd(x2, op_t(x3, i(x2))), x2)} \\
= \quad & \text{by Lemma 1078 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, x2, rd(rd(x3, x2), x2))}} \\
= \quad & \text{by Lemma 286 LR with } \{x3 \leftarrow rd(x3, x2), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, x2, rd(x3, x2))}} \\
= \quad & \text{by Lemma 286 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x2, x3)}
\end{aligned}
$$

Lemma 1080: $op_l(x1, x2, x3) = op_l(x1, x3, op_l(i(rd(x2, x3)), x2, x3))$

$$
\begin{aligned}
& \underbrace{op_l(x1, x2, x3)} \\
= \quad & \text{by Lemma 292 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, rd(x2, x3), x3)}} \\
= \quad & \text{by Lemma 1061 RL with } \{x3 \leftarrow rd(x2, x3), x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x3, \underbrace{op_l(i(rd(x2, x3)), rd(x2, x3), x3)})} \\
= \quad & \text{by Lemma 292 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow i(rd(x2, x3))\} \\
& op_l(x1, x3, \overbrace{op_l(i(rd(x2, x3)), x2, x3)})
\end{aligned}
$$

Lemma 1081: $op_l(x1, x2, op_l(rd(x2, x3), x3, x2)) = op_l(x1, x3, x2)$

$$
\begin{aligned}
& op_l(x1, x2, op_l(\underbrace{rd(x2, x3)}, x3, x2)) \\
= \quad & \text{by Lemma 10 RL with } \{x2 \leftarrow x2, x1 \leftarrow x3\} \\
& \underbrace{op_l(x1, x2, op_l(\overbrace{i(rd(x3, x2))}, x3, x2))} \\
= \quad & \text{by Lemma 1080 RL with } \{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x3, x2)}
\end{aligned}
$$

Lemma 1082: $op_l(x1, x3, x2) = op_l(x1, rd(x2, op_l(i(x3), x3, x2)), op_l(x2, x3, x2))$

$$
\begin{aligned}
& \overbrace{op_l(x1, x3, x2)} \\
= & \quad \text{by Lemma 1061 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x2, \underbrace{op_l(i(x3), x3, x2)})} \\
= & \quad \text{by Lemma 965 LR with } \{x3 \leftarrow op_l(i(x3), x3, x2), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, rd(x2, op_l(i(x3), x3, x2)), \underbrace{op_l(x2, x2, op_l(i(x3), x3, x2))})} \\
= & \quad \text{by Lemma 1061 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x2\} \\
& op_l(x1, rd(x2, op_l(i(x3), x3, x2)), \overbrace{op_l(x2, x3, x2)})
\end{aligned}
$$

Lemma 1083: $op_l(x1, mult(x2, x3), op_l(x2, x3, x2)) = op_l(x1, x3, x2)$

$$
\begin{aligned}
& op_l(x1, \underbrace{mult(x2, x3)}, op_l(x2, x3, x2)) \\
= & \quad \text{by Lemma 231 LR with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& \underbrace{op_l(x1, \overbrace{rd(x2, op_l(i(x3), x3, x2))}, op_l(x2, x3, x2))} \\
= & \quad \text{by Lemma 1082 RL with } \{x2 \leftarrow x2, x3 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x3, x2)}
\end{aligned}
$$

Lemma 1084: $asoc(x1, mult(i(op_t(x1, x2)), x3), x1) = asoc(x1, x3, x1)$

$$
\begin{aligned}
& \underbrace{asoc(x1, mult(i(op_t(x1, x2)), x3), x1)} \\
= & \quad \text{by Lemma 617 RL with } \{x2 \leftarrow mult(i(op_t(x1, x2)), x3), x1 \leftarrow x1\} \\
& \overbrace{\underbrace{rd(x1, op_l(x1, mult(i(op_t(x1, x2)), x3), x1))}} \\
= & \quad \text{by Lemma 1065 RL with } \{x3 \leftarrow mult(i(op_t(x1, x2)), x3), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{asoc(x1, \underbrace{mult(op_t(x1, x2), mult(i(op_t(x1, x2)), x3))}, x1)} \\
= & \quad \text{by Lemma 12 LR with } \{x2 \leftarrow x3, x1 \leftarrow op_t(x1, x2)\} \\
& asoc(x1, \overbrace{x3}, x1)
\end{aligned}
$$

Lemma 1085: $op_l(i(op_l(x1, x1, x2)), x2, op_t(x1, x3)) = i(x1)$

$$
\begin{aligned}
& \underbrace{op_l(i(op_l(x1, x1, x2)), x2, op_t(x1, x3))} \\
= & \quad \text{by Lemma 1049 RL with } \{x3 \leftarrow op_t(x1, x3), x2 \leftarrow x2, x1 \leftarrow op_l(x1, x1, x2)\} \\
& \overbrace{i(op_l(\underbrace{op_l(x1, x1, x2)}, x2, op_t(x1, x3)))} \\
= & \quad \text{by Lemma 1036 RL with } \{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x1\} \\
& i(\underbrace{op_l(\overbrace{op_l(x1, op_t(x1, x3), x2)}, x2, op_t(x1, x3)))} \\
= & \quad \text{by Lemma 138 LR with } \{x3 \leftarrow x2, x2 \leftarrow op_t(x1, x3), x1 \leftarrow x1\} \\
& i(\overbrace{x1})
\end{aligned}
$$

Lemma 1086: $mult(mult(x1,x2),mult(i(x1),x3)) = mult(x1,mult(rd(x2,x1),x3))$

$$
\begin{aligned}
& \underbrace{mult(mult(x1,x2),mult(i(x1),x3))} \\
= \quad & \text{by Lemma 54 RL with } \{x3 \leftarrow mult(i(x1),x3), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{mult(x1,mult(rd(x2,x1),\underbrace{mult(x1,mult(i(x1),x3))}))} \\
= \quad & \text{by Lemma 12 LR with } \{x2 \leftarrow x3, x1 \leftarrow x1\} \\
& mult(x1,mult(rd(x2,x1),\overbrace{x3}))
\end{aligned}
$$

Lemma 1087: $asoc(mult(x1,x2),i(x1),x3) = mult(i(mult(x1,mult(rd(x2,x1),x3))),mult(mult(x1,rd(x2,x1)),x3))$

$$
\begin{aligned}
& \underbrace{asoc(mult(x1,x2),i(x1),x3)} \\
= \quad & \text{by Lemma 108 RL with } \{x3 \leftarrow x3, x2 \leftarrow i(x1), x1 \leftarrow mult(x1,x2)\} \\
& \overbrace{mult(i(\underbrace{mult(mult(x1,x2),mult(i(x1),x3))}),mult(mult(mult(x1,x2),i(x1)),x3))} \\
= \quad & \text{by Lemma 1086 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& mult(i(\overbrace{mult(x1,mult(rd(x2,x1),x3))}),mult(\underbrace{mult(mult(x1,x2),i(x1))},x3)) \\
= \quad & \text{by Lemma 55 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& mult(i(mult(x1,mult(rd(x2,x1),x3))),mult(\overbrace{mult(x1,rd(x2,x1))},x3))
\end{aligned}
$$

Lemma 1088: $asoc(mult(x1,x2),i(x1),x3) = asoc(x1,rd(x2,x1),x3)$

$$
\begin{aligned}
& \underbrace{asoc(mult(x1,x2),i(x1),x3)} \\
= \quad & \text{by Lemma 1087 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{mult(i(mult(x1,mult(rd(x2,x1),x3))),mult(mult(x1,rd(x2,x1)),x3))} \\
= \quad & \text{by Lemma 108 LR with } \{x3 \leftarrow x3, x2 \leftarrow rd(x2,x1), x1 \leftarrow x1\} \\
& \overbrace{asoc(x1,rd(x2,x1),x3)}
\end{aligned}
$$

Lemma 1089: $asoc(x1,rd(x2,x1),mult(x1,x2)) = rd(x1,op_t(x1,x2))$

$$
\begin{aligned}
& \underbrace{asoc(x1,rd(x2,x1),mult(x1,x2))} \\
= \quad & \text{by Lemma 1088 RL with } \{x3 \leftarrow mult(x1,x2), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{asoc(mult(x1,x2),i(x1),mult(x1,x2))}} \\
= \quad & \text{by Lemma 756 LR with } \{x2 \leftarrow x1, x1 \leftarrow mult(x1,x2)\} \\
& \overbrace{\underbrace{asoc(x1,mult(x1,x2),mult(x1,x2))}} \\
= \quad & \text{by Lemma 860 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{rd(x1,op_t(x1,x2))}
\end{aligned}
$$

Lemma 1090: $rd(x1, mult(i(x1), rd(x2, x3))) = mult(x1, mult(rd(x3, x2), x1))$

$$
\begin{aligned}
& \underbrace{rd(x1, \underbrace{mult(i(x1), rd(x2, x3))}_{}}_{} \\
= \quad & \text{by Lemma 45 RL with } \{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \underbrace{rd(x1, \overbrace{i(\overbrace{mult(x1, rd(x3, x2))}^{})}^{})}_{} \\
= \quad & \text{by Lemma 58 LR with } \{x2 \leftarrow rd(x3, x2), x1 \leftarrow x1\} \\
& \overbrace{mult(x1, \underbrace{mult(rd(x3, x2), x1)}_{})}^{}
\end{aligned}
$$

Lemma 1091: $mult(rd(x1, x2), mult(x2, x3)) = mult(x1, op_l(x3, x2, x1))$

$$
\begin{aligned}
& \underbrace{mult(rd(x1, x2), mult(x2, x3))}_{} \\
= \quad & \text{by Lemma 132 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow rd(x1, x2)\} \\
& \overbrace{mult(\underbrace{mult(rd(x1, x2), x2)}_{}, op_l(x3, x2, rd(x1, x2)))}^{} \\
= \quad & \text{by Axiom 4 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& mult(\overbrace{x1}^{}, \underbrace{op_l(x3, x2, rd(x1, x2))}_{}) \\
= \quad & \text{by Lemma 286 LR with } \{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x3\} \\
& mult(x1, \overbrace{op_l(x3, x2, x1)}^{})
\end{aligned}
$$

Lemma 1092: $mult(i(x1), mult(op_t(x1, x2), x3)) = mult(rd(op_t(x1, x2), x1), x3)$

$$
\begin{aligned}
& mult(\underbrace{i(x1)}_{}, mult(op_t(x1, x2), x3)) \\
= \quad & \text{by Axiom 5 LR with } \{x2 \leftarrow i(x2), x1 \leftarrow i(x1)\} \\
& mult(\overbrace{rd(\underbrace{mult(i(x1), i(x2))}_{}, i(x2))}^{}, mult(op_t(x1, x2), x3)) \\
= \quad & \text{by Lemma 166 RL with } \{x3 \leftarrow x3, x2 \leftarrow i(x2), x1 \leftarrow i(x1)\} \\
& mult(\underbrace{rd(\overbrace{mult(op_r(i(x1), i(\underbrace{op_t(i(x1), i(x2))}_{})), x3), i(x2))}^{}, i(x2))}_{}, mult(op_t(x1, x2), x3)) \\
= \quad & \text{by Axiom 5 RL with } \{x2 \leftarrow i(x2), x1 \leftarrow op_r(i(x1), i(op_t(i(x1), i(x2))), x3)\} \\
& mult(\overbrace{op_r(i(x1), i(\underbrace{op_t(i(x1), i(x2))}_{}), x3)}^{}, mult(op_t(x1, x2), x3)) \\
= \quad & \text{by Lemma 15 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& mult(op_r(i(x1), \underbrace{i(\overbrace{i(op_t(x1, x2))}^{})}_{}, x3), mult(op_t(x1, x2), x3)) \\
= \quad & \text{by Lemma 3 LR with } \{x1 \leftarrow op_t(x1, x2)\} \\
& \underbrace{mult(op_r(i(x1), \overbrace{op_t(x1, x2)}^{}, x3), mult(op_t(x1, x2), x3))}_{} \\
= \quad & \text{by Lemma 91 LR with } \{x3 \leftarrow x3, x2 \leftarrow op_t(x1, x2), x1 \leftarrow i(x1)\} \\
& \overbrace{mult(\underbrace{mult(i(x1), op_t(x1, x2))}_{}, x3)}^{} \\
= \quad & \text{by Lemma 96 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& mult(\overbrace{rd(op_t(x1, x2), x1)}^{}, x3)
\end{aligned}
$$

Lemma 1093: $op_l(x1, i(op_r(x2, x3, x4)), x2) = op_l(x1, op_t(x2, op_r(x2, x3, x4)), op_r(x2, x3, x4))$

$\underbrace{op_l(x1, i(op_r(x2, x3, x4)), x2)}$

$=$ by Lemma 286 RL with $\{x3 \leftarrow x2, x2 \leftarrow i(op_r(x2, x3, x4)), x1 \leftarrow x1\}$

$\overbrace{op_l(x1, i(op_r(x2, x3, x4)), \underbrace{rd(x2, i(op_r(x2, x3, x4)))}}$

$=$ by Lemma 121 LR with $\{x3 \leftarrow x4, x2 \leftarrow x3, x1 \leftarrow x2\}$

$\underbrace{op_l(x1, i(op_r(x2, x3, x4)), \overbrace{mult(x2, op_r(x2, x3, x4))})}$

$=$ by Lemma 246 LR with $\{x3 \leftarrow x2, x2 \leftarrow op_r(x2, x3, x4), x1 \leftarrow x1\}$

$\overbrace{op_l(x1, \underbrace{op_t(x2, op_r(x2, x3, x4))}, op_r(x2, x3, x4))}$

Lemma 1094: $op_l(x1, op_r(x2, x3, x4), i(x2)) = op_l(x1, x2, op_r(x2, x3, x4))$

$\underbrace{op_l(x1, op_r(x2, x3, x4), i(x2))}$

$=$ by Lemma 1050 RL with $\{x3 \leftarrow x2, x2 \leftarrow op_r(x2, x3, x4), x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_l(x1, i(op_r(x2, x3, x4)), x2)}}$

$=$ by Lemma 1093 LR with $\{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_l(x1, \underbrace{op_t(x2, op_r(x2, x3, x4))}, op_r(x2, x3, x4))}$

$=$ by Lemma 120 LR with $\{x3 \leftarrow x4, x2 \leftarrow x3, x1 \leftarrow x2\}$

$op_l(x1, \overbrace{x2}, op_r(x2, x3, x4))$

Lemma 1095: $mult(i(op_t(x1, x2)), mult(x1, x3)) = mult(rd(x1, op_t(x1, x2)), x3)$

$\underbrace{mult(i(op_t(x1, x2)), mult(x1, x3))}$

$=$ by Lemma 132 RL with $\{x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow i(op_t(x1, x2))\}$

$\overbrace{mult(mult(i(op_t(x1, x2)), x1), \underbrace{op_l(x3, x1, i(op_t(x1, x2)))})}$

$=$ by Lemma 289 LR with $\{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x3\}$

$mult(\underbrace{mult(i(op_t(x1, x2)), x1)}, \overbrace{x3})$

$=$ by Lemma 99 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$mult(\overbrace{rd(x1, op_t(x1, x2))}, x3)$

Lemma 1096: $mult(rd(op_t(x1, x2), x1), x3) = mult(op_t(x1, x2), mult(i(x1), x3))$

$mult(\underbrace{rd(op_t(x1, x2), x1)}, x3)$

$=$ by Lemma 97 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{mult(\overbrace{mult(op_t(x1, x2)}, i(x1)), x3)}$

$=$ by Lemma 137 RL with $\{x3 \leftarrow x3, x2 \leftarrow i(x1), x1 \leftarrow op_t(x1, x2)\}$

$\overbrace{mult(op_t(x1, x2), mult(i(x1), \underbrace{op_l(x3, op_t(x1, x2), i(x1))}))}$

$=$ by Lemma 290 RL with $\{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x3\}$

$mult(op_t(x1, x2), mult(i(x1), \overbrace{x3}))$

Lemma 1097: $mult(mult(i(x1), x2), i(x1)) = rd(op_t(mult(x2, i(x1)), x1), x1)$

$\underbrace{mult(mult(i(x1), x2), i(x1))}$

$=$ by Lemma 104 RL with $\{x2 \leftarrow x1, x1 \leftarrow mult(i(x1), x2)\}$

$\overbrace{op_r(rd(mult(i(x1), x2), x1), x1, i(x1))}$

$=$ by Lemma 317 RL with $\{x2 \leftarrow rd(mult(i(x1), x2), x1), x1 \leftarrow x1\}$

$\overbrace{rd(op_t(\underbrace{mult(x1, rd(mult(i(x1), x2), x1))}, x1), x1)}$

$=$ by Lemma 84 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$rd(op_t(\overbrace{mult(x2, i(x1))}, x1), x1)$

Lemma 1098: $op_t(op_t(mult(x1, x2), x1), x1) = mult(x1, op_t(x2, mult(x1, x1)))$

$\underbrace{op_t(op_t(mult(x1, x2), x1), x1)}$

$=$ by Axiom 10 RL with $\{x2 \leftarrow op_t(mult(x1, x2), x1), x1 \leftarrow x1\}$

$\overbrace{mult(i(x1), \underbrace{mult(op_t(mult(x1, x2), x1), x1)})}$

$=$ by Lemma 350 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$mult(i(x1), \overbrace{mult(x2, \underbrace{mult(x1, x1)})})$

$=$ by Lemma 42 RL with $\{x1 \leftarrow x1\}$

$mult(i(x1), mult(x2, \overbrace{\underbrace{rd(x1, i(x1))}}))$

$=$ by Lemma 10 RL with $\{x2 \leftarrow x1, x1 \leftarrow i(x1)\}$

$\underbrace{mult(i(x1), mult(x2, \overbrace{i(rd(i(x1), x1))}))}$

$=$ by Axiom 2 LR with $\{x2 \leftarrow i(x1), x1 \leftarrow mult(i(x1), mult(x2, i(rd(i(x1), x1))))\}$

$\overbrace{mult(i(i(x1)), \underbrace{mult(i(x1), mult(i(x1), mult(x2, i(rd(i(x1), x1)))))})}$

$=$ by Lemma 24 RL with $\{x2 \leftarrow mult(x2, i(rd(i(x1), x1))), x1 \leftarrow i(x1)\}$

$mult(i(i(x1)), \overbrace{mult(\underbrace{mult(i(x1), i(x1))}, mult(x2, i(rd(i(x1), x1)))))})$

$=$ by Axiom 11 LR with $\{x2 \leftarrow x1, x1 \leftarrow x1\}$

$mult(i(i(x1)), mult(\overbrace{i(\underbrace{mult(x1, x1)})}, mult(x2, i(rd(i(x1), x1)))))$

$=$ by Lemma 43 LR with $\{x1 \leftarrow x1\}$

$mult(\underbrace{i(i(x1))}, mult(\overbrace{rd(i(x1), x1)}, mult(x2, i(rd(i(x1), x1)))))$

$=$ by Lemma 3 LR with $\{x1 \leftarrow x1\}$

$mult(\overbrace{x1}, \underbrace{mult(rd(i(x1), x1), mult(x2, i(rd(i(x1), x1))))})$

$=$ by Lemma 30 LR with $\{x2 \leftarrow x2, x1 \leftarrow rd(i(x1), x1)\}$

$mult(x1, \overbrace{op_t(x2, \underbrace{i(rd(i(x1), x1))})})$

$=$ by Lemma 10 LR with $\{x2 \leftarrow x1, x1 \leftarrow i(x1)\}$

$mult(x1, op_t(x2, \overbrace{\underbrace{rd(x1, i(x1))}}))$

$=$ by Lemma 42 LR with $\{x1 \leftarrow x1\}$

$mult(x1, op_t(x2, \overbrace{mult(x1, x1)}))$

Lemma 1099: $mult(i(x1), op_r(i(x3), x2, x2)) = i(mult(x1, op_r(x3, x2, x2)))$

$$
\begin{aligned}
& mult(i(x1), \underbrace{op_r(i(x3), x2, x2)}) \\
= & \quad \text{by Lemma 354 RL with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& \underbrace{mult(i(x1), \overbrace{rd(rd(x2, x3), x2)})} \\
= & \quad \text{by Lemma 45 RL with } \{x3 \leftarrow rd(x2, x3), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{i(mult(x1, \underbrace{rd(x2, rd(x2, x3))}))} \\
= & \quad \text{by Lemma 353 LR with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& i(mult(x1, \overbrace{op_r(x3, x2, x2)}))
\end{aligned}
$$

Lemma 1100: $mult(i(x1), op_r(x3, x2, x2)) = i(mult(x1, op_r(i(x3), x2, x2)))$

$$
\begin{aligned}
& mult(i(x1), \underbrace{op_r(x3, x2, x2)}) \\
= & \quad \text{by Lemma 353 RL with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& \underbrace{mult(i(x1), \overbrace{rd(x2, rd(x2, x3))})} \\
= & \quad \text{by Lemma 45 RL with } \{x3 \leftarrow x2, x2 \leftarrow rd(x2, x3), x1 \leftarrow x1\} \\
& \overbrace{i(mult(x1, \underbrace{rd(rd(x2, x3), x2)}))} \\
= & \quad \text{by Lemma 354 LR with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& i(mult(x1, \overbrace{op_r(i(x3), x2, x2)}))
\end{aligned}
$$

Lemma 1101: $mult(mult(mult(i(x1), x2), x2), x2) = op_t(mult(x2, mult(rd(x2, x1), x2)), x2)$

$$
\begin{aligned}
& mult(mult(\underbrace{mult(i(x1), x2)}, x2), x2) \\
= & \quad \text{by Lemma 17 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{mult(mult(\overbrace{i(mult(x1, i(x2)))}, x2), x2)} \\
= & \quad \text{by Lemma 406 RL with } \{x2 \leftarrow mult(x1, i(x2)), x1 \leftarrow x2\} \\
& \overbrace{op_t(mult(x2, \underbrace{rd(x2, mult(x1, i(x2)))}), x2)} \\
= & \quad \text{by Lemma 334 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& op_t(mult(x2, \overbrace{mult(rd(x2, x1), x2)}), x2)
\end{aligned}
$$

Lemma 1102: $op_l(x1, op_t(x2, mult(x2, x3)), x3) = op_l(x1, i(x3), op_t(x2, x3))$

$$
\begin{aligned}
& \underbrace{op_l(x1, op_t(x2, mult(x2, x3)), x3)} \\
= & \quad \text{by Lemma 292 RL with } \{x3 \leftarrow x3, x2 \leftarrow op_t(x2, mult(x2, x3)), x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, \underbrace{rd(op_t(x2, mult(x2, x3)), x3)}, x3)} \\
= & \quad \text{by Lemma 459 LR with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& \underbrace{op_l(x1, \overbrace{mult(i(x3), op_t(x2, x3))}, x3)} \\
= & \quad \text{by Lemma 244 LR with } \{x3 \leftarrow op_t(x2, x3), x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, i(x3), op_t(x2, x3))}
\end{aligned}
$$

Lemma 1103: $op_l(x1, mult(x2, i(x3)), x1) = op_l(x1, mult(i(x2), x3), i(x1))$

$$
\begin{aligned}
& \underbrace{op_l(x1, mult(x2, i(x3)), x1)} \\
= \quad & \text{by Lemma 499 RL with } \{x2 \leftarrow mult(x2, i(x3)), x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, \underbrace{rd(mult(x2, i(x3)), i(x1))}, x1)} \\
= \quad & \text{by Lemma 61 LR with } \{x3 \leftarrow x1, x2 \leftarrow x3, x1 \leftarrow x2\} \\
& \underbrace{op_l(x1, \overbrace{rd(x1, mult(i(x2), x3))}, x1)} \\
= \quad & \text{by Lemma 494 LR with } \{x2 \leftarrow mult(i(x2), x3), x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, mult(i(x2), x3), i(x1))}
\end{aligned}
$$

Lemma 1104: $op_l(x1, mult(i(x2), x3), x1) = op_l(x1, mult(x2, i(x3)), i(x1))$

$$
\begin{aligned}
& \underbrace{op_l(x1, mult(i(x2), x3), x1)} \\
= \quad & \text{by Lemma 499 RL with } \{x2 \leftarrow mult(i(x2), x3), x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, \underbrace{rd(mult(i(x2), x3), i(x1))}, x1)} \\
= \quad & \text{by Lemma 63 LR with } \{x3 \leftarrow x1, x2 \leftarrow x3, x1 \leftarrow x2\} \\
& \underbrace{op_l(x1, \overbrace{rd(x1, mult(x2, i(x3)))}, x1)} \\
= \quad & \text{by Lemma 494 LR with } \{x2 \leftarrow mult(x2, i(x3)), x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, mult(x2, i(x3)), i(x1))}
\end{aligned}
$$

Lemma 1105: $op_l(x1, op_t(i(x2), x3), x1) = op_l(x1, op_t(x2, i(x3)), i(x1))$

$$
\begin{aligned}
& \underbrace{op_l(x1, op_t(i(x2), x3), x1)} \\
= \quad & \text{by Lemma 499 RL with } \{x2 \leftarrow op_t(i(x2), x3), x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, \underbrace{rd(op_t(i(x2), x3), i(x1))}, x1)} \\
= \quad & \text{by Lemma 67 LR with } \{x3 \leftarrow x1, x2 \leftarrow x3, x1 \leftarrow x2\} \\
& \underbrace{op_l(x1, \overbrace{rd(x1, op_t(x2, i(x3)))}, x1)} \\
= \quad & \text{by Lemma 494 LR with } \{x2 \leftarrow op_t(x2, i(x3)), x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, op_t(x2, i(x3)), i(x1))}
\end{aligned}
$$

Lemma 1106: $op_l(op_l(x1, x2, x3), x4, i(x1)) = op_l(op_l(x1, x1, x4), x2, x3)$

$$
\begin{aligned}
& \underbrace{op_l(op_l(x1, x2, x3), x4, i(x1))} \\
= \quad & \text{by Axiom 19 RL with } \{x5 \leftarrow x3, x4 \leftarrow x2, x3 \leftarrow i(x1), x2 \leftarrow x4, x1 \leftarrow x1\} \\
& \overbrace{op_l(\underbrace{op_l(x1, x4, i(x1))}, x2, x3)} \\
= \quad & \text{by Lemma 606 LR with } \{x2 \leftarrow x4, x1 \leftarrow x1\} \\
& op_l(\overbrace{op_l(x1, x1, x4)}, x2, x3)
\end{aligned}
$$

Lemma 1107: $op_l(op_l(x1, x2, x3), x1, i(x4)) = op_l(op_l(x1, x4, x1), x2, x3)$

$$
\begin{aligned}
& \overbrace{op_l(op_l(x1, x2, x3), x1, i(x4))} \\
= & \quad \text{by Axiom 19 RL with } \{x5 \leftarrow x3, x4 \leftarrow x2, x3 \leftarrow i(x4), x2 \leftarrow x1, x1 \leftarrow x1\} \\
& \overbrace{op_l(\underbrace{op_l(x1, x1, i(x4))}, x2, x3)} \\
= & \quad \text{by Lemma 608 LR with } \{x2 \leftarrow x4, x1 \leftarrow x1\} \\
& op_l(\overbrace{op_l(x1, x4, x1)}, x2, x3)
\end{aligned}
$$

Lemma 1108: $rd(mult(x2, x1), x2) = rd(x1, rd(op_t(x1, x2), x1))$

$$
\begin{aligned}
& \overbrace{rd(mult(x2, x1), x2)} \\
= & \quad \text{by Lemma 823 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_r(x1, mult(x1, x2), mult(x1, x2))}} \\
= & \quad \text{by Lemma 361 RL with } \{x2 \leftarrow mult(x1, x2), x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_r(x1, i(mult(x1, x2)), i(mult(x1, x2)))}} \\
= & \quad \text{by Lemma 698 RL with } \{x2 \leftarrow i(mult(x1, x2)), x1 \leftarrow x1\} \\
& \overbrace{mult(\underbrace{rd(x1, i(mult(x1, x2)))}, op_l(i(mult(x1, x2)), x1, i(mult(x1, x2))))} \\
= & \quad \text{by Lemma 58 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{mult(\overbrace{mult(x1, mult(x2, x1))}, op_l(i(mult(x1, x2)), x1, i(mult(x1, x2))))} \\
= & \quad \text{by Axiom 6 RL with } \{x3 \leftarrow op_l(i(mult(x1, x2)), x1, i(mult(x1, x2))), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{mult(x1, mult(x2, \underbrace{mult(x1, op_l(i(mult(x1, x2)), x1, i(mult(x1, x2))))}))} \\
= & \quad \text{by Lemma 616 LR with } \{x2 \leftarrow i(mult(x1, x2)), x1 \leftarrow x1\} \\
& mult(x1, mult(x2, \overbrace{rd(x1, \underbrace{i(i(mult(x1, x2)))})})) \\
= & \quad \text{by Lemma 3 LR with } \{x1 \leftarrow mult(x1, x2)\} \\
& mult(x1, \underbrace{mult(x2, rd(x1, \overbrace{mult(x1, x2)}))}) \\
= & \quad \text{by Lemma 151 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& mult(x1, \overbrace{rd(i(rd(mult(x1, x2), x1)), i(x2))}) \\
= & \quad \text{by Lemma 23 LR with } \{x2 \leftarrow x2, x1 \leftarrow rd(mult(x1, x2), x1)\} \\
& mult(x1, \overbrace{rd(x2, rd(mult(x1, x2), x1))}) \\
= & \quad \text{by Lemma 396 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{mult(x1, \overbrace{rd(x1, op_t(x1, x2))})} \\
= & \quad \text{by Lemma 130 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{rd(x1, rd(op_t(x1, x2), x1))}
\end{aligned}
$$

Lemma 1109: $mult(mult(x1, rd(x2, i(x1))), x1) = mult(x1, mult(mult(x2, x1), x1))$

$$
\begin{aligned}
& \underbrace{mult(mult(x1, rd(x2, i(x1))), x1)} \\
= \quad & \text{by Lemma 13 RL with } \{x2 \leftarrow mult(x1, rd(x2, i(x1))), x1 \leftarrow x1\} \\
& \overbrace{mult(x1, \underbrace{op_t(mult(x1, rd(x2, i(x1))), x1)})} \\
= \quad & \text{by Lemma 320 RL with } \{x2 \leftarrow x1, x1 \leftarrow rd(x2, i(x1))\} \\
& mult(x1, \overbrace{op_r(\underbrace{mult(rd(x2, i(x1)), x1)}, x1, i(x1))}) \\
= \quad & \text{by Lemma 336 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& mult(x1, \underbrace{op_r(\overbrace{rd(x1, i(mult(x2, x1)))}, x1, i(x1))}) \\
= \quad & \text{by Lemma 330 LR with } \{x2 \leftarrow i(mult(x2, x1)), x1 \leftarrow x1\} \\
& mult(x1, \overbrace{mult(\underbrace{i(i(mult(x2, x1)))}, x1)}) \\
= \quad & \text{by Lemma 3 LR with } \{x1 \leftarrow mult(x2, x1)\} \\
& mult(x1, mult(\overbrace{mult(x2, x1)}, x1))
\end{aligned}
$$

Lemma 1110: $op_r(x1, op_t(i(x2), x1), i(x3)) = op_r(i(i(x1)), mult(x2, i(x1)), x3)$

$$
\begin{aligned}
& op_r(x1, \underbrace{op_t(i(x2), x1)}, i(x3)) \\
= \quad & \text{by Lemma 19 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \underbrace{op_r(x1, \overbrace{i(op_t(x2, i(x1)))}, i(x3))} \\
= \quad & \text{by Lemma 813 RL with } \{x3 \leftarrow x3, x2 \leftarrow op_t(x2, i(x1)), x1 \leftarrow x1\} \\
& \overbrace{\underbrace{i(op_r(i(x1), op_t(x2, i(x1)), x3))}} \\
= \quad & \text{by Lemma 742 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow i(x1)\} \\
& \overbrace{op_r(i(i(x1)), mult(x2, i(x1)), x3)}
\end{aligned}
$$

Lemma 1111: $mult(rd(x1, mult(x2, x2)), i(x2)) = rd(rd(rd(x1, x2), x2), x2)$

$$
\begin{aligned}
& mult(\underbrace{rd(x1, mult(x2, x2))}, i(x2)) \\
= \quad & \text{by Lemma 839 RL with } \{x3 \leftarrow x2, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{mult(\overbrace{op_r(rd(rd(x1, x2), x2), x2, x2)}, i(x2))} \\
= \quad & \text{by Lemma 352 LR with } \{x2 \leftarrow x2, x1 \leftarrow rd(rd(x1, x2), x2)\} \\
& \overbrace{rd(rd(rd(x1, x2), x2), x2)}
\end{aligned}
$$

Lemma 1112: $rd(x2, op_t(i(mult(i(x1), i(x2))), x1)) = mult(i(op_t(x2, x1)), rd(i(x1), i(x2)))$

$$
\begin{aligned}
& \underbrace{rd(x2, op_t(i(mult(i(x1), i(x2))), x1))} \\
= \quad & \text{by Lemma 65 RL with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow mult(i(x1), i(x2))\} \\
& \overbrace{\underbrace{rd(op_t(mult(i(x1), i(x2)), i(x1)), i(x2))}} \\
= \quad & \text{by Lemma 895 RL with } \{x2 \leftarrow i(x1), x1 \leftarrow i(x2)\} \\
& \overbrace{mult(\underbrace{op_t(i(x2), i(x1))}, rd(i(x1), i(x2)))} \\
= \quad & \text{by Lemma 15 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& mult(\overbrace{i(op_t(x2, x1))}, rd(i(x1), i(x2)))
\end{aligned}
$$

Lemma 1113: $rd(x2, op_t(mult(x1, x2), x1)) = mult(i(op_t(x2, x1)), rd(i(x1), i(x2)))$

$$
\begin{aligned}
& rd(x2, op_t(mult(x1, \underbrace{x2}), x1)) \\
= & \quad \text{by Lemma 3 RL with } \{x1 \leftarrow x2\} \\
& rd(x2, \underbrace{op_t(mult(x1, \overbrace{i(i(x2))}), x1)}) \\
= & \quad \text{by Lemma 16 RL with } \{x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& \underbrace{rd(x2, op_t(\overbrace{i(mult(i(x1), i(x2)))}, x1))} \\
= & \quad \text{by Lemma 1112 LR with } \{x1 \leftarrow x1, x2 \leftarrow x2\} \\
& \overbrace{mult(i(op_t(x2, x1)), rd(i(x1), i(x2)))}
\end{aligned}
$$

Lemma 1114: $op_t(x1, mult(op_t(x1, x2), x3)) = op_t(x1, mult(x1, x3))$

$$
\begin{aligned}
& \underbrace{op_t(x1, mult(op_t(x1, x2), x3))} \\
= & \quad \text{by Axiom 10 RL with } \{x2 \leftarrow x1, x1 \leftarrow mult(op_t(x1, x2), x3)\} \\
& \overbrace{mult(i(mult(op_t(x1, x2), x3)), \underbrace{mult(x1, mult(op_t(x1, x2), x3))})} \\
= & \quad \text{by Lemma 161 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& mult(i(mult(op_t(x1, x2), x3)), \overbrace{mult(op_t(x1, x2), \underbrace{mult(x1, x3)})}) \\
= & \quad \text{by Lemma 13 RL with } \{x2 \leftarrow x1, x1 \leftarrow x3\} \\
& \underbrace{mult(i(mult(op_t(x1, x2), x3)), mult(op_t(x1, x2), \overbrace{mult(x3, op_t(x1, x3))}))} \\
= & \quad \text{by Axiom 12 LR with } \{x3 \leftarrow op_t(x1, x3), x2 \leftarrow x3, x1 \leftarrow op_t(x1, x2)\} \\
& \underbrace{\overbrace{op_l(op_t(x1, x3), x3, op_t(x1, x2))}} \\
= & \quad \text{by Axiom 15 RL with } \{x4 \leftarrow x3, x3 \leftarrow op_t(x1, x2), x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{op_t(\underbrace{op_l(x1, x3, op_t(x1, x2))}, x3)} \\
= & \quad \text{by Lemma 1024 LR with } \{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \underbrace{op_t(\overbrace{op_l(x1, x3, x1)}, x3)} \\
= & \quad \text{by Axiom 15 LR with } \{x4 \leftarrow x3, x3 \leftarrow x1, x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \underbrace{\overbrace{op_l(op_t(x1, x3), x3, x1)}} \\
= & \quad \text{by Lemma 251 LR with } \{x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, mult(x1, x3))}
\end{aligned}
$$

Lemma 1115: $op_t(x1, mult(asoc(x1, x2, x2), x3)) = op_t(x1, x3)$

$op_t(x1, mult(\underbrace{asoc(x1, x2, x2)}, x3))$

$=$ by Lemma 777 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$op_t(x1, mult(\overbrace{rd(x1, \underbrace{op_r(x1, x2, i(x2))})}, x3))$

$=$ by Lemma 547 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$op_t(x1, mult(\underbrace{rd(x1, \overbrace{op_t(x1, rd(x2, x1))})}, x3))$

$=$ by Lemma 98 LR with $\{x2 \leftarrow rd(x2, x1), x1 \leftarrow x1\}$

$op_t(x1, \underbrace{mult(\overbrace{mult(x1, i(\underbrace{op_t(x1, rd(x2, x1))}))}, x3)})$

$=$ by Lemma 91 RL with $\{x3 \leftarrow x3, x2 \leftarrow i(op_t(x1, rd(x2, x1))), x1 \leftarrow x1\}$

$op_t(x1, \overbrace{mult(\underbrace{op_r(x1, i(op_t(x1, rd(x2, x1))), x3)}, mult(i(op_t(x1, rd(x2, x1))), x3))})$

$=$ by Axiom 5 LR with $\{x2 \leftarrow rd(x2, x1), x1 \leftarrow op_r(x1, i(op_t(x1, rd(x2, x1))), x3)\}$

$op_t(x1, mult(\overbrace{rd(\underbrace{mult(op_r(x1, i(op_t(x1, rd(x2, x1))), x3), rd(x2, x1))}, rd(x2, x1))}, mult(i(op_t(x1, rd(x2, x1))), x3)))$

$=$ by Lemma 166 LR with $\{x3 \leftarrow x3, x2 \leftarrow rd(x2, x1), x1 \leftarrow x1\}$

$op_t(x1, mult(\underbrace{rd(\overbrace{mult(x1, rd(x2, x1))}, rd(x2, x1))}, mult(i(op_t(x1, rd(x2, x1))), x3)))$

$=$ by Axiom 5 RL with $\{x2 \leftarrow rd(x2, x1), x1 \leftarrow x1\}$

$\underbrace{op_t(x1, mult(\overbrace{x1}, mult(i(op_t(x1, rd(x2, x1))), x3)))}$

$=$ by Lemma 1114 RL with $\{x3 \leftarrow mult(i(op_t(x1, rd(x2, x1))), x3), x2 \leftarrow rd(x2, x1), x1 \leftarrow x1\}$

$\overbrace{op_t(x1, \underbrace{mult(op_t(x1, rd(x2, x1)), mult(i(op_t(x1, rd(x2, x1))), x3))})}$

$=$ by Lemma 12 LR with $\{x2 \leftarrow x3, x1 \leftarrow op_t(x1, rd(x2, x1))\}$

$op_t(x1, \overbrace{x3})$

Lemma 1116: $op_t(x1, mult(asoc(x2, x1, x2), x3)) = op_t(x1, x3)$

$op_t(x1, mult(\underbrace{asoc(x2, x1, x2)}, x3))$

$=$ by Lemma 759 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{op_t(x1, mult(\overbrace{i(asoc(x1, x2, x2))}, x3))}$

$=$ by Lemma 1115 RL with $\{x3 \leftarrow mult(i(asoc(x1, x2, x2)), x3), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_t(x1, \underbrace{mult(asoc(x1, x2, x2), mult(i(asoc(x1, x2, x2)), x3))})}$

$=$ by Lemma 12 LR with $\{x2 \leftarrow x3, x1 \leftarrow asoc(x1, x2, x2)\}$

$op_t(x1, \overbrace{x3})$

Lemma 1117: $op_t(x1, mult(op_t(x2, x1), x3)) = op_t(x1, mult(x2, op_l(x3, op_t(x2, x1), x2)))$

$$
\begin{aligned}
& \underbrace{op_t(x1, mult(op_t(x2, x1), x3))} \\
= \quad & \text{by Lemma 1116 RL with } \{x3 \leftarrow mult(op_t(x2, x1), x3), x2 \leftarrow mult(x2, x1), x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, mult(\underbrace{asoc(mult(x2, x1), x1, mult(x2, x1))}, mult(op_t(x2, x1), x3)))} \\
= \quad & \text{by Lemma 838 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& op_t(x1, \underbrace{mult(\overbrace{rd(x2, op_t(x2, x1))}, mult(op_t(x2, x1), x3))}) \\
= \quad & \text{by Lemma 1091 LR with } \{x3 \leftarrow x3, x2 \leftarrow op_t(x2, x1), x1 \leftarrow x2\} \\
& op_t(x1, \overbrace{mult(x2, op_l(x3, op_t(x2, x1), x2))}))
\end{aligned}
$$

Lemma 1118: $op_t(x1, mult(op_t(x2, x1), x3)) = op_t(x1, mult(x2, x3))$

$$
\begin{aligned}
& \underbrace{op_t(x1, mult(op_t(x2, x1), x3))} \\
= \quad & \text{by Lemma 1117 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, mult(x2, \underbrace{op_l(x3, op_t(x2, x1), x2)}))} \\
= \quad & \text{by Lemma 163 LR with } \{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x3\} \\
& op_t(x1, mult(x2, \overbrace{x3}^{}))
\end{aligned}
$$

Lemma 1119: $op_t(x1, rd(x1, mult(op_t(x1, x2), x3))) = op_t(x1, i(x3))$

$$
\begin{aligned}
& \underbrace{op_t(x1, rd(x1, mult(op_t(x1, x2), x3)))} \\
= \quad & \text{by Lemma 564 RL with } \{x2 \leftarrow mult(op_t(x1, x2), x3), x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, mult(x1, i(mult(op_t(x1, x2), x3))))} \\
= \quad & \text{by Lemma 1114 RL with } \{x3 \leftarrow i(mult(op_t(x1, x2), x3)), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, \underbrace{mult(op_t(x1, x2), i(mult(op_t(x1, x2), x3)))})} \\
= \quad & \text{by Lemma 20 LR with } \{x2 \leftarrow x3, x1 \leftarrow op_t(x1, x2)\} \\
& op_t(x1, \overbrace{i(x3)}^{})
\end{aligned}
$$

Lemma 1120: $op_t(x1, i(mult(op_t(x1, x2), x3))) = op_t(x1, rd(x3, x1))$

$$
\begin{aligned}
& \underbrace{op_t(x1, i(mult(op_t(x1, x2), x3)))} \\
= \quad & \text{by Lemma 571 RL with } \{x2 \leftarrow mult(op_t(x1, x2), x3), x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, rd(x1, \underbrace{mult(x1, mult(op_t(x1, x2), x3))}))} \\
= \quad & \text{by Lemma 161 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{op_t(x1, rd(x1, \overbrace{mult(op_t(x1, x2), mult(x1, x3))}))} \\
= \quad & \text{by Lemma 1119 LR with } \{x3 \leftarrow mult(x1, x3), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_t(x1, i(mult(x1, x3)))}} \\
= \quad & \text{by Lemma 708 LR with } \{x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, rd(x3, x1))}
\end{aligned}
$$

Lemma 1121: $op_t(x1, rd(x2, x1)) = op_t(x1, mult(i(op_t(x1, x3)), x2))$

$$
\begin{aligned}
& \underbrace{op_t(x1, rd(x2, x1))} \\
= & \quad \text{by Lemma 712 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_t(x1, rd(i(x2), x1))}} \\
= & \quad \text{by Lemma 1120 RL with } \{x3 \leftarrow i(x2), x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, \underbrace{i(mult(op_t(x1, x3), i(x2)))})} \\
= & \quad \text{by Lemma 17 LR with } \{x2 \leftarrow x2, x1 \leftarrow op_t(x1, x3)\} \\
& op_t(x1, \overbrace{mult(i(op_t(x1, x3)), x2)})
\end{aligned}
$$

Lemma 1122: $op_t(x1, rd(mult(op_t(x1, x2), x3), x1)) = op_t(x1, i(i(x3)))$

$$
\begin{aligned}
& \underbrace{op_t(x1, rd(mult(op_t(x1, x2), x3), x1))} \\
= & \quad \text{by Lemma 712 RL with } \{x2 \leftarrow mult(op_t(x1, x2), x3), x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_t(x1, rd(i(mult(op_t(x1, x2), x3)), x1))}} \\
= & \quad \text{by Lemma 1120 RL with } \{x3 \leftarrow i(mult(op_t(x1, x2), x3)), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, i(\underbrace{mult(op_t(x1, x2), i(mult(op_t(x1, x2), x3)))}))} \\
= & \quad \text{by Lemma 20 LR with } \{x2 \leftarrow x3, x1 \leftarrow op_t(x1, x2)\} \\
& op_t(x1, i(\overbrace{i(x3)}))
\end{aligned}
$$

Lemma 1123: $op_t(x1, rd(mult(op_t(x1, x2), x3), x1)) = op_t(x1, x3)$

$$
\begin{aligned}
& \underbrace{op_t(x1, rd(mult(op_t(x1, x2), x3), x1))} \\
= & \quad \text{by Lemma 1122 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, \underbrace{i(i(x3))})} \\
= & \quad \text{by Lemma 3 LR with } \{x1 \leftarrow x3\} \\
& op_t(x1, \overbrace{x3}) \\
\end{aligned}
$$

Lemma 1124: $op_t(x1, asoc(x3, op_t(x1,x2), op_t(x1,x2))) = x1$

$op_t(x1, \underbrace{asoc(x3, op_t(x1,x2), op_t(x1,x2))})$

$=$ by Lemma 757 RL with $\{x2 \leftarrow x3, x1 \leftarrow op_t(x1,x2)\}$

$\underbrace{op_t(x1, \overbrace{i(asoc(op_t(x1,x2), x3, op_t(x1,x2)))})}$

$=$ by Lemma 1119 RL with $\{x3 \leftarrow asoc(op_t(x1,x2), x3, op_t(x1,x2)), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_t(x1, rd(x1, \underbrace{mult(op_t(x1,x2), asoc(op_t(x1,x2), x3, op_t(x1,x2)))}))}$

$=$ by Lemma 158 LR with $\{x2 \leftarrow x3, x1 \leftarrow op_t(x1,x2)\}$

$op_t(x1, rd(x1, \underbrace{\overbrace{op_l(op_t(x1,x2), op_t(x1,x2), x3)}}))$

$=$ by Axiom 15 RL with $\{x4 \leftarrow x2, x3 \leftarrow x3, x2 \leftarrow op_t(x1,x2), x1 \leftarrow x1\}$

$op_t(x1, rd(x1, \overbrace{op_t(\underbrace{op_l(x1, op_t(x1,x2), x3)}, x2)}))$

$=$ by Lemma 1036 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$op_t(x1, rd(x1, \underbrace{op_t(\overbrace{op_l(x1,x1,x3)}, x2)}))$

$=$ by Axiom 15 LR with $\{x4 \leftarrow x2, x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow x1\}$

$\underbrace{op_t(x1, rd(x1, \overbrace{op_l(op_t(x1,x2), x1, x3)}))}$

$=$ by Lemma 901 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{\overbrace{op_l(x1, x1, op_l(op_t(x1,x2), x1, x3))}}$

$=$ by Lemma 601 RL with $\{x2 \leftarrow op_l(op_t(x1,x2), x1, x3), x1 \leftarrow x1\}$

$\overbrace{op_r(x1, \underbrace{op_l(op_t(x1,x2), x1, x3)}, x1)}$

$=$ by Axiom 15 RL with $\{x4 \leftarrow x2, x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow x1\}$

$op_r(x1, \overbrace{\underbrace{op_t(\underbrace{op_l(x1,x1,x3)}, x2)}}, x1)$

$=$ by Lemma 427 LR with $\{x2 \leftarrow x3, x1 \leftarrow x1\}$

$op_r(\underbrace{x1}, op_t(\overbrace{mult(x3, rd(x1,x3))}, x2), x1)$

$=$ by Lemma 843 LR with $\{x2 \leftarrow x3, x1 \leftarrow x1\}$

$\underbrace{op_r(\overbrace{op_r(mult(x3, rd(x1,x3)), x3, i(x1))}, op_t(mult(x3, rd(x1,x3)), x2), x1)}$

$=$ by Axiom 17 RL with $\{x5 \leftarrow i(x1), x4 \leftarrow x3, x3 \leftarrow x1, x2 \leftarrow op_t(mult(x3, rd(x1,x3)), x2), x1 \leftarrow mult(x3, rd(x1,x3))\}$

$\overbrace{op_r(\underbrace{op_r(mult(x3, rd(x1,x3)), op_t(\overbrace{mult(x3, rd(x1,x3))}, x2), x1)}, x3, i(x1))}$

$=$ by Lemma 157 LR with $\{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow mult(x3, rd(x1,x3))\}$

$\underbrace{op_r(\overbrace{mult(x3, rd(x1,x3))}, x3, i(x1))}$

$=$ by Lemma 843 RL with $\{x2 \leftarrow x3, x1 \leftarrow x1\}$

$\overbrace{x1}$

Lemma 1125: $mult(rd(x1, op_t(i(x2), x1)), x2) = mult(x1, rd(x2, op_t(i(x2), i(x1))))$

$mult(rd(x1, \overbrace{op_t(i(x2), x1)}), x2)$

$=$ by Axiom 10 RL with $\{x2 \leftarrow i(x2), x1 \leftarrow x1\}$

$mult(\overbrace{rd(x1, \overbrace{mult(i(x1), mult(i(x2), x1))})}, x2)$

$=$ by Lemma 61 RL with $\{x3 \leftarrow x1, x2 \leftarrow mult(i(x2), x1), x1 \leftarrow x1\}$

$mult(\overbrace{rd(mult(x1, i(\overbrace{mult(i(x2), x1)})), i(x1))}, x2)$

$=$ by Lemma 64 RL with $\{x3 \leftarrow mult(i(x2), x1), x2 \leftarrow x1, x1 \leftarrow i(x1)\}$

$mult(\overbrace{rd(i(i(x1)), \overbrace{mult(i(x1), mult(i(x2), x1))})}, x2)$

$=$ by Lemma 59 RL with $\{x2 \leftarrow mult(i(x2), x1), x1 \leftarrow i(x1)\}$

$\overbrace{mult(i(\overbrace{mult(i(x1), mult(mult(i(x2), x1), i(x1)))}), x2)}$

$=$ by Lemma 17 RL with $\{x2 \leftarrow x2, x1 \leftarrow mult(i(x1), mult(mult(i(x2), x1), i(x1)))\}$

$\overbrace{i(\overbrace{mult(mult(i(x1), mult(mult(i(x2), x1), i(x1))), i(x2))})}$

$=$ by Axiom 6 RL with $\{x3 \leftarrow i(x2), x2 \leftarrow mult(i(x2), x1), x1 \leftarrow i(x1)\}$

$i(\overbrace{mult(i(x1), \overbrace{mult(mult(i(x2), x1), mult(i(x1), i(x2)))})})$

$=$ by Lemma 924 LR with $\{x2 \leftarrow x1, x1 \leftarrow i(x2)\}$

$i(mult(i(x1), \overbrace{mult(i(x2), op_t(i(x2), i(x1)))}))$

$=$ by Lemma 103 LR with $\{x2 \leftarrow x1, x1 \leftarrow i(x2)\}$

$\overbrace{i(mult(i(x1), \overbrace{rd(i(x2), op_t(i(i(x2)), x1))}))}$

$=$ by Lemma 16 LR with $\{x2 \leftarrow rd(i(x2), op_t(i(i(x2)), x1)), x1 \leftarrow x1\}$

$\overbrace{mult(x1, \overbrace{i(rd(i(x2), op_t(i(i(x2)), x1)))})}$

$=$ by Lemma 10 LR with $\{x2 \leftarrow op_t(i(i(x2)), x1), x1 \leftarrow i(x2)\}$

$mult(x1, \overbrace{rd(op_t(i(i(x2)), x1), i(x2))})$

$=$ by Lemma 67 LR with $\{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow i(x2)\}$

$mult(x1, \overbrace{rd(x2, op_t(i(x2), i(x1)))})$

Lemma 1126: $rd(op_t(x1, x2), op_t(x2, x1)) = mult(op_l(op_t(x1, x2), mult(x1, x2), op_t(x1, x2)), i(x2))$

$rd(op_t(x1, x2), \overbrace{op_t(x2, x1)})$

$=$ by Lemma 384 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{rd(op_t(x1, x2), \overbrace{mult(i(op_t(x1, x2)), mult(x1, x2))})}$

$=$ by Lemma 952 RL with $\{x2 \leftarrow mult(x1, x2), x1 \leftarrow op_t(x1, x2)\}$

$\overbrace{mult(op_l(op_t(x1, x2), mult(x1, x2), op_t(x1, x2)), \overbrace{rd(op_t(x1, x2), mult(x1, x2))})}$

$=$ by Lemma 36 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$mult(op_l(op_t(x1, x2), mult(x1, x2), op_t(x1, x2)), \overbrace{i(x2)})$

Lemma 1127: $rd(op_t(x1,x2),op_t(x2,x1)) = mult(op_l(op_t(x1,x2),x2,op_t(x1,x2)),i(x2))$

$$
\begin{aligned}
& \overbrace{rd(op_t(x1,x2),op_t(x2,x1))} \\
= \quad & \text{by Lemma 1126 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{mult(op_l(op_t(x1,x2),\underbrace{mult(x1,x2)}, op_t(x1,x2)),i(x2))} \\
= \quad & \text{by Lemma 13 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& mult(\underbrace{op_l(op_t(x1,x2),\overbrace{mult(x2,op_t(x1,x2))}, op_t(x1,x2))}, i(x2)) \\
= \quad & \text{by Lemma 285 LR with } \{x3 \leftarrow op_t(x1,x2), x2 \leftarrow x2, x1 \leftarrow op_t(x1,x2)\} \\
& mult(\overbrace{op_l(op_t(x1,x2),x2,op_t(x1,x2))}, i(x2))
\end{aligned}
$$

Lemma 1128: $mult(rd(x1,x2),op_t(x1,x2)) = rd(mult(i(x2),x1),i(mult(rd(x1,x2),rd(mult(x1,x2),x1))))$

$mult(rd(x1,x2),\underbrace{op_t(x1,x2)})$

$=$ by Axiom 2 LR with $\{x2 \leftarrow rd(x2,x1), x1 \leftarrow op_t(x1,x2)\}$

$mult(rd(x1,x2),\overbrace{mult(i(rd(x2,x1)),\underbrace{mult(rd(x2,x1),op_t(x1,x2))})})$

$=$ by Lemma 650 LR with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$mult(rd(x1,x2),mult(\underbrace{i(rd(x2,x1))},\overbrace{rd(mult(x1,x2),x1)}))$

$=$ by Lemma 10 LR with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$mult(\underbrace{rd(x1,x2)},mult(\overbrace{rd(x1,x2)},rd(mult(x1,x2),x1)))$

$=$ by Axiom 5 LR with $\{x2 \leftarrow rd(mult(x1,x2),x1), x1 \leftarrow rd(x1,x2)\}$

$\underbrace{mult(\overbrace{rd(mult(rd(x1,x2),rd(mult(x1,x2),x1)),rd(mult(x1,x2),x1))},mult(rd(x1,x2),rd(mult(x1,x2),x1)))}$

$=$ by Lemma 334 RL with $\{x2 \leftarrow rd(mult(x1,x2),x1), x1 \leftarrow mult(rd(x1,x2),rd(mult(x1,x2),x1))\}$

$\overbrace{rd(mult(rd(x1,x2),rd(mult(x1,x2),x1)),\underbrace{mult(rd(mult(x1,x2),x1),i(mult(rd(x1,x2),rd(mult(x1,x2),x1))))})}$

$=$ by Lemma 31 LR with $\{x2 \leftarrow rd(x1,x2), x1 \leftarrow rd(mult(x1,x2),x1)\}$

$\underbrace{rd(mult(rd(x1,x2),rd(mult(x1,x2),x1)),\overbrace{i(op_t(rd(x1,x2),rd(mult(x1,x2),x1)))})}$

$=$ by Lemma 38 LR with $\{x1 \leftarrow op_t(rd(x1,x2),rd(mult(x1,x2),x1)), x2 \leftarrow mult(rd(x1,x2),rd(mult(x1,x2),x1))\}$

$\overbrace{rd(\underbrace{op_t(rd(x1,x2),rd(mult(x1,x2),x1))},i(mult(rd(x1,x2),rd(mult(x1,x2),x1))))}$

$=$ by Lemma 568 RL with $\{x2 \leftarrow rd(mult(x1,x2),x1), x1 \leftarrow rd(x1,x2)\}$

$rd(\overbrace{op_t(rd(x1,x2),op_t(rd(mult(x1,x2),x1),mult(\underbrace{rd(x1,x2)},rd(mult(x1,x2),x1))))},i(mult(rd(x1,x2),rd(mult(x1,x2),x1$

$=$ by Lemma 10 RL with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$rd(op_t(rd(x1,x2),op_t(rd(mult(x1,x2),x1),mult(\overbrace{i(rd(x2,x1))},\underbrace{rd(mult(x1,x2),x1)}))),i(mult(rd(x1,x2),rd(mult(x1,x2),$

$=$ by Lemma 650 RL with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$rd(op_t(rd(x1,x2),op_t(rd(mult(x1,x2),x1),\underbrace{mult(i(rd(x2,x1)),\overbrace{mult(rd(x2,x1),op_t(x1,x2))})})),i(mult(rd(x1,x2),rd(mult$

$=$ by Axiom 2 RL with $\{x2 \leftarrow rd(x2,x1), x1 \leftarrow op_t(x1,x2)\}$

$rd(op_t(rd(x1,x2),\underbrace{op_t(rd(mult(x1,x2),x1),\overbrace{op_t(x1,x2)})}),i(mult(rd(x1,x2),rd(mult(x1,x2),x1))))$

$=$ by Lemma 400 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$rd(\underbrace{op_t(rd(x1,x2),\overbrace{x2})},i(mult(rd(x1,x2),rd(mult(x1,x2),x1))))$

$=$ by Lemma 14 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$rd(\overbrace{mult(i(x2),x1)},i(mult(rd(x1,x2),rd(mult(x1,x2),x1))))$

Lemma 1129: $op_t(mult(x1, mult(i(x1), x2)), x1) = op_t(op_l(x2, mult(x1, x1), x2), i(x1))$

$op_t(mult(x1, \underbrace{mult(i(x1), x2)}), x1)$

$=$ by Axiom 2 LR with $\{x2 \leftarrow i(x1), x1 \leftarrow mult(i(x1), x2)\}$

$op_t(mult(x1, \overbrace{mult(i(i(x1)), \underbrace{mult(i(x1), mult(i(x1), x2))}}), x1)$

$=$ by Lemma 24 RL with $\{x2 \leftarrow x2, x1 \leftarrow i(x1)\}$

$op_t(mult(x1, mult(i(i(x1)), \overbrace{mult(\underbrace{mult(i(x1), i(x1))}, x2)})), x1)$

$=$ by Axiom 11 LR with $\{x2 \leftarrow x1, x1 \leftarrow x1\}$

$op_t(mult(x1, mult(i(i(x1)), mult(\overbrace{\underbrace{i(mult(x1, x1))}}, x2))), x1)$

$=$ by Lemma 43 LR with $\{x1 \leftarrow x1\}$

$op_t(mult(x1, mult(\underbrace{i(i(x1))}, mult(\overbrace{rd(i(x1), x1)}, x2))), x1)$

$=$ by Lemma 3 LR with $\{x1 \leftarrow x1\}$

$op_t(mult(x1, mult(\overbrace{x1}, mult(\underbrace{rd(i(x1), x1)}, x2))), x1)$

$=$ by Lemma 43 RL with $\{x1 \leftarrow x1\}$

$\underbrace{op_t(mult(x1, mult(x1, mult(\overbrace{i(mult(x1, x1))}, x2))), x1)}$

$=$ by Lemma 320 RL with $\{x2 \leftarrow x1, x1 \leftarrow mult(x1, mult(i(mult(x1, x1)), x2))\}$

$\overbrace{\underbrace{op_r(mult(mult(x1, mult(i(\overbrace{\underbrace{mult(x1, x1)}}), x2)), x1), x1, i(x1))}}$

$=$ by Lemma 321 RL with $\{x2 \leftarrow x1, x1 \leftarrow mult(mult(x1, mult(i(mult(x1, x1)), x2)), x1)\}$

$\overbrace{\underbrace{op_r(mult(mult(x1, mult(i(\overbrace{\underbrace{mult(x1, x1)}}), x2)), x1), i(x1), x1)}}$

$=$ by Lemma 828 RL with $\{x2 \leftarrow x1, x1 \leftarrow mult(x1, mult(i(mult(x1, x1)), x2))\}$

$\overbrace{op_t(mult(mult(x1, mult(i(\overbrace{\underbrace{mult(x1, x1)}}), x2)), x1), \underbrace{i(op_t(mult(x1, mult(i(\overbrace{\underbrace{mult(x1, x1)}}), x2)), x1))})}$

$=$ by Lemma 410 RL with $\{x2 \leftarrow mult(i(mult(x1, x1)), x2), x1 \leftarrow x1\}$

$op_t(\underbrace{mult(mult(x1, mult(i(\overbrace{\underbrace{mult(x1, x1)}}), x2)), x1)}, \overbrace{rd(x1, mult(mult(i(\overbrace{\underbrace{mult(x1, x1)}}), x2), mult(x1, x1))))}$

$=$ by Lemma 13 RL with $\{x2 \leftarrow mult(x1, mult(i(mult(x1, x1)), x2)), x1 \leftarrow x1\}$

$op_t(\overbrace{mult(x1, \underbrace{op_t(mult(x1, mult(i(\overbrace{\underbrace{mult(x1, x1)}}), x2)), x1)})}, rd(x1, mult(mult(i(\overbrace{\underbrace{mult(x1, x1)}}), x2), mult(x1, x1))))$

$=$ by Axiom 5 LR with $\{x2 \leftarrow x1, x1 \leftarrow op_t(mult(x1, mult(i(\overbrace{\underbrace{mult(x1, x1)}}), x2)), x1)\}$

$op_t(mult(x1, \overbrace{rd(\underbrace{mult(op_t(mult(x1, mult(i(\overbrace{\underbrace{mult(x1, x1)}}), x2)), x1), x1)}, x1)}), rd(x1, mult(mult(i(\overbrace{\underbrace{mult(x1, x1)}}), x2), mult($

$=$ by Lemma 350 LR with $\{x2 \leftarrow mult(i(mult(x1, x1)), x2), x1 \leftarrow x1\}$

$\underbrace{op_t(mult(x1, rd(\overbrace{mult(mult(i(\overbrace{\underbrace{mult(x1, x1)}}), x2), mult(x1, x1))}, x1)), rd(x1, mult(mult(i(\overbrace{\underbrace{mult(x1, x1)}}), x2), mult(x1, x1)))}$

$=$ by Lemma 1006 LR with $\{x2 \leftarrow mult(mult(i(mult(x1, x1)), x2), mult(x1, x1)), x1 \leftarrow x1\}$

$\overbrace{op_t(\underbrace{mult(mult(i(\overbrace{\underbrace{mult(x1, x1)}}), x2), mult(x1, x1))}, i(x1))}$

$=$ by Lemma 744 RL with $\{x2 \leftarrow mult(x1, x1), x1 \leftarrow x2\}$

$op_t(\overbrace{op_l(x2, mult(x1, x1), x2)}, i(x1))$

Lemma 1130: $op_l(op_t(x1, x2), x1, mult(x2, x2)) = op_t(x1, i(x2))$

$op_l(op_t(x1, \underbrace{x2}), x1, mult(x2, x2))$

$=$ by Lemma 3 RL with $\{x1 \leftarrow x2\}$

$\underbrace{op_l(op_t(x1, \overbrace{i(i(x2))}), x1, mult(x2, x2))}$

$=$ by Axiom 15 RL with $\{x4 \leftarrow i(i(x2)), x3 \leftarrow mult(x2, x2), x2 \leftarrow x1, x1 \leftarrow x1\}$

$\overbrace{op_t(\underbrace{op_l(x1, x1, mult(x2, x2))}, i(i(x2)))}$

$=$ by Lemma 606 RL with $\{x2 \leftarrow mult(x2, x2), x1 \leftarrow x1\}$

$\underbrace{op_t(\overbrace{op_l(x1, mult(x2, x2), i(x1))}, i(i(x2)))}$

$=$ by Axiom 15 LR with $\{x4 \leftarrow i(i(x2)), x3 \leftarrow i(x1), x2 \leftarrow mult(x2, x2), x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_l(op_t(x1, i(i(x2))), mult(x2, x2), i(x1))}}$

$=$ by Lemma 1050 RL with $\{x3 \leftarrow x1, x2 \leftarrow mult(x2, x2), x1 \leftarrow op_t(x1, i(i(x2)))\}$

$\overbrace{op_l(op_t(x1, i(i(x2))), \underbrace{i(mult(x2, x2))}, x1)}$

$=$ by Axiom 11 RL with $\{x2 \leftarrow x2, x1 \leftarrow x2\}$

$\underbrace{op_l(op_t(x1, i(i(x2))), \overbrace{mult(i(x2), i(x2))}, x1)}$

$=$ by Axiom 15 RL with $\{x4 \leftarrow i(i(x2)), x3 \leftarrow x1, x2 \leftarrow mult(i(x2), i(x2)), x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_t(op_l(x1, mult(i(x2), i(x2)), x1), i(i(x2)))}}$

$=$ by Lemma 1129 RL with $\{x2 \leftarrow x1, x1 \leftarrow i(x2)\}$

$\overbrace{op_t(\underbrace{mult(i(x2), mult(i(i(x2)), x1))}, i(x2))}$

$=$ by Lemma 12 LR with $\{x2 \leftarrow x1, x1 \leftarrow i(x2)\}$

$op_t(\overbrace{x1}, i(x2))$

Lemma 1131: $mult(op_t(op_r(mult(i(i(x1)), x2), x1, x1), x3), x1) = mult(x1, op_t(mult(x2, i(i(x1))), x3))$

$mult(op_t(\underbrace{op_r(mult(i(i(x1)), x2), x1, x1)}, x3), x1)$

$=$ by Lemma 358 RL with $\{x2 \leftarrow x1, x1 \leftarrow mult(i(i(x1)), x2)\}$

$\underbrace{mult(op_t(\overbrace{rd(rd(mult(i(i(x1)), x2), i(x1)), x1)}, x3), x1)}$

$=$ by Lemma 1014 RL with $\{x3 \leftarrow x3, x2 \leftarrow rd(mult(i(i(x1)), x2), i(x1)), x1 \leftarrow x1\}$

$\overbrace{mult(x1, op_t(\underbrace{mult(i(x1), rd(mult(i(i(x1)), x2), i(x1)))}, x3))}$

$=$ by Lemma 84 LR with $\{x2 \leftarrow x2, x1 \leftarrow i(x1)\}$

$mult(x1, op_t(\overbrace{mult(x2, i(i(x1)))}, x3))$

Lemma 1132: $rd(mult(x1, x2), i(op_t(x1, x3))) = mult(x1, mult(x2, op_t(x1, x3)))$

$$
\begin{aligned}
& \overbrace{rd(mult(x1, x2), i(op_t(x1, x3)))} \\
= & \quad \text{by Lemma 1016 RL with } \{x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow mult(x1, x2)\} \\
& \overbrace{mult(mult(x1, x2), \underbrace{op_l(op_t(x1, x3), mult(x1, x2), x1)}_{})} \\
= & \quad \text{by Axiom 15 RL with } \{x4 \leftarrow x3, x3 \leftarrow x1, x2 \leftarrow mult(x1, x2), x1 \leftarrow x1\} \\
& mult(mult(x1, x2), \overbrace{op_t(\underbrace{op_l(x1, mult(x1, x2), x1)}_{}, x3)}) \\
= & \quad \text{by Lemma 619 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& mult(mult(x1, x2), \underbrace{op_t(\overbrace{op_l(x1, x2, x1)}, x3)}_{}) \\
= & \quad \text{by Axiom 15 LR with } \{x4 \leftarrow x3, x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{mult(mult(x1, x2), \overbrace{op_l(op_t(x1, x3), x2, x1)})}_{} \\
= & \quad \text{by Lemma 132 LR with } \{x3 \leftarrow op_t(x1, x3), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{mult(x1, mult(x2, op_t(x1, x3)))}
\end{aligned}
$$

Lemma 1133: $i(mult(x1, op_l(i(x2), x3, x4))) = mult(i(x1), op_l(x2, x3, x4))$

$$
\begin{aligned}
& i(mult(x1, \underbrace{op_l(i(x2), x3, x4)})) \\
= & \quad \text{by Lemma 1049 RL with } \{x3 \leftarrow x4, x2 \leftarrow x3, x1 \leftarrow x2\} \\
& \underbrace{i(mult(x1, \overbrace{i(op_l(x2, x3, x4))}))} \\
= & \quad \text{by Lemma 17 LR with } \{x2 \leftarrow op_l(x2, x3, x4), x1 \leftarrow x1\} \\
& \overbrace{mult(i(x1), op_l(x2, x3, x4))}
\end{aligned}
$$

Lemma 1134: $op_l(x1, x1, op_l(i(x2), x3, x4)) = op_l(x1, op_l(x2, x3, x4), x1)$

$$
\begin{aligned}
& op_l(x1, x1, \underbrace{op_l(i(x2), x3, x4)}) \\
= & \quad \text{by Lemma 1049 RL with } \{x3 \leftarrow x4, x2 \leftarrow x3, x1 \leftarrow x2\} \\
& \underbrace{op_l(x1, x1, \overbrace{i(op_l(x2, x3, x4))})} \\
= & \quad \text{by Lemma 608 LR with } \{x2 \leftarrow op_l(x2, x3, x4), x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, op_l(x2, x3, x4), x1)}
\end{aligned}
$$

Lemma 1135: $op_l(x1, mult(i(x2), x3), i(x4)) = op_l(x1, mult(x2, i(x3)), x4)$

$$
\begin{aligned}
& \underbrace{op_l(x1, mult(i(x2), x3), i(x4))} \\
= & \quad \text{by Lemma 1050 RL with } \{x3 \leftarrow x4, x2 \leftarrow mult(i(x2), x3), x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, \underbrace{i(mult(i(x2), x3))}, x4)} \\
= & \quad \text{by Lemma 16 LR with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& op_l(x1, \overbrace{mult(x2, i(x3))}, x4)
\end{aligned}
$$

Lemma 1136: $op_l(x1, op_t(i(x2), x3), i(x4)) = op_l(x1, op_t(x2, i(x3)), x4)$

$$
\begin{aligned}
& \underbrace{op_l(x1, op_t(i(x2), x3), i(x4))} \\
= \quad & \text{by Lemma 1050 RL with } \{x3 \leftarrow x4, x2 \leftarrow op_t(i(x2), x3), x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, \underbrace{i(op_t(i(x2), x3))}, x4)} \\
= \quad & \text{by Lemma 18 LR with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& op_l(x1, \overbrace{op_t(x2, i(x3))}, x4)
\end{aligned}
$$

Lemma 1137: $op_l(x1, op_t(x2, i(x3)), i(x4)) = op_l(x1, op_t(i(x2), x3), x4)$

$$
\begin{aligned}
& \underbrace{op_l(x1, op_t(x2, i(x3)), i(x4))} \\
= \quad & \text{by Lemma 1050 RL with } \{x3 \leftarrow x4, x2 \leftarrow op_t(x2, i(x3)), x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, \underbrace{i(op_t(x2, i(x3)))}, x4)} \\
= \quad & \text{by Lemma 19 LR with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& op_l(x1, \overbrace{op_t(i(x2), x3)}, x4)
\end{aligned}
$$

Lemma 1138: $op_l(x1, op_r(x2, x3, x3), x3) = op_l(x1, x3, op_r(i(mult(x3, x2)), x3, x3))$

$$
\begin{aligned}
& \underbrace{op_l(x1, op_r(x2, x3, x3), x3)} \\
= \quad & \text{by Lemma 1051 RL with } \{x3 \leftarrow op_r(x2, x3, x3), x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x3, \underbrace{i(mult(x3, op_r(x2, x3, x3)))})} \\
= \quad & \text{by Lemma 383 LR with } \{x2 \leftarrow x2, x1 \leftarrow x3\} \\
& op_l(x1, x3, \underbrace{i(\overbrace{op_r(mult(x3, x2), x3, x3)})}) \\
= \quad & \text{by Lemma 356 LR with } \{x2 \leftarrow x3, x1 \leftarrow mult(x3, x2)\} \\
& op_l(x1, x3, \overbrace{op_r(i(mult(x3, x2)), x3, x3)})
\end{aligned}
$$

Lemma 1139: $op_l(x1, op_t(rd(x2, x3), x2), x3) = op_l(x1, x3, rd(mult(x3, i(x2)), x3))$

$$
\begin{aligned}
& \underbrace{op_l(x1, op_t(rd(x2, x3), x2), x3)} \\
= \quad & \text{by Lemma 1051 RL with } \{x3 \leftarrow op_t(rd(x2, x3), x2), x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x3, \underbrace{i(mult(x3, op_t(rd(x2, x3), x2)))})} \\
= \quad & \text{by Lemma 465 LR with } \{x2 \leftarrow x2, x1 \leftarrow x3\} \\
& op_l(x1, x3, \underbrace{i(\overbrace{rd(x3, mult(x3, i(x2)))})}) \\
= \quad & \text{by Lemma 10 LR with } \{x2 \leftarrow mult(x3, i(x2)), x1 \leftarrow x3\} \\
& op_l(x1, x3, \overbrace{rd(mult(x3, i(x2)), x3)})
\end{aligned}
$$

Lemma 1140: $op_l(x1, rd(x2, mult(x2, x3)), x2) = op_l(x1, x2, op_r(x3, x2, x2))$

$$
\begin{aligned}
& \overbrace{op_l(x1, rd(x2, mult(x2, x3)), x2)} \\
= \quad & \text{by Lemma 1051 RL with } \{x3 \leftarrow rd(x2, mult(x2, x3)), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x2, \underbrace{i(\underbrace{mult(x2, \underbrace{rd(x2, mult(x2, x3))}_{x2})}_{x3})}_{x2})}^{x3} \\
= \quad & \text{by Lemma 807 RL with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& op_l(x1, x2, \underbrace{i(\overbrace{op_r(rd(x2, x3), x2, x2)}^{x3})}_{x2}) \\
= \quad & \text{by Lemma 356 LR with } \{x2 \leftarrow x2, x1 \leftarrow rd(x2, x3)\} \\
& \underbrace{op_l(x1, x2, \overbrace{op_r(i(rd(x2, x3)), x2, x2)}^{x3})}_{x2} \\
= \quad & \text{by Lemma 907 LR with } \{x3 \leftarrow rd(x2, x3), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x2, \underbrace{rd(x2, rd(x2, x3))}_{x3})}^{x2} \\
= \quad & \text{by Lemma 353 LR with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& op_l(x1, x2, \overbrace{op_r(x3, x2, x2)}^{x3})
\end{aligned}
$$

Lemma 1141: $op_l(x1, x2, i(i(mult(op_t(i(x2), x3), x3)))) = op_l(x1, rd(i(x2), mult(i(x2), x3)), x2)$

$$
\begin{aligned}
& \underbrace{op_l(x1, x2, i(i(mult(op_t(i(x2), x3), x3))))}_{x2} \\
= \quad & \text{by Lemma 1054 RL with } \{x3 \leftarrow i(mult(op_t(i(x2), x3), x3)), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, mult(i(x2), \underbrace{i(mult(op_t(i(x2), x3), x3))}_{x3}), x2)}^{x3} \\
= \quad & \text{by Lemma 408 RL with } \{x2 \leftarrow x3, x1 \leftarrow i(x2)\} \\
& op_l(x1, \underbrace{mult(i(x2), \overbrace{op_t(rd(i(i(x2)), x3), i(x3))}^{x3})}_{x2}, x2) \\
= \quad & \text{by Lemma 812 RL with } \{x2 \leftarrow x3, x1 \leftarrow i(x2)\} \\
& op_l(x1, \overbrace{rd(i(x2), mult(i(x2), x3))}^{x3}, x2)
\end{aligned}
$$

Lemma 1142: $op_l(x1, x2, op_t(rd(x3, x2), x3)) = op_l(x1, rd(i(x2), mult(i(x2), x3)), x2)$

$$
\begin{aligned}
& op_l(x1, x2, \underbrace{op_t(rd(x3, x2), x3)}_{x2}) \\
= \quad & \text{by Lemma 340 RL with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& op_l(x1, x2, \underbrace{\overbrace{mult(op_t(i(x2), x3), x3)}^{x3}}_{x2}) \\
= \quad & \text{by Lemma 3 RL with } \{x1 \leftarrow mult(op_t(i(x2), x3), x3)\} \\
& \underbrace{op_l(x1, x2, \overbrace{i(i(mult(op_t(i(x2), x3), x3))))}^{x3})}_{x2} \\
= \quad & \text{by Lemma 1141 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, rd(i(x2), mult(i(x2), x3)), x2)}^{x3}
\end{aligned}
$$

Lemma 1143: $op_l(x1, x2, op_t(rd(x3, x2), x3)) = op_l(x1, mult(x2, i(x3)), x2)$

$$
\begin{aligned}
& \underbrace{op_l(x1, x2, op_t(rd(x3, x2), x3))} \\
= \quad & \text{by Lemma 1142 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, \underbrace{rd(i(x2), mult(i(x2), x3))}, x2)} \\
= \quad & \text{by Lemma 64 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x2\} \\
& \underbrace{op_l(x1, \overbrace{rd(mult(x2, i(x3)), x2)}, x2)} \\
= \quad & \text{by Lemma 292 LR with } \{x3 \leftarrow x2, x2 \leftarrow mult(x2, i(x3)), x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, \underbrace{mult(x2, i(x3))}, x2)}
\end{aligned}
$$

Lemma 1144: $op_l(x1, x2, rd(x3, i(x2))) = op_l(x1, rd(x2, mult(x2, x3)), x2)$

$$
\begin{aligned}
& \underbrace{op_l(x1, x2, rd(x3, i(x2)))} \\
= \quad & \text{by Lemma 1079 RL with } \{x3 \leftarrow rd(x3, i(x2)), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, rd(x2, \underbrace{op_t(rd(x3, i(x2)), i(x2))}), x2)} \\
= \quad & \text{by Lemma 14 LR with } \{x2 \leftarrow i(x2), x1 \leftarrow x3\} \\
& op_l(x1, rd(x2, \overbrace{mult(\underbrace{i(i(x2))}, x3)}), x2) \\
= \quad & \text{by Lemma 3 LR with } \{x1 \leftarrow x2\} \\
& op_l(x1, rd(x2, mult(\overbrace{\ x2\ }, x3)), x2)
\end{aligned}
$$

Lemma 1145: $op_t(rd(mult(x2, x2), x1), i(i(x2))) = mult(i(x2), i(rd(rd(rd(x1, x2), x2), x2)))$

$$
\begin{aligned}
& \underbrace{op_t(rd(mult(x2, x2), x1), i(i(x2)))} \\
= \quad & \text{by Lemma 46 RL with } \{x3 \leftarrow i(x2), x2 \leftarrow mult(x2, x2), x1 \leftarrow x1\} \\
& \overbrace{\underbrace{i(op_t(rd(x1, mult(x2, x2)), i(x2)))}} \\
= \quad & \text{by Lemma 31 RL with } \{x2 \leftarrow rd(x1, mult(x2, x2)), x1 \leftarrow i(x2)\} \\
& \overbrace{mult(i(x2), \underbrace{i(mult(rd(x1, mult(x2, x2)), i(x2)))})} \\
= \quad & \text{by Lemma 1111 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& mult(i(x2), i(\overbrace{rd(rd(rd(x1, x2), x2), x2)}))
\end{aligned}
$$

Lemma 1146: $op_t(x1, mult(mult(i(x1), x2), x3)) = op_t(x1, mult(rd(x2, x1), x3))$

$$
\begin{aligned}
& op_t(x1, mult(\underbrace{mult(i(x1), x2)}, x3)) \\
= \quad & \text{by Lemma 14 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \underbrace{op_t(x1, mult(\overbrace{op_t(rd(x2, x1), x1)}, x3))} \\
= \quad & \text{by Lemma 1118 LR with } \{x3 \leftarrow x3, x2 \leftarrow rd(x2, x1), x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, mult(rd(x2, x1), x3))}
\end{aligned}
$$

Lemma 1147: $op_t(x1, mult(op_r(x2, x1, x1), x3)) = op_t(x1, mult(op_l(x2, x2, i(x1)), x3))$

$$
\begin{aligned}
& op_t(x1, mult(\overbrace{op_r(x2, x1, x1)}, x3)) \\
= & \quad \text{by Lemma 358 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \underbrace{op_t(x1, mult(\overbrace{rd(rd(x2, i(x1)), x1)}, x3))} \\
= & \quad \text{by Lemma 1146 RL with } \{x3 \leftarrow x3, x2 \leftarrow rd(x2, i(x1)), x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, mult(\underbrace{mult(i(x1), rd(x2, i(x1)))}, x3))} \\
= & \quad \text{by Lemma 427 RL with } \{x2 \leftarrow i(x1), x1 \leftarrow x2\} \\
& op_t(x1, mult(\overbrace{op_l(x2, x2, i(x1))}, x3))
\end{aligned}
$$

Lemma 1148: $op_t(x1, mult(op_r(x2, x1, x1), x3)) = op_t(x1, mult(x2, x3))$

$$
\begin{aligned}
& \underbrace{op_t(x1, mult(op_r(x2, x1, x1), x3))} \\
= & \quad \text{by Lemma 1147 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, mult(\underbrace{op_l(x2, x2, i(x1))}, x3))} \\
= & \quad \text{by Lemma 608 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& op_t(x1, \underbrace{mult(\overbrace{op_l(x2, x1, x2)}, x3))} \\
= & \quad \text{by Lemma 959 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{op_t(x1, \overbrace{mult(asoc(x1, x2, x2), mult(x2, x3))})} \\
= & \quad \text{by Lemma 1115 LR with } \{x3 \leftarrow mult(x2, x3), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, mult(x2, x3))}
\end{aligned}
$$

Lemma 1149: $mult(i(x1), op_t(mult(x1, x2), x3)) = rd(op_t(mult(x2, x1), x3), x1)$

$\overbrace{mult(i(x1), op_t(mult(x1, x2), x3))}$

$=$ by Lemma 17 RL with $\{x2 \leftarrow op_t(mult(x1, x2), x3), x1 \leftarrow x1\}$

$\overbrace{i(\underbrace{mult(x1, i(\overbrace{op_t(mult(x1, x2), x3)}))})}$

$=$ by Lemma 85 RL with $\{x2 \leftarrow i(op_t(mult(x1, x2), x3)), x1 \leftarrow x1\}$

$\overbrace{rd(mult(i(x1), rd(x1, \underbrace{i(op_t(mult(x1, x2), x3))})), x1)}$

$=$ by Lemma 15 RL with $\{x2 \leftarrow x3, x1 \leftarrow mult(x1, x2)\}$

$rd(mult(i(x1), rd(x1, \overbrace{op_t(\underbrace{i(mult(x1, x2))}, i(x3))})), x1)$

$=$ by Axiom 11 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$rd(mult(i(x1), rd(x1, op_t(\overbrace{\underbrace{mult(i(x1), i(x2))}, i(x3)}))), x1)$

$=$ by Lemma 16 RL with $\{x2 \leftarrow x2, x1 \leftarrow i(x1)\}$

$rd(mult(i(x1), rd(x1, \underbrace{op_t(\overbrace{i(mult(i(\underbrace{i(x1)}), x2))}, i(x3))})), x1)$

$=$ by Lemma 15 LR with $\{x2 \leftarrow x3, x1 \leftarrow mult(i(i(x1)), x2)\}$

$rd(mult(i(x1), \underbrace{rd(x1, \overbrace{i(op_t(mult(i(\underbrace{i(x1)}), x2), x3))})}), x1)$

$=$ by Lemma 357 RL with $\{x2 \leftarrow x1, x1 \leftarrow op_t(mult(i(i(x1)), x2), x3)\}$

$rd(mult(i(x1), \overbrace{mult(\underbrace{op_r(op_t(mult(i(\underbrace{i(x1)}), x2), x3), x1, x1)}, x1)}), x1)$

$=$ by Axiom 14 RL with $\{x4 \leftarrow x3, x3 \leftarrow x1, x2 \leftarrow x1, x1 \leftarrow mult(i(i(x1)), x2)\}$

$rd(mult(i(x1), \underbrace{mult(\overbrace{op_t(op_r(mult(i(\underbrace{i(x1)}), x2), x1, x1), x3)}, x1)}), x1)$

$=$ by Lemma 1131 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$rd(mult(i(x1), \overbrace{mult(x1, op_t(mult(x2, \underbrace{i(i(x1))}), x3))}), x1)$

$=$ by Lemma 3 LR with $\{x1 \leftarrow x1\}$

$rd(\underbrace{mult(i(x1), mult(x1, op_t(mult(x2, \overbrace{x1}), x3)))}, x1)$

$=$ by Axiom 2 RL with $\{x2 \leftarrow x1, x1 \leftarrow op_t(mult(x2, x1), x3)\}$

$rd(\overbrace{op_t(mult(x2, x1), x3)}, x1)$

Lemma 1150: $mult(x1, i(op_t(mult(x1, x2), x3))) = rd(x1, op_t(mult(x2, x1), x3))$

$\overbrace{mult(x1, i(op_t(mult(x1, x2), x3)))}$

$=$ by Lemma 16 RL with $\{x2 \leftarrow op_t(mult(x1, x2), x3), x1 \leftarrow x1\}$

$\overbrace{i(\underbrace{mult(i(x1), op_t(mult(x1, x2), x3))})}$

$=$ by Lemma 1149 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{i(\overbrace{rd(op_t(mult(x2, x1), x3), x1)})}$

$=$ by Lemma 10 LR with $\{x2 \leftarrow x1, x1 \leftarrow op_t(mult(x2, x1), x3)\}$

$\overbrace{rd(x1, op_t(mult(x2, x1), x3))}$

Lemma 1151: $op_t(x1, mult(rd(x2, i(x1)), x3)) = op_t(x1, mult(mult(x1, x2), x3))$

$$
\begin{aligned}
& \underbrace{op_t(x1, mult(rd(x2, i(x1)), x3))} \\
= & \quad \text{by Lemma 1116 RL with } \{x3 \leftarrow mult(rd(x2, i(x1)), x3), x2 \leftarrow rd(x2, i(x1)), x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, \underbrace{mult(asoc(rd(x2, i(x1)), x1, rd(x2, i(x1))), mult(rd(x2, i(x1)), x3))})} \\
= & \quad \text{by Lemma 957 LR with } \{x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow rd(x2, i(x1))\} \\
& op_t(x1, \overbrace{mult(\underbrace{op_l(rd(x2, i(x1)), rd(x2, i(x1)), x1)}, x3)}) \\
= & \quad \text{by Lemma 684 LR with } \{x2 \leftarrow x1, x1 \leftarrow rd(x2, i(x1))\} \\
& op_t(x1, mult(\overbrace{rd(rd(rd(x2, i(x1)), op_t(i(x1), rd(x2, i(x1)))), x1)}, x3)) \\
= & \quad \text{by Lemma 364 RL with } \{x2 \leftarrow x1, x1 \leftarrow rd(rd(x2, i(x1)), op_t(i(x1), rd(x2, i(x1))))\} \\
& \underbrace{op_t(x1, mult(\overbrace{op_r(mult(rd(rd(x2, i(x1)), op_t(i(x1), rd(x2, i(x1)))), i(x1)), x1, x1)}, x3))} \\
= & \quad \text{by Lemma 1148 LR with } \{x3 \leftarrow x3, x2 \leftarrow mult(rd(rd(x2, i(x1)), op_t(i(x1), rd(x2, i(x1)))), i(x1)), x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, mult(\underbrace{mult(rd(rd(x2, i(x1)), op_t(i(x1), rd(x2, i(x1)))), i(x1))}, x3))} \\
= & \quad \text{by Lemma 597 LR with } \{x2 \leftarrow i(x1), x1 \leftarrow rd(x2, i(x1))\} \\
& op_t(x1, mult(\overbrace{op_t(rd(x2, i(x1)), i(x1))}, x3)) \\
= & \quad \text{by Lemma 14 LR with } \{x2 \leftarrow i(x1), x1 \leftarrow x2\} \\
& op_t(x1, mult(\overbrace{mult(\underbrace{i(i(x1))}, x2)}, x3)) \\
= & \quad \text{by Lemma 3 LR with } \{x1 \leftarrow x1\} \\
& op_t(x1, mult(mult(\overbrace{x1}, x2), x3))
\end{aligned}
$$

Lemma 1152: $mult(x1, mult(rd(x2, mult(x1, x3)), x1)) = rd(mult(x1, x2), x3)$

$$
\begin{aligned}
& \underbrace{mult(x1, mult(rd(x2, mult(x1, x3)), x1))} \\
= & \quad \text{by Lemma 76 RL with } \{x3 \leftarrow x3, x2 \leftarrow rd(x2, mult(x1, x3)), x1 \leftarrow x1\} \\
& \overbrace{rd(mult(x1, \underbrace{mult(rd(x2, mult(x1, x3)), mult(x1, x3))}), x3)} \\
= & \quad \text{by Axiom 4 RL with } \{x2 \leftarrow mult(x1, x3), x1 \leftarrow x2\} \\
& rd(mult(x1, \overbrace{x2}), x3)
\end{aligned}
$$

Lemma 1153: $mult(i(x1), rd(mult(x1, x2), x3)) = mult(rd(x2, mult(x1, x3)), x1)$

$$
\begin{aligned}
& mult(i(x1), \underbrace{rd(mult(x1, x2), x3)}) \\
= & \quad \text{by Lemma 1152 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{mult(i(x1), \overbrace{mult(x1, mult(rd(x2, mult(x1, x3)), x1))})} \\
= & \quad \text{by Axiom 2 RL with } \{x2 \leftarrow x1, x1 \leftarrow mult(rd(x2, mult(x1, x3)), x1)\} \\
& \overbrace{mult(rd(x2, mult(x1, x3)), x1)}
\end{aligned}
$$

Lemma 1154: $mult(rd(mult(x2,x3),x1),i(x2)) = mult(x2,i(rd(mult(x2,x1),x3)))$

$$
\begin{aligned}
& \underbrace{mult(rd(mult(x2,x3),x1),i(x2))} \\
= \quad & \text{by Lemma 44 RL with } \{x3 \leftarrow x2, x2 \leftarrow mult(x2,x3), x1 \leftarrow x1\} \\
& \overbrace{\underbrace{i(mult(rd(x1,mult(x2,x3)),x2))}} \\
= \quad & \text{by Lemma 20 RL with } \{x2 \leftarrow mult(rd(x1,mult(x2,x3)),x2), x1 \leftarrow x2\} \\
& \overbrace{mult(x2,\underbrace{i(mult(x2,mult(rd(x1,mult(x2,x3)),x2)))})} \\
= \quad & \text{by Lemma 1152 LR with } \{x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow x2\} \\
& mult(x2,i(\overbrace{rd(mult(x2,x1),x3)}))
\end{aligned}
$$

Lemma 1155: $mult(rd(mult(x1,x2),x3),i(x1)) = mult(x1,rd(x2,mult(x1,x3)))$

$$
\begin{aligned}
& \underbrace{mult(rd(mult(x1,x2),x3),i(x1))} \\
= \quad & \text{by Lemma 1154 LR with } \{x1 \leftarrow x3, x3 \leftarrow x2, x2 \leftarrow x1\} \\
& \overbrace{mult(x1,\underbrace{i(rd(mult(x1,x3),x2))})} \\
= \quad & \text{by Lemma 10 LR with } \{x2 \leftarrow x2, x1 \leftarrow mult(x1,x3)\} \\
& mult(x1,\overbrace{rd(x2,mult(x1,x3))})
\end{aligned}
$$

Lemma 1156: $rd(x1,mult(x1,x2)) = op_r(mult(i(x1),rd(x1,x2)),x1,x1)$

$$
\begin{aligned}
& rd(x1,\underbrace{mult(x1,x2)}) \\
= \quad & \text{by Lemma 3 RL with } \{x1 \leftarrow mult(x1,x2)\} \\
& \underbrace{rd(x1,\overbrace{i(i(mult(x1,x2)))})} \\
= \quad & \text{by Lemma 359 RL with } \{x2 \leftarrow x1, x1 \leftarrow i(mult(x1,x2))\} \\
& \overbrace{op_r(\underbrace{mult(i(mult(x1,x2)),x1)},x1,x1)} \\
= \quad & \text{by Lemma 56 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_r(\overbrace{mult(i(x1),rd(x1,x2))},x1,x1)
\end{aligned}
$$

Lemma 1157: $mult(op_t(x1,op_t(x2,op_t(x1,x3))),x3) = mult(op_t(x1,x2),x3)$

$$
\begin{aligned}
& \underbrace{mult(op_t(x1,op_t(x2,op_t(x1,x3))),x3)} \\
= \quad & \text{by Lemma 50 RL with } \{x3 \leftarrow op_t(x2,op_t(x1,x3)), x2 \leftarrow x1, x1 \leftarrow x3\} \\
& \overbrace{mult(x3,\underbrace{op_t(op_t(x1,x3),op_t(x2,op_t(x1,x3)))})} \\
= \quad & \text{by Lemma 374 LR with } \{x2 \leftarrow x2, x1 \leftarrow op_t(x1,x3)\} \\
& \underbrace{mult(x3,\overbrace{op_t(op_t(x1,x3),x2)})} \\
= \quad & \text{by Lemma 50 LR with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x3\} \\
& \overbrace{mult(op_t(x1,x2),x3)}
\end{aligned}
$$

Lemma 1158: $op_t(x1, rd(x2, op_t(x1, x3))) = op_t(x1, rd(x2, x1))$

$$
\begin{aligned}
& \underbrace{op_t(x1, rd(x2, op_t(x1, x3)))} \\
= \quad & \text{by Axiom 5 LR with } \{x2 \leftarrow x3, x1 \leftarrow op_t(x1, rd(x2, op_t(x1, x3)))\} \\
& \overbrace{rd(\underbrace{mult(op_t(x1, rd(x2, op_t(x1, x3))), x3)}, x3)} \\
= \quad & \text{by Lemma 1157 RL with } \{x3 \leftarrow x3, x2 \leftarrow rd(x2, op_t(x1, x3)), x1 \leftarrow x1\} \\
& \underbrace{rd(\overbrace{mult(op_t(x1, op_t(rd(x2, op_t(x1, x3)), op_t(x1, x3))), x3)}, x3)} \\
= \quad & \text{by Axiom 5 RL with } \{x2 \leftarrow x3, x1 \leftarrow op_t(x1, op_t(rd(x2, op_t(x1, x3)), op_t(x1, x3)))\} \\
& \overbrace{op_t(x1, \underbrace{op_t(rd(x2, op_t(x1, x3)), op_t(x1, x3))})} \\
= \quad & \text{by Lemma 14 LR with } \{x2 \leftarrow op_t(x1, x3), x1 \leftarrow x2\} \\
& \underbrace{op_t(x1, \overbrace{mult(i(op_t(x1, x3)), x2)})} \\
= \quad & \text{by Lemma 1121 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, rd(x2, x1))}
\end{aligned}
$$

Lemma 1159: $op_t(x1, rd(op_t(i(x1), x3), x2)) = i(op_t(i(x1), rd(x2, i(x1))))$

$$
\begin{aligned}
& op_t(x1, \underbrace{rd(op_t(i(x1), x3), x2)}) \\
= \quad & \text{by Lemma 10 RL with } \{x2 \leftarrow op_t(i(x1), x3), x1 \leftarrow x2\} \\
& \underbrace{op_t(x1, \overbrace{i(rd(x2, op_t(i(x1), x3)))})} \\
= \quad & \text{by Lemma 18 RL with } \{x2 \leftarrow rd(x2, op_t(i(x1), x3)), x1 \leftarrow x1\} \\
& \overbrace{i(\underbrace{op_t(i(x1), rd(x2, op_t(i(x1), x3)))})} \\
= \quad & \text{by Lemma 1158 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow i(x1)\} \\
& i(\overbrace{op_t(i(x1), rd(x2, i(x1)))})
\end{aligned}
$$

Lemma 1160: $op_t(x1, rd(op_t(i(x1), x3), x2)) = op_t(x1, rd(i(x1), x2))$

$$
\begin{aligned}
& \underbrace{op_t(x1, rd(op_t(i(x1), x3), x2))} \\
= \quad & \text{by Lemma 1159 LR with } \{x2 \leftarrow x2, x3 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{i(op_t(i(x1), rd(x2, i(x1))))}} \\
= \quad & \text{by Lemma 18 LR with } \{x2 \leftarrow rd(x2, i(x1)), x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, \underbrace{i(rd(x2, i(x1)))})} \\
= \quad & \text{by Lemma 10 LR with } \{x2 \leftarrow i(x1), x1 \leftarrow x2\} \\
& op_t(x1, \overbrace{rd(i(x1), x2)})
\end{aligned}
$$

Lemma 1161: $op_t(x1, rd(op_t(i(x1), x2), x3)) = op_t(x1, rd(x3, x1))$

$$
\begin{aligned}
& \underbrace{op_t(x1, rd(op_t(i(x1), x2), x3))} \\
= \quad & \text{by Lemma 1160 LR with } \{x2 \leftarrow x3, x3 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_t(x1, rd(i(x1), x3))}} \\
= \quad & \text{by Lemma 707 LR with } \{x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, rd(x3, x1))}
\end{aligned}
$$

Lemma 1162: $op_t(x1, rd(mult(x2, op_t(x1, x3)), x1)) = op_t(x1, x2)$

$$
\begin{aligned}
& \underbrace{op_t(x1, rd(mult(x2, op_t(x1, x3)), x1))} \\
= & \quad \text{by Lemma 1158 RL with } \{x3 \leftarrow x3, x2 \leftarrow mult(x2, op_t(x1, x3)), x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, \underbrace{rd(mult(x2, op_t(x1, x3)), op_t(x1, x3))})} \\
= & \quad \text{by Axiom 5 RL with } \{x2 \leftarrow op_t(x1, x3), x1 \leftarrow x2\} \\
& op_t(x1, \overbrace{x2}^{})
\end{aligned}
$$

Lemma 1163: $op_t(x1, rd(x1, rd(x1, mult(x2, op_t(x1, x3))))) = rd(op_t(x1, x2), asoc(x1, rd(mult(x2, op_t(x1, x3)), x1), x1))$

$$
\begin{aligned}
& \underbrace{op_t(x1, rd(x1, rd(x1, mult(x2, op_t(x1, x3)))))} \\
= & \quad \text{by Lemma 737 RL with } \{x3 \leftarrow x1, x2 \leftarrow mult(x2, op_t(x1, x3)), x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_t(x1, mult(rd(mult(x2, op_t(x1, x3)), x1), x1))}} \\
= & \quad \text{by Lemma 879 RL with } \{x2 \leftarrow rd(mult(x2, op_t(x1, x3)), x1), x1 \leftarrow x1\} \\
& \overbrace{rd(\underbrace{op_t(x1, rd(mult(x2, op_t(x1, x3)), x1))}, asoc(x1, rd(mult(x2, op_t(x1, x3)), x1), x1))} \\
= & \quad \text{by Lemma 1162 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& rd(\overbrace{op_t(x1, x2)}, asoc(x1, rd(mult(x2, op_t(x1, x3)), x1), x1))
\end{aligned}
$$

Lemma 1164: $op_t(x1, mult(x2, op_t(x1, x3))) = rd(op_t(x1, x2), asoc(x1, rd(mult(x2, op_t(x1, x3)), x1), x1))$

$$
\begin{aligned}
& \underbrace{op_t(x1, mult(x2, op_t(x1, x3)))} \\
= & \quad \text{by Lemma 550 RL with } \{x2 \leftarrow mult(x2, op_t(x1, x3)), x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, \underbrace{op_r(mult(x2, op_t(x1, x3)), x1, x1)})} \\
= & \quad \text{by Lemma 353 RL with } \{x2 \leftarrow mult(x2, op_t(x1, x3)), x1 \leftarrow x1\} \\
& \underbrace{op_t(x1, \overbrace{rd(x1, rd(x1, mult(x2, op_t(x1, x3))))})} \\
= & \quad \text{by Lemma 1163 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{rd(op_t(x1, x2), asoc(x1, rd(mult(x2, op_t(x1, x3)), x1), x1))}
\end{aligned}
$$

Lemma 1165: $op_t(x1, mult(x2, op_t(x1, x3))) = op_t(x1, mult(x2, x1))$

$\underbrace{op_t(x1, mult(x2, op_t(x1, x3)))}$

$=$ by Lemma 1164 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{rd(op_t(x1, x2), \underbrace{asoc(x1, rd(mult(x2, op_t(x1, x3)), x1), x1)})}$

$=$ by Lemma 263 LR with $\{x2 \leftarrow mult(x2, op_t(x1, x3)), x1 \leftarrow x1\}$

$rd(op_t(x1, x2), \overbrace{\underbrace{asoc(x1, mult(x2, op_t(x1, x3)), x1)}})$

$=$ by Lemma 617 RL with $\{x2 \leftarrow mult(x2, op_t(x1, x3)), x1 \leftarrow x1\}$

$rd(op_t(x1, x2), \overbrace{\underbrace{rd(x1, op_l(x1, mult(x2, op_t(x1, x3)), x1))}})$

$=$ by Lemma 176 LR with $\{x3 \leftarrow x1, x2 \leftarrow mult(x2, op_t(x1, x3)), x1 \leftarrow x1\}$

$rd(op_t(x1, x2), \overbrace{mult(x1, i(\underbrace{op_l(x1, mult(x2, op_t(x1, x3)), x1)}))})$

$=$ by Lemma 1025 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$rd(op_t(x1, x2), \underbrace{mult(x1, i(\overbrace{op_l(x1, x2, x1)}))})$

$=$ by Lemma 176 RL with $\{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x1\}$

$rd(op_t(x1, x2), \overbrace{\underbrace{rd(x1, op_l(x1, x2, x1))}})$

$=$ by Lemma 617 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{rd(op_t(x1, x2), \overbrace{asoc(x1, x2, x1)})}$

$=$ by Lemma 879 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_t(x1, mult(x2, x1))}$

Lemma 1166: $op_t(x1, rd(x3, i(op_t(x1, x2)))) = op_t(x1, mult(x3, x1))$

$\underbrace{op_t(x1, rd(x3, i(op_t(x1, x2))))}$

$=$ by Lemma 1123 RL with $\{x3 \leftarrow rd(x3, i(op_t(x1, x2))), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_t(x1, \underbrace{rd(mult(op_t(x1, x2), rd(x3, i(op_t(x1, x2)))), x1)})}$

$=$ by Lemma 1120 RL with $\{x3 \leftarrow mult(op_t(x1, x2), rd(x3, i(op_t(x1, x2)))), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_t(x1, \underbrace{i(mult(op_t(x1, x2), mult(op_t(x1, x2), rd(x3, i(op_t(x1, x2))))))})$

$=$ by Axiom 4 LR with $\{x2 \leftarrow op_t(x1, x2), x1 \leftarrow i(mult(op_t(x1, x2), mult(op_t(x1, x2), rd(x3, i(op_t(x1, x2))))))\}$

$\underbrace{op_t(x1, \overbrace{mult(rd(i(mult(op_t(x1, x2), mult(op_t(x1, x2), rd(x3, i(op_t(x1, x2)))))), op_t(x1, x2)), op_t(x1, x2)})}$

$=$ by Lemma 1165 LR with $\{x3 \leftarrow x2, x2 \leftarrow rd(i(mult(op_t(x1, x2), mult(op_t(x1, x2), rd(x3, i(op_t(x1, x2)))))), op_t(x1, x2)), x1 \leftarrow$

$\overbrace{op_t(x1, \underbrace{mult(rd(i(mult(op_t(x1, x2), mult(op_t(x1, x2), rd(x3, i(op_t(x1, x2)))))), op_t(x1, x2)), x1)})}$

$=$ by Lemma 737 LR with $\{x3 \leftarrow op_t(x1, x2), x2 \leftarrow i(mult(op_t(x1, x2), mult(op_t(x1, x2), rd(x3, i(op_t(x1, x2))))))), x1 \leftarrow x1\}$

$\overbrace{op_t(x1, rd(x1, \underbrace{rd(op_t(x1, x2), i(mult(op_t(x1, x2), mult(op_t(x1, x2), rd(x3, i(op_t(x1, x2))))))))})}$

$=$ by Lemma 58 LR with $\{x2 \leftarrow mult(op_t(x1, x2), rd(x3, i(op_t(x1, x2)))), x1 \leftarrow op_t(x1, x2)\}$

$\underbrace{op_t(x1, rd(x1, \overbrace{mult(op_t(x1, x2), mult(mult(op_t(x1, x2), rd(x3, i(op_t(x1, x2)))), op_t(x1, x2)))}))}$

$=$ by Lemma 1119 LR with $\{x3 \leftarrow mult(mult(op_t(x1, x2), rd(x3, i(op_t(x1, x2)))), op_t(x1, x2)), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_t(x1, \underbrace{i(mult(mult(op_t(x1, x2), rd(x3, i(op_t(x1, x2)))), op_t(x1, x2)))})$

$=$ by Lemma 1109 LR with $\{x2 \leftarrow x3, x1 \leftarrow op_t(x1, x2)\}$

$\underbrace{op_t(x1, i(\overbrace{mult(op_t(x1, x2), mult(mult(x3, op_t(x1, x2)), op_t(x1, x2)))}))}$

$=$ by Lemma 1120 LR with $\{x3 \leftarrow mult(mult(x3, op_t(x1, x2)), op_t(x1, x2)), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_t(x1, \underbrace{rd(mult(mult(x3, op_t(x1, x2)), op_t(x1, x2)), x1)})}$

$=$ by Lemma 1162 LR with $\{x3 \leftarrow x2, x2 \leftarrow mult(x3, op_t(x1, x2)), x1 \leftarrow x1\}$

$\overbrace{op_t(x1, \underbrace{mult(x3, op_t(x1, x2))})}$

$=$ by Lemma 1165 LR with $\{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x1\}$

$\overbrace{op_t(x1, \underbrace{mult(x3, x1)})}$

Lemma 1167: $op_t(x1, rd(x1, mult(op_t(x1, x2), x3))) = op_t(x1, mult(i(op_t(x1, x2)), rd(op_t(x1, x2), x3)))$

$\underbrace{op_t(x1, rd(x1, mult(op_t(x1, x2), x3)))}$

$=$ by Lemma 554 RL with $\{x2 \leftarrow mult(op_t(x1, x2), x3), x1 \leftarrow x1\}$

$\overbrace{op_t(x1, \underbrace{mult(i(mult(op_t(x1, x2), x3)), x1)})}$

$=$ by Lemma 1165 RL with $\{x3 \leftarrow x2, x2 \leftarrow i(mult(op_t(x1, x2), x3)), x1 \leftarrow x1\}$

$\overbrace{op_t(x1, \underbrace{mult(i(mult(op_t(x1, x2), x3)), op_t(x1, x2))})}$

$=$ by Lemma 56 LR with $\{x2 \leftarrow x3, x1 \leftarrow op_t(x1, x2)\}$

$op_t(x1, \overbrace{mult(i(op_t(x1, x2)), rd(op_t(x1, x2), x3))})$

Lemma 1168: $mult(op_t(x2, x1), x3) = mult(rd(x1, op_t(x1, x2)), mult(x2, x3))$

$$
\begin{aligned}
& mult(\underbrace{op_t(x2, x1)}, x3) \\
= \quad & \text{by Lemma 388 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{mult(\overbrace{mult(rd(x1, op_t(x1, x2)), x2)}, x3)} \\
= \quad & \text{by Lemma 137 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow rd(x1, op_t(x1, x2))\} \\
& \overbrace{mult(rd(x1, op_t(x1, x2)), mult(x2, \underbrace{op_l(x3, rd(x1, op_t(x1, x2)), x2)}))} \\
= \quad & \text{by Lemma 392 LR with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x3\} \\
& mult(rd(x1, op_t(x1, x2)), mult(x2, \overbrace{x3}))
\end{aligned}
$$

Lemma 1169: $op_t(op_r(x1,x2,x2),mult(x1,x2)) = mult(i(mult(x1,mult(x2,x2))),mult(mult(x1,x2),mult(x1,x2)))$

$op_t(\underbrace{op_r(x1,x2,x2)},mult(x1,x2))$

$=$ by Lemma 520 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$op_t(\overbrace{rd(\underbrace{op_t(op_r(mult(x2,x1),x2,x2),x2)},op_l(x2,x2,x1))},mult(x1,x2))$

$=$ by Axiom 14 LR with $\{x4 \leftarrow x2, x3 \leftarrow x2, x2 \leftarrow x2, x1 \leftarrow mult(x2,x1)\}$

$op_t(rd(\overbrace{op_r(op_t(mult(x2,x1),x2),x2,x2)},op_l(x2,x2,x1)),mult(x1,x2))$

$=$ by Lemma 385 LR with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$op_t(rd(\overbrace{mult(x1,x2)},\underbrace{op_l(x2,x2,x1)}),mult(x1,x2))$

$=$ by Lemma 427 LR with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$op_t(rd(mult(x1,x2),\overbrace{mult(x1,rd(x2,x1))}),\underbrace{mult(x1,x2)})$

$=$ by Lemma 3 RL with $\{x1 \leftarrow mult(x1,x2)\}$

$op_t(\underbrace{rd(mult(x1,x2),mult(x1,rd(x2,x1)))},\overbrace{i(i(mult(x1,x2)))})$

$=$ by Lemma 23 RL with $\{x2 \leftarrow mult(x1,x2), x1 \leftarrow mult(x1,rd(x2,x1))\}$

$\underbrace{op_t(\overbrace{rd(i(mult(x1,rd(x2,x1))),i(mult(x1,x2)))},i(i(mult(x1,x2))))}$

$=$ by Lemma 199 RL with $\{x2 \leftarrow i(mult(x1,rd(x2,x1))), x1 \leftarrow i(mult(x1,x2))\}$

$\overbrace{mult(\underbrace{mult(i(mult(x1,x2)),i(mult(x1,rd(x2,x1))))},rd(i(i(mult(x1,x2))),i(mult(x1,x2))))}$

$=$ by Axiom 11 LR with $\{x2 \leftarrow mult(x1,rd(x2,x1)), x1 \leftarrow mult(x1,x2)\}$

$mult(\overbrace{i(mult(mult(x1,x2),mult(x1,rd(x2,x1))))},\underbrace{rd(i(i(mult(x1,x2))),i(mult(x1,x2))))}$

$=$ by Lemma 43 RL with $\{x1 \leftarrow i(mult(x1,x2))\}$

$\underbrace{mult(i(mult(mult(x1,x2),mult(x1,rd(x2,x1)))),\overbrace{i(mult(i(mult(x1,x2)),i(mult(x1,x2))))})}$

$=$ by Axiom 11 LR with $\{x2 \leftarrow mult(i(mult(x1,x2)),i(mult(x1,x2))), x1 \leftarrow mult(mult(x1,x2),mult(x1,rd(x2,x1)))\}$

$\overbrace{\underbrace{i(mult(mult(mult(x1,x2),mult(x1,rd(x2,x1))),mult(i(mult(x1,x2)),i(mult(x1,x2)))))}}$

$=$ by Lemma 87 LR with $\{x3 \leftarrow i(mult(x1,x2)), x2 \leftarrow mult(x1,x2), x1 \leftarrow mult(mult(x1,x2),mult(x1,rd(x2,x1)))\}$

$\overbrace{mult(i(mult(mult(x1,x2),mult(x1,rd(x2,x1)))),mult(mult(x1,x2),\underbrace{i(i(mult(x1,x2)))}))}$

$=$ by Lemma 3 LR with $\{x1 \leftarrow mult(x1,x2)\}$

$mult(i(mult(mult(x1,x2),\underbrace{mult(x1,rd(x2,x1))})),mult(mult(x1,x2),\overbrace{mult(x1,x2)}))$

$=$ by Lemma 427 RL with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$mult(i(\underbrace{mult(mult(x1,x2),\overbrace{op_l(x2,x2,x1)})}),mult(mult(x1,x2),mult(x1,x2)))$

$=$ by Lemma 132 LR with $\{x3 \leftarrow x2, x2 \leftarrow x2, x1 \leftarrow x1\}$

$mult(i(\overbrace{mult(x1,mult(x2,x2))}),mult(mult(x1,x2),mult(x1,x2)))$

Lemma 1170: $op_r(op_t(x1, mult(x1, x2)), x2, x2) = op_l(x1, mult(x2, x2), x1)$

$\overbrace{op_r(op_t(x1, mult(x1, x2)), x2, x2)}$

$=$ by Axiom 14 RL with $\{x4 \leftarrow mult(x1, x2), x3 \leftarrow x2, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_t(op_r(x1, x2, x2), mult(x1, x2))}}$

$=$ by Lemma 1169 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{mult(i(mult(x1, mult(x2, x2))), \underbrace{mult(mult(x1, x2), mult(x1, x2))})}$

$=$ by Lemma 530 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$mult(i(mult(x1, mult(x2, x2))), \overbrace{mult(x1, \underbrace{mult(x2, mult(x2, x1))})})$

$=$ by Lemma 24 RL with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$\underbrace{mult(i(mult(x1, mult(x2, x2))), mult(x1, \overbrace{mult(mult(x2, x2), x1)}))}$

$=$ by Axiom 12 LR with $\{x3 \leftarrow x1, x2 \leftarrow mult(x2, x2), x1 \leftarrow x1\}$

$\overbrace{op_l(x1, mult(x2, x2), x1)}$

Lemma 1171: $op_l(op_l(x1,x2,x1),x2,op_l(x1,x2,x1)) = op_t(rd(mult(x2,op_l(x1,x2,x1)),x2),rd(x2,i(op_t(x1,x2))))$

$\overbrace{op_l(op_l(x1,x2,x1),x2,op_l(x1,x2,x1))}$

$=$ by Lemma 972 RL with $\{x2 \leftarrow op_l(x1,x2,x1), x1 \leftarrow x2\}$

$\overbrace{op_t(rd(mult(x2,op_l(x1,x2,x1)),x2),mult(x2,op_l(x1,x2,x1)))}$

$=$ by Lemma 340 RL with $\{x2 \leftarrow mult(x2,op_l(x1,x2,x1)), x1 \leftarrow x2\}$

$\overbrace{mult(op_t(i(x2),mult(x2,op_l(x1,x2,x1))),mult(x2,op_l(x1,x2,x1)))}$

$=$ by Lemma 337 RL with $\{x2 \leftarrow mult(x2,op_l(x1,x2,x1)), x1 \leftarrow i(x2)\}$

$\overbrace{op_t(rd(i(x2),i(mult(x2,op_l(x1,x2,x1)))),mult(x2,op_l(x1,x2,x1)))}$

$=$ by Lemma 553 LR with $\{x2 \leftarrow mult(x2,op_l(x1,x2,x1)), x1 \leftarrow rd(i(x2),i(mult(x2,op_l(x1,x2,x1))))\}$

$\overbrace{op_t(rd(i(x2),i(mult(x2,op_l(x1,x2,x1)))),op_l(mult(x2,op_l(x1,x2,x1)),mult(x2,op_l(x1,x2,x1)),\underbrace{rd(i(x2),i(mult(x2,op_l(}$

$=$ by Lemma 38 RL with $\{x1 \leftarrow i(x2), x2 \leftarrow mult(x2,op_l(x1,x2,x1))\}$

$op_t(rd(i(x2),i(mult(x2,op_l(x1,x2,x1)))),\underbrace{op_l(mult(x2,op_l(x1,x2,x1)),mult(x2,op_l(x1,x2,x1)),\overbrace{rd(mult(x2,op_l(x1,x2,x}$

$=$ by Lemma 496 LR with $\{x2 \leftarrow i(i(x2)), x1 \leftarrow mult(x2,op_l(x1,x2,x1))\}$

$op_t(rd(i(x2),i(mult(x2,op_l(x1,x2,x1)))),\overbrace{op_l(mult(x2,op_l(x1,x2,x1)),mult(x2,op_l(x1,x2,x1)),\underbrace{i(i(i(x2)))}}$

$=$ by Lemma 3 LR with $\{x1 \leftarrow i(x2)\}$

$op_t(rd(i(x2),i(mult(x2,op_l(x1,x2,x1)))),\underbrace{op_l(mult(x2,op_l(x1,x2,x1)),mult(x2,op_l(x1,x2,x1)),\overbrace{i(x2)}}$

$=$ by Lemma 242 LR with $\{x3 \leftarrow op_l(x1,x2,x1), x2 \leftarrow x2, x1 \leftarrow mult(x2,op_l(x1,x2,x1))\}$

$op_t(\underbrace{rd(i(x2),i(mult(x2,op_l(x1,x2,x1))))},\overbrace{op_l(mult(x2,op_l(x1,x2,x1)),x2,op_l(x1,x2,x1))})$

$=$ by Lemma 23 LR with $\{x2 \leftarrow mult(x2,op_l(x1,x2,x1)), x1 \leftarrow x2\}$

$op_t(\overbrace{rd(mult(x2,op_l(x1,x2,x1)),x2)},\underbrace{op_l(mult(x2,op_l(x1,x2,x1)),x2,op_l(x1,x2,x1))})$

$=$ by Lemma 435 LR with $\{x2 \leftarrow op_l(x1,x2,x1), x1 \leftarrow x2\}$

$op_t(rd(mult(x2,op_l(x1,x2,x1)),x2),\overbrace{mult(op_l(x1,x2,x1),x2)})$

$=$ by Lemma 460 LR with $\{x1 \leftarrow x2, x2 \leftarrow x1\}$

$op_t(rd(mult(x2,op_l(x1,x2,x1)),x2),\overbrace{rd(x2,\underbrace{mult(i(x2),mult(x2,i(op_t(x1,x2))))})})$

$=$ by Axiom 2 RL with $\{x2 \leftarrow x2, x1 \leftarrow i(op_t(x1,x2))\}$

$op_t(rd(mult(x2,op_l(x1,x2,x1)),x2),rd(x2,\overbrace{i(op_t(x1,x2))}))$

Lemma 1172: $op_l(op_l(x1,x2,x1),x2,x1) = op_t(rd(rd(x2,i(x1)),x2),rd(x2,i(op_t(x1,x2))))$

$$
\begin{aligned}
& \underbrace{op_l(op_l(x1,x2,x1),x2,x1)} \\
= & \quad \text{by Lemma 1107 RL with } \{x4 \leftarrow x2, x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(op_l(x1,x2,x1),x1,i(x2))}} \\
= & \quad \text{by Lemma 651 LR with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow op_l(x1,x2,x1)\} \\
& \overbrace{\underbrace{op_l(op_l(x1,x2,x1),x2,op_l(x1,x2,x1))}} \\
= & \quad \text{by Lemma 1171 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_t(rd(\underbrace{mult(x2,op_l(x1,x2,x1))},x2),rd(x2,i(op_t(x1,x2))))} \\
= & \quad \text{by Lemma 616 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& op_t(rd(\overbrace{rd(x2,i(x1))},x2),rd(x2,i(op_t(x1,x2))))
\end{aligned}
$$

Lemma 1173: $op_l(op_l(x1,x2,x1),x2,x1) = op_l(x1,mult(x2,x2),x1)$

$$
\begin{aligned}
& \underbrace{op_l(op_l(x1,x2,x1),x2,x1)} \\
= & \quad \text{by Lemma 1172 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_t(\underbrace{rd(rd(x2,i(x1)),x2)},rd(x2,i(op_t(x1,x2))))} \\
= & \quad \text{by Lemma 354 LR with } \{x2 \leftarrow i(x1), x1 \leftarrow x2\} \\
& \underbrace{op_t(\overbrace{op_r(i(i(x1)),x2,x2)},rd(x2,i(op_t(x1,x2))))} \\
= & \quad \text{by Axiom 14 LR with } \{x4 \leftarrow rd(x2,i(op_t(x1,x2))), x3 \leftarrow x2, x2 \leftarrow x2, x1 \leftarrow i(i(x1))\} \\
& \overbrace{op_r(op_t(\underbrace{i(i(x1))},rd(x2,i(op_t(x1,x2)))),x2,x2)} \\
= & \quad \text{by Lemma 3 LR with } \{x1 \leftarrow x1\} \\
& \underbrace{op_r(op_t(\overbrace{x1},rd(x2,i(op_t(x1,x2))))},x2,x2) \\
= & \quad \text{by Lemma 1166 LR with } \{x2 \leftarrow x2, x3 \leftarrow x2, x1 \leftarrow x1\} \\
& op_r(\overbrace{\underbrace{op_t(x1,mult(x2,x1))}},x2,x2) \\
= & \quad \text{by Lemma 455 RL with } \{x2 \leftarrow mult(x2,x1), x1 \leftarrow x1\} \\
& op_r(\overbrace{op_l(op_t(x1,\underbrace{rd(mult(x2,x1),x1)}),mult(x2,x1),x1)},x2,x2) \\
= & \quad \text{by Axiom 5 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& op_r(\underbrace{op_l(op_t(x1,\overbrace{x2}),mult(x2,x1),x1)},x2,x2) \\
= & \quad \text{by Lemma 285 LR with } \{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow op_t(x1,x2)\} \\
& op_r(\overbrace{\underbrace{op_l(op_t(x1,x2),x2,x1)}},x2,x2) \\
= & \quad \text{by Lemma 251 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{op_r(\overbrace{op_t(x1,mult(x1,x2))},x2,x2)} \\
= & \quad \text{by Lemma 1170 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1,mult(x2,x2),x1)}
\end{aligned}
$$

Lemma 1174: $op_l(op_l(x1, x1, x2), x1, x2) = op_l(x1, x1, mult(x2, x2))$

$$
\begin{aligned}
& \overbrace{op_l(op_l(x1, x1, x2), x1, x2)} \\
= \quad & \text{by Lemma 1106 RL with } \{x4 \leftarrow x2, x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x1\} \\
& \overbrace{op_l(op_l(x1, x1, x2), x2, i(x1))} \\
= \quad & \text{by Lemma 1050 RL with } \{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow op_l(x1, x1, x2)\} \\
& \overbrace{op_l(op_l(x1, x1, x2), i(x2), x1)} \\
= \quad & \text{by Lemma 1106 RL with } \{x4 \leftarrow x2, x3 \leftarrow x1, x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& \overbrace{op_l(op_l(x1, i(x2), x1), x2, i(x1))} \\
= \quad & \text{by Lemma 1050 RL with } \{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow op_l(x1, i(x2), x1)\} \\
& \overbrace{op_l(op_l(x1, i(x2), x1), i(x2), x1)} \\
= \quad & \text{by Lemma 1173 LR with } \{x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, \underbrace{mult(i(x2), i(x2))}, x1)} \\
= \quad & \text{by Axiom 11 LR with } \{x2 \leftarrow x2, x1 \leftarrow x2\} \\
& op_l(x1, \overbrace{i(\underbrace{mult(x2, x2)})}, x1) \\
= \quad & \text{by Lemma 43 LR with } \{x1 \leftarrow x2\} \\
& \overbrace{op_l(x1, \overbrace{rd(i(x2), x2)}, x1)} \\
= \quad & \text{by Lemma 1066 LR with } \{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, mult(x2, x2), i(x1))} \\
= \quad & \text{by Lemma 606 LR with } \{x2 \leftarrow mult(x2, x2), x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x1, mult(x2, x2))}
\end{aligned}
$$

Lemma 1175: $op_l(i(x1), x1, mult(mult(x1, x2), mult(x1, x2))) = op_l(op_l(i(x1), mult(x1, x2), i(x1)), x1, x2)$

$$
\begin{aligned}
& \overbrace{op_l(i(x1), x1, mult(mult(x1, x2), mult(x1, x2)))} \\
= \quad & \text{by Lemma 633 RL with } \{x2 \leftarrow mult(mult(x1, x2), mult(x1, x2)), x1 \leftarrow x1\} \\
& \overbrace{op_l(i(x1), mult(mult(x1, x2), mult(x1, x2)), i(x1))} \\
= \quad & \text{by Lemma 1173 RL with } \{x2 \leftarrow mult(x1, x2), x1 \leftarrow i(x1)\} \\
& \overbrace{op_l(op_l(i(x1), mult(x1, x2), i(x1)), mult(x1, x2), i(x1))} \\
= \quad & \text{by Lemma 242 LR with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow op_l(i(x1), mult(x1, x2), i(x1))\} \\
& \overbrace{op_l(op_l(i(x1), mult(x1, x2), i(x1)), x1, x2)}
\end{aligned}
$$

Lemma 1176: $op_l(i(x1), x1, mult(x2, mult(x2, x1))) = op_l(op_l(i(x1), mult(x1, x2), i(x1)), x1, x2)$

$$
\begin{aligned}
& \underbrace{op_l(i(x1), x1, mult(x2, mult(x2, x1)))} \\
= \quad & \text{by Lemma 652 RL with } \{x2 \leftarrow mult(x2, mult(x2, x1)), x1 \leftarrow x1\} \\
& \overbrace{op_l(i(x1), x1, \underbrace{mult(x1, mult(x2, mult(x2, x1)))})} \\
= \quad & \text{by Lemma 530 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{op_l(i(x1), x1, \overbrace{mult(mult(x1, x2), mult(x1, x2))})} \\
= \quad & \text{by Lemma 1175 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(op_l(i(x1), \underbrace{mult(x1, x2)}, i(x1)), x1, x2)}
\end{aligned}
$$

Lemma 1177: $op_l(i(x1), x1, mult(x2, x2)) = op_l(op_l(i(x1), x1, mult(x1, x2)), x1, x2)$

$$
\begin{aligned}
& \underbrace{op_l(i(x1), x1, \underbrace{mult(x2, x2)})} \\
= \quad & \text{by Lemma 313 RL with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow i(x1)\} \\
& \overbrace{\underbrace{op_l(i(x1), x1, mult(x2, mult(x2, x1)))}} \\
= \quad & \text{by Lemma 1176 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(\underbrace{op_l(i(x1), mult(x1, x2), i(x1))}, x1, x2)} \\
= \quad & \text{by Lemma 633 LR with } \{x2 \leftarrow mult(x1, x2), x1 \leftarrow x1\} \\
& op_l(\overbrace{op_l(i(x1), x1, mult(x1, x2))}, x1, x2)
\end{aligned}
$$

Lemma 1178: $mult(mult(x1, x2), op_t(x1, mult(x2, x1))) = mult(x1, mult(x1, x2))$

$$
\begin{aligned}
& \underbrace{mult(mult(x1, x2), op_t(x1, mult(x2, x1)))} \\
= \quad & \text{by Lemma 137 RL with } \{x3 \leftarrow op_t(x1, mult(x2, x1)), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{mult(x1, mult(x2, \underbrace{op_l(op_t(x1, mult(x2, x1)), x1, x2)}))} \\
= \quad & \text{by Lemma 583 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& mult(x1, \underbrace{mult(x2, \overbrace{op_t(x1, x2)})}) \\
= \quad & \text{by Lemma 13 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& mult(x1, \overbrace{mult(x1, x2)})
\end{aligned}
$$

Lemma 1179: $op_t(mult(x1, i(x2)), op_l(x2, x1, x2)) = rd(x1, op_t(x2, x1))$

$$
\begin{aligned}
& \underbrace{op_t(mult(x1, i(x2)), op_l(x2, x1, x2))} \\
= \quad & \text{by Lemma 194 RL with } \{x3 \leftarrow op_l(x2, x1, x2), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_t(\underbrace{op_t(rd(x1, x2), op_l(x2, x1, x2))}, i(x1))} \\
= \quad & \text{by Lemma 587 RL with } \{x1 \leftarrow x1, x2 \leftarrow x2\} \\
& \underbrace{op_t(\overbrace{mult(i(x2), x1)}, i(x1))} \\
= \quad & \text{by Lemma 326 LR with } \{x2 \leftarrow x1, x1 \leftarrow i(x2)\} \\
& \overbrace{rd(x1, op_t(\underbrace{i(i(x2))}, x1))} \\
= \quad & \text{by Lemma 3 LR with } \{x1 \leftarrow x2\} \\
& rd(x1, op_t(\overbrace{x2}, x1))
\end{aligned}
$$

Lemma 1180: $mult(op_t(x1,x2),mult(x2,i(x1))) = op_l(x2,x2,mult(x1,x1))$

$mult(\underbrace{op_t(x1,x2)}, mult(x2,i(x1)))$

$=$ by Lemma 455 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$mult(\overbrace{op_l(\underbrace{op_t(x1,rd(x2,x1))}, x2, x1)}, mult(x2,i(x1)))$

$=$ by Lemma 551 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$mult(\underbrace{op_l(\overbrace{op_t(x1,mult(x2,i(x1)))}, x2, x1)}, mult(x2,i(x1)))$

$=$ by Axiom 15 RL with $\{x4 \leftarrow mult(x2,i(x1)), x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{mult(\overbrace{op_t(op_l(x1,x2,x1), mult(x2,i(x1)))}, mult(x2,i(x1)))}$

$=$ by Lemma 825 LR with $\{x2 \leftarrow mult(x2,i(x1)), x1 \leftarrow op_l(x1,x2,x1)\}$

$\overbrace{op_t(rd(op_l(x1,x2,x1), i(mult(x2,i(x1)))), \underbrace{op_t(mult(x2,i(x1)), op_l(x1,x2,x1))})}$

$=$ by Lemma 1179 LR with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$op_t(rd(op_l(x1,x2,x1), \underbrace{i(mult(x2,i(x1)))}), \overbrace{rd(x2,op_t(x1,x2))})$

$=$ by Lemma 17 LR with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$op_t(\underbrace{rd(op_l(x1,x2,x1), \overbrace{mult(i(x2),x1)})}, rd(x2,op_t(x1,x2)))$

$=$ by Lemma 983 RL with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$\underbrace{op_t(\overbrace{op_r(x2,x1,x1)}, rd(x2,op_t(x1,x2)))}$

$=$ by Axiom 14 LR with $\{x4 \leftarrow rd(x2,op_t(x1,x2)), x3 \leftarrow x1, x2 \leftarrow x1, x1 \leftarrow x2\}$

$\overbrace{op_r(\underbrace{op_t(x2,rd(x2,op_t(x1,x2)))}, x1, x1)}$

$=$ by Lemma 578 RL with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$op_r(\overbrace{\underbrace{op_t(x2,rd(x2,x1))}}, x1, x1)$

$=$ by Lemma 564 RL with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$\underbrace{op_r(\overbrace{op_t(x2,mult(x2,i(x1)))}, x1, x1)}$

$=$ by Lemma 361 RL with $\{x2 \leftarrow x1, x1 \leftarrow op_t(x2,mult(x2,i(x1)))\}$

$\overbrace{\underbrace{op_r(op_t(x2,mult(x2,i(x1))), i(x1), i(x1))}}$

$=$ by Lemma 1170 LR with $\{x2 \leftarrow i(x1), x1 \leftarrow x2\}$

$\overbrace{op_l(x2,\underbrace{mult(i(x1),i(x1))}, x2)}$

$=$ by Axiom 11 LR with $\{x2 \leftarrow x1, x1 \leftarrow x1\}$

$op_l(x2,\overbrace{\underbrace{i(mult(x1,x1))}}, x2)$

$=$ by Lemma 43 LR with $\{x1 \leftarrow x1\}$

$\underbrace{op_l(x2,\overbrace{rd(i(x1),x1)}, x2)}$

$=$ by Lemma 1066 LR with $\{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x2\}$

$\overbrace{\underbrace{op_l(x2,mult(x1,x1),i(x2))}}$

$=$ by Lemma 606 LR with $\{x2 \leftarrow mult(x1,x1), x1 \leftarrow x2\}$

$\overbrace{op_l(x2,x2,mult(x1,x1))}$

Lemma 1181: $rd(x1, op_t(x1, x2)) = rd(op_l(x2, x1, x2), op_r(i(i(x2)), x1, x1))$

$$
\begin{aligned}
& rd(x1, \underbrace{op_t(x1, x2)}) \\
= & \quad \text{by Lemma 560 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{rd(x1, \overbrace{op_t(x1, op_l(x2, x1, x2))})} \\
= & \quad \text{by Lemma 396 LR with } \{x2 \leftarrow op_l(x2, x1, x2), x1 \leftarrow x1\} \\
& \overbrace{rd(op_l(x2, x1, x2), rd(\underbrace{mult(x1, op_l(x2, x1, x2))}, x1))} \\
= & \quad \text{by Lemma 616 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& rd(op_l(x2, x1, x2), \underbrace{rd(\overbrace{rd(x1, i(x2))}, x1)}) \\
= & \quad \text{by Lemma 354 LR with } \{x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& rd(op_l(x2, x1, x2), \overbrace{op_r(i(i(x2)), x1, x1)})
\end{aligned}
$$

Lemma 1182: $rd(op_l(x1, x2, x1), op_r(x1, x2, x2)) = rd(x2, op_t(x2, x1))$

$$
\begin{aligned}
& rd(op_l(x1, x2, x1), op_r(\underbrace{x1}, x2, x2)) \\
= & \quad \text{by Lemma 3 RL with } \{x1 \leftarrow x1\} \\
& \underbrace{rd(op_l(x1, x2, x1), op_r(\overbrace{i(i(x1))}, x2, x2))} \\
= & \quad \text{by Lemma 1181 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \overbrace{rd(x2, op_t(x2, x1))}
\end{aligned}
$$

Lemma 1183: $op_r(i(x1), x2, x2) = mult(op_l(x2, x2, x1), i(mult(x2, x1)))$

$$
\begin{aligned}
& \underbrace{op_r(i(x1), x2, x2)} \\
= & \quad \text{by Lemma 356 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{\overbrace{i(op_r(x1, x2, x2))}} \\
= & \quad \text{by Lemma 22 RL with } \{x2 \leftarrow op_r(x1, x2, x2), x1 \leftarrow mult(x2, x1)\} \\
& \overbrace{mult(\underbrace{rd(mult(x2, x1), op_r(x1, x2, x2))}, i(mult(x2, x1)))} \\
= & \quad \text{by Lemma 722 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& mult(\overbrace{op_l(x2, x2, x1)}, i(mult(x2, x1)))
\end{aligned}
$$

Lemma 1184: $mult(op_r(x1,x2,x2),i(op_t(x2,x1))) = rd(x1,op_t(x2,x1))$

$\underbrace{mult(op_r(x1,x2,x2),i(op_t(x2,x1)))}$

$=$ by Lemma 372 RL with $\{x2 \leftarrow op_t(x2,x1), x1 \leftarrow op_r(x1,x2,x2)\}$

$\overbrace{rd(op_t(op_r(x1,x2,x2),rd(op_t(x2,x1),\underbrace{op_r(x1,x2,x2)})),op_t(x2,x1))}$

$=$ by Lemma 605 RL with $\{x2 \leftarrow x2, x1 \leftarrow op_r(x1,x2,x2)\}$

$rd(op_t(op_r(x1,x2,x2),rd(op_t(x2,x1),\overbrace{mult(rd(op_t(op_r(x1,x2,x2),x2),x2),\underbrace{op_t(x2,op_r(x1,x2,x2))})})),op_t(x2,x1))$

$=$ by Lemma 550 LR with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$rd(op_t(op_r(x1,x2,x2),rd(op_t(x2,x1),mult(rd(\underbrace{op_t(op_r(x1,x2,x2),x2)},x2),\overbrace{op_t(x2,x1)}))),op_t(x2,x1))$

$=$ by Axiom 14 LR with $\{x4 \leftarrow x2, x3 \leftarrow x2, x2 \leftarrow x2, x1 \leftarrow x1\}$

$rd(op_t(op_r(x1,x2,x2),rd(op_t(x2,x1),mult(\underbrace{rd(\overbrace{op_r(op_t(x1,x2),x2,x2)},x2)},op_t(x2,x1)))),op_t(x2,x1))$

$=$ by Lemma 370 RL with $\{x1 \leftarrow x2, x2 \leftarrow op_t(x1,x2)\}$

$rd(op_t(op_r(x1,x2,x2),rd(op_t(x2,x1),mult(\overbrace{op_r(rd(op_t(x1,x2),x2),x2,x2)},op_t(x2,x1)))),op_t(x2,x1))$

$=$ by Lemma 467 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$rd(op_t(op_r(x1,x2,x2),\underbrace{rd(op_t(x2,x1),mult(\overbrace{mult(i(x2),x1)},op_t(x2,x1))))},op_t(x2,x1))$

$=$ by Lemma 9 LR with $\{x2 \leftarrow mult(i(x2),x1), x1 \leftarrow op_t(x2,x1)\}$

$rd(op_t(op_r(x1,x2,x2),\underbrace{\overbrace{i(mult(i(x2),x1))}}),op_t(x2,x1))$

$=$ by Lemma 16 LR with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$rd(\underbrace{op_t(op_r(x1,x2,x2),\overbrace{mult(x2,i(x1))})},op_t(x2,x1))$

$=$ by Axiom 14 LR with $\{x4 \leftarrow mult(x2,i(x1)), x3 \leftarrow x2, x2 \leftarrow x2, x1 \leftarrow x1\}$

$rd(\overbrace{op_r(\underbrace{op_t(x1,mult(x2,i(x1)))},x2,x2)},op_t(x2,x1))$

$=$ by Lemma 551 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$rd(\underbrace{op_r(\overbrace{op_t(x1,rd(x2,x1))},x2,x2)},op_t(x2,x1))$

$=$ by Lemma 368 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$rd(\overbrace{x1},op_t(x2,x1))$

Lemma 1185: $rd(op_t(i(x1),x2),asoc(x1,x2,x2)) = op_l(i(x1),x1,x2)$

$rd(op_t(i(x1),x2),\underbrace{asoc(x1,x2,x2)})$

$=$ by Lemma 756 RL with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$\underbrace{rd(op_t(i(x1),x2),\overbrace{asoc(x2,i(x1),x2)})}$

$=$ by Lemma 735 RL with $\{x1 \leftarrow x2, x2 \leftarrow i(x1)\}$

$\underbrace{\overbrace{op_l(i(x1),x2,i(x1))}}$

$=$ by Lemma 633 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_l(i(x1),x1,x2)}$

Lemma 1186: $op_l(i(x1), x1, mult(x1, i(x2))) = mult(op_r(rd(i(x1), x2), i(x1), i(x1)), mult(x1, mult(i(x1), x2)))$

$$
\begin{aligned}
& \underbrace{op_l(i(x1), x1, mult(x1, i(x2)))} \\
= \quad & \text{by Lemma 633 RL with } \{x2 \leftarrow mult(x1, i(x2)), x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(i(x1), mult(x1, i(x2)), i(x1))}} \\
= \quad & \text{by Lemma 508 RL with } \{x2 \leftarrow mult(x1, i(x2)), x1 \leftarrow x1\} \\
& \overbrace{op_l(i(x1), \underbrace{rd(x1, mult(x1, i(x2)))}, x1)} \\
= \quad & \text{by Lemma 63 RL with } \{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{op_l(i(x1), \overbrace{rd(mult(i(x1), x2), i(x1))}, x1)} \\
= \quad & \text{by Lemma 280 LR with } \{x3 \leftarrow x1, x2 \leftarrow mult(i(x1), x2), x1 \leftarrow i(x1)\} \\
& \overbrace{mult(\underbrace{mult(i(x1), rd(i(x1), mult(i(x1), x2)))}, mult(x1, mult(i(x1), x2)))} \\
= \quad & \text{by Lemma 807 RL with } \{x2 \leftarrow x2, x1 \leftarrow i(x1)\} \\
& mult(\overbrace{op_r(rd(i(x1), x2), i(x1), i(x1))}, mult(x1, mult(i(x1), x2)))
\end{aligned}
$$

Lemma 1187: $op_l(i(x1), x2, x1) = mult(op_r(rd(i(x1), x2), i(x1), i(x1)), mult(x1, mult(i(x1), x2)))$

$$
\begin{aligned}
& \underbrace{op_l(i(x1), x2, x1)} \\
= \quad & \text{by Lemma 629 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(i(x1), x1, i(x2))}} \\
= \quad & \text{by Lemma 652 RL with } \{x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(i(x1), x1, mult(x1, i(x2)))}} \\
= \quad & \text{by Lemma 1186 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{mult(op_r(rd(i(x1), x2), i(x1), i(x1)), mult(x1, mult(i(x1), x2)))}
\end{aligned}
$$

Lemma 1188: $op_r(x1, x2, rd(x1, x2)) = op_t(x1, i(x2))$

$$
\begin{aligned}
& \underbrace{op_r(x1, x2, rd(x1, x2))} \\
= \quad & \text{by Lemma 138 RL with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow op_r(x1, x2, rd(x1, x2))\} \\
& \overbrace{op_l(op_l(op_r(x1, \underbrace{x2}, rd(x1, x2)), x1, x2), x2, x1)} \\
= \quad & \text{by Lemma 3 RL with } \{x1 \leftarrow x2\} \\
& op_l(\underbrace{op_l(op_r(x1, \overbrace{i(i(x2))}, rd(x1, x2)), x1, x2)}, x2, x1) \\
= \quad & \text{by Axiom 18 RL with } \{x5 \leftarrow rd(x1, x2), x4 \leftarrow i(i(x2)), x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x1\} \\
& op_l(\overbrace{op_r(op_l(x1, x1, x2), i(i(x2)), \underbrace{rd(x1, x2)})}, x2, x1) \\
= \quad & \text{by Lemma 440 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& op_l(\underbrace{op_r(op_l(x1, x1, x2), i(i(x2)), \overbrace{mult(i(x2), op_l(x1, x1, x2))})}, x2, x1) \\
= \quad & \text{by Lemma 809 LR with } \{x2 \leftarrow i(x2), x1 \leftarrow op_l(x1, x1, x2)\} \\
& op_l(\overbrace{\underbrace{op_t(op_l(x1, x1, x2), i(x2))}}, x2, x1) \\
= \quad & \text{by Axiom 15 LR with } \{x4 \leftarrow i(x2), x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x1\} \\
& op_l(\overbrace{\underbrace{op_l(op_t(x1, i(x2)), x1, x2)}}, x2, x1) \\
= \quad & \text{by Lemma 580 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{op_l(\overbrace{op_t(x1, rd(x1, x2))}, x2, x1)} \\
= \quad & \text{by Lemma 681 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, i(x2))}
\end{aligned}
$$

Lemma 1189: $i(op_t(x1, x2)) = op_r(i(x1), x2, rd(i(x2), x1))$

$$
\begin{aligned}
& \underbrace{i(op_t(x1, x2))} \\
= \quad & \text{by Lemma 15 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_t(i(x1), i(x2))}} \\
= \quad & \text{by Lemma 1188 RL with } \{x2 \leftarrow x2, x1 \leftarrow i(x1)\} \\
& \overbrace{op_r(i(x1), x2, \underbrace{rd(i(x1), x2)})} \\
= \quad & \text{by Lemma 37 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_r(i(x1), x2, \overbrace{rd(i(x2), x1)})
\end{aligned}
$$

Lemma 1190: $op_t(mult(i(x1), x2), i(x2)) = op_r(i(mult(x1, i(x2))), x2, i(x1))$

$$
\begin{aligned}
& \underbrace{op_t(mult(i(x1), x2), i(x2))} \\
= \quad & \text{by Lemma 89 RL with } \{x3 \leftarrow x2, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{i(op_t(mult(x1, i(x2)), x2))}} \\
= \quad & \text{by Lemma 1189 LR with } \{x2 \leftarrow x2, x1 \leftarrow mult(x1, i(x2))\} \\
& \overbrace{op_r(i(mult(x1, i(x2))), x2, \underbrace{rd(i(x2), mult(x1, i(x2)))})} \\
= \quad & \text{by Lemma 9 LR with } \{x2 \leftarrow x1, x1 \leftarrow i(x2)\} \\
& op_r(i(mult(x1, i(x2))), x2, \overbrace{i(x1)})
\end{aligned}
$$

Lemma 1191: $rd(x2, op_t(x1, x2)) = op_r(i(mult(x1, i(x2))), x2, i(x1))$

$$
\begin{aligned}
& rd(x2, op_t(\underbrace{x1}, x2)) \\
= & \quad \text{by Lemma 3 RL with } \{x1 \leftarrow x1\} \\
& \underbrace{rd(x2, op_t(\overbrace{i(i(x1))}, x2))} \\
= & \quad \text{by Lemma 326 RL with } \{x2 \leftarrow x2, x1 \leftarrow i(x1)\} \\
& \underbrace{\overbrace{op_t(mult(i(x1), x2), i(x2))}} \\
= & \quad \text{by Lemma 1190 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_r(i(mult(x1, i(x2))), x2, i(x1))}
\end{aligned}
$$

Lemma 1192: $i(op_r(mult(x1, x2), x2, x1)) = rd(op_t(i(x1), x2), x2)$

$$
\begin{aligned}
& \underbrace{i(op_r(mult(x1, x2), x2, x1))} \\
= & \quad \text{by Lemma 815 RL with } \{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow mult(x1, x2)\} \\
& \overbrace{op_r(\underbrace{i(mult(x1, x2))}, i(x2), i(x1))} \\
= & \quad \text{by Axiom 11 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_r(\overbrace{mult(i(x1), i(x2))}, i(x2), i(x1)) \\
= & \quad \text{by Lemma 17 RL with } \{x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& \underbrace{op_r(\overbrace{i(mult(x1, i(i(x2))))}, i(x2), i(x1))} \\
= & \quad \text{by Lemma 1191 RL with } \{x1 \leftarrow x1, x2 \leftarrow i(x2)\} \\
& \underbrace{\overbrace{rd(i(x2), op_t(x1, i(x2)))}} \\
= & \quad \text{by Lemma 66 LR with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \overbrace{rd(op_t(i(x1), x2), x2)}
\end{aligned}
$$

Lemma 1193: $op_r(rd(op_t(mult(x1, x2), x1), x2), x2, i(x2)) = rd(op_t(op_t(mult(x2, x1), x1), x2), x2)$

$$
\begin{aligned}
& \underbrace{op_r(rd(op_t(mult(x1, x2), x1), x2), x2, i(x2))} \\
= & \quad \text{by Lemma 317 RL with } \{x2 \leftarrow rd(op_t(mult(x1, x2), x1), x2), x1 \leftarrow x2\} \\
& \overbrace{rd(op_t(mult(x2, \underbrace{rd(op_t(mult(x1, x2), x1), x2)}), x2), x2)} \\
= & \quad \text{by Lemma 895 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& rd(op_t(\underbrace{mult(x2, \overbrace{mult(op_t(x2, x1), rd(x1, x2))})}, x2), x2) \\
= & \quad \text{by Lemma 161 RL with } \{x3 \leftarrow rd(x1, x2), x2 \leftarrow x1, x1 \leftarrow x2\} \\
& rd(op_t(\overbrace{mult(op_t(x2, x1), \underbrace{mult(x2, rd(x1, x2))})}, x2), x2) \\
= & \quad \text{by Lemma 427 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& rd(op_t(\underbrace{mult(op_t(x2, x1), \overbrace{op_l(x1, x1, x2)})}, x2), x2) \\
= & \quad \text{by Lemma 834 RL with } \{x1 \leftarrow x1, x2 \leftarrow x2\} \\
& rd(op_t(\overbrace{op_t(mult(x2, x1), x1)}, x2), x2)
\end{aligned}
$$

Lemma 1194: $op_l(x1, x1, mult(x2, x2)) = rd(op_t(op_t(mult(x2, x1), x1), x2), x2)$

$\underbrace{op_l(x1, x1, mult(x2, x2))}$

$=$ by Lemma 313 RL with $\{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_l(x1, x1, mult(x2, mult(x2, x1)))}}$

$=$ by Lemma 624 RL with $\{x2 \leftarrow mult(x2, mult(x2, x1)), x1 \leftarrow x1\}$

$\overbrace{op_l(x1, x1, \underbrace{mult(x1, mult(x2, mult(x2, x1)))})}$

$=$ by Lemma 530 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{op_l(x1, x1, \overbrace{mult(mult(x1, x2), mult(x1, x2))})}$

$=$ by Lemma 1180 RL with $\{x2 \leftarrow x1, x1 \leftarrow mult(x1, x2)\}$

$\overbrace{mult(op_t(mult(x1, x2), x1), \underbrace{mult(x1, i(mult(x1, x2)))})}$

$=$ by Lemma 20 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{mult(op_t(mult(x1, x2), x1), \overbrace{i(x2)})}$

$=$ by Lemma 104 RL with $\{x2 \leftarrow x2, x1 \leftarrow op_t(mult(x1, x2), x1)\}$

$\overbrace{\underbrace{op_r(rd(op_t(mult(x1, x2), x1), x2), x2, i(x2))}}$

$=$ by Lemma 1193 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{rd(op_t(op_t(mult(x2, x1), x1), x2), x2)}$

Lemma 1195: $op_l(mult(x1, x2), x2, x1) = rd(mult(x1, x2), rd(x1, op_t(x1, x2)))$

$\underbrace{op_l(mult(x1, x2), x2, x1)}$

$=$ by Lemma 291 RL with $\{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow mult(x1, x2)\}$

$\overbrace{\underbrace{op_l(mult(x1, x2), x2, mult(x1, x2))}}$

$=$ by Lemma 620 RL with $\{x2 \leftarrow x2, x1 \leftarrow mult(x1, x2)\}$

$\overbrace{rd(mult(x1, x2), \underbrace{asoc(mult(x1, x2), x2, mult(x1, x2))})}$

$=$ by Lemma 838 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$rd(mult(x1, x2), \overbrace{rd(x1, op_t(x1, x2))})$

Lemma 1196: $rd(mult(x1,x2),mult(x1,mult(x2,x1))) = i(op_r(x1,mult(x1,x2),mult(x1,mult(x2,unit()))))$

$rd(mult(x1,\underbrace{x2}),mult(x1,mult(x2,x1)))$

$=$ by Axiom 1 LR with $\{x1 \leftarrow x2\}$

$rd(mult(x1,\overbrace{mult(x2,\underbrace{unit()})}),mult(x1,mult(x2,x1)))$

$=$ by Axiom 7 RL with $\{x1 \leftarrow x1\}$

$rd(\underbrace{mult(x1,mult(x2,\overbrace{mult(x1,i(x1))}))},mult(x1,mult(x2,x1)))$

$=$ by Axiom 6 LR with $\{x3 \leftarrow i(x1), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{rd(\overbrace{mult(mult(x1,mult(x2,x1)),i(x1))},mult(x1,mult(x2,x1)))}$

$=$ by Lemma 800 RL with $\{x2 \leftarrow mult(x1,mult(x2,x1)), x1 \leftarrow i(x1)\}$

$\overbrace{\underbrace{op_r(i(x1),mult(mult(x1,mult(x2,x1)),i(x1)),mult(mult(x1,mult(x2,x1)),i(x1)))}}$

$=$ by Lemma 361 RL with $\{x2 \leftarrow mult(mult(x1,mult(x2,x1)),i(x1)), x1 \leftarrow i(x1)\}$

$\overbrace{\underbrace{op_r(i(x1),i(mult(mult(x1,mult(x2,x1)),i(x1))),i(mult(mult(x1,mult(x2,x1)),i(x1))))}}$

$=$ by Lemma 820 LR with $\{x3 \leftarrow i(mult(mult(x1,mult(x2,x1)),i(x1))), x2 \leftarrow mult(x1,mult(x2,x1)), x1 \leftarrow i(x1)\}$

$\overbrace{op_r(i(x1),op_t(mult(x1,mult(x2,x1)),i(x1)),\underbrace{i(i(mult(mult(x1,mult(x2,x1)),i(x1))))})$

$=$ by Lemma 3 LR with $\{x1 \leftarrow mult(mult(x1,mult(x2,x1)),i(x1))\}$

$op_r(i(x1),\underbrace{op_t(mult(x1,mult(x2,x1)),i(x1))},\overbrace{mult(mult(x1,mult(x2,x1)),i(x1))})$

$=$ by Lemma 134 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{op_r(i(x1),\overbrace{mult(x1,mult(x1,x2))},mult(mult(x1,mult(x2,x1)),i(x1)))}$

$=$ by Lemma 654 LR with $\{x3 \leftarrow mult(mult(x1,mult(x2,x1)),i(x1)), x2 \leftarrow mult(x1,x2), x1 \leftarrow x1\}$

$\overbrace{i(op_r(x1,mult(x1,x2),\underbrace{mult(mult(x1,mult(x2,x1)),i(x1))}))}$

$=$ by Axiom 6 RL with $\{x3 \leftarrow i(x1), x2 \leftarrow x2, x1 \leftarrow x1\}$

$i(op_r(x1,mult(x1,x2),\overbrace{mult(x1,mult(x2,\underbrace{mult(x1,i(x1))}))}))$

$=$ by Axiom 7 LR with $\{x1 \leftarrow x1\}$

$i(op_r(x1,mult(x1,x2),mult(x1,mult(x2,\overbrace{unit()}))))$

Lemma 1197: $op_t(op_t(x1, rd(x2, x1)), mult(x1, x2)) = op_t(op_t(x1, x2), x2)$

$$
\begin{aligned}
& op_t(\underbrace{op_t(x1, rd(x2, x1))}, mult(x1, x2)) \\
= & \quad \text{by Lemma 708 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_t(\overbrace{op_t(x1, i(mult(x1, x2)))}, mult(x1, x2)) \\
= & \quad \text{by Lemma 18 RL with } \{x2 \leftarrow mult(x1, x2), x1 \leftarrow x1\} \\
& op_t(\overbrace{i(op_t(i(x1), mult(x1, x2)))}, mult(x1, x2)) \\
= & \quad \text{by Lemma 15 RL with } \{x2 \leftarrow mult(x1, x2), x1 \leftarrow i(x1)\} \\
& op_t(\overbrace{op_t(i(i(x1)), i(mult(x1, x2)))}, mult(x1, x2)) \\
= & \quad \text{by Lemma 345 RL with } \{x2 \leftarrow i(x1), x1 \leftarrow i(mult(x1, x2))\} \\
& \underbrace{op_t(\overbrace{rd(op_t(rd(i(mult(x1, x2)), i(x1)), i(mult(x1, x2))), i(mult(x1, x2)))}, mult(x1, x2))} \\
= & \quad \text{by Lemma 337 LR with } \{x2 \leftarrow mult(x1, x2), x1 \leftarrow op_t(rd(i(mult(x1, x2)), i(x1)), i(mult(x1, x2)))\} \\
& \overbrace{mult(\underbrace{op_t(op_t(rd(i(mult(x1, x2)), i(x1)), i(mult(x1, x2))), mult(x1, x2))}, mult(x1, x2))} \\
= & \quad \text{by Axiom 13 RL with } \{x3 \leftarrow i(mult(x1, x2)), x2 \leftarrow mult(x1, x2), x1 \leftarrow rd(i(mult(x1, x2)), i(x1))\} \\
& mult(\overbrace{op_t(\underbrace{op_t(rd(i(mult(x1, x2)), i(x1)), mult(x1, x2))}, i(mult(x1, x2)))}, mult(x1, x2)) \\
= & \quad \text{by Lemma 33 LR with } \{x2 \leftarrow i(x1), x1 \leftarrow mult(x1, x2)\} \\
& mult(\underbrace{op_t(\overbrace{i(mult(mult(x1, x2), i(x1)))}, i(mult(x1, x2)))}, mult(x1, x2)) \\
= & \quad \text{by Lemma 15 LR with } \{x2 \leftarrow mult(x1, x2), x1 \leftarrow mult(mult(x1, x2), i(x1))\} \\
& mult(\overbrace{i(op_t(\underbrace{mult(mult(x1, x2), i(x1))}, mult(x1, x2)))}, mult(x1, x2)) \\
= & \quad \text{by Lemma 55 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& mult(i(\underbrace{op_t(\overbrace{mult(x1, rd(x2, x1))}, mult(x1, x2))}), mult(x1, x2)) \\
= & \quad \text{by Lemma 887 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{mult(i(\overbrace{op_t(x2, x1)}), mult(x1, x2))} \\
= & \quad \text{by Lemma 803 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_t(op_t(x1, x2), x2)}
\end{aligned}
$$

Lemma 1198: $op_l(x1, mult(i(x2), x3), x2) = op_l(x1, rd(x2, i(op_t(x3, x2))), x2)$

$$
\begin{aligned}
& \underbrace{op_l(x1, mult(i(x2), x3), x2)} \\
= & \quad \text{by Lemma 285 RL with } \{x3 \leftarrow x2, x2 \leftarrow mult(i(x2), x3), x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, mult(mult(i(x2), x3), x2), x2)}} \\
= & \quad \text{by Lemma 285 RL with } \{x3 \leftarrow x2, x2 \leftarrow mult(mult(i(x2), x3), x2), x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, \underbrace{mult(mult(mult(i(x2), x3), x2), x2)}, x2)} \\
= & \quad \text{by Lemma 921 LR with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& op_l(x1, \overbrace{rd(x2, i(op_t(x3, x2)))}, x2)
\end{aligned}
$$

Lemma 1199: $op_l(x1, x2, i(x3)) = op_l(x1, rd(x2, i(op_t(x3, x2))), x2)$

$$
\begin{aligned}
& \overbrace{op_l(x1, x2, i(x3))} \\
= \quad & \text{by Lemma 1050 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, i(x2), x3)} \\
= \quad & \text{by Lemma 244 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, mult(i(x2), x3), x2)} \\
= \quad & \text{by Lemma 1198 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, rd(x2, i(op_t(x3, x2))), x2)}
\end{aligned}
$$

Lemma 1200: $op_t(x1, i(mult(op_t(x2, x1), i(x1)))) = rd(asoc(i(x1), op_t(x2, x1), i(x1)), op_t(i(x1), x2))$

$$
\begin{aligned}
& \overbrace{op_t(x1, i(mult(op_t(x2, x1), i(x1))))} \\
= \quad & \text{by Lemma 18 RL with } \{x2 \leftarrow mult(op_t(x2, x1), i(x1)), x1 \leftarrow x1\} \\
& \overbrace{i(op_t(i(x1), mult(op_t(x2, x1), i(x1))))} \\
= \quad & \text{by Lemma 953 RL with } \{x1 \leftarrow op_t(x2, x1), x2 \leftarrow i(x1)\} \\
& \overbrace{rd(asoc(i(x1), op_t(x2, x1), i(x1)), \underbrace{op_t(i(x1), op_t(x2, x1))})} \\
= \quad & \text{by Lemma 561 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& rd(asoc(i(x1), op_t(x2, x1), i(x1)), \overbrace{op_t(i(x1), x2)})
\end{aligned}
$$

Lemma 1201: $op_t(x1, rd(x1, op_t(x2, x1))) = rd(asoc(i(x1), op_t(x2, x1), i(x1)), op_t(i(x1), x2))$

$$
\begin{aligned}
& \overbrace{op_t(x1, rd(x1, op_t(x2, x1)))} \\
= \quad & \text{by Lemma 554 RL with } \{x2 \leftarrow op_t(x2, x1), x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, \underbrace{mult(i(op_t(x2, x1)), x1)})} \\
= \quad & \text{by Lemma 17 RL with } \{x2 \leftarrow x1, x1 \leftarrow op_t(x2, x1)\} \\
& \underbrace{op_t(x1, \overbrace{i(mult(op_t(x2, x1), i(x1)))})} \\
= \quad & \text{by Lemma 1200 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{rd(asoc(i(x1), op_t(x2, x1), i(x1)), op_t(i(x1), x2))}
\end{aligned}
$$

Lemma 1202: $rd(asoc(x1, x2, x1), op_t(i(x1), x2)) = op_t(x1, rd(x1, x2))$

$$
\begin{aligned}
& rd(\overbrace{asoc(x1, x2, x1)}, op_t(i(x1), x2)) \\
= & \quad \text{by Lemma 611 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& rd(\overbrace{asoc(x2, x1, i(x1))}, op_t(i(x1), x2)) \\
= & \quad \text{by Lemma 492 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& rd(\overbrace{asoc(mult(x2, x1), x1, i(x1))}, op_t(i(x1), x2)) \\
= & \quad \text{by Lemma 504 RL with } \{x2 \leftarrow mult(x2, x1), x1 \leftarrow x1\} \\
& rd(\overbrace{asoc(x1, \underbrace{mult(i(x1), mult(x2, x1))}, x1)}, op_t(i(x1), x2)) \\
= & \quad \text{by Axiom 10 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& rd(\underbrace{asoc(x1, \overbrace{op_t(x2, x1)}, x1)}, op_t(i(x1), x2)) \\
= & \quad \text{by Lemma 613 LR with } \{x2 \leftarrow op_t(x2, x1), x1 \leftarrow x1\} \\
& \underbrace{rd(\overbrace{asoc(i(x1), op_t(x2, x1), i(x1))}, op_t(i(x1), x2))} \\
= & \quad \text{by Lemma 1201 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_t(x1, rd(x1, op_t(x2, x1)))}} \\
= & \quad \text{by Lemma 578 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, rd(x1, x2))}
\end{aligned}
$$

Lemma 1203: $op_r(x1, mult(x1, x2), mult(i(i(x1)), x2)) = op_r(x1, op_l(x2, i(x1), x2), mult(i(i(x1)), x2))$

$$
\begin{aligned}
& op_r(x1, mult(\underbrace{x1}, x2), mult(i(i(x1)), x2)) \\
= & \quad \text{by Lemma 3 RL with } \{x1 \leftarrow x1\} \\
& \underbrace{op_r(x1, mult(\overbrace{i(i(x1))}, x2), mult(i(i(x1)), x2))} \\
= & \quad \text{by Lemma 997 LR with } \{x2 \leftarrow mult(i(i(x1)), x2), x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, \underbrace{mult(mult(i(i(x1)), x2), i(x1))}, mult(i(i(x1)), x2))} \\
= & \quad \text{by Lemma 744 RL with } \{x2 \leftarrow i(x1), x1 \leftarrow x2\} \\
& op_r(x1, \overbrace{op_l(x2, i(x1), x2)}, mult(i(i(x1)), x2))
\end{aligned}
$$

Lemma 1204: $rd(mult(x2, x1), x2) = op_r(x1, op_l(x2, i(x1), x2), mult(i(i(x1)), x2))$

$$
\begin{aligned}
& \underbrace{rd(mult(x2, x1), x2)} \\
= & \quad \text{by Lemma 823 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, mult(x1, x2), mult(\underbrace{x1}, x2))} \\
= & \quad \text{by Lemma 3 RL with } \{x1 \leftarrow x1\} \\
& \underbrace{op_r(x1, mult(x1, x2), mult(\overbrace{i(i(x1))}, x2))} \\
= & \quad \text{by Lemma 1203 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, op_l(x2, i(x1), x2), mult(i(i(x1)), x2))}
\end{aligned}
$$

Lemma 1205: $rd(mult(x2, x1), x2) = op_r(x1, op_l(x2, x2, x1), mult(x1, x2))$

$\underbrace{rd(mult(x2, x1), x2)}$

$=$ by Lemma 1204 LR with $\{x1 \leftarrow x1, x2 \leftarrow x2\}$

$\overbrace{op_r(x1, \underbrace{op_l(x2, i(x1), x2)}, mult(i(i(x1)), x2))}$

$=$ by Lemma 607 LR with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$op_r(x1, \overbrace{op_l(x2, x2, x1)}, mult(\underbrace{i(i(x1))}, x2))$

$=$ by Lemma 3 LR with $\{x1 \leftarrow x1\}$

$op_r(x1, op_l(x2, x2, x1), mult(\overbrace{x1}, x2))$

Lemma 1206: $op_t(rd(x1, mult(x1, x2)), rd(x2, x1)) = rd(rd(x1, x2), op_t(x1, i(x2)))$

$op_t(rd(x1, mult(x1, x2)), \underbrace{rd(x2, x1)})$

$=$ by Lemma 10 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{op_t(rd(x1, mult(x1, x2)), \overbrace{i(rd(x1, x2))})}$

$=$ by Lemma 344 RL with $\{x2 \leftarrow rd(x1, mult(x1, x2)), x1 \leftarrow rd(x1, x2)\}$

$\overbrace{rd(rd(x1, x2), op_t(rd(rd(x1, x2), rd(x1, \underbrace{mult(x1, x2)})), rd(x1, x2)))}$

$=$ by Lemma 231 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$rd(rd(x1, x2), op_t(rd(rd(x1, x2), \underbrace{rd(x1, \overbrace{rd(x1, op_l(i(x2), x2, x1))})}), rd(x1, x2)))$

$=$ by Lemma 353 LR with $\{x2 \leftarrow op_l(i(x2), x2, x1), x1 \leftarrow x1\}$

$rd(rd(x1, x2), op_t(rd(rd(x1, x2), \overbrace{\underbrace{op_r(op_l(i(x2), x2, x1), x1, x1)}}), rd(x1, x2)))$

$=$ by Axiom 18 LR with $\{x5 \leftarrow x1, x4 \leftarrow x1, x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow i(x2)\}$

$rd(rd(x1, x2), op_t(rd(rd(x1, x2), \overbrace{op_l(\underbrace{op_r(i(x2), x1, x1)}, x2, x1)}), rd(x1, x2)))$

$=$ by Lemma 356 RL with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$rd(rd(x1, x2), op_t(rd(rd(x1, x2), op_l(\underbrace{i(op_r(x2, x1, x1))}, x2, x1)), rd(x1, x2)))$

$=$ by Lemma 815 RL with $\{x3 \leftarrow x1, x2 \leftarrow x1, x1 \leftarrow x2\}$

$rd(rd(x1, x2), op_t(rd(rd(x1, x2), \underbrace{op_l(\overbrace{op_r(i(x2), i(x1), i(x1))}, x2, x1)}), rd(x1, x2)))$

$=$ by Axiom 18 RL with $\{x5 \leftarrow i(x1), x4 \leftarrow i(x1), x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow i(x2)\}$

$rd(rd(x1, x2), op_t(\underbrace{rd(rd(x1, x2), \overbrace{op_r(op_l(i(x2), x2, x1), i(x1), i(x1))})}, rd(x1, x2)))$

$=$ by Lemma 998 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$rd(rd(x1, x2), \underbrace{op_t(\overbrace{op_l(x1, x2, x1)}, rd(x1, x2))})$

$=$ by Axiom 15 LR with $\{x4 \leftarrow rd(x1, x2), x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x1\}$

$rd(rd(x1, x2), \underbrace{\overbrace{op_l(op_t(x1, rd(x1, x2)), x2, x1)}})$

$=$ by Lemma 681 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$rd(rd(x1, x2), \overbrace{op_t(x1, i(x2))})$

Lemma 1207: $op_t(x1, mult(x2, op_t(x3, x1))) = op_t(x1, mult(x2, x3))$

$$
\begin{aligned}
& op_t(x1, mult(x2, op_t(\underbrace{x3}, x1))) \\
= \quad & \text{by Axiom 2 LR with } \{x2 \leftarrow x2, x1 \leftarrow x3\} \\
& op_t(x1, \underbrace{mult(x2, op_t(\overbrace{mult(i(x2), mult(x2, x3))}, x1)))}) \\
= \quad & \text{by Lemma 1014 LR with } \{x3 \leftarrow x1, x2 \leftarrow mult(x2, x3), x1 \leftarrow x2\} \\
& \underbrace{op_t(x1, \overbrace{mult(op_t(rd(mult(x2, x3), x2), x1), x2)})} \\
= \quad & \text{by Lemma 1118 LR with } \{x3 \leftarrow x2, x2 \leftarrow rd(mult(x2, x3), x2), x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, \underbrace{mult(rd(mult(x2, x3), x2), x2)})} \\
= \quad & \text{by Axiom 4 RL with } \{x2 \leftarrow x2, x1 \leftarrow mult(x2, x3)\} \\
& op_t(x1, \overbrace{mult(x2, x3)})
\end{aligned}
$$

Lemma 1208: $op_t(rd(mult(x1, x2), x1), x3) = rd(mult(x1, op_t(x2, x3)), x1)$

$$
\begin{aligned}
& \underbrace{op_t(rd(mult(x1, x2), x1), x3)} \\
= \quad & \text{by Axiom 5 LR with } \{x2 \leftarrow x1, x1 \leftarrow op_t(rd(mult(x1, x2), x1), x3)\} \\
& \overbrace{rd(\underbrace{mult(op_t(rd(mult(x1, x2), x1), x3), x1)}, x1)} \\
= \quad & \text{by Lemma 1014 RL with } \{x3 \leftarrow x3, x2 \leftarrow mult(x1, x2), x1 \leftarrow x1\} \\
& rd(\overbrace{mult(x1, op_t(\underbrace{mult(i(x1), mult(x1, x2))}, x3))}, x1) \\
= \quad & \text{by Axiom 2 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& rd(mult(x1, op_t(\overbrace{x2}, x3)), x1)
\end{aligned}
$$

Lemma 1209: $mult(x1, rd(op_t(x2, x3), x1)) = op_t(mult(x1, rd(x2, x1)), x3)$

$mult(x1, rd(\underbrace{op_t(x2, x3)}, x1))$

$=$ by Lemma 3 RL with $\{x1 \leftarrow op_t(x2, x3)\}$

$\underbrace{mult(x1, rd(\overbrace{i(i(op_t(x2, x3)))}, x1))}$

$=$ by Lemma 413 RL with $\{x2 \leftarrow x1, x1 \leftarrow i(op_t(x2, x3))\}$

$\overbrace{\underbrace{op_r(i(op_t(i(op_t(x2, x3)), x1)), x1, x1)}}$

$=$ by Lemma 354 RL with $\{x2 \leftarrow op_t(i(op_t(x2, x3)), x1), x1 \leftarrow x1\}$

$\overbrace{rd(\underbrace{rd(x1, op_t(i(op_t(x2, x3)), x1))}, x1)}$

$=$ by Lemma 402 RL with $\{x2 \leftarrow op_t(x2, x3), x1 \leftarrow x1\}$

$rd(\overbrace{mult(x1, \underbrace{op_r(op_t(x2, x3), x1, i(x1))})}, x1)$

$=$ by Axiom 14 RL with $\{x4 \leftarrow x3, x3 \leftarrow i(x1), x2 \leftarrow x1, x1 \leftarrow x2\}$

$\underbrace{rd(mult(x1, \overbrace{op_t(op_r(x2, x1, i(x1)), x3)}), x1)}$

$=$ by Lemma 1208 RL with $\{x3 \leftarrow x3, x2 \leftarrow op_r(x2, x1, i(x1)), x1 \leftarrow x1\}$

$\overbrace{op_t(rd(\underbrace{mult(x1, op_r(x2, x1, i(x1)))}, x1), x3)}$

$=$ by Lemma 402 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$op_t(\underbrace{rd(\overbrace{rd(x1, op_t(i(x2), x1))}, x1)}, x3)$

$=$ by Lemma 354 LR with $\{x2 \leftarrow op_t(i(x2), x1), x1 \leftarrow x1\}$

$op_t(\underbrace{\overbrace{op_r(i(op_t(i(x2), x1)), x1, x1)}}, x3)$

$=$ by Lemma 413 LR with $\{x2 \leftarrow x1, x1 \leftarrow i(x2)\}$

$op_t(\overbrace{mult(x1, rd(\underbrace{i(i(x2))}, x1))}, x3)$

$=$ by Lemma 3 LR with $\{x1 \leftarrow x2\}$

$op_t(mult(x1, rd(\overbrace{x2}, x1)), x3)$

Lemma 1210: $op_t(x1, rd(op_t(x2, x1), x3)) = op_t(x1, rd(x2, x3))$

$op_t(x1, rd(op_t(\underbrace{x2}, x1), x3))$

$=$ by Axiom 4 LR with $\{x2 \leftarrow x3, x1 \leftarrow x2\}$

$op_t(x1, \underbrace{rd(op_t(\overbrace{mult(rd(x2, x3), x3)}, x1), x3)})$

$=$ by Lemma 1149 RL with $\{x3 \leftarrow x1, x2 \leftarrow rd(x2, x3), x1 \leftarrow x3\}$

$\underbrace{op_t(x1, \overbrace{mult(i(x3), op_t(mult(x3, rd(x2, x3)), x1))})}$

$=$ by Lemma 1207 LR with $\{x3 \leftarrow mult(x3, rd(x2, x3)), x2 \leftarrow i(x3), x1 \leftarrow x1\}$

$\overbrace{op_t(x1, \underbrace{mult(i(x3), mult(x3, rd(x2, x3)))})}$

$=$ by Axiom 2 RL with $\{x2 \leftarrow x3, x1 \leftarrow rd(x2, x3)\}$

$op_t(x1, \overbrace{rd(x2, x3)})$

Lemma 1211: $op_t(x1, rd(x3, op_t(x2, i(x1)))) = i(op_t(i(x1), rd(x2, x3)))$

$$
\begin{aligned}
& \underbrace{op_t(x1, \overbrace{rd(x3, op_t(x2, i(x1)))}^{\text{by Lemma 10 RL with } \{x2 \leftarrow x3, x1 \leftarrow op_t(x2, i(x1))\}})} \\
= & \quad \underbrace{op_t(x1, \overbrace{i(rd(op_t(x2, i(x1)), x3))}^{\text{by Lemma 18 RL with } \{x2 \leftarrow rd(op_t(x2, i(x1)), x3), x1 \leftarrow x1\}})} \\
= & \quad \overbrace{i(\underbrace{op_t(i(x1), rd(op_t(x2, i(x1)), x3))}^{\text{by Lemma 1210 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow i(x1)\}})} \\
& i(\overbrace{op_t(i(x1), rd(x2, x3))})
\end{aligned}
$$

Lemma 1212: $op_t(x1, rd(x3, op_t(x2, i(x1)))) = op_t(x1, rd(x3, x2))$

$$
\begin{aligned}
& \underbrace{op_t(x1, rd(x3, op_t(x2, i(x1))))}^{\text{by Lemma 1211 LR with } \{x2 \leftarrow x2, x3 \leftarrow x3, x1 \leftarrow x1\}} \\
= & \quad \overbrace{\underbrace{i(op_t(i(x1), rd(x2, x3)))}^{\text{by Lemma 18 LR with } \{x2 \leftarrow rd(x2, x3), x1 \leftarrow x1\}}} \\
= & \quad \overbrace{op_t(x1, \underbrace{i(rd(x2, x3))}^{\text{by Lemma 10 LR with } \{x2 \leftarrow x3, x1 \leftarrow x2\}})} \\
& op_t(x1, \overbrace{rd(x3, x2)})
\end{aligned}
$$

Lemma 1213: $op_t(i(x1), rd(x3, op_t(x2, x1))) = op_t(i(op_t(x1, rd(x2, x3))), x1)$

$$
\begin{aligned}
& \underbrace{op_t(i(x1), rd(x3, op_t(x2, x1)))}^{\text{by Lemma 47 RL with } \{x3 \leftarrow x3, x2 \leftarrow op_t(x2, x1), x1 \leftarrow x1\}} \\
= & \quad \overbrace{\underbrace{i(op_t(x1, rd(op_t(x2, x1), x3)))}^{\text{by Lemma 35 RL with } \{x2 \leftarrow rd(op_t(x2, x1), x3), x1 \leftarrow x1\}}} \\
= & \quad \overbrace{op_t(i(\underbrace{op_t(x1, rd(op_t(x2, x1), x3))}^{\text{by Lemma 1210 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}}), x1)} \\
& op_t(i(\overbrace{op_t(x1, rd(x2, x3))}), x1)
\end{aligned}
$$

Lemma 1214: $op_t(i(x1), rd(x3, op_t(x2, x1))) = op_t(i(x1), rd(x3, x2))$

$$
\begin{aligned}
& \underbrace{op_t(i(x1), rd(x3, op_t(x2, x1)))}^{\text{by Lemma 1213 LR with } \{x2 \leftarrow x2, x3 \leftarrow x3, x1 \leftarrow x1\}} \\
= & \quad \overbrace{\underbrace{op_t(i(op_t(x1, rd(x2, x3))), x1)}^{\text{by Lemma 35 LR with } \{x2 \leftarrow rd(x2, x3), x1 \leftarrow x1\}}} \\
= & \quad \overbrace{\underbrace{i(op_t(x1, rd(x2, x3)))}^{\text{by Lemma 47 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}}} \\
& \overbrace{op_t(i(x1), rd(x3, x2))}
\end{aligned}
$$

Lemma 1215: $op_t(x1, rd(mult(i(x1), x2), x3)) = op_t(x1, rd(rd(x2, x1), x3))$

$$
\begin{aligned}
& op_t(x1, rd(\underbrace{mult(i(x1), x2)}, x3)) \\
= \quad & \text{by Lemma 14 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \underbrace{op_t(x1, rd(\overbrace{op_t(rd(x2, x1), x1)}, x3))} \\
= \quad & \text{by Lemma 1210 LR with } \{x3 \leftarrow x3, x2 \leftarrow rd(x2, x1), x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, rd(rd(x2, x1), x3))}
\end{aligned}
$$

Lemma 1216: $op_t(x1, rd(x2, rd(x3, i(x1)))) = op_t(x1, rd(x2, mult(x1, x3)))$

$$
\begin{aligned}
& \underbrace{op_t(x1, rd(x2, rd(x3, i(x1))))} \\
= \quad & \text{by Lemma 1212 RL with } \{x2 \leftarrow rd(x3, i(x1)), x3 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, rd(x2, \underbrace{op_t(rd(x3, i(x1)), i(x1))}))} \\
= \quad & \text{by Lemma 14 LR with } \{x2 \leftarrow i(x1), x1 \leftarrow x3\} \\
& op_t(x1, rd(x2, \overbrace{mult(\underbrace{i(i(x1))}, x3)})) \\
= \quad & \text{by Lemma 3 LR with } \{x1 \leftarrow x1\} \\
& op_t(x1, rd(x2, mult(\overbrace{x1}, x3)))
\end{aligned}
$$

Lemma 1217: $op_t(x1, rd(x2, rd(x1, x3))) = op_t(x1, rd(x2, mult(x1, i(x3))))$

$$
\begin{aligned}
& op_t(x1, rd(x2, rd(\underbrace{x1}, x3))) \\
= \quad & \text{by Lemma 3 RL with } \{x1 \leftarrow x1\} \\
& \underbrace{op_t(x1, rd(x2, rd(\overbrace{i(i(x1))}, x3)))} \\
= \quad & \text{by Lemma 1212 RL with } \{x2 \leftarrow rd(i(i(x1)), x3), x3 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, rd(x2, \underbrace{op_t(rd(i(i(x1)), x3), i(x1))}))} \\
= \quad & \text{by Lemma 33 LR with } \{x2 \leftarrow x3, x1 \leftarrow i(x1)\} \\
& op_t(x1, rd(x2, \overbrace{i(\underbrace{mult(i(x1), x3)})})) \\
= \quad & \text{by Lemma 16 LR with } \{x2 \leftarrow x3, x1 \leftarrow x1\} \\
& op_t(x1, rd(x2, \overbrace{mult(x1, i(x3))})))
\end{aligned}
$$

Lemma 1218: $op_t(x1, rd(x3, rd(x1, i(x2)))) = op_t(x1, rd(x3, mult(i(i(x1)), x2)))$

$$
\begin{aligned}
& op_t(x1, \underbrace{rd(x3, rd(x1, i(x2)))}) \\
= \quad & \text{by Lemma 39 RL with } \{x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow i(x2)\} \\
& \underbrace{op_t(x1, \overbrace{rd(rd(i(x2), x1), i(x3))})} \\
= \quad & \text{by Lemma 1215 RL with } \{x3 \leftarrow i(x3), x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, \underbrace{rd(mult(i(x1), i(x2)), i(x3))})} \\
= \quad & \text{by Lemma 61 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow i(x1)\} \\
& op_t(x1, \overbrace{rd(x3, mult(i(i(x1)), x2))})
\end{aligned}
$$

Lemma 1219: $op_t(x1, rd(x2, rd(x1, i(x3)))) = op_t(x1, rd(x2, mult(x1, x3)))$

$$
\begin{aligned}
& \overbrace{op_t(x1, rd(x2, rd(x1, i(x3))))} \\
= & \quad \text{by Lemma 1218 LR with } \{x2 \leftarrow x3, x3 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, rd(x2, mult(\underbrace{i(i(x1))}, x3)))} \\
= & \quad \text{by Lemma 3 LR with } \{x1 \leftarrow x1\} \\
& op_t(x1, rd(x2, mult(\overbrace{x1}, x3)))
\end{aligned}
$$

Lemma 1220: $asoc(x1, rd(mult(i(x1), x2), x1), x3) = asoc(x2, i(x1), x3)$

$$
\begin{aligned}
& \overbrace{asoc(x1, rd(mult(i(x1), x2), x1), x3)} \\
= & \quad \text{by Lemma 1088 RL with } \{x3 \leftarrow x3, x2 \leftarrow mult(i(x1), x2), x1 \leftarrow x1\} \\
& \overbrace{asoc(\underbrace{mult(x1, mult(i(x1), x2))}, i(x1), x3)} \\
= & \quad \text{by Lemma 12 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& asoc(\overbrace{x2}, i(x1), x3)
\end{aligned}
$$

Lemma 1221: $mult(x1, op_l(mult(i(x2), x3), x2, x1)) = mult(rd(x1, x2), x3)$

$$
\begin{aligned}
& \overbrace{mult(x1, op_l(mult(i(x2), x3), x2, x1))} \\
= & \quad \text{by Lemma 1091 RL with } \{x3 \leftarrow mult(i(x2), x3), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{mult(rd(x1, x2), \underbrace{mult(x2, mult(i(x2), x3))})} \\
= & \quad \text{by Lemma 12 LR with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& mult(rd(x1, x2), \overbrace{x3})
\end{aligned}
$$

Lemma 1222: $mult(rd(op_t(x1, x2), x3), mult(x3, op_l(i(x1), x1, x3))) = mult(op_t(x1, x2), i(x1))$

$$
\begin{aligned}
& \overbrace{mult(rd(op_t(x1, x2), x3), mult(x3, op_l(i(x1), x1, x3)))} \\
= & \quad \text{by Lemma 1091 LR with } \{x3 \leftarrow op_l(i(x1), x1, x3), x2 \leftarrow x3, x1 \leftarrow op_t(x1, x2)\} \\
& \overbrace{mult(op_t(x1, x2), op_l(\underbrace{op_l(i(x1), x1, x3)}, x3, op_t(x1, x2)))} \\
= & \quad \text{by Lemma 1049 RL with } \{x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow x1\} \\
& mult(op_t(x1, x2), \underbrace{op_l(\overbrace{i(op_l(x1, x1, x3))}, x3, op_t(x1, x2)))} \\
= & \quad \text{by Lemma 1085 LR with } \{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x1\} \\
& mult(op_t(x1, x2), \overbrace{i(x1)})
\end{aligned}
$$

Lemma 1223: $mult(rd(op_t(x1,x2),x3),rd(x3,x1)) = rd(op_t(x1,x2),x1)$

$$
\begin{aligned}
& mult(rd(op_t(x1,x2),x3),\underbrace{rd(x3,x1)}) \\
= & \quad \text{by Lemma 232 RL with } \{x2 \leftarrow x1, x1 \leftarrow x3\} \\
& \underbrace{mult(rd(op_t(x1,x2),x3),\overbrace{mult(x3,op_l(i(x1),x1,x3))})} \\
= & \quad \text{by Lemma 1222 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{mult(op_t(x1,x2),i(x1))}} \\
= & \quad \text{by Lemma 97 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{rd(op_t(x1,x2),x1)}
\end{aligned}
$$

Lemma 1224: $mult(op_t(x1,x2),mult(x2,op_t(x2,x1))) = mult(op_t(mult(i(i(x2)),x1),i(i(x2))),x2)$

$$
\begin{aligned}
& mult(op_t(x1,x2),mult(x2,\underbrace{op_t(x2,x1)})) \\
= & \quad \text{by Lemma 374 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& mult(op_t(x1,x2),mult(x2,\overbrace{op_t(x2,op_t(x1,\underbrace{x2}))})) \\
= & \quad \text{by Lemma 3 RL with } \{x1 \leftarrow x2\} \\
& mult(op_t(x1,\underbrace{x2}),mult(x2,op_t(x2,op_t(x1,\overbrace{i(i(x2))})))) \\
= & \quad \text{by Lemma 3 RL with } \{x1 \leftarrow x2\} \\
& mult(op_t(x1,\overbrace{i(i(x2))}),mult(x2,\underbrace{op_t(x2,op_t(x1,i(i(x2))))})) \\
= & \quad \text{by Axiom 4 LR with } \{x2 \leftarrow x2, x1 \leftarrow op_t(x2,op_t(x1,i(i(x2))))\} \\
& mult(op_t(x1,i(i(x2))),\underbrace{mult(x2,\overbrace{mult(rd(op_t(x2,op_t(x1,i(i(x2)))),x2),x2)})}) \\
= & \quad \text{by Lemma 58 RL with } \{x2 \leftarrow rd(op_t(x2,op_t(x1,i(i(x2)))),x2), x1 \leftarrow x2\} \\
& mult(op_t(x1,i(i(x2))),\overbrace{rd(x2,i(\underbrace{mult(x2,rd(op_t(x2,op_t(x1,i(i(x2)))),x2)}))})) \\
= & \quad \text{by Lemma 95 LR with } \{x2 \leftarrow op_t(x1,i(i(x2))), x1 \leftarrow x2\} \\
& mult(op_t(x1,i(i(x2))),rd(x2,\underbrace{i(\overbrace{op_t(x2,op_t(x1,i(i(x2))))})})) \\
= & \quad \text{by Lemma 15 RL with } \{x2 \leftarrow op_t(x1,i(i(x2))), x1 \leftarrow x2\} \\
& \underbrace{mult(op_t(x1,i(i(x2))),rd(x2,\overbrace{op_t(i(x2),i(op_t(x1,i(i(x2)))))}))} \\
= & \quad \text{by Lemma 1125 RL with } \{x2 \leftarrow x2, x1 \leftarrow op_t(x1,i(i(x2)))\} \\
& \overbrace{mult(rd(op_t(x1,i(i(x2))),\underbrace{op_t(i(x2),op_t(x1,i(i(x2))))}),x2)} \\
= & \quad \text{by Lemma 559 LR with } \{x2 \leftarrow x1, x1 \leftarrow i(x2)\} \\
& mult(rd(op_t(x1,i(i(x2))),\overbrace{op_t(\underbrace{i(x2)},x1)}),x2) \\
= & \quad \text{by Lemma 3 RL with } \{x1 \leftarrow i(x2)\} \\
& mult(\underbrace{rd(op_t(x1,i(i(x2))),op_t(\overbrace{i(i(i(x2)))},x1))},x2) \\
= & \quad \text{by Lemma 883 RL with } \{x2 \leftarrow x1, x1 \leftarrow i(i(x2))\} \\
& mult(\overbrace{op_t(mult(i(i(x2)),x1),i(i(x2)))},x2)
\end{aligned}
$$

Lemma 1225: $mult(op_t(x1,x2),mult(x2,op_t(x2,x1))) = mult(op_t(mult(x2,x1),x2),x2)$

$$
\begin{aligned}
& \overbrace{mult(op_t(x1,x2),mult(x2,op_t(x2,x1)))} \\
= \quad & \text{by Lemma 1224 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{mult(op_t(mult(\underbrace{i(i(x2))},x1),i(i(x2))),x2)} \\
= \quad & \text{by Lemma 3 LR with } \{x1 \leftarrow x2\} \\
& mult(op_t(mult(\overbrace{x2},x1),\underbrace{i(i(x2))}),x2) \\
= \quad & \text{by Lemma 3 LR with } \{x1 \leftarrow x2\} \\
& mult(op_t(mult(x2,x1),\overbrace{x2}),x2)
\end{aligned}
$$

Lemma 1226: $mult(x1,rd(x2,op_t(x2,x3))) = rd(mult(x1,x2),op_t(x2,x3))$

$$
\begin{aligned}
& mult(x1,\underbrace{rd(x2,op_t(x2,x3))}) \\
= \quad & \text{by Lemma 98 LR with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& \underbrace{mult(x1,\overbrace{mult(x2,i(op_t(x2,x3)))})} \\
= \quad & \text{by Lemma 132 RL with } \{x3 \leftarrow i(op_t(x2,x3)), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{mult(mult(x1,x2),\underbrace{op_l(i(op_t(x2,x3)),x2,x1)})} \\
= \quad & \text{by Lemma 291 RL with } \{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow i(op_t(x2,x3))\} \\
& mult(mult(x1,x2),\overbrace{op_l(i(op_t(x2,x3)),x2,mult(x1,x2))}) \\
= \quad & \text{by Lemma 1049 RL with } \{x3 \leftarrow mult(x1,x2), x2 \leftarrow x2, x1 \leftarrow op_t(x2,x3)\} \\
& mult(mult(x1,x2),\overbrace{i(\underbrace{op_l(op_t(x2,x3),x2,mult(x1,x2))})}) \\
= \quad & \text{by Axiom 15 RL with } \{x4 \leftarrow x3, x3 \leftarrow mult(x1,x2), x2 \leftarrow x2, x1 \leftarrow x2\} \\
& mult(mult(x1,x2),i(\overbrace{op_t(\underbrace{op_l(x2,x2,mult(x1,x2))},x3)})) \\
= \quad & \text{by Lemma 427 LR with } \{x2 \leftarrow mult(x1,x2), x1 \leftarrow x2\} \\
& \underbrace{mult(mult(x1,x2),i(op_t(\overbrace{mult(mult(x1,x2),rd(x2,mult(x1,x2)))},x3)))} \\
= \quad & \text{by Lemma 1150 LR with } \{x3 \leftarrow x3, x2 \leftarrow rd(x2,mult(x1,x2)), x1 \leftarrow mult(x1,x2)\} \\
& \overbrace{rd(mult(x1,x2),op_t(\underbrace{mult(rd(x2,mult(x1,x2)),mult(x1,x2))},x3))} \\
= \quad & \text{by Axiom 4 RL with } \{x2 \leftarrow mult(x1,x2), x1 \leftarrow x2\} \\
& rd(mult(x1,x2),op_t(\overbrace{x2},x3))
\end{aligned}
$$

Lemma 1227: $op_r(x1, rd(x2, op_t(x2, x3)), op_t(x2, x3)) = x1$

$$
\begin{aligned}
& \underbrace{op_r(x1, rd(x2, op_t(x2, x3)), op_t(x2, x3))} \\
= & \quad \text{by Lemma 693 RL with } \{x3 \leftarrow op_t(x2, x3), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{rd(mult(\underbrace{mult(x1, rd(x2, op_t(x2, x3)))}, op_t(x2, x3)), x2)} \\
= & \quad \text{by Lemma 1226 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& rd(\underbrace{mult(\overbrace{rd(mult(x1, x2), op_t(x2, x3))}, op_t(x2, x3))}, x2) \\
= & \quad \text{by Axiom 4 RL with } \{x2 \leftarrow op_t(x2, x3), x1 \leftarrow mult(x1, x2)\} \\
& \underbrace{rd(\overbrace{mult(x1, x2)}, x2)} \\
= & \quad \text{by Axiom 5 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{x1}
\end{aligned}
$$

Lemma 1228: $x1 = op_r(x1, op_r(i(rd(op_t(x2, x3), x2)), x2, x2), op_t(rd(x2, rd(op_t(x2, x3), x2)), x3))$

$$
\begin{aligned}
& \underbrace{x1} \\
= & \quad \text{by Lemma 1227 RL with } \{x3 \leftarrow x3, x2 \leftarrow rd(x2, rd(op_t(x2, x3), x2)), x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, rd(rd(x2, rd(op_t(x2, x3), x2)), \underbrace{op_t(rd(x2, rd(op_t(x2, x3), x2)), x3)}), op_t(rd(x2, rd(op_t(x2, x3), x2)), x3))} \\
= & \quad \text{by Lemma 401 LR with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& op_r(x1, \underbrace{rd(rd(x2, rd(op_t(x2, x3), x2)), \overbrace{x2})}, op_t(rd(x2, rd(op_t(x2, x3), x2)), x3)) \\
= & \quad \text{by Lemma 354 LR with } \{x2 \leftarrow rd(op_t(x2, x3), x2), x1 \leftarrow x2\} \\
& op_r(x1, \overbrace{op_r(i(rd(op_t(x2, x3), x2)), x2, x2)}, op_t(rd(x2, rd(op_t(x2, x3), x2)), x3))
\end{aligned}
$$

Lemma 1229: $x1 = op_r(x1, rd(x2, op_t(x2, x3)), op_t(rd(x2, rd(op_t(x2, x3), x2)), x3))$

$$
\begin{aligned}
& \underbrace{x1} \\
= & \quad \text{by Lemma 1228 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, op_r(\underbrace{i(rd(op_t(x2, x3), x2))}, x2, x2), op_t(rd(x2, rd(op_t(x2, x3), x2)), x3))} \\
= & \quad \text{by Lemma 10 LR with } \{x2 \leftarrow x2, x1 \leftarrow op_t(x2, x3)\} \\
& op_r(x1, \underbrace{op_r(\overbrace{rd(x2, op_t(x2, x3))}, x2, x2)}, op_t(rd(x2, rd(op_t(x2, x3), x2)), x3)) \\
= & \quad \text{by Lemma 1005 RL with } \{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x2\} \\
& op_r(x1, \overbrace{rd(x2, op_t(x2, x3))}, op_t(rd(x2, rd(op_t(x2, x3), x2)), x3))
\end{aligned}
$$

Lemma 1230: $op_r(x1, rd(x2, op_t(x2, x3)), x2) = x1$

$$
\begin{aligned}
& op_r(x1, rd(x2, op_t(x2, x3)), \underbrace{x2}) \\
= & \quad \text{by Lemma 401 RL with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& \underbrace{op_r(x1, rd(x2, op_t(x2, x3)), \overbrace{op_t(rd(x2, rd(op_t(x2, x3), x2)), x3)})} \\
= & \quad \text{by Lemma 1229 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{x1}
\end{aligned}
$$

Lemma 1231: $x1 = op_r(x1, asoc(x2, x3, x3), x2)$

$$
\begin{aligned}
& \overbrace{x1} \\
= \quad & \text{by Lemma 1230 RL with } \{x3 \leftarrow rd(x3, x2), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, rd(x2, \underbrace{op_t(x2, rd(x3, x2))}), x2)} \\
= \quad & \text{by Lemma 547 LR with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& op_r(x1, \underbrace{rd(x2, \overbrace{op_r(x2, x3, i(x3))})}, x2) \\
= \quad & \text{by Lemma 777 LR with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& op_r(x1, \overbrace{asoc(x2, x3, x3)}, x2)
\end{aligned}
$$

Lemma 1232: $x1 = op_r(x1, rd(i(x2), op_r(i(x2), x3, i(x3))), i(x2))$

$$
\begin{aligned}
& \overbrace{x1} \\
= \quad & \text{by Lemma 1230 RL with } \{x3 \leftarrow rd(x2, x3), x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, rd(i(x2), \underbrace{op_t(i(x2), rd(x2, x3))}), i(x2))} \\
= \quad & \text{by Lemma 714 RL with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& op_r(x1, rd(i(x2), \overbrace{\underbrace{op_t(i(x2), mult(x2, x3))}}), i(x2)) \\
= \quad & \text{by Lemma 902 RL with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& op_r(x1, rd(i(x2), \overbrace{op_r(i(x2), x3, i(x3))}), i(x2))
\end{aligned}
$$

Lemma 1233: $x1 = op_r(x1, asoc(x3, x2, x3), i(x2))$

$$
\begin{aligned}
& \overbrace{x1} \\
= \quad & \text{by Lemma 1232 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, \underbrace{rd(i(x2), op_r(i(x2), x3, i(x3)))}, i(x2))} \\
= \quad & \text{by Lemma 777 LR with } \{x2 \leftarrow x3, x1 \leftarrow i(x2)\} \\
& op_r(x1, \underbrace{\overbrace{asoc(i(x2), x3, x3)}}, i(x2)) \\
= \quad & \text{by Lemma 758 LR with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& op_r(x1, \overbrace{asoc(x3, x2, x3)}, i(x2))
\end{aligned}
$$

Lemma 1234: $x1 = op_r(x1, rd(x2, op_t(x2, x3)), i(x3))$

$\overbrace{x1}$

$=$ by Lemma 1227 RL with $\{x3 \leftarrow op_t(i(x2), x3), x2 \leftarrow rd(x2, mult(x2, x3)), x1 \leftarrow x1\}$

$\overbrace{op_r(x1, rd(rd(x2, mult(x2, x3)), \underbrace{op_t(rd(x2, mult(x2, x3)), op_t(i(x2), x3))}), op_t(rd(x2, mult(x2, x3)), op_t(i(x2), x3)))}$

$=$ by Lemma 1013 RL with $\{x1 \leftarrow x2, x2 \leftarrow x3\}$

$op_r(x1, \underbrace{rd(rd(x2, mult(x2, x3)), \overbrace{i(x3)})}, op_t(rd(x2, mult(x2, x3)), op_t(i(x2), x3)))$

$=$ by Lemma 39 LR with $\{x3 \leftarrow x3, x2 \leftarrow mult(x2, x3), x1 \leftarrow x2\}$

$op_r(x1, \underbrace{\overbrace{rd(x3, rd(mult(x2, x3), x2))}}, op_t(rd(x2, mult(x2, x3)), op_t(i(x2), x3)))$

$=$ by Lemma 396 RL with $\{x2 \leftarrow x3, x1 \leftarrow x2\}$

$op_r(x1, \overbrace{rd(x2, op_t(x2, x3))}, \underbrace{op_t(rd(x2, mult(x2, x3)), op_t(i(x2), x3))})$

$=$ by Lemma 1013 RL with $\{x1 \leftarrow x2, x2 \leftarrow x3\}$

$op_r(x1, rd(x2, op_t(x2, x3)), \overbrace{i(x3)})$

Lemma 1235: $rd(mult(x1, op_t(x2, x3)), x2) = rd(x1, rd(x2, op_t(x2, x3)))$

$rd(mult(x1, op_t(x2, x3)), \underbrace{x2})$

$=$ by Axiom 4 LR with $\{x2 \leftarrow op_t(x2, x3), x1 \leftarrow x2\}$

$\underbrace{rd(mult(x1, op_t(x2, x3)), \overbrace{mult(rd(x2, op_t(x2, x3)), op_t(x2, x3))})}$

$=$ by Lemma 676 RL with $\{x3 \leftarrow op_t(x2, x3), x2 \leftarrow rd(x2, op_t(x2, x3)), x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_r(rd(x1, rd(x2, op_t(x2, x3))), rd(x2, op_t(x2, x3)), op_t(x2, x3))}}$

$=$ by Lemma 1227 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow rd(x1, rd(x2, op_t(x2, x3)))\}$

$\overbrace{rd(x1, rd(x2, op_t(x2, x3)))}$

Lemma 1236: $rd(x1, mult(x2, op_t(x1, x3))) = rd(rd(x1, op_t(x1, x3)), x2)$

$\underbrace{rd(x1, mult(x2, op_t(x1, x3)))}$

$=$ by Lemma 10 RL with $\{x2 \leftarrow x1, x1 \leftarrow mult(x2, op_t(x1, x3))\}$

$\overbrace{i(\underbrace{rd(mult(x2, op_t(x1, x3)), x1)})}$

$=$ by Lemma 1235 LR with $\{x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow x2\}$

$\underbrace{i(\overbrace{rd(x2, rd(x1, op_t(x1, x3)))})}$

$=$ by Lemma 10 LR with $\{x2 \leftarrow rd(x1, op_t(x1, x3)), x1 \leftarrow x2\}$

$\overbrace{rd(rd(x1, op_t(x1, x3)), x2)}$

Lemma 1237: $op_t(x1, rd(x2, rd(x1, op_t(x1, x3)))) = op_t(x1, x2)$

$op_t(x1, \underbrace{rd(x2, rd(x1, op_t(x1, x3)))})$

$=$ by Lemma 1235 RL with $\{x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow x2\}$

$\underbrace{op_t(x1, \overbrace{rd(mult(x2, op_t(x1, x3)), x1)})}$

$=$ by Lemma 1162 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_t(x1, x2)}$

Lemma 1238: $op_t(x1, x2) = op_t(x1, rd(x2, asoc(x1, x3, x3)))$

$\overbrace{op_t(x1, x2)}$

= by Lemma 1237 RL with $\{x3 \leftarrow rd(x3, x1), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_t(x1, rd(x2, rd(x1, \underbrace{op_t(x1, rd(x3, x1))}))))}$

= by Lemma 547 LR with $\{x2 \leftarrow x3, x1 \leftarrow x1\}$

$op_t(x1, rd(x2, \underbrace{rd(x1, \overbrace{op_r(x1, x3, i(x3))})}))$

= by Lemma 777 LR with $\{x2 \leftarrow x3, x1 \leftarrow x1\}$

$op_t(x1, rd(x2, \overbrace{asoc(x1, x3, x3)}))$

Lemma 1239: $op_t(x1, mult(x2, asoc(x3, x1, x3))) = op_t(x1, x2)$

$op_t(x1, \underbrace{mult(x2, asoc(x3, x1, x3))})$

= by Axiom 5 LR with $\{x2 \leftarrow asoc(x1, x3, x3), x1 \leftarrow mult(x2, asoc(x3, x1, x3))\}$

$\underbrace{op_t(x1, \overbrace{rd(mult(mult(x2, asoc(x3, x1, x3)), asoc(x1, x3, x3)), asoc(x1, x3, x3))})}$

= by Lemma 1238 RL with $\{x3 \leftarrow x3, x2 \leftarrow mult(mult(x2, asoc(x3, x1, x3)), asoc(x1, x3, x3)), x1 \leftarrow x1\}$

$\overbrace{op_t(x1, \underbrace{mult(mult(x2, asoc(x3, x1, x3)), asoc(x1, x3, x3))})}$

= by Axiom 5 LR with $\{x2 \leftarrow asoc(x1, x3, x3), x1 \leftarrow mult(mult(x2, asoc(x3, x1, x3)), asoc(x1, x3, x3))\}$

$\underbrace{op_t(x1, \overbrace{rd(mult(mult(mult(x2, asoc(x3, x1, x3)), asoc(x1, x3, x3)), asoc(x1, x3, x3)), asoc(x1, x3, x3))})}$

= by Lemma 1238 RL with $\{x3 \leftarrow x3, x2 \leftarrow mult(mult(mult(x2, asoc(x3, x1, x3)), asoc(x1, x3, x3)), asoc(x1, x3, x3)), x1 \leftarrow x1$

$\overbrace{op_t(x1, \underbrace{mult(mult(mult(x2, asoc(x3, x1, x3)), asoc(x1, x3, x3)), asoc(x1, x3, x3))})}$

= by Lemma 1115 RL with $\{x3 \leftarrow mult(mult(mult(x2, asoc(x3, x1, x3)), asoc(x1, x3, x3)), asoc(x1, x3, x3)), x2 \leftarrow x3, x1 \leftarrow x1$

$\overbrace{op_t(x1, \underbrace{mult(asoc(x1, x3, x3), mult(mult(mult(x2, asoc(x3, x1, x3)), asoc(x1, x3, x3)), asoc(x1, x3, x3)))})}$

= by Lemma 1109 RL with $\{x2 \leftarrow mult(x2, asoc(x3, x1, x3)), x1 \leftarrow asoc(x1, x3, x3)\}$

$\underbrace{op_t(x1, \overbrace{mult(mult(asoc(x1, x3, x3), rd(mult(x2, asoc(x3, x1, x3)), i(asoc(x1, x3, x3)))), asoc(x1, x3, x3))})}$

= by Lemma 1238 LR with $\{x3 \leftarrow x3, x2 \leftarrow mult(mult(asoc(x1, x3, x3), rd(mult(x2, asoc(x3, x1, x3)), i(asoc(x1, x3, x3)))), asc$

$\overbrace{op_t(x1, \underbrace{rd(mult(mult(asoc(x1, x3, x3), rd(mult(x2, asoc(x3, x1, x3)), i(asoc(x1, x3, x3)))), asoc(x1, x3, x3)), asoc(x1, x3, x3}$

= by Axiom 5 RL with $\{x2 \leftarrow asoc(x1, x3, x3), x1 \leftarrow mult(asoc(x1, x3, x3), rd(mult(x2, asoc(x3, x1, x3)), i(asoc(x1, x3, x3))))\}$

$\underbrace{op_t(x1, \overbrace{mult(asoc(x1, x3, x3), rd(mult(x2, asoc(x3, x1, x3)), i(asoc(x1, x3, x3))))})}$

= by Lemma 1115 LR with $\{x3 \leftarrow rd(mult(x2, asoc(x3, x1, x3)), i(asoc(x1, x3, x3))), x2 \leftarrow x3, x1 \leftarrow x1\}$

$\overbrace{op_t(x1, rd(mult(x2, asoc(x3, x1, x3)), \underbrace{i(asoc(x1, x3, x3))}))}$

= by Lemma 759 LR with $\{x2 \leftarrow x3, x1 \leftarrow x1\}$

$op_t(x1, \underbrace{rd(mult(x2, asoc(x3, x1, x3)), \overbrace{asoc(x3, x1, x3)})})$

= by Axiom 5 RL with $\{x2 \leftarrow asoc(x3, x1, x3), x1 \leftarrow x2\}$

$op_t(x1, \overbrace{x2})$

Lemma 1240: $op_t(x1, x2) = op_t(x1, rd(mult(x2, x3), op_t(x3, x1)))$

$$
\begin{aligned}
& \underbrace{op_t(x1, x2)} \\
= \quad & \text{by Lemma 1239 RL with } \{x3 \leftarrow mult(x3, x1), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, mult(x2, \underbrace{asoc(mult(x3, x1), x1, mult(x3, x1))}))} \\
= \quad & \text{by Lemma 838 LR with } \{x2 \leftarrow x1, x1 \leftarrow x3\} \\
& op_t(x1, \underbrace{mult(x2, \overbrace{rd(x3, op_t(x3, x1))})}) \\
= \quad & \text{by Lemma 1226 LR with } \{x3 \leftarrow x1, x2 \leftarrow x3, x1 \leftarrow x2\} \\
& op_t(x1, \overbrace{rd(mult(x2, x3), op_t(x3, x1))})
\end{aligned}
$$

Lemma 1241: $x1 = op_r(x1, rd(mult(x2, x3), rd(x3, i(x2))), rd(i(x2), x3))$

$$
\begin{aligned}
& \underbrace{x1} \\
= \quad & \text{by Lemma 1230 RL with } \{x3 \leftarrow x2, x2 \leftarrow rd(i(x2), x3), x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, rd(rd(i(x2), x3), \underbrace{op_t(rd(i(x2), x3), x2)}), rd(i(x2), x3))} \\
= \quad & \text{by Lemma 33 LR with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& op_r(x1, \underbrace{rd(rd(i(x2), x3), \overbrace{i(mult(x2, x3))})}, rd(i(x2), x3)) \\
= \quad & \text{by Lemma 39 LR with } \{x3 \leftarrow mult(x2, x3), x2 \leftarrow x3, x1 \leftarrow i(x2)\} \\
& op_r(x1, \overbrace{rd(mult(x2, x3), rd(x3, i(x2)))}, rd(i(x2), x3))
\end{aligned}
$$

Lemma 1242: $rd(mult(x1, x2), op_r(x2, x3, x3)) = rd(x1, asoc(x2, x3, x3))$

$$
\begin{aligned}
& rd(mult(x1, x2), \underbrace{op_r(x2, x3, x3)}) \\
= \quad & \text{by Lemma 767 RL with } \{x1 \leftarrow x3, x2 \leftarrow x2\} \\
& \underbrace{rd(mult(x1, x2), \overbrace{mult(asoc(x2, x3, x3), x2)})} \\
= \quad & \text{by Lemma 676 RL with } \{x3 \leftarrow x2, x2 \leftarrow asoc(x2, x3, x3), x1 \leftarrow x1\} \\
& \underbrace{\overbrace{op_r(rd(x1, asoc(x2, x3, x3)), asoc(x2, x3, x3), x2)}} \\
= \quad & \text{by Lemma 1231 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow rd(x1, asoc(x2, x3, x3))\} \\
& \overbrace{rd(x1, asoc(x2, x3, x3))}
\end{aligned}
$$

Lemma 1243: $rd(rd(x1, x2), asoc(x2, x3, x3)) = rd(x1, op_r(x2, x3, x3))$

$$
\begin{aligned}
& \underbrace{rd(rd(x1, x2), asoc(x2, x3, x3))} \\
= \quad & \text{by Lemma 1231 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow rd(rd(x1, x2), asoc(x2, x3, x3))\} \\
& \underbrace{\overbrace{op_r(rd(rd(x1, x2), asoc(x2, x3, x3)), asoc(x2, x3, x3), x2)}} \\
= \quad & \text{by Lemma 839 LR with } \{x3 \leftarrow asoc(x2, x3, x3), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{rd(x1, \underbrace{mult(asoc(x2, x3, x3), x2)})} \\
= \quad & \text{by Lemma 767 LR with } \{x1 \leftarrow x3, x2 \leftarrow x2\} \\
& rd(x1, \overbrace{op_r(x2, x3, x3)})
\end{aligned}
$$

Lemma 1244: $op_t(x1, rd(x2, op_r(x1, x3, x3))) = op_t(x1, rd(x2, x1))$

$$
\begin{aligned}
& op_t(x1, \underbrace{rd(x2, op_r(x1, x3, x3))}) \\
= & \quad \text{by Lemma 1243 RL with } \{x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \underbrace{op_t(x1, \overbrace{rd(rd(x2, x1), asoc(x1, x3, x3))})} \\
= & \quad \text{by Lemma 1238 RL with } \{x3 \leftarrow x3, x2 \leftarrow rd(x2, x1), x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, rd(x2, x1))}
\end{aligned}
$$

Lemma 1245: $rd(op_r(x1, x2, x2), mult(x3, x1)) = rd(asoc(x1, x2, x2), x3)$

$$
\begin{aligned}
& rd(\underbrace{op_r(x1, x2, x2)}, mult(x3, x1)) \\
= & \quad \text{by Lemma 767 RL with } \{x1 \leftarrow x2, x2 \leftarrow x1\} \\
& rd(\overbrace{mult(\underbrace{asoc(x1, x2, x2)}, x1)}, mult(x3, x1)) \\
= & \quad \text{by Lemma 757 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \underbrace{rd(mult(\overbrace{i(asoc(x2, x1, x2))}, x1), mult(x3, x1))} \\
= & \quad \text{by Lemma 62 RL with } \{x3 \leftarrow x1, x2 \leftarrow asoc(x2, x1, x2), x1 \leftarrow mult(x3, x1)\} \\
& \overbrace{rd(\underbrace{i(mult(x3, x1))}, mult(asoc(x2, x1, x2), i(x1)))} \\
= & \quad \text{by Axiom 11 RL with } \{x2 \leftarrow x1, x1 \leftarrow x3\} \\
& rd(\overbrace{mult(\underbrace{i(x3)}, i(x1))}, mult(asoc(x2, x1, x2), i(x1))) \\
= & \quad \text{by Lemma 21 RL with } \{x2 \leftarrow x3, x1 \leftarrow asoc(x2, x1, x2)\} \\
& \underbrace{rd(mult(\overbrace{mult(rd(i(asoc(x2, x1, x2)), x3), asoc(x2, x1, x2))}, i(x1)), mult(asoc(x2, x1, x2), i(x1)))} \\
= & \quad \text{by Axiom 16 RL with } \{x3 \leftarrow i(x1), x2 \leftarrow asoc(x2, x1, x2), x1 \leftarrow rd(i(asoc(x2, x1, x2)), x3)\} \\
& \overbrace{\underbrace{op_r(rd(i(asoc(x2, x1, x2)), x3), asoc(x2, x1, x2), i(x1))}} \\
= & \quad \text{by Lemma 1233 RL with } \{x2 \leftarrow x1, x3 \leftarrow x2, x1 \leftarrow rd(i(asoc(x2, x1, x2)), x3)\} \\
& \overbrace{rd(\underbrace{i(asoc(x2, x1, x2))}, x3)} \\
= & \quad \text{by Lemma 757 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& rd(\overbrace{asoc(x1, x2, x2)}, x3)
\end{aligned}
$$

Lemma 1246: $rd(mult(x1, op_r(x2, x3, x3)), x2) = mult(x1, asoc(x2, x3, x3))$

$$
\begin{aligned}
& rd(mult(x1, \underbrace{op_r(x2, x3, x3)}), x2) \\
= & \quad \text{by Lemma 767 RL with } \{x1 \leftarrow x3, x2 \leftarrow x2\} \\
& rd(mult(\underbrace{x1}, \overbrace{mult(asoc(x2, x3, x3), x2)}), x2) \\
= & \quad \text{by Lemma 1231 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& rd(\underbrace{mult(\overbrace{op_r(x1, asoc(x2, x3, x3), x2)}, mult(asoc(x2, x3, x3), x2))}, x2) \\
= & \quad \text{by Lemma 91 LR with } \{x3 \leftarrow x2, x2 \leftarrow asoc(x2, x3, x3), x1 \leftarrow x1\} \\
& \underbrace{rd(\overbrace{mult(mult(x1, asoc(x2, x3, x3)), x2)}, x2)} \\
= & \quad \text{by Axiom 5 RL with } \{x2 \leftarrow x2, x1 \leftarrow mult(x1, asoc(x2, x3, x3))\} \\
& \overbrace{mult(x1, asoc(x2, x3, x3))}
\end{aligned}
$$

Lemma 1247: $rd(rd(x1,x2),asoc(x3,x1,x3)) = rd(i(x2),op_r(i(x1),x3,x3))$

$$
\begin{aligned}
& rd(rd(x1,x2),\underbrace{asoc(x3,x1,x3)}) \\
= & \quad \text{by Lemma 758 RL with } \{x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \underbrace{rd(rd(x1,x2),\overbrace{asoc(i(x1),x3,x3)})} \\
= & \quad \text{by Lemma 1242 RL with } \{x3 \leftarrow x3, x2 \leftarrow i(x1), x1 \leftarrow rd(x1,x2)\} \\
& \overbrace{rd(\underbrace{mult(rd(x1,x2),i(x1))},op_r(i(x1),x3,x3))} \\
= & \quad \text{by Lemma 22 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& rd(\overbrace{i(x2)},op_r(i(x1),x3,x3))
\end{aligned}
$$

Lemma 1248: $rd(rd(x1,x2),asoc(x3,x1,x3)) = rd(op_r(x1,x3,x3),x2)$

$$
\begin{aligned}
& \underbrace{rd(rd(x1,x2),asoc(x3,x1,x3))} \\
= & \quad \text{by Lemma 1247 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{\overbrace{rd(i(x2),op_r(i(x1),x3,x3))}} \\
= & \quad \text{by Lemma 918 RL with } \{x3 \leftarrow x2, x1 \leftarrow x3, x2 \leftarrow x1\} \\
& \overbrace{rd(op_r(x1,x3,x3),x2)}
\end{aligned}
$$

Lemma 1249: $rd(asoc(x1,x2,x2),rd(x3,x1)) = rd(op_r(x1,x2,x2),x3)$

$$
\begin{aligned}
& \underbrace{rd(asoc(x1,x2,x2),rd(x3,x1))} \\
= & \quad \text{by Lemma 10 RL with } \{x2 \leftarrow asoc(x1,x2,x2), x1 \leftarrow rd(x3,x1)\} \\
& \overbrace{i(\underbrace{rd(rd(x3,x1),asoc(x1,x2,x2))})} \\
= & \quad \text{by Lemma 1243 LR with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x3\} \\
& \underbrace{i(\overbrace{rd(x3,op_r(x1,x2,x2))})} \\
= & \quad \text{by Lemma 10 LR with } \{x2 \leftarrow op_r(x1,x2,x2), x1 \leftarrow x3\} \\
& \overbrace{rd(op_r(x1,x2,x2),x3)}
\end{aligned}
$$

Lemma 1250: $rd(asoc(x2,x1,x2),rd(x1,x3)) = rd(op_r(i(x1),x2,x2),i(x3))$

$$
\begin{aligned}
& rd(\underbrace{asoc(x2,x1,x2)},rd(x1,x3)) \\
= & \quad \text{by Lemma 758 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{rd(\overbrace{asoc(i(x1),x2,x2)},rd(x1,x3))} \\
= & \quad \text{by Lemma 1245 RL with } \{x3 \leftarrow rd(x1,x3), x2 \leftarrow x2, x1 \leftarrow i(x1)\} \\
& \overbrace{rd(op_r(i(x1),x2,x2),\underbrace{mult(rd(x1,x3),i(x1))})} \\
= & \quad \text{by Lemma 22 LR with } \{x2 \leftarrow x3, x1 \leftarrow x1\} \\
& rd(op_r(i(x1),x2,x2),\overbrace{i(x3)})
\end{aligned}
$$

Lemma 1251: $rd(asoc(x1,x2,x1),rd(x2,x3)) = rd(x3,op_r(x2,x1,x1))$

$$
\begin{aligned}
& \underbrace{rd(asoc(x1,x2,x1),rd(x2,x3))} \\
= & \quad \text{by Lemma 1250 LR with } \{x3 \leftarrow x3, x1 \leftarrow x2, x2 \leftarrow x1\} \\
& \overbrace{\underbrace{rd(op_r(i(x2),x1,x1),i(x3))}} \\
= & \quad \text{by Lemma 917 RL with } \{x2 \leftarrow x1, x3 \leftarrow x2, x1 \leftarrow x3\} \\
& \overbrace{rd(x3,op_r(x2,x1,x1))}
\end{aligned}
$$

Lemma 1252: $op_l(x1,x2,x1) = op_l(x1,x1,rd(asoc(x1,x3,x3),x2))$

$$
\begin{aligned}
& \underbrace{op_l(x1,x2,x1)} \\
= & \quad \text{by Lemma 285 RL with } \{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1,mult(x2,x1),x1)}} \\
= & \quad \text{by Lemma 770 LR with } \{x2 \leftarrow mult(x2,x1), x1 \leftarrow x1\} \\
& \overbrace{\underbrace{mult(x1,asoc(mult(x2,x1),x1,x1))}} \\
= & \quad \text{by Lemma 1044 RL with } \{x3 \leftarrow mult(x2,x1), x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1,x1,\underbrace{rd(op_r(x1,x3,x3),mult(x2,x1))})} \\
= & \quad \text{by Lemma 1245 LR with } \{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x1\} \\
& op_l(x1,x1,\overbrace{rd(asoc(x1,x3,x3),x2)})
\end{aligned}
$$

Lemma 1253: $mult(mult(x1,x2),i(op_t(x1,x3))) = mult(x1,rd(x2,op_t(x1,x3)))$

$$
\begin{aligned}
& \underbrace{mult(mult(x1,x2),i(op_t(x1,x3)))} \\
= & \quad \text{by Lemma 137 RL with } \{x3 \leftarrow i(op_t(x1,x3)), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{mult(x1,mult(x2,\underbrace{op_l(i(op_t(x1,x3)),x1,x2)}))} \\
= & \quad \text{by Lemma 1049 RL with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow op_t(x1,x3)\} \\
& mult(x1,mult(x2,\overbrace{i(\underbrace{op_l(op_t(x1,x3),x1,x2)})})) \\
= & \quad \text{by Axiom 15 RL with } \{x4 \leftarrow x3, x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x1\} \\
& mult(x1,mult(x2,i(\overbrace{op_t(\underbrace{op_l(x1,x1,x2)},x3)}))) \\
= & \quad \text{by Lemma 427 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& mult(x1,\underbrace{mult(x2,i(op_t(\overbrace{mult(x2,rd(x1,x2))},x3)))}) \\
= & \quad \text{by Lemma 1150 LR with } \{x3 \leftarrow x3, x2 \leftarrow rd(x1,x2), x1 \leftarrow x2\} \\
& mult(x1,\overbrace{rd(x2,op_t(\underbrace{mult(rd(x1,x2),x2)},x3))}) \\
= & \quad \text{by Axiom 4 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& mult(x1,rd(x2,op_t(\overbrace{x1},x3)))
\end{aligned}
$$

Lemma 1254: $rd(mult(x1, i(x2)), i(op_t(x2, x3))) = mult(x1, rd(i(x2), i(op_t(x2, x3))))$

$$
\begin{aligned}
& rd(mult(x1, i(x2)), \underbrace{i(op_t(x2, x3))}) \\
= & \quad \text{by Lemma 15 RL with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& \underbrace{rd(mult(x1, i(x2)), \overbrace{op_t(i(x2), i(x3))})} \\
= & \quad \text{by Lemma 1226 RL with } \{x3 \leftarrow i(x3), x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& \overbrace{mult(x1, rd(i(x2), \underbrace{op_t(i(x2), i(x3))}))} \\
= & \quad \text{by Lemma 15 LR with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& mult(x1, rd(i(x2), \overbrace{i(op_t(x2, x3))}))
\end{aligned}
$$

Lemma 1255: $rd(op_t(x2, x3), mult(i(x1), x2)) = mult(x1, rd(op_t(x2, x3), x2))$

$$
\begin{aligned}
& \underbrace{rd(op_t(x2, x3), mult(i(x1), x2))} \\
= & \quad \text{by Lemma 61 RL with } \{x3 \leftarrow op_t(x2, x3), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{rd(mult(x1, i(x2)), i(op_t(x2, x3)))}} \\
= & \quad \text{by Lemma 1254 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{mult(x1, \underbrace{rd(i(x2), i(op_t(x2, x3)))})} \\
= & \quad \text{by Lemma 23 LR with } \{x2 \leftarrow op_t(x2, x3), x1 \leftarrow x2\} \\
& mult(x1, \overbrace{rd(op_t(x2, x3), x2)})
\end{aligned}
$$

Lemma 1256: $rd(x1, rd(op_t(x2, i(i(x3))), x2)) = rd(mult(x1, i(op_t(x2, x3))), i(x2))$

$$
\begin{aligned}
& rd(x1, \underbrace{rd(op_t(x2, i(i(x3))), x2)}) \\
= & \quad \text{by Lemma 68 RL with } \{x3 \leftarrow i(x3), x2 \leftarrow x2, x1 \leftarrow x2\} \\
& \underbrace{rd(x1, \overbrace{rd(i(x2), op_t(i(x2), i(x3)))})} \\
= & \quad \text{by Lemma 1235 RL with } \{x3 \leftarrow i(x3), x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& \overbrace{rd(mult(x1, \underbrace{op_t(i(x2), i(x3))}), i(x2))} \\
= & \quad \text{by Lemma 15 LR with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& rd(mult(x1, \overbrace{i(op_t(x2, x3))}), i(x2))
\end{aligned}
$$

Lemma 1257: $rd(x1, rd(op_t(x2, x3), x2)) = rd(x2, mult(i(x1), op_t(x2, x3)))$

$$
\begin{aligned}
& rd(x1, rd(op_t(x2, \underbrace{x3}), x2)) \\
= & \quad \text{by Lemma 3 RL with } \{x1 \leftarrow x3\} \\
& \underbrace{rd(x1, rd(op_t(x2, \overbrace{i(i(x3))}), x2))} \\
= & \quad \text{by Lemma 1256 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{rd(mult(x1, i(op_t(x2, x3))), i(x2))}} \\
= & \quad \text{by Lemma 61 LR with } \{x3 \leftarrow x2, x2 \leftarrow op_t(x2, x3), x1 \leftarrow x1\} \\
& \overbrace{rd(x2, mult(i(x1), op_t(x2, x3)))}
\end{aligned}
$$

Lemma 1258: $op_t(x1, rd(x2, mult(i(x1), x3))) = op_t(x1, rd(x2, rd(x3, x1)))$

$op_t(x1, rd(x2, \underbrace{mult(i(x1), x3)}))$

$=$ by Lemma 14 RL with $\{x2 \leftarrow x1, x1 \leftarrow x3\}$

$op_t(x1, rd(\underbrace{x2}, \overbrace{op_t(rd(x3, x1), x1)}))$

$=$ by Axiom 4 LR with $\{x2 \leftarrow rd(x3, x1), x1 \leftarrow x2\}$

$\underbrace{op_t(x1, rd(\overbrace{mult(rd(x2, rd(x3, x1)), rd(x3, x1))}, op_t(rd(x3, x1), x1)))}$

$=$ by Lemma 1240 RL with $\{x3 \leftarrow rd(x3, x1), x2 \leftarrow rd(x2, rd(x3, x1)), x1 \leftarrow x1\}$

$\overbrace{op_t(x1, rd(x2, rd(x3, x1)))}$

Lemma 1259: $op_t(x1, rd(rd(x1, i(x3)), x2)) = op_t(x1, rd(mult(i(i(x1)), x3), x2))$

$op_t(x1, \underbrace{rd(rd(x1, i(x3)), x2)})$

$=$ by Lemma 40 RL with $\{x3 \leftarrow x1, x2 \leftarrow i(x3), x1 \leftarrow x2\}$

$\underbrace{op_t(x1, \overbrace{rd(i(x2), rd(i(x3), x1))})}$

$=$ by Lemma 1258 RL with $\{x3 \leftarrow i(x3), x2 \leftarrow i(x2), x1 \leftarrow x1\}$

$\overbrace{op_t(x1, \underbrace{rd(i(x2), mult(i(x1), i(x3)))})}$

$=$ by Lemma 62 LR with $\{x3 \leftarrow x3, x2 \leftarrow i(x1), x1 \leftarrow x2\}$

$op_t(x1, \overbrace{rd(mult(i(i(x1)), x3), x2)})$

Lemma 1260: $op_t(x1, rd(rd(x1, i(x2)), x3)) = op_t(x1, rd(mult(x1, x2), x3))$

$\underbrace{op_t(x1, rd(rd(x1, i(x2)), x3))}$

$=$ by Lemma 1259 LR with $\{x2 \leftarrow x3, x3 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_t(x1, rd(mult(\underbrace{i(i(x1))}, x2), x3))}$

$=$ by Lemma 3 LR with $\{x1 \leftarrow x1\}$

$op_t(x1, rd(mult(\overbrace{x1}, x2), x3))$

Lemma 1261: $mult(rd(x1, rd(x2, op_t(x2, x3))), x2) = mult(x1, op_t(x2, x3))$

$mult(rd(x1, \underbrace{rd(x2, op_t(x2, x3))}), x2)$

$=$ by Lemma 98 LR with $\{x2 \leftarrow x3, x1 \leftarrow x2\}$

$\underbrace{mult(rd(x1, \overbrace{mult(x2, i(op_t(x2, x3)))}), x2)}$

$=$ by Lemma 1153 RL with $\{x3 \leftarrow i(op_t(x2, x3)), x2 \leftarrow x1, x1 \leftarrow x2\}$

$\overbrace{mult(i(x2), \underbrace{rd(mult(x2, x1), i(op_t(x2, x3)))})}$

$=$ by Lemma 1132 LR with $\{x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow x2\}$

$\underbrace{mult(i(x2), \overbrace{mult(x2, mult(x1, op_t(x2, x3)))})}$

$=$ by Axiom 2 RL with $\{x2 \leftarrow x2, x1 \leftarrow mult(x1, op_t(x2, x3))\}$

$\overbrace{mult(x1, op_t(x2, x3))}$

Lemma 1262: $mult(x1, op_t(x2, rd(x3, x2))) = mult(rd(x1, asoc(x2, x3, x3)), x2)$

$\underbrace{mult(x1, op_t(x2, rd(x3, x2)))}$

$=$ by Lemma 1261 RL with $\{x3 \leftarrow rd(x3, x2), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{mult(rd(x1, rd(x2, \underbrace{op_t(x2, rd(x3, x2))})), x2)}$

$=$ by Lemma 547 LR with $\{x2 \leftarrow x3, x1 \leftarrow x2\}$

$mult(rd(x1, \underbrace{rd(x2, \overbrace{op_r(x2, x3, i(x3))})}), x2)$

$=$ by Lemma 777 LR with $\{x2 \leftarrow x3, x1 \leftarrow x2\}$

$mult(rd(x1, \overbrace{asoc(x2, x3, x3)}), x2)$

Lemma 1263: $mult(op_t(x1, x2), mult(op_t(x2, x1), x3)) = mult(x1, mult(x2, x3))$

$mult(op_t(\underbrace{x1}, x2), mult(op_t(x2, x1), x3))$

$=$ by Lemma 8 LR with $\{x1 \leftarrow x1\}$

$mult(\underbrace{op_t(\overbrace{op_t(x1, i(x1))}, x2)}, mult(op_t(x2, x1), x3))$

$=$ by Axiom 13 LR with $\{x3 \leftarrow x2, x2 \leftarrow i(x1), x1 \leftarrow x1\}$

$mult(\overbrace{op_t(op_t(x1, x2), i(x1))}, mult(op_t(x2, x1), x3))$

$=$ by Lemma 391 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$mult(\overbrace{mult(x1, rd(x2, op_t(x2, x1)))}, \underbrace{mult(op_t(x2, x1), x3)})$

$=$ by Lemma 392 RL with $\{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow mult(op_t(x2, x1), x3)\}$

$\underbrace{mult(mult(x1, rd(x2, op_t(x2, x1))), \overbrace{op_l(mult(op_t(x2, x1), x3), rd(x2, op_t(x2, x1)), x1)})}$

$=$ by Lemma 132 LR with $\{x3 \leftarrow mult(op_t(x2, x1), x3), x2 \leftarrow rd(x2, op_t(x2, x1)), x1 \leftarrow x1\}$

$\overbrace{mult(x1, \underbrace{mult(rd(x2, op_t(x2, x1)), mult(op_t(x2, x1), x3))})}$

$=$ by Lemma 1091 LR with $\{x3 \leftarrow x3, x2 \leftarrow op_t(x2, x1), x1 \leftarrow x2\}$

$mult(x1, \overbrace{mult(x2, \underbrace{op_l(x3, op_t(x2, x1), x2)})})$

$=$ by Lemma 163 LR with $\{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x3\}$

$mult(x1, mult(x2, \overbrace{x3}))$

Lemma 1264: $op_l(x1, op_t(x2, x3), op_t(x3, x2)) = op_l(x1, x2, x3)$

$\underbrace{op_l(x1, op_t(x2, x3), op_t(x3, x2))}$

$=$ by Axiom 12 RL with $\{x3 \leftarrow x1, x2 \leftarrow op_t(x2, x3), x1 \leftarrow op_t(x3, x2)\}$

$\overbrace{mult(i(mult(op_t(x3, x2), op_t(x2, x3))), \underbrace{mult(op_t(x3, x2), mult(op_t(x2, x3), x1))})}$

$=$ by Lemma 1263 LR with $\{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x3\}$

$mult(i(\underbrace{mult(op_t(x3, x2), op_t(x2, x3))}), \overbrace{mult(x3, mult(x2, x1))})$

$=$ by Lemma 377 RL with $\{x1 \leftarrow x2, x2 \leftarrow x3\}$

$\underbrace{mult(i(\overbrace{mult(x3, x2)}), mult(x3, mult(x2, x1)))}$

$=$ by Axiom 12 LR with $\{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x3\}$

$\overbrace{op_l(x1, x2, x3)}$

Lemma 1265: $mult(x1, mult(x2, op_t(x2, mult(x1, x2)))) = mult(mult(x1, x2), op_t(x2, x1))$

$\underbrace{mult(x1, mult(x2, op_t(x2, mult(x1, x2))))}$

$=$ by Lemma 1068 RL with $\{x3 \leftarrow op_t(x2, mult(x1, x2)), x2 \leftarrow x1, x1 \leftarrow x2\}$

$\overbrace{op_l(\underbrace{mult(mult(x2, x1), op_t(x2, mult(x1, x2)))}, x2, x1)}$

$=$ by Lemma 1178 LR with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$op_l(\overbrace{mult(x2, \underbrace{mult(x2, x1)})}, x2, x1)$

$=$ by Lemma 13 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{op_l(mult(x2, \overbrace{mult(x1, op_t(x2, x1))}), x2, x1)}$

$=$ by Lemma 1047 LR with $\{x3 \leftarrow op_t(x2, x1), x2 \leftarrow x1, x1 \leftarrow x2\}$

$\overbrace{mult(mult(x1, x2), op_t(x2, x1))}$

Lemma 1266: $op_t(x1, rd(rd(x2, x1), mult(i(i(x1)), x3))) = op_t(x1, mult(i(x1), mult(rd(x2, x3), i(x1))))$

$\underbrace{op_t(x1, rd(rd(x2, x1), mult(i(i(x1)), x3)))}$

$=$ by Lemma 1215 RL with $\{x3 \leftarrow mult(i(i(x1)), x3), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_t(x1, \underbrace{rd(mult(i(x1), x2), mult(i(i(x1)), x3))})}$

$=$ by Lemma 1152 RL with $\{x3 \leftarrow mult(i(i(x1)), x3), x2 \leftarrow x2, x1 \leftarrow i(x1)\}$

$op_t(x1, \overbrace{mult(i(x1), mult(rd(x2, \underbrace{mult(i(x1), mult(i(i(x1)), x3))}), i(x1))))}$

$=$ by Lemma 12 LR with $\{x2 \leftarrow x3, x1 \leftarrow i(x1)\}$

$op_t(x1, mult(i(x1), mult(rd(x2, \overbrace{x3}), i(x1))))$

Lemma 1267: $op_t(x1, rd(rd(x2, x1), mult(x1, x3))) = op_t(x1, rd(mult(rd(x2, x3), i(x1)), x1))$

$op_t(x1, rd(rd(x2, x1), mult(\underbrace{x1}, x3)))$

$=$ by Lemma 3 RL with $\{x1 \leftarrow x1\}$

$\underbrace{op_t(x1, rd(rd(x2, x1), mult(\overbrace{i(i(x1))}, x3)))}$

$=$ by Lemma 1266 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_t(x1, mult(i(x1), mult(rd(x2, x3), i(x1))))}}$

$=$ by Lemma 375 LR with $\{x2 \leftarrow mult(rd(x2, x3), i(x1)), x1 \leftarrow x1\}$

$\overbrace{op_t(x1, rd(mult(rd(x2, x3), i(x1)), x1))}$

Lemma 1268: $op_t(x1, rd(rd(x2, x1), mult(x1, x3))) = op_t(x1, i(rd(x2, x3)))$

$\underbrace{op_t(x1, rd(rd(x2, x1), mult(x1, x3)))}$

$=$ by Lemma 1267 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_t(x1, rd(mult(rd(x2, x3), i(x1)), x1))}}$

$=$ by Lemma 707 RL with $\{x2 \leftarrow mult(rd(x2, x3), i(x1)), x1 \leftarrow x1\}$

$\overbrace{op_t(x1, \underbrace{rd(i(x1), mult(rd(x2, x3), i(x1)))})}$

$=$ by Lemma 9 LR with $\{x2 \leftarrow rd(x2, x3), x1 \leftarrow i(x1)\}$

$op_t(x1, \overbrace{i(rd(x2, x3))})$

Lemma 1269: $op_t(x1, rd(rd(x2, x1), mult(x1, x3))) = op_t(x1, rd(x3, x2))$

$$
\begin{aligned}
& \underbrace{op_t(x1, rd(rd(x2, x1), mult(x1, x3)))} \\
= \quad & \text{by Lemma 1268 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, \underbrace{i(rd(x2, x3))})} \\
= \quad & \text{by Lemma 10 LR with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& op_t(x1, \overbrace{rd(x3, x2)})
\end{aligned}
$$

Lemma 1270: $op_t(x1, rd(x2, op_l(x3, x1, x3))) = op_t(x1, rd(rd(x3, x1), mult(x1, x2)))$

$$
\begin{aligned}
& \underbrace{op_t(x1, rd(x2, op_l(x3, x1, x3)))} \\
= \quad & \text{by Lemma 1269 RL with } \{x3 \leftarrow x2, x2 \leftarrow op_l(x3, x1, x3), x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, rd(\underbrace{rd(op_l(x3, x1, x3), x1)}, mult(x1, x2)))} \\
= \quad & \text{by Lemma 615 LR with } \{x2 \leftarrow x1, x1 \leftarrow x3\} \\
& \underbrace{op_t(x1, rd(\overbrace{mult(i(x1), x3)}, mult(x1, x2)))} \\
= \quad & \text{by Lemma 1215 LR with } \{x3 \leftarrow mult(x1, x2), x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, rd(rd(x3, x1), mult(x1, x2)))}
\end{aligned}
$$

Lemma 1271: $op_t(x1, rd(op_l(x2, x1, x2), x3)) = op_t(x1, rd(rd(x3, x1), mult(x1, x2)))$

$$
\begin{aligned}
& \underbrace{op_t(x1, rd(op_l(x2, x1, x2), x3))} \\
= \quad & \text{by Lemma 1269 RL with } \{x3 \leftarrow op_l(x2, x1, x2), x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, rd(rd(x3, x1), \underbrace{mult(x1, op_l(x2, x1, x2))}))} \\
= \quad & \text{by Lemma 616 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{op_t(x1, rd(rd(x3, x1), \overbrace{rd(x1, i(x2))}))} \\
= \quad & \text{by Lemma 1219 LR with } \{x3 \leftarrow x2, x2 \leftarrow rd(x3, x1), x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, rd(rd(x3, x1), mult(x1, x2)))}
\end{aligned}
$$

Lemma 1272: $op_t(x1, i(rd(x2, op_l(x3, i(x1), x3)))) = i(op_t(i(x1), rd(x2, x3)))$

$$
\begin{aligned}
& \underbrace{op_t(x1, i(rd(x2, op_l(x3, i(x1), x3))))} \\
= \quad & \text{by Lemma 18 RL with } \{x2 \leftarrow rd(x2, op_l(x3, i(x1), x3)), x1 \leftarrow x1\} \\
& \overbrace{i(\underbrace{op_t(i(x1), rd(x2, op_l(x3, i(x1), x3)))})} \\
= \quad & \text{by Lemma 1270 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow i(x1)\} \\
& i(\overbrace{\underbrace{op_t(i(x1), rd(rd(x3, i(x1)), mult(i(x1), x2)))}}) \\
= \quad & \text{by Lemma 1269 LR with } \{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow i(x1)\} \\
& i(\overbrace{op_t(i(x1), rd(x2, x3))})
\end{aligned}
$$

Lemma 1273: $op_t(x1, rd(op_l(x3, x1, i(x3)), x2)) = i(op_t(i(x1), rd(x2, x3)))$

$$
\begin{aligned}
& op_t(x1, rd(\underbrace{op_l(x3, x1, i(x3))}, x2)) \\
= & \quad \text{by Lemma 1050 RL with } \{x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow x3\} \\
& op_t(x1, \underbrace{rd(\overbrace{op_l(x3, i(x1), x3)}, x2)}) \\
= & \quad \text{by Lemma 10 RL with } \{x2 \leftarrow op_l(x3, i(x1), x3), x1 \leftarrow x2\} \\
& \underbrace{op_t(x1, \overbrace{i(rd(x2, op_l(x3, i(x1), x3)))})} \\
= & \quad \text{by Lemma 1272 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{i(op_t(i(x1), rd(x2, x3)))}
\end{aligned}
$$

Lemma 1274: $op_t(x1, rd(op_l(x3, x3, x1), x2)) = op_t(x1, i(rd(x2, x3)))$

$$
\begin{aligned}
& op_t(x1, rd(\underbrace{op_l(x3, x3, x1)}, x2)) \\
= & \quad \text{by Lemma 606 RL with } \{x2 \leftarrow x1, x1 \leftarrow x3\} \\
& \underbrace{op_t(x1, rd(\overbrace{op_l(x3, x1, i(x3))}, x2))} \\
= & \quad \text{by Lemma 1273 LR with } \{x2 \leftarrow x2, x3 \leftarrow x3, x1 \leftarrow x1\} \\
& \underbrace{\overbrace{i(op_t(i(x1), rd(x2, x3)))}} \\
= & \quad \text{by Lemma 18 LR with } \{x2 \leftarrow rd(x2, x3), x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, i(rd(x2, x3)))}
\end{aligned}
$$

Lemma 1275: $op_t(x1, i(rd(op_l(x2, i(x1), x2), x3))) = i(op_t(i(x1), rd(x2, x3)))$

$$
\begin{aligned}
& \underbrace{op_t(x1, i(rd(op_l(x2, i(x1), x2), x3)))} \\
= & \quad \text{by Lemma 18 RL with } \{x2 \leftarrow rd(op_l(x2, i(x1), x2), x3), x1 \leftarrow x1\} \\
& \overbrace{i(\underbrace{op_t(i(x1), rd(op_l(x2, i(x1), x2), x3))})} \\
= & \quad \text{by Lemma 1271 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow i(x1)\} \\
& i(\overbrace{\underbrace{op_t(i(x1), rd(rd(x3, i(x1)), mult(i(x1), x2)))}}) \\
= & \quad \text{by Lemma 1269 LR with } \{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow i(x1)\} \\
& i(\overbrace{op_t(i(x1), rd(x2, x3))})
\end{aligned}
$$

Lemma 1276: $op_t(x1, rd(x3, op_l(x2, x1, i(x2)))) = i(op_t(i(x1), rd(x2, x3)))$

$$
\begin{aligned}
& op_t(x1, rd(x3, \underbrace{op_l(x2, x1, i(x2))})) \\
= & \quad \text{by Lemma 1050 RL with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x2\} \\
& op_t(x1, \underbrace{rd(x3, \overbrace{op_l(x2, i(x1), x2)})}) \\
= & \quad \text{by Lemma 10 RL with } \{x2 \leftarrow x3, x1 \leftarrow op_l(x2, i(x1), x2)\} \\
& \underbrace{op_t(x1, \overbrace{i(rd(op_l(x2, i(x1), x2), x3))})} \\
= & \quad \text{by Lemma 1275 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{i(op_t(i(x1), rd(x2, x3)))}
\end{aligned}
$$

Lemma 1277: $op_t(x1, rd(x3, op_l(x2, x2, x1))) = op_t(x1, i(rd(x2, x3)))$

$$
\begin{aligned}
& op_t(x1, rd(x3, \underbrace{op_l(x2, x2, x1)})) \\
= & \quad \text{by Lemma 606 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \underbrace{op_t(x1, rd(x3, \overbrace{op_l(x2, x1, i(x2))}))} \\
= & \quad \text{by Lemma 1276 LR with } \{x2 \leftarrow x2, x3 \leftarrow x3, x1 \leftarrow x1\} \\
& \underbrace{\overbrace{i(op_t(i(x1), rd(x2, x3)))}} \\
= & \quad \text{by Lemma 18 LR with } \{x2 \leftarrow rd(x2, x3), x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, i(rd(x2, x3)))}
\end{aligned}
$$

Lemma 1278: $op_t(x1, rd(x2, op_l(x3, x3, x1))) = op_t(x1, rd(x2, x3))$

$$
\begin{aligned}
& \underbrace{op_t(x1, rd(x2, op_l(x3, x3, x1)))} \\
= & \quad \text{by Lemma 1277 LR with } \{x2 \leftarrow x3, x3 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, \underbrace{i(rd(x3, x2))})} \\
= & \quad \text{by Lemma 10 LR with } \{x2 \leftarrow x2, x1 \leftarrow x3\} \\
& op_t(x1, \overbrace{rd(x2, x3)})
\end{aligned}
$$

Lemma 1279: $op_t(x1, rd(i(mult(x1, x2)), x3)) = op_t(x1, rd(x2, rd(x1, x3)))$

$$
\begin{aligned}
& \underbrace{op_t(x1, rd(i(mult(x1, x2)), x3))} \\
= & \quad \text{by Lemma 1269 RL with } \{x3 \leftarrow i(mult(x1, x2)), x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, rd(rd(x3, x1), \underbrace{mult(x1, i(mult(x1, x2)))}))} \\
= & \quad \text{by Lemma 20 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_t(x1, \underbrace{rd(rd(x3, x1), \overbrace{i(x2)})}) \\
= & \quad \text{by Lemma 39 LR with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x3\} \\
& op_t(x1, \overbrace{rd(x2, rd(x1, x3))})
\end{aligned}
$$

Lemma 1280: $op_t(x1, rd(x2, op_r(x3, x1, x1))) = op_t(x1, rd(x2, x3))$

$$
\begin{aligned}
& op_t(x1, rd(x2, \underbrace{op_r(x3, x1, x1)})) \\
= & \quad \text{by Lemma 353 RL with } \{x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \underbrace{op_t(x1, rd(x2, \overbrace{rd(x1, rd(x1, x3))}))} \\
= & \quad \text{by Lemma 1279 RL with } \{x3 \leftarrow rd(x1, x3), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_t(x1, rd(i(mult(x1, x2)), rd(x1, x3)))}} \\
= & \quad \text{by Lemma 1279 RL with } \{x3 \leftarrow x3, x2 \leftarrow i(mult(x1, x2)), x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, rd(\underbrace{i(mult(x1, i(mult(x1, x2))))}, x3))} \\
= & \quad \text{by Lemma 20 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_t(x1, rd(\underbrace{i(\overbrace{i(x2)})}, x3)) \\
= & \quad \text{by Lemma 3 LR with } \{x1 \leftarrow x2\} \\
& op_t(x1, rd(\overbrace{x2}, x3))
\end{aligned}
$$

Lemma 1281: $op_t(x1, rd(x2, asoc(x1, x3, x1))) = op_t(x1, x2)$

$op_t(x1, rd(x2, \underbrace{asoc(x1, x3, x1)}))$

$=$ by Lemma 758 RL with $\{x2 \leftarrow x1, x1 \leftarrow x3\}$

$op_t(x1, \underbrace{rd(x2, \overbrace{asoc(i(x3), x1, x1)})})$

$=$ by Lemma 1242 RL with $\{x3 \leftarrow x1, x2 \leftarrow i(x3), x1 \leftarrow x2\}$

$\underbrace{op_t(x1, \overbrace{rd(mult(x2, i(x3)), op_r(i(x3), x1, x1))})}$

$=$ by Lemma 1280 LR with $\{x3 \leftarrow i(x3), x2 \leftarrow mult(x2, i(x3)), x1 \leftarrow x1\}$

$\overbrace{op_t(x1, \underbrace{rd(mult(x2, i(x3)), i(x3))})}$

$=$ by Axiom 5 RL with $\{x2 \leftarrow i(x3), x1 \leftarrow x2\}$

$op_t(x1, \overbrace{x2})$

Lemma 1282: $op_t(x1, rd(mult(x2, i(x1)), x3)) = op_t(x1, rd(rd(x3, x1), x2))$

$\underbrace{op_t(x1, rd(mult(x2, i(x1)), x3))}$

$=$ by Lemma 1269 RL with $\{x3 \leftarrow mult(x2, i(x1)), x2 \leftarrow x3, x1 \leftarrow x1\}$

$\overbrace{op_t(x1, rd(rd(x3, x1), \underbrace{mult(x1, mult(x2, i(x1)))}))}$

$=$ by Lemma 30 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{op_t(x1, rd(rd(x3, x1), \overbrace{op_t(x2, i(x1))}))}$

$=$ by Lemma 1212 LR with $\{x2 \leftarrow x2, x3 \leftarrow rd(x3, x1), x1 \leftarrow x1\}$

$\overbrace{op_t(x1, rd(rd(x3, x1), x2))}$

Lemma 1283: $op_t(x1, rd(mult(i(x1), x2), x3)) = op_t(x1, rd(op_l(mult(i(x1), x2), x1, i(x2)), x3))$

$op_t(x1, \underbrace{rd(mult(i(x1), x2), x3)})$

$=$ by Lemma 10 RL with $\{x2 \leftarrow mult(i(x1), x2), x1 \leftarrow x3\}$

$\underbrace{op_t(x1, \overbrace{i(rd(x3, mult(i(x1), x2)))})}$

$=$ by Lemma 1274 RL with $\{x2 \leftarrow x3, x3 \leftarrow mult(i(x1), x2), x1 \leftarrow x1\}$

$\overbrace{op_t(x1, rd(\underbrace{op_l(mult(i(x1), x2), mult(i(x1), x2), x1)}, x3))}$

$=$ by Lemma 1054 LR with $\{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow mult(i(x1), x2)\}$

$op_t(x1, rd(\overbrace{op_l(mult(i(x1), x2), x1, i(x2))}, x3))$

Lemma 1284: $op_t(x1, rd(rd(x2, x1), x3)) = op_t(x1, rd(mult(x2, i(x1)), x3))$

$\underbrace{op_t(x1, rd(rd(x2, x1), x3))}$

$=$ by Lemma 1215 RL with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_t(x1, rd(mult(i(x1), x2), x3))}}$

$=$ by Lemma 1283 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_t(x1, rd(\underbrace{op_l(mult(i(x1), x2), x1, i(x2))}, x3))}$

$=$ by Lemma 197 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$op_t(x1, rd(\overbrace{mult(x2, i(x1))}, x3))$

Lemma 1285: $op_t(x1, rd(x2, rd(x3, x1))) = op_t(x1, rd(x2, op_l(mult(i(x1), x3), x1, i(x3))))$

$$
\begin{aligned}
& \overbrace{op_t(x1, rd(x2, rd(x3, x1)))} \\
= & \quad \text{by Lemma 1258 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, \underbrace{rd(x2, mult(i(x1), x3))})} \\
= & \quad \text{by Lemma 1278 RL with } \{x3 \leftarrow mult(i(x1), x3), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, rd(x2, \underbrace{op_l(mult(i(x1), x3), mult(i(x1), x3), x1)}))} \\
= & \quad \text{by Lemma 1054 LR with } \{x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow mult(i(x1), x3)\} \\
& op_t(x1, rd(x2, \overbrace{op_l(mult(i(x1), x3), x1, i(x3))}))
\end{aligned}
$$

Lemma 1286: $op_t(x1, rd(x3, rd(x1, x2))) = i(op_t(i(x1), rd(x2, rd(i(x1), x3))))$

$$
\begin{aligned}
& \overbrace{op_t(x1, rd(x3, rd(x1, x2)))} \\
= & \quad \text{by Lemma 1217 LR with } \{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, rd(x3, \underbrace{mult(x1, i(x2))}))} \\
= & \quad \text{by Lemma 16 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_t(x1, \underbrace{rd(x3, \overbrace{i(mult(i(x1), x2))})}) \\
= & \quad \text{by Lemma 10 RL with } \{x2 \leftarrow x3, x1 \leftarrow i(mult(i(x1), x2))\} \\
& \underbrace{op_t(x1, \overbrace{i(rd(i(mult(i(x1), x2)), x3))})} \\
= & \quad \text{by Lemma 18 RL with } \{x2 \leftarrow rd(i(mult(i(x1), x2)), x3), x1 \leftarrow x1\} \\
& \overbrace{i(\underbrace{op_t(i(x1), rd(i(mult(i(x1), x2)), x3))})} \\
= & \quad \text{by Lemma 1279 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow i(x1)\} \\
& i(\overbrace{op_t(i(x1), rd(x2, rd(i(x1), x3)))})
\end{aligned}
$$

Lemma 1287: $op_t(x1, rd(x3, rd(x1, x2))) = op_t(x1, rd(rd(i(x1), x3), x2))$

$$
\begin{aligned}
& \overbrace{op_t(x1, rd(x3, rd(x1, x2)))} \\
= & \quad \text{by Lemma 1286 LR with } \{x2 \leftarrow x2, x3 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{i(\underbrace{op_t(i(x1), rd(x2, rd(i(x1), x3)))})} \\
= & \quad \text{by Lemma 18 LR with } \{x2 \leftarrow rd(x2, rd(i(x1), x3)), x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, \underbrace{i(rd(x2, rd(i(x1), x3)))})} \\
= & \quad \text{by Lemma 10 LR with } \{x2 \leftarrow rd(i(x1), x3), x1 \leftarrow x2\} \\
& op_t(x1, \overbrace{rd(rd(i(x1), x3), x2)})
\end{aligned}
$$

Lemma 1288: $op_t(x1, rd(i(x2), mult(x3, x1))) = op_t(x1, rd(x3, rd(x1, x2)))$

$$
\begin{aligned}
& op_t(x1, \underbrace{rd(i(x2), mult(x3, x1))}) \\
= & \quad \text{by Lemma 37 RL with } \{x2 \leftarrow x2, x1 \leftarrow mult(x3, x1)\} \\
& \underbrace{op_t(x1, \overbrace{rd(i(mult(x3, x1)), x2)})} \\
= & \quad \text{by Lemma 1269 RL with } \{x3 \leftarrow i(mult(x3, x1)), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, rd(rd(x2, x1), \underbrace{mult(x1, i(mult(x3, x1)))}))} \\
= & \quad \text{by Lemma 31 LR with } \{x2 \leftarrow x3, x1 \leftarrow x1\} \\
& op_t(x1, \underbrace{rd(rd(x2, x1), \overbrace{i(op_t(x3, x1))})}) \\
= & \quad \text{by Lemma 38 RL with } \{x1 \leftarrow rd(x2, x1), x2 \leftarrow op_t(x3, x1)\} \\
& \underbrace{op_t(x1, \overbrace{rd(op_t(x3, x1), i(rd(x2, x1)))})} \\
= & \quad \text{by Lemma 1210 LR with } \{x3 \leftarrow i(rd(x2, x1)), x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, rd(x3, \underbrace{i(rd(x2, x1))}))} \\
= & \quad \text{by Lemma 10 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& op_t(x1, rd(x3, \overbrace{rd(x1, x2)}))
\end{aligned}
$$

Lemma 1289: $op_t(x1, rd(mult(x3, i(x1)), i(x2))) = i(op_t(i(x1), rd(x3, rd(i(x1), x2))))$

$$
\begin{aligned}
& op_t(x1, \underbrace{rd(mult(x3, i(x1)), i(x2))}) \\
= & \quad \text{by Lemma 10 RL with } \{x2 \leftarrow mult(x3, i(x1)), x1 \leftarrow i(x2)\} \\
& \underbrace{op_t(x1, \overbrace{i(rd(i(x2), mult(x3, i(x1))))})} \\
= & \quad \text{by Lemma 18 RL with } \{x2 \leftarrow rd(i(x2), mult(x3, i(x1))), x1 \leftarrow x1\} \\
& \overbrace{i(\underbrace{op_t(i(x1), rd(i(x2), mult(x3, i(x1))))})} \\
= & \quad \text{by Lemma 1288 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow i(x1)\} \\
& i(\overbrace{op_t(i(x1), rd(x3, rd(i(x1), x2)))})
\end{aligned}
$$

Lemma 1290: $op_t(x1, rd(x2, mult(i(x3), x1))) = op_t(x1, i(rd(x3, rd(i(x1), x2))))$

$$
\begin{aligned}
& op_t(x1, \underbrace{rd(x2, mult(i(x3), x1))}) \\
= & \quad \text{by Lemma 61 RL with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x3\} \\
& \underbrace{op_t(x1, \overbrace{rd(mult(x3, i(x1)), i(x2))})} \\
= & \quad \text{by Lemma 1289 LR with } \{x2 \leftarrow x2, x3 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{i(op_t(i(x1), rd(x3, rd(i(x1), x2))))}} \\
= & \quad \text{by Lemma 18 LR with } \{x2 \leftarrow rd(x3, rd(i(x1), x2)), x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, i(rd(x3, rd(i(x1), x2))))}
\end{aligned}
$$

Lemma 1291: $rd(rd(op_t(x1,x2),x1),x3) = rd(op_t(x1,x2),mult(x3,x1))$

$$
\begin{aligned}
& rd(rd(op_t(x1,x2),x1),\underbrace{x3}) \\
= \quad & \text{by Axiom 5 LR with } \{x2 \leftarrow x1, x1 \leftarrow x3\} \\
& rd(\underbrace{rd(op_t(x1,x2),x1)},\overbrace{rd(mult(x3,x1),x1)}) \\
= \quad & \text{by Lemma 1223 RL with } \{x3 \leftarrow mult(x3,x1), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{rd(\overbrace{mult(rd(op_t(x1,x2),mult(x3,x1)),rd(mult(x3,x1),x1))},rd(mult(x3,x1),x1))} \\
= \quad & \text{by Axiom 5 RL with } \{x2 \leftarrow rd(mult(x3,x1),x1), x1 \leftarrow rd(op_t(x1,x2),mult(x3,x1))\} \\
& \overbrace{rd(op_t(x1,x2),mult(x3,x1))}
\end{aligned}
$$

Lemma 1292: $rd(x1,rd(op_t(x2,x3),x2)) = rd(mult(x1,x2),op_t(x2,x3))$

$$
\begin{aligned}
& \underbrace{rd(x1,rd(op_t(x2,x3),x2))} \\
= \quad & \text{by Lemma 10 RL with } \{x2 \leftarrow x1, x1 \leftarrow rd(op_t(x2,x3),x2)\} \\
& \overbrace{i(\underbrace{rd(rd(op_t(x2,x3),x2),x1)})} \\
= \quad & \text{by Lemma 1291 LR with } \{x3 \leftarrow x1, x2 \leftarrow x3, x1 \leftarrow x2\} \\
& \underbrace{i(\overbrace{rd(op_t(x2,x3),mult(x1,x2))})} \\
= \quad & \text{by Lemma 10 LR with } \{x2 \leftarrow mult(x1,x2), x1 \leftarrow op_t(x2,x3)\} \\
& \overbrace{rd(mult(x1,x2),op_t(x2,x3))}
\end{aligned}
$$

Lemma 1293: $mult(x1,rd(x2,op_t(x2,x3))) = rd(x1,rd(op_t(x2,x3),x2))$

$$
\begin{aligned}
& \underbrace{mult(x1,rd(x2,op_t(x2,x3)))} \\
= \quad & \text{by Lemma 1226 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{rd(mult(x1,x2),op_t(x2,x3))}} \\
= \quad & \text{by Lemma 1292 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{rd(x1,rd(op_t(x2,x3),x2))}
\end{aligned}
$$

Lemma 1294: $rd(x1,rd(mult(i(x3),x2),rd(x2,x3))) = mult(x1,rd(rd(x2,x3),mult(i(x3),x2)))$

$$
\begin{aligned}
& rd(x1,rd(\underbrace{mult(i(x3),x2)},rd(x2,x3))) \\
= \quad & \text{by Lemma 14 RL with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& \underbrace{rd(x1,rd(\overbrace{op_t(rd(x2,x3),x3)},rd(x2,x3)))} \\
= \quad & \text{by Lemma 1293 RL with } \{x3 \leftarrow x3, x2 \leftarrow rd(x2,x3), x1 \leftarrow x1\} \\
& \overbrace{mult(x1,rd(rd(x2,x3),\underbrace{op_t(rd(x2,x3),x3)}))} \\
= \quad & \text{by Lemma 14 LR with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& mult(x1,rd(rd(x2,x3),\overbrace{mult(i(x3),x2)}))
\end{aligned}
$$

Lemma 1295: $mult(x1, asoc(x2, x3, x2)) = rd(x1, asoc(x3, x2, x2))$

$$
\begin{aligned}
& mult(x1, \underbrace{asoc(x2, x3, x2)}) \\
= & \quad \text{by Lemma 704 RL with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& \underbrace{mult(x1, \overbrace{rd(rd(x2, x3), mult(i(x3), x2))})} \\
= & \quad \text{by Lemma 1294 RL with } \{x2 \leftarrow x2, x3 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{rd(x1, \underbrace{rd(mult(i(x3), x2), rd(x2, x3))})} \\
= & \quad \text{by Lemma 794 RL with } \{x2 \leftarrow x2, x1 \leftarrow x3\} \\
& rd(x1, \overbrace{asoc(x3, x2, x2)})
\end{aligned}
$$

Lemma 1296: $op_t(x1, rd(asoc(x2, x3, x2), x1)) = x1$

$$
\begin{aligned}
& \underbrace{op_t(x1, rd(asoc(x2, x3, x2), x1))} \\
= & \quad \text{by Lemma 375 RL with } \{x2 \leftarrow asoc(x2, x3, x2), x1 \leftarrow x1\} \\
& \underbrace{\overbrace{op_t(x1, mult(i(x1), asoc(x2, x3, x2)))}} \\
= & \quad \text{by Lemma 376 RL with } \{x2 \leftarrow mult(i(x1), asoc(x2, x3, x2)), x1 \leftarrow x1\} \\
& \overbrace{rd(\underbrace{mult(x1, mult(i(x1), asoc(x2, x3, x2)))}, op_t(mult(i(x1), asoc(x2, x3, x2)), x1))} \\
= & \quad \text{by Lemma 12 LR with } \{x2 \leftarrow asoc(x2, x3, x2), x1 \leftarrow x1\} \\
& rd(\overbrace{asoc(x2, x3, x2)}, op_t(\underbrace{mult(i(x1), asoc(x2, x3, x2))}, x1)) \\
= & \quad \text{by Lemma 1295 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow i(x1)\} \\
& rd(asoc(x2, x3, x2), \underbrace{op_t(\overbrace{rd(i(x1), asoc(x3, x2, x2))}, x1)}) \\
= & \quad \text{by Lemma 33 LR with } \{x2 \leftarrow asoc(x3, x2, x2), x1 \leftarrow x1\} \\
& \underbrace{rd(asoc(x2, x3, x2), \overbrace{i(mult(x1, asoc(x3, x2, x2)))})} \\
= & \quad \text{by Lemma 38 RL with } \{x1 \leftarrow asoc(x2, x3, x2), x2 \leftarrow mult(x1, asoc(x3, x2, x2))\} \\
& \overbrace{rd(mult(x1, asoc(x3, x2, x2)), \underbrace{i(asoc(x2, x3, x2))})} \\
= & \quad \text{by Lemma 757 LR with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& \underbrace{rd(mult(x1, asoc(x3, x2, x2)), \overbrace{asoc(x3, x2, x2)})} \\
= & \quad \text{by Axiom 5 RL with } \{x2 \leftarrow asoc(x3, x2, x2), x1 \leftarrow x1\} \\
& \overbrace{x1}
\end{aligned}
$$

Lemma 1297: $op_t(x1, rd(i(x1), asoc(x2, x3, x2))) = i(i(x1))$

$$
\begin{aligned}
& op_t(x1, \underbrace{rd(i(x1), asoc(x2, x3, x2))}) \\
= & \quad \text{by Lemma 10 RL with } \{x2 \leftarrow i(x1), x1 \leftarrow asoc(x2, x3, x2)\} \\
& \underbrace{op_t(x1, \overbrace{i(rd(asoc(x2, x3, x2), i(x1)))})} \\
= & \quad \text{by Lemma 18 RL with } \{x2 \leftarrow rd(asoc(x2, x3, x2), i(x1)), x1 \leftarrow x1\} \\
& \overbrace{i(\underbrace{op_t(i(x1), rd(asoc(x2, x3, x2), i(x1)))})} \\
= & \quad \text{by Lemma 1296 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow i(x1)\} \\
& i(\overbrace{i(x1)})
\end{aligned}
$$

Lemma 1298: $op_t(x1, rd(asoc(x3, x2, x2), x1)) = x1$

$$
\begin{aligned}
& op_t(x1, \underbrace{rd(asoc(x3, x2, x2), x1)}) \\
= & \quad \text{by Lemma 785 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{op_t(x1, \overbrace{rd(i(x1), asoc(x2, x3, x2))})} \\
= & \quad \text{by Lemma 1297 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{\overbrace{i(i(x1))}} \\
= & \quad \text{by Lemma 3 LR with } \{x1 \leftarrow x1\} \\
& \overbrace{x1}
\end{aligned}
$$

Lemma 1299: $rd(x1, rd(mult(x2, i(x3)), rd(x2, x3))) = mult(x1, rd(rd(x2, x3), mult(x2, i(x3))))$

$$
\begin{aligned}
& rd(x1, rd(\underbrace{mult(x2, i(x3))}, rd(x2, x3))) \\
= & \quad \text{by Lemma 34 RL with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& \underbrace{rd(x1, rd(\overbrace{op_t(rd(x2, x3), i(x2))}, rd(x2, x3)))} \\
= & \quad \text{by Lemma 1293 RL with } \{x3 \leftarrow i(x2), x2 \leftarrow rd(x2, x3), x1 \leftarrow x1\} \\
& \overbrace{mult(x1, rd(rd(x2, x3), \underbrace{op_t(rd(x2, x3), i(x2))}))} \\
= & \quad \text{by Lemma 34 LR with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& mult(x1, rd(rd(x2, x3), \overbrace{mult(x2, i(x3))}))
\end{aligned}
$$

Lemma 1300: $rd(x1, asoc(x3, x2, x3)) = mult(x1, asoc(x2, x3, x3))$

$$
\begin{aligned}
& rd(x1, \underbrace{asoc(x3, x2, x3)}) \\
= & \quad \text{by Lemma 701 LR with } \{x2 \leftarrow x2, x1 \leftarrow x3\} \\
& \underbrace{rd(x1, \overbrace{rd(mult(x2, i(x3)), rd(x2, x3))})} \\
= & \quad \text{by Lemma 1299 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{mult(x1, rd(rd(x2, x3), \underbrace{mult(x2, i(x3))}))} \\
= & \quad \text{by Lemma 104 RL with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& mult(x1, \underbrace{rd(rd(x2, x3), \overbrace{op_r(rd(x2, x3), x3, i(x3))})}) \\
= & \quad \text{by Lemma 777 LR with } \{x2 \leftarrow x3, x1 \leftarrow rd(x2, x3)\} \\
& mult(x1, \underbrace{\overbrace{asoc(rd(x2, x3), x3, x3)}}) \\
= & \quad \text{by Lemma 774 LR with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& mult(x1, \overbrace{asoc(x2, x3, x3)})
\end{aligned}
$$

Lemma 1301: $op_t(x1, i(asoc(x2, x3, x2))) = mult(rd(x1, asoc(x3, x2, x2)), op_r(i(asoc(x2, x3, x2)), x1, x1))$

$$
\begin{aligned}
& \underbrace{op_t(x1, i(asoc(x2, x3, x2)))} \\
= \quad & \text{by Lemma 980 LR with } \{x2 \leftarrow asoc(x2, x3, x2), x1 \leftarrow x1\} \\
& \overbrace{mult(mult(x1, asoc(x2, x3, x2)), \underbrace{i(op_r(asoc(x2, x3, x2), x1, x1))})} \\
= \quad & \text{by Lemma 356 LR with } \{x2 \leftarrow x1, x1 \leftarrow asoc(x2, x3, x2)\} \\
& mult(\underbrace{mult(x1, asoc(x2, x3, x2))}, \overbrace{op_r(i(asoc(x2, x3, x2)), x1, x1)}) \\
= \quad & \text{by Lemma 1295 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& mult(\overbrace{rd(x1, asoc(x3, x2, x2))}, op_r(i(asoc(x2, x3, x2)), x1, x1))
\end{aligned}
$$

Lemma 1302: $op_t(x1, asoc(x3, x2, x2)) = mult(rd(x1, asoc(x3, x2, x2)), op_r(asoc(x3, x2, x2), x1, x1))$

$$
\begin{aligned}
& op_t(x1, \underbrace{asoc(x3, x2, x2)}) \\
= \quad & \text{by Lemma 757 RL with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& \underbrace{op_t(x1, \overbrace{i(asoc(x2, x3, x2))})} \\
= \quad & \text{by Lemma 1301 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{mult(rd(x1, asoc(x3, x2, x2)), op_r(\underbrace{i(asoc(x2, x3, x2))}, x1, x1))} \\
= \quad & \text{by Lemma 757 LR with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& mult(rd(x1, asoc(x3, x2, x2)), op_r(\overbrace{asoc(x3, x2, x2)}, x1, x1))
\end{aligned}
$$

Lemma 1303: $asoc(x1, x2, x1) = mult(x3, rd(asoc(x1, x2, x1), x3))$

$$
\begin{aligned}
& \underbrace{asoc(x1, x2, x1)} \\
= \quad & \text{by Lemma 12 RL with } \{x2 \leftarrow asoc(x1, x2, x1), x1 \leftarrow x3\} \\
& \overbrace{mult(x3, \underbrace{mult(i(x3), asoc(x1, x2, x1))})} \\
= \quad & \text{by Lemma 1295 LR with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow i(x3)\} \\
& mult(x3, \overbrace{\underbrace{rd(i(x3), asoc(x2, x1, x1))}}) \\
= \quad & \text{by Lemma 784 LR with } \{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x3\} \\
& mult(x3, \overbrace{rd(asoc(x1, x2, x1), x3)})
\end{aligned}
$$

Lemma 1304: $x1 = op_l(rd(mult(i(asoc(x2, x3, x2)), x1), asoc(x3, x2, x2)), asoc(x3, x2, x2), x1)$

$$
\begin{aligned}
& \underbrace{x1} \\
= \quad & \text{by Lemma 226 LR with } \{x1 \leftarrow asoc(x2, x3, x2), x2 \leftarrow x1\} \\
& \overbrace{op_l(\underbrace{mult(mult(i(asoc(x2, x3, x2)), x1), asoc(x2, x3, x2))}, asoc(x2, x3, x2), i(x1))} \\
= \quad & \text{by Lemma 1295 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow mult(i(asoc(x2, x3, x2)), x1)\} \\
& \underbrace{op_l(\overbrace{rd(mult(i(asoc(x2, x3, x2)), x1), asoc(x3, x2, x2))}, asoc(x2, x3, x2), i(x1))} \\
= \quad & \text{by Lemma 1050 RL with } \{x3 \leftarrow x1, x2 \leftarrow asoc(x2, x3, x2), x1 \leftarrow rd(mult(i(asoc(x2, x3, x2)), x1), asoc(x3, x2, x2))\} \\
& \overbrace{op_l(rd(mult(i(asoc(x2, x3, x2)), x1), asoc(x3, x2, x2)), \underbrace{i(asoc(x2, x3, x2))}, x1)} \\
= \quad & \text{by Lemma 757 LR with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& op_l(rd(mult(i(asoc(x2, x3, x2)), x1), asoc(x3, x2, x2)), \overbrace{asoc(x3, x2, x2)}, x1)
\end{aligned}
$$

Lemma 1305: $x1 = op_r(x1, asoc(x3, x2, x2), asoc(x3, x2, x2))$

$$
\begin{aligned}
& \overbrace{x1} \\
= \quad & \text{by Lemma 1304 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(rd(mult(\underbrace{i(asoc(x2, x3, x2))}, x1), asoc(x3, x2, x2)), asoc(x3, x2, x2), x1)} \\
= \quad & \text{by Lemma 757 LR with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& \underbrace{op_l(rd(mult(\overbrace{asoc(x3, x2, x2)}, x1), asoc(x3, x2, x2)), asoc(x3, x2, x2), x1)} \\
= \quad & \text{by Lemma 485 LR with } \{x2 \leftarrow x1, x1 \leftarrow asoc(x3, x2, x2)\} \\
& \overbrace{op_r(x1, asoc(x3, x2, x2), asoc(x3, x2, x2))}
\end{aligned}
$$

Lemma 1306: $op_r(asoc(x1, x2, x1), x3, i(x3)) = op_t(asoc(x1, x2, x1), x3)$

$$
\begin{aligned}
& \underbrace{op_r(asoc(x1, x2, x1), x3, i(x3))} \\
= \quad & \text{by Lemma 317 RL with } \{x2 \leftarrow asoc(x1, x2, x1), x1 \leftarrow x3\} \\
& \overbrace{rd(op_t(\underbrace{mult(x3, asoc(x1, x2, x1))}, x3), x3)} \\
= \quad & \text{by Lemma 1295 LR with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x3\} \\
& \underbrace{rd(op_t(\overbrace{rd(x3, asoc(x2, x1, x1))}, x3), x3)} \\
= \quad & \text{by Lemma 345 LR with } \{x2 \leftarrow asoc(x2, x1, x1), x1 \leftarrow x3\} \\
& \overbrace{op_t(\underbrace{i(asoc(x2, x1, x1))}, x3)} \\
= \quad & \text{by Lemma 759 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& op_t(\overbrace{asoc(x1, x2, x1)}, x3)
\end{aligned}
$$

Lemma 1307: $op_t(asoc(x1, x2, x1), i(x3)) = op_r(asoc(x1, x2, x1), x3, i(x3))$

$$
\begin{aligned}
& \underbrace{op_t(asoc(x1, x2, x1), i(x3))} \\
= \quad & \text{by Lemma 1306 RL with } \{x3 \leftarrow i(x3), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_r(asoc(x1, x2, x1), i(x3), \underbrace{i(i(x3))})} \\
= \quad & \text{by Lemma 3 LR with } \{x1 \leftarrow x3\} \\
& \underbrace{op_r(asoc(x1, x2, x1), i(x3), \overbrace{x3})} \\
= \quad & \text{by Lemma 321 LR with } \{x2 \leftarrow x3, x1 \leftarrow asoc(x1, x2, x1)\} \\
& \overbrace{op_r(asoc(x1, x2, x1), x3, i(x3))}
\end{aligned}
$$

Lemma 1308: $op_t(asoc(x1, x2, x1), i(x3)) = op_t(asoc(x1, x2, x1), x3)$

$$
\begin{aligned}
& \underbrace{op_t(asoc(x1, x2, x1), i(x3))} \\
= \quad & \text{by Lemma 1307 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_r(asoc(x1, x2, x1), x3, i(x3))}} \\
= \quad & \text{by Lemma 1306 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_t(asoc(x1, x2, x1), x3)}
\end{aligned}
$$

Lemma 1309: $i(op_t(asoc(x1,x2,x2),x3)) = op_t(asoc(x2,x1,x2),x3)$

$$
\begin{aligned}
& i(op_t(\underbrace{asoc(x1,x2,x2)}, x3)) \\
= & \quad \text{by Lemma 757 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \underbrace{i(op_t(\overbrace{i(asoc(x2,x1,x2))}, x3))} \\
= & \quad \text{by Lemma 18 LR with } \{x2 \leftarrow x3, x1 \leftarrow asoc(x2,x1,x2)\} \\
& \overbrace{\underbrace{op_t(asoc(x2,x1,x2), i(x3))}} \\
= & \quad \text{by Lemma 1308 LR with } \{x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \overbrace{op_t(asoc(x2,x1,x2), x3)}
\end{aligned}
$$

Lemma 1310: $mult(i(x1), rd(x1, asoc(x2,x3,x2))) = mult(rd(asoc(x3,x2,x2),x1),x1)$

$$
\begin{aligned}
& \underbrace{mult(i(x1), rd(x1, asoc(x2,x3,x2)))} \\
= & \quad \text{by Lemma 56 RL with } \{x2 \leftarrow asoc(x2,x3,x2), x1 \leftarrow x1\} \\
& \overbrace{mult(i(\underbrace{mult(x1, asoc(x2,x3,x2))}), x1)} \\
= & \quad \text{by Lemma 1295 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& mult(\underbrace{i(\overbrace{rd(x1, asoc(x3,x2,x2))})}, x1) \\
= & \quad \text{by Lemma 10 LR with } \{x2 \leftarrow asoc(x3,x2,x2), x1 \leftarrow x1\} \\
& mult(\overbrace{rd(asoc(x3,x2,x2), x1)}, x1)
\end{aligned}
$$

Lemma 1311: $op_l(x1, op_t(x2, x3), i(mult(x2, x3))) = mult(i(mult(x2, x3)), mult(op_t(x2, x3), mult(op_t(x3, x2), x1)))$

$op_l(x1, op_t(x2, x3), \underbrace{i(mult(x2, x3))}$

$=$ by Lemma 682 LR with $\{x2 \leftarrow x3, x1 \leftarrow x2\}$

$op_l(x1, op_t(x2, x3), \overbrace{op_t(\underbrace{rd(i(x2), x3)}, op_t(x2, x3))})$

$=$ by Lemma 10 RL with $\{x2 \leftarrow i(x2), x1 \leftarrow x3\}$

$\underbrace{op_l(x1, op_t(x2, x3), op_t(\overbrace{i(rd(x3, i(x2)))}, op_t(x2, x3)))}$

$=$ by Lemma 1067 RL with $\{x3 \leftarrow op_t(x2, x3), x2 \leftarrow rd(x3, i(x2)), x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_l(x1, \overbrace{mult(rd(x3, i(x2)), i(op_t(x2, x3)))}, op_t(x2, x3))}}$

$=$ by Axiom 12 RL with $\{x3 \leftarrow x1, x2 \leftarrow mult(rd(x3, i(x2)), i(op_t(x2, x3))), x1 \leftarrow op_t(x2, x3)\}$

$\overbrace{mult(i(\underbrace{mult(op_t(x2, x3), mult(rd(x3, i(x2)), i(op_t(x2, x3))))}), mult(op_t(x2, x3), mult(mult(rd(x3, i(x2)), i(op_t(x2, x3))), x$

$=$ by Lemma 30 LR with $\{x2 \leftarrow rd(x3, i(x2)), x1 \leftarrow op_t(x2, x3)\}$

$mult(\underbrace{i(\overbrace{op_t(rd(x3, i(x2)), i(op_t(x2, x3)))})}, mult(op_t(x2, x3), mult(mult(rd(x3, i(x2)), i(op_t(x2, x3))), x1)))$

$=$ by Lemma 19 LR with $\{x2 \leftarrow op_t(x2, x3), x1 \leftarrow rd(x3, i(x2))\}$

$mult(\overbrace{op_t(i(rd(x3, i(x2))), op_t(x2, x3))}, mult(op_t(x2, x3), mult(mult(rd(x3, i(x2)), \underbrace{i(op_t(x2, x3))}), x1)))$

$=$ by Lemma 15 RL with $\{x2 \leftarrow x3, x1 \leftarrow x2\}$

$mult(op_t(i(rd(x3, i(x2))), op_t(x2, x3)), mult(op_t(x2, x3), mult(\underbrace{mult(rd(x3, i(x2)), \overbrace{op_t(i(x2), i(x3))})}, x1)))$

$=$ by Lemma 680 RL with $\{x2 \leftarrow i(x2), x1 \leftarrow x3\}$

$mult(op_t(i(rd(x3, i(x2))), op_t(x2, x3)), mult(op_t(x2, x3), mult(\overbrace{op_t(x3, \underbrace{i(i(x2))})}, x1)))$

$=$ by Lemma 3 LR with $\{x1 \leftarrow x2\}$

$mult(op_t(\underbrace{i(rd(x3, i(x2)))}, op_t(x2, x3)), mult(op_t(x2, x3), mult(op_t(x3, \overbrace{x2}), x1)))$

$=$ by Lemma 10 LR with $\{x2 \leftarrow i(x2), x1 \leftarrow x3\}$

$mult(\underbrace{op_t(\overbrace{rd(i(x2), x3)}, op_t(x2, x3))}, mult(op_t(x2, x3), mult(op_t(x3, x2), x1)))$

$=$ by Lemma 682 RL with $\{x2 \leftarrow x3, x1 \leftarrow x2\}$

$mult(\overbrace{i(mult(x2, x3))}, mult(op_t(x2, x3), mult(op_t(x3, x2), x1)))$

Lemma 1312: $op_l(x1, op_t(x2, x3), i(mult(x2, x3))) = op_l(x1, x3, x2)$

$\underbrace{op_l(x1, op_t(x2, x3), i(mult(x2, x3)))}$

$=$ by Lemma 1311 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{mult(i(mult(x2, x3)), \underbrace{mult(op_t(x2, x3), mult(op_t(x3, x2), x1))})}$

$=$ by Lemma 1263 LR with $\{x3 \leftarrow x1, x2 \leftarrow x3, x1 \leftarrow x2\}$

$\underbrace{mult(i(mult(x2, x3)), \overbrace{mult(x2, mult(x3, x1))})}$

$=$ by Axiom 12 LR with $\{x3 \leftarrow x1, x2 \leftarrow x3, x1 \leftarrow x2\}$

$\overbrace{op_l(x1, x3, x2)}$

Lemma 1313: $op_l(x1, x2, i(x3)) = op_l(x1, op_t(rd(i(x2), x3), x2), i(i(x3)))$

$$
\begin{aligned}
& \underbrace{op_l(x1, x2, i(x3))} \\
= \quad & \text{by Lemma 296 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, x2, rd(i(x2), x3))}} \\
= \quad & \text{by Lemma 1312 RL with } \{x3 \leftarrow x2, x2 \leftarrow rd(i(x2), x3), x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, op_t(rd(i(x2), x3), x2), \underbrace{i(mult(rd(i(x2), x3), x2))})} \\
= \quad & \text{by Lemma 21 LR with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& op_l(x1, op_t(rd(i(x2), x3), x2), \overbrace{i(i(x3))})
\end{aligned}
$$

Lemma 1314: $op_l(x1, x2, i(x3)) = op_l(x1, mult(x2, x3), i(i(i(x3))))$

$$
\begin{aligned}
& \underbrace{op_l(x1, x2, i(x3))} \\
= \quad & \text{by Lemma 1313 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, \underbrace{op_t(rd(i(x2), x3), x2)}, i(i(x3)))} \\
= \quad & \text{by Lemma 33 LR with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& \underbrace{op_l(x1, \overbrace{i(mult(x2, x3))}, i(i(x3)))} \\
= \quad & \text{by Lemma 1050 LR with } \{x3 \leftarrow i(i(x3)), x2 \leftarrow mult(x2, x3), x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, mult(x2, x3), i(i(i(x3))))}
\end{aligned}
$$

Lemma 1315: $op_l(x1, x2, op_t(x3, x2)) = op_l(x1, x3, i(x2))$

$$
\begin{aligned}
& op_l(x1, \underbrace{x2}, op_t(x3, x2)) \\
= \quad & \text{by Lemma 3 RL with } \{x1 \leftarrow x2\} \\
& op_l(x1, \overbrace{i(i(x2))}, \underbrace{op_t(x3, x2)}) \\
= \quad & \text{by Axiom 10 RL with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& \underbrace{op_l(x1, i(i(x2)), \overbrace{mult(i(x2), mult(x3, x2))})} \\
= \quad & \text{by Lemma 241 LR with } \{x3 \leftarrow mult(x3, x2), x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, mult(x3, x2), \underbrace{i(x2)})} \\
= \quad & \text{by Lemma 3 RL with } \{x1 \leftarrow i(x2)\} \\
& \underbrace{op_l(x1, mult(x3, x2), \overbrace{i(i(i(x2)))})} \\
= \quad & \text{by Lemma 1314 RL with } \{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x3, i(x2))}
\end{aligned}
$$

Lemma 1316: $op_l(x1, rd(x3, x2), x3) = op_l(x1, x3, mult(i(x3), x2))$

$$
\begin{aligned}
& \underbrace{op_l(x1, rd(x3, x2), x3)} \\
= & \quad \text{by Lemma 1060 RL with } \{x4 \leftarrow x3, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{\overbrace{op_l(x1, rd(x2, x3), i(x3))}} \\
= & \quad \text{by Lemma 1315 RL with } \{x3 \leftarrow rd(x2, x3), x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x3, \underbrace{op_t(rd(x2, x3), x3)})} \\
= & \quad \text{by Lemma 14 LR with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& op_l(x1, x3, \overbrace{mult(i(x3), x2)})
\end{aligned}
$$

Lemma 1317: $op_l(x1, rd(x2, x3), x2) = op_l(x1, x3, i(x2))$

$$
\begin{aligned}
& \underbrace{op_l(x1, rd(x2, x3), x2)} \\
= & \quad \text{by Lemma 1316 LR with } \{x2 \leftarrow x3, x3 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{\overbrace{op_l(x1, x2, mult(i(x2), x3))}} \\
= & \quad \text{by Lemma 243 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x3, i(x2))}
\end{aligned}
$$

Lemma 1318: $op_l(x1, op_t(x3, x2), i(i(x2))) = op_l(x1, x2, i(x3))$

$$
\begin{aligned}
& \underbrace{op_l(x1, op_t(x3, x2), i(i(x2)))} \\
= & \quad \text{by Lemma 1050 RL with } \{x3 \leftarrow i(x2), x2 \leftarrow op_t(x3, x2), x1 \leftarrow x1\} \\
& \underbrace{\overbrace{op_l(x1, i(op_t(x3, x2)), i(x2))}} \\
= & \quad \text{by Lemma 1317 RL with } \{x3 \leftarrow i(op_t(x3, x2)), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{\overbrace{op_l(x1, rd(x2, i(op_t(x3, x2))), x2)}} \\
= & \quad \text{by Lemma 1199 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x2, i(x3))}
\end{aligned}
$$

Lemma 1319: $op_l(x1, op_t(x2, x3), x3) = op_l(x1, x3, i(x2))$

$$
\begin{aligned}
& op_l(x1, op_t(x2, x3), \underbrace{x3}) \\
= & \quad \text{by Lemma 3 RL with } \{x1 \leftarrow x3\} \\
& \underbrace{op_l(x1, op_t(x2, x3), \overbrace{i(i(x3))})} \\
= & \quad \text{by Lemma 1318 LR with } \{x2 \leftarrow x3, x3 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x3, i(x2))}
\end{aligned}
$$

Lemma 1320: $op_l(x1, x2, rd(x2, x3)) = op_l(x1, x2, mult(mult(i(x3), x2), x2))$

$$
\begin{aligned}
& \underbrace{op_l(x1, x2, rd(x2, x3))} \\
= \quad & \text{by Lemma 242 RL with } \{x3 \leftarrow rd(x2, x3), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, mult(x2, rd(x2, x3)), i(x2))}} \\
= \quad & \text{by Lemma 1315 RL with } \{x3 \leftarrow mult(x2, rd(x2, x3)), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x2, \underbrace{op_t(mult(x2, rd(x2, x3)), x2)})} \\
= \quad & \text{by Lemma 406 LR with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& op_l(x1, x2, \overbrace{mult(mult(i(x3), x2), x2)})
\end{aligned}
$$

Lemma 1321: $op_l(x1, x2, rd(x2, x3)) = op_l(x1, x2, i(x3))$

$$
\begin{aligned}
& \underbrace{op_l(x1, x2, rd(x2, x3))} \\
= \quad & \text{by Lemma 1320 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, x2, mult(mult(i(x3), x2), x2))}} \\
= \quad & \text{by Lemma 291 LR with } \{x3 \leftarrow mult(i(x3), x2), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, x2, mult(i(x3), x2))}} \\
= \quad & \text{by Lemma 291 LR with } \{x3 \leftarrow i(x3), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x2, i(x3))}
\end{aligned}
$$

Lemma 1322: $op_l(x1, mult(x2, i(x3)), x3) = op_l(x1, x2, i(i(x3)))$

$$
\begin{aligned}
& \underbrace{op_l(x1, mult(x2, i(x3)), x3)} \\
= \quad & \text{by Lemma 1067 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, x3, op_t(i(x2), x3))}} \\
= \quad & \text{by Lemma 1315 LR with } \{x3 \leftarrow i(x2), x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, i(x2), i(x3))}} \\
= \quad & \text{by Lemma 1050 LR with } \{x3 \leftarrow i(x3), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x2, i(i(x3)))}
\end{aligned}
$$

Lemma 1323: $op_l(x1, mult(x2, i(x3)), x3) = op_l(x1, x2, x3)$

$$
\begin{aligned}
& \underbrace{op_l(x1, mult(x2, i(x3)), x3)} \\
= \quad & \text{by Lemma 1322 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x2, \underbrace{i(i(x3))})} \\
= \quad & \text{by Lemma 3 LR with } \{x1 \leftarrow x3\} \\
& op_l(x1, x2, \overbrace{x3}^{})
\end{aligned}
$$

Lemma 1324: $op_l(x1, x3, i(i(x2))) = op_l(x1, x3, mult(x2, i(x3)))$

$$
\begin{aligned}
& \underbrace{op_l(x1, x3, i(i(x2)))} \\
= & \quad \text{by Lemma 1319 RL with } \{x3 \leftarrow x3, x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, op_t(i(x2), x3), x3)}} \\
= & \quad \text{by Lemma 1059 RL with } \{x3 \leftarrow i(x2), x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x3, \underbrace{i(mult(i(x2), x3))})} \\
= & \quad \text{by Lemma 16 LR with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& op_l(x1, x3, \overbrace{mult(x2, i(x3))})
\end{aligned}
$$

Lemma 1325: $op_l(x1, x2, mult(x3, i(x2))) = op_l(x1, x2, x3)$

$$
\begin{aligned}
& \underbrace{op_l(x1, x2, mult(x3, i(x2)))} \\
= & \quad \text{by Lemma 1324 RL with } \{x2 \leftarrow x3, x3 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x2, \underbrace{i(i(x3))})} \\
= & \quad \text{by Lemma 3 LR with } \{x1 \leftarrow x3\} \\
& op_l(x1, x2, \overbrace{x3}^{})
\end{aligned}
$$

Lemma 1326: $op_l(x1, op_r(x2, x3, x3), x3) = op_l(x1, x2, x3)$

$$
\begin{aligned}
& \underbrace{op_l(x1, op_r(x2, x3, x3), x3)} \\
= & \quad \text{by Lemma 1138 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, x3, op_r(i(mult(x3, x2)), x3, x3))}} \\
= & \quad \text{by Lemma 907 LR with } \{x3 \leftarrow mult(x3, x2), x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, x3, rd(x3, mult(x3, x2)))}} \\
= & \quad \text{by Lemma 1321 LR with } \{x3 \leftarrow mult(x3, x2), x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, x3, i(mult(x3, x2)))}} \\
= & \quad \text{by Lemma 1051 LR with } \{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x2, x3)}
\end{aligned}
$$

Lemma 1327: $op_l(x1, x2, x3) = op_l(x1, x2, op_r(x3, x2, x2))$

$$
\begin{aligned}
& \underbrace{op_l(x1, x2, x3)} \\
= & \quad \text{by Lemma 242 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, mult(x2, x3), i(x2))}} \\
= & \quad \text{by Lemma 1317 RL with } \{x3 \leftarrow mult(x2, x3), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, rd(x2, mult(x2, x3)), x2)}} \\
= & \quad \text{by Lemma 1140 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x2, op_r(x3, x2, x2))}
\end{aligned}
$$

Lemma 1328: $op_l(x1, x2, rd(x3, i(x2))) = op_l(x1, x2, x3)$

$$
\begin{aligned}
& \overbrace{op_l(x1, x2, rd(x3, i(x2)))} \\
= \quad & \text{by Lemma 1144 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, rd(x2, mult(x2, x3)), x2)}} \\
= \quad & \text{by Lemma 1140 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, x2, op_r(x3, x2, x2))}} \\
= \quad & \text{by Lemma 1327 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x2, x3)}
\end{aligned}
$$

Lemma 1329: $op_l(x1, x2, i(op_t(x3, x2))) = op_l(x1, x3, x2)$

$$
\begin{aligned}
& \overbrace{op_l(x1, x2, i(op_t(x3, x2)))} \\
= \quad & \text{by Lemma 1315 RL with } \{x3 \leftarrow x2, x2 \leftarrow op_t(x3, x2), x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, op_t(x3, x2), \underbrace{op_t(x2, op_t(x3, x2))})} \\
= \quad & \text{by Lemma 374 LR with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& \underbrace{op_l(x1, op_t(x3, x2), \overbrace{op_t(x2, x3)})} \\
= \quad & \text{by Lemma 1264 LR with } \{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x3, x2)}
\end{aligned}
$$

Lemma 1330: $op_l(x1, x2, mult(x2, x3)) = op_l(x1, x3, i(x2))$

$$
\begin{aligned}
& op_l(x1, x2, \underbrace{mult(x2, x3)}) \\
= \quad & \text{by Lemma 3 RL with } \{x1 \leftarrow mult(x2, x3)\} \\
& op_l(x1, x2, \overbrace{i(\underbrace{i(mult(x2, x3))})}) \\
= \quad & \text{by Lemma 33 RL with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& \underbrace{op_l(x1, x2, i(\overbrace{op_t(rd(i(x2), x3), x2)}))} \\
= \quad & \text{by Lemma 1329 LR with } \{x3 \leftarrow rd(i(x2), x3), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, rd(i(x2), x3), x2)}} \\
= \quad & \text{by Lemma 1055 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x3, i(x2))}
\end{aligned}
$$

Lemma 1331: $op_l(x1, mult(x2, x3), x2) = op_l(x1, x2, i(x3))$

$$
\begin{aligned}
& op_l(x1, mult(x2, x3), \underbrace{x2}) \\
= & \quad \text{by Lemma 3 RL with } \{x1 \leftarrow x2\} \\
& \underbrace{op_l(x1, mult(x2, x3), \overbrace{i(i(x2))})} \\
= & \quad \text{by Lemma 1330 RL with } \{x3 \leftarrow mult(x2, x3), x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, i(x2), \underbrace{mult(i(x2), mult(x2, x3))})} \\
= & \quad \text{by Axiom 2 RL with } \{x2 \leftarrow x2, x1 \leftarrow x3\} \\
& \underbrace{op_l(x1, i(x2), \overbrace{x3})} \\
= & \quad \text{by Lemma 1050 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x2, i(x3))}
\end{aligned}
$$

Lemma 1332: $op_l(x1, op_t(x2, x3), i(x3)) = op_l(x1, x3, x2)$

$$
\begin{aligned}
& \underbrace{op_l(x1, op_t(x2, x3), i(x3))} \\
= & \quad \text{by Lemma 1319 RL with } \{x3 \leftarrow op_t(x2, x3), x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, \underbrace{op_t(x3, op_t(x2, x3))}, op_t(x2, x3))} \\
= & \quad \text{by Lemma 374 LR with } \{x2 \leftarrow x2, x1 \leftarrow x3\} \\
& \underbrace{op_l(x1, \overbrace{op_t(x3, x2)}, op_t(x2, x3))} \\
= & \quad \text{by Lemma 1264 LR with } \{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x3, x2)}
\end{aligned}
$$

Lemma 1333: $op_l(x1, x2, i(mult(x3, x2))) = op_l(x1, x2, i(x3))$

$$
\begin{aligned}
& \underbrace{op_l(x1, x2, i(mult(x3, x2)))} \\
= & \quad \text{by Lemma 1059 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, op_t(x3, x2), x2)}} \\
= & \quad \text{by Lemma 1319 LR with } \{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x2, i(x3))}
\end{aligned}
$$

Lemma 1334: $op_l(x1, rd(x3, x2), i(x2)) = op_l(x1, i(x2), op_l(i(x3), x3, x2))$

$$
\begin{aligned}
& \underbrace{op_l(x1, rd(x3, x2), i(x2))} \\
= & \quad \text{by Lemma 1060 RL with } \{x4 \leftarrow i(x2), x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, rd(x2, x3), i(i(x2)))}} \\
= & \quad \text{by Lemma 1330 RL with } \{x3 \leftarrow rd(x2, x3), x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, i(x2), \underbrace{mult(i(x2), rd(x2, x3))})} \\
= & \quad \text{by Lemma 310 RL with } \{x2 \leftarrow x2, x1 \leftarrow x3\} \\
& op_l(x1, i(x2), \overbrace{op_l(i(x3), x3, x2)})
\end{aligned}
$$

Lemma 1335: $op_l(x1, x3, i(x2)) = op_l(x1, i(x2), op_l(i(x3), x3, x2))$

$$
\begin{aligned}
& \underbrace{op_l(x1, x3, i(x2))} \\
= \quad & \text{by Lemma 1317 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{\overbrace{op_l(x1, rd(x2, x3), x2)}} \\
= \quad & \text{by Lemma 1060 RL with } \{x4 \leftarrow x2, x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \underbrace{\overbrace{op_l(x1, rd(x3, x2), i(x2))}} \\
= \quad & \text{by Lemma 1334 LR with } \{x2 \leftarrow x2, x3 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, i(x2), op_l(i(x3), x3, x2))}
\end{aligned}
$$

Lemma 1336: $op_l(x1, x3, i(x2)) = op_l(x1, x2, op_l(i(i(x3)), x3, x2))$

$$
\begin{aligned}
& \underbrace{op_l(x1, x3, i(x2))} \\
= \quad & \text{by Lemma 1335 LR with } \{x2 \leftarrow x2, x3 \leftarrow x3, x1 \leftarrow x1\} \\
& \underbrace{\overbrace{op_l(x1, i(x2), op_l(i(x3), x3, x2))}} \\
= \quad & \text{by Lemma 1050 LR with } \{x3 \leftarrow op_l(i(x3), x3, x2), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x2, \underbrace{i(op_l(i(x3), x3, x2))})} \\
= \quad & \text{by Lemma 1049 LR with } \{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow i(x3)\} \\
& op_l(x1, x2, \overbrace{op_l(i(i(x3)), x3, x2)})
\end{aligned}
$$

Lemma 1337: $op_l(x1, op_l(x2, x2, x3), x3) = op_l(x1, i(x3), rd(x2, x3))$

$$
\begin{aligned}
& op_l(x1, op_l(x2, x2, x3), \underbrace{x3}) \\
= \quad & \text{by Lemma 3 RL with } \{x1 \leftarrow x3\} \\
& \underbrace{op_l(x1, op_l(x2, x2, x3), \overbrace{i(i(x3))})} \\
= \quad & \text{by Lemma 1330 RL with } \{x3 \leftarrow op_l(x2, x2, x3), x2 \leftarrow i(x3), x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, i(x3), \underbrace{mult(i(x3), op_l(x2, x2, x3))})} \\
= \quad & \text{by Lemma 440 LR with } \{x2 \leftarrow x2, x1 \leftarrow x3\} \\
& op_l(x1, i(x3), \overbrace{rd(x2, x3)})
\end{aligned}
$$

Lemma 1338: $op_l(x1, op_l(x2, x2, x3), x3) = op_l(x1, x3, rd(x3, x2))$

$$
\begin{aligned}
& \underbrace{op_l(x1, op_l(x2, x2, x3), x3)} \\
= \quad & \text{by Lemma 1337 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{\overbrace{op_l(x1, i(x3), rd(x2, x3))}} \\
= \quad & \text{by Lemma 1050 LR with } \{x3 \leftarrow rd(x2, x3), x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x3, \underbrace{i(rd(x2, x3))})} \\
= \quad & \text{by Lemma 10 LR with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& op_l(x1, x3, \overbrace{rd(x3, x2)})
\end{aligned}
$$

Lemma 1339: $op_l(x1, op_l(x2, x2, x3), x3) = op_l(x1, x3, i(x2))$

$$
\begin{aligned}
& \underbrace{op_l(x1, op_l(x2, x2, x3), x3)} \\
= & \quad \text{by Lemma 1338 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, x3, rd(x3, x2))}} \\
= & \quad \text{by Lemma 1321 LR with } \{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x3, i(x2))}
\end{aligned}
$$

Lemma 1340: $op_l(x1, x2, op_r(x3, x2, i(x2))) = op_l(x1, x2, x3)$

$$
\begin{aligned}
& op_l(x1, x2, \underbrace{op_r(x3, x2, i(x2))}) \\
= & \quad \text{by Lemma 323 RL with } \{x2 \leftarrow x2, x1 \leftarrow x3\} \\
& \underbrace{op_l(x1, x2, \overbrace{mult(mult(x3, i(x2)), x2)})} \\
= & \quad \text{by Lemma 291 LR with } \{x3 \leftarrow mult(x3, i(x2)), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, x2, mult(x3, i(x2)))}} \\
= & \quad \text{by Lemma 1325 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x2, x3)}
\end{aligned}
$$

Lemma 1341: $op_l(x1, x2, op_t(x3, rd(x2, x3))) = op_l(x1, x2, x3)$

$$
\begin{aligned}
& \underbrace{op_l(x1, x2, op_t(x3, rd(x2, x3)))} \\
= & \quad \text{by Lemma 286 RL with } \{x3 \leftarrow op_t(x3, rd(x2, x3)), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x2, \underbrace{rd(op_t(x3, rd(x2, x3)), x2)})} \\
= & \quad \text{by Lemma 372 LR with } \{x2 \leftarrow x2, x1 \leftarrow x3\} \\
& \underbrace{op_l(x1, x2, \overbrace{mult(x3, i(x2))})} \\
= & \quad \text{by Lemma 1325 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x2, x3)}
\end{aligned}
$$

Lemma 1342: $op_l(x1, op_t(rd(x2, x3), x2), x3) = op_l(x1, i(x2), i(x3))$

$$
\begin{aligned}
& \underbrace{op_l(x1, op_t(rd(x2, x3), x2), x3)} \\
= & \quad \text{by Lemma 1139 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, x3, rd(mult(x3, i(x2)), x3))}} \\
= & \quad \text{by Lemma 286 LR with } \{x3 \leftarrow mult(x3, i(x2)), x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, x3, mult(x3, i(x2)))}} \\
= & \quad \text{by Lemma 1330 LR with } \{x3 \leftarrow i(x2), x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, i(x2), i(x3))}
\end{aligned}
$$

Lemma 1343: $op_l(x1, op_t(rd(x2, x3), x2), x3) = op_l(x1, x2, x3)$

$\underbrace{op_l(x1, op_t(rd(x2, x3), x2), x3)}$

$=$ by Lemma 1342 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_l(x1, i(x2), i(x3))}}$

$=$ by Lemma 1050 LR with $\{x3 \leftarrow i(x3), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_l(x1, x2, \underbrace{i(i(x3))})}$

$=$ by Lemma 3 LR with $\{x1 \leftarrow x3\}$

$op_l(x1, x2, \overbrace{x3})$

Lemma 1344: $op_l(x1, x2, op_t(rd(x3, x2), x3)) = op_l(x1, x2, x3)$

$\underbrace{op_l(x1, x2, op_t(rd(x3, x2), x3))}$

$=$ by Lemma 1143 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_l(x1, mult(x2, i(x3)), x2)}}$

$=$ by Lemma 1331 LR with $\{x3 \leftarrow i(x3), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_l(x1, x2, \underbrace{i(i(x3))})}$

$=$ by Lemma 3 LR with $\{x1 \leftarrow x3\}$

$op_l(x1, x2, \overbrace{x3})$

Lemma 1345: $op_l(x1, x2, x3) = op_l(x1, op_t(x2, rd(x3, rd(x2, x2))), i(i(mult(x3, x2))))$

$\underbrace{op_l(x1, x2, x3)}$

$=$ by Lemma 242 RL with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_l(x1, mult(x2, x3), i(x2))}}$

$=$ by Lemma 1055 RL with $\{x3 \leftarrow mult(x2, x3), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_l(x1, rd(i(x2), mult(x2, x3)), x2)}}$

$=$ by Lemma 1312 RL with $\{x3 \leftarrow rd(i(x2), mult(x2, x3)), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_l(x1, op_t(x2, rd(i(x2), mult(x2, x3))), i(\underbrace{mult(x2, rd(i(x2), mult(x2, x3)))}))}$

$=$ by Lemma 37 RL with $\{x2 \leftarrow x2, x1 \leftarrow mult(x2, x3)\}$

$op_l(x1, op_t(x2, rd(i(x2), mult(x2, x3))), i(\underbrace{mult(x2, \overbrace{rd(i(mult(x2, x3)), x2)})}))$

$=$ by Lemma 55 RL with $\{x2 \leftarrow i(mult(x2, x3)), x1 \leftarrow x2\}$

$op_l(x1, op_t(x2, rd(i(x2), mult(x2, x3))), i(\overbrace{mult(\underbrace{mult(x2, i(mult(x2, x3)))}, i(x2))}))$

$=$ by Lemma 20 LR with $\{x2 \leftarrow x3, x1 \leftarrow x2\}$

$op_l(x1, op_t(x2, rd(i(x2), mult(x2, x3))), i(\underbrace{mult(\overbrace{i(x3)}, i(x2))}))$

$=$ by Axiom 11 LR with $\{x2 \leftarrow x2, x1 \leftarrow x3\}$

$op_l(x1, \underbrace{op_t(x2, rd(i(x2), mult(x2, x3)))}, i(\overbrace{i(mult(x3, x2))}))$

$=$ by Lemma 1210 RL with $\{x3 \leftarrow mult(x2, x3), x2 \leftarrow i(x2), x1 \leftarrow x2\}$

$op_l(x1, \overbrace{op_t(x2, rd(\underbrace{op_t(i(x2), x2)}, mult(x2, x3)))}, i(i(mult(x3, x2))))$

$=$ by Lemma 345 RL with $\{x2 \leftarrow x2, x1 \leftarrow x2\}$

$op_l(x1, \underbrace{op_t(x2, rd(\overbrace{rd(op_t(rd(x2, x2), x2), x2)}, mult(x2, x3)))}, i(i(mult(x3, x2))))$

$=$ by Lemma 1269 LR with $\{x3 \leftarrow x3, x2 \leftarrow op_t(rd(x2, x2), x2), x1 \leftarrow x2\}$

$op_l(x1, \overbrace{op_t(x2, rd(\underbrace{\ x3\ }, op_t(rd(x2, x2), x2)))}, i(i(mult(x3, x2))))$

$=$ by Axiom 4 LR with $\{x2 \leftarrow rd(x2, x2), x1 \leftarrow x3\}$

$op_l(x1, \underbrace{op_t(x2, rd(\overbrace{mult(rd(x3, rd(x2, x2)), rd(x2, x2))}, op_t(rd(x2, x2), x2)))}, i(i(mult(x3, x2))))$

$=$ by Lemma 1240 RL with $\{x3 \leftarrow rd(x2, x2), x2 \leftarrow rd(x3, rd(x2, x2)), x1 \leftarrow x2\}$

$op_l(x1, \overbrace{op_t(x2, rd(x3, rd(x2, x2)))}, i(i(mult(x3, x2))))$

Lemma 1346: $op_l(x1, x2, x3) = op_l(x1, op_t(x2, x3), i(i(mult(x3, x2))))$

$\underbrace{op_l(x1, x2, x3)}$

$=$ by Lemma 1345 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_l(x1, op_t(x2, rd(x3, \underbrace{rd(x2, x2)})), i(i(mult(x3, x2))))}$

$=$ by Lemma 2 LR with $\{x1 \leftarrow x2\}$

$op_l(x1, op_t(x2, \underbrace{rd(x3, \overbrace{unit()})}), i(i(mult(x3, x2))))$

$=$ by Lemma 1 LR with $\{x1 \leftarrow x3\}$

$op_l(x1, op_t(x2, \overbrace{\ x3\ }), i(i(mult(x3, x2))))$

Lemma 1347: $op_l(x1, op_t(x2, x3), mult(x3, x2)) = op_l(x1, x2, x3)$

$$
\begin{aligned}
& op_l(x1, op_t(x2, x3), \underbrace{mult(x3, x2)}) \\
= & \quad \text{by Lemma 3 RL with } \{x1 \leftarrow mult(x3, x2)\} \\
& \underbrace{op_l(x1, op_t(x2, x3), \overbrace{i(i(mult(x3, x2)))})} \\
= & \quad \text{by Lemma 1346 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x2, x3)}
\end{aligned}
$$

Lemma 1348: $op_l(x1, x3, x2) = op_l(x1, x2, op_t(rd(mult(x2, x2), x3), x2))$

$$
\begin{aligned}
& \underbrace{op_l(x1, x3, x2)} \\
= & \quad \text{by Lemma 292 RL with } \{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \underbrace{\overbrace{op_l(x1, rd(x3, x2), x2)}} \\
= & \quad \text{by Lemma 1060 RL with } \{x4 \leftarrow x2, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{\overbrace{op_l(x1, rd(x2, x3), i(x2))}} \\
= & \quad \text{by Lemma 1315 RL with } \{x3 \leftarrow rd(x2, x3), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x2, op_t(rd(x2, x3), \underbrace{x2}))} \\
= & \quad \text{by Lemma 3 RL with } \{x1 \leftarrow x2\} \\
& op_l(x1, x2, \underbrace{op_t(rd(x2, x3), \overbrace{i(i(x2))})}) \\
= & \quad \text{by Lemma 46 RL with } \{x3 \leftarrow i(x2), x2 \leftarrow x2, x1 \leftarrow x3\} \\
& \underbrace{op_l(x1, x2, \overbrace{i(op_t(rd(x3, x2), i(x2)))})} \\
= & \quad \text{by Lemma 1321 RL with } \{x3 \leftarrow op_t(rd(x3, x2), i(x2)), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x2, \underbrace{rd(x2, op_t(rd(x3, x2), i(x2)))})} \\
= & \quad \text{by Lemma 923 RL with } \{x2 \leftarrow rd(x3, x2), x1 \leftarrow x2\} \\
& op_l(x1, x2, \overbrace{\underbrace{mult(i(x2), rd(x2, rd(rd(x3, x2), x2)))}}) \\
= & \quad \text{by Lemma 45 RL with } \{x3 \leftarrow x2, x2 \leftarrow rd(rd(x3, x2), x2), x1 \leftarrow x2\} \\
& op_l(x1, x2, \overbrace{\underbrace{i(mult(x2, rd(rd(rd(x3, x2), x2), x2)))}}) \\
= & \quad \text{by Axiom 11 RL with } \{x2 \leftarrow rd(rd(rd(x3, x2), x2), x2), x1 \leftarrow x2\} \\
& op_l(x1, x2, \overbrace{\underbrace{mult(i(x2), i(rd(rd(rd(x3, x2), x2), x2)))}}) \\
= & \quad \text{by Lemma 1145 RL with } \{x1 \leftarrow x3, x2 \leftarrow x2\} \\
& op_l(x1, x2, \overbrace{op_t(rd(mult(x2, x2), x3), \underbrace{i(i(x2))})}) \\
= & \quad \text{by Lemma 3 LR with } \{x1 \leftarrow x2\} \\
& op_l(x1, x2, op_t(rd(mult(x2, x2), x3), \overbrace{x2})) \\
\end{aligned}
$$

Lemma 1349: $op_l(x1, x3, x2) = op_l(x1, rd(x3, mult(x2, x2)), x2)$

$\underbrace{op_l(x1, x3, x2)}$

$=$ by Lemma 1348 LR with $\{x2 \leftarrow x2, x3 \leftarrow x3, x1 \leftarrow x1\}$

$\overbrace{op_l(x1, x2, \underbrace{op_t(rd(mult(x2, x2), x3), x2)})}$

$=$ by Lemma 1315 LR with $\{x3 \leftarrow rd(mult(x2, x2), x3), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_l(x1, rd(mult(x2, x2), x3), i(x2))}}$

$=$ by Lemma 1060 LR with $\{x4 \leftarrow x2, x3 \leftarrow x3, x2 \leftarrow mult(x2, x2), x1 \leftarrow x1\}$

$\overbrace{op_l(x1, rd(x3, mult(x2, x2)), x2)}$

Lemma 1350: $op_l(x1, x2, rd(x3, mult(x2, x2))) = op_l(x1, x2, rd(rd(x3, x2), x2))$

$\underbrace{op_l(x1, x2, rd(x3, mult(x2, x2)))}$

$=$ by Lemma 1325 RL with $\{x3 \leftarrow rd(x3, mult(x2, x2)), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_l(x1, x2, \underbrace{mult(rd(x3, mult(x2, x2)), i(x2))})}$

$=$ by Lemma 1111 LR with $\{x2 \leftarrow x2, x1 \leftarrow x3\}$

$\underbrace{op_l(x1, x2, \overbrace{rd(rd(rd(x3, x2), x2), x2)})}$

$=$ by Lemma 286 LR with $\{x3 \leftarrow rd(rd(x3, x2), x2), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_l(x1, x2, rd(rd(x3, x2), x2))}$

Lemma 1351: $op_l(x1, x2, rd(x3, mult(x2, x2))) = op_l(x1, x2, x3)$

$\underbrace{op_l(x1, x2, rd(x3, mult(x2, x2)))}$

$=$ by Lemma 1350 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_l(x1, x2, rd(rd(x3, x2), x2))}}$

$=$ by Lemma 286 LR with $\{x3 \leftarrow rd(x3, x2), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_l(x1, x2, rd(x3, x2))}}$

$=$ by Lemma 286 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_l(x1, x2, x3)}$

Lemma 1352: $op_l(x1, op_t(x2, rd(x2, x3)), x3) = op_l(x1, i(x3), op_l(rd(x2, x3), x2, x3))$

$op_l(x1, op_t(x2, rd(x2, x3)), \underbrace{x3})$

$=$ by Lemma 3 RL with $\{x1 \leftarrow x3\}$

$\underbrace{op_l(x1, op_t(x2, rd(x2, x3)), \overbrace{i(i(x3))})}$

$=$ by Lemma 1330 RL with $\{x3 \leftarrow op_t(x2, rd(x2, x3)), x2 \leftarrow i(x3), x1 \leftarrow x1\}$

$\overbrace{op_l(x1, i(x3), \underbrace{mult(i(x3), op_t(x2, rd(x2, x3)))})}$

$=$ by Lemma 314 LR with $\{x2 \leftarrow x2, x1 \leftarrow x3\}$

$op_l(x1, i(x3), \overbrace{op_l(rd(x2, x3), x2, x3)})$

Lemma 1353: $op_l(x1, op_t(x2, rd(x2, x3)), x3) = op_l(x1, x3, op_l(i(rd(x2, x3)), x2, x3))$

$$
\begin{aligned}
& \underbrace{op_l(x1, op_t(x2, rd(x2, x3)), x3)} \\
= & \quad \text{by Lemma 1352 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, i(x3), op_l(rd(x2, x3), x2, x3))}} \\
= & \quad \text{by Lemma 1050 LR with } \{x3 \leftarrow op_l(rd(x2, x3), x2, x3), x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x3, \underbrace{i(op_l(rd(x2, x3), x2, x3))})} \\
= & \quad \text{by Lemma 1049 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow rd(x2, x3)\} \\
& op_l(x1, x3, \overbrace{op_l(i(rd(x2, x3)), x2, x3)})
\end{aligned}
$$

Lemma 1354: $op_l(x1, op_t(x2, rd(x2, x3)), x3) = op_l(x1, x2, x3)$

$$
\begin{aligned}
& \underbrace{op_l(x1, op_t(x2, rd(x2, x3)), x3)} \\
= & \quad \text{by Lemma 1353 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x3, op_l(\underbrace{i(rd(x2, x3))}, x2, x3))} \\
= & \quad \text{by Lemma 10 LR with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& \underbrace{op_l(x1, x3, op_l(\overbrace{rd(x3, x2)}, x2, x3))} \\
= & \quad \text{by Lemma 1081 LR with } \{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x2, x3)}
\end{aligned}
$$

Lemma 1355: $op_l(x1, i(x2), i(x3)) = op_l(x1, x3, op_l(i(x2), x3, x2))$

$$
\begin{aligned}
& \underbrace{op_l(x1, i(x2), i(x3))} \\
= & \quad \text{by Lemma 1317 RL with } \{x3 \leftarrow i(x2), x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, rd(x3, i(x2)), x3)}} \\
= & \quad \text{by Lemma 1060 RL with } \{x4 \leftarrow x3, x3 \leftarrow x3, x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, rd(i(x2), x3), i(x3))}} \\
= & \quad \text{by Lemma 1330 RL with } \{x3 \leftarrow rd(i(x2), x3), x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x3, \underbrace{mult(x3, rd(i(x2), x3))})} \\
= & \quad \text{by Lemma 904 LR with } \{x2 \leftarrow x2, x1 \leftarrow x3\} \\
& op_l(x1, x3, \overbrace{op_l(i(x2), x3, x2)})
\end{aligned}
$$

Lemma 1356: $op_l(x1, x2, x3) = op_l(x1, x3, op_l(i(x2), x3, x2))$

$$
\begin{aligned}
& op_l(x1, x2, \underbrace{x3}) \\
= & \quad \text{by Lemma 3 RL with } \{x1 \leftarrow x3\} \\
& \underbrace{op_l(x1, x2, \overbrace{i(i(x3))})} \\
= & \quad \text{by Lemma 1050 RL with } \{x3 \leftarrow i(x3), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, i(x2), i(x3))}} \\
= & \quad \text{by Lemma 1355 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x3, op_l(i(x2), x3, x2))}
\end{aligned}
$$

Lemma 1357: $op_l(x1, x2, rd(op_t(x2, x3), x3)) = op_l(x1, x2, i(op_t(x3, x2)))$

$$
\begin{aligned}
& op_l(x1, x2, \underbrace{rd(op_t(x2, x3), x3)}) \\
= & \quad \text{by Lemma 10 RL with } \{x2 \leftarrow op_t(x2, x3), x1 \leftarrow x3\} \\
& \underbrace{op_l(x1, x2, \overbrace{i(rd(x3, op_t(x2, x3)))})} \\
= & \quad \text{by Lemma 1333 RL with } \{x3 \leftarrow rd(x3, op_t(x2, x3)), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x2, i(\underbrace{mult(rd(x3, op_t(x2, x3)), x2)}))} \\
= & \quad \text{by Lemma 597 LR with } \{x2 \leftarrow x2, x1 \leftarrow x3\} \\
& op_l(x1, x2, i(\overbrace{op_t(x3, x2)}))
\end{aligned}
$$

Lemma 1358: $op_l(x1, x2, x3) = op_l(x1, i(op_t(x2, op_l(x3, x3, x2))), rd(x2, x3))$

$$
\begin{aligned}
& \underbrace{op_l(x1, x2, x3)} \\
= & \quad \text{by Lemma 286 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x2, \underbrace{rd(x3, x2)})} \\
= & \quad \text{by Lemma 10 RL with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& \underbrace{op_l(x1, x2, \overbrace{i(rd(x2, x3))})} \\
= & \quad \text{by Lemma 1050 RL with } \{x3 \leftarrow rd(x2, x3), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, i(x2), rd(x2, x3))}} \\
= & \quad \text{by Lemma 1329 RL with } \{x3 \leftarrow i(x2), x2 \leftarrow rd(x2, x3), x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, rd(x2, x3), i(op_t(i(x2), rd(x2, x3))))}} \\
= & \quad \text{by Lemma 1050 RL with } \{x3 \leftarrow op_t(i(x2), rd(x2, x3)), x2 \leftarrow rd(x2, x3), x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, i(rd(x2, x3)), op_t(i(x2), rd(x2, x3)))}} \\
= & \quad \text{by Lemma 1102 RL with } \{x3 \leftarrow rd(x2, x3), x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, op_t(i(x2), \underbrace{mult(i(x2), rd(x2, x3))}), rd(x2, x3))} \\
= & \quad \text{by Lemma 310 RL with } \{x2 \leftarrow x2, x1 \leftarrow x3\} \\
& op_l(x1, op_t(i(x2), \overbrace{\underbrace{op_l(i(x3), x3, x2)}}), rd(x2, x3)) \\
= & \quad \text{by Lemma 1049 RL with } \{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x3\} \\
& op_l(x1, \underbrace{op_t(i(x2), \overbrace{i(op_l(x3, x3, x2))})}, rd(x2, x3)) \\
= & \quad \text{by Lemma 15 LR with } \{x2 \leftarrow op_l(x3, x3, x2), x1 \leftarrow x2\} \\
& op_l(x1, \overbrace{i(op_t(x2, op_l(x3, x3, x2)))}, rd(x2, x3))
\end{aligned}
$$

Lemma 1359: $op_l(x1, op_l(x2, x2, x3), op_t(x3, x2)) = op_l(x1, x2, x3)$

$$
\begin{aligned}
& op_l(x1, op_l(x2, x2, x3), \underbrace{op_t(x3, x2)}) \\
= & \quad \text{by Lemma 553 LR with } \{x2 \leftarrow x2, x1 \leftarrow x3\} \\
& \underbrace{op_l(x1, op_l(x2, x2, x3), \overbrace{op_t(x3, op_l(x2, x2, x3))})} \\
= & \quad \text{by Lemma 1315 LR with } \{x3 \leftarrow x3, x2 \leftarrow op_l(x2, x2, x3), x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x3, \underbrace{i(op_l(x2, x2, x3))})} \\
= & \quad \text{by Lemma 1049 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x2\} \\
& \underbrace{op_l(x1, x3, \overbrace{op_l(i(x2), x2, x3)})} \\
= & \quad \text{by Lemma 1061 LR with } \{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x2, x3)}
\end{aligned}
$$

Lemma 1360: $op_l(x1, x2, x3) = op_l(x1, x2, op_t(mult(x3, x2), i(x3)))$

$$
\begin{aligned}
& \underbrace{op_l(x1, x2, x3)} \\
= & \quad \text{by Lemma 242 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, mult(x2, x3), i(x2))}} \\
= & \quad \text{by Lemma 1315 RL with } \{x3 \leftarrow mult(x2, x3), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x2, \underbrace{op_t(mult(x2, x3), x2)})} \\
= & \quad \text{by Lemma 919 RL with } \{x2 \leftarrow x2, x1 \leftarrow x3\} \\
& op_l(x1, x2, \overbrace{op_t(mult(x3, x2), i(x3))})
\end{aligned}
$$

Lemma 1361: $op_l(x1, rd(x3, x2), op_r(x3, x2, x2)) = op_l(x1, op_r(x3, x2, x2), x2)$

$$
\begin{aligned}
& \underbrace{op_l(x1, rd(x3, x2), op_r(x3, x2, x2))} \\
= & \quad \text{by Lemma 1060 RL with } \{x4 \leftarrow op_r(x3, x2, x2), x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, rd(x2, x3), i(op_r(x3, x2, x2)))}} \\
= & \quad \text{by Lemma 1330 RL with } \{x3 \leftarrow rd(x2, x3), x2 \leftarrow op_r(x3, x2, x2), x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, op_r(x3, x2, x2), \underbrace{mult(op_r(x3, x2, x2), rd(x2, x3))})} \\
= & \quad \text{by Lemma 366 LR with } \{x2 \leftarrow x2, x1 \leftarrow x3\} \\
& op_l(x1, op_r(x3, x2, x2), \overbrace{x2}^{})
\end{aligned}
$$

Lemma 1362: $op_l(x1, rd(x2, x3), op_r(x2, x3, x3)) = op_l(x1, x2, x3)$

$$
\begin{aligned}
& \underbrace{op_l(x1, rd(x2, x3), op_r(x2, x3, x3))} \\
= & \quad \text{by Lemma 1361 LR with } \{x2 \leftarrow x3, x3 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{\overbrace{op_l(x1, op_r(x2, x3, x3), x3)}} \\
= & \quad \text{by Lemma 1326 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x2, x3)}
\end{aligned}
$$

Lemma 1363: $op_l(x1, mult(x2, rd(i(x3), x3)), x3) = op_l(x1, x2, x3)$

$op_l(x1, mult(x2, \underbrace{rd(i(x3), x3)}), x3)$

$=$ by Lemma 43 RL with $\{x1 \leftarrow x3\}$

$\underbrace{op_l(x1, mult(x2, \overbrace{i(mult(x3, x3))}), x3)}$

$=$ by Lemma 1135 RL with $\{x4 \leftarrow x3, x3 \leftarrow mult(x3, x3), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_l(x1, \underbrace{mult(i(x2), mult(x3, x3))}, i(x3))}$

$=$ by Lemma 216 RL with $\{x2 \leftarrow x2, x1 \leftarrow x3\}$

$\underbrace{op_l(x1, \overbrace{op_t(rd(x3, mult(i(x3), x2)), x3)}, i(x3))}$

$=$ by Lemma 1332 LR with $\{x3 \leftarrow x3, x2 \leftarrow rd(x3, mult(i(x3), x2)), x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_l(x1, x3, rd(x3, mult(i(x3), x2)))}}$

$=$ by Lemma 1321 LR with $\{x3 \leftarrow mult(i(x3), x2), x2 \leftarrow x3, x1 \leftarrow x1\}$

$\overbrace{op_l(x1, x3, \underbrace{i(mult(i(x3), x2))})}$

$=$ by Lemma 16 LR with $\{x2 \leftarrow x2, x1 \leftarrow x3\}$

$\underbrace{op_l(x1, x3, \overbrace{mult(x3, i(x2))})}$

$=$ by Lemma 1330 LR with $\{x3 \leftarrow i(x2), x2 \leftarrow x3, x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_l(x1, i(x2), i(x3))}}$

$=$ by Lemma 1050 LR with $\{x3 \leftarrow i(x3), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_l(x1, x2, \underbrace{i(i(x3))})}$

$=$ by Lemma 3 LR with $\{x1 \leftarrow x3\}$

$op_l(x1, x2, \overbrace{x3})$

Lemma 1364: $op_l(x1, x2, op_l(i(x3), x3, x2)) = op_l(x1, op_l(op_l(x3, x3, x2), x2, i(x3)), x2)$

$op_l(x1, x2, \underbrace{op_l(i(x3), x3, x2)})$

$=$ by Lemma 1049 RL with $\{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x3\}$

$\underbrace{op_l(x1, x2, \overbrace{i(op_l(x3, x3, x2))})}$

$=$ by Lemma 1339 RL with $\{x3 \leftarrow x2, x2 \leftarrow op_l(x3, x3, x2), x1 \leftarrow x1\}$

$\overbrace{op_l(x1, \underbrace{op_l(op_l(x3, x3, x2), op_l(x3, x3, x2), x2)}, x2)}$

$=$ by Lemma 1339 LR with $\{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow op_l(x3, x3, x2)\}$

$op_l(x1, \overbrace{op_l(op_l(x3, x3, x2), x2, i(x3))}, x2)$

Lemma 1365: $op_l(x1, x3, x2) = op_l(x1, op_l(op_l(x3, x3, x2), x3, x2), x2)$

$\overbrace{op_l(x1, x3, x2)}$

$=$ by Lemma 1061 RL with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_l(x1, x2, \underbrace{op_l(i(x3), x3, x2)})}$

$=$ by Lemma 1364 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_l(x1, \underbrace{op_l(op_l(x3, x3, x2), x2, i(x3))}, x2)}$

$=$ by Lemma 1106 LR with $\{x4 \leftarrow x2, x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x3\}$

$op_l(x1, \overbrace{op_l(op_l(x3, x3, x2), x3, x2)}, x2)$

Lemma 1366: $op_l(x1, mult(i(rd(i(x2), x2)), x3), x2) = op_l(x1, rd(rd(i(x2), x2), i(op_t(x3, rd(i(x2), x2)))), x2)$

$\overbrace{op_l(x1, mult(i(rd(i(x2), x2)), x3), x2)}$

$=$ by Lemma 1363 RL with $\{x3 \leftarrow x2, x2 \leftarrow mult(i(rd(i(x2), x2)), x3), x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_l(x1, mult(mult(i(rd(i(x2), x2)), x3), rd(i(x2), x2)), x2)}}$

$=$ by Lemma 1363 RL with $\{x3 \leftarrow x2, x2 \leftarrow mult(mult(i(rd(i(x2), x2)), x3), rd(i(x2), x2)), x1 \leftarrow x1\}$

$\overbrace{op_l(x1, \underbrace{mult(mult(mult(i(rd(i(x2), x2)), x3), rd(i(x2), x2)), rd(i(x2), x2))}, x2)}$

$=$ by Lemma 921 LR with $\{x2 \leftarrow x3, x1 \leftarrow rd(i(x2), x2)\}$

$op_l(x1, \overbrace{rd(rd(i(x2), x2), i(op_t(x3, rd(i(x2), x2))))}, x2)$

Lemma 1367: $op_l(x1, mult(mult(x2, x2), x3), x2) = op_l(x1, rd(rd(i(x2), x2), i(op_t(x3, rd(i(x2), x2)))), x2)$

$op_l(x1, mult(\underbrace{mult(x2, x2)}, x3), x2)$

$=$ by Lemma 42 RL with $\{x1 \leftarrow x2\}$

$op_l(x1, mult(\overbrace{rd(x2, i(x2))}, x3), x2)$

$=$ by Lemma 10 RL with $\{x2 \leftarrow x2, x1 \leftarrow i(x2)\}$

$\underbrace{op_l(x1, mult(\overbrace{i(rd(i(x2), x2))}, x3), x2)}$

$=$ by Lemma 1366 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_l(x1, rd(rd(i(x2), x2), i(op_t(x3, rd(i(x2), x2)))), x2)}$

Lemma 1368: $op_l(x1, x2, i(mult(x2, x3))) = op_l(x1, rd(rd(i(x2), x2), i(op_t(x3, rd(i(x2), x2)))), x2)$

$\overbrace{op_l(x1, x2, i(mult(x2, x3)))}$

$=$ by Lemma 1331 RL with $\{x3 \leftarrow mult(x2, x3), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_l(x1, \underbrace{mult(x2, mult(x2, x3))}, x2)}$

$=$ by Lemma 24 RL with $\{x2 \leftarrow x3, x1 \leftarrow x2\}$

$\underbrace{op_l(x1, \overbrace{mult(mult(x2, x2), x3)}, x2)}$

$=$ by Lemma 1367 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_l(x1, rd(rd(i(x2), x2), i(op_t(x3, rd(i(x2), x2)))), x2)}$

Lemma 1369: $op_l(x1, x3, x2) = op_l(x1, rd(op_t(x3, rd(i(x2), x2)), rd(x2, i(x2))), x2)$

$$
\begin{aligned}
& \overbrace{op_l(x1, x3, x2)} \\
= \quad & \text{by Lemma 1051 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, x2, i(mult(x2, x3)))}} \\
= \quad & \text{by Lemma 1368 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, \underbrace{rd(rd(i(x2), x2), i(op_t(x3, rd(i(x2), x2))))}, x2)} \\
= \quad & \text{by Lemma 39 LR with } \{x3 \leftarrow op_t(x3, rd(i(x2), x2)), x2 \leftarrow x2, x1 \leftarrow i(x2)\} \\
& op_l(x1, \overbrace{rd(op_t(x3, rd(i(x2), x2)), rd(x2, i(x2)))}, x2)
\end{aligned}
$$

Lemma 1370: $op_l(x1, x3, x2) = op_l(x1, op_t(x3, rd(i(x2), x2)), x2)$

$$
\begin{aligned}
& \overbrace{op_l(x1, x3, x2)} \\
= \quad & \text{by Lemma 1369 LR with } \{x2 \leftarrow x2, x3 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, rd(op_t(x3, rd(i(x2), x2)), \underbrace{rd(x2, i(x2))}), x2)} \\
= \quad & \text{by Lemma 42 LR with } \{x1 \leftarrow x2\} \\
& \underbrace{op_l(x1, rd(op_t(x3, rd(i(x2), x2)), \overbrace{mult(x2, x2)}), x2)} \\
= \quad & \text{by Lemma 1349 RL with } \{x2 \leftarrow x2, x3 \leftarrow op_t(x3, rd(i(x2), x2)), x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, op_t(x3, \underbrace{rd(i(x2), x2)}), x2)}
\end{aligned}
$$

Lemma 1371: $op_l(x1, mult(i(mult(x2, x2)), x3), x2) = op_l(x1, op_l(mult(i(mult(x2, x2)), x3), mult(x2, x2), i(x3)), x2)$

$$
\begin{aligned}
& \underbrace{op_l(x1, mult(i(mult(x2, x2)), x3), x2)} \\
= \quad & \text{by Lemma 1365 LR with } \{x2 \leftarrow x2, x3 \leftarrow mult(i(mult(x2, x2)), x3), x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, \underbrace{op_l(op_l(mult(i(mult(x2, x2)), x3), mult(i(mult(x2, x2)), x3), x2), mult(i(mult(x2, x2)), x3), x2)}, x2)} \\
= \quad & \text{by Lemma 1174 LR with } \{x2 \leftarrow x2, x1 \leftarrow mult(i(mult(x2, x2)), x3)\} \\
& op_l(x1, \overbrace{\underbrace{op_l(mult(i(mult(x2, x2)), x3), mult(i(mult(x2, x2)), x3), mult(x2, x2))}, x2)} \\
= \quad & \text{by Lemma 1054 LR with } \{x3 \leftarrow x3, x2 \leftarrow mult(x2, x2), x1 \leftarrow mult(i(mult(x2, x2)), x3)\} \\
& op_l(x1, \overbrace{op_l(mult(i(mult(x2, x2)), x3), mult(x2, x2), i(x3))}, x2)
\end{aligned}
$$

Lemma 1372: $op_l(x1, mult(rd(i(x2), x2), x3), x2) = op_l(x1, mult(x3, i(mult(x2, x2))), x2)$

$$
\begin{aligned}
& op_l(x1, mult(\underbrace{rd(i(x2), x2)}, x3), x2) \\
= \quad & \text{by Lemma 43 RL with } \{x1 \leftarrow x2\} \\
& \underbrace{op_l(x1, mult(\overbrace{i(mult(x2, x2))}, x3), x2)} \\
= \quad & \text{by Lemma 1371 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, \underbrace{op_l(mult(i(mult(x2, x2)), x3), mult(x2, x2), i(x3))}, x2)} \\
= \quad & \text{by Lemma 197 LR with } \{x2 \leftarrow x3, x1 \leftarrow mult(x2, x2)\} \\
& op_l(x1, \overbrace{mult(x3, i(mult(x2, x2)))}, x2)
\end{aligned}
$$

Lemma 1373: $op_l(x1, mult(rd(i(x2), x2), x3), x2) = op_l(x1, x3, x2)$

$\overbrace{op_l(x1, mult(rd(i(x2), x2), x3), x2)}$

$=$ by Lemma 1372 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_l(x1, mult(x3, \underbrace{i(mult(x2, x2))}), x2)}$

$=$ by Lemma 43 LR with $\{x1 \leftarrow x2\}$

$\underbrace{op_l(x1, mult(x3, \overbrace{rd(i(x2), x2)}), x2)}$

$=$ by Lemma 1363 LR with $\{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x1\}$

$\overbrace{op_l(x1, x3, x2)}$

Lemma 1374: $mult(rd(x1, op_t(x2, x3)), rd(x2, x1)) = rd(x2, op_t(x2, x3))$

$mult(rd(x1, op_t(x2, x3)), \underbrace{rd(x2, x1)})$

$=$ by Lemma 10 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{mult(rd(x1, op_t(x2, x3)), \overbrace{i(rd(x1, x2))})}$

$=$ by Lemma 44 RL with $\{x3 \leftarrow rd(x1, x2), x2 \leftarrow x1, x1 \leftarrow op_t(x2, x3)\}$

$\overbrace{i(\underbrace{mult(rd(op_t(x2, x3), x1), rd(x1, x2))})}$

$=$ by Lemma 1223 LR with $\{x3 \leftarrow x1, x2 \leftarrow x3, x1 \leftarrow x2\}$

$\underbrace{i(\overbrace{rd(op_t(x2, x3), x2)})}$

$=$ by Lemma 10 LR with $\{x2 \leftarrow x2, x1 \leftarrow op_t(x2, x3)\}$

$\overbrace{rd(x2, op_t(x2, x3))}$

Lemma 1375: $mult(x1, rd(x2, mult(x1, op_t(x2, x3)))) = rd(x2, op_t(x2, x3))$

$\underbrace{mult(x1, rd(x2, mult(x1, op_t(x2, x3))))}$

$=$ by Lemma 1155 RL with $\{x3 \leftarrow op_t(x2, x3), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{mult(\underbrace{rd(mult(x1, x2), op_t(x2, x3))}, i(x1))}$

$=$ by Lemma 1292 RL with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{mult(\overbrace{rd(x1, rd(op_t(x2, x3), x2))}, i(x1))}$

$=$ by Lemma 22 LR with $\{x2 \leftarrow rd(op_t(x2, x3), x2), x1 \leftarrow x1\}$

$\overbrace{\underbrace{i(rd(op_t(x2, x3), x2))}}$

$=$ by Lemma 10 LR with $\{x2 \leftarrow x2, x1 \leftarrow op_t(x2, x3)\}$

$\overbrace{rd(x2, op_t(x2, x3))}$

Lemma 1376: $op_l(x1, rd(i(x2), rd(x1, op_t(x1, x3))), x2) = mult(rd(x1, op_t(x1, x3)), mult(x2, mult(i(x2), op_t(x1, x3))))$

$op_l(x1, \underbrace{rd(i(x2), rd(x1, op_t(x1, x3)))}, x2)$

$=$ by Lemma 1235 RL with $\{x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow i(x2)\}$

$\underbrace{op_l(x1, \overbrace{rd(mult(i(x2), op_t(x1, x3)), x1)}, x2)}$

$=$ by Lemma 280 LR with $\{x3 \leftarrow x2, x2 \leftarrow mult(i(x2), op_t(x1, x3)), x1 \leftarrow x1\}$

$\overbrace{mult(\underbrace{mult(i(x2), rd(x1, mult(i(x2), op_t(x1, x3))))}, mult(x2, mult(i(x2), op_t(x1, x3))))}$

$=$ by Lemma 1375 LR with $\{x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow i(x2)\}$

$mult(\overbrace{rd(x1, op_t(x1, x3))}, mult(x2, mult(i(x2), op_t(x1, x3))))$

Lemma 1377: $op_l(x1, rd(op_t(x1, x3), mult(x2, x1)), x2) = mult(rd(x1, op_t(x1, x3)), mult(x2, mult(i(x2), op_t(x1, x3))))$

$op_l(x1, \underbrace{rd(op_t(x1, x3), mult(x2, x1))}, x2)$

$=$ by Lemma 1291 RL with $\{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x1\}$

$op_l(x1, \overbrace{\underbrace{rd(rd(op_t(x1, x3), x1), x2)}, x2})$

$=$ by Lemma 40 RL with $\{x3 \leftarrow op_t(x1, x3), x2 \leftarrow x1, x1 \leftarrow x2\}$

$\underbrace{op_l(x1, \overbrace{rd(i(x2), rd(x1, op_t(x1, x3)))}, x2)}$

$=$ by Lemma 1376 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{mult(rd(x1, op_t(x1, x3)), mult(x2, mult(i(x2), op_t(x1, x3))))}$

Lemma 1378: $x1 = op_l(x1, x2, rd(i(x2), rd(op_t(x1, x3), x1)))$

$\overbrace{x1}$

$=$ by Axiom 2 LR with $\{x2 \leftarrow rd(op_t(x1, x3), mult(i(x2), x1)), x1 \leftarrow x1\}$

$\overbrace{mult(\underbrace{i(rd(op_t(x1, x3), mult(i(x2), x1)))}, mult(rd(op_t(x1, x3), mult(i(x2), x1)), x1))}$

$=$ by Axiom 2 LR with $\{x2 \leftarrow x2, x1 \leftarrow i(rd(op_t(x1, x3), mult(i(x2), x1)))\}$

$\underbrace{mult(\overbrace{mult(i(x2), mult(x2, \underbrace{i(rd(op_t(x1, x3), mult(i(x2), x1)))}))}, mult(rd(op_t(x1, x3), mult(i(x2), x1)), x1))}$

$=$ by Lemma 1047 RL with $\{x3 \leftarrow mult(rd(op_t(x1, x3), mult(i(x2), x1)), x1), x2 \leftarrow i(x2), x1 \leftarrow mult(x2, i(rd(op_t(x1, x3), mult$

$\overbrace{op_l(mult(\underbrace{mult(x2, \underbrace{i(rd(op_t(x1, x3), mult(i(x2), x1)))})}, mult(i(x2), mult(rd(op_t(x1, x3), mult(i(x2), x1)), x1))), mult(x2, i($

$=$ by Lemma 16 RL with $\{x2 \leftarrow rd(op_t(x1, x3), mult(i(x2), x1)), x1 \leftarrow x2\}$

$op_l(\underbrace{mult(\overbrace{i(mult(i(x2), rd(op_t(x1, x3), mult(i(x2), x1))))}, mult(i(x2), mult(rd(op_t(x1, x3), mult(i(x2), x1)), x1)))}, mult(x2$

$=$ by Axiom 12 LR with $\{x3 \leftarrow x1, x2 \leftarrow rd(op_t(x1, x3), mult(i(x2), x1)), x1 \leftarrow i(x2)\}$

$\underbrace{op_l(\overbrace{op_l(x1, rd(op_t(x1, x3), mult(i(x2), x1)), i(x2))}, mult(x2, i(rd(op_t(x1, x3), mult(i(x2), x1)))), i(x2))}$

$=$ by Lemma 242 LR with $\{x3 \leftarrow i(rd(op_t(x1, x3), mult(i(x2), x1))), x2 \leftarrow x2, x1 \leftarrow op_l(x1, rd(op_t(x1, x3), mult(i(x2), x1)), i(x$

$\overbrace{op_l(\underbrace{op_l(x1, rd(op_t(x1, x3), mult(i(x2), x1)), i(x2))}, x2, i(rd(op_t(x1, x3), mult(i(x2), x1))))}$

$=$ by Lemma 1377 LR with $\{x2 \leftarrow i(x2), x3 \leftarrow x3, x1 \leftarrow x1\}$

$op_l(\overbrace{mult(rd(x1, op_t(x1, x3)), \underbrace{mult(i(x2), mult(i(i(x2)), op_t(x1, x3)))})}, x2, i(rd(op_t(x1, x3), mult(i(x2), x1))))$

$=$ by Lemma 12 LR with $\{x2 \leftarrow op_t(x1, x3), x1 \leftarrow i(x2)\}$

$op_l(\underbrace{mult(rd(x1, op_t(x1, x3)), \overbrace{op_t(x1, x3)})}, x2, i(rd(op_t(x1, x3), mult(i(x2), x1))))$

$=$ by Axiom 4 RL with $\{x2 \leftarrow op_t(x1, x3), x1 \leftarrow x1\}$

$op_l(\overbrace{x1}, x2, \underbrace{i(rd(op_t(x1, x3), mult(i(x2), x1)))})$

$=$ by Lemma 10 LR with $\{x2 \leftarrow mult(i(x2), x1), x1 \leftarrow op_t(x1, x3)\}$

$op_l(x1, x2, \overbrace{rd(mult(i(x2), x1), op_t(x1, x3))})$

$=$ by Lemma 1292 RL with $\{x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow i(x2)\}$

$op_l(x1, x2, \overbrace{rd(i(x2), rd(op_t(x1, x3), x1))})$

Lemma 1379: $op_l(x1, x2, i(x3)) = op_l(x1, op_t(x2, mult(i(x2), x3)), i(x3))$

$\underbrace{op_l(x1, x2, i(x3))}$

$=$ by Lemma 1054 RL with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{\overbrace{op_l(x1, mult(i(x2), x3), x2)}}$

$=$ by Lemma 1312 RL with $\{x3 \leftarrow mult(i(x2), x3), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_l(x1, op_t(x2, mult(i(x2), x3)), \underbrace{i(mult(x2, mult(i(x2), x3)))})}$

$=$ by Lemma 12 LR with $\{x2 \leftarrow x3, x1 \leftarrow x2\}$

$op_l(x1, op_t(x2, mult(i(x2), x3)), i(\overbrace{x3}))$

Lemma 1380: $op_l(x1, x3, i(x2)) = op_l(x1, op_l(x2, x3, x2), mult(x3, x2))$

$\overbrace{op_l(x1, x3, i(x2))}$

$=$ by Lemma 1315 RL with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_l(x1, x2, \underbrace{op_t(x3, x2)})}$

$=$ by Lemma 1077 LR with $\{x3 \leftarrow op_t(x3, x2), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_l(x1, \underbrace{op_l(x2, op_t(x3, x2), x2)}, mult(x2, op_t(x3, x2)))}$

$=$ by Lemma 1319 LR with $\{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x2\}$

$op_l(x1, \overbrace{\underbrace{op_l(x2, x2, i(x3))}, mult(x2, op_t(x3, x2))})$

$=$ by Lemma 608 LR with $\{x2 \leftarrow x3, x1 \leftarrow x2\}$

$op_l(x1, \overbrace{op_l(x2, x3, x2)}, \underbrace{mult(x2, op_t(x3, x2))})$

$=$ by Lemma 13 LR with $\{x2 \leftarrow x3, x1 \leftarrow x2\}$

$op_l(x1, op_l(x2, x3, x2), \overbrace{mult(x3, x2)})$

Lemma 1381: $op_l(x1, mult(x2, x3), i(i(op_l(x3, x3, x2)))) = op_l(x1, i(x2), i(mult(x2, x3)))$

$\overbrace{op_l(x1, mult(x2, x3), i(i(op_l(x3, x3, x2))))}$

$=$ by Lemma 1050 RL with $\{x3 \leftarrow i(op_l(x3, x3, x2)), x2 \leftarrow mult(x2, x3), x1 \leftarrow x1\}$

$\overbrace{op_l(x1, i(mult(x2, x3)), i(op_l(x3, x3, x2)))}$

$=$ by Lemma 1331 RL with $\{x3 \leftarrow op_l(x3, x3, x2), x2 \leftarrow i(mult(x2, x3)), x1 \leftarrow x1\}$

$\overbrace{op_l(x1, \underbrace{mult(i(mult(x2, x3)), op_l(x3, x3, x2))}, i(mult(x2, x3)))}$

$=$ by Lemma 529 LR with $\{x2 \leftarrow x3, x1 \leftarrow x2\}$

$op_l(x1, \overbrace{i(x2)}, i(mult(x2, x3)))$

Lemma 1382: $op_l(x1, mult(x2, x3), op_l(x3, x3, x2)) = op_l(x1, x2, i(i(mult(x2, x3))))$

$op_l(x1, mult(x2, x3), \underbrace{op_l(x3, x3, x2)})$

$=$ by Lemma 3 RL with $\{x1 \leftarrow op_l(x3, x3, x2)\}$

$\underbrace{op_l(x1, mult(x2, x3), \overbrace{i(i(op_l(x3, x3, x2)))})}$

$=$ by Lemma 1381 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{\overbrace{op_l(x1, i(x2), i(mult(x2, x3)))}}$

$=$ by Lemma 1050 LR with $\{x3 \leftarrow i(mult(x2, x3)), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_l(x1, x2, i(i(mult(x2, x3))))}$

Lemma 1383: $op_l(x1, mult(x2, x3), op_l(x3, x3, x2)) = op_l(x1, x3, i(x2))$

$$
\begin{aligned}
& \underbrace{op_l(x1, mult(x2, x3), op_l(x3, x3, x2))} \\
= & \quad \text{by Lemma 1382 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x2, \underbrace{i(i(mult(x2, x3)))})} \\
= & \quad \text{by Lemma 3 LR with } \{x1 \leftarrow mult(x2, x3)\} \\
& \underbrace{op_l(x1, x2, \overbrace{mult(x2, x3)})} \\
= & \quad \text{by Lemma 1330 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x3, i(x2))}
\end{aligned}
$$

Lemma 1384: $op_l(x1, x2, op_t(i(mult(x3, x2)), x3)) = op_l(x1, mult(mult(x2, x3), x2), x2)$

$$
\begin{aligned}
& op_l(x1, x2, \underbrace{op_t(i(mult(x3, x2)), x3)}) \\
= & \quad \text{by Lemma 19 RL with } \{x2 \leftarrow x3, x1 \leftarrow mult(x3, x2)\} \\
& \underbrace{op_l(x1, x2, \overbrace{i(op_t(mult(x3, x2), i(x3)))})} \\
= & \quad \text{by Lemma 1331 RL with } \{x3 \leftarrow op_t(mult(x3, x2), i(x3)), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, \underbrace{mult(x2, op_t(mult(x3, x2), i(x3)))}, x2)} \\
= & \quad \text{by Lemma 810 RL with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& op_l(x1, \overbrace{mult(mult(x2, x3), x2)}, x2)
\end{aligned}
$$

Lemma 1385: $op_l(x1, x2, op_t(i(mult(x3, x2)), x3)) = op_l(x1, x2, i(x3))$

$$
\begin{aligned}
& \underbrace{op_l(x1, x2, op_t(i(mult(x3, x2)), x3))} \\
= & \quad \text{by Lemma 1384 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, mult(mult(x2, x3), x2), x2)}} \\
= & \quad \text{by Lemma 285 LR with } \{x3 \leftarrow x2, x2 \leftarrow mult(x2, x3), x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, mult(x2, x3), x2)}} \\
= & \quad \text{by Lemma 1331 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x2, i(x3))}
\end{aligned}
$$

Lemma 1386: $op_l(x1, x2, rd(op_t(x2, x3), i(x3))) = op_l(x1, op_r(i(x3), x2, x2), x2)$

$$
\begin{aligned}
& op_l(x1, x2, \underbrace{rd(op_t(x2, x3), i(x3))}) \\
= & \quad \text{by Lemma 10 RL with } \{x2 \leftarrow op_t(x2, x3), x1 \leftarrow i(x3)\} \\
& \underbrace{op_l(x1, x2, \overbrace{i(rd(i(x3), op_t(x2, x3)))})} \\
= & \quad \text{by Lemma 1331 RL with } \{x3 \leftarrow rd(i(x3), op_t(x2, x3)), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, \underbrace{mult(x2, rd(i(x3), op_t(x2, x3)))}, x2)} \\
= & \quad \text{by Lemma 913 LR with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& op_l(x1, \overbrace{op_r(i(x3), x2, x2)}, x2)
\end{aligned}
$$

Lemma 1387: $op_l(x1, x2, rd(op_t(x2, x3), i(x3))) = op_l(x1, x3, i(x2))$

$$
\begin{aligned}
& \underbrace{op_l(x1, x2, rd(op_t(x2, x3), i(x3)))} \\
= & \quad \text{by Lemma 1386 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, op_r(i(x3), x2, x2), x2)}} \\
= & \quad \text{by Lemma 1326 LR with } \{x3 \leftarrow x2, x2 \leftarrow i(x3), x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, i(x3), x2)}} \\
= & \quad \text{by Lemma 1050 LR with } \{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x3, i(x2))}
\end{aligned}
$$

Lemma 1388: $op_l(x1, x2, i(op_t(x3, mult(x2, x2)))) = op_l(x1, x2, i(op_t(mult(x2, x3), x2)))$

$$
\begin{aligned}
& \underbrace{op_l(x1, x2, i(op_t(x3, mult(x2, x2))))} \\
= & \quad \text{by Lemma 1331 RL with } \{x3 \leftarrow op_t(x3, mult(x2, x2)), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, \underbrace{mult(x2, op_t(x3, mult(x2, x2)))}, x2)} \\
= & \quad \text{by Lemma 1098 RL with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& \underbrace{op_l(x1, \overbrace{op_t(op_t(mult(x2, x3), x2), x2)}, x2)} \\
= & \quad \text{by Lemma 1319 LR with } \{x3 \leftarrow x2, x2 \leftarrow op_t(mult(x2, x3), x2), x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x2, i(op_t(mult(x2, x3), x2)))}
\end{aligned}
$$

Lemma 1389: $op_l(x1, x2, i(rd(op_t(i(x2), x3), x3))) = op_l(x1, x2, i(op_t(i(x3), x2)))$

$$
\begin{aligned}
& \underbrace{op_l(x1, x2, i(rd(op_t(i(x2), x3), x3)))} \\
= & \quad \text{by Lemma 1333 RL with } \{x3 \leftarrow rd(op_t(i(x2), x3), x3), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x2, i(\underbrace{mult(rd(op_t(i(x2), x3), x3), x2)}))} \\
= & \quad \text{by Lemma 692 LR with } \{x1 \leftarrow x3, x2 \leftarrow x2\} \\
& op_l(x1, x2, i(\overbrace{\underbrace{i(op_t(x3, i(x2)))}})) \\
= & \quad \text{by Lemma 19 LR with } \{x2 \leftarrow x2, x1 \leftarrow x3\} \\
& op_l(x1, x2, i(\overbrace{op_t(i(x3), x2)}))
\end{aligned}
$$

Lemma 1390: $rd(op_t(x2, x3), x1) = rd(rd(x2, x1), rd(x2, op_t(x2, x3)))$

$$
\begin{aligned}
& \underbrace{rd(op_t(x2, x3), x1)} \\
= & \quad \text{by Lemma 10 RL with } \{x2 \leftarrow op_t(x2, x3), x1 \leftarrow x1\} \\
& \overbrace{\underbrace{i(rd(x1, op_t(x2, x3)))}} \\
= & \quad \text{by Lemma 9 RL with } \{x2 \leftarrow rd(x1, op_t(x2, x3)), x1 \leftarrow rd(x2, x1)\} \\
& \overbrace{rd(rd(x2, x1), \underbrace{mult(rd(x1, op_t(x2, x3)), rd(x2, x1))})} \\
= & \quad \text{by Lemma 1374 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& rd(rd(x2, x1), \overbrace{rd(x2, op_t(x2, x3))})
\end{aligned}
$$

Lemma 1391: $rd(rd(x1,x2),asoc(x1,x3,x3)) = rd(op_t(x1,rd(x3,x1)),x2)$

$$
\begin{aligned}
& rd(rd(x1,x2),\underbrace{asoc(x1,x3,x3)}) \\
= & \quad \text{by Lemma 777 RL with } \{x2 \leftarrow x3, x1 \leftarrow x1\} \\
& rd(rd(x1,x2),\overbrace{rd(x1,\underbrace{op_r(x1,x3,i(x3))})}) \\
= & \quad \text{by Lemma 547 RL with } \{x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \underbrace{rd(rd(x1,x2),rd(x1,\overbrace{op_t(x1,rd(x3,x1))}))} \\
= & \quad \text{by Lemma 1390 RL with } \{x1 \leftarrow x2, x3 \leftarrow rd(x3,x1), x2 \leftarrow x1\} \\
& \overbrace{rd(op_t(x1,rd(x3,x1)),x2)}
\end{aligned}
$$

Lemma 1392: $rd(asoc(x1,x2,x2),rd(x1,x3)) = rd(x3,op_t(x1,rd(x2,x1)))$

$$
\begin{aligned}
& \underbrace{rd(asoc(x1,x2,x2),rd(x1,x3))} \\
= & \quad \text{by Lemma 10 RL with } \{x2 \leftarrow asoc(x1,x2,x2), x1 \leftarrow rd(x1,x3)\} \\
& \overbrace{i(\underbrace{rd(rd(x1,x3),asoc(x1,x2,x2))})} \\
= & \quad \text{by Lemma 1391 LR with } \{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \underbrace{i(\overbrace{rd(op_t(x1,rd(x2,x1)),x3)})} \\
= & \quad \text{by Lemma 10 LR with } \{x2 \leftarrow x3, x1 \leftarrow op_t(x1,rd(x2,x1))\} \\
& \overbrace{rd(x3,op_t(x1,rd(x2,x1)))}
\end{aligned}
$$

Lemma 1393: $rd(x1, mult(rd(x2, x3), op_t(x1, x3))) = rd(op_t(x3, x1), x2)$

$\overbrace{rd(x1, mult(rd(x2, x3), op_t(x1, x3)))}$

$=$ by Lemma 1236 LR with $\{x3 \leftarrow x3, x2 \leftarrow rd(x2, x3), x1 \leftarrow x1\}$

$\overbrace{rd(\underbrace{rd(x1, op_t(x1, x3))}, rd(x2, x3))}$

$=$ by Lemma 10 RL with $\{x2 \leftarrow rd(x1, op_t(x1, x3)), x1 \leftarrow rd(x2, x3)\}$

$\overbrace{i(\underbrace{rd(rd(x2, x3), rd(x1, op_t(x1, x3)))})}$

$=$ by Axiom 5 LR with $\{x2 \leftarrow rd(op_t(x3, x1), x3), x1 \leftarrow rd(rd(x2, x3), rd(x1, op_t(x1, x3)))\}$

$i(\overbrace{rd(mult(rd(rd(x2, x3), rd(x1, op_t(x1, x3))), rd(op_t(x3, x1), x3)), rd(op_t(x3, x1), x3))})$

$=$ by Lemma 1292 LR with $\{x3 \leftarrow x1, x2 \leftarrow x3, x1 \leftarrow mult(rd(rd(x2, x3), rd(x1, op_t(x1, x3))), rd(op_t(x3, x1), x3))\}$

$i(\overbrace{rd(mult(mult(rd(rd(x2, x3), rd(x1, op_t(x1, x3))), rd(op_t(x3, x1), x3)), x3), op_t(x3, x1)))})$

$=$ by Lemma 693 LR with $\{x3 \leftarrow x3, x2 \leftarrow op_t(x3, x1), x1 \leftarrow rd(rd(x2, x3), rd(x1, op_t(x1, x3)))\}$

$i(\overbrace{op_r(rd(rd(x2, x3), rd(x1, op_t(x1, x3))), \underbrace{rd(op_t(x3, x1), x3)}, x3)})$

$=$ by Lemma 395 LR with $\{x1 \leftarrow x1, x2 \leftarrow x3\}$

$i(\underbrace{op_r(rd(rd(x2, x3), rd(x1, op_t(x1, x3))), \overbrace{rd(x1, op_t(x1, x3))}, x3)})$

$=$ by Lemma 839 LR with $\{x3 \leftarrow rd(x1, op_t(x1, x3)), x2 \leftarrow x3, x1 \leftarrow x2\}$

$i(\overbrace{rd(x2, \underbrace{mult(rd(x1, op_t(x1, x3)), x3)})})$

$=$ by Lemma 388 LR with $\{x2 \leftarrow x3, x1 \leftarrow x1\}$

$\underbrace{i(rd(x2, \overbrace{op_t(x3, x1)}))}$

$=$ by Lemma 10 LR with $\{x2 \leftarrow op_t(x3, x1), x1 \leftarrow x2\}$

$\overbrace{rd(op_t(x3, x1), x2)}$

Lemma 1394: $x1 = op_l(x1, x2, i(rd(x2, rd(x3, op_t(x3, x1)))))$

$\overbrace{x1}$

$=$ by Lemma 1378 LR with $\{x3 \leftarrow x3, x2 \leftarrow rd(mult(i(x2), x3), x3), x1 \leftarrow x1\}$

$\overbrace{op_l(x1, rd(mult(i(x2), x3), x3), \underbrace{rd(i(rd(mult(i(x2), x3), x3)), rd(op_t(x1, x3), x1))}}$

$=$ by Lemma 40 LR with $\{x3 \leftarrow x1, x2 \leftarrow op_t(x1, x3), x1 \leftarrow rd(mult(i(x2), x3), x3)\}$

$op_l(x1, rd(mult(i(x2), x3), x3), \overbrace{\underbrace{rd(rd(x1, op_t(x1, x3)), rd(mult(i(x2), x3), x3))}})$

$=$ by Lemma 1236 RL with $\{x3 \leftarrow x3, x2 \leftarrow rd(mult(i(x2), x3), x3), x1 \leftarrow x1\}$

$op_l(x1, rd(mult(i(x2), x3), x3), \overbrace{\underbrace{rd(x1, mult(rd(mult(i(x2), x3), x3), op_t(x1, x3)))}})$

$=$ by Lemma 1393 LR with $\{x3 \leftarrow x3, x2 \leftarrow mult(i(x2), x3), x1 \leftarrow x1\}$

$op_l(x1, rd(mult(i(x2), x3), x3), \overbrace{\underbrace{rd(op_t(x3, x1), mult(i(x2), x3))}})$

$=$ by Lemma 1291 RL with $\{x3 \leftarrow i(x2), x2 \leftarrow x1, x1 \leftarrow x3\}$

$op_l(x1, rd(mult(i(x2), x3), x3), \overbrace{\underbrace{rd(rd(op_t(x3, x1), x3), i(x2))}})$

$=$ by Lemma 38 LR with $\{x1 \leftarrow x2, x2 \leftarrow rd(op_t(x3, x1), x3)\}$

$op_l(x1, rd(mult(i(x2), x3), x3), \overbrace{rd(x2, \underbrace{i(rd(op_t(x3, x1), x3))})})$

$=$ by Lemma 10 LR with $\{x2 \leftarrow x3, x1 \leftarrow op_t(x3, x1)\}$

$op_l(x1, \underbrace{rd(mult(i(x2), x3), x3)}, rd(x2, \overbrace{rd(x3, op_t(x3, x1))}))$

$=$ by Axiom 5 RL with $\{x2 \leftarrow x3, x1 \leftarrow i(x2)\}$

$\underbrace{op_l(x1, \overbrace{i(x2)}, rd(x2, rd(x3, op_t(x3, x1))))}$

$=$ by Lemma 1050 LR with $\{x3 \leftarrow rd(x2, rd(x3, op_t(x3, x1))), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_l(x1, x2, i(rd(x2, rd(x3, op_t(x3, x1)))))}$

Lemma 1395: $asoc(rd(x1, x2), x3, x3) = asoc(x3, rd(op_t(x2, x3), x1), x3)$

$\underbrace{asoc(rd(x1, x2), x3, x3)}$

$=$ by Lemma 1028 RL with $\{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow rd(x1, x2)\}$

$\overbrace{\underbrace{asoc(mult(rd(x1, x2), op_t(x3, x2)), x3, x3)}}$

$=$ by Lemma 762 RL with $\{x2 \leftarrow mult(rd(x1, x2), op_t(x3, x2)), x1 \leftarrow x3\}$

$\overbrace{asoc(x3, \underbrace{rd(x3, mult(rd(x1, x2), op_t(x3, x2)))}, x3)}$

$=$ by Lemma 1393 LR with $\{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x3\}$

$asoc(x3, \overbrace{rd(op_t(x2, x3), x1)}, x3)$

Lemma 1396: $asoc(x2, rd(x3, op_t(x1, x2)), x2) = asoc(x2, rd(x3, x1), x2)$

$\underbrace{asoc(x2, rd(x3, op_t(x1, x2)), x2)}$

$=$ by Lemma 938 RL with $\{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow op_t(x1, x2)\}$

$\overbrace{\underbrace{asoc(rd(op_t(x1, x2), x3), x2, x2)}}$

$=$ by Lemma 757 RL with $\{x2 \leftarrow rd(op_t(x1, x2), x3), x1 \leftarrow x2\}$

$\overbrace{i(\underbrace{asoc(x2, rd(op_t(x1, x2), x3), x2)})}$

$=$ by Lemma 1395 RL with $\{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x3\}$

$\underbrace{i(\overbrace{asoc(rd(x3, x1), x2, x2)})}$

$=$ by Lemma 759 LR with $\{x2 \leftarrow x2, x1 \leftarrow rd(x3, x1)\}$

$\overbrace{asoc(x2, rd(x3, x1), x2)}$

Lemma 1397: $op_l(x1, x1, rd(x2, op_t(x3, x1))) = op_l(x1, x1, rd(x2, x3))$

$\underbrace{op_l(x1, x1, rd(x2, op_t(x3, x1)))}$

$=$ by Lemma 173 RL with $\{x2 \leftarrow rd(x2, op_t(x3, x1)), x1 \leftarrow x1\}$

$\overbrace{mult(\underbrace{asoc(x1, rd(x2, op_t(x3, x1)), x1)}, x1)}$

$=$ by Lemma 1396 LR with $\{x1 \leftarrow x3, x3 \leftarrow x2, x2 \leftarrow x1\}$

$\underbrace{mult(\overbrace{asoc(x1, rd(x2, x3), x1)}, x1)}$

$=$ by Lemma 173 LR with $\{x2 \leftarrow rd(x2, x3), x1 \leftarrow x1\}$

$\overbrace{op_l(x1, x1, rd(x2, x3))}$

Lemma 1398: $rd(op_t(x1, x2), mult(rd(x2, x3), x1)) = rd(x3, op_t(x2, x1))$

$\underbrace{rd(op_t(x1, x2), mult(rd(x2, x3), x1))}$

$=$ by Lemma 1291 RL with $\{x3 \leftarrow rd(x2, x3), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{rd(\underbrace{rd(op_t(x1, x2), x1)}, rd(x2, x3))}$

$=$ by Lemma 858 RL with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$\underbrace{rd(\overbrace{asoc(x2, mult(x1, x2), mult(x1, x2))}, rd(x2, x3))}$

$=$ by Lemma 1392 LR with $\{x3 \leftarrow x3, x2 \leftarrow mult(x1, x2), x1 \leftarrow x2\}$

$\overbrace{rd(x3, op_t(x2, \underbrace{rd(mult(x1, x2), x2)}))}$

$=$ by Axiom 5 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$rd(x3, op_t(x2, \overbrace{x1}))$

Lemma 1399: $mult(rd(x2,x1),i(mult(x2,x3))) = mult(x2,i(mult(mult(x2,x1),x3)))$

$$
\begin{aligned}
& \overbrace{mult(rd(x2,x1),i(mult(x2,x3)))} \\
= & \quad \text{by Lemma 44 RL with } \{x3 \leftarrow mult(x2,x3), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{i(mult(rd(x1,x2),mult(x2,x3)))} \\
= & \quad \text{by Lemma 20 RL with } \{x2 \leftarrow mult(rd(x1,x2),mult(x2,x3)), x1 \leftarrow x2\} \\
& \overbrace{mult(x2,i(\underbrace{mult(x2,mult(rd(x1,x2),mult(x2,x3)))}))} \\
= & \quad \text{by Lemma 54 LR with } \{x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow x2\} \\
& mult(x2,i(\overbrace{mult(mult(x2,x1),x3)}))
\end{aligned}
$$

Lemma 1400: $mult(mult(x1,x2),i(mult(x1,x3))) = mult(x1,mult(rd(x2,x1),i(x3)))$

$$
\begin{aligned}
& \overbrace{mult(mult(x1,x2),i(mult(x1,x3)))} \\
= & \quad \text{by Lemma 54 RL with } \{x3 \leftarrow i(mult(x1,x3)), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{mult(x1,mult(rd(x2,x1),\underbrace{mult(x1,i(mult(x1,x3)))}))} \\
= & \quad \text{by Lemma 20 LR with } \{x2 \leftarrow x3, x1 \leftarrow x1\} \\
& mult(x1,mult(rd(x2,x1),\overbrace{i(x3)}))
\end{aligned}
$$

Lemma 1401: $mult(mult(x1,op_r(x1,x2,x3)),x4) = mult(op_r(x1,x2,x3),mult(x1,x4))$

$$
\begin{aligned}
& mult(\underbrace{mult(x1,op_r(x1,x2,x3))},x4) \\
= & \quad \text{by Lemma 119 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{mult(\overbrace{mult(op_r(x1,x2,x3),x1)},x4)} \\
= & \quad \text{by Lemma 91 RL with } \{x3 \leftarrow x4, x2 \leftarrow x1, x1 \leftarrow op_r(x1,x2,x3)\} \\
& \overbrace{mult(\underbrace{op_r(op_r(x1,x2,x3),x1,x4)},mult(x1,x4))} \\
= & \quad \text{by Axiom 17 RL with } \{x5 \leftarrow x3, x4 \leftarrow x2, x3 \leftarrow x4, x2 \leftarrow x1, x1 \leftarrow x1\} \\
& mult(\overbrace{op_r(\underbrace{op_r(x1,x1,x4)},x2,x3)},mult(x1,x4)) \\
= & \quad \text{by Lemma 26 LR with } \{x2 \leftarrow x4, x1 \leftarrow x1\} \\
& mult(op_r(\overbrace{x1},x2,x3),mult(x1,x4))
\end{aligned}
$$

Lemma 1402: $mult(x1,rd(x2,op_t(i(x2),x1))) = op_t(mult(x2,mult(x1,x2)),i(x1))$

$$
\begin{aligned}
& mult(x1,\underbrace{rd(x2,op_t(i(x2),x1))}) \\
= & \quad \text{by Lemma 10 RL with } \{x2 \leftarrow x2, x1 \leftarrow op_t(i(x2),x1)\} \\
& \underbrace{mult(x1,\overbrace{i(\overbrace{rd(op_t(i(x2),x1),x2)})})} \\
= & \quad \text{by Lemma 34 RL with } \{x2 \leftarrow rd(op_t(i(x2),x1),x2), x1 \leftarrow x1\} \\
& \overbrace{op_t(\underbrace{rd(x1,rd(op_t(i(x2),x1),x2))},i(x1))} \\
= & \quad \text{by Lemma 140 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_t(\overbrace{mult(x2,mult(x1,x2))},i(x1))
\end{aligned}
$$

Lemma 1403: $mult(op_t(i(x1),x2),mult(x1,x3)) = mult(x1,mult(op_t(i(x1),x2),x3))$

$mult(op_t(\underbrace{i(x1)}, x2), mult(x1,x3))$

$=$ by Lemma 9 RL with $\{x2 \leftarrow x1, x1 \leftarrow x1\}$

$mult(op_t(\overbrace{rd(x1,mult(x1,x1))}, x2), mult(x1,x3))$

$=$ by Lemma 51 RL with $\{x3 \leftarrow x3, x2 \leftarrow mult(x1,x1), x1 \leftarrow x1\}$

$mult(op_t(\overbrace{op_r(rd(x1,mult(x1,x1)), \underbrace{rd(mult(x1,x1),x1)}, x3)}, x2), mult(x1,x3))$

$=$ by Axiom 5 RL with $\{x2 \leftarrow x1, x1 \leftarrow x1\}$

$mult(op_t(op_r(\underbrace{rd(x1,mult(x1,x1))}, \overbrace{x1}, x3), x2), mult(x1,x3))$

$=$ by Lemma 9 LR with $\{x2 \leftarrow x1, x1 \leftarrow x1\}$

$mult(\underbrace{op_t(op_r(\overbrace{i(x1)}, x1, x3), x2)}, mult(x1,x3))$

$=$ by Axiom 14 LR with $\{x4 \leftarrow x2, x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow i(x1)\}$

$\underbrace{mult(\overbrace{op_r(op_t(i(x1),x2),x1,x3)}, mult(x1,x3))}$

$=$ by Lemma 91 LR with $\{x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow op_t(i(x1),x2)\}$

$\overbrace{mult(\underbrace{mult(op_t(i(x1),x2),x1)}, x3)}$

$=$ by Lemma 13 RL with $\{x2 \leftarrow op_t(i(x1),x2), x1 \leftarrow x1\}$

$mult(\overbrace{mult(x1,\underbrace{op_t(op_t(i(x1),x2),x1)})}, x3)$

$=$ by Axiom 13 RL with $\{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow i(x1)\}$

$mult(mult(x1,\overbrace{op_t(\underbrace{op_t(i(x1),x1)}, x2)}), x3)$

$=$ by Lemma 7 LR with $\{x1 \leftarrow x1\}$

$\underbrace{mult(mult(x1,op_t(\overbrace{i(x1)}, x2)), x3)}$

$=$ by Lemma 91 RL with $\{x3 \leftarrow x3, x2 \leftarrow op_t(i(x1),x2), x1 \leftarrow x1\}$

$\overbrace{mult(op_r(x1,\underbrace{op_t(i(x1),x2)}, x3), mult(op_t(i(x1),x2),x3))}$

$=$ by Lemma 19 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$mult(\underbrace{op_r(x1,\overbrace{i(op_t(x1,i(x2)))}, x3)}, mult(op_t(i(x1),x2),x3))$

$=$ by Axiom 5 LR with $\{x2 \leftarrow i(x2), x1 \leftarrow op_r(x1,i(op_t(x1,i(x2))),x3)\}$

$mult(\overbrace{rd(\underbrace{mult(op_r(x1,i(op_t(x1,i(x2)))),x3)}, i(x2))}, i(x2))}, mult(op_t(i(x1),x2),x3))$

$=$ by Lemma 166 LR with $\{x3 \leftarrow x3, x2 \leftarrow i(x2), x1 \leftarrow x1\}$

$mult(\underbrace{rd(\overbrace{mult(x1,i(x2))}, i(x2))}, mult(op_t(i(x1),x2),x3))$

$=$ by Axiom 5 RL with $\{x2 \leftarrow i(x2), x1 \leftarrow x1\}$

$mult(\overbrace{x1}, mult(op_t(i(x1),x2),x3))$

Lemma 1404: $rd(mult(x1, i(op_l(x1, x2, x3))), i(x1)) = i(mult(i(x1), rd(op_l(x1, x2, x3), x1)))$

$$
\begin{aligned}
& \underbrace{rd(mult(x1, i(op_l(x1, x2, x3))), i(x1))} \\
= & \quad \text{by Lemma 64 RL with } \{x3 \leftarrow op_l(x1, x2, x3), x2 \leftarrow x1, x1 \leftarrow i(x1)\} \\
& \overbrace{\underbrace{rd(i(i(x1)), mult(i(x1), op_l(x1, x2, x3)))}} \\
= & \quad \text{by Lemma 59 RL with } \{x2 \leftarrow op_l(x1, x2, x3), x1 \leftarrow i(x1)\} \\
& \overbrace{i(mult(i(x1), \underbrace{mult(op_l(x1, x2, x3), i(x1))}))} \\
= & \quad \text{by Lemma 178 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& i(mult(i(x1), \overbrace{rd(op_l(x1, x2, x3), x1)}))
\end{aligned}
$$

Lemma 1405: $rd(x1, rd(op_l(x1, x2, x3), x1)) = i(mult(i(x1), rd(op_l(x1, x2, x3), x1)))$

$$
\begin{aligned}
& rd(x1, \underbrace{rd(op_l(x1, x2, x3), x1)}) \\
= & \quad \text{by Lemma 171 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{rd(x1, \overbrace{mult(i(x1), op_l(x1, x2, x3))})} \\
= & \quad \text{by Lemma 61 RL with } \{x3 \leftarrow x1, x2 \leftarrow op_l(x1, x2, x3), x1 \leftarrow x1\} \\
& \overbrace{\underbrace{rd(mult(x1, i(op_l(x1, x2, x3))), i(x1))}} \\
= & \quad \text{by Lemma 1404 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{i(mult(i(x1), rd(op_l(x1, x2, x3), x1)))}
\end{aligned}
$$

Lemma 1406: $rd(x1, rd(op_l(x1, x2, x3), x1)) = mult(x1, rd(x1, op_l(x1, x2, x3)))$

$$
\begin{aligned}
& \underbrace{rd(x1, rd(op_l(x1, x2, x3), x1))} \\
= & \quad \text{by Lemma 1405 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{i(mult(i(x1), rd(op_l(x1, x2, x3), x1)))}} \\
= & \quad \text{by Lemma 16 LR with } \{x2 \leftarrow rd(op_l(x1, x2, x3), x1), x1 \leftarrow x1\} \\
& \overbrace{mult(x1, \underbrace{i(rd(op_l(x1, x2, x3), x1))})} \\
= & \quad \text{by Lemma 10 LR with } \{x2 \leftarrow x1, x1 \leftarrow op_l(x1, x2, x3)\} \\
& mult(x1, \overbrace{rd(x1, op_l(x1, x2, x3))})
\end{aligned}
$$

Lemma 1407: $op_l(x1, mult(x2, mult(x3, x2)), x3) = op_l(x1, x2, mult(x3, mult(x2, x3)))$

$$
\begin{aligned}
& \underbrace{op_l(x1, mult(x2, mult(x3, x2)), x3)} \\
= \quad & \text{by Axiom 12 RL with } \{x3 \leftarrow x1, x2 \leftarrow mult(x2, mult(x3, x2)), x1 \leftarrow x3\} \\
& \overbrace{mult(i(mult(x3, mult(x2, mult(x3, x2)))), mult(x3, \underbrace{mult(mult(x2, mult(x3, x2)), x1)}))} \\
= \quad & \text{by Axiom 6 RL with } \{x3 \leftarrow x1, x2 \leftarrow x3, x1 \leftarrow x2\} \\
& mult(i(mult(x3, mult(x2, mult(x3, x2)))), \underbrace{mult(x3, \overbrace{mult(x2, mult(x3, mult(x2, x1)))})}) \\
= \quad & \text{by Axiom 6 LR with } \{x3 \leftarrow mult(x2, x1), x2 \leftarrow x2, x1 \leftarrow x3\} \\
& mult(i(\underbrace{mult(x3, mult(x2, mult(x3, x2)))}), \overbrace{mult(mult(x3, mult(x2, x3)), mult(x2, x1))}) \\
= \quad & \text{by Axiom 6 LR with } \{x3 \leftarrow x2, x2 \leftarrow x2, x1 \leftarrow x3\} \\
& \underbrace{mult(i(\overbrace{mult(mult(x3, mult(x2, x3)), x2)}), mult(mult(x3, mult(x2, x3)), mult(x2, x1)))} \\
= \quad & \text{by Axiom 12 LR with } \{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow mult(x3, mult(x2, x3))\} \\
& \overbrace{op_l(x1, x2, mult(x3, mult(x2, x3)))}
\end{aligned}
$$

Lemma 1408: $mult(rd(x1, op_r(x1, x2, x3)), x1) = rd(x1, mult(i(x1), op_r(i(i(x1)), x2, x3)))$

$$
\begin{aligned}
& mult(rd(x1, op_r(\underbrace{x1}, x2, x3)), x1) \\
= \quad & \text{by Lemma 3 RL with } \{x1 \leftarrow x1\} \\
& \underbrace{mult(rd(x1, op_r(\overbrace{i(i(x1))}, x2, x3)), x1)} \\
= \quad & \text{by Lemma 334 RL with } \{x2 \leftarrow op_r(i(i(x1)), x2, x3), x1 \leftarrow x1\} \\
& \overbrace{rd(x1, \underbrace{mult(op_r(i(i(x1)), x2, x3), i(x1))})} \\
= \quad & \text{by Lemma 159 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow i(x1)\} \\
& rd(x1, \overbrace{mult(i(x1), op_r(i(i(x1)), x2, x3))})
\end{aligned}
$$

Lemma 1409: $mult(rd(x1, op_r(x1, x2, x3)), x1) = rd(x1, rd(op_r(x1, x2, x3), x1))$

$$
\begin{aligned}
& \underbrace{mult(rd(x1, op_r(x1, x2, x3)), x1)} \\
= \quad & \text{by Lemma 1408 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{rd(x1, mult(i(x1), op_r(\underbrace{i(i(x1))}, x2, x3)))} \\
= \quad & \text{by Lemma 3 LR with } \{x1 \leftarrow x1\} \\
& rd(x1, \underbrace{mult(i(x1), op_r(\overbrace{x1}, x2, x3))}) \\
= \quad & \text{by Lemma 117 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& rd(x1, \overbrace{rd(op_r(x1, x2, x3), x1)})
\end{aligned}
$$

Lemma 1410: $op_t(x1, rd(mult(i(x2), x3), x1)) = op_t(x1, rd(mult(x2, i(x3)), x1))$

$$
\begin{aligned}
& op_t(x1, \underbrace{rd(mult(i(x2), x3), x1)}) \\
= \quad & \text{by Lemma 62 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{op_t(x1, \overbrace{rd(i(x1), mult(x2, i(x3)))})} \\
= \quad & \text{by Lemma 707 LR with } \{x2 \leftarrow mult(x2, i(x3)), x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, rd(mult(x2, i(x3)), x1))}
\end{aligned}
$$

Lemma 1411: $mult(rd(x2, mult(i(x1), i(x1))), x1) = i(rd(i(x1), rd(rd(x2, i(x1)), i(x1))))$

$mult(\underbrace{rd(x2, mult(i(x1), i(x1)))}, x1)$

$=$ by Lemma 10 RL with $\{x2 \leftarrow x2, x1 \leftarrow mult(i(x1), i(x1))\}$

$\underbrace{mult(\overbrace{i(rd(mult(i(x1), i(x1)), x2))}, x1)}$

$=$ by Lemma 17 RL with $\{x2 \leftarrow x1, x1 \leftarrow rd(mult(i(x1), i(x1)), x2)\}$

$\overbrace{i(\underbrace{mult(rd(mult(i(x1), i(x1)), x2), i(x1))})}$

$=$ by Lemma 869 RL with $\{x2 \leftarrow x2, x1 \leftarrow i(x1)\}$

$i(\overbrace{rd(i(x1), rd(rd(x2, i(x1)), i(x1)))})$

Lemma 1412: $op_r(x1, i(x2), mult(asoc(x2, x1, x2), i(mult(x1, i(x2))))) = rd(mult(i(i(x2)), mult(asoc(x1, x2, x2), mult(x1, i(x2)$

$op_r(x1, i(x2), \underbrace{mult(asoc(x2, x1, x2), i(mult(x1, i(x2))))})$

$=$ by Lemma 16 RL with $\{x2 \leftarrow mult(x1, i(x2)), x1 \leftarrow asoc(x2, x1, x2)\}$

$op_r(x1, i(x2), \overbrace{i(mult(\underbrace{i(asoc(x2, x1, x2))}, mult(x1, i(x2)))))})$

$=$ by Lemma 757 LR with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$\underbrace{op_r(x1, i(x2), i(mult(\overbrace{asoc(x1, x2, x2)}, mult(x1, i(x2)))))})$

$=$ by Axiom 16 LR with $\{x3 \leftarrow i(mult(asoc(x1, x2, x2), mult(x1, i(x2)))), x2 \leftarrow i(x2), x1 \leftarrow x1\}$

$\overbrace{rd(\underbrace{mult(mult(x1, i(x2)), i(mult(asoc(x1, x2, x2), mult(x1, i(x2)))))}, mult(i(x2), i(mult(asoc(x1, x2, x2), mult(x1, i(x2)$

$=$ by Lemma 31 LR with $\{x2 \leftarrow asoc(x1, x2, x2), x1 \leftarrow mult(x1, i(x2))\}$

$\underbrace{rd(\overbrace{i(op_t(asoc(x1, x2, x2), mult(x1, i(x2))))}, mult(i(x2), i(mult(asoc(x1, x2, x2), mult(x1, i(x2))))))}$

$=$ by Lemma 62 LR with $\{x3 \leftarrow mult(asoc(x1, x2, x2), mult(x1, i(x2))), x2 \leftarrow i(x2), x1 \leftarrow op_t(asoc(x1, x2, x2), mult(x1, i(x2)$

$\overbrace{rd(mult(i(i(x2)), mult(asoc(x1, x2, x2), mult(x1, i(x2)))), op_t(\underbrace{asoc(x1, x2, x2)}, mult(x1, i(x2))))}$

$=$ by Lemma 779 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$rd(mult(i(i(x2)), mult(asoc(x1, x2, x2), mult(x1, i(x2)))), op_t(\overbrace{asoc(mult(x1, i(x2)), x2, x2)}, mult(x1, i(x2))))$

$=$ by Lemma 756 RL with $\{x2 \leftarrow mult(x1, i(x2)), x1 \leftarrow x2\}$

$rd(mult(i(i(x2)), mult(asoc(x1, x2, x2), mult(x1, i(x2)))), \underbrace{op_t(\overbrace{asoc(x2, i(mult(x1, i(x2))), x2)}, mult(x1, i(x2))))}$

$=$ by Lemma 376 RL with $\{x2 \leftarrow mult(x1, i(x2)), x1 \leftarrow asoc(x2, i(mult(x1, i(x2))), x2)\}$

$rd(mult(i(i(x2)), mult(asoc(x1, x2, x2), mult(x1, i(x2)))), \overbrace{rd(mult(asoc(x2, i(mult(x1, i(x2))), x2), mult(x1, i(x2))), \underbrace{op_t(m}$

$=$ by Lemma 539 LR with $\{x2 \leftarrow x2, x1 \leftarrow mult(x1, i(x2))\}$

$rd(mult(i(i(x2)), mult(asoc(x1, x2, x2), mult(x1, i(x2)))), \underbrace{rd(mult(asoc(x2, i(mult(x1, i(x2))), x2), mult(x1, i(x2))), \overbrace{mult($

$=$ by Axiom 5 RL with $\{x2 \leftarrow mult(x1, i(x2)), x1 \leftarrow asoc(x2, i(mult(x1, i(x2))), x2)\}$

$rd(mult(i(i(x2)), mult(asoc(x1, x2, x2), mult(x1, i(x2)))), \underbrace{\overbrace{asoc(x2, i(mult(x1, i(x2))), x2)})}$

$=$ by Lemma 756 LR with $\{x2 \leftarrow mult(x1, i(x2)), x1 \leftarrow x2\}$

$rd(mult(i(i(x2)), mult(asoc(x1, x2, x2), mult(x1, i(x2)))), \underbrace{\overbrace{asoc(mult(x1, i(x2)), x2, x2)})}$

$=$ by Lemma 779 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$rd(mult(i(i(x2)), mult(asoc(x1, x2, x2), mult(x1, i(x2)))), \overbrace{asoc(x1, x2, x2)})$

Lemma 1413: $op_r(x1, i(x2), rd(x2, x1)) = rd(mult(i(i(x2)), mult(asoc(x1, x2, x2), mult(x1, i(x2)))), asoc(x1, x2, x2))$

$$
\begin{aligned}
& op_r(x1, i(x2), rd(x2, \underbrace{x1}_{})) \\
= \quad & \text{by Lemma 3 RL with } \{x1 \leftarrow x1\} \\
& op_r(x1, i(x2), \underbrace{rd(x2, \overbrace{i(i(x1))}^{})}_{}) \\
= \quad & \text{by Lemma 359 RL with } \{x2 \leftarrow x2, x1 \leftarrow i(x1)\} \\
& op_r(x1, i(x2), \overbrace{op_r(\underbrace{mult(i(x1), x2)}_{}, x2, x2)}^{}) \\
= \quad & \text{by Lemma 17 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_r(x1, i(x2), \underbrace{op_r(\overbrace{i(mult(x1, i(x2)))}^{}, x2, x2)}_{}) \\
= \quad & \text{by Lemma 642 RL with } \{x2 \leftarrow mult(x1, i(x2)), x1 \leftarrow x2\} \\
& op_r(x1, i(x2), \overbrace{\underbrace{rd(asoc(x2, mult(x1, i(x2)), x2), mult(x1, i(x2)))}_{}}^{}) \\
= \quad & \text{by Lemma 542 RL with } \{x2 \leftarrow mult(x1, i(x2)), x1 \leftarrow x2\} \\
& op_r(x1, i(x2), \overbrace{mult(asoc(x2, \underbrace{mult(x1, i(x2))}_{}, x2), i(mult(x1, i(x2))))}^{}) \\
= \quad & \text{by Lemma 3 RL with } \{x1 \leftarrow mult(x1, i(x2))\} \\
& op_r(x1, i(x2), mult(\underbrace{asoc(x2, \overbrace{i(i(mult(x1, i(x2))))}^{}, x2)}_{}, i(mult(x1, i(x2))))) \\
= \quad & \text{by Lemma 498 RL with } \{x2 \leftarrow i(mult(x1, i(x2))), x1 \leftarrow x2\} \\
& op_r(x1, i(x2), mult(\overbrace{asoc(x2, \underbrace{rd(x2, i(mult(x1, i(x2))))}_{}, x2)}^{}, i(mult(x1, i(x2))))) \\
= \quad & \text{by Lemma 38 LR with } \{x1 \leftarrow mult(x1, i(x2)), x2 \leftarrow x2\} \\
& op_r(x1, i(x2), mult(asoc(x2, \overbrace{\underbrace{rd(mult(x1, i(x2)), i(x2))}_{}}^{}, x2), i(mult(x1, i(x2))))) \\
= \quad & \text{by Axiom 5 RL with } \{x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& \underbrace{op_r(x1, i(x2), mult(asoc(x2, \overbrace{x1}^{}, x2), i(mult(x1, i(x2)))))}_{} \\
= \quad & \text{by Lemma 1412 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{rd(mult(i(i(x2)), mult(asoc(x1, x2, x2), mult(x1, i(x2)))), asoc(x1, x2, x2))}^{}
\end{aligned}
$$

Lemma 1414: $op_r(x1, i(x2), rd(x2, x1)) = rd(mult(op_l(x2, x1, x2), mult(x1, i(x2))), asoc(x1, x2, x2))$

$$
\begin{aligned}
& \underbrace{op_r(x1, i(x2), rd(x2, x1))}_{} \\
= \quad & \text{by Lemma 1413 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{rd(mult(\underbrace{i(i(x2))}_{}, mult(asoc(x1, x2, x2), mult(x1, i(x2)))), asoc(x1, x2, x2))}^{} \\
= \quad & \text{by Lemma 3 LR with } \{x1 \leftarrow x2\} \\
& rd(\underbrace{mult(\overbrace{x2}^{}, mult(asoc(x1, x2, x2), mult(x1, i(x2))))}_{}, asoc(x1, x2, x2)) \\
= \quad & \text{by Lemma 943 LR with } \{x3 \leftarrow mult(x1, i(x2)), x2 \leftarrow x1, x1 \leftarrow x2\} \\
& rd(\overbrace{mult(op_l(x2, x1, x2), mult(x1, i(x2)))}^{}, asoc(x1, x2, x2))
\end{aligned}
$$

Lemma 1415: $op_r(x1, i(x2), rd(x2, x1)) = op_t(x1, i(x2))$

$$
\begin{aligned}
& \underbrace{op_r(x1, i(x2), rd(x2, x1))} \\
= \quad & \text{by Lemma 1414 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{rd(\underbrace{mult(op_l(x2, x1, x2), mult(x1, i(x2)))}, asoc(x1, x2, x2))} \\
= \quad & \text{by Lemma 979 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \underbrace{rd(\overbrace{op_l(x1, x1, x2)}, asoc(x1, x2, x2))} \\
= \quad & \text{by Lemma 988 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, i(x2))}
\end{aligned}
$$

Lemma 1416: $op_t(x1, x2) = op_r(x1, x2, rd(i(x2), x1))$

$$
\begin{aligned}
& op_t(x1, \underbrace{x2}) \\
= \quad & \text{by Lemma 3 RL with } \{x1 \leftarrow x2\} \\
& \underbrace{op_t(x1, \overbrace{i(i(x2))})} \\
= \quad & \text{by Lemma 1415 RL with } \{x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, \underbrace{i(i(x2))}, rd(i(x2), x1))} \\
= \quad & \text{by Lemma 3 LR with } \{x1 \leftarrow x2\} \\
& op_r(x1, \overbrace{x2}, rd(i(x2), x1))
\end{aligned}
$$

Lemma 1417: $mult(i(mult(x1, x2)), mult(x1, x3)) = mult(i(x1), mult(rd(x1, x2), x3))$

$$
\begin{aligned}
& mult(\underbrace{i(mult(x1, x2))}, mult(x1, x3)) \\
= \quad & \text{by Axiom 11 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{mult(\overbrace{mult(i(x1), i(x2))}, mult(x1, x3))} \\
= \quad & \text{by Lemma 982 RL with } \{x3 \leftarrow mult(x1, x3), x2 \leftarrow x2, x1 \leftarrow i(x1)\} \\
& \overbrace{mult(i(x1), mult(rd(i(i(x1)), x2), \underbrace{mult(i(x1), mult(x1, x3))}))} \\
= \quad & \text{by Axiom 2 RL with } \{x2 \leftarrow x1, x1 \leftarrow x3\} \\
& mult(i(x1), mult(rd(\underbrace{i(i(x1))}, x2), \overbrace{x3})) \\
= \quad & \text{by Lemma 3 LR with } \{x1 \leftarrow x1\} \\
& mult(i(x1), mult(rd(\overbrace{x1}, x2), x3))
\end{aligned}
$$

Lemma 1418: $mult(rd(x1, x2), mult(i(x1), x3)) = mult(x1, mult(mult(i(x1), i(x2)), x3))$

$$
\begin{aligned}
& mult(rd(\underbrace{x1}, x2), mult(i(x1), x3)) \\
= \quad & \text{by Lemma 3 RL with } \{x1 \leftarrow x1\} \\
& \underbrace{mult(rd(\overbrace{i(i(x1))}, x2), mult(i(x1), x3))} \\
= \quad & \text{by Lemma 12 RL with } \{x2 \leftarrow mult(rd(i(i(x1)), x2), mult(i(x1), x3)), x1 \leftarrow x1\} \\
& \overbrace{mult(x1, \underbrace{mult(i(x1), mult(rd(i(i(x1)), x2), mult(i(x1), x3)))})} \\
= \quad & \text{by Lemma 982 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow i(x1)\} \\
& mult(x1, \overbrace{mult(mult(i(x1), i(x2)), x3)})
\end{aligned}
$$

Lemma 1419: $op_l(x1, op_l(i(x2), x3, x4), x5) = op_l(x1, op_l(x2, x3, x4), i(x5))$

$$
\begin{aligned}
& op_l(x1, \underbrace{op_l(i(x2), x3, x4)}, x5) \\
= & \quad \text{by Lemma 1049 RL with } \{x3 \leftarrow x4, x2 \leftarrow x3, x1 \leftarrow x2\} \\
& \underbrace{op_l(x1, \overbrace{i(op_l(x2, x3, x4))}, x5)} \\
= & \quad \text{by Lemma 1050 LR with } \{x3 \leftarrow x5, x2 \leftarrow op_l(x2, x3, x4), x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, op_l(x2, x3, x4), i(x5))}
\end{aligned}
$$

Lemma 1420: $mult(mult(x1, op_l(x1, x2, x3)), x4) = mult(x1, mult(op_l(x1, x2, x3), x4))$

$$
\begin{aligned}
& \underbrace{mult(mult(x1, op_l(x1, x2, x3)), x4)} \\
= & \quad \text{by Lemma 91 RL with } \{x3 \leftarrow x4, x2 \leftarrow op_l(x1, x2, x3), x1 \leftarrow x1\} \\
& \overbrace{mult(\underbrace{op_r(x1, op_l(x1, x2, x3), x4)}, mult(op_l(x1, x2, x3), x4))} \\
= & \quad \text{by Lemma 138 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow op_r(x1, op_l(x1, x2, x3), x4)\} \\
& mult(\overbrace{op_l(\underbrace{op_l(op_r(x1, op_l(x1, x2, x3), x4), x2, x3)}, x3, x2)}, mult(op_l(x1, x2, x3), x4)) \\
= & \quad \text{by Axiom 18 RL with } \{x5 \leftarrow x4, x4 \leftarrow op_l(x1, x2, x3), x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& mult(op_l(\overbrace{op_r(op_l(x1, x2, x3), op_l(x1, x2, x3), x4)}, x3, x2), mult(op_l(x1, x2, x3), x4)) \\
= & \quad \text{by Lemma 26 LR with } \{x2 \leftarrow x4, x1 \leftarrow op_l(x1, x2, x3)\} \\
& mult(\underbrace{op_l(\overbrace{op_l(x1, x2, x3)}, x3, x2)}, mult(op_l(x1, x2, x3), x4)) \\
= & \quad \text{by Lemma 138 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& mult(\overbrace{x1}, mult(op_l(x1, x2, x3), x4))
\end{aligned}
$$

Lemma 1421: $op_l(x1, op_l(x2, x3, x4), x2) = x1$

$$
\begin{aligned}
& op_l(x1, op_l(x2, x3, x4), \underbrace{x2}) \\
= & \quad \text{by Lemma 3 RL with } \{x1 \leftarrow x2\} \\
& \underbrace{op_l(x1, op_l(x2, x3, x4), \overbrace{i(i(x2))})} \\
= & \quad \text{by Lemma 1419 RL with } \{x5 \leftarrow i(x2), x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{\overbrace{op_l(x1, op_l(i(x2), x3, x4), i(x2))}} \\
= & \quad \text{by Lemma 987 RL with } \{x3 \leftarrow x1, x2 \leftarrow op_l(i(x2), x3, x4), x1 \leftarrow x2\} \\
& \overbrace{\underbrace{mult(x2, mult(rd(i(\overbrace{op_l(i(x2), x3, x4)}), x2), mult(op_l(i(x2), x3, x4), x1)))}} \\
= & \quad \text{by Lemma 1086 RL with } \{x3 \leftarrow mult(op_l(i(x2), x3, x4), x1), x2 \leftarrow i(op_l(i(x2), x3, x4)), x1 \leftarrow x2\} \\
& \overbrace{mult(mult(x2, i(op_l(i(x2), x3, x4))), \underbrace{mult(i(x2), mult(op_l(i(x2), x3, x4), x1))})} \\
= & \quad \text{by Lemma 1420 RL with } \{x4 \leftarrow x1, x3 \leftarrow x4, x2 \leftarrow x3, x1 \leftarrow i(x2)\} \\
& mult(\underbrace{mult(x2, i(op_l(i(x2), x3, x4)))}, \overbrace{mult(mult(i(x2), op_l(i(x2), x3, x4)), x1)}) \\
= & \quad \text{by Lemma 16 RL with } \{x2 \leftarrow op_l(i(x2), x3, x4), x1 \leftarrow x2\} \\
& \underbrace{mult(\overbrace{i(mult(i(x2), op_l(i(x2), x3, x4)))}, mult(mult(i(x2), op_l(i(x2), x3, x4)), x1))} \\
= & \quad \text{by Axiom 2 RL with } \{x2 \leftarrow mult(i(x2), op_l(i(x2), x3, x4)), x1 \leftarrow x1\} \\
& \overbrace{x1}
\end{aligned}
$$

Lemma 1422: $op_l(x1, x2, op_l(x2, x3, x4)) = x1$

$$
\begin{aligned}
& op_l(\underbrace{x1}, x2, op_l(x2, x3, x4)) \\
= & \quad \text{by Lemma 1421 RL with } \{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{op_l(\overbrace{op_l(x1, op_l(x2, x3, x4), x2)}, x2, op_l(x2, x3, x4))} \\
= & \quad \text{by Lemma 138 LR with } \{x3 \leftarrow x2, x2 \leftarrow op_l(x2, x3, x4), x1 \leftarrow x1\} \\
& \overbrace{x1}
\end{aligned}
$$

Lemma 1423: $mult(op_t(x1, x2), mult(i(x2), x1)) = rd(op_t(x1, x2), mult(i(x1), x2))$

$$
\begin{aligned}
& mult(op_t(x1, x2), \underbrace{mult(i(x2), x1)}) \\
= & \quad \text{by Lemma 17 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \underbrace{mult(op_t(x1, x2), \overbrace{i(\overbrace{mult(x2, i(x1))})})} \\
= & \quad \text{by Lemma 34 RL with } \{x2 \leftarrow mult(x2, i(x1)), x1 \leftarrow op_t(x1, x2)\} \\
& \overbrace{op_t(rd(\underbrace{op_t(x1, x2)}, mult(x2, i(x1))), i(op_t(x1, x2)))} \\
= & \quad \text{by Axiom 2 LR with } \{x2 \leftarrow rd(x2, x1), x1 \leftarrow op_t(x1, x2)\} \\
& op_t(rd(\overbrace{mult(i(\overbrace{rd(x2, x1)}), \underbrace{mult(rd(x2, x1), op_t(x1, x2))})}, mult(x2, i(x1))), i(op_t(x1, x2))) \\
= & \quad \text{by Lemma 650 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& op_t(rd(mult(\underbrace{i(\overbrace{rd(x2, x1)})}, \overbrace{rd(mult(x1, x2), x1)}), mult(x2, i(x1))), i(op_t(x1, x2))) \\
= & \quad \text{by Lemma 10 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& op_t(\underbrace{rd(mult(\overbrace{rd(x1, x2)}, rd(mult(x1, x2), x1)), mult(x2, i(x1)))}, i(op_t(x1, x2))) \\
= & \quad \text{by Lemma 63 RL with } \{x3 \leftarrow mult(rd(x1, x2), rd(mult(x1, x2), x1)), x2 \leftarrow x1, x1 \leftarrow x2\} \\
& op_t(\overbrace{rd(mult(i(x2), x1), i(\underbrace{mult(rd(x1, x2), rd(mult(x1, x2), x1))}))}, i(op_t(x1, x2))) \\
= & \quad \text{by Lemma 1128 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{op_t(\overbrace{mult(rd(x1, x2), op_t(x1, x2))}, i(op_t(x1, x2)))} \\
= & \quad \text{by Lemma 326 LR with } \{x2 \leftarrow op_t(x1, x2), x1 \leftarrow rd(x1, x2)\} \\
& \overbrace{rd(op_t(x1, x2), op_t(\underbrace{i(rd(x1, x2))}, op_t(x1, x2)))} \\
= & \quad \text{by Lemma 10 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& rd(op_t(x1, x2), op_t(\overbrace{rd(x2, x1)}, \underbrace{op_t(x1, x2)})) \\
= & \quad \text{by Lemma 553 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& rd(op_t(x1, x2), \underbrace{op_t(rd(x2, x1), \overbrace{op_t(x1, op_l(x2, x2, x1))})}) \\
= & \quad \text{by Lemma 573 RL with } \{x1 \leftarrow x2, x2 \leftarrow x1\} \\
& rd(op_t(x1, x2), \overbrace{mult(i(x1), x2)})
\end{aligned}
$$

Lemma 1424: $i(mult(x1, mult(x2, op_t(x1, x3)))) = rd(i(op_t(x1, x3)), mult(x1, x2))$

$$
\begin{aligned}
& \underbrace{i(mult(x1, mult(x2, op_t(x1, x3))))} \\
= & \quad \text{by Lemma 1132 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{i(\overbrace{rd(mult(x1, x2), i(op_t(x1, x3)))})} \\
= & \quad \text{by Lemma 10 LR with } \{x2 \leftarrow i(op_t(x1, x3)), x1 \leftarrow mult(x1, x2)\} \\
& \overbrace{rd(i(op_t(x1, x3)), mult(x1, x2))}
\end{aligned}
$$

Lemma 1425: $rd(x1, op_t(mult(i(x2), x1), x3)) = mult(op_t(rd(x2, x1), i(x3)), x1)$

$$
\begin{aligned}
& \underbrace{rd(x1, op_t(mult(i(x2), x1), x3))} \\
= & \quad \text{by Lemma 1150 RL with } \{x3 \leftarrow x3, x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& \overbrace{mult(x1, \underbrace{i(op_t(mult(x1, i(x2)), x3))})} \\
= & \quad \text{by Lemma 89 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{mult(x1, \overbrace{op_t(mult(i(x1), x2), i(x3))})} \\
= & \quad \text{by Lemma 1014 LR with } \{x3 \leftarrow i(x3), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{mult(op_t(rd(x2, x1), i(x3)), x1)}
\end{aligned}
$$

Lemma 1426: $mult(rd(x2, x1), i(x3)) = op_t(rd(rd(x2, x1), x3), rd(x1, x2))$

$$
\begin{aligned}
& \underbrace{mult(rd(x2, x1), i(x3))} \\
= & \quad \text{by Lemma 44 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{\overbrace{i(mult(rd(x1, x2), x3))}} \\
= & \quad \text{by Lemma 33 RL with } \{x2 \leftarrow x3, x1 \leftarrow rd(x1, x2)\} \\
& \overbrace{op_t(rd(\underbrace{i(rd(x1, x2))}, x3), rd(x1, x2))} \\
= & \quad \text{by Lemma 10 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_t(rd(\overbrace{rd(x2, x1)}, x3), rd(x1, x2))
\end{aligned}
$$

Lemma 1427: $rd(mult(i(x1), x2), mult(x1, x3)) = rd(i(x1), mult(x1, rd(x3, x2)))$

$\overbrace{rd(mult(i(x1), x2), mult(x1, x3))}$

$=$ by Lemma 1152 RL with $\{x3 \leftarrow mult(x1, x3), x2 \leftarrow x2, x1 \leftarrow i(x1)\}$

$\overbrace{mult(i(x1), mult(rd(x2, \underbrace{mult(i(x1), mult(x1, x3))}), i(x1)))}$

$=$ by Axiom 2 RL with $\{x2 \leftarrow x1, x1 \leftarrow x3\}$

$\underbrace{mult(i(x1), mult(rd(x2, \overbrace{x3}), i(x1)))}$

$=$ by Lemma 73 LR with $\{x2 \leftarrow rd(x2, x3), x1 \leftarrow x1\}$

$\overbrace{\underbrace{rd(mult(i(x1), \overbrace{rd(x2, x3)}), x1)}}$

$=$ by Lemma 10 RL with $\{x2 \leftarrow mult(i(x1), rd(x2, x3)), x1 \leftarrow x1\}$

$\overbrace{i(\underbrace{rd(x1, mult(i(x1), rd(x2, x3)))})}$

$=$ by Lemma 60 RL with $\{x2 \leftarrow rd(x2, x3), x1 \leftarrow x1\}$

$i(\overbrace{\underbrace{mult(x1, mult(i(\overbrace{rd(x2, x3)}), x1))}})$

$=$ by Lemma 76 RL with $\{x3 \leftarrow rd(x2, x3), x2 \leftarrow i(rd(x2, x3)), x1 \leftarrow x1\}$

$i(\overbrace{rd(mult(x1, \underbrace{mult(i(rd(x2, x3)), mult(x1, rd(x2, x3)))}), rd(x2, x3))})$

$=$ by Axiom 10 LR with $\{x2 \leftarrow x1, x1 \leftarrow rd(x2, x3)\}$

$\underbrace{i(rd(mult(x1, \overbrace{op_t(x1, rd(x2, x3))}), rd(x2, x3)))}$

$=$ by Lemma 10 LR with $\{x2 \leftarrow rd(x2, x3), x1 \leftarrow mult(x1, op_t(x1, rd(x2, x3)))\}$

$\overbrace{rd(rd(x2, x3), mult(x1, \underbrace{op_t(x1, rd(x2, x3))}))}$

$=$ by Axiom 10 RL with $\{x2 \leftarrow x1, x1 \leftarrow rd(x2, x3)\}$

$rd(rd(x2, x3), mult(x1, \overbrace{mult(\underbrace{i(rd(x2, x3))}, mult(x1, rd(x2, x3)))}))$

$=$ by Lemma 10 LR with $\{x2 \leftarrow x3, x1 \leftarrow x2\}$

$rd(rd(x2, x3), \underbrace{mult(x1, mult(\overbrace{rd(x3, x2)}, mult(x1, rd(x2, x3))))})$

$=$ by Axiom 6 LR with $\{x3 \leftarrow rd(x2, x3), x2 \leftarrow rd(x3, x2), x1 \leftarrow x1\}$

$\underbrace{rd(rd(x2, x3), \overbrace{mult(mult(x1, mult(rd(x3, x2), x1)), rd(x2, x3))})}$

$=$ by Lemma 9 LR with $\{x2 \leftarrow mult(x1, mult(rd(x3, x2), x1)), x1 \leftarrow rd(x2, x3)\}$

$\overbrace{\underbrace{i(mult(x1, mult(rd(x3, x2), x1)))}}$

$=$ by Lemma 59 LR with $\{x2 \leftarrow rd(x3, x2), x1 \leftarrow x1\}$

$\overbrace{rd(i(x1), mult(x1, rd(x3, x2)))}$

Lemma 1428: $op_l(x1, x2, rd(x3, mult(x2, x4))) = op_l(x1, rd(mult(x2, x3), x4), i(x2))$

$$
\begin{aligned}
& \underbrace{op_l(x1, x2, rd(x3, mult(x2, x4)))} \\
= \quad & \text{by Lemma 291 RL with } \{x3 \leftarrow rd(x3, mult(x2, x4)), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, x2, mult(rd(x3, mult(x2, x4)), x2))}} \\
= \quad & \text{by Lemma 242 RL with } \{x3 \leftarrow mult(rd(x3, mult(x2, x4)), x2), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, \underbrace{mult(x2, mult(rd(x3, mult(x2, x4)), x2))}, i(x2))} \\
= \quad & \text{by Lemma 1152 LR with } \{x3 \leftarrow x4, x2 \leftarrow x3, x1 \leftarrow x2\} \\
& op_l(x1, \overbrace{rd(mult(x2, x3), x4)}, i(x2))
\end{aligned}
$$

Lemma 1429: $op_l(x1, rd(x2, mult(x3, x4)), x3) = op_l(x1, x3, rd(x4, mult(x3, x2)))$

$$
\begin{aligned}
& \underbrace{op_l(x1, rd(x2, mult(x3, x4)), x3)} \\
= \quad & \text{by Lemma 1060 RL with } \{x4 \leftarrow x3, x3 \leftarrow x2, x2 \leftarrow mult(x3, x4), x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, rd(mult(x3, x4), x2), i(x3))}} \\
= \quad & \text{by Lemma 1428 RL with } \{x4 \leftarrow x2, x3 \leftarrow x4, x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x3, rd(x4, mult(x3, x2)))}
\end{aligned}
$$

Lemma 1430: $rd(mult(x1, i(op_t(x2, x1))), x2) = mult(x1, rd(op_t(i(x2), x1), x2))$

$\underbrace{rd(mult(x1, i(op_t(x2, x1))), x2)}$

$=$ by Lemma 1152 RL with $\{x3 \leftarrow x2, x2 \leftarrow i(op_t(x2, x1)), x1 \leftarrow x1\}$

$\overbrace{mult(x1, mult(rd(\underbrace{i(op_t(x2, x1))}, mult(x1, x2)), x1))}$

$=$ by Lemma 15 RL with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$mult(x1, mult(rd(\overbrace{op_t(i(x2), \underbrace{i(x1)})}, mult(x1, x2)), x1))$

$=$ by Lemma 9 RL with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$mult(x1, mult(\underbrace{rd(op_t(i(x2), \overbrace{rd(x2, mult(x1, x2))})}, mult(x1, x2)), x1))$

$=$ by Lemma 10 RL with $\{x2 \leftarrow op_t(i(x2), rd(x2, mult(x1, x2))), x1 \leftarrow mult(x1, x2)\}$

$mult(x1, mult(\overbrace{i(\underbrace{rd(mult(x1, x2), op_t(i(x2), rd(x2, mult(x1, x2))))})}, x1))$

$=$ by Lemma 386 LR with $\{x2 \leftarrow x2, x1 \leftarrow mult(x1, x2)\}$

$mult(x1, mult(\underbrace{i(\overbrace{mult(x2, mult(x1, x2))})}, x1))$

$=$ by Lemma 59 LR with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$mult(x1, mult(\overbrace{rd(i(x2), mult(\underbrace{x2}, x1))}, x1))$

$=$ by Lemma 3 RL with $\{x1 \leftarrow x2\}$

$mult(x1, mult(\underbrace{rd(i(x2), mult(\overbrace{i(i(x2))}, x1))}, x1))$

$=$ by Lemma 60 RL with $\{x2 \leftarrow x1, x1 \leftarrow i(x2)\}$

$mult(x1, mult(\underbrace{\overbrace{mult(i(x2), mult(i(x1), i(x2)))}}, x1))$

$=$ by Lemma 76 RL with $\{x3 \leftarrow x1, x2 \leftarrow i(x1), x1 \leftarrow i(x2)\}$

$mult(x1, mult(\overbrace{rd(mult(i(x2), \underbrace{mult(i(x1), mult(i(x2), x1))}), x1)}, x1))$

$=$ by Axiom 10 LR with $\{x2 \leftarrow i(x2), x1 \leftarrow x1\}$

$mult(x1, \underbrace{mult(rd(mult(i(x2), \overbrace{op_t(i(x2), x1)}), x1), x1)})$

$=$ by Axiom 4 RL with $\{x2 \leftarrow x1, x1 \leftarrow mult(i(x2), op_t(i(x2), x1))\}$

$mult(x1, \underbrace{\overbrace{mult(i(x2), op_t(i(x2), x1))}})$

$=$ by Lemma 111 LR with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$mult(x1, \overbrace{rd(op_t(i(x2), x1), x2)})$

Lemma 1431: $rd(mult(x1, op_t(x2, x1)), op_t(i(x1), x2)) = mult(x1, op_t(op_t(mult(x1, x2), x1), i(x2)))$

$rd(mult(x1, op_t(x2, x1)), \underbrace{op_t(i(x1), x2)})$

$=$ by Lemma 19 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{rd(mult(x1, op_t(x2, x1)), \overbrace{i(op_t(x1, i(x2)))})}$

$=$ by Lemma 1132 LR with $\{x3 \leftarrow i(x2), x2 \leftarrow op_t(x2, x1), x1 \leftarrow x1\}$

$\overbrace{mult(x1, mult(\underbrace{op_t(x2, x1)}, op_t(x1, i(x2))))}$

$=$ by Lemma 559 RL with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$mult(x1, \underbrace{mult(\overbrace{op_t(x2, op_t(x1, i(x2)))}, op_t(x1, i(x2)))})$

$=$ by Lemma 341 RL with $\{x2 \leftarrow x2, x1 \leftarrow op_t(x1, i(x2))\}$

$mult(x1, \overbrace{op_t(\underbrace{rd(op_t(x1, i(x2)), i(x2))}, op_t(x1, i(x2)))})$

$=$ by Lemma 359 RL with $\{x2 \leftarrow op_t(x1, i(x2)), x1 \leftarrow x2\}$

$mult(x1, \underbrace{op_t(\overbrace{op_r(mult(x2, op_t(x1, i(x2))), op_t(x1, i(x2)), op_t(x1, i(x2)))}, op_t(x1, i(x2)))})$

$=$ by Axiom 14 LR with $\{x4 \leftarrow op_t(x1, i(x2)), x3 \leftarrow op_t(x1, i(x2)), x2 \leftarrow op_t(x1, i(x2)), x1 \leftarrow mult(x2, op_t(x1, i(x2)))\}$

$mult(x1, \overbrace{op_r(\underbrace{op_t(mult(x2, op_t(x1, i(x2))), op_t(x1, i(x2)))}, op_t(x1, i(x2)), op_t(x1, i(x2)))})$

$=$ by Lemma 751 LR with $\{x2 \leftarrow op_t(x1, i(x2)), x1 \leftarrow x2\}$

$mult(x1, \underbrace{op_r(\overbrace{op_t(mult(op_t(x2, op_t(x1, i(x2))), op_t(x1, i(x2))), i(x2))}, op_t(x1, i(x2)), op_t(x1, i(x2)))})$

$=$ by Axiom 14 RL with $\{x4 \leftarrow i(x2), x3 \leftarrow op_t(x1, i(x2)), x2 \leftarrow op_t(x1, i(x2)), x1 \leftarrow mult(op_t(x2, op_t(x1, i(x2))), op_t(x1, i(x2$

$mult(x1, \overbrace{op_t(\underbrace{op_r(mult(op_t(x2, op_t(x1, i(x2))), op_t(x1, i(x2))), op_t(x1, i(x2)), op_t(x1, i(x2)))}, i(x2)))})$

$=$ by Lemma 359 LR with $\{x2 \leftarrow op_t(x1, i(x2)), x1 \leftarrow op_t(x2, op_t(x1, i(x2)))\}$

$mult(x1, op_t(\overbrace{rd(op_t(x1, i(x2)), i(\underbrace{op_t(x2, op_t(x1, i(x2)))}))}, i(x2)))$

$=$ by Lemma 559 LR with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$mult(x1, op_t(\underbrace{rd(op_t(x1, i(x2)), i(\overbrace{op_t(x2, x1)}))}, i(x2)))$

$=$ by Lemma 65 LR with $\{x3 \leftarrow op_t(x2, x1), x2 \leftarrow x2, x1 \leftarrow x1\}$

$mult(x1, op_t(\overbrace{rd(op_t(x2, x1), op_t(i(x1), x2))}, i(x2)))$

$=$ by Lemma 883 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$mult(x1, op_t(\overbrace{op_t(mult(x1, x2), x1)}, i(x2)))$

Lemma 1432: $rd(mult(x2, x1), op_t(i(x1), x2)) = mult(op_t(mult(x1, x2), i(x2)), x1)$

$rd(\underbrace{mult(x2, x1)}, op_t(i(x1), x2))$

$=$ by Lemma 13 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{rd(\overbrace{mult(x1, op_t(x2, x1))}, op_t(i(x1), x2))}$

$=$ by Lemma 1431 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{\underbrace{mult(x1, op_t(op_t(mult(x1, x2), x1), i(x2)))}}$

$=$ by Lemma 50 LR with $\{x3 \leftarrow i(x2), x2 \leftarrow mult(x1, x2), x1 \leftarrow x1\}$

$\overbrace{mult(op_t(mult(x1, x2), i(x2)), x1)}$

Lemma 1433: $rd(mult(op_r(x1,x2,x2),i(op_t(x2,x1))),x2) = rd(rd(x1,x2),op_t(i(i(x2)),op_r(x1,x2,x2)))$

$rd(mult(op_r(x1,x2,x2),i(\underbrace{op_t(x2,x1)}))),x2)$

$=$ by Lemma 550 RL with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$\underbrace{rd(mult(op_r(x1,x2,x2),i(\overbrace{op_t(x2,op_r(x1,x2,x2))})),x2)}$

$=$ by Lemma 1430 LR with $\{x2 \leftarrow x2, x1 \leftarrow op_r(x1,x2,x2)\}$

$\overbrace{mult(op_r(x1,x2,x2),\underbrace{rd(op_t(i(x2),op_r(x1,x2,x2)),x2)})}$

$=$ by Lemma 111 RL with $\{x2 \leftarrow op_r(x1,x2,x2), x1 \leftarrow x2\}$

$mult(op_r(x1,x2,x2),\overbrace{mult(i(x2),\underbrace{op_t(i(x2),op_r(x1,x2,x2))})})$

$=$ by Axiom 4 LR with $\{x2 \leftarrow i(x2), x1 \leftarrow op_t(i(x2),op_r(x1,x2,x2))\}$

$mult(op_r(x1,x2,x2),\underbrace{mult(i(x2),\overbrace{mult(rd(op_t(i(x2),op_r(x1,x2,x2)),i(x2)),i(x2))})})$

$=$ by Lemma 58 RL with $\{x2 \leftarrow rd(op_t(i(x2),op_r(x1,x2,x2)),i(x2)), x1 \leftarrow i(x2)\}$

$mult(op_r(x1,x2,x2),\overbrace{rd(i(x2),i(\underbrace{mult(i(x2),rd(op_t(i(x2),op_r(x1,x2,x2)),i(x2))}))})$

$=$ by Lemma 95 LR with $\{x2 \leftarrow op_r(x1,x2,x2), x1 \leftarrow i(x2)\}$

$mult(op_r(x1,x2,x2),rd(i(x2),\underbrace{i(\overbrace{op_t(i(x2),op_r(x1,x2,x2))})}))$

$=$ by Lemma 15 RL with $\{x2 \leftarrow op_r(x1,x2,x2), x1 \leftarrow i(x2)\}$

$\underbrace{mult(op_r(x1,x2,x2),rd(i(x2),\overbrace{op_t(i(i(x2)),i(op_r(x1,x2,x2)))}))}$

$=$ by Lemma 1125 RL with $\{x2 \leftarrow i(x2), x1 \leftarrow op_r(x1,x2,x2)\}$

$\overbrace{mult(\underbrace{rd(op_r(x1,x2,x2),op_t(i(i(x2)),op_r(x1,x2,x2)))},i(x2))}$

$=$ by Lemma 326 RL with $\{x2 \leftarrow op_r(x1,x2,x2), x1 \leftarrow i(x2)\}$

$\underbrace{mult(\overbrace{op_t(mult(i(x2),op_r(x1,x2,x2)),i(op_r(x1,x2,x2)))},i(x2))}$

$=$ by Lemma 1432 RL with $\{x1 \leftarrow i(x2), x2 \leftarrow op_r(x1,x2,x2)\}$

$\overbrace{rd(\underbrace{mult(op_r(x1,x2,x2),i(x2))},op_t(i(i(x2)),op_r(x1,x2,x2)))}$

$=$ by Lemma 352 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$rd(\overbrace{rd(x1,x2)},op_t(i(i(x2)),op_r(x1,x2,x2)))$

Lemma 1434: $rd(rd(x1,op_t(x2,x1)),x2) = rd(rd(x1,x2),op_t(x2,op_r(x1,x2,x2)))$

$\underbrace{rd(rd(x1,op_t(x2,x1)),x2)}$

$=$ by Lemma 1184 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{rd(\overbrace{mult(op_r(x1,x2,x2),i(op_t(x2,x1)))},x2)}$

$=$ by Lemma 1433 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{rd(rd(x1,x2),op_t(\underbrace{i(i(x2))},op_r(x1,x2,x2)))}$

$=$ by Lemma 3 LR with $\{x1 \leftarrow x2\}$

$rd(rd(x1,x2),op_t(\overbrace{x2},op_r(x1,x2,x2)))$

Lemma 1435: $op_t(x1, rd(op_t(x2, asoc(x1, x3, x1)), asoc(x1, x3, x1))) = op_t(x1, mult(mult(i(asoc(x1, x3, x1)), x2), i(asoc(x1, x3, x$

$$
\begin{aligned}
& \overbrace{op_t(x1, rd(op_t(x2, asoc(x1, x3, x1)), asoc(x1, x3, x1)))} \\
= \quad & \text{by Lemma 1281 RL with } \{x3 \leftarrow x3, x2 \leftarrow rd(op_t(x2, asoc(x1, x3, x1)), asoc(x1, x3, x1)), x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, rd(\underbrace{rd(op_t(x2, asoc(x1, x3, x1)), asoc(x1, x3, x1))}, asoc(x1, x3, x1)))} \\
= \quad & \text{by Lemma 324 RL with } \{x2 \leftarrow asoc(x1, x3, x1), x1 \leftarrow x2\} \\
& op_t(x1, \underbrace{rd(\overbrace{op_t(mult(x2, i(asoc(x1, x3, x1))), asoc(x1, x3, x1))}, asoc(x1, x3, x1))}) \\
= \quad & \text{by Lemma 1097 RL with } \{x2 \leftarrow x2, x1 \leftarrow asoc(x1, x3, x1)\} \\
& op_t(x1, \overbrace{mult(mult(\underbrace{i(asoc(x1, x3, x1))}, x2), i(asoc(x1, x3, x1)))})
\end{aligned}
$$

Lemma 1436: $op_t(x1, op_t(x2, asoc(x1, x3, x1))) = op_t(x1, mult(mult(asoc(x3, x1, x1), x2), i(asoc(x1, x3, x1))))$

$$
\begin{aligned}
& \overbrace{op_t(x1, op_t(x2, asoc(x1, x3, x1)))} \\
= \quad & \text{by Lemma 1281 RL with } \{x3 \leftarrow x3, x2 \leftarrow op_t(x2, asoc(x1, x3, x1)), x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, rd(\underbrace{op_t(x2, asoc(x1, x3, x1))}, asoc(x1, x3, x1)))} \\
= \quad & \text{by Lemma 1435 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, mult(mult(\underbrace{i(asoc(x1, x3, x1))}, x2), i(asoc(x1, x3, x1))))} \\
= \quad & \text{by Lemma 757 LR with } \{x2 \leftarrow x3, x1 \leftarrow x1\} \\
& op_t(x1, mult(mult(\overbrace{asoc(x3, x1, x1)}, x2), i(asoc(x1, x3, x1))))
\end{aligned}
$$

Lemma 1437: $mult(rd(x1, x2), asoc(x4, x3, x3)) = i(rd(rd(x2, x1), asoc(x4, x3, x3)))$

$$
\begin{aligned}
& mult(rd(x1, x2), \underbrace{asoc(x4, x3, x3)}) \\
= \quad & \text{by Lemma 757 RL with } \{x2 \leftarrow x4, x1 \leftarrow x3\} \\
& \underbrace{mult(rd(x1, x2), \overbrace{i(asoc(x3, x4, x3))})} \\
= \quad & \text{by Lemma 44 RL with } \{x3 \leftarrow asoc(x3, x4, x3), x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \overbrace{i(\underbrace{mult(rd(x2, x1), asoc(x3, x4, x3))})} \\
= \quad & \text{by Lemma 1295 LR with } \{x3 \leftarrow x4, x2 \leftarrow x3, x1 \leftarrow rd(x2, x1)\} \\
& i(\overbrace{rd(rd(x2, x1), asoc(x4, x3, x3))})
\end{aligned}
$$

Lemma 1438: $op_l(x1, mult(mult(x2, x3), x3), x2) = op_l(x1, x2, i(rd(x3, rd(x2, x3))))$

$op_l(x1, mult(mult(x2, x3), x3), \underbrace{x2}_{})$

$=$ by Lemma 3 RL with $\{x1 \leftarrow x2\}$

$\underbrace{op_l(x1, mult(mult(x2, x3), x3), \overbrace{i(i(x2))}^{})}_{}$

$=$ by Lemma 1315 RL with $\{x3 \leftarrow mult(mult(x2, x3), x3), x2 \leftarrow i(x2), x1 \leftarrow x1\}$

$\overbrace{op_l(x1, \underbrace{i(x2)}_{}, op_t(mult(mult(x2, x3), x3), i(x2)))}^{}$

$=$ by Lemma 3 RL with $\{x1 \leftarrow i(x2)\}$

$op_l(x1, \overbrace{i(i(i(x2)))}^{}, \underbrace{op_t(mult(mult(x2, x3), x3), i(x2))}_{})$

$=$ by Axiom 10 RL with $\{x2 \leftarrow mult(mult(x2, x3), x3), x1 \leftarrow i(x2)\}$

$\underbrace{op_l(x1, i(i(i(x2))), \overbrace{mult(i(i(x2)), mult(mult(mult(x2, x3), x3), i(x2)))}^{})}_{}$

$=$ by Lemma 241 LR with $\{x3 \leftarrow mult(mult(mult(x2, x3), x3), i(x2)), x2 \leftarrow i(i(x2)), x1 \leftarrow x1\}$

$\overbrace{op_l(x1, mult(mult(mult(x2, x3), x3), \underbrace{i(x2)}_{}), i(i(x2)))}^{}$

$=$ by Lemma 9 RL with $\{x2 \leftarrow x2, x1 \leftarrow x3\}$

$op_l(x1, \underbrace{mult(mult(mult(x2, x3), x3), \overbrace{rd(x3, mult(x2, x3))}^{})}_{}, i(i(x2)))$

$=$ by Lemma 309 LR with $\{x2 \leftarrow x3, x1 \leftarrow mult(x2, x3)\}$

$op_l(x1, \overbrace{mult(mult(x2, x3), rd(x3, \underbrace{rd(mult(x2, x3), x3)}_{}))}^{}, i(i(x2)))$

$=$ by Axiom 5 RL with $\{x2 \leftarrow x3, x1 \leftarrow x2\}$

$op_l(x1, \underbrace{mult(mult(x2, x3), rd(x3, \overbrace{x2}^{}))}_{}, i(i(x2)))$

$=$ by Lemma 309 LR with $\{x2 \leftarrow x3, x1 \leftarrow x2\}$

$op_l(x1, \overbrace{mult(x2, rd(x3, rd(x2, x3)))}^{}, \underbrace{i(i(x2))}_{})$

$=$ by Lemma 3 LR with $\{x1 \leftarrow x2\}$

$\underbrace{op_l(x1, mult(x2, rd(x3, rd(x2, x3))), \overbrace{x2}^{})}_{}$

$=$ by Lemma 1331 LR with $\{x3 \leftarrow rd(x3, rd(x2, x3)), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_l(x1, x2, i(rd(x3, rd(x2, x3))))}^{}$

Lemma 1439: $op_l(x1, mult(mult(x2, x3), x3), x2) = op_l(x1, x2, rd(rd(x2, x3), x3))$

$\underbrace{op_l(x1, mult(mult(x2, x3), x3), x2)}_{}$

$=$ by Lemma 1438 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_l(x1, x2, \underbrace{i(rd(x3, rd(x2, x3)))}_{})}^{}$

$=$ by Lemma 10 LR with $\{x2 \leftarrow rd(x2, x3), x1 \leftarrow x3\}$

$op_l(x1, x2, \overbrace{rd(rd(x2, x3), x3)}^{})$

Lemma 1440: $mult(mult(op_r(x1, x2, x3), x1), x4) = mult(x1, mult(op_r(x1, x2, x3), x4))$

$$
\begin{aligned}
& \underbrace{mult(mult(op_r(x1, x2, x3), x1), x4)} \\
= \quad & \text{by Lemma 137 RL with } \{x3 \leftarrow x4, x2 \leftarrow x1, x1 \leftarrow op_r(x1, x2, x3)\} \\
& \overbrace{\underbrace{mult(op_r(x1, x2, x3), mult(x1, op_l(x4, op_r(x1, x2, x3), x1)))}} \\
= \quad & \text{by Lemma 1401 RL with } \{x4 \leftarrow op_l(x4, op_r(x1, x2, x3), x1), x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{mult(mult(x1, op_r(x1, x2, x3)), op_l(x4, op_r(x1, x2, x3), x1))}} \\
= \quad & \text{by Lemma 132 LR with } \{x3 \leftarrow x4, x2 \leftarrow op_r(x1, x2, x3), x1 \leftarrow x1\} \\
& \overbrace{mult(x1, \underbrace{mult(op_r(x1, x2, x3), x4)})}
\end{aligned}
$$

Lemma 1441: $mult(op_r(x1, x2, x3), mult(x1, x4)) = mult(x1, mult(op_r(x1, x2, x3), x4))$

$$
\begin{aligned}
& \underbrace{mult(op_r(x1, x2, x3), mult(x1, x4))} \\
= \quad & \text{by Lemma 1401 RL with } \{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{mult(\underbrace{mult(x1, op_r(x1, x2, x3))}, x4)} \\
= \quad & \text{by Lemma 119 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{mult(\overbrace{mult(op_r(x1, x2, x3), x1)}, x4)} \\
= \quad & \text{by Lemma 1440 LR with } \{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{mult(x1, \underbrace{mult(op_r(x1, x2, x3), x4)})}
\end{aligned}
$$

Lemma 1442: $mult(mult(x1, op_r(x1, x2, x3)), x4) = mult(x1, mult(op_r(x1, x2, x3), x4))$

$$
\begin{aligned}
& \underbrace{mult(mult(x1, op_r(x1, x2, x3)), x4)} \\
= \quad & \text{by Lemma 1401 LR with } \{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{mult(op_r(x1, x2, x3), mult(x1, x4))}} \\
= \quad & \text{by Lemma 1441 LR with } \{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{mult(x1, \underbrace{mult(op_r(x1, x2, x3), x4)})}
\end{aligned}
$$

Lemma 1443: $op_l(x1, op_r(x2, x3, x4), x2) = x1$

$$
\begin{aligned}
& op_l(x1, op_r(x2, x3, x4), \underbrace{x2}) \\
= \quad & \text{by Lemma 3 RL with } \{x1 \leftarrow x2\} \\
& op_l(x1, op_r(\underbrace{x2}, x3, x4), \overbrace{i(i(x2))}) \\
= \quad & \text{by Lemma 3 RL with } \{x1 \leftarrow x2\} \\
& \underbrace{op_l(x1, op_r(\overbrace{i(i(x2))}, x3, x4), i(i(x2)))} \\
= \quad & \text{by Lemma 987 RL with } \{x3 \leftarrow x1, x2 \leftarrow op_r(i(i(x2)), x3, x4), x1 \leftarrow i(x2)\} \\
& \overbrace{\underbrace{mult(i(x2), mult(rd(i(op_r(i(i(x2)), x3, x4)), i(x2)), mult(op_r(i(i(x2)), x3, x4), x1)))}} \\
= \quad & \text{by Lemma 1086 RL with } \{x3 \leftarrow mult(op_r(i(i(x2)), x3, x4), x1), x2 \leftarrow i(op_r(i(i(x2)), x3, x4)), x1 \leftarrow i(x2)\} \\
& \overbrace{mult(mult(i(x2), i(op_r(i(i(x2)), x3, x4))), \underbrace{mult(i(i(x2)), mult(op_r(i(i(x2)), x3, x4), x1))})} \\
= \quad & \text{by Lemma 1442 RL with } \{x4 \leftarrow x1, x3 \leftarrow x4, x2 \leftarrow x3, x1 \leftarrow i(i(x2))\} \\
& mult(\underbrace{mult(i(x2), i(op_r(i(i(x2)), x3, x4)))}, \overbrace{mult(mult(i(i(x2)), op_r(i(i(x2)), x3, x4)), x1)}) \\
= \quad & \text{by Lemma 16 RL with } \{x2 \leftarrow op_r(i(i(x2)), x3, x4), x1 \leftarrow i(x2)\} \\
& \underbrace{mult(\overbrace{i(mult(i(i(x2)), op_r(i(i(x2)), x3, x4)))}, mult(mult(i(i(x2)), op_r(i(i(x2)), x3, x4)), x1))} \\
= \quad & \text{by Axiom 2 RL with } \{x2 \leftarrow mult(i(i(x2)), op_r(i(i(x2)), x3, x4)), x1 \leftarrow x1\} \\
& \overbrace{x1}
\end{aligned}
$$

Lemma 1444: $op_l(x1, x2, op_r(x2, x3, x4)) = x1$

$$
\begin{aligned}
& op_l(\underbrace{x1}, x2, op_r(x2, x3, x4)) \\
= \quad & \text{by Lemma 1443 RL with } \{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{op_l(\overbrace{op_l(x1, op_r(x2, x3, x4), x2)}, x2, op_r(x2, x3, x4))} \\
= \quad & \text{by Lemma 138 LR with } \{x3 \leftarrow x2, x2 \leftarrow op_r(x2, x3, x4), x1 \leftarrow x1\} \\
& \overbrace{x1}
\end{aligned}
$$

Lemma 1445: $op_l(x1, op_r(x2, x3, x4), i(x2)) = x1$

$$
\begin{aligned}
& \underbrace{op_l(x1, op_r(x2, x3, x4), i(x2))} \\
= \quad & \text{by Lemma 1094 LR with } \{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, x2, op_r(x2, x3, x4))}} \\
= \quad & \text{by Lemma 1444 LR with } \{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{x1}
\end{aligned}
$$

Lemma 1446: $op_l(x1, rd(x2, x3), rd(mult(i(x2), rd(x2, x3)), x2)) = op_l(x1, mult(rd(x2, x3), op_l(i(x3), x3, x2)), i(x2))$

$$
\begin{aligned}
& op_l(x1, rd(x2, x3), \underbrace{rd(mult(i(x2), rd(x2, x3)), x2)}) \\
= & \quad \text{by Lemma 73 RL with } \{x2 \leftarrow rd(x2, x3), x1 \leftarrow x2\} \\
& \underbrace{op_l(x1, rd(x2, x3), \overbrace{mult(i(x2), mult(rd(x2, x3), i(x2)))})} \\
= & \quad \text{by Lemma 1407 RL with } \{x3 \leftarrow i(x2), x2 \leftarrow rd(x2, x3), x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, mult(rd(x2, x3), \underbrace{mult(i(x2), rd(x2, x3))}), i(x2))} \\
= & \quad \text{by Lemma 310 RL with } \{x2 \leftarrow x2, x1 \leftarrow x3\} \\
& op_l(x1, mult(rd(x2, x3), \overbrace{op_l(i(x3), x3, x2)}), i(x2))
\end{aligned}
$$

Lemma 1447: $op_l(x1, rd(x3, x2), mult(x2, x3)) = op_l(x1, mult(rd(x2, x3), op_l(i(x3), x3, x2)), i(x2))$

$$
\begin{aligned}
& \underbrace{op_l(x1, rd(x3, x2), mult(x2, x3))} \\
= & \quad \text{by Lemma 1060 RL with } \{x4 \leftarrow mult(x2, x3), x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, rd(x2, x3), \underbrace{i(mult(x2, x3))})} \\
= & \quad \text{by Lemma 85 RL with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& \underbrace{op_l(x1, rd(x2, x3), \overbrace{rd(mult(i(x2), rd(x2, x3)), x2)})} \\
= & \quad \text{by Lemma 1446 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, mult(rd(x2, x3), op_l(i(x3), x3, x2)), i(x2))}
\end{aligned}
$$

Lemma 1448: $op_l(x1, rd(x3, x2), mult(x2, x3)) = op_l(x1, rd(rd(x2, x3), x3), i(x2))$

$$
\begin{aligned}
& \underbrace{op_l(x1, rd(x3, x2), mult(x2, x3))} \\
= & \quad \text{by Lemma 1447 LR with } \{x2 \leftarrow x2, x3 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, mult(rd(x2, x3), \underbrace{op_l(i(x3), x3, x2)}), i(x2))} \\
= & \quad \text{by Lemma 286 RL with } \{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow i(x3)\} \\
& op_l(x1, \underbrace{mult(rd(x2, x3), \overbrace{op_l(i(x3), x3, rd(x2, x3))})}, i(x2)) \\
= & \quad \text{by Lemma 232 LR with } \{x2 \leftarrow x3, x1 \leftarrow rd(x2, x3)\} \\
& op_l(x1, \overbrace{rd(rd(x2, x3), x3)}, i(x2))
\end{aligned}
$$

Lemma 1449: $op_t(x1, rd(mult(x2, op_l(i(i(x3)), x3, i(x2))), x1)) = op_t(x1, rd(rd(i(x2), x3), x1))$

$$
\begin{aligned}
& op_t(x1, rd(mult(x2, \underbrace{op_l(i(i(x3)), x3, i(x2))}), x1)) \\
= & \quad \text{by Lemma 1049 RL with } \{x3 \leftarrow i(x2), x2 \leftarrow x3, x1 \leftarrow i(x3)\} \\
& \underbrace{op_t(x1, rd(mult(x2, \overbrace{i(op_l(i(x3), x3, i(x2)))}), x1))} \\
= & \quad \text{by Lemma 1410 RL with } \{x3 \leftarrow op_l(i(x3), x3, i(x2)), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, rd(\underbrace{mult(i(x2), op_l(i(x3), x3, i(x2)))}, x1))} \\
= & \quad \text{by Lemma 232 LR with } \{x2 \leftarrow x3, x1 \leftarrow i(x2)\} \\
& op_t(x1, rd(\overbrace{rd(i(x2), x3)}, x1))
\end{aligned}
$$

Lemma 1450: $op_t(x1, rd(mult(x2, op_l(x3, x2, x3)), x1)) = op_t(x1, rd(rd(i(x2), x3), x1))$

$op_t(x1, rd(mult(x2, \underbrace{op_l(x3, x2, x3)}), x1))$

$=$ by Lemma 608 RL with $\{x2 \leftarrow x2, x1 \leftarrow x3\}$

$op_t(x1, rd(mult(x2, \overbrace{op_l(\underbrace{x3}, x3, i(x2))}), x1))$

$=$ by Lemma 3 RL with $\{x1 \leftarrow x3\}$

$\underbrace{op_t(x1, rd(mult(x2, op_l(\overbrace{i(i(x3))}, x3, i(x2))), x1))}$

$=$ by Lemma 1449 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_t(x1, rd(rd(i(x2), x3), x1))}$

Lemma 1451: $op_l(x1, x1, rd(mult(x1, x2), x3)) = op_l(x1, x1, rd(x2, mult(x3, x1)))$

$\underbrace{op_l(x1, x1, rd(mult(x1, x2), x3))}$

$=$ by Lemma 773 LR with $\{x2 \leftarrow rd(mult(x1, x2), x3), x1 \leftarrow x1\}$

$\overbrace{rd(x1, \underbrace{asoc(rd(mult(x1, x2), x3), x1, x1)})}$

$=$ by Lemma 1395 LR with $\{x3 \leftarrow x1, x2 \leftarrow x3, x1 \leftarrow mult(x1, x2)\}$

$\underbrace{rd(x1, \overbrace{asoc(x1, rd(op_t(x3, x1), mult(x1, x2)), x1)})}$

$=$ by Lemma 620 LR with $\{x2 \leftarrow rd(op_t(x3, x1), mult(x1, x2)), x1 \leftarrow x1\}$

$\underbrace{\overbrace{op_l(x1, rd(op_t(x3, x1), mult(x1, x2)), x1)}}$

$=$ by Lemma 1429 LR with $\{x4 \leftarrow x2, x3 \leftarrow x1, x2 \leftarrow op_t(x3, x1), x1 \leftarrow x1\}$

$\overbrace{op_l(x1, x1, rd(x2, \underbrace{mult(x1, op_t(x3, x1))}))}$

$=$ by Lemma 13 LR with $\{x2 \leftarrow x3, x1 \leftarrow x1\}$

$op_l(x1, x1, rd(x2, \overbrace{mult(x3, x1)}))$

Lemma 1452: $rd(rd(op_l(x1, x2, x1), x2), op_t(x2, op_l(x1, x2, x1))) = rd(rd(op_l(x1, x2, x1), op_t(x2, x1)), x2)$

$rd(rd(op_l(x1, x2, x1), x2), \underbrace{op_t(x2, op_l(x1, x2, x1))})$

$=$ by Lemma 550 RL with $\{x2 \leftarrow op_l(x1, x2, x1), x1 \leftarrow x2\}$

$\underbrace{rd(rd(op_l(x1, x2, x1), x2), \overbrace{op_t(x2, op_r(op_l(x1, x2, x1), x2, x2))})}$

$=$ by Lemma 1434 RL with $\{x2 \leftarrow x2, x1 \leftarrow op_l(x1, x2, x1)\}$

$\overbrace{rd(rd(op_l(x1, x2, x1), \underbrace{op_t(x2, op_l(x1, x2, x1))}), x2)}$

$=$ by Lemma 560 LR with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$rd(rd(op_l(x1, x2, x1), \overbrace{op_t(x2, x1)}), x2)$

Lemma 1453: $rd(mult(i(x2),x1),op_t(x2,x1)) = rd(rd(op_l(x1,x2,x1),op_t(x2,x1)),x2)$

$$
\begin{aligned}
& rd(mult(i(x2),x1),\underbrace{op_t(x2,x1)}) \\
= & \quad \text{by Lemma 560 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& rd(\underbrace{mult(i(x2),x1)},\overbrace{op_t(x2,op_l(x1,x2,x1))}) \\
= & \quad \text{by Lemma 615 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{rd(\overbrace{rd(op_l(x1,x2,x1),x2)},op_t(x2,op_l(x1,x2,x1)))} \\
= & \quad \text{by Lemma 1452 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{rd(rd(\underbrace{op_l(x1,x2,x1)},op_t(x2,x1)),x2)}
\end{aligned}
$$

Lemma 1454: $rd(mult(i(x2),x1),op_t(x2,x1)) = rd(mult(i(op_t(x2,x1)),x1),x2)$

$$
\begin{aligned}
& \underbrace{rd(mult(i(x2),x1),op_t(x2,x1))} \\
= & \quad \text{by Lemma 1453 LR with } \{x1 \leftarrow x1, x2 \leftarrow x2\} \\
& \overbrace{rd(rd(\underbrace{op_l(x1,x2,x1)},op_t(x2,x1)),x2)} \\
= & \quad \text{by Lemma 623 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& rd(\underbrace{rd(\overbrace{op_l(x1,op_t(x2,x1),x1)},op_t(x2,x1))},x2) \\
= & \quad \text{by Lemma 615 LR with } \{x2 \leftarrow op_t(x2,x1), x1 \leftarrow x1\} \\
& rd(\overbrace{mult(i(op_t(x2,x1)),x1)},x2)
\end{aligned}
$$

Lemma 1455: $op_t(x1,mult(asoc(x1,x2,x1),x3)) = op_t(x1,op_t(x3,asoc(x2,x1,x1)))$

$$
\begin{aligned}
& op_t(x1,mult(\underbrace{asoc(x1,x2,x1)},x3)) \\
= & \quad \text{by Lemma 758 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& op_t(x1,\underbrace{mult(\overbrace{asoc(i(x2),x1,x1)},x3)}) \\
= & \quad \text{by Axiom 5 LR with } \{x2 \leftarrow op_r(i(x2),x1,x1), x1 \leftarrow mult(asoc(i(x2),x1,x1),x3)\} \\
& \underbrace{op_t(x1,\overbrace{rd(mult(mult(asoc(i(x2),x1,x1),x3),op_r(i(x2),x1,x1)),op_r(i(x2),x1,x1))})} \\
= & \quad \text{by Lemma 1280 LR with } \{x3 \leftarrow i(x2), x2 \leftarrow mult(mult(asoc(i(x2),x1,x1),x3),op_r(i(x2),x1,x1)), x1 \leftarrow x1\} \\
& \overbrace{op_t(x1,\underbrace{rd(mult(mult(asoc(i(x2),x1,x1),x3),op_r(i(x2),x1,x1)),i(x2))})} \\
= & \quad \text{by Lemma 1246 LR with } \{x3 \leftarrow x1, x2 \leftarrow i(x2), x1 \leftarrow mult(asoc(i(x2),x1,x1),x3)\} \\
& op_t(x1,\overbrace{mult(mult(asoc(i(x2),x1,x1),x3),\underbrace{asoc(i(x2),x1,x1)})}) \\
= & \quad \text{by Lemma 757 RL with } \{x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& \underbrace{op_t(x1,mult(mult(asoc(i(x2),x1,x1),x3),\overbrace{i(asoc(x1,i(x2),x1))}))} \\
= & \quad \text{by Lemma 1436 RL with } \{x3 \leftarrow i(x2), x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{op_t(x1,op_t(x3,\underbrace{asoc(x1,i(x2),x1)}))} \\
= & \quad \text{by Lemma 756 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_t(x1,op_t(x3,\overbrace{asoc(x2,x1,x1)}))
\end{aligned}
$$

Lemma 1456: $rd(x1, mult(i(op_t(x1, x2)), x2)) = rd(op_t(x1, x2), mult(i(x1), x2))$

$$
\begin{aligned}
& \overbrace{rd(x1, mult(i(op_t(x1, x2)), x2))} \\
= & \quad \text{by Lemma 10 RL with } \{x2 \leftarrow x1, x1 \leftarrow mult(i(op_t(x1, x2)), x2)\} \\
& \overbrace{i(\underbrace{rd(mult(i(op_t(x1, x2)), x2), x1)})} \\
= & \quad \text{by Lemma 1454 RL with } \{x1 \leftarrow x2, x2 \leftarrow x1\} \\
& \underbrace{i(\overbrace{rd(mult(i(x1), x2), op_t(x1, x2))})} \\
= & \quad \text{by Lemma 10 LR with } \{x2 \leftarrow op_t(x1, x2), x1 \leftarrow mult(i(x1), x2)\} \\
& \overbrace{rd(op_t(x1, x2), mult(i(x1), x2))}
\end{aligned}
$$

Lemma 1457: $i(asoc(op_t(x1, x2), i(x2), x1)) = mult(rd(x1, mult(i(op_t(x1, x2)), x2)), i(mult(mult(op_t(x1, x2), i(x2)), x1)))$

$$
\begin{aligned}
& \underbrace{i(asoc(op_t(x1, x2), i(x2), x1))} \\
= & \quad \text{by Lemma 964 RL with } \{x3 \leftarrow x1, x2 \leftarrow i(x2), x1 \leftarrow op_t(x1, x2)\} \\
& \overbrace{mult(\underbrace{mult(op_t(x1, x2), mult(i(x2), x1))}, i(mult(mult(op_t(x1, x2), i(x2)), x1)))} \\
= & \quad \text{by Lemma 1423 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& mult(\overbrace{rd(op_t(x1, x2), mult(i(x1), x2))}, i(mult(mult(op_t(x1, x2), i(x2)), x1))) \\
= & \quad \text{by Lemma 1456 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& mult(\overbrace{rd(x1, mult(i(op_t(x1, x2)), x2))}, i(mult(mult(op_t(x1, x2), i(x2)), x1)))
\end{aligned}
$$

Lemma 1458: $asoc(op_t(x1, x2), i(x2), x1) = asoc(x2, x1, x1)$

$$
\begin{aligned}
& \underbrace{asoc(op_t(x1, x2), i(x2), x1)} \\
= & \quad \text{by Lemma 3 RL with } \{x1 \leftarrow asoc(op_t(x1, x2), i(x2), x1)\} \\
& \overbrace{i(\underbrace{i(asoc(op_t(x1, x2), i(x2), x1))})} \\
= & \quad \text{by Lemma 1457 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& i(\overbrace{mult(rd(x1, mult(i(op_t(x1, x2)), x2)), \underbrace{i(mult(mult(op_t(x1, x2), i(x2)), x1))})}) \\
= & \quad \text{by Lemma 88 LR with } \{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow op_t(x1, x2)\} \\
& i(mult(\underbrace{rd(x1, mult(i(op_t(x1, x2)), x2))}, \overbrace{mult(mult(i(op_t(x1, x2)), x2), i(x1))})) \\
= & \quad \text{by Lemma 10 RL with } \{x2 \leftarrow x1, x1 \leftarrow mult(i(op_t(x1, x2)), x2)\} \\
& i(mult(\overbrace{i(\underbrace{rd(mult(i(op_t(x1, x2)), x2), x1)})}, \underbrace{mult(mult(i(op_t(x1, x2)), x2), i(x1))})) \\
= & \quad \text{by Lemma 699 LR with } \{x2 \leftarrow x1, x1 \leftarrow mult(i(op_t(x1, x2)), x2)\} \\
& i(\underbrace{mult(i(rd(mult(i(op_t(x1, x2)), x2), x1))}, \overbrace{mult(rd(mult(i(op_t(x1, x2)), x2), x1), asoc(x1, mult(i(op_t(x1, x2)), x2), x1))})) \\
= & \quad \text{by Axiom 2 RL with } \{x2 \leftarrow rd(mult(i(op_t(x1, x2)), x2), x1), x1 \leftarrow asoc(x1, mult(i(op_t(x1, x2)), x2), x1)\} \\
& i(\overbrace{asoc(x1, mult(i(\underbrace{op_t(x1, x2)}), x2), x1)}) \\
= & \quad \text{by Lemma 1084 LR with } \{x3 \leftarrow x2, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{i(\overbrace{asoc(x1, x2, x1)})} \\
= & \quad \text{by Lemma 757 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{asoc(x2, x1, x1)}
\end{aligned}
$$

Lemma 1459: $asoc(x1, x2, x2) = asoc(op_t(x2, x1), i(op_t(x1, i(x2))), x2)$

$$
\begin{aligned}
& \overbrace{asoc(x1, x2, x2)} \\
= & \quad \text{by Lemma 757 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \overbrace{i(\overbrace{asoc(x2, x1, x2)})} \\
= & \quad \text{by Lemma 781 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\overbrace{asoc(op_t(x1, i(x2)), x2, x2)}} \\
= & \quad \text{by Lemma 1458 RL with } \{x2 \leftarrow op_t(x1, i(x2)), x1 \leftarrow x2\} \\
& \overbrace{asoc(\overbrace{op_t(x2, op_t(x1, i(x2)))}, i(op_t(x1, i(x2))), x2)} \\
= & \quad \text{by Lemma 559 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& asoc(\overbrace{op_t(x2, x1)}, i(op_t(x1, i(x2))), x2)
\end{aligned}
$$

Lemma 1460: $mult(mult(op_t(x1, x2), mult(x2, i(x1))), x2) = op_l(mult(x2, x2), mult(x2, i(x1)), op_t(x1, x2))$

$$
\begin{aligned}
& \overbrace{mult(mult(op_t(x1, x2), mult(x2, i(x1))), x2)} \\
= & \quad \text{by Lemma 1047 RL with } \{x3 \leftarrow x2, x2 \leftarrow op_t(x1, x2), x1 \leftarrow mult(x2, i(x1))\} \\
& \overbrace{op_l(mult(mult(x2, i(x1)), \underbrace{mult(op_t(x1, x2), x2)}), mult(x2, i(x1)), op_t(x1, x2))} \\
= & \quad \text{by Lemma 341 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& op_l(\underbrace{mult(mult(x2, i(x1)), \overbrace{op_t(rd(x2, i(x1)), x2)})}, mult(x2, i(x1)), op_t(x1, x2)) \\
= & \quad \text{by Lemma 204 LR with } \{x2 \leftarrow i(x1), x1 \leftarrow x2\} \\
& op_l(\overbrace{mult(x2, x2)}, mult(x2, i(x1)), op_t(x1, x2))
\end{aligned}
$$

Lemma 1461: $mult(x2, op_l(x2, x2, mult(x1, x1))) = op_l(mult(x2, x2), x2, x1)$

$\overbrace{mult(x2, op_l(x2, x2, mult(x1, x1)))}$

$=$ by Lemma 168 RL with $\{x3 \leftarrow mult(x1, x1), x2 \leftarrow x2, x1 \leftarrow x2\}$

$\overbrace{mult(\underbrace{op_l(x2, x2, mult(x1, x1))}, x2)}$

$=$ by Lemma 1180 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{mult(\overbrace{mult(op_t(x1, x2), mult(x2, i(x1)))}, x2)}$

$=$ by Lemma 1460 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_l(mult(x2, x2), mult(x2, i(x1)), op_t(x1, x2))}}$

$=$ by Lemma 1135 RL with $\{x4 \leftarrow op_t(x1, x2), x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow mult(x2, x2)\}$

$\overbrace{\underbrace{op_l(mult(x2, x2), mult(i(x2), x1), i(op_t(x1, x2)))}}$

$=$ by Lemma 1050 RL with $\{x3 \leftarrow op_t(x1, x2), x2 \leftarrow mult(i(x2), x1), x1 \leftarrow mult(x2, x2)\}$

$\overbrace{\underbrace{op_l(mult(x2, x2), i(mult(i(x2), x1)), op_t(x1, x2))}}$

$=$ by Lemma 1329 RL with $\{x3 \leftarrow i(mult(i(x2), x1)), x2 \leftarrow op_t(x1, x2), x1 \leftarrow mult(x2, x2)\}$

$\overbrace{op_l(mult(x2, x2), op_t(x1, x2), \underbrace{i(op_t(i(mult(i(x2), x1)), op_t(x1, x2))))}}$

$=$ by Lemma 18 LR with $\{x2 \leftarrow op_t(x1, x2), x1 \leftarrow mult(i(x2), x1)\}$

$op_l(mult(x2, x2), op_t(x1, x2), \overbrace{\underbrace{op_t(mult(i(x2), x1)}, i(op_t(x1, x2))))}$

$=$ by Lemma 14 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$op_l(mult(x2, x2), op_t(x1, x2), \underbrace{op_t(\overbrace{op_t(rd(x1, x2), x2)}, i(op_t(x1, x2))))}$

$=$ by Axiom 13 LR with $\{x3 \leftarrow i(op_t(x1, x2)), x2 \leftarrow x2, x1 \leftarrow rd(x1, x2)\}$

$op_l(mult(x2, x2), op_t(x1, x2), \overbrace{\underbrace{op_t(rd(x1, x2), i(op_t(x1, x2)))}, x2)}$

$=$ by Lemma 591 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$op_l(mult(x2, x2), op_t(x1, x2), \underbrace{op_t(\overbrace{mult(x1, i(x2))}, x2)})$

$=$ by Lemma 324 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{op_l(mult(x2, x2), op_t(x1, x2), \overbrace{rd(op_t(x1, x2), x2)})}$

$=$ by Lemma 1321 LR with $\{x3 \leftarrow x2, x2 \leftarrow op_t(x1, x2), x1 \leftarrow mult(x2, x2)\}$

$\overbrace{\underbrace{op_l(mult(x2, x2), op_t(x1, x2), i(x2))}}$

$=$ by Lemma 1332 LR with $\{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow mult(x2, x2)\}$

$\overbrace{op_l(mult(x2, x2), x2, x1)}$

Lemma 1462: $op_l(mult(x1, x1), x2, i(x1)) = mult(x1, op_l(x1, x1, mult(x2, x2)))$

$\underbrace{op_l(mult(x1, x1), x2, i(x1))}$

$=$ by Lemma 1315 RL with $\{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow mult(x1, x1)\}$

$\overbrace{\underbrace{op_l(mult(x1, x1), x1, op_t(x2, x1))}}$

$=$ by Lemma 1461 RL with $\{x1 \leftarrow op_t(x2, x1), x2 \leftarrow x1\}$

$\overbrace{mult(x1, \underbrace{op_l(x1, x1, mult(op_t(x2, x1), op_t(x2, x1)))})}$

$=$ by Lemma 1063 RL with $\{x3 \leftarrow mult(op_t(x2, x1), op_t(x2, x1)), x2 \leftarrow x2, x1 \leftarrow x1\}$

$mult(x1, \overbrace{op_l(x1, x1, \underbrace{mult(op_t(x1, x2), mult(op_t(x2, x1), op_t(x2, x1)))})})$

$=$ by Lemma 1263 LR with $\{x3 \leftarrow op_t(x2, x1), x2 \leftarrow x2, x1 \leftarrow x1\}$

$mult(x1, \underbrace{op_l(x1, x1, \overbrace{mult(x1, mult(x2, op_t(x2, x1)))})})$

$=$ by Lemma 624 LR with $\{x2 \leftarrow mult(x2, op_t(x2, x1)), x1 \leftarrow x1\}$

$mult(x1, \underbrace{op_l(x1, x1, mult(x2, op_t(x2, x1)))})$

$=$ by Lemma 1063 RL with $\{x3 \leftarrow mult(x2, op_t(x2, x1)), x2 \leftarrow x2, x1 \leftarrow x1\}$

$mult(x1, \overbrace{op_l(x1, x1, \underbrace{mult(op_t(x1, x2), mult(x2, op_t(x2, x1)))})})$

$=$ by Lemma 1225 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$mult(x1, op_l(x1, x1, \overbrace{\underbrace{mult(op_t(mult(x2, x1), x2), x2)}}))$

$=$ by Lemma 350 LR with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$mult(x1, \underbrace{op_l(x1, x1, \overbrace{mult(x1, mult(x2, x2))})})$

$=$ by Lemma 624 LR with $\{x2 \leftarrow mult(x2, x2), x1 \leftarrow x1\}$

$mult(x1, \overbrace{op_l(x1, x1, mult(x2, x2))})$

Lemma 1463: $op_l(mult(x1, x1), x1, i(x2)) = op_l(mult(x1, x1), x2, x1)$

$\underbrace{op_l(mult(x1, x1), x1, i(x2))}$

$=$ by Lemma 1461 RL with $\{x1 \leftarrow i(x2), x2 \leftarrow x1\}$

$\overbrace{\underbrace{mult(x1, op_l(x1, x1, mult(i(x2), i(x2))))}}$

$=$ by Lemma 1462 RL with $\{x2 \leftarrow i(x2), x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_l(mult(x1, x1), i(x2), i(x1))}}$

$=$ by Lemma 1050 LR with $\{x3 \leftarrow i(x1), x2 \leftarrow x2, x1 \leftarrow mult(x1, x1)\}$

$\overbrace{op_l(mult(x1, x1), x2, \underbrace{i(i(x1))})}$

$=$ by Lemma 3 LR with $\{x1 \leftarrow x1\}$

$op_l(mult(x1, x1), x2, \overbrace{x1})$

Lemma 1464: $rd(x1, x2) = rd(op_r(x1, x2, i(x2)), op_l(x2, x2, x1))$

$$
\begin{aligned}
& \underbrace{rd(x1, x2)} \\
= \quad & \text{by Lemma 283 RL with } \{x2 \leftarrow rd(x1, x2), x1 \leftarrow x2\} \\
& \overbrace{rd(\underbrace{op_t(mult(x2, rd(x1, x2)), x2)}, op_l(x2, x2, rd(x1, x2)))} \\
= \quad & \text{by Lemma 349 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& rd(\overbrace{op_r(x1, x2, i(x2))}, \underbrace{op_l(x2, x2, rd(x1, x2))}) \\
= \quad & \text{by Lemma 286 LR with } \{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x2\} \\
& rd(op_r(x1, x2, i(x2)), \overbrace{op_l(x2, x2, x1)})
\end{aligned}
$$

Lemma 1465: $op_t(x1, x2) = rd(op_t(rd(x1, x2), i(op_t(x1, x2))), rd(x1, mult(x1, x2)))$

$$
\begin{aligned}
& \underbrace{op_t(x1, x2)} \\
= \quad & \text{by Lemma 3 RL with } \{x1 \leftarrow op_t(x1, x2)\} \\
& \overbrace{\underbrace{i(i(op_t(x1, x2)))}} \\
= \quad & \text{by Lemma 36 RL with } \{x2 \leftarrow i(op_t(x1, x2)), x1 \leftarrow rd(x1, x2)\} \\
& \overbrace{rd(op_t(rd(x1, x2), i(op_t(x1, x2))), \underbrace{mult(rd(x1, x2), i(op_t(x1, x2)))})} \\
= \quad & \text{by Lemma 833 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& rd(op_t(rd(x1, x2), i(op_t(x1, x2))), \overbrace{rd(x1, mult(x1, x2))})
\end{aligned}
$$

Lemma 1466: $rd(x1, mult(i(op_t(x2, x1)), x1)) = mult(op_t(mult(x1, x2), x1), i(op_t(x1, op_t(i(op_t(x2, x1)), x1))))$

$$
\begin{aligned}
& rd(x1, \underbrace{mult(i(op_t(x2, x1)), x1)}) \\
= \quad & \text{by Lemma 13 RL with } \{x2 \leftarrow i(op_t(x2, x1)), x1 \leftarrow x1\} \\
& \underbrace{rd(x1, \overbrace{mult(x1, op_t(i(op_t(x2, x1)), x1))})} \\
= \quad & \text{by Lemma 833 RL with } \{x2 \leftarrow op_t(i(op_t(x2, x1)), x1), x1 \leftarrow x1\} \\
& \overbrace{mult(\underbrace{rd(x1, op_t(i(op_t(x2, x1)), x1))}, i(op_t(x1, op_t(i(op_t(x2, x1)), x1))))} \\
= \quad & \text{by Lemma 750 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& mult(\overbrace{op_t(mult(x1, x2), x1)}, i(op_t(x1, op_t(i(op_t(x2, x1)), x1))))
\end{aligned}
$$

Lemma 1467: $op_t(x2, x1) = mult(op_t(mult(x1, x2), x1), i(op_t(x1, op_t(i(op_t(x2, x1)), x1))))$

$$
\begin{aligned}
& \underbrace{op_t(x2, x1)} \\
= \quad & \text{by Lemma 3 RL with } \{x1 \leftarrow op_t(x2, x1)\} \\
& \overbrace{\underbrace{i(i(op_t(x2, x1)))}} \\
= \quad & \text{by Lemma 9 RL with } \{x2 \leftarrow i(op_t(x2, x1)), x1 \leftarrow x1\} \\
& \overbrace{\underbrace{rd(x1, mult(i(op_t(x2, x1)), x1))}} \\
= \quad & \text{by Lemma 1466 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{mult(op_t(mult(x1, x2), x1), i(op_t(x1, op_t(i(op_t(x2, x1)), x1))))}
\end{aligned}
$$

Lemma 1468: $op_t(x2, x1) = mult(op_t(mult(x1, x2), x1), op_t(i(x1), op_t(x2, x1)))$

$$
\begin{aligned}
& \underbrace{op_t(x2, x1)} \\
= \quad & \text{by Lemma 1467 LR with } \{x1 \leftarrow x1, x2 \leftarrow x2\} \\
& \overbrace{mult(op_t(mult(x1, x2), x1), \underbrace{i(op_t(x1, op_t(i(op_t(x2, x1)), x1)))})} \\
= \quad & \text{by Lemma 90 LR with } \{x3 \leftarrow x1, x2 \leftarrow op_t(x2, x1), x1 \leftarrow x1\} \\
& mult(op_t(mult(x1, x2), x1), \overbrace{\underbrace{op_t(i(x1), op_t(op_t(x2, x1), i(x1)))}}) \\
= \quad & \text{by Lemma 374 LR with } \{x2 \leftarrow op_t(x2, x1), x1 \leftarrow i(x1)\} \\
& mult(op_t(mult(x1, x2), x1), \overbrace{op_t(i(x1), op_t(x2, x1))})
\end{aligned}
$$

Lemma 1469: $op_r(rd(op_t(x2, x1), i(x1)), x2, i(x2)) = rd(x2, op_t(op_r(i(x1), x2, x2), x2))$

$$
\begin{aligned}
& op_r(\underbrace{rd(op_t(x2, x1), i(x1))}, x2, i(x2)) \\
= \quad & \text{by Lemma 10 RL with } \{x2 \leftarrow op_t(x2, x1), x1 \leftarrow i(x1)\} \\
& \underbrace{op_r(\overbrace{i(rd(i(x1), op_t(x2, x1)))}, x2, i(x2))} \\
= \quad & \text{by Lemma 404 RL with } \{x2 \leftarrow rd(i(x1), op_t(x2, x1)), x1 \leftarrow x2\} \\
& \overbrace{rd(x2, op_t(\underbrace{mult(x2, rd(i(x1), op_t(x2, x1)))}, x2))} \\
= \quad & \text{by Lemma 913 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& rd(x2, op_t(\overbrace{op_r(i(x1), x2, x2)}, x2))
\end{aligned}
$$

Lemma 1470: $rd(x1, rd(x2, i(op_t(x1, x2)))) = rd(op_l(x1, x1, rd(x2, i(op_t(x1, x2)))), mult(x1, x2))$

$\overbrace{rd(x1, rd(x2, i(op_t(x1, x2))))}$

$=$ by Lemma 10 RL with $\{x2 \leftarrow x1, x1 \leftarrow rd(x2, i(op_t(x1, x2)))\}$

$\overbrace{i(rd(rd(x2, i(op_t(x1, x2))), x1))}$

$=$ by Lemma 20 RL with $\{x2 \leftarrow rd(rd(x2, i(op_t(x1, x2))), x1), x1 \leftarrow x1\}$

$\overbrace{mult(x1, i(mult(x1, rd(rd(x2, i(op_t(x1, x2))), x1))))}$

$=$ by Lemma 16 RL with $\{x2 \leftarrow mult(x1, rd(rd(x2, i(op_t(x1, x2))), x1)), x1 \leftarrow x1\}$

$\overbrace{i(mult(i(x1), mult(x1, rd(rd(x2, i(op_t(x1, x2))), x1))))}$

$=$ by Axiom 5 LR with $\{x2 \leftarrow mult(i(x1), mult(mult(x1, rd(rd(x2, i(op_t(x1, x2))), x1)), x1)), x1 \leftarrow i(mult(i(x1), mult(x1, rd(r$

$\overbrace{rd(mult(i(mult(i(x1), mult(x1, rd(rd(x2, i(op_t(x1, x2))), x1)))), mult(i(x1), mult(mult(x1, rd(rd(x2, i(op_t(x1, x2))), x1)),}$

$=$ by Axiom 12 LR with $\{x3 \leftarrow x1, x2 \leftarrow mult(x1, rd(rd(x2, i(op_t(x1, x2))), x1)), x1 \leftarrow i(x1)\}$

$rd(\overbrace{op_l(x1, mult(x1, rd(rd(x2, i(op_t(x1, x2))), x1)), i(x1))}, mult(i(x1), mult(mult(x1, rd(rd(x2, i(op_t(x1, x2))), x1)), x1)))$

$=$ by Lemma 606 LR with $\{x2 \leftarrow mult(x1, rd(rd(x2, i(op_t(x1, x2))), x1)), x1 \leftarrow x1\}$

$rd(\overbrace{op_l(x1, x1, mult(x1, rd(rd(x2, i(op_t(x1, x2))), x1)))}, \underbrace{mult(i(x1), mult(mult(x1, rd(rd(x2, i(op_t(x1, x2))), x1)), x1))}$

$=$ by Axiom 10 LR with $\{x2 \leftarrow mult(x1, rd(rd(x2, i(op_t(x1, x2))), x1)), x1 \leftarrow x1\}$

$rd(op_l(x1, x1, mult(x1, rd(rd(x2, i(op_t(x1, x2))), x1))), \overbrace{op_t(mult(x1, rd(rd(x2, i(op_t(x1, x2))), x1)), x1)})$

$=$ by Lemma 349 LR with $\{x2 \leftarrow rd(x2, i(op_t(x1, x2))), x1 \leftarrow x1\}$

$rd(\underbrace{op_l(x1, x1, mult(x1, rd(rd(x2, i(op_t(x1, x2))), x1)))}, \overbrace{op_r(rd(x2, i(op_t(x1, x2))), x1, i(x1))})$

$=$ by Lemma 624 LR with $\{x2 \leftarrow rd(rd(x2, i(op_t(x1, x2))), x1), x1 \leftarrow x1\}$

$rd(\overbrace{op_l(x1, x1, rd(rd(x2, i(op_t(x1, x2))), x1))}, op_r(rd(x2, i(op_t(x1, x2))), x1, i(x1)))$

$=$ by Lemma 286 LR with $\{x3 \leftarrow rd(x2, i(op_t(x1, x2))), x2 \leftarrow x1, x1 \leftarrow x1\}$

$rd(\overbrace{op_l(x1, x1, rd(x2, i(op_t(x1, x2))))}, op_r(\underbrace{rd(x2, i(op_t(x1, x2)))}, x1, i(x1)))$

$=$ by Lemma 38 RL with $\{x1 \leftarrow x2, x2 \leftarrow op_t(x1, x2)\}$

$rd(op_l(x1, x1, rd(x2, i(op_t(x1, x2)))), \underbrace{op_r(\overbrace{rd(op_t(x1, x2), i(x2))}, x1, i(x1))})$

$=$ by Lemma 1469 LR with $\{x1 \leftarrow x2, x2 \leftarrow x1\}$

$rd(op_l(x1, x1, rd(x2, i(op_t(x1, x2)))), \overbrace{rd(x1, op_t(\underbrace{op_r(i(x2), x1, x1)}, x1))})$

$=$ by Lemma 356 RL with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$rd(op_l(x1, x1, rd(x2, i(op_t(x1, x2)))), rd(x1, \underbrace{op_t(\overbrace{i(op_r(x2, x1, x1))}, x1)}))$

$=$ by Lemma 411 LR with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$rd(op_l(x1, x1, rd(x2, i(op_t(x1, x2)))), rd(x1, \underbrace{mult(i(x1), \overbrace{i(rd(x2, x1))})}))$

$=$ by Axiom 11 LR with $\{x2 \leftarrow rd(x2, x1), x1 \leftarrow x1\}$

$rd(op_l(x1, x1, rd(x2, i(op_t(x1, x2)))), rd(x1, \underbrace{\overbrace{i(mult(x1, rd(x2, x1)))}}))$

$=$ by Lemma 45 LR with $\{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x1\}$

$rd(op_l(x1, x1, rd(x2, i(op_t(x1, x2)))), \underbrace{rd(x1, \overbrace{mult(i(x1), rd(x1, x2))})})$

$=$ by Lemma 57 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$rd(op_l(x1, x1, rd(x2, i(op_t(x1, x2)))), \overbrace{mult(x1, x2)})$

Lemma 1471: $op_l(mult(x1, mult(rd(x2, x1), x3)), x1, x2) = mult(x2, x3)$

$$
\begin{aligned}
& \underbrace{op_l(mult(x1, mult(rd(x2, x1), x3)), x1, x2)} \\
= \quad & \text{by Lemma 286 RL with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow mult(x1, mult(rd(x2, x1), x3))\} \\
& \overbrace{\underbrace{op_l(mult(x1, mult(rd(x2, x1), x3)), x1, rd(x2, x1))}} \\
= \quad & \text{by Lemma 1047 LR with } \{x3 \leftarrow x3, x2 \leftarrow rd(x2, x1), x1 \leftarrow x1\} \\
& \overbrace{mult(\underbrace{mult(rd(x2, x1), x1)}, x3)} \\
= \quad & \text{by Axiom 4 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& mult(\overbrace{x2}, x3)
\end{aligned}
$$

Lemma 1472: $rd(mult(x1, op_l(x2, x3, x1)), mult(x3, x2)) = rd(x1, x3)$

$$
\begin{aligned}
& rd(\underbrace{mult(x1, op_l(x2, x3, x1))}, mult(x3, x2)) \\
= \quad & \text{by Lemma 1091 RL with } \{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \underbrace{rd(\overbrace{mult(rd(x1, x3), mult(x3, x2))}, mult(x3, x2))} \\
= \quad & \text{by Axiom 5 RL with } \{x2 \leftarrow mult(x3, x2), x1 \leftarrow rd(x1, x3)\} \\
& \overbrace{rd(x1, x3)}
\end{aligned}
$$

Lemma 1473: $rd(x2, x1) = rd(mult(x2, x3), mult(x1, op_l(x3, x2, x1)))$

$$
\begin{aligned}
& \underbrace{rd(x2, x1)} \\
= \quad & \text{by Lemma 10 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{i(rd(x1, x2))}} \\
= \quad & \text{by Lemma 9 RL with } \{x2 \leftarrow rd(x1, x2), x1 \leftarrow mult(x2, x3)\} \\
& \overbrace{rd(mult(x2, x3), \underbrace{mult(rd(x1, x2), mult(x2, x3))})} \\
= \quad & \text{by Lemma 1091 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& rd(mult(x2, x3), \overbrace{mult(x1, op_l(x3, x2, x1))})
\end{aligned}
$$

Lemma 1474: $mult(i(x1), x2) = rd(op_t(x2, mult(x1, i(x2))), op_t(x1, i(x2)))$

$$
\begin{aligned}
& \underbrace{mult(i(x1), x2)} \\
= \quad & \text{by Lemma 17 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{i(mult(x1, i(x2)))}} \\
= \quad & \text{by Lemma 36 RL with } \{x2 \leftarrow mult(x1, i(x2)), x1 \leftarrow x2\} \\
& \overbrace{rd(op_t(x2, mult(x1, i(x2))), \underbrace{mult(x2, mult(x1, i(x2)))})} \\
= \quad & \text{by Lemma 30 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& rd(op_t(x2, mult(x1, i(x2))), \overbrace{op_t(x1, i(x2))})
\end{aligned}
$$

Lemma 1475: $rd(rd(x1, x2), rd(x3, x4)) = rd(rd(x4, x3), rd(x2, x1))$

$$
\begin{aligned}
& \underbrace{rd(rd(x1, x2), rd(x3, x4))} \\
= & \quad \text{by Lemma 39 RL with } \{x3 \leftarrow rd(x1, x2), x2 \leftarrow x3, x1 \leftarrow x4\} \\
& \overbrace{rd(rd(x4, x3), \underbrace{i(rd(x1, x2))})} \\
= & \quad \text{by Lemma 10 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& rd(rd(x4, x3), \overbrace{rd(x2, x1)})
\end{aligned}
$$

Lemma 1476: $x1 = mult(op_l(x2, x2, x1), rd(x1, op_t(x2, i(x1))))$

$$
\begin{aligned}
& \underbrace{x1} \\
= & \quad \text{by Lemma 3 RL with } \{x1 \leftarrow x1\} \\
& \overbrace{\underbrace{i(i(x1))}} \\
= & \quad \text{by Lemma 20 RL with } \{x2 \leftarrow i(x1), x1 \leftarrow op_l(x2, x2, x1)\} \\
& \overbrace{mult(op_l(x2, x2, x1), \underbrace{i(\overbrace{mult(op_l(x2, x2, x1), i(x1))})})} \\
= & \quad \text{by Lemma 457 RL with } \{x1 \leftarrow x1, x2 \leftarrow x2\} \\
& mult(op_l(x2, x2, x1), \underbrace{i(\overbrace{rd(op_t(x2, i(x1)), x1)})}) \\
= & \quad \text{by Lemma 10 LR with } \{x2 \leftarrow x1, x1 \leftarrow op_t(x2, i(x1))\} \\
& mult(op_l(x2, x2, x1), \overbrace{rd(x1, \overbrace{op_t(x2, i(x1))})})
\end{aligned}
$$

Lemma 1477: $rd(op_l(x2, x2, x1), rd(x2, x1)) = op_r(op_t(x1, i(x2)), rd(x1, x2), rd(x1, x2))$

$$
\begin{aligned}
& \underbrace{rd(op_l(x2, x2, x1), rd(x2, x1))} \\
= & \quad \text{by Lemma 39 RL with } \{x3 \leftarrow op_l(x2, x2, x1), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{\overbrace{rd(rd(x1, x2), i(op_l(x2, x2, x1)))}} \\
= & \quad \text{by Lemma 359 RL with } \{x2 \leftarrow rd(x1, x2), x1 \leftarrow op_l(x2, x2, x1)\} \\
& \overbrace{op_r(\underbrace{mult(op_l(x2, x2, x1), rd(x1, x2))}, rd(x1, x2), rd(x1, x2))} \\
= & \quad \text{by Lemma 688 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& op_r(\overbrace{op_t(x1, i(x2))}, rd(x1, x2), rd(x1, x2))
\end{aligned}
$$

Lemma 1478: $asoc(rd(x1,x2),i(x3),i(x4)) = i(asoc(rd(x2,x1),x3,x4))$

$\underbrace{asoc(rd(x1,x2),i(x3),i(x4))}$

$=$ by Lemma 108 RL with $\{x3 \leftarrow i(x4), x2 \leftarrow i(x3), x1 \leftarrow rd(x1,x2)\}$

$\overbrace{mult(i(mult(rd(x1,x2),mult(i(x3),i(x4)))),mult(\underbrace{mult(rd(x1,x2),i(x3))},i(x4)))}$

$=$ by Lemma 44 RL with $\{x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow x2\}$

$mult(i(mult(rd(x1,x2),mult(i(x3),i(x4)))),\underbrace{mult(\overbrace{i(mult(rd(x2,x1),x3))},i(x4)))}$

$=$ by Axiom 11 LR with $\{x2 \leftarrow x4, x1 \leftarrow mult(rd(x2,x1),x3)\}$

$\underbrace{mult(i(mult(rd(x1,x2),mult(i(x3),i(x4)))),\overbrace{i(mult(mult(rd(x2,x1),x3),x4))})}$

$=$ by Axiom 11 LR with $\{x2 \leftarrow mult(mult(rd(x2,x1),x3),x4), x1 \leftarrow mult(rd(x1,x2),mult(i(x3),i(x4)))\}$

$\overbrace{i(mult(mult(rd(x1,x2),\underbrace{mult(i(x3),i(x4))}),mult(mult(rd(x2,x1),x3),x4)))}$

$=$ by Axiom 11 LR with $\{x2 \leftarrow x4, x1 \leftarrow x3\}$

$\underbrace{i(mult(mult(rd(x1,x2),\overbrace{i(mult(x3,x4))}),mult(mult(rd(x2,x1),x3),x4)))}$

$=$ by Lemma 88 LR with $\{x3 \leftarrow mult(mult(rd(x2,x1),x3),x4), x2 \leftarrow mult(x3,x4), x1 \leftarrow rd(x1,x2)\}$

$\overbrace{mult(mult(\underbrace{i(rd(x1,x2))},mult(x3,x4)),i(mult(mult(rd(x2,x1),x3),x4)))}$

$=$ by Lemma 10 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{mult(mult(\overbrace{rd(x2,x1)},mult(x3,x4)),i(mult(mult(rd(x2,x1),x3),x4)))}$

$=$ by Lemma 964 LR with $\{x3 \leftarrow x4, x2 \leftarrow x3, x1 \leftarrow rd(x2,x1)\}$

$\overbrace{i(asoc(rd(x2,x1),x3,x4))}$

Lemma 1479: $i(asoc(i(x1),x2,x3)) = asoc(x1,i(x2),i(x3))$

$i(asoc(\underbrace{i(x1)},x2,x3))$

$=$ by Lemma 9 RL with $\{x2 \leftarrow x1, x1 \leftarrow x1\}$

$\underbrace{i(asoc(\overbrace{rd(x1,mult(x1,x1))},x2,x3))}$

$=$ by Lemma 1478 RL with $\{x4 \leftarrow x3, x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow mult(x1,x1)\}$

$\overbrace{asoc(\underbrace{rd(mult(x1,x1),x1)},i(x2),i(x3))}$

$=$ by Axiom 5 RL with $\{x2 \leftarrow x1, x1 \leftarrow x1\}$

$asoc(\overbrace{x1},i(x2),i(x3))$

Lemma 1480: $i(asoc(x1,x2,x3)) = asoc(i(x1),i(x2),i(x3))$

$i(asoc(\underbrace{x1},x2,x3))$

$=$ by Lemma 1 RL with $\{x1 \leftarrow x1\}$

$\underbrace{i(asoc(\overbrace{rd(x1,unit())},x2,x3))}$

$=$ by Lemma 1478 RL with $\{x4 \leftarrow x3, x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow unit()\}$

$\overbrace{asoc(\underbrace{rd(unit(),x1)},i(x2),i(x3))}$

$=$ by Lemma 5 LR with $\{x1 \leftarrow x1\}$

$asoc(\overbrace{i(x1)},i(x2),i(x3))$

Lemma 1481: $op_t(x1, rd(asoc(x2, x3, x2), x1)) = op_t(op_t(x1, rd(rd(asoc(x2, x3, x2), x1), x1)), asoc(x2, x3, x2))$

$op_t(\underbrace{x1}, rd(asoc(x2, x3, x2), x1))$

$=$ by Lemma 1296 RL with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{op_t(\overbrace{op_t(x1, rd(asoc(x2, x3, x2), x1))}, rd(asoc(x2, x3, x2), x1))}$

$=$ by Lemma 1197 RL with $\{x2 \leftarrow rd(asoc(x2, x3, x2), x1), x1 \leftarrow x1\}$

$\overbrace{op_t(op_t(x1, rd(rd(asoc(x2, x3, x2), x1), x1)), \underbrace{mult(x1, rd(asoc(x2, x3, x2), x1))})}$

$=$ by Lemma 1303 RL with $\{x3 \leftarrow x1, x2 \leftarrow x3, x1 \leftarrow x2\}$

$op_t(op_t(x1, rd(rd(asoc(x2, x3, x2), x1), x1)), \overbrace{asoc(x2, x3, x2)})$

Lemma 1482: $op_t(x1, rd(mult(rd(x1, x2), mult(x2, x3)), x3)) = x1$

$op_t(x1, rd(\underbrace{mult(rd(x1, x2), mult(x2, x3))}, x3))$

$=$ by Lemma 1045 RL with $\{x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow x2\}$

$op_t(x1, \underbrace{rd(\overbrace{mult(i(x2), mult(mult(x2, x1), x3))}, x3)})$

$=$ by Lemma 62 RL with $\{x3 \leftarrow mult(mult(x2, x1), x3), x2 \leftarrow x2, x1 \leftarrow x3\}$

$op_t(x1, \overbrace{rd(\underbrace{i(x3)}, mult(x2, i(mult(mult(x2, x1), x3))))})$

$=$ by Lemma 20 RL with $\{x2 \leftarrow x3, x1 \leftarrow mult(x2, x1)\}$

$op_t(x1, \underbrace{rd(\overbrace{mult(mult(x2, x1), i(mult(mult(x2, x1), x3)))}, mult(x2, i(mult(mult(x2, x1), x3)))))}$

$=$ by Lemma 676 RL with $\{x3 \leftarrow i(mult(mult(x2, x1), x3)), x2 \leftarrow x2, x1 \leftarrow mult(x2, x1)\}$

$op_t(\underbrace{x1}, \overbrace{op_r(rd(mult(x2, x1), x2), x2, i(mult(mult(x2, x1), x3)))})$

$=$ by Lemma 400 RL with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$\underbrace{op_t(\overbrace{op_t(rd(mult(x2, x1), x2), op_t(x2, x1))}, op_r(rd(mult(x2, x1), x2), x2, i(mult(mult(x2, x1), x3))))}$

$=$ by Axiom 13 RL with $\{x3 \leftarrow op_t(x2, x1), x2 \leftarrow op_r(rd(mult(x2, x1), x2), x2, i(mult(mult(x2, x1), x3))), x1 \leftarrow rd(mult(x2, x1$

$\overbrace{op_t(\underbrace{op_t(rd(mult(x2, x1), x2), op_r(rd(mult(x2, x1), x2), x2, i(mult(mult(x2, x1), x3))))}, op_t(x2, x1))}$

$=$ by Lemma 120 LR with $\{x3 \leftarrow i(mult(mult(x2, x1), x3)), x2 \leftarrow x2, x1 \leftarrow rd(mult(x2, x1), x2)\}$

$\underbrace{op_t(\overbrace{rd(mult(x2, x1), x2)}, op_t(x2, x1))}$

$=$ by Lemma 400 LR with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$\overbrace{x1}$

Lemma 1483: $op_t(x1, rd(mult(x1, op_l(x2, x3, x1)), x2)) = x1$

$op_t(x1, rd(\underbrace{mult(x1, op_l(x2, x3, x1))}, x2))$

$=$ by Lemma 1091 RL with $\{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x1\}$

$\underbrace{op_t(x1, rd(\overbrace{mult(rd(x1, x3), mult(x3, x2))}, x2))}$

$=$ by Lemma 1482 LR with $\{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x1\}$

$\overbrace{x1}$

Lemma 1484: $op_t(x1, rd(op_l(x2, x1, x3), rd(x2, x1))) = x1$

$\overbrace{op_t(x1, rd(op_l(x2, x1, x3), rd(x2, x1)))}$

$=$ by Lemma 1258 RL with $\{x3 \leftarrow x2, x2 \leftarrow op_l(x2, x1, x3), x1 \leftarrow x1\}$

$\overbrace{op_t(x1, rd(op_l(\underbrace{x2}, x1, x3), mult(i(x1), x2)))}$

$=$ by Lemma 12 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$op_t(x1, \underbrace{rd(op_l(\overbrace{mult(x1, mult(i(x1), x2))}, x1, x3), mult(i(x1), x2)))}$

$=$ by Lemma 1074 RL with $\{x4 \leftarrow x3, x3 \leftarrow x1, x2 \leftarrow mult(x1, mult(i(x1), x2)), x1 \leftarrow mult(i(x1), x2)\}$

$op_t(x1, \overbrace{rd(\underbrace{i(mult(i(x1), x2))}, op_l(i(mult(x1, mult(i(x1), x2))), x1, x3))})$

$=$ by Lemma 20 RL with $\{x2 \leftarrow mult(i(x1), x2), x1 \leftarrow x1\}$

$op_t(x1, rd(\overbrace{mult(x1, i(mult(x1, mult(i(x1), x2))))}), \underbrace{op_l(i(mult(x1, mult(i(x1), x2))), x1, x3)})$

$=$ by Lemma 292 RL with $\{x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow i(mult(x1, mult(i(x1), x2)))\}$

$op_t(x1, rd(mult(\underbrace{x1}, i(mult(x1, mult(i(x1), x2)))), \overbrace{op_l(i(mult(x1, mult(i(x1), x2))), rd(x1, x3), x3)}))$

$=$ by Axiom 4 LR with $\{x2 \leftarrow x3, x1 \leftarrow x1\}$

$op_t(x1, rd(\underbrace{mult(\overbrace{mult(rd(x1, x3), x3)}, i(mult(x1, mult(i(x1), x2))))}, op_l(i(mult(x1, mult(i(x1), x2))), rd(x1, x3), x3)))$

$=$ by Lemma 137 RL with $\{x3 \leftarrow i(mult(x1, mult(i(x1), x2))), x2 \leftarrow x3, x1 \leftarrow rd(x1, x3)\}$

$\underbrace{op_t(x1, rd(\overbrace{mult(rd(x1, x3), mult(x3, op_l(i(mult(x1, mult(i(x1), x2))), rd(x1, x3), x3)))}, op_l(i(mult(x1, mult(i(x1), x2))), rd$

$=$ by Lemma 1482 LR with $\{x3 \leftarrow op_l(i(mult(x1, mult(i(x1), x2))), rd(x1, x3), x3), x2 \leftarrow x3, x1 \leftarrow x1\}$

$\overbrace{x1}$

Lemma 1485: $x1 = op_t(x1, rd(x3, mult(rd(x1, x2), mult(x2, x3))))$

$\overbrace{x1}$

$=$ by Lemma 400 RL with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$\overbrace{op_t(\underbrace{rd(mult(x2, x1), x2)}, op_t(x2, x1))}$

$=$ by Axiom 2 LR with $\{x2 \leftarrow op_r(i(rd(mult(x2, x1), x2)), mult(x2, x1), i(mult(mult(x2, x1), x3))), x1 \leftarrow rd(mult(x2, x1), x2)\}$

$op_t(\overbrace{mult(i(op_r(i(rd(mult(x2, x1), x2)), mult(x2, x1), i(mult(mult(x2, x1), x3)))), \underbrace{mult(op_r(i(rd(mult(x2, x1), x2)), mult(x}$

$=$ by Lemma 159 LR with $\{x3 \leftarrow i(mult(mult(x2, x1), x3)), x2 \leftarrow mult(x2, x1), x1 \leftarrow rd(mult(x2, x1), x2)\}$

$op_t(\underbrace{mult(i(op_r(i(rd(mult(x2, x1), x2)), mult(x2, x1), i(mult(mult(x2, x1), x3)))), \overbrace{mult(rd(mult(x2, x1), x2), op_r(i(rd(mul}$

$=$ by Axiom 10 LR with $\{x2 \leftarrow rd(mult(x2, x1), x2), x1 \leftarrow op_r(i(rd(mult(x2, x1), x2)), mult(x2, x1), i(mult(mult(x2, x1), x3)))\}$

$\underbrace{op_t(\overbrace{op_t(rd(mult(x2, x1), x2), op_r(i(rd(mult(x2, x1), x2)), mult(x2, x1), i(mult(mult(x2, x1), x3))))}, op_t(x2, x1))}$

$=$ by Axiom 13 LR with $\{x3 \leftarrow op_t(x2, x1), x2 \leftarrow op_r(i(rd(mult(x2, x1), x2)), mult(x2, x1), i(mult(mult(x2, x1), x3))), x1 \leftarrow r$

$\overbrace{op_t(\underbrace{op_t(rd(mult(x2, x1), x2), op_t(x2, x1))}, op_r(i(rd(mult(x2, x1), x2)), mult(x2, x1), i(mult(mult(x2, x1), x3))))}$

$=$ by Lemma 400 LR with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$op_t(\overbrace{x1}, op_r(\underbrace{i(rd(mult(x2, x1), x2))}, mult(x2, x1), i(mult(mult(x2, x1), x3))))$

$=$ by Lemma 10 LR with $\{x2 \leftarrow x2, x1 \leftarrow mult(x2, x1)\}$

$op_t(x1, \underbrace{op_r(\overbrace{rd(x2, mult(x2, x1))}, mult(x2, x1), i(mult(mult(x2, x1), x3)))})$

$=$ by Lemma 676 LR with $\{x3 \leftarrow i(mult(mult(x2, x1), x3)), x2 \leftarrow mult(x2, x1), x1 \leftarrow x2\}$

$op_t(x1, \overbrace{rd(mult(x2, i(mult(mult(x2, x1), x3))), \underbrace{mult(mult(x2, x1), i(mult(mult(x2, x1), x3)))}})$

$=$ by Lemma 20 LR with $\{x2 \leftarrow x3, x1 \leftarrow mult(x2, x1)\}$

$op_t(x1, \underbrace{rd(mult(x2, i(mult(mult(x2, x1), x3))), \overbrace{i(x3)}})$

$=$ by Lemma 61 LR with $\{x3 \leftarrow x3, x2 \leftarrow mult(mult(x2, x1), x3), x1 \leftarrow x2\}$

$op_t(x1, \overbrace{rd(x3, \underbrace{mult(i(x2), mult(mult(x2, x1), x3))})})$

$=$ by Lemma 1045 LR with $\{x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow x2\}$

$op_t(x1, rd(x3, \overbrace{mult(rd(x1, x2), mult(x2, x3))}))$

Lemma 1486: $op_t(mult(x1, x2), i(op_l(x2, x2, x1))) = mult(op_l(x2, x2, x1), i(i(x1)))$

$\underbrace{op_t(mult(x1, x2), i(op_l(x2, x2, x1)))}$

$=$ by Lemma 18 RL with $\{x2 \leftarrow op_l(x2, x2, x1), x1 \leftarrow mult(x1, x2)\}$

$\overbrace{\underbrace{i(op_t(i(mult(x1, x2)), op_l(x2, x2, x1)))}}$

$=$ by Lemma 31 RL with $\{x2 \leftarrow i(mult(x1, x2)), x1 \leftarrow op_l(x2, x2, x1)\}$

$\overbrace{mult(op_l(x2, x2, x1), i(\underbrace{mult(i(mult(x1, x2)), op_l(x2, x2, x1))}))}$

$=$ by Lemma 529 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$mult(op_l(x2, x2, x1), i(\overbrace{i(x1)}))$

Lemma 1487: $op_r(rd(rd(x1,x2),x3),x3,i(x1)) = rd(mult(i(x3),x1),x2)$

$$
\begin{aligned}
& \overbrace{op_r(rd(rd(x1,x2),x3),x3,i(x1))} \\
= & \quad \text{by Lemma 676 LR with } \{x3 \leftarrow i(x1), x2 \leftarrow x3, x1 \leftarrow rd(x1,x2)\} \\
& \overbrace{rd(\underbrace{mult(rd(x1,x2),i(x1))}, mult(x3,i(x1)))} \\
= & \quad \text{by Lemma 22 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{rd(\overbrace{i(x2)}, mult(x3,i(x1)))} \\
= & \quad \text{by Lemma 62 LR with } \{x3 \leftarrow x1, x2 \leftarrow x3, x1 \leftarrow x2\} \\
& \overbrace{rd(mult(i(x3),x1),x2)}
\end{aligned}
$$

Lemma 1488: $mult(i(mult(x1,x2)),x2) = rd(op_t(x2,mult(x1,x2)),op_l(mult(x1,x2),x1,x2))$

$$
\begin{aligned}
& \underbrace{mult(i(mult(x1,x2)),x2)} \\
= & \quad \text{by Lemma 17 RL with } \{x2 \leftarrow x2, x1 \leftarrow mult(x1,x2)\} \\
& \overbrace{\underbrace{i(mult(mult(x1,x2),i(x2)))}} \\
= & \quad \text{by Lemma 721 LR with } \{x2 \leftarrow x2, x1 \leftarrow mult(x1,x2)\} \\
& \overbrace{rd(op_t(x2,mult(x1,x2)),\underbrace{op_l(mult(x1,x2),mult(x1,x2),x2)})} \\
= & \quad \text{by Lemma 285 LR with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow mult(x1,x2)\} \\
& rd(op_t(x2,mult(x1,x2)),\overbrace{op_l(mult(x1,x2),x1,x2)})
\end{aligned}
$$

Lemma 1489: $op_r(i(x1),x2,i(x2)) = rd(op_t(x2,mult(x1,x2)),mult(x2,x1))$

$$
\begin{aligned}
& \underbrace{op_r(i(x1),x2,i(x2))} \\
= & \quad \text{by Lemma 325 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{mult(i(mult(x1,x2)),x2)}} \\
= & \quad \text{by Lemma 1488 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{rd(op_t(x2,mult(x1,x2)),\underbrace{op_l(mult(x1,x2),x1,x2)})} \\
= & \quad \text{by Lemma 435 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& rd(op_t(x2,mult(x1,x2)),\overbrace{mult(x2,x1)})
\end{aligned}
$$

Lemma 1490: $rd(rd(i(x1),x2),x1) = rd(i(mult(x1,op_t(x2,x1))),op_l(x1,x1,rd(rd(i(x1),x2),x1)))$

$\underbrace{rd(rd(i(x1),x2),x1)}$

$=$ by Lemma 283 RL with $\{x2 \leftarrow rd(rd(i(x1),x2),x1), x1 \leftarrow x1\}$

$\overbrace{rd(op_t(\underbrace{mult(x1,rd(rd(i(x1),x2),x1))}, x1), op_l(x1,x1,rd(rd(i(x1),x2),x1)))}$

$=$ by Lemma 381 RL with $\{x2 \leftarrow x1, x1 \leftarrow rd(i(x1),x2)\}$

$rd(op_t(\overbrace{op_r(op_t(rd(i(x1),x2),i(x1)),x1,x1)}, x1), op_l(x1,x1,rd(rd(i(x1),x2),x1)))$

$=$ by Lemma 361 RL with $\{x2 \leftarrow x1, x1 \leftarrow op_t(rd(i(x1),x2),i(x1))\}$

$rd(op_t(\overbrace{op_r(op_t(rd(i(x1),x2),i(x1)),i(x1),i(x1))}, x1), op_l(x1,x1,rd(rd(i(x1),x2),x1)))$

$=$ by Axiom 14 RL with $\{x4 \leftarrow i(x1), x3 \leftarrow i(x1), x2 \leftarrow i(x1), x1 \leftarrow rd(i(x1),x2)\}$

$rd(op_t(\overbrace{op_t(op_r(rd(i(x1),x2),i(x1),i(x1)),i(x1))}, x1), op_l(x1,x1,rd(rd(i(x1),x2),x1)))$

$=$ by Lemma 922 RL with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$rd(op_t(\overbrace{i(mult(\underbrace{op_t(op_r(x2,i(x1),i(x1)),x1)}, x1))}, x1), op_l(x1,x1,rd(rd(i(x1),x2),x1)))$

$=$ by Axiom 14 LR with $\{x4 \leftarrow x1, x3 \leftarrow i(x1), x2 \leftarrow i(x1), x1 \leftarrow x2\}$

$rd(op_t(i(mult(\overbrace{op_r(op_t(x2,x1),i(x1),i(x1))}, x1))), x1), op_l(x1,x1,rd(rd(i(x1),x2),x1)))$

$=$ by Lemma 361 LR with $\{x2 \leftarrow x1, x1 \leftarrow op_t(x2,x1)\}$

$rd(op_t(i(mult(\overbrace{op_r(op_t(x2,x1),x1,x1)}, x1))), x1), op_l(x1,x1,rd(rd(i(x1),x2),x1)))$

$=$ by Lemma 357 LR with $\{x2 \leftarrow x1, x1 \leftarrow op_t(x2,x1)\}$

$rd(op_t(\underbrace{i(\overbrace{rd(x1,i(op_t(x2,x1)))})}, x1), op_l(x1,x1,rd(rd(i(x1),x2),x1)))$

$=$ by Lemma 10 LR with $\{x2 \leftarrow i(op_t(x2,x1)), x1 \leftarrow x1\}$

$rd(op_t(\overbrace{rd(i(op_t(x2,x1)),x1)}, x1), op_l(x1,x1,rd(rd(i(x1),x2),x1)))$

$=$ by Lemma 37 LR with $\{x2 \leftarrow x1, x1 \leftarrow op_t(x2,x1)\}$

$rd(\underbrace{op_t(\overbrace{rd(i(x1),op_t(x2,x1))}, x1)}, op_l(x1,x1,rd(rd(i(x1),x2),x1)))$

$=$ by Lemma 33 LR with $\{x2 \leftarrow op_t(x2,x1), x1 \leftarrow x1\}$

$\overbrace{i(mult(x1,op_t(x2,x1)))}, op_l(x1,x1,rd(rd(i(x1),x2),x1)))$

Lemma 1491: $rd(rd(i(x1),x2),x1) = rd(i(mult(x2,x1)),op_l(x1,x1,rd(i(x1),x2)))$

$\underbrace{rd(rd(i(x1),x2),x1)}$

$=$ by Lemma 1490 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{rd(i(\underbrace{mult(x1,op_t(x2,x1))}), op_l(x1,x1,rd(rd(i(x1),x2),x1)))}$

$=$ by Lemma 13 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$rd(i(\overbrace{mult(x2,x1)}), \underbrace{op_l(x1,x1,rd(rd(i(x1),x2),x1))})$

$=$ by Lemma 286 LR with $\{x3 \leftarrow rd(i(x1),x2), x2 \leftarrow x1, x1 \leftarrow x1\}$

$rd(i(mult(x2,x1)), \overbrace{op_l(x1,x1,rd(i(x1),x2))})$

Lemma 1492: $rd(rd(i(x1),x2),x1) = rd(i(mult(x2,x1)),op_l(x1,x2,x1))$

$\underbrace{rd(rd(i(x1),x2),x1)}$

$=$ by Lemma 1491 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{rd(i(mult(x2,x1)),\underbrace{op_l(x1,x1,rd(i(x1),x2))})}$

$=$ by Lemma 296 LR with $\{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x1\}$

$rd(i(mult(x2,x1)),\overbrace{\underbrace{op_l(x1,x1,i(x2))}})$

$=$ by Lemma 608 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$rd(i(mult(x2,x1)),\overbrace{op_l(x1,x2,x1)})$

Lemma 1493: $op_t(i(x1),mult(asoc(x2,x1,x1),x3)) = op_t(i(x1),x3)$

$op_t(i(x1),mult(\underbrace{asoc(x2,x1,x1)},x3))$

$=$ by Lemma 757 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$op_t(i(x1),\underbrace{mult(\overbrace{i(asoc(x1,x2,x1))},x3)})$

$=$ by Lemma 17 RL with $\{x2 \leftarrow x3, x1 \leftarrow asoc(x1,x2,x1)\}$

$\underbrace{op_t(i(x1),\overbrace{i(mult(asoc(x1,x2,x1),i(x3)))})}$

$=$ by Lemma 15 LR with $\{x2 \leftarrow mult(asoc(x1,x2,x1),i(x3)), x1 \leftarrow x1\}$

$\overbrace{i(op_t(x1,\underbrace{mult(asoc(x1,x2,x1)},i(x3))))}$

$=$ by Lemma 758 RL with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$i(op_t(x1,mult(\overbrace{asoc(i(x2),x1,x1)},\underbrace{i(x3)})))$

$=$ by Axiom 2 LR with $\{x2 \leftarrow x2, x1 \leftarrow i(x3)\}$

$i(op_t(x1,\underbrace{mult(asoc(i(x2),x1,x1)},\overbrace{mult(i(x2),mult(x2,i(x3)))})))$

$=$ by Lemma 941 LR with $\{x3 \leftarrow mult(x2,i(x3)), x2 \leftarrow x1, x1 \leftarrow i(x2)\}$

$i(\underbrace{op_t(x1,\overbrace{mult(op_r(i(x2),x1,x1),mult(x2,i(x3)))})})$

$=$ by Lemma 1148 LR with $\{x3 \leftarrow mult(x2,i(x3)), x2 \leftarrow i(x2), x1 \leftarrow x1\}$

$i(\overbrace{op_t(x1,\underbrace{mult(i(x2),mult(x2,i(x3)))})})$

$=$ by Axiom 2 RL with $\{x2 \leftarrow x2, x1 \leftarrow i(x3)\}$

$\underbrace{i(op_t(x1,\overbrace{i(x3)}))}$

$=$ by Lemma 19 LR with $\{x2 \leftarrow x3, x1 \leftarrow x1\}$

$\overbrace{op_t(i(x1),x3)}$

Lemma 1494: $rd(mult(x1, i(x2)), asoc(x2, x1, x1)) = rd(op_t(x1, i(x2)), x2)$

$$
\begin{aligned}
& \overbrace{rd(mult(x1, i(x2)), asoc(x2, x1, x1))} \\
= & \quad \text{by Lemma 1295 RL with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow mult(x1, i(x2))\} \\
& \overbrace{mult(\overbrace{mult(x1, i(x2)), asoc(x1, x2, x1)})} \\
= & \quad \text{by Lemma 13 RL with } \{x2 \leftarrow mult(x1, i(x2)), x1 \leftarrow asoc(x1, x2, x1)\} \\
& \overbrace{mult(asoc(x1, x2, x1), \overbrace{op_t(mult(x1, i(x2)), asoc(x1, x2, x1))})} \\
= & \quad \text{by Lemma 967 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{mult(asoc(x1, x2, x1), \overbrace{mult(x1, i(x2))})} \\
= & \quad \text{by Lemma 957 LR with } \{x3 \leftarrow i(x2), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{mult(op_l(x1, x1, x2), i(x2))} \\
= & \quad \text{by Lemma 457 RL with } \{x1 \leftarrow x2, x2 \leftarrow x1\} \\
& \overbrace{rd(op_t(x1, i(x2)), x2)}
\end{aligned}
$$

Lemma 1495: $op_r(rd(x2, x1), x2, x2) = op_r(i(rd(x1, x2)), op_l(x1, x1, x2), x2)$

$$
\begin{aligned}
& op_r(\overbrace{rd(x2, x1)}, x2, x2) \\
= & \quad \text{by Lemma 10 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_r(\overbrace{i(rd(x1, x2))}, x2, x2)} \\
= & \quad \text{by Lemma 996 RL with } \{x2 \leftarrow x2, x1 \leftarrow rd(x1, x2)\} \\
& \overbrace{op_r(i(rd(x1, x2)), \overbrace{mult(x2, rd(x1, x2))}, x2)} \\
= & \quad \text{by Lemma 427 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_r(i(rd(x1, x2)), \overbrace{op_l(x1, x1, x2)}, x2)
\end{aligned}
$$

Lemma 1496: $op_l(op_t(x1, x2), x3, x1) = op_l(op_t(x1, x2), x3, rd(rd(mult(x1, x2), x1), rd(x2, x1)))$

$$
\begin{aligned}
& \overbrace{op_l(op_t(x1, x2), x3, x1)} \\
= & \quad \text{by Lemma 1020 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(op_t(x1, x2), x3, \overbrace{op_t(x1, x2)})} \\
= & \quad \text{by Lemma 1022 RL with } \{x3 \leftarrow rd(x2, x1), x2 \leftarrow x3, x1 \leftarrow op_t(x1, x2)\} \\
& \overbrace{op_l(op_t(x1, x2), x3, rd(\overbrace{mult(rd(x2, x1), op_t(x1, x2))}, rd(x2, x1)))} \\
= & \quad \text{by Lemma 650 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& op_l(op_t(x1, x2), x3, rd(\overbrace{rd(mult(x1, x2), x1)}, rd(x2, x1)))
\end{aligned}
$$

Lemma 1497: $asoc(x1, rd(i(x1), mult(x1, x2)), x3) = asoc(i(x2), i(x1), x3)$

$$
\begin{aligned}
& asoc(x1, \underbrace{rd(i(x1), mult(x1, x2))}, x3) \\
= & \quad \text{by Lemma 37 RL with } \{x2 \leftarrow x1, x1 \leftarrow mult(x1, x2)\} \\
& \underbrace{asoc(x1, \overbrace{rd(i(mult(x1, x2)), x1)}, x3)} \\
= & \quad \text{by Lemma 1088 RL with } \{x3 \leftarrow x3, x2 \leftarrow i(mult(x1, x2)), x1 \leftarrow x1\} \\
& \overbrace{asoc(\underbrace{mult(x1, i(mult(x1, x2)))}, i(x1), x3)} \\
= & \quad \text{by Lemma 20 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& asoc(\overbrace{i(x2)}, i(x1), x3)
\end{aligned}
$$

Lemma 1498: $mult(rd(x1, x2), mult(x2, op_t(x3, x1))) = mult(op_l(x3, x2, x1), x1)$

$$
\begin{aligned}
& \underbrace{mult(rd(x1, x2), mult(x2, op_t(x3, x1)))} \\
= & \quad \text{by Lemma 1091 LR with } \{x3 \leftarrow op_t(x3, x1), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{\overbrace{mult(x1, op_l(op_t(x3, x1), x2, x1))}} \\
= & \quad \text{by Lemma 167 LR with } \{x4 \leftarrow x1, x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{mult(op_l(x3, x2, x1), x1)}
\end{aligned}
$$

Lemma 1499: $mult(op_r(mult(x1, x2), x2, x1), i(x1)) = op_t(x2, i(x1))$

$$
\begin{aligned}
& \underbrace{mult(op_r(mult(x1, x2), x2, x1), i(x1))} \\
= & \quad \text{by Lemma 155 RL with } \{x4 \leftarrow x1, x3 \leftarrow x2, x2 \leftarrow mult(x1, x2), x1 \leftarrow i(x1)\} \\
& \overbrace{mult(i(x1), op_r(\underbrace{op_t(mult(x1, x2), i(x1))}, x2, x1))} \\
= & \quad \text{by Lemma 919 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& mult(i(x1), \underbrace{op_r(\overbrace{op_t(mult(x2, x1), x2)}, x2, x1))} \\
= & \quad \text{by Axiom 14 RL with } \{x4 \leftarrow x2, x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow mult(x2, x1)\} \\
& mult(i(x1), \overbrace{op_t(\underbrace{op_r(mult(x2, x1), x2, x1)}, x2)}) \\
= & \quad \text{by Lemma 991 RL with } \{x1 \leftarrow x2, x2 \leftarrow x1\} \\
& mult(i(x1), \underbrace{op_t(\overbrace{op_t(mult(x1, x2), x1)}, x2))} \\
= & \quad \text{by Lemma 456 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{mult(i(x1), \overbrace{mult(op_t(x1, i(x2)), x2)})} \\
= & \quad \text{by Lemma 1092 LR with } \{x3 \leftarrow x2, x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& \underbrace{\overbrace{mult(rd(op_t(x1, i(x2)), x1), x2)}} \\
= & \quad \text{by Lemma 471 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{\overbrace{i(op_t(i(x2), x1))}} \\
= & \quad \text{by Lemma 18 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \overbrace{op_t(x2, i(x1))}
\end{aligned}
$$

Lemma 1500: $rd(op_r(mult(x1,x2),x2,x1),x1) = op_l(x2,x2,x1)$

$\underbrace{rd(op_r(mult(x1,x2),x2,x1),x1)}$

$=$ by Lemma 23 RL with $\{x2 \leftarrow op_r(mult(x1,x2),x2,x1), x1 \leftarrow x1\}$

$\overbrace{rd(i(x1),i(op_r(mult(x1,x2),x2,x1)))}$

$=$ by Axiom 5 LR with $\{x2 \leftarrow op_t(i(x1),op_r(mult(x1,x2),x2,x1)), x1 \leftarrow rd(i(x1),i(op_r(mult(x1,x2),x2,x1)))\}$

$\overbrace{rd(mult(rd(i(x1),i(op_r(mult(x1,x2),x2,x1))),\underbrace{op_t(i(x1),op_r(mult(x1,x2),x2,x1))}),op_t(i(x1),op_r(mult(x1,x2),x2,x1)))}$

$=$ by Lemma 550 RL with $\{x2 \leftarrow op_r(mult(x1,x2),x2,x1), x1 \leftarrow i(x1)\}$

$rd(\underbrace{mult(rd(i(x1),i(op_r(mult(x1,x2),x2,x1))),\overbrace{op_t(i(x1),op_r(op_r(mult(x1,x2),x2,x1),i(x1),i(x1)))}),op_t(i(x1),op_r(mult$

$=$ by Lemma 872 RL with $\{x2 \leftarrow op_r(mult(x1,x2),x2,x1), x1 \leftarrow i(x1)\}$

$rd(\overbrace{mult(i(x1),\underbrace{mult(op_r(mult(x1,x2),x2,x1),i(x1))})},op_t(i(x1),op_r(mult(x1,x2),x2,x1)))$

$=$ by Lemma 1499 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$rd(\underbrace{mult(i(x1),\overbrace{op_t(x2,i(x1))})},op_t(i(x1),op_r(mult(x1,x2),x2,x1)))$

$=$ by Lemma 13 LR with $\{x2 \leftarrow x2, x1 \leftarrow i(x1)\}$

$\underbrace{rd(\overbrace{mult(x2,i(x1))},op_t(i(x1),op_r(mult(x1,x2),x2,x1)))}$

$=$ by Lemma 1292 RL with $\{x3 \leftarrow op_r(mult(x1,x2),x2,x1), x2 \leftarrow i(x1), x1 \leftarrow x2\}$

$\overbrace{rd(x2,\underbrace{rd(op_t(i(x1),op_r(mult(x1,x2),x2,x1)),i(x1))})}$

$=$ by Lemma 67 LR with $\{x3 \leftarrow x1, x2 \leftarrow op_r(mult(x1,x2),x2,x1), x1 \leftarrow x1\}$

$rd(x2,\overbrace{rd(x1,op_t(x1,\underbrace{i(op_r(mult(x1,x2),x2,x1))}))})$

$=$ by Lemma 1192 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$rd(x2,rd(x1,\underbrace{op_t(x1,\overbrace{rd(op_t(i(x1),x2),x2)})}))$

$=$ by Lemma 1161 LR with $\{x3 \leftarrow x2, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{rd(x2,rd(x1,\overbrace{op_t(x1,rd(x2,x1))}))}$

$=$ by Lemma 1007 LR with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$\overbrace{rd(x2,\underbrace{rd(op_l(x2,x1,x2),x2)})}$

$=$ by Lemma 765 LR with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$\underbrace{rd(x2,\overbrace{asoc(x1,x2,x2)})}$

$=$ by Lemma 773 RL with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$\overbrace{op_l(x2,x2,x1)}$

Lemma 1501: $rd(x1, op_r(mult(x1, x2), x2, x1)) = op_l(i(x2), x2, x1)$

$\overbrace{rd(x1, op_r(mult(x1, x2), x2, x1))}$

$=$ by Lemma 10 RL with $\{x2 \leftarrow x1, x1 \leftarrow op_r(mult(x1, x2), x2, x1)\}$

$\overbrace{i(\underbrace{rd(op_r(mult(x1, x2), x2, x1), x1)})}$

$=$ by Lemma 1500 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{i(\overbrace{op_l(x2, x2, x1)})}$

$=$ by Lemma 1049 LR with $\{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x2\}$

$\overbrace{op_l(i(x2), x2, x1)}$

Lemma 1502: $op_l(x1, x2, op_r(mult(x2, x3), x3, x2)) = op_l(x1, op_t(x2, mult(x2, x3)), rd(x2, i(op_r(mult(x2, x3), x3, x2))))$

$\overbrace{op_l(x1, x2, op_r(mult(x2, x3), x3, x2))}$

$=$ by Lemma 1264 RL with $\{x3 \leftarrow op_r(mult(x2, x3), x3, x2), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_l(x1, op_t(x2, op_r(mult(x2, x3), x3, x2)), \underbrace{op_t(op_r(mult(x2, x3), x3, x2), x2)})}$

$=$ by Lemma 3 RL with $\{x1 \leftarrow op_t(op_r(mult(x2, x3), x3, x2), x2)\}$

$\underbrace{op_l(x1, op_t(x2, op_r(mult(x2, x3), x3, x2)), \overbrace{i(\overbrace{i(op_t(op_r(mult(x2, x3), x3, x2), x2))})})}$

$=$ by Lemma 1321 RL with $\{x3 \leftarrow i(op_t(op_r(mult(x2, x3), x3, x2), x2)), x2 \leftarrow op_t(x2, op_r(mult(x2, x3), x3, x2)), x1 \leftarrow x1\}$

$\overbrace{op_l(x1, op_t(x2, op_r(mult(x2, x3), x3, x2)), \underbrace{rd(op_t(x2, op_r(mult(x2, x3), x3, x2)), i(op_t(op_r(mult(x2, x3), x3, x2), x2)))})}$

$=$ by Lemma 732 RL with $\{x1 \leftarrow op_r(mult(x2, x3), x3, x2), x2 \leftarrow x2\}$

$op_l(x1, \underbrace{op_t(x2, op_r(mult(x2, x3), x3, x2))}, \overbrace{rd(x2, i(\overbrace{op_r(mult(x2, x3), x3, x2)}))})$

$=$ by Lemma 559 RL with $\{x2 \leftarrow op_r(mult(x2, x3), x3, x2), x1 \leftarrow x2\}$

$op_l(x1, \overbrace{op_t(x2, \underbrace{op_t(op_r(mult(x2, x3), x3, x2), i(x2))})}, rd(x2, i(op_r(mult(x2, x3), x3, x2))))$

$=$ by Axiom 14 LR with $\{x4 \leftarrow i(x2), x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow mult(x2, x3)\}$

$op_l(x1, op_t(x2, \overbrace{op_r(\underbrace{op_t(mult(x2, x3), i(x2))}, x3, x2)}), rd(x2, i(op_r(mult(x2, x3), x3, x2))))$

$=$ by Lemma 919 LR with $\{x2 \leftarrow x3, x1 \leftarrow x2\}$

$op_l(x1, op_t(x2, \underbrace{op_r(\overbrace{op_t(mult(x3, x2), x3)}, x3, x2)}), rd(x2, i(op_r(mult(x2, x3), x3, x2))))$

$=$ by Axiom 14 RL with $\{x4 \leftarrow x3, x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow mult(x3, x2)\}$

$op_l(x1, op_t(x2, \overbrace{op_t(\underbrace{op_r(mult(x3, x2), x3, x2)}, x3)}), rd(x2, i(op_r(mult(x2, x3), x3, x2))))$

$=$ by Lemma 991 RL with $\{x1 \leftarrow x3, x2 \leftarrow x2\}$

$op_l(x1, op_t(x2, \underbrace{op_t(\overbrace{op_t(mult(x2, x3), x2)}, x3)}), rd(x2, i(op_r(mult(x2, x3), x3, x2))))$

$=$ by Lemma 456 LR with $\{x2 \leftarrow x3, x1 \leftarrow x2\}$

$op_l(x1, \underbrace{op_t(x2, \overbrace{mult(op_t(x2, i(x3)), x3)})}, rd(x2, i(op_r(mult(x2, x3), x3, x2))))$

$=$ by Lemma 1114 LR with $\{x3 \leftarrow x3, x2 \leftarrow i(x3), x1 \leftarrow x2\}$

$op_l(x1, \overbrace{op_t(x2, mult(x2, x3))}, rd(x2, i(op_r(mult(x2, x3), x3, x2))))$

Lemma 1503: $op_l(x1, x2, op_r(mult(x2, x3), x3, x2)) = op_l(x1, op_t(x2, mult(x2, x3)), mult(mult(x2, x3), x2))$

$op_l(x1, x2, op_r(mult(x2, x3), x3, x2))$

$=$ by Lemma 1502 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$op_l(x1, op_t(x2, mult(x2, x3)), rd(x2, i(op_r(mult(x2, x3), x3, x2))))$

$=$ by Lemma 1192 LR with $\{x2 \leftarrow x3, x1 \leftarrow x2\}$

$op_l(x1, op_t(x2, mult(x2, x3)), rd(x2, rd(op_t(i(x2), x3), x3)))$

$=$ by Lemma 832 LR with $\{x2 \leftarrow x3, x1 \leftarrow x2\}$

$op_l(x1, op_t(x2, mult(x2, x3)), mult(mult(x2, x3), x2))$

Lemma 1504: $op_l(x1, x2, op_r(mult(x2, x3), x3, x2)) = op_l(x1, x3, i(x2))$

$op_l(x1, x2, op_r(mult(x2, x3), x3, x2))$

$=$ by Lemma 1503 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$op_l(x1, op_t(x2, mult(x2, x3)), mult(mult(x2, x3), x2))$

$=$ by Lemma 1347 LR with $\{x3 \leftarrow mult(x2, x3), x2 \leftarrow x2, x1 \leftarrow x1\}$

$op_l(x1, x2, mult(x2, x3))$

$=$ by Lemma 1330 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$op_l(x1, x3, i(x2))$

Lemma 1505: $mult(op_r(mult(x1, x2), x2, x1), i(x2)) = op_t(x1, i(x2))$

$mult(op_r(mult(x1, x2), x2, x1), i(x2))$

$=$ by Lemma 741 RL with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$mult(op_r(mult(x1, x2), x2, x1), op_l(op_l(i(x2), x2, x1), x2, i(x1)))$

$=$ by Lemma 1501 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$mult(op_r(mult(x1, x2), x2, x1), op_l(rd(x1, op_r(mult(x1, x2), x2, x1)), x2, i(x1)))$

$=$ by Lemma 1504 RL with $\{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow rd(x1, op_r(mult(x1, x2), x2, x1))\}$

$mult(op_r(mult(x1, x2), x2, x1), op_l(rd(x1, op_r(mult(x1, x2), x2, x1)), x1, op_r(mult(x1, x2), x2, x1)))$

$=$ by Lemma 292 RL with $\{x3 \leftarrow op_r(mult(x1, x2), x2, x1), x2 \leftarrow x1, x1 \leftarrow rd(x1, op_r(mult(x1, x2), x2, x1))\}$

$mult(op_r(mult(x1, x2), x2, x1), op_l(rd(x1, op_r(mult(x1, x2), x2, x1)), rd(x1, op_r(mult(x1, x2), x2, x1)), op_r(mult(x1, x2), x$

$=$ by Lemma 264 RL with $\{x2 \leftarrow rd(x1, op_r(mult(x1, x2), x2, x1)), x1 \leftarrow op_r(mult(x1, x2), x2, x1)\}$

$op_t(mult(rd(x1, op_r(mult(x1, x2), x2, x1)), op_r(mult(x1, x2), x2, x1)), rd(x1, op_r(mult(x1, x2), x2, x1)))$

$=$ by Axiom 4 RL with $\{x2 \leftarrow op_r(mult(x1, x2), x2, x1), x1 \leftarrow x1\}$

$op_t(x1, rd(x1, op_r(mult(x1, x2), x2, x1)))$

$=$ by Lemma 1501 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$op_t(x1, op_l(i(x2), x2, x1))$

$=$ by Lemma 572 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$op_t(x1, i(x2))$

Lemma 1506: $rd(op_r(mult(x2,x1),x1,x2),x1) = rd(x2,rd(x1,op_t(x1,rd(op_t(x2,i(x1)),x1))))$

$\overbrace{rd(op_r(mult(x2,x1),x1,x2),x1)}$

$=$ by Lemma 23 RL with $\{x2 \leftarrow op_r(mult(x2,x1),x1,x2), x1 \leftarrow x1\}$

$\overbrace{rd(i(x1),i(op_r(mult(x2,x1),x1,x2)))}$

$=$ by Axiom 5 LR with $\{x2 \leftarrow op_t(i(x1),op_r(mult(x2,x1),x1,x2)), x1 \leftarrow rd(i(x1),i(op_r(mult(x2,x1),x1,x2)))\}$

$\overbrace{rd(mult(rd(i(x1),i(op_r(mult(x2,x1),x1,x2))),\underbrace{op_t(i(x1),op_r(mult(x2,x1),x1,x2))}),op_t(i(x1),op_r(mult(x2,x1),x1,x2)))}$

$=$ by Lemma 550 RL with $\{x2 \leftarrow op_r(mult(x2,x1),x1,x2), x1 \leftarrow i(x1)\}$

$rd(\underbrace{mult(rd(i(x1),i(op_r(mult(x2,x1),x1,x2))),\overbrace{op_t(i(x1),op_r(op_r(mult(x2,x1),x1,x2),i(x1),i(x1)))}),op_t(i(x1),op_r(mult}$

$=$ by Lemma 872 RL with $\{x2 \leftarrow op_r(mult(x2,x1),x1,x2), x1 \leftarrow i(x1)\}$

$rd(\overbrace{mult(i(x1),\underbrace{mult(op_r(mult(x2,x1),x1,x2),i(x1))}),op_t(i(x1),op_r(mult(x2,x1),x1,x2)))}$

$=$ by Lemma 1505 LR with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$rd(\underbrace{mult(i(x1),\overbrace{op_t(x2,i(x1))}),op_t(i(x1),op_r(mult(x2,x1),x1,x2)))}$

$=$ by Lemma 13 LR with $\{x2 \leftarrow x2, x1 \leftarrow i(x1)\}$

$\underbrace{rd(\overbrace{mult(x2,i(x1))},op_t(i(x1),op_r(mult(x2,x1),x1,x2)))}$

$=$ by Lemma 1292 RL with $\{x3 \leftarrow op_r(mult(x2,x1),x1,x2), x2 \leftarrow i(x1), x1 \leftarrow x2\}$

$\overbrace{rd(x2,\underbrace{rd(op_t(i(x1),op_r(mult(x2,x1),x1,x2)),i(x1))})}$

$=$ by Lemma 67 LR with $\{x3 \leftarrow x1, x2 \leftarrow op_r(mult(x2,x1),x1,x2), x1 \leftarrow x1\}$

$rd(x2,\overbrace{rd(x1,op_t(x1,\underbrace{i(op_r(mult(x2,x1),x1,x2))}))})$

$=$ by Lemma 1192 LR with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$rd(x2,rd(x1,op_t(x1,\overbrace{rd(op_t(i(x2),x1),x1)})))$

$=$ by Lemma 66 RL with $\{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x1\}$

$rd(x2,rd(x1,\underbrace{op_t(x1,\overbrace{rd(i(x1),op_t(x2,i(x1)))})}))$

$=$ by Lemma 707 LR with $\{x2 \leftarrow op_t(x2,i(x1)), x1 \leftarrow x1\}$

$rd(x2,rd(x1,\overbrace{op_t(x1,rd(op_t(x2,i(x1)),x1))}))$

Lemma 1507: $rd(op_r(mult(x1,x2),x2,x1),x2) = op_l(x1,x1,x2)$

$\underbrace{rd(op_r(mult(x1,x2),x2,x1),x2)}$

$=$ by Lemma 1506 LR with $\{x1 \leftarrow x2, x2 \leftarrow x1\}$

$\overbrace{rd(x1,rd(x2,op_t(x2,\underbrace{rd(op_t(x1,i(x2)),x2)})))}$

$=$ by Lemma 10 RL with $\{x2 \leftarrow op_t(x1,i(x2)), x1 \leftarrow x2\}$

$rd(x1,rd(x2,\underbrace{op_t(x2,\overbrace{i(rd(x2,op_t(x1,i(x2))))})}))$

$=$ by Lemma 18 RL with $\{x2 \leftarrow rd(x2,op_t(x1,i(x2))), x1 \leftarrow x2\}$

$rd(x1,rd(x2,\overbrace{i(op_t(i(x2),rd(\underbrace{x2},op_t(x1,i(x2)))))}))$

$=$ by Axiom 4 LR with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$rd(x1,rd(x2,i(\underbrace{op_t(i(x2),rd(\overbrace{mult(rd(x2,x1),x1)},op_t(x1,i(x2)))))}))$

$=$ by Lemma 1240 RL with $\{x3 \leftarrow x1, x2 \leftarrow rd(x2,x1), x1 \leftarrow i(x2)\}$

$rd(x1,rd(x2,\underbrace{i(\overbrace{op_t(i(x2),rd(x2,x1))})}))$

$=$ by Lemma 18 LR with $\{x2 \leftarrow rd(x2,x1), x1 \leftarrow x2\}$

$rd(x1,rd(x2,\overbrace{op_t(x2,\underbrace{i(rd(x2,x1))})}))$

$=$ by Lemma 10 LR with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$\underbrace{rd(x1,rd(x2,op_t(x2,\overbrace{rd(x1,x2)})))}$

$=$ by Lemma 1007 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{rd(x1,\underbrace{rd(op_l(x1,x2,x1),x1)})}$

$=$ by Lemma 765 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{rd(x1,\overbrace{asoc(x2,x1,x1)})}$

$=$ by Lemma 773 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_l(x1,x1,x2)}$

Lemma 1508: $op_t(x1,rd(op_t(i(x1),x2),x2)) = mult(op_l(rd(x1,op_r(mult(x1,x2),x2,x1)),x2,i(x1)),op_r(mult(x1,x2),x2,x1))$

$op_t(x1,\underbrace{rd(op_t(i(x1),x2),x2)})$

$=$ by Lemma 1192 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{op_t(x1,\overbrace{i(op_r(mult(x1,x2),x2,x1))})}$

$=$ by Lemma 481 RL with $\{x2 \leftarrow op_r(mult(x1,x2),x2,x1), x1 \leftarrow x1\}$

$\overbrace{mult(\underbrace{op_l(rd(x1,op_r(mult(x1,x2),x2,x1)),x1,op_r(mult(x1,x2),x2,x1))},op_r(mult(x1,x2),x2,x1))}$

$=$ by Lemma 1504 LR with $\{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow rd(x1,op_r(mult(x1,x2),x2,x1))\}$

$mult(\overbrace{op_l(rd(x1,op_r(mult(x1,x2),x2,x1)),x2,i(x1))},op_r(mult(x1,x2),x2,x1))$

Lemma 1509: $op_t(x1, rd(x2, x1)) = mult(i(x2), op_r(mult(x1, x2), x2, x1))$

$\underbrace{op_t(x1, rd(x2, x1))}$

$=$ by Lemma 1161 RL with $\{x3 \leftarrow x2, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_t(x1, \underbrace{rd(op_t(i(x1), x2), x2)})}$

$=$ by Lemma 1508 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{mult(op_l(\underbrace{rd(x1, op_r(mult(x1, x2), x2, x1))}, x2, i(x1)), op_r(mult(x1, x2), x2, x1))}$

$=$ by Lemma 1501 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$mult(\underbrace{op_l(\overbrace{op_l(i(x2), x2, x1)}, x2, i(x1))}, op_r(mult(x1, x2), x2, x1))$

$=$ by Lemma 741 LR with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$mult(\overbrace{i(x2)}, op_r(mult(x1, x2), x2, x1))$

Lemma 1510: $x1 = mult(op_r(mult(x1, x2), x2, x1), i(op_t(x2, i(x1))))$

$\underbrace{x1}$

$=$ by Lemma 3 RL with $\{x1 \leftarrow x1\}$

$\overbrace{\underbrace{i(i(x1))}}$

$=$ by Lemma 20 RL with $\{x2 \leftarrow i(x1), x1 \leftarrow op_r(mult(x1, x2), x2, x1)\}$

$\overbrace{mult(op_r(mult(x1, x2), x2, x1), i(\underbrace{mult(op_r(mult(x1, x2), x2, x1), i(x1))}))}$

$=$ by Lemma 1499 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$mult(op_r(mult(x1, x2), x2, x1), i(\overbrace{op_t(x2, i(x1))}))$

Lemma 1511: $mult(op_r(mult(x1, x2), x2, x1), op_t(i(x2), x1)) = x1$

$mult(op_r(mult(x1, x2), x2, x1), \underbrace{op_t(i(x2), x1)})$

$=$ by Lemma 19 RL with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$\underbrace{mult(op_r(mult(x1, x2), x2, x1), \overbrace{i(op_t(x2, i(x1)))})}$

$=$ by Lemma 1510 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{x1}$

Lemma 1512: $op_l(x1, rd(x2, i(mult(x1, x2))), x1) = op_l(op_l(x1, x1, rd(i(x1), x2)), x1, i(x2))$

$op_l(x1, rd(x2, \underbrace{i(mult(x1, x2))}), x1)$

$=$ by Axiom 11 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$op_l(x1, \underbrace{rd(x2, \overbrace{mult(i(x1), i(x2))})}, x1)$

$=$ by Lemma 61 RL with $\{x3 \leftarrow x2, x2 \leftarrow i(x2), x1 \leftarrow x1\}$

$op_l(x1, \overbrace{rd(mult(x1, \underbrace{i(i(x2))}), i(x2))}, x1)$

$=$ by Lemma 332 RL with $\{x2 \leftarrow i(x2), x1 \leftarrow x1\}$

$op_l(x1, \overbrace{mult(\underbrace{rd(x1, i(x2))}, i(i(x2)))}, x1)$

$=$ by Lemma 38 LR with $\{x1 \leftarrow x2, x2 \leftarrow x1\}$

$op_l(x1, mult(\overbrace{rd(x2, i(x1))}, \underbrace{i(i(x2))}), x1)$

$=$ by Lemma 3 LR with $\{x1 \leftarrow x2\}$

$\underbrace{op_l(x1, mult(rd(x2, i(x1)), \overbrace{x2}), x1)}$

$=$ by Lemma 608 RL with $\{x2 \leftarrow mult(rd(x2, i(x1)), x2), x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_l(x1, x1, i(mult(rd(x2, i(x1)), x2)))}}$

$=$ by Lemma 296 RL with $\{x3 \leftarrow mult(rd(x2, i(x1)), x2), x2 \leftarrow x1, x1 \leftarrow x1\}$

$\overbrace{op_l(x1, x1, rd(i(x1), mult(\underbrace{rd(x2, i(x1))}, x2)))}$

$=$ by Lemma 10 RL with $\{x2 \leftarrow x2, x1 \leftarrow i(x1)\}$

$op_l(x1, x1, rd(\underbrace{i(x1)}), mult(\overbrace{i(rd(i(x1), x2))}, x2)))$

$=$ by Axiom 4 LR with $\{x2 \leftarrow x2, x1 \leftarrow i(x1)\}$

$op_l(x1, x1, rd(\overbrace{mult(rd(i(x1), x2), \underbrace{x2})}), mult(i(rd(i(x1), x2)), x2)))$

$=$ by Lemma 12 RL with $\{x2 \leftarrow x2, x1 \leftarrow rd(i(x1), x2)\}$

$op_l(x1, x1, \underbrace{rd(mult(rd(i(x1), x2), \overbrace{mult(rd(i(x1), x2), mult(i(rd(i(x1), x2)), x2))}), mult(i(rd(i(x1), x2)), x2)))}$

$=$ by Lemma 41 LR with $\{x2 \leftarrow mult(i(rd(i(x1), x2)), x2), x1 \leftarrow rd(i(x1), x2)\}$

$\underbrace{op_l(x1, x1, \overbrace{mult(rd(i(x1), x2), rd(i(x1), x2))})}$

$=$ by Lemma 1174 RL with $\{x2 \leftarrow rd(i(x1), x2), x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_l(op_l(x1, x1, rd(i(x1), x2)), x1, rd(i(x1), x2))}}$

$=$ by Lemma 296 LR with $\{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow op_l(x1, x1, rd(i(x1), x2))\}$

$\overbrace{op_l(op_l(x1, x1, rd(i(x1), x2)), x1, i(x2))}$

Lemma 1513: $op_l(x1, rd(x2, i(mult(x1, x2))), x1) = op_l(op_l(x1, x2, x1), x1, i(x2))$

$$
\begin{aligned}
& \underbrace{op_l(x1, rd(x2, i(mult(x1, x2))), x1)} \\
= & \quad \text{by Lemma 1512 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(op_l(x1, x1, rd(i(x1), x2)), x1, i(x2))}} \\
= & \quad \text{by Lemma 1107 LR with } \{x4 \leftarrow x2, x3 \leftarrow rd(i(x1), x2), x2 \leftarrow x1, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(op_l(x1, x2, x1), x1, rd(i(x1), x2))}} \\
= & \quad \text{by Lemma 296 LR with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow op_l(x1, x2, x1)\} \\
& \overbrace{op_l(op_l(x1, x2, x1), x1, i(x2))}
\end{aligned}
$$

Lemma 1514: $op_l(x1, rd(x2, i(mult(x1, x2))), x1) = op_l(x1, mult(x2, x2), x1)$

$$
\begin{aligned}
& \underbrace{op_l(x1, rd(x2, i(mult(x1, x2))), x1)} \\
= & \quad \text{by Lemma 1513 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(op_l(x1, x2, x1), x1, i(x2))}} \\
= & \quad \text{by Lemma 1107 LR with } \{x4 \leftarrow x2, x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(op_l(x1, x2, x1), x2, x1)}} \\
= & \quad \text{by Lemma 1173 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, mult(x2, x2), x1)}
\end{aligned}
$$

Lemma 1515: $op_l(x2, x2, mult(x1, x1)) = rd(op_l(x2, x2, x1), asoc(x1, x2, x2))$

$\underbrace{op_l(x2, x2, mult(x1, x1))}$

$=$ by Lemma 1180 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{mult(\underbrace{op_t(x1, x2)}, mult(x2, i(x1)))}$

$=$ by Lemma 455 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$mult(\overbrace{op_l(\underbrace{op_t(x1, rd(x2, x1))}, x2, x1)}, mult(x2, i(x1)))$

$=$ by Lemma 551 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$mult(\underbrace{op_l(\overbrace{op_t(x1, mult(x2, i(x1)))}, x2, x1)}, mult(x2, i(x1)))$

$=$ by Axiom 15 RL with $\{x4 \leftarrow mult(x2, i(x1)), x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{mult(\overbrace{op_t(op_l(x1, x2, x1), mult(x2, i(x1)))}, mult(x2, i(x1)))}$

$=$ by Lemma 636 RL with $\{x2 \leftarrow mult(x2, i(x1)), x1 \leftarrow op_l(x1, x2, x1)\}$

$\overbrace{\underbrace{op_l(mult(op_l(x1, x2, x1), mult(x2, i(x1))), mult(x2, i(x1)), op_l(x1, x2, x1))}}$

$=$ by Lemma 1195 LR with $\{x2 \leftarrow mult(x2, i(x1)), x1 \leftarrow op_l(x1, x2, x1)\}$

$\overbrace{rd(\underbrace{mult(op_l(x1, x2, x1), mult(x2, i(x1)))}, rd(op_l(x1, x2, x1), op_t(op_l(x1, x2, x1), mult(x2, i(x1)))))}$

$=$ by Lemma 979 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$rd(\overbrace{op_l(x2, x2, x1)}, rd(op_l(x1, x2, x1), \underbrace{op_t(op_l(x1, x2, x1), mult(x2, i(x1)))}))$

$=$ by Axiom 15 LR with $\{x4 \leftarrow mult(x2, i(x1)), x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x1\}$

$rd(op_l(x2, x2, x1), rd(op_l(x1, x2, x1), \overbrace{op_l(\underbrace{op_t(x1, mult(x2, i(x1)))}, x2, x1)}))$

$=$ by Lemma 551 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$rd(op_l(x2, x2, x1), rd(op_l(x1, x2, x1), \underbrace{op_l(\overbrace{op_t(x1, rd(x2, x1))}, x2, x1)}))$

$=$ by Lemma 455 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$rd(op_l(x2, x2, x1), \underbrace{rd(op_l(x1, x2, x1), \overbrace{op_t(x1, x2)})})$

$=$ by Lemma 795 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$rd(op_l(x2, x2, x1), \overbrace{asoc(x1, x2, x2)})$

Lemma 1516: $rd(x1, op_t(i(x2), x1)) = op_r(mult(x2, x1), x1, x2)$

$\overbrace{rd(x1, op_t(i(x2), x1))}$

$=$ by Lemma 424 RL with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$\overbrace{mult(op_l(x2, x2, x1), \overbrace{x1})}$

$=$ by Lemma 3 RL with $\{x1 \leftarrow x1\}$

$\overbrace{mult(op_l(x2, x2, x1), \overbrace{i(i(x1))})}$

$=$ by Lemma 1486 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_t(mult(x1, x2), \overbrace{i(op_l(x2, x2, x1))})}$

$=$ by Lemma 430 LR with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$op_t(mult(x1, x2), \overbrace{op_l(i(x2), x2, x1)})$

$=$ by Lemma 633 RL with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$op_t(mult(x1, x2), \overbrace{op_l(i(x2), x1, i(x2))})$

$=$ by Lemma 1050 RL with $\{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow i(x2)\}$

$op_t(\overbrace{mult(x1, x2)}, \overbrace{op_l(i(x2), i(x1), x2)})$

$=$ by Axiom 5 LR with $\{x2 \leftarrow mult(x1, x2), x1 \leftarrow mult(x1, x2)\}$

$op_t(\overbrace{rd(\overbrace{mult(mult(x1, x2), mult(x1, x2))}, mult(x1, x2))}, op_l(i(x2), i(x1), x2))$

$=$ by Axiom 4 LR with $\{x2 \leftarrow x2, x1 \leftarrow mult(mult(x1, x2), mult(x1, x2))\}$

$op_t(\overbrace{rd(\overbrace{mult(rd(mult(mult(x1, x2), mult(x1, x2)), x2), x2)}, mult(x1, x2))}, op_l(i(x2), i(x1), x2))$

$=$ by Lemma 999 RL with $\{x2 \leftarrow x2, x1 \leftarrow mult(x1, x2)\}$

$op_t(\overbrace{op_r(op_r(mult(x1, x2), x2, x2), \underbrace{rd(mult(x1, x2), x2)}, x2)}, op_l(i(x2), i(x1), x2))$

$=$ by Axiom 5 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$op_t(op_r(\underbrace{op_r(mult(x1, x2), x2, x2)}, \overbrace{x1}, x2), op_l(i(x2), i(x1), x2))$

$=$ by Lemma 359 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{op_t(op_r(\overbrace{rd(x2, i(x1))}, x1, x2), op_l(i(x2), i(x1), x2))}$

$=$ by Axiom 14 LR with $\{x4 \leftarrow op_l(i(x2), i(x1), x2), x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow rd(x2, i(x1))\}$

$\overbrace{op_r(\underbrace{op_t(rd(x2, i(x1)), op_l(i(x2), i(x1), x2))}, x1, x2)}$

$=$ by Lemma 969 RL with $\{x2 \leftarrow i(x1), x1 \leftarrow x2\}$

$op_r(\overbrace{mult(x2, \underbrace{i(i(x1))})}, x1, x2)$

$=$ by Lemma 3 LR with $\{x1 \leftarrow x1\}$

$op_r(mult(x2, \overbrace{x1}), x1, x2)$

Lemma 1517: $op_l(x1, op_t(op_t(x2, x3), x4), i(x3)) = op_l(x1, x3, op_t(x2, x4))$

$$
\begin{aligned}
& \underbrace{op_l(x1, op_t(op_t(x2, x3), x4), i(x3))} \\
= \quad & \text{by Lemma 1330 RL with } \{x3 \leftarrow op_t(op_t(x2, x3), x4), x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x3, \underbrace{mult(x3, op_t(op_t(x2, x3), x4))})} \\
= \quad & \text{by Lemma 50 LR with } \{x3 \leftarrow x4, x2 \leftarrow x2, x1 \leftarrow x3\} \\
& \underbrace{op_l(x1, x3, \overbrace{mult(op_t(x2, x4), x3)})} \\
= \quad & \text{by Lemma 291 LR with } \{x3 \leftarrow op_t(x2, x4), x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x3, op_t(x2, x4))}
\end{aligned}
$$

Lemma 1518: $mult(rd(op_t(x1, x2), x3), op_t(x2, x1)) = mult(rd(x1, x3), x2)$

$$
\begin{aligned}
& mult(\underbrace{rd(op_t(x1, x2), x3)}, op_t(x2, x1)) \\
= \quad & \text{by Lemma 10 RL with } \{x2 \leftarrow op_t(x1, x2), x1 \leftarrow x3\} \\
& \underbrace{mult(\overbrace{i(rd(x3, op_t(x1, x2)))}, op_t(x2, x1))} \\
= \quad & \text{by Lemma 1261 RL with } \{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow i(rd(x3, op_t(x1, x2)))\} \\
& \overbrace{mult(rd(i(rd(x3, op_t(x1, x2))), \underbrace{rd(x2, op_t(x2, x1))}), x2)} \\
= \quad & \text{by Lemma 395 RL with } \{x1 \leftarrow x2, x2 \leftarrow x1\} \\
& mult(\underbrace{rd(i(rd(x3, op_t(x1, x2))), \overbrace{rd(op_t(x1, x2), x1)})}, x2) \\
= \quad & \text{by Lemma 37 LR with } \{x2 \leftarrow rd(op_t(x1, x2), x1), x1 \leftarrow rd(x3, op_t(x1, x2))\} \\
& mult(\overbrace{rd(\underbrace{i(rd(op_t(x1, x2), x1))}, rd(x3, op_t(x1, x2)))}, x2) \\
= \quad & \text{by Lemma 10 LR with } \{x2 \leftarrow x1, x1 \leftarrow op_t(x1, x2)\} \\
& mult(\underbrace{rd(\overbrace{rd(x1, op_t(x1, x2))}, rd(x3, op_t(x1, x2)))}, x2) \\
= \quad & \text{by Lemma 1236 RL with } \{x3 \leftarrow x2, x2 \leftarrow rd(x3, op_t(x1, x2)), x1 \leftarrow x1\} \\
& mult(\overbrace{rd(x1, \underbrace{mult(rd(x3, op_t(x1, x2)), op_t(x1, x2))})}, x2) \\
= \quad & \text{by Axiom 4 RL with } \{x2 \leftarrow op_t(x1, x2), x1 \leftarrow x3\} \\
& mult(rd(x1, \overbrace{x3}), x2)
\end{aligned}
$$

Lemma 1519: $mult(rd(x1, op_t(op_t(x2, x1), x1)), x2) = op_t(op_t(x1, x2), x2)$

$$
\begin{aligned}
& mult(rd(x1, op_t(\underbrace{op_t(x2, x1)}, x1)), x2) \\
= & \quad \text{by Lemma 374 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& mult(rd(x1, \underbrace{op_t(\overbrace{op_t(x2, op_t(x1, x2))}, x1)}), x2) \\
= & \quad \text{by Axiom 13 LR with } \{x3 \leftarrow x1, x2 \leftarrow op_t(x1, x2), x1 \leftarrow x2\} \\
& \underbrace{mult(rd(x1, \overbrace{op_t(op_t(x2, x1), op_t(x1, x2))}), x2)} \\
= & \quad \text{by Lemma 1518 RL with } \{x3 \leftarrow op_t(op_t(x2, x1), op_t(x1, x2)), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{mult(rd(op_t(x1, x2), op_t(op_t(x2, x1), op_t(x1, x2))), op_t(x2, x1))}} \\
= & \quad \text{by Lemma 597 LR with } \{x2 \leftarrow op_t(x2, x1), x1 \leftarrow op_t(x1, x2)\} \\
& \overbrace{\underbrace{op_t(op_t(x1, x2), op_t(x2, x1))}} \\
= & \quad \text{by Axiom 13 RL with } \{x3 \leftarrow x2, x2 \leftarrow op_t(x2, x1), x1 \leftarrow x1\} \\
& \overbrace{op_t(\underbrace{op_t(x1, op_t(x2, x1))}, x2)} \\
= & \quad \text{by Lemma 374 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_t(\overbrace{op_t(x1, x2)}, x2)
\end{aligned}
$$

Lemma 1520: $op_l(x1, x1, rd(x3, x2)) = op_l(x1, x1, rd(op_t(x3, i(x1)), x2))$

$$
\begin{aligned}
& op_l(x1, x1, \underbrace{rd(x3, x2)}) \\
= & \quad \text{by Lemma 23 RL with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& \underbrace{op_l(x1, x1, \overbrace{rd(i(x2), i(x3))})} \\
= & \quad \text{by Lemma 1397 RL with } \{x3 \leftarrow i(x3), x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x1, \underbrace{rd(i(x2), op_t(i(x3), x1))})} \\
= & \quad \text{by Lemma 68 LR with } \{x3 \leftarrow x1, x2 \leftarrow x3, x1 \leftarrow x2\} \\
& op_l(x1, x1, \overbrace{rd(op_t(x3, i(x1)), x2)})
\end{aligned}
$$

Lemma 1521: $op_l(x1, mult(x1, mult(x1, x2)), i(x2)) = mult(rd(i(mult(x2, x1)), mult(x2, x1)), rd(mult(x2, x1), rd(rd(i(mult(x2$

$\overbrace{op_l(x1, mult(x1, mult(x1, x2)), i(x2))}$

$=$ by Lemma 1330 RL with $\{x3 \leftarrow mult(x1, mult(x1, x2)), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_l(x1, x2, \underbrace{mult(x2, mult(x1, mult(x1, x2)))})}$

$=$ by Lemma 530 RL with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$op_l(x1, x2, \overbrace{\underbrace{mult(mult(x2, x1), mult(x2, x1))}})$

$=$ by Lemma 42 RL with $\{x1 \leftarrow mult(x2, x1)\}$

$op_l(x1, x2, \overbrace{\underbrace{rd(mult(x2, x1), i(mult(x2, x1)))}})$

$=$ by Lemma 10 RL with $\{x2 \leftarrow mult(x2, x1), x1 \leftarrow i(mult(x2, x1))\}$

$\overbrace{op_l(x1, x2, \underbrace{i(rd(i(mult(x2, x1)), mult(x2, x1)))})}$

$=$ by Lemma 987 RL with $\{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow rd(i(mult(x2, x1)), mult(x2, x1))\}$

$\overbrace{mult(rd(i(mult(x2, x1)), mult(x2, x1)), mult(rd(i(x2), \underbrace{rd(i(mult(x2, x1)), mult(x2, x1))}), mult(x2, x1)))}$

$=$ by Lemma 43 RL with $\{x1 \leftarrow mult(x2, x1)\}$

$mult(rd(i(mult(x2, x1)), mult(x2, x1)), mult(rd(i(x2), \underbrace{i(mult(mult(x2, x1), mult(x2, x1)))}), mult(x2, x1)))$

$=$ by Axiom 11 RL with $\{x2 \leftarrow mult(x2, x1), x1 \leftarrow mult(x2, x1)\}$

$mult(rd(i(mult(x2, x1)), mult(x2, x1)), \underbrace{mult(rd(i(x2), \overbrace{mult(i(mult(x2, x1)), i(mult(x2, x1)))}), mult(x2, x1))})$

$=$ by Lemma 1411 LR with $\{x1 \leftarrow mult(x2, x1), x2 \leftarrow i(x2)\}$

$mult(rd(i(mult(x2, x1)), mult(x2, x1)), \overbrace{i(rd(i(mult(x2, x1)), rd(rd(i(x2), i(mult(x2, x1))), i(mult(x2, x1)))))})$

$=$ by Lemma 10 LR with $\{x2 \leftarrow rd(rd(i(x2), i(mult(x2, x1))), i(mult(x2, x1))), x1 \leftarrow i(mult(x2, x1))\}$

$mult(rd(i(mult(x2, x1)), mult(x2, x1)), \overbrace{\underbrace{rd(rd(rd(i(x2), i(mult(x2, x1))), i(mult(x2, x1))), i(mult(x2, x1)))}})$

$=$ by Lemma 39 LR with $\{x3 \leftarrow mult(x2, x1), x2 \leftarrow i(mult(x2, x1)), x1 \leftarrow rd(i(x2), i(mult(x2, x1)))\}$

$mult(rd(i(mult(x2, x1)), mult(x2, x1)), \overbrace{rd(mult(x2, x1), \underbrace{rd(i(mult(x2, x1)), rd(i(x2), i(mult(x2, x1))))})})$

$=$ by Lemma 40 LR with $\{x3 \leftarrow i(mult(x2, x1)), x2 \leftarrow i(x2), x1 \leftarrow mult(x2, x1)\}$

$mult(rd(i(mult(x2, x1)), mult(x2, x1)), rd(mult(x2, x1), \overbrace{rd(rd(i(mult(x2, x1)), i(x2)), mult(x2, x1))}))$

Lemma 1522: $op_l(x1, mult(x1, mult(x1, x2)), i(x2)) = op_t(mult(i(mult(x2, x1)), rd(mult(x2, x1), rd(x2, mult(x2, x1)))), mult(x$

$\overbrace{op_l(x1, mult(x1, mult(x1, x2)), i(x2))}$

$=$ by Lemma 1521 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{mult(\underbrace{rd(i(mult(x2, x1)), mult(x2, x1))}, rd(mult(x2, x1), rd(rd(i(mult(x2, x1)), i(x2)), mult(x2, x1))))}$

$=$ by Lemma 43 RL with $\{x1 \leftarrow mult(x2, x1)\}$

$mult(\overbrace{i(mult(mult(x2, x1), mult(x2, x1)))}, rd(mult(x2, x1), rd(rd(i(mult(x2, x1)), i(x2)), mult(x2, x1))))$

$=$ by Axiom 11 RL with $\{x2 \leftarrow mult(x2, x1), x1 \leftarrow mult(x2, x1)\}$

$\underbrace{mult(\overbrace{mult(i(mult(x2, x1)), i(mult(x2, x1)))}, rd(mult(x2, x1), rd(rd(i(mult(x2, x1)), i(x2)), mult(x2, x1))))}$

$=$ by Lemma 24 LR with $\{x2 \leftarrow rd(mult(x2, x1), rd(rd(i(mult(x2, x1)), i(x2)), mult(x2, x1))), x1 \leftarrow i(mult(x2, x1))\}$

$\overbrace{mult(i(mult(x2, x1)), mult(i(mult(x2, x1)), \underbrace{rd(mult(x2, x1), rd(rd(i(mult(x2, x1)), i(x2)), mult(x2, x1)))}))}$

$=$ by Lemma 23 RL with $\{x2 \leftarrow mult(x2, x1), x1 \leftarrow rd(rd(i(mult(x2, x1)), i(x2)), mult(x2, x1))\}$

$\underbrace{mult(i(mult(x2, x1)), mult(i(mult(x2, x1)), \overbrace{rd(i(rd(rd(i(mult(x2, x1)), i(x2)), mult(x2, x1))), i(mult(x2, x1)))}))}$

$=$ by Lemma 70 LR with $\{x2 \leftarrow i(rd(rd(i(mult(x2, x1)), i(x2)), mult(x2, x1))), x1 \leftarrow i(mult(x2, x1))\}$

$\overbrace{op_t(\underbrace{mult(i(mult(x2, x1)), i(rd(rd(i(mult(x2, x1)), i(x2)), mult(x2, x1))))}, i(i(mult(x2, x1))))}$

$=$ by Axiom 11 LR with $\{x2 \leftarrow rd(rd(i(mult(x2, x1)), i(x2)), mult(x2, x1)), x1 \leftarrow mult(x2, x1)\}$

$\underbrace{op_t(\overbrace{i(mult(mult(x2, x1), rd(rd(i(mult(x2, x1)), i(x2)), mult(x2, x1))))}, i(i(mult(x2, x1))))}$

$=$ by Lemma 15 LR with $\{x2 \leftarrow i(mult(x2, x1)), x1 \leftarrow mult(mult(x2, x1), rd(rd(i(mult(x2, x1)), i(x2)), mult(x2, x1)))\}$

$\overbrace{i(op_t(mult(mult(x2, x1), rd(rd(i(mult(x2, x1)), i(x2)), mult(x2, x1))), i(mult(x2, x1))))}$

$=$ by Lemma 19 LR with $\{x2 \leftarrow mult(x2, x1), x1 \leftarrow mult(mult(x2, x1), rd(rd(i(mult(x2, x1)), i(x2)), mult(x2, x1)))\}$

$\overbrace{op_t(\underbrace{i(mult(mult(x2, x1), rd(rd(i(mult(x2, x1)), i(x2)), mult(x2, x1))))}, mult(x2, x1))}$

$=$ by Lemma 45 LR with $\{x3 \leftarrow mult(x2, x1), x2 \leftarrow rd(i(mult(x2, x1)), i(x2)), x1 \leftarrow mult(x2, x1)\}$

$op_t(\overbrace{mult(i(mult(x2, x1)), rd(mult(x2, x1), \underbrace{rd(i(mult(x2, x1)), i(x2))}))}, mult(x2, x1))$

$=$ by Lemma 23 LR with $\{x2 \leftarrow x2, x1 \leftarrow mult(x2, x1)\}$

$op_t(mult(i(mult(x2, x1)), rd(mult(x2, x1), \overbrace{rd(x2, mult(x2, x1))})), mult(x2, x1))$

Lemma 1523: $op_l(x1, mult(x1, mult(x1, x2)), i(x2)) = op_t(rd(mult(x2, x1), op_t(x2, rd(x1, x2))), mult(x2, x1))$

$\overbrace{op_l(x1, mult(x1, mult(x1, x2)), i(x2))}$

$=$ by Lemma 1522 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_t(\underbrace{mult(i(mult(x2, x1)), rd(mult(x2, x1), rd(x2, mult(x2, x1))))}, mult(x2, x1))}$

$=$ by Lemma 923 LR with $\{x2 \leftarrow x2, x1 \leftarrow mult(x2, x1)\}$

$op_t(\overbrace{rd(mult(x2, x1), \underbrace{op_t(x2, i(mult(x2, x1)))})}, mult(x2, x1))$

$=$ by Lemma 708 LR with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$op_t(rd(mult(x2, x1), \overbrace{op_t(x2, rd(x1, x2))}), mult(x2, x1))$

Lemma 1524: $mult(x1, op_l(x1, mult(x1, mult(x1, i(x2))), x2)) = rd(x1, op_l(i(x1), mult(x2, x2), i(x1)))$

$mult(x1, op_l(x1, mult(x1, \underbrace{mult(x1, i(x2))}), x2))$

$=$ by Lemma 16 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$mult(x1, \underbrace{op_l(x1, mult(x1, \overbrace{i(mult(i(x1), x2))}), x2)})$

$=$ by Lemma 1135 RL with $\{x4 \leftarrow x2, x3 \leftarrow mult(i(x1), x2), x2 \leftarrow x1, x1 \leftarrow x1\}$

$\underbrace{mult(x1, \overbrace{op_l(x1, mult(i(x1), mult(i(x1), x2)), i(x2))})}$

$=$ by Lemma 1057 RL with $\{x3 \leftarrow i(x2), x2 \leftarrow mult(i(x1), mult(i(x1), x2)), x1 \leftarrow x1\}$

$\overbrace{rd(x1, \underbrace{op_l(i(x1), mult(i(x1), mult(i(x1), x2)), i(x2))})}$

$=$ by Lemma 1523 LR with $\{x2 \leftarrow x2, x1 \leftarrow i(x1)\}$

$rd(x1, \overbrace{op_t(\underbrace{rd(mult(x2, i(x1)), op_t(x2, rd(i(x1), x2)))}, mult(x2, i(x1)))})$

$=$ by Lemma 985 LR with $\{x2 \leftarrow i(x1), x1 \leftarrow x2\}$

$rd(x1, \underbrace{op_t(\overbrace{op_r(i(x1), x2, x2)}, mult(x2, i(x1)))})$

$=$ by Axiom 14 LR with $\{x4 \leftarrow mult(x2, i(x1)), x3 \leftarrow x2, x2 \leftarrow x2, x1 \leftarrow i(x1)\}$

$rd(x1, \overbrace{op_r(\underbrace{op_t(i(x1), mult(x2, i(x1)))}, x2, x2)})$

$=$ by Lemma 455 RL with $\{x2 \leftarrow mult(x2, i(x1)), x1 \leftarrow i(x1)\}$

$rd(x1, op_r(\overbrace{op_l(op_t(i(x1), \underbrace{rd(mult(x2, i(x1)), i(x1))}), mult(x2, i(x1)), i(x1))}, x2, x2))$

$=$ by Axiom 5 RL with $\{x2 \leftarrow i(x1), x1 \leftarrow x2\}$

$rd(x1, op_r(\underbrace{op_l(op_t(i(x1), \overbrace{x2}), mult(x2, i(x1)), i(x1))}, x2, x2))$

$=$ by Lemma 285 LR with $\{x3 \leftarrow i(x1), x2 \leftarrow x2, x1 \leftarrow op_t(i(x1), x2)\}$

$rd(x1, op_r(\overbrace{op_l(op_t(i(x1), x2), x2, i(x1))}, x2, x2))$

$=$ by Lemma 251 LR with $\{x2 \leftarrow x2, x1 \leftarrow i(x1)\}$

$rd(x1, \underbrace{op_r(\overbrace{op_t(i(x1), mult(i(x1), x2))}, x2, x2)})$

$=$ by Lemma 1170 LR with $\{x2 \leftarrow x2, x1 \leftarrow i(x1)\}$

$rd(x1, \overbrace{op_l(i(x1), mult(x2, x2), i(x1))})$

Lemma 1525: $mult(x1, op_l(x1, mult(x1, x1), x2)) = rd(x1, op_l(i(x1), mult(x2, x2), i(x1)))$

$mult(x1, \underbrace{op_l(x1, mult(x1, x1), x2)})$

$=$ by Lemma 1323 RL with $\{x3 \leftarrow x2, x2 \leftarrow mult(x1, x1), x1 \leftarrow x1\}$

$mult(x1, \overbrace{op_l(x1, \underbrace{mult(mult(x1, x1), i(x2))}, x2)})$

$=$ by Lemma 24 LR with $\{x2 \leftarrow i(x2), x1 \leftarrow x1\}$

$\underbrace{mult(x1, op_l(x1, \overbrace{mult(x1, mult(x1, i(x2)))}, x2))}$

$=$ by Lemma 1524 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{rd(x1, \underbrace{op_l(i(x1), mult(x2, x2), i(x1))})}$

Lemma 1526: $mult(x1, op_l(x1, mult(x1, x1), x2)) = mult(x1, op_l(x1, x1, mult(x2, x2)))$

$$
\begin{aligned}
& \overbrace{mult(x1, op_l(x1, mult(x1, x1), x2))} \\
= \quad & \text{by Lemma 1525 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{rd(x1, \underbrace{op_l(i(x1), mult(x2, x2), i(x1))})} \\
= \quad & \text{by Lemma 1057 LR with } \{x3 \leftarrow i(x1), x2 \leftarrow mult(x2, x2), x1 \leftarrow x1\} \\
& \overbrace{mult(x1, \underbrace{op_l(x1, mult(x2, x2), i(x1))})} \\
= \quad & \text{by Lemma 606 LR with } \{x2 \leftarrow mult(x2, x2), x1 \leftarrow x1\} \\
& mult(x1, \overbrace{op_l(x1, x1, mult(x2, x2))})
\end{aligned}
$$

Lemma 1527: $mult(x1, op_l(x1, mult(x1, x1), x2)) = op_l(mult(x1, x1), x1, x2)$

$$
\begin{aligned}
& \overbrace{mult(x1, op_l(x1, mult(x1, x1), x2))} \\
= \quad & \text{by Lemma 1526 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{mult(x1, op_l(x1, x1, mult(x2, x2)))}} \\
= \quad & \text{by Lemma 1461 LR with } \{x1 \leftarrow x2, x2 \leftarrow x1\} \\
& \overbrace{op_l(mult(x1, x1), x1, x2)}
\end{aligned}
$$

Lemma 1528: $op_l(x1, mult(x1, x1), x2) = rd(op_l(mult(x1, x1), x1, x2), x1)$

$$
\begin{aligned}
& \underbrace{op_l(x1, mult(x1, x1), x2)} \\
= \quad & \text{by Lemma 29 RL with } \{x2 \leftarrow op_l(x1, mult(x1, x1), x2), x1 \leftarrow x1\} \\
& \overbrace{rd(\underbrace{mult(x1, op_l(x1, mult(x1, x1), x2))}, op_t(x1, op_l(x1, mult(x1, x1), x2)))} \\
= \quad & \text{by Lemma 1527 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& rd(\overbrace{op_l(mult(x1, x1), x1, x2)}, \underbrace{op_t(x1, op_l(x1, mult(x1, x1), x2))}) \\
= \quad & \text{by Lemma 169 LR with } \{x3 \leftarrow x2, x2 \leftarrow mult(x1, x1), x1 \leftarrow x1\} \\
& rd(op_l(mult(x1, x1), x1, x2), \overbrace{x1})
\end{aligned}
$$

Lemma 1529: $op_l(x1, mult(x1, x1), i(x2)) = op_l(x1, x2, mult(x1, x1))$

$\overbrace{op_l(x1, mult(x1, x1), i(x2))}$

$=$ by Lemma 1528 LR with $\{x2 \leftarrow i(x2), x1 \leftarrow x1\}$

$\overbrace{rd(\underbrace{op_l(mult(x1, x1), x1, i(x2))}, x1)}$

$=$ by Lemma 1331 RL with $\{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow mult(x1, x1)\}$

$rd(\overbrace{op_l(mult(x1, x1), \underbrace{mult(x1, x2)}, x1)}, x1)$

$=$ by Lemma 1329 RL with $\{x3 \leftarrow mult(x1, x2), x2 \leftarrow x1, x1 \leftarrow mult(x1, x1)\}$

$rd(\overbrace{op_l(mult(x1, x1), x1, \underbrace{i(op_t(mult(x1, x2), x1))})}, x1)$

$=$ by Lemma 1388 RL with $\{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow mult(x1, x1)\}$

$\underbrace{rd(\overbrace{op_l(mult(x1, x1), x1, i(\underbrace{op_t(x2, mult(x1, x1))}))}, x1)}$

$=$ by Lemma 1528 RL with $\{x2 \leftarrow i(op_t(x2, mult(x1, x1))), x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_l(x1, mult(x1, x1), i(op_t(x2, mult(x1, x1))))}}$

$=$ by Lemma 1329 LR with $\{x3 \leftarrow x2, x2 \leftarrow mult(x1, x1), x1 \leftarrow x1\}$

$\overbrace{op_l(x1, x2, mult(x1, x1))}$

Lemma 1530: $op_l(mult(x1, x1), mult(x1, mult(x1, x2)), x1) = mult(x1, op_l(x1, x2, mult(x1, x1)))$

$op_l(mult(x1, x1), \underbrace{mult(x1, mult(x1, x2))}, x1)$

$=$ by Lemma 24 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{op_l(mult(x1, x1), \overbrace{mult(mult(x1, x1), x2)}, x1)}$

$=$ by Lemma 1463 RL with $\{x2 \leftarrow mult(mult(x1, x1), x2), x1 \leftarrow x1\}$

$\underbrace{\overbrace{op_l(mult(x1, x1), x1, i(\underbrace{mult(mult(x1, x1), x2)}))}}$

$=$ by Lemma 1527 RL with $\{x2 \leftarrow i(mult(mult(x1, x1), x2)), x1 \leftarrow x1\}$

$\overbrace{mult(x1, \underbrace{op_l(x1, mult(x1, x1), i(\underbrace{mult(mult(x1, x1), x2)}))})}$

$=$ by Lemma 1051 LR with $\{x3 \leftarrow x2, x2 \leftarrow mult(x1, x1), x1 \leftarrow x1\}$

$mult(x1, \overbrace{op_l(x1, x2, mult(x1, x1))})$

Lemma 1531: $op_l(mult(x1, x1), x1, i(x2)) = mult(x1, op_l(x1, x2, mult(x1, x1)))$

$\underbrace{op_l(mult(x1, x1), x1, i(x2))}$

$=$ by Lemma 1331 RL with $\{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow mult(x1, x1)\}$

$\underbrace{\overbrace{op_l(mult(x1, x1), mult(x1, x2), x1)}}$

$=$ by Lemma 1463 RL with $\{x2 \leftarrow mult(x1, x2), x1 \leftarrow x1\}$

$\underbrace{\overbrace{op_l(mult(x1, x1), x1, i(mult(x1, x2)))}}$

$=$ by Lemma 1331 RL with $\{x3 \leftarrow mult(x1, x2), x2 \leftarrow x1, x1 \leftarrow mult(x1, x1)\}$

$\overbrace{\underbrace{op_l(mult(x1, x1), mult(x1, mult(x1, x2)), x1)}}$

$=$ by Lemma 1530 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{mult(x1, op_l(x1, x2, mult(x1, x1)))}$

Lemma 1532: $op_l(x1, mult(x1, x1), mult(x2, x1)) = op_l(x1, mult(x1, x1), x2)$

$$
\begin{aligned}
& \overbrace{op_l(x1, mult(x1, x1), mult(x2, x1))} \\
= \quad & \text{by Lemma 1528 LR with } \{x2 \leftarrow mult(x2, x1), x1 \leftarrow x1\} \\
& \overbrace{rd(\underbrace{op_l(mult(x1, x1), x1, mult(x2, x1))}, x1)} \\
= \quad & \text{by Lemma 291 LR with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow mult(x1, x1)\} \\
& \underbrace{rd(\overbrace{op_l(mult(x1, x1), x1, x2)}, x1)} \\
= \quad & \text{by Lemma 1528 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, mult(x1, x1), x2)}
\end{aligned}
$$

Lemma 1533: $op_l(x1, mult(x1, x1), mult(mult(i(x2), x1), x1)) = op_l(x1, mult(x1, x1), mult(rd(x1, x2), mult(x1, x1)))$

$$
\begin{aligned}
& \underbrace{op_l(x1, mult(x1, x1), mult(mult(i(x2), x1), x1))} \\
= \quad & \text{by Lemma 1532 RL with } \{x2 \leftarrow mult(mult(i(x2), x1), x1), x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, mult(x1, x1), \underbrace{mult(mult(mult(i(x2), x1), x1), x1)})} \\
= \quad & \text{by Lemma 1101 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& op_l(x1, mult(x1, x1), \overbrace{op_t(mult(x1, mult(rd(x1, x2), x1)), x1)}) \\
= \quad & \text{by Lemma 200 LR with } \{x2 \leftarrow rd(x1, x2), x1 \leftarrow x1\} \\
& op_l(x1, mult(x1, x1), \overbrace{mult(rd(x1, x2), mult(x1, x1))})
\end{aligned}
$$

Lemma 1534: $op_l(x1, mult(x1, x1), i(x2)) = op_l(x1, mult(x1, x1), mult(rd(x1, x2), mult(x1, x1)))$

$$
\begin{aligned}
& \underbrace{op_l(x1, mult(x1, x1), i(x2))} \\
= \quad & \text{by Lemma 1532 RL with } \{x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, mult(x1, x1), \underbrace{mult(i(x2), x1)})} \\
= \quad & \text{by Lemma 1532 RL with } \{x2 \leftarrow mult(i(x2), x1), x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, mult(x1, x1), \underbrace{mult(mult(i(x2), x1), x1)})} \\
= \quad & \text{by Lemma 1533 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, mult(x1, x1), mult(rd(x1, x2), mult(x1, x1)))}
\end{aligned}
$$

Lemma 1535: $op_l(x1, x2, mult(x1, x1)) = op_l(x1, mult(x1, x1), rd(x1, x2))$

$$
\begin{aligned}
& \underbrace{op_l(x1, x2, mult(x1, x1))} \\
= \quad & \text{by Lemma 1529 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, mult(x1, x1), i(x2))}} \\
= \quad & \text{by Lemma 1534 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, mult(x1, x1), mult(rd(x1, x2), mult(x1, x1)))}} \\
= \quad & \text{by Lemma 291 LR with } \{x3 \leftarrow rd(x1, x2), x2 \leftarrow mult(x1, x1), x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, mult(x1, x1), rd(x1, x2))}
\end{aligned}
$$

Lemma 1536: $mult(op_l(x1,x1,x2),op_l(x1,x1,x2)) = rd(op_l(op_l(x1,x1,x2),op_t(x2,x1),op_l(x1,x1,x2)),op_l(i(op_l(x1,x1,x2)),x1$

$\underbrace{mult(op_l(x1,x1,x2),op_l(x1,x1,x2))}$

$=$ by Lemma 42 RL with $\{x1 \leftarrow op_l(x1,x1,x2)\}$

$\overbrace{rd(op_l(x1,x1,x2),\underbrace{i(op_l(x1,x1,x2))})}$

$=$ by Lemma 797 RL with $\{x2 \leftarrow op_l(x1,x1,x2), x1 \leftarrow op_t(x2,x1)\}$

$\underbrace{rd(op_l(x1,x1,x2),\overbrace{rd(asoc(op_t(x2,x1),op_l(x1,x1,x2),op_l(x1,x1,x2)),op_l(op_l(x1,x1,x2),op_t(x2,x1),op_l(x1,x1,x2))))}$

$=$ by Lemma 39 RL with $\{x3 \leftarrow op_l(x1,x1,x2), x2 \leftarrow asoc(op_t(x2,x1),op_l(x1,x1,x2),op_l(x1,x1,x2)), x1 \leftarrow op_l(op_l(x1,x1,x2)$

$\overbrace{rd(rd(op_l(op_l(x1,x1,x2),op_t(x2,x1),op_l(x1,x1,x2)),asoc(op_t(x2,x1),op_l(x1,x1,x2),op_l(x1,x1,x2))),\underbrace{i(op_l(x1,x1,x2))})}$

$=$ by Lemma 797 RL with $\{x2 \leftarrow op_l(x1,x1,x2), x1 \leftarrow op_t(x2,x1)\}$

$\underbrace{rd(rd(op_l(op_l(x1,x1,x2),op_t(x2,x1),op_l(x1,x1,x2)),asoc(op_t(x2,x1),op_l(x1,x1,x2),op_l(x1,x1,x2))),\overbrace{rd(asoc(op_t(x2,x1$

$=$ by Lemma 40 RL with $\{x3 \leftarrow op_l(op_l(x1,x1,x2),op_t(x2,x1),op_l(x1,x1,x2)), x2 \leftarrow asoc(op_t(x2,x1),op_l(x1,x1,x2),op_l(x1,$

$\overbrace{rd(i(rd(asoc(op_t(x2,x1),op_l(x1,x1,x2),op_l(x1,x1,x2)),op_l(op_l(x1,x1,x2),op_t(x2,x1),op_l(x1,x1,x2)))),rd(asoc(op_t(x2,$

$=$ by Lemma 49 LR with $\{x2 \leftarrow op_l(op_l(x1,x1,x2),op_t(x2,x1),op_l(x1,x1,x2)), x1 \leftarrow rd(asoc(op_t(x2,x1),op_l(x1,x1,x2),op_l(x$

$\overbrace{rd(op_l(op_l(x1,x1,x2),op_t(x2,x1),op_l(x1,x1,x2)),mult(rd(asoc(op_t(x2,x1),op_l(x1,x1,x2),op_l(x1,x1,x2)),op_l(op_l(x1,x1$

$=$ by Axiom 4 RL with $\{x2 \leftarrow op_l(op_l(x1,x1,x2),op_t(x2,x1),op_l(x1,x1,x2)), x1 \leftarrow asoc(op_t(x2,x1),op_l(x1,x1,x2),op_l(x1,x1$

$rd(op_l(op_l(x1,x1,x2),op_t(x2,x1),op_l(x1,x1,x2)),mult(\underbrace{rd(asoc(op_t(x2,x1),op_l(x1,x1,x2),op_l(x1,x1,x2)),op_l(op_l(x1,x1$

$=$ by Lemma 797 LR with $\{x2 \leftarrow op_l(x1,x1,x2), x1 \leftarrow op_t(x2,x1)\}$

$rd(op_l(op_l(x1,x1,x2),op_t(x2,x1),op_l(x1,x1,x2)),\underbrace{mult(\overbrace{i(op_l(x1,x1,x2))},asoc(op_t(x2,x1),op_l(x1,x1,x2),op_l(x1,x1,x2))}$

$=$ by Lemma 783 RL with $\{x2 \leftarrow op_t(x2,x1), x1 \leftarrow op_l(x1,x1,x2)\}$

$rd(op_l(op_l(x1,x1,x2),op_t(x2,x1),op_l(x1,x1,x2)),\overbrace{op_l(i(op_l(x1,x1,x2)),\underbrace{i(op_t(x2,x1)),op_l(x1,x1,x2)})})$

$=$ by Lemma 634 LR with $\{x2 \leftarrow op_t(x2,x1), x1 \leftarrow op_l(x1,x1,x2)\}$

$rd(op_l(op_l(x1,x1,x2),op_t(x2,x1),op_l(x1,x1,x2)),\overbrace{op_l(i(op_l(x1,x1,x2)),op_l(x1,x1,x2),op_t(x2,x1))})$

$=$ by Lemma 1359 LR with $\{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow i(op_l(x1,x1,x2))\}$

$rd(op_l(op_l(x1,x1,x2),op_t(x2,x1),op_l(x1,x1,x2)),\overbrace{op_l(i(op_l(x1,x1,x2)),x1,x2)})$

Lemma 1537: $mult(op_l(x1,x1,x2),op_l(x1,x1,x2)) = op_l(mult(x1,x1),x1,x2)$

$\underbrace{mult(op_l(x1,x1,x2),op_l(x1,x1,x2))}$

$=$ by Lemma 1536 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{rd(op_l(op_l(x1,x1,x2),\underbrace{op_t(x2,x1)},op_l(x1,x1,x2)),op_l(i(op_l(x1,x1,x2)),x1,x2))}$

$=$ by Lemma 553 LR with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$rd(\underbrace{op_l(op_l(x1,x1,x2),\overbrace{op_t(x2,op_l(x1,x1,x2))},op_l(x1,x1,x2))},op_l(i(op_l(x1,x1,x2)),x1,x2))$

$=$ by Lemma 1319 LR with $\{x3 \leftarrow op_l(x1,x1,x2), x2 \leftarrow x2, x1 \leftarrow op_l(x1,x1,x2)\}$

$rd(\overbrace{\underbrace{op_l(op_l(x1,x1,x2),op_l(x1,x1,x2),i(x2))}},op_l(i(op_l(x1,x1,x2)),x1,x2))$

$=$ by Lemma 484 RL with $\{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow op_l(x1,x1,x2)\}$

$rd(\overbrace{\underbrace{op_l(op_l(x1,x1,x2),x2,x1)}},op_l(i(op_l(x1,x1,x2)),x1,x2))$

$=$ by Lemma 138 LR with $\{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x1\}$

$rd(\overbrace{x1},op_l(\underbrace{i(op_l(x1,x1,x2))},x1,x2))$

$=$ by Lemma 1049 LR with $\{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x1\}$

$rd(x1,op_l(\overbrace{\underbrace{op_l(i(x1),x1,x2)}},x1,x2))$

$=$ by Lemma 652 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$rd(x1,\underbrace{op_l(\overbrace{op_l(i(x1),x1,mult(x1,x2))},x1,x2))}$

$=$ by Lemma 1177 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{rd(x1,\overbrace{op_l(i(x1),x1,mult(x2,x2))})}$

$=$ by Lemma 1057 LR with $\{x3 \leftarrow mult(x2,x2), x2 \leftarrow x1, x1 \leftarrow x1\}$

$\overbrace{\underbrace{mult(x1,op_l(x1,x1,mult(x2,x2)))}}$

$=$ by Lemma 1461 LR with $\{x1 \leftarrow x2, x2 \leftarrow x1\}$

$\overbrace{op_l(mult(x1,x1),x1,x2)}$

Lemma 1538: $rd(mult(x1,op_t(x1,mult(x2,x1))),x2) = mult(x1,op_t(mult(x1,i(x2)),x1))$

$\underbrace{rd(mult(x1,op_t(x1,mult(x2,x1))),x2)}$

$=$ by Lemma 1152 RL with $\{x3 \leftarrow x2, x2 \leftarrow op_t(x1,mult(x2,x1)), x1 \leftarrow x1\}$

$\overbrace{mult(x1,mult(\underbrace{rd(op_t(x1,mult(x2,x1)),mult(x1,x2))},x1))}$

$=$ by Lemma 1489 RL with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$mult(x1,\underbrace{mult(\overbrace{op_r(i(x2),x1,i(x1))},x1)})$

$=$ by Lemma 318 LR with $\{x2 \leftarrow x1, x1 \leftarrow i(x2)\}$

$mult(x1,\overbrace{op_t(mult(x1,i(x2)),x1)})$

Lemma 1539: $mult(mult(i(x1), mult(x2, op_t(x2, mult(i(x1), x2)))), x1) = mult(i(x1), mult(mult(x2, i(i(x1))), x2))$

$\overbrace{mult(mult(i(x1), mult(x2, op_t(x2, mult(i(x1), x2)))), x1)}$

$=$ by Lemma 69 RL with $\{x2 \leftarrow mult(x2, op_t(x2, mult(i(x1), x2))), x1 \leftarrow x1\}$

$\overbrace{mult(i(x1), \underbrace{rd(mult(x2, op_t(x2, mult(i(x1), x2))), i(x1))})}$

$=$ by Lemma 1538 LR with $\{x2 \leftarrow i(x1), x1 \leftarrow x2\}$

$mult(i(x1), \overbrace{\underbrace{mult(x2, op_t(mult(x2, i(i(x1))), x2))}})$

$=$ by Lemma 13 LR with $\{x2 \leftarrow mult(x2, i(i(x1))), x1 \leftarrow x2\}$

$mult(i(x1), \overbrace{mult(mult(x2, i(i(x1))), x2)})$

Lemma 1540: $mult(mult(mult(x2, i(x1)), x2), x1) = mult(rd(x2, x1), mult(x2, x1))$

$mult(\underbrace{mult(mult(x2, i(x1)), x2)}, x1)$

$=$ by Lemma 871 RL with $\{x2 \leftarrow x2, x1 \leftarrow i(x1)\}$

$mult(\overbrace{\underbrace{mult(mult(i(x1), x2), op_t(x2, i(x1)))}}, x1)$

$=$ by Lemma 1265 RL with $\{x2 \leftarrow x2, x1 \leftarrow i(x1)\}$

$\underbrace{mult(\overbrace{mult(i(x1), mult(x2, op_t(x2, mult(i(x1), x2))))}, x1)}$

$=$ by Lemma 1539 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{mult(i(x1), mult(mult(x2, \underbrace{i(i(x1))}), x2))}$

$=$ by Lemma 3 LR with $\{x1 \leftarrow x1\}$

$mult(i(x1), \underbrace{mult(mult(x2, \overbrace{x1}), x2)})$

$=$ by Lemma 871 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{mult(i(x1), \overbrace{mult(mult(x1, x2), op_t(x2, x1))})}$

$=$ by Lemma 1045 LR with $\{x3 \leftarrow op_t(x2, x1), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{mult(rd(x2, x1), \underbrace{mult(x1, op_t(x2, x1))})}$

$=$ by Lemma 13 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$mult(rd(x2, x1), \overbrace{mult(x2, x1)})$

Lemma 1541: $op_r(mult(i(x1),x2),x2,x1) = rd(x2,op_t(x1,i(x2)))$

$op_r(\underbrace{mult(i(x1),x2)},x2,x1)$

$=$ by Lemma 435 RL with $\{x2 \leftarrow i(x1), x1 \leftarrow x2\}$

$\underbrace{op_r(\overbrace{op_l(mult(x2,i(x1)),x2,i(x1))},x2,x1)}$

$=$ by Axiom 18 LR with $\{x5 \leftarrow x1, x4 \leftarrow x2, x3 \leftarrow i(x1), x2 \leftarrow x2, x1 \leftarrow mult(x2,i(x1))\}$

$\overbrace{op_l(\underbrace{op_r(mult(x2,i(x1)),x2,x1)},x2,i(x1))}$

$=$ by Axiom 16 LR with $\{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow mult(x2,i(x1))\}$

$op_l(\overbrace{rd(\underbrace{mult(mult(mult(x2,i(x1)),x2),x1)},mult(x2,x1))},x2,i(x1))$

$=$ by Lemma 1540 LR with $\{x1 \leftarrow x1, x2 \leftarrow x2\}$

$op_l(\underbrace{rd(\overbrace{mult(rd(x2,x1),mult(x2,x1))},mult(x2,x1))},x2,i(x1))$

$=$ by Axiom 5 RL with $\{x2 \leftarrow mult(x2,x1), x1 \leftarrow rd(x2,x1)\}$

$op_l(\overbrace{rd(x2,\underbrace{x1})},x2,i(x1))$

$=$ by Lemma 3 RL with $\{x1 \leftarrow x1\}$

$op_l(\underbrace{rd(x2,\overbrace{i(i(x1))})},x2,i(x1))$

$=$ by Lemma 359 RL with $\{x2 \leftarrow x2, x1 \leftarrow i(x1)\}$

$\underbrace{op_l(\overbrace{op_r(mult(i(x1),x2),x2,x2)},x2,i(x1))}$

$=$ by Lemma 291 RL with $\{x3 \leftarrow i(x1), x2 \leftarrow x2, x1 \leftarrow op_r(mult(i(x1),x2),x2,x2)\}$

$\overbrace{\underbrace{op_l(op_r(mult(i(x1),x2),x2,x2),x2,mult(i(x1),x2))}}$

$=$ by Axiom 18 RL with $\{x5 \leftarrow x2, x4 \leftarrow x2, x3 \leftarrow mult(i(x1),x2), x2 \leftarrow x2, x1 \leftarrow mult(i(x1),x2)\}$

$\overbrace{op_r(\underbrace{op_l(mult(i(x1),x2),x2,mult(i(x1),x2))},x2,x2)}$

$=$ by Lemma 353 RL with $\{x2 \leftarrow op_l(mult(i(x1),x2),x2,mult(i(x1),x2)), x1 \leftarrow x2\}$

$\overbrace{rd(x2,\underbrace{rd(x2,op_l(mult(i(x1),x2),x2,mult(i(x1),x2)))})}$

$=$ by Lemma 614 LR with $\{x2 \leftarrow mult(i(x1),x2), x1 \leftarrow x2\}$

$rd(x2,\overbrace{\underbrace{mult(x2,i(mult(i(x1),x2)))})$

$=$ by Lemma 31 LR with $\{x2 \leftarrow i(x1), x1 \leftarrow x2\}$

$rd(x2,\underbrace{\overbrace{i(op_t(i(x1),x2))})$

$=$ by Lemma 18 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$rd(x2,\overbrace{op_t(x1,i(x2))})$

Lemma 1542: $rd(mult(x2, i(x1)), x2) = op_r(i(x1), i(x2), rd(x1, x2))$

$rd(mult(x2, \underbrace{i(x1)}), x2)$

$=$ by Lemma 3 RL with $\{x1 \leftarrow i(x1)\}$

$\underbrace{rd(mult(x2, \overbrace{i(i(i(x1)))}), x2)}$

$=$ by Lemma 64 RL with $\{x3 \leftarrow i(i(x1)), x2 \leftarrow x2, x1 \leftarrow x2\}$

$\overbrace{\underbrace{rd(i(x2), mult(i(x2), i(i(x1))))}}$

$=$ by Lemma 465 RL with $\{x2 \leftarrow i(x1), x1 \leftarrow i(x2)\}$

$\overbrace{mult(i(x2), op_t(\underbrace{rd(i(x1), i(x2))}, i(x1)))}$

$=$ by Lemma 10 RL with $\{x2 \leftarrow i(x1), x1 \leftarrow i(x2)\}$

$mult(\underbrace{i(x2)}, op_t(\overbrace{i(rd(i(x2), i(x1)))}, i(x1)))$

$=$ by Axiom 4 LR with $\{x2 \leftarrow i(x1), x1 \leftarrow i(x2)\}$

$\underbrace{mult(\overbrace{mult(rd(i(x2), i(x1)), i(x1))}, op_t(i(rd(i(x2), i(x1))), i(x1)))}$

$=$ by Lemma 695 LR with $\{x2 \leftarrow i(x1), x1 \leftarrow rd(i(x2), i(x1))\}$

$\overbrace{op_r(i(x1), \underbrace{rd(i(x2), i(x1))}, rd(i(x2), i(x1)))}$

$=$ by Lemma 23 LR with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$op_r(i(x1), \overbrace{rd(x1, x2)}, \underbrace{rd(i(x2), i(x1))})$

$=$ by Lemma 23 LR with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$\underbrace{op_r(i(x1), rd(x1, x2), \overbrace{rd(x1, x2)})}$

$=$ by Lemma 985 RL with $\{x2 \leftarrow i(x1), x1 \leftarrow rd(x1, x2)\}$

$\overbrace{rd(mult(rd(x1, x2), i(x1)), \underbrace{op_t(rd(x1, x2), rd(i(x1), rd(x1, x2)))})$

$=$ by Lemma 708 RL with $\{x2 \leftarrow i(x1), x1 \leftarrow rd(x1, x2)\}$

$\underbrace{rd(mult(rd(x1, x2), i(x1)), \overbrace{op_t(rd(x1, x2), i(mult(rd(x1, x2), i(x1))))})$

$=$ by Lemma 1541 RL with $\{x2 \leftarrow mult(rd(x1, x2), i(x1)), x1 \leftarrow rd(x1, x2)\}$

$\overbrace{op_r(\underbrace{mult(i(rd(x1, x2)), mult(rd(x1, x2), i(x1)))}, mult(rd(x1, x2), i(x1)), rd(x1, x2))}$

$=$ by Axiom 2 RL with $\{x2 \leftarrow rd(x1, x2), x1 \leftarrow i(x1)\}$

$op_r(\overbrace{i(x1)}, \underbrace{mult(rd(x1, x2), i(x1))}, rd(x1, x2))$

$=$ by Lemma 22 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$op_r(i(x1), \overbrace{i(x2)}, rd(x1, x2))$

Lemma 1543: $rd(mult(x2, i(x1)), x2) = i(op_r(x1, x2, rd(x2, x1)))$

$$
\begin{aligned}
& \underbrace{rd(mult(x2, i(x1)), x2)} \\
= \quad & \text{by Lemma 1542 LR with } \{x1 \leftarrow x1, x2 \leftarrow x2\} \\
& \overbrace{\underbrace{op_r(i(x1), i(x2), rd(x1, x2))}} \\
= \quad & \text{by Lemma 814 RL with } \{x3 \leftarrow rd(x1, x2), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{i(op_r(x1, x2, \underbrace{i(rd(x1, x2))}))} \\
= \quad & \text{by Lemma 10 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& i(op_r(x1, x2, \overbrace{rd(x2, x1)}))
\end{aligned}
$$

Lemma 1544: $op_t(x1, i(i(op_r(rd(x1, x2), x1, x2)))) = op_t(x1, i(rd(x2, x1)))$

$$
\begin{aligned}
& \underbrace{op_t(x1, i(i(op_r(rd(x1, x2), x1, x2))))} \\
= \quad & \text{by Lemma 562 RL with } \{x2 \leftarrow i(op_r(rd(x1, x2), x1, x2)), x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, i(\underbrace{op_t(i(op_r(rd(x1, x2), x1, x2)), x1)}))} \\
= \quad & \text{by Lemma 19 RL with } \{x2 \leftarrow x1, x1 \leftarrow op_r(rd(x1, x2), x1, x2)\} \\
& op_t(x1, i(\overbrace{i(\underbrace{op_t(op_r(rd(x1, x2), x1, x2), i(x1))})})) \\
= \quad & \text{by Axiom 14 LR with } \{x4 \leftarrow i(x1), x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow rd(x1, x2)\} \\
& op_t(x1, i(\overbrace{i(\underbrace{op_r(op_t(rd(x1, x2), i(x1)), x1, x2)})})) \\
= \quad & \text{by Lemma 815 RL with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow op_t(rd(x1, x2), i(x1))\} \\
& op_t(x1, i(\overbrace{op_r(i(\underbrace{op_t(rd(x1, x2), i(x1))}), i(x1), \underbrace{i(x2)})})) \\
= \quad & \text{by Lemma 22 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_t(x1, i(op_r(\underbrace{i(op_t(rd(x1, x2), i(x1)))}, i(x1), \overbrace{mult(rd(x1, x2), i(x1))}))) \\
= \quad & \text{by Lemma 31 RL with } \{x2 \leftarrow rd(x1, x2), x1 \leftarrow i(x1)\} \\
& op_t(x1, i(\underbrace{op_r(\overbrace{mult(i(x1), i(mult(rd(x1, x2), i(x1))))}, i(x1), mult(rd(x1, x2), i(x1)))})) \\
= \quad & \text{by Axiom 16 LR with } \{x3 \leftarrow mult(rd(x1, x2), i(x1)), x2 \leftarrow i(x1), x1 \leftarrow mult(i(x1), i(mult(rd(x1, x2), i(x1))))\} \\
& op_t(x1, i(\overbrace{rd(\underbrace{mult(mult(mult(i(x1), i(mult(rd(x1, x2), i(x1)))), i(x1)), mult(rd(x1, x2), i(x1)))}, mult(i(x1), mult(rd(x1, x2
= \quad & \text{by Lemma 1540 LR with } \{x1 \leftarrow mult(rd(x1, x2), i(x1)), x2 \leftarrow i(x1)\} \\
& op_t(x1, i(\overbrace{rd(\overbrace{mult(rd(i(x1), mult(rd(x1, x2), i(x1))), mult(i(x1), mult(rd(x1, x2), i(x1))))}, mult(i(x1), mult(rd(x1, x2), i(x
= \quad & \text{by Axiom 5 RL with } \{x2 \leftarrow mult(i(x1), mult(rd(x1, x2), i(x1))), x1 \leftarrow rd(i(x1), mult(rd(x1, x2), i(x1)))\} \\
& op_t(x1, i(\overbrace{rd(i(x1), mult(rd(x1, x2), i(x1)))})) \\
= \quad & \text{by Lemma 9 LR with } \{x2 \leftarrow rd(x1, x2), x1 \leftarrow i(x1)\} \\
& op_t(x1, i(\overbrace{i(rd(x1, x2))})) \\
= \quad & \text{by Lemma 10 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_t(x1, i(\overbrace{rd(x2, x1)}))
\end{aligned}
$$

Lemma 1545: $op_t(x1, op_r(rd(x1, x2), x1, x2)) = op_t(x1, rd(x1, x2))$

$op_t(x1, \underbrace{op_r(rd(x1, x2), x1, x2)})$

$=$ by Lemma 3 RL with $\{x1 \leftarrow op_r(rd(x1, x2), x1, x2)\}$

$\underbrace{op_t(x1, \overbrace{i(i(op_r(rd(x1, x2), x1, x2))))}}$

$=$ by Lemma 1544 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_t(x1, \underbrace{i(rd(x2, x1)))}}$

$=$ by Lemma 10 LR with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$op_t(x1, \overbrace{rd(x1, x2)})$

Lemma 1546: $op_t(x1, op_r(x2, x1, x1)) = op_t(x1, op_r(op_r(x2, x1, x1), x1, rd(x1, x2)))$

$op_t(x1, \underbrace{op_r(x2, x1, x1)})$

$=$ by Lemma 353 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{op_t(x1, \overbrace{rd(x1, rd(x1, x2)))})}$

$=$ by Lemma 1545 RL with $\{x2 \leftarrow rd(x1, x2), x1 \leftarrow x1\}$

$\overbrace{op_t(x1, op_r(\underbrace{rd(x1, rd(x1, x2))}, x1, rd(x1, x2)))}$

$=$ by Lemma 353 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$op_t(x1, op_r(\overbrace{op_r(x2, x1, x1)}, x1, rd(x1, x2)))$

Lemma 1547: $op_r(op_t(x1,x2),i(x2),rd(i(x2),x1)) = mult(op_t(mult(x2,rd(mult(x2,x1),x2)),x2),op_t(i(x2),x1))$

$\overbrace{op_r(op_t(x1,x2),i(x2),rd(i(x2),x1))}$

$=$ by Axiom 14 RL with $\{x4 \leftarrow x2, x3 \leftarrow rd(i(x2),x1), x2 \leftarrow i(x2), x1 \leftarrow x1\}$

$\overbrace{op_t(op_r(x1,i(x2),rd(i(x2),x1)),x2)}$

$=$ by Lemma 1468 LR with $\{x1 \leftarrow x2, x2 \leftarrow op_r(x1,i(x2),rd(i(x2),x1))\}$

$\overbrace{mult(op_t(mult(x2,op_r(x1,i(x2),rd(i(x2),x1))),x2),\underbrace{op_t(i(x2),op_t(op_r(x1,i(x2),rd(i(x2),x1)),x2))}}$

$=$ by Lemma 561 LR with $\{x2 \leftarrow op_r(x1,i(x2),rd(i(x2),x1)), x1 \leftarrow x2\}$

$mult(op_t(mult(x2,op_r(x1,i(x2),rd(i(x2),x1))),x2),\overbrace{\underbrace{op_t(i(x2),op_r(x1,i(x2),rd(i(x2),x1)))}})$

$=$ by Lemma 550 RL with $\{x2 \leftarrow op_r(x1,i(x2),rd(i(x2),x1)), x1 \leftarrow i(x2)\}$

$mult(op_t(mult(x2,op_r(x1,i(x2),rd(i(x2),x1))),x2),\overbrace{\underbrace{op_t(i(x2),\underbrace{op_r(op_r(x1,i(x2),rd(i(x2),x1)),i(x2),i(x2))})}})$

$=$ by Axiom 17 LR with $\{x5 \leftarrow i(x2), x4 \leftarrow i(x2), x3 \leftarrow rd(i(x2),x1), x2 \leftarrow i(x2), x1 \leftarrow x1\}$

$mult(op_t(mult(x2,op_r(x1,i(x2),rd(i(x2),x1))),x2),\underbrace{op_t(i(x2),\overbrace{op_r(op_r(x1,i(x2),i(x2)),i(x2),rd(i(x2),x1))})})$

$=$ by Lemma 1546 RL with $\{x2 \leftarrow x1, x1 \leftarrow i(x2)\}$

$mult(op_t(mult(x2,op_r(x1,i(x2),rd(i(x2),x1))),x2),\overbrace{\underbrace{op_t(i(x2),op_r(x1,i(x2),i(x2)))}})$

$=$ by Lemma 550 LR with $\{x2 \leftarrow x1, x1 \leftarrow i(x2)\}$

$mult(op_t(mult(x2,op_r(x1,i(x2),\underbrace{rd(i(x2),x1)})),x2),\overbrace{op_t(i(x2),x1)})$

$=$ by Lemma 10 RL with $\{x2 \leftarrow i(x2), x1 \leftarrow x1\}$

$mult(op_t(mult(x2,\underbrace{op_r(x1,i(x2),\overbrace{i(\overbrace{rd(x1,i(x2))})})}),x2),op_t(i(x2),x1))$

$=$ by Lemma 813 RL with $\{x3 \leftarrow rd(x1,i(x2)), x2 \leftarrow x2, x1 \leftarrow x1\}$

$mult(op_t(mult(x2,\overbrace{i(\overbrace{op_r(i(x1),x2,\underbrace{rd(x1,i(x2))})})}),x2),op_t(i(x2),x1))$

$=$ by Lemma 38 RL with $\{x1 \leftarrow x1, x2 \leftarrow x2\}$

$mult(op_t(mult(x2,\underbrace{i(op_r(i(x1),x2,\overbrace{rd(x2,i(x1))}))}),x2),op_t(i(x2),x1))$

$=$ by Lemma 1543 RL with $\{x1 \leftarrow i(x1), x2 \leftarrow x2\}$

$mult(op_t(mult(x2,\overbrace{rd(mult(x2,\underbrace{i(i(x1))},x2)}),x2),op_t(i(x2),x1))$

$=$ by Lemma 3 LR with $\{x1 \leftarrow x1\}$

$mult(op_t(mult(x2,rd(mult(x2,\overbrace{x1}),x2)),x2),op_t(i(x2),x1))$

Lemma 1548: $op_r(op_t(x1, i(x2)), x2, rd(x2, x1)) = x1$

$$
\begin{aligned}
& op_r(op_t(x1, i(x2)), x2, rd(\underbrace{x2}, x1)) \\
= & \quad \text{by Lemma 3 RL with } \{x1 \leftarrow x2\} \\
& op_r(op_t(x1, i(x2)), \underbrace{x2}, rd(\overbrace{i(i(x2))}, x1)) \\
= & \quad \text{by Lemma 3 RL with } \{x1 \leftarrow x2\} \\
& \underbrace{op_r(op_t(x1, i(x2)), \overbrace{i(i(x2))}, rd(i(i(x2)), x1))} \\
= & \quad \text{by Lemma 1547 LR with } \{x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& \overbrace{mult(\underbrace{op_t(mult(i(x2), rd(mult(i(x2), x1), i(x2))), i(x2))}, op_t(i(i(x2)), x1))} \\
= & \quad \text{by Lemma 349 LR with } \{x2 \leftarrow mult(i(x2), x1), x1 \leftarrow i(x2)\} \\
& mult(\overbrace{\underbrace{op_r(mult(i(x2), x1), i(x2), i(i(x2)))}}, op_t(i(i(x2)), x1)) \\
= & \quad \text{by Lemma 405 LR with } \{x2 \leftarrow x1, x1 \leftarrow i(x2)\} \\
& mult(\overbrace{rd(i(x2), \underbrace{op_t(i(x1), i(x2))})}, op_t(i(i(x2)), x1)) \\
= & \quad \text{by Lemma 19 RL with } \{x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& mult(\underbrace{rd(i(x2), \overbrace{i(op_t(x1, i(i(x2))))})}, op_t(i(i(x2)), x1)) \\
= & \quad \text{by Lemma 38 RL with } \{x1 \leftarrow i(x2), x2 \leftarrow op_t(x1, i(i(x2)))\} \\
& \underbrace{mult(\overbrace{rd(op_t(x1, i(i(x2))), i(i(x2)))}, op_t(i(i(x2)), x1))} \\
= & \quad \text{by Lemma 605 LR with } \{x2 \leftarrow i(i(x2)), x1 \leftarrow x1\} \\
& \overbrace{x1}
\end{aligned}
$$

Lemma 1549: $mult(i(x1), x2) = op_r(rd(op_t(i(x1), i(x2)), i(x2)), x2, rd(x2, i(mult(x1, i(x2)))))$

$$
\begin{aligned}
& \underbrace{mult(i(x1), x2)} \\
= & \quad \text{by Lemma 17 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{\overbrace{i(mult(x1, i(x2)))}} \\
= & \quad \text{by Lemma 1548 RL with } \{x2 \leftarrow x2, x1 \leftarrow i(mult(x1, i(x2)))\} \\
& \overbrace{op_r(\underbrace{op_t(i(mult(x1, i(x2))), i(x2))}, x2, rd(x2, i(mult(x1, i(x2)))))} \\
= & \quad \text{by Lemma 329 LR with } \{x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& op_r(\overbrace{rd(op_t(i(x1), \overbrace{i(x2)}), i(x2))}, x2, rd(x2, i(mult(x1, i(x2)))))
\end{aligned}
$$

Lemma 1550: $mult(i(x1), x2) = op_r(rd(x2, op_t(x1, x2)), x2, rd(x2, i(mult(x1, i(x2)))))$

$$
\begin{aligned}
& \underbrace{mult(i(x1), x2)} \\
= & \quad \text{by Lemma 1549 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_r(\underbrace{rd(op_t(i(x1), i(x2)), i(x2))}, x2, rd(x2, i(mult(x1, i(x2)))))} \\
= & \quad \text{by Lemma 67 LR with } \{x3 \leftarrow x2, x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& op_r(\overbrace{rd(x2, op_t(x1, \underbrace{i(i(x2))}))}, x2, rd(x2, i(mult(x1, i(x2))))) \\
= & \quad \text{by Lemma 3 LR with } \{x1 \leftarrow x2\} \\
& op_r(rd(x2, op_t(x1, \overbrace{x2})), x2, rd(x2, i(mult(x1, i(x2)))))
\end{aligned}
$$

Lemma 1551: $mult(i(x1), x2) = op_r(rd(x2, op_t(x1, x2)), x2, i(i(x1)))$

$\underbrace{mult(i(x1), x2)}$

$=$ by Lemma 1550 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_r(rd(x2, op_t(x1, x2)), x2, rd(x2, \underbrace{i(mult(x1, i(x2)))}))}$

$=$ by Lemma 17 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$op_r(rd(x2, op_t(x1, x2)), x2, \underbrace{rd(x2, \overbrace{mult(i(x1), x2)})})$

$=$ by Lemma 9 LR with $\{x2 \leftarrow i(x1), x1 \leftarrow x2\}$

$op_r(rd(x2, op_t(x1, x2)), x2, \overbrace{i(i(x1))})$

Lemma 1552: $asoc(op_r(x1, x2, rd(x2, x1)), x2, x2) = asoc(x1, x2, x2)$

$\underbrace{asoc(op_r(x1, x2, rd(x2, x1)), x2, x2)}$

$=$ by Lemma 1459 LR with $\{x2 \leftarrow x2, x1 \leftarrow op_r(x1, x2, rd(x2, x1))\}$

$\overbrace{asoc(op_t(x2, op_r(x1, x2, rd(x2, x1))), \underbrace{i(op_t(op_r(x1, x2, rd(x2, x1)), i(x2)))}, x2)}$

$=$ by Lemma 19 LR with $\{x2 \leftarrow x2, x1 \leftarrow op_r(x1, x2, rd(x2, x1))\}$

$asoc(\underbrace{op_t(x2, op_r(x1, x2, rd(x2, x1)))}, \overbrace{op_t(i(op_r(x1, x2, rd(x2, x1))), x2)}, x2)$

$=$ by Lemma 550 RL with $\{x2 \leftarrow op_r(x1, x2, rd(x2, x1)), x1 \leftarrow x2\}$

$asoc(\overbrace{op_t(x2, \underbrace{op_r(op_r(x1, x2, rd(x2, x1)), x2, x2)})}, op_t(i(op_r(x1, x2, rd(x2, x1))), x2), x2)$

$=$ by Axiom 17 LR with $\{x5 \leftarrow x2, x4 \leftarrow x2, x3 \leftarrow rd(x2, x1), x2 \leftarrow x2, x1 \leftarrow x1\}$

$asoc(\underbrace{op_t(x2, \overbrace{op_r(op_r(x1, x2, x2), x2, rd(x2, x1))})}, op_t(i(op_r(x1, x2, rd(x2, x1))), x2), x2)$

$=$ by Lemma 1546 RL with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$asoc(\overbrace{\underbrace{op_t(x2, op_r(x1, x2, x2))}}, op_t(i(op_r(x1, x2, rd(x2, x1))), x2), x2)$

$=$ by Lemma 550 LR with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$asoc(\overbrace{op_t(x2, x1)}, op_t(\underbrace{i(op_r(x1, x2, rd(x2, x1)))}, x2), x2)$

$=$ by Lemma 1543 RL with $\{x1 \leftarrow x1, x2 \leftarrow x2\}$

$asoc(op_t(x2, x1), \underbrace{op_t(\overbrace{rd(mult(x2, i(x1)), x2)}, x2)}, x2)$

$=$ by Lemma 1208 LR with $\{x3 \leftarrow x2, x2 \leftarrow i(x1), x1 \leftarrow x2\}$

$asoc(op_t(x2, x1), \overbrace{rd(\underbrace{mult(x2, op_t(i(x1), x2))}, x2)}, x2)$

$=$ by Lemma 13 LR with $\{x2 \leftarrow i(x1), x1 \leftarrow x2\}$

$asoc(op_t(x2, x1), \underbrace{rd(\overbrace{mult(i(x1), x2)}, x2)}, x2)$

$=$ by Axiom 5 RL with $\{x2 \leftarrow x2, x1 \leftarrow i(x1)\}$

$\underbrace{asoc(op_t(x2, x1), \overbrace{i(x1)}, x2)}$

$=$ by Lemma 1458 LR with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$\overbrace{asoc(x1, x2, x2)}$

Lemma 1553: $op_t(i(x1), rd(x1, x2)) = op_t(i(x1), mult(op_l(x1, x3, x1), x2))$

$$
\begin{aligned}
& \underbrace{op_t(i(x1), rd(x1, x2))} \\
= \quad & \text{by Lemma 714 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_t(i(x1), mult(x1, x2))} \\
= \quad & \text{by Lemma 1493 RL with } \{x3 \leftarrow mult(x1, x2), x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{op_t(i(x1), \underbrace{mult(asoc(x3, x1, x1), mult(x1, x2))})} \\
= \quad & \text{by Lemma 959 LR with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x3\} \\
& op_t(i(x1), \overbrace{mult(op_l(x1, x3, x1), x2)})
\end{aligned}
$$

Lemma 1554: $op_t(x1, rd(i(x1), x2)) = op_t(x1, mult(op_l(i(x1), x3, i(x1)), x2))$

$$
\begin{aligned}
& op_t(\underbrace{x1}, rd(i(x1), x2)) \\
= \quad & \text{by Lemma 3 RL with } \{x1 \leftarrow x1\} \\
& \underbrace{op_t(\overbrace{i(i(x1))}, rd(i(x1), x2))} \\
= \quad & \text{by Lemma 1553 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow i(x1)\} \\
& \overbrace{op_t(\underbrace{i(i(x1))}, mult(op_l(i(x1), x3, i(x1)), x2))} \\
= \quad & \text{by Lemma 3 LR with } \{x1 \leftarrow x1\} \\
& op_t(\overbrace{x1}, mult(op_l(i(x1), x3, i(x1)), x2))
\end{aligned}
$$

Lemma 1555: $op_t(x1, rd(x2, x1)) = op_t(x1, mult(op_l(i(x1), x1, x3), x2))$

$$
\begin{aligned}
& \underbrace{op_t(x1, rd(x2, x1))} \\
= \quad & \text{by Lemma 707 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{\overbrace{op_t(x1, rd(i(x1), x2))}} \\
= \quad & \text{by Lemma 1554 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, mult(\underbrace{op_l(i(x1), x3, i(x1))}, x2))} \\
= \quad & \text{by Lemma 633 LR with } \{x2 \leftarrow x3, x1 \leftarrow x1\} \\
& op_t(x1, mult(\overbrace{op_l(i(x1), x1, x3)}, x2))
\end{aligned}
$$

Lemma 1556: $op_t(x1, rd(mult(op_l(i(x1), x1, x2), i(x3)), x1)) = op_t(x1, x3)$

$$
\begin{aligned}
& \underbrace{op_t(x1, rd(mult(op_l(i(x1), x1, x2), i(x3)), x1))} \\
= \quad & \text{by Lemma 1410 RL with } \{x3 \leftarrow x3, x2 \leftarrow op_l(i(x1), x1, x2), x1 \leftarrow x1\} \\
& \underbrace{\overbrace{op_t(x1, rd(mult(i(op_l(i(x1), x1, x2)), x3), x1))}} \\
= \quad & \text{by Lemma 1555 LR with } \{x3 \leftarrow x2, x2 \leftarrow mult(i(op_l(i(x1), x1, x2)), x3), x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, \underbrace{mult(op_l(i(x1), x1, x2), mult(i(op_l(i(x1), x1, x2)), x3))})} \\
= \quad & \text{by Lemma 12 LR with } \{x2 \leftarrow x3, x1 \leftarrow op_l(i(x1), x1, x2)\} \\
& op_t(x1, \overbrace{x3})
\end{aligned}
$$

Lemma 1557: $asoc(x1, mult(op_l(x1, x1, x2), x3), x1) = rd(op_t(x1, rd(x3, x1)), op_t(x1, x3))$

$asoc(x1, mult(op_l(x1, x1, x2), x3), x1)$

$=$ by Lemma 263 RL with $\{x2 \leftarrow mult(op_l(x1, x1, x2), x3), x1 \leftarrow x1\}$

$asoc(x1, rd(mult(op_l(x1, x1, x2), x3), x1), x1)$

$=$ by Lemma 986 LR with $\{x2 \leftarrow rd(mult(op_l(x1, x1, x2), x3), x1), x1 \leftarrow x1\}$

$rd(mult(rd(mult(op_l(x1, x1, x2), x3), x1), rd(x1, op_t(rd(mult(op_l(x1, x1, x2), x3), x1), x1))), op_t(x1, rd(mult(op_l(x1, x1, x2$

$=$ by Lemma 604 LR with $\{x2 \leftarrow x1, x1 \leftarrow rd(mult(op_l(x1, x1, x2), x3), x1)\}$

$rd(op_r(x1, rd(mult(op_l(x1, x1, x2), x3), x1), i(rd(mult(op_l(x1, x1, x2), x3), x1))), op_t(x1, rd(mult(op_l(x1, x1, x2), x3), x1)))$

$=$ by Lemma 712 RL with $\{x2 \leftarrow mult(op_l(x1, x1, x2), x3), x1 \leftarrow x1\}$

$rd(op_r(x1, rd(mult(op_l(x1, x1, x2), x3), x1), i(rd(mult(op_l(x1, x1, x2), x3), x1))), op_t(x1, rd(i(mult(op_l(x1, x1, x2), x3)), x1$

$=$ by Axiom 11 RL with $\{x2 \leftarrow x3, x1 \leftarrow op_l(x1, x1, x2)\}$

$rd(op_r(x1, rd(mult(op_l(x1, x1, x2), x3), x1), i(rd(mult(op_l(x1, x1, x2), x3), x1))), op_t(x1, rd(mult(i(op_l(x1, x1, x2)), i(x3)),$

$=$ by Lemma 1049 LR with $\{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x1\}$

$rd(op_r(x1, rd(mult(op_l(x1, x1, x2), x3), x1), i(rd(mult(op_l(x1, x1, x2), x3), x1))), op_t(x1, rd(mult(op_l(i(x1), x1, x2), i(x3)),$

$=$ by Lemma 1556 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$rd(op_r(x1, rd(mult(op_l(x1, x1, x2), x3), x1), i(rd(mult(op_l(x1, x1, x2), x3), x1))), op_t(x1, x3))$

$=$ by Lemma 10 LR with $\{x2 \leftarrow x1, x1 \leftarrow mult(op_l(x1, x1, x2), x3)\}$

$rd(op_r(x1, rd(mult(op_l(x1, x1, x2), x3), x1), rd(x1, mult(op_l(x1, x1, x2), x3))), op_t(x1, x3))$

$=$ by Lemma 586 LR with $\{x2 \leftarrow mult(op_l(x1, x1, x2), x3), x1 \leftarrow x1\}$

$rd(op_t(x1, i(mult(op_l(x1, x1, x2), x3))), op_t(x1, x3))$

$=$ by Lemma 606 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$rd(op_t(x1, i(mult(op_l(x1, x2, i(x1)), x3))), op_t(x1, x3))$

$=$ by Lemma 18 RL with $\{x2 \leftarrow mult(op_l(x1, x2, i(x1)), x3), x1 \leftarrow x1\}$

$rd(i(op_t(i(x1), mult(op_l(x1, x2, i(x1)), x3))), op_t(x1, x3))$

$=$ by Lemma 15 RL with $\{x2 \leftarrow mult(op_l(x1, x2, i(x1)), x3), x1 \leftarrow i(x1)\}$

$rd(op_t(i(i(x1)), i(mult(op_l(x1, x2, i(x1)), x3))), op_t(x1, x3))$

$=$ by Axiom 11 RL with $\{x2 \leftarrow x3, x1 \leftarrow op_l(x1, x2, i(x1))\}$

$rd(op_t(i(i(x1)), mult(i(op_l(x1, x2, i(x1))), i(x3))), op_t(x1, x3))$

$=$ by Lemma 1049 LR with $\{x3 \leftarrow i(x1), x2 \leftarrow x2, x1 \leftarrow x1\}$

$rd(op_t(i(i(x1)), mult(op_l(i(x1), x2, i(x1)), i(x3))), op_t(x1, x3))$

$=$ by Lemma 1553 RL with $\{x3 \leftarrow x2, x2 \leftarrow i(x3), x1 \leftarrow i(x1)\}$

$rd(op_t(i(i(x1)), rd(i(x1), i(x3))), op_t(x1, x3))$

$=$ by Lemma 714 RL with $\{x2 \leftarrow i(x3), x1 \leftarrow i(x1)\}$

$rd(op_t(i(i(x1)), mult(i(x1), i(x3))), op_t(x1, x3))$

$=$ by Lemma 414 LR with $\{x2 \leftarrow x3, x1 \leftarrow i(x1)\}$

$rd(op_t(i(i(x1)), rd(i(x1), x3)), op_t(x1, x3))$

$=$ by Lemma 3 LR with $\{x1 \leftarrow x1\}$

$rd(op_t(x1, rd(i(x1), x3)), op_t(x1, x3))$

$=$ by Lemma 707 LR with $\{x2 \leftarrow x3, x1 \leftarrow x1\}$

$rd(op_t(x1, rd(x3, x1)), op_t(x1, x3))$

Lemma 1558: $op_l(x1, x1, x2) = op_l(x1, x1, mult(op_l(x1, x3, x1), x2))$

$\overbrace{op_l(x1, x1, x2)}$

$=$ by Lemma 173 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{mult(\underbrace{asoc(x1, x2, x1)}, x1)}$

$=$ by Lemma 1009 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$mult(\overbrace{\underbrace{rd(op_t(x1, rd(x2, x1)), op_t(x1, x2))}}, x1)$

$=$ by Lemma 1557 RL with $\{x3 \leftarrow x2, x2 \leftarrow i(x3), x1 \leftarrow x1\}$

$\underbrace{mult(\overbrace{asoc(x1, mult(op_l(x1, x1, i(x3)), x2), x1)}, x1)}$

$=$ by Lemma 173 LR with $\{x2 \leftarrow mult(op_l(x1, x1, i(x3)), x2), x1 \leftarrow x1\}$

$\overbrace{op_l(x1, x1, mult(\underbrace{op_l(x1, x1, i(x3))}, x2))}$

$=$ by Lemma 608 LR with $\{x2 \leftarrow x3, x1 \leftarrow x1\}$

$op_l(x1, x1, mult(\overbrace{op_l(x1, x3, x1)}, x2))$

Lemma 1559: $op_l(x1, x1, x2) = op_l(x1, x2, i(op_l(x1, x3, x1)))$

$\overbrace{op_l(x1, x1, x2)}$

$=$ by Lemma 606 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_l(x1, x2, i(x1))}$

$=$ by Lemma 1050 RL with $\{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_l(x1, i(x2), x1)}$

$=$ by Axiom 5 LR with $\{x2 \leftarrow x3, x1 \leftarrow op_l(x1, i(x2), x1)\}$

$\overbrace{rd(\underbrace{mult(op_l(x1, i(x2), x1), x3)}, x3)}$

$=$ by Lemma 167 RL with $\{x4 \leftarrow x1, x3 \leftarrow i(x2), x2 \leftarrow x1, x1 \leftarrow x3\}$

$rd(\overbrace{mult(x3, \underbrace{op_l(op_t(x1, x3), i(x2), x1)})}, x3)$

$=$ by Lemma 1496 LR with $\{x3 \leftarrow i(x2), x2 \leftarrow x3, x1 \leftarrow x1\}$

$rd(mult(x3, \overbrace{op_l(op_t(x1, x3), i(x2), \underbrace{rd(\underbrace{rd(mult(x1, x3), x1)}, rd(x3, x1))})}), x3)$

$=$ by Lemma 453 RL with $\{x2 \leftarrow x1, x1 \leftarrow x3\}$

$rd(mult(x3, op_l(op_t(x1, x3), i(x2), rd(\overbrace{op_l(op_r(x3, x1, x1), x3, x1)}, rd(x3, x1)))), x3)$

$=$ by Axiom 18 RL with $\{x5 \leftarrow x1, x4 \leftarrow x1, x3 \leftarrow x1, x2 \leftarrow x3, x1 \leftarrow x3\}$

$rd(mult(x3, op_l(op_t(x1, x3), i(x2), \underbrace{rd(\overbrace{op_r(op_l(x3, x3, x1), x1, x1)}, rd(x3, x1))})), x3)$

$=$ by Lemma 984 RL with $\{x2 \leftarrow x3, x1 \leftarrow x1\}$

$rd(\underbrace{mult(x3, op_l(op_t(x1, x3), i(x2), \overbrace{op_l(x1, x3, x1)}))}, x3)$

$=$ by Lemma 167 LR with $\{x4 \leftarrow op_l(x1, x3, x1), x3 \leftarrow i(x2), x2 \leftarrow x1, x1 \leftarrow x3\}$

$\underbrace{rd(\overbrace{mult(op_l(x1, i(x2), op_l(x1, x3, x1)), x3)}, x3)}$

$=$ by Axiom 5 RL with $\{x2 \leftarrow x3, x1 \leftarrow op_l(x1, i(x2), op_l(x1, x3, x1))\}$

$\overbrace{op_l(x1, i(x2), op_l(x1, x3, x1))}$

$=$ by Lemma 1050 LR with $\{x3 \leftarrow op_l(x1, x3, x1), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_l(x1, x2, i(op_l(x1, x3, x1)))}$

Lemma 1560: $op_l(x1, x1, mult(i(op_l(i(x1), x2, x1)), x3)) = op_l(x1, op_l(i(x1), x2, x1), i(x3))$

$\underbrace{op_l(x1, x1, mult(i(op_l(i(x1), x2, x1)), x3))}$

$=$ by Lemma 1559 LR with $\{x3 \leftarrow x2, x2 \leftarrow mult(i(op_l(i(x1), x2, x1)), x3), x1 \leftarrow x1\}$

$\overbrace{op_l(x1, \underbrace{mult(i(op_l(i(x1), x2, x1)), x3)}, \underbrace{i(op_l(x1, x2, x1))})}$

$=$ by Lemma 1049 LR with $\{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{op_l(x1, \underbrace{mult(i(op_l(i(x1), x2, x1)), x3)}, \overbrace{op_l(i(x1), x2, x1)})}$

$=$ by Lemma 1054 LR with $\{x3 \leftarrow x3, x2 \leftarrow op_l(i(x1), x2, x1), x1 \leftarrow x1\}$

$\overbrace{op_l(x1, op_l(i(x1), x2, x1), i(x3))}$

Lemma 1561: $op_l(x1, x1, mult(op_l(x1, x2, x1), x3)) = op_l(x1, op_l(i(x1), x2, x1), i(x3))$

$$
\begin{aligned}
& op_l(x1, x1, mult(op_l(\underbrace{x1}, x2, x1), x3)) \\
= & \quad \text{by Lemma 3 RL with } \{x1 \leftarrow x1\} \\
& op_l(x1, x1, mult(\underbrace{op_l(\overbrace{i(i(x1))}, x2, x1)}, x3)) \\
= & \quad \text{by Lemma 1049 RL with } \{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow i(x1)\} \\
& \underbrace{op_l(x1, x1, mult(\overbrace{i(op_l(i(x1), x2, x1))}, x3))} \\
= & \quad \text{by Lemma 1560 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, op_l(i(x1), x2, x1), i(x3))}
\end{aligned}
$$

Lemma 1562: $op_l(x1, x1, x3) = op_l(x1, op_l(x1, x2, x1), i(i(x3)))$

$$
\begin{aligned}
& \underbrace{op_l(x1, x1, x3)} \\
= & \quad \text{by Lemma 1558 LR with } \{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \underbrace{\overbrace{op_l(x1, x1, mult(op_l(x1, x2, x1), x3))}} \\
= & \quad \text{by Lemma 1561 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{\overbrace{op_l(x1, op_l(i(x1), x2, x1), i(x3))}} \\
= & \quad \text{by Lemma 1419 LR with } \{x5 \leftarrow i(x3), x4 \leftarrow x1, x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, op_l(x1, x2, x1), i(i(x3)))}
\end{aligned}
$$

Lemma 1563: $op_l(x1, op_l(x1, x2, x1), x3) = op_l(x1, x1, x3)$

$$
\begin{aligned}
& op_l(x1, op_l(x1, x2, x1), \underbrace{x3}) \\
= & \quad \text{by Lemma 3 RL with } \{x1 \leftarrow x3\} \\
& \underbrace{op_l(x1, op_l(x1, x2, x1), \overbrace{i(i(x3))})} \\
= & \quad \text{by Lemma 1562 RL with } \{x2 \leftarrow x2, x3 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x1, x3)}
\end{aligned}
$$

Lemma 1564: $op_l(i(x1), op_l(x1, x2, x1), x3) = op_l(i(x1), x1, x3)$

$$
\begin{aligned}
& \underbrace{op_l(i(x1), op_l(x1, x2, x1), x3)} \\
= & \quad \text{by Lemma 1049 RL with } \{x3 \leftarrow x3, x2 \leftarrow op_l(x1, x2, x1), x1 \leftarrow x1\} \\
& \overbrace{i(\underbrace{op_l(x1, op_l(x1, x2, x1), x3)})} \\
= & \quad \text{by Lemma 1563 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{i(\overbrace{op_l(x1, x1, x3)})} \\
= & \quad \text{by Lemma 1049 LR with } \{x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow x1\} \\
& \overbrace{op_l(i(x1), x1, x3)}
\end{aligned}
$$

Lemma 1565: $op_l(i(x1), op_l(x1, x1, x2), x3) = op_l(i(x1), x1, x3)$

$\underbrace{op_l(i(x1), op_l(x1, x1, x2), x3)}$

$=$ by Lemma 1049 RL with $\{x3 \leftarrow x3, x2 \leftarrow op_l(x1, x1, x2), x1 \leftarrow x1\}$

$\overbrace{i(op_l(x1, \underbrace{op_l(x1, x1, x2)}, x3))}$

$=$ by Lemma 773 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$i(op_l(x1, \overbrace{rd(x1, asoc(x2, x1, x1))}, x3))$

$=$ by Lemma 1031 RL with $\{x3 \leftarrow x2, x2 \leftarrow u, x1 \leftarrow x1\}$

$i(\underbrace{op_l(x1, \overbrace{op_l(x1, rd(op_t(x1, u), x2), x1)}, x3)})$

$=$ by Lemma 1563 LR with $\{x3 \leftarrow x3, x2 \leftarrow rd(op_t(x1, u), x2), x1 \leftarrow x1\}$

$\underbrace{i(\overbrace{op_l(x1, x1, x3)})}$

$=$ by Lemma 1049 LR with $\{x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow x1\}$

$\overbrace{op_l(i(x1), x1, x3)}$

Lemma 1566: $rd(x1, op_l(x2, x3, x2)) = rd(rd(x1, x2), mult(op_l(x2, x3, x2), i(x2)))$

$\underbrace{rd(x1, op_l(x2, x3, x2))}$

$=$ by Lemma 1472 RL with $\{x3 \leftarrow op_l(x2, x3, x2), x2 \leftarrow i(x2), x1 \leftarrow x1\}$

$\overbrace{rd(mult(x1, \underbrace{op_l(i(x2), op_l(x2, x3, x2), x1)}), mult(op_l(x2, x3, x2), i(x2)))}$

$=$ by Lemma 1564 LR with $\{x3 \leftarrow x1, x2 \leftarrow x3, x1 \leftarrow x2\}$

$rd(\underbrace{mult(x1, \overbrace{op_l(i(x2), x2, x1)})}, mult(op_l(x2, x3, x2), i(x2)))$

$=$ by Lemma 232 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$rd(\overbrace{rd(x1, x2)}, mult(op_l(x2, x3, x2), i(x2)))$

Lemma 1567: $rd(x1, op_l(x2, x3, x2)) = rd(rd(x1, x2), asoc(x3, x2, x2))$

$\underbrace{rd(x1, op_l(x2, x3, x2))}$

$=$ by Lemma 1566 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{rd(rd(x1, x2), \underbrace{mult(op_l(x2, x3, x2), i(x2))})}$

$=$ by Lemma 178 LR with $\{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x2\}$

$rd(rd(x1, x2), \overbrace{\underbrace{rd(op_l(x2, x3, x2), x2)}})$

$=$ by Lemma 765 LR with $\{x2 \leftarrow x3, x1 \leftarrow x2\}$

$rd(rd(x1, x2), \overbrace{asoc(x3, x2, x2)})$

Lemma 1568: $rd(op_l(x1,x2,x1),x3) = rd(asoc(x2,x1,x1),mult(x3,op_l(i(x1),x1,x3)))$

$$
\begin{aligned}
& \underbrace{rd(op_l(x1,x2,x1),x3)} \\
= \quad & \text{by Lemma 1473 LR with } \{x3 \leftarrow i(x1), x1 \leftarrow x3, x2 \leftarrow op_l(x1,x2,x1)\} \\
& \overbrace{rd(mult(op_l(x1,x2,x1),i(x1)),mult(x3,\underbrace{op_l(i(x1),op_l(x1,x2,x1),x3)}))} \\
= \quad & \text{by Lemma 1564 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& rd(\underbrace{mult(op_l(x1,x2,x1),i(x1))},mult(x3,\overbrace{op_l(i(x1),x1,x3)})) \\
= \quad & \text{by Lemma 178 LR with } \{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& rd(\overbrace{\underbrace{rd(op_l(x1,x2,x1),x1)}},mult(x3,op_l(i(x1),x1,x3))) \\
= \quad & \text{by Lemma 765 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& rd(\overbrace{asoc(x2,x1,x1)},mult(x3,op_l(i(x1),x1,x3)))
\end{aligned}
$$

Lemma 1569: $rd(asoc(x1,x2,x2),rd(x3,x2)) = rd(op_l(x2,x1,x2),x3)$

$$
\begin{aligned}
& rd(asoc(x1,x2,x2),\underbrace{rd(x3,x2)}) \\
= \quad & \text{by Lemma 232 RL with } \{x2 \leftarrow x2, x1 \leftarrow x3\} \\
& \underbrace{rd(asoc(x1,x2,x2),\overbrace{mult(x3,op_l(i(x2),x2,x3))})} \\
= \quad & \text{by Lemma 1568 RL with } \{x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \overbrace{rd(op_l(x2,x1,x2),x3)}
\end{aligned}
$$

Lemma 1570: $op_t(x1,rd(asoc(x2,x3,x3),x1)) = op_t(x1,rd(op_l(x3,x2,x3),mult(x3,x1)))$

$$
\begin{aligned}
& \underbrace{op_t(x1,rd(asoc(x2,x3,x3),x1))} \\
= \quad & \text{by Lemma 1244 RL with } \{x3 \leftarrow mult(x3,x1), x2 \leftarrow asoc(x2,x3,x3), x1 \leftarrow x1\} \\
& \overbrace{op_t(x1,rd(asoc(x2,x3,x3),\underbrace{op_r(x1,mult(x3,x1),mult(x3,x1))}))} \\
= \quad & \text{by Lemma 800 LR with } \{x2 \leftarrow x3, x1 \leftarrow x1\} \\
& op_t(x1,\underbrace{rd(asoc(x2,x3,x3),\overbrace{rd(mult(x3,x1),x3)})}) \\
= \quad & \text{by Lemma 1569 LR with } \{x3 \leftarrow mult(x3,x1), x2 \leftarrow x3, x1 \leftarrow x2\} \\
& op_t(x1,\overbrace{rd(op_l(x3,x2,x3),mult(x3,x1))})
\end{aligned}
$$

Lemma 1571: $op_t(x1,i(rd(op_l(x2,x3,x2),mult(x2,i(x1))))) = i(i(x1))$

$$
\begin{aligned}
& \underbrace{op_t(x1,i(rd(op_l(x2,x3,x2),mult(x2,i(x1)))))} \\
= \quad & \text{by Lemma 18 RL with } \{x2 \leftarrow rd(op_l(x2,x3,x2),mult(x2,i(x1))), x1 \leftarrow x1\} \\
& \overbrace{i(\underbrace{op_t(i(x1),rd(op_l(x2,x3,x2),mult(x2,i(x1))))})} \\
= \quad & \text{by Lemma 1570 RL with } \{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow i(x1)\} \\
& i(\overbrace{op_t(i(x1),rd(asoc(x3,x2,x2),i(x1)))}) \\
= \quad & \text{by Lemma 1298 LR with } \{x2 \leftarrow x2, x3 \leftarrow x3, x1 \leftarrow i(x1)\} \\
& i(\overbrace{i(x1)})
\end{aligned}
$$

Lemma 1572: $op_t(x1, rd(rd(op_l(x2, x3, x2), x1), x2)) = i(i(x1))$

$$
\begin{aligned}
& \underbrace{op_t(x1, rd(rd(op_l(x2, x3, x2), x1), x2))} \\
= & \quad \text{by Lemma 1282 RL with } \{x3 \leftarrow op_l(x2, x3, x2), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, \underbrace{rd(mult(x2, i(x1)), op_l(x2, x3, x2))})} \\
= & \quad \text{by Lemma 10 RL with } \{x2 \leftarrow mult(x2, i(x1)), x1 \leftarrow op_l(x2, x3, x2)\} \\
& \underbrace{op_t(x1, \overbrace{i(rd(op_l(x2, x3, x2), mult(x2, i(x1))))})} \\
= & \quad \text{by Lemma 1571 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{i(i(x1))}
\end{aligned}
$$

Lemma 1573: $op_t(x1, rd(rd(x2, x1), op_l(x2, x3, x2))) = x1$

$$
\begin{aligned}
& \underbrace{op_t(x1, rd(rd(x2, x1), op_l(x2, x3, x2)))} \\
= & \quad \text{by Lemma 1282 RL with } \{x3 \leftarrow x2, x2 \leftarrow op_l(x2, x3, x2), x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_t(x1, rd(mult(op_l(x2, x3, x2), i(x1)), x2))}} \\
= & \quad \text{by Lemma 1284 RL with } \{x3 \leftarrow x2, x2 \leftarrow op_l(x2, x3, x2), x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_t(x1, rd(rd(op_l(x2, x3, x2), x1), x2))}} \\
= & \quad \text{by Lemma 1572 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{\overbrace{i(i(x1))}} \\
= & \quad \text{by Lemma 3 LR with } \{x1 \leftarrow x1\} \\
& \overbrace{x1}
\end{aligned}
$$

Lemma 1574: $rd(x1, op_l(x2, x2, x3)) = rd(rd(x1, x2), mult(op_l(x2, x2, x3), i(x2)))$

$$
\begin{aligned}
& \underbrace{rd(x1, op_l(x2, x2, x3))} \\
= & \quad \text{by Lemma 1472 RL with } \{x3 \leftarrow op_l(x2, x2, x3), x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& \overbrace{rd(mult(x1, \underbrace{op_l(i(x2), op_l(x2, x2, x3), x1)}), mult(op_l(x2, x2, x3), i(x2)))} \\
= & \quad \text{by Lemma 1565 LR with } \{x3 \leftarrow x1, x2 \leftarrow x3, x1 \leftarrow x2\} \\
& rd(\underbrace{mult(x1, \overbrace{op_l(i(x2), x2, x1)})}, mult(op_l(x2, x2, x3), i(x2))) \\
= & \quad \text{by Lemma 232 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& rd(\overbrace{rd(x1, x2)}, mult(op_l(x2, x2, x3), i(x2)))
\end{aligned}
$$

Lemma 1575: $rd(x1, op_l(x2, x2, x3)) = rd(rd(x1, x2), asoc(x2, x3, x2))$

$$
\begin{aligned}
& \underbrace{rd(x1, op_l(x2, x2, x3))} \\
= & \quad \text{by Lemma 1574 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{rd(rd(x1, x2), \underbrace{mult(op_l(x2, x2, x3), i(x2))})} \\
= & \quad \text{by Lemma 178 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x2\} \\
& rd(rd(x1, x2), \overbrace{\underbrace{rd(op_l(x2, x2, x3), x2)}}) \\
= & \quad \text{by Lemma 172 LR with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& rd(rd(x1, x2), \overbrace{asoc(x2, x3, x2)})
\end{aligned}
$$

Lemma 1576: $rd(op_l(x1, x1, x2), x3) = rd(rd(op_l(x1, x1, x2), x1), mult(x3, op_l(i(x1), x1, x3)))$

$\underbrace{rd(op_l(x1, x1, x2), x3)}$

$=$ by Lemma 1473 LR with $\{x3 \leftarrow i(x1), x1 \leftarrow x3, x2 \leftarrow op_l(x1, x1, x2)\}$

$\overbrace{rd(mult(op_l(x1, x1, x2), i(x1)), mult(x3, \underbrace{op_l(i(x1), op_l(x1, x1, x2), x3)}))}$

$=$ by Lemma 1565 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$rd(\underbrace{mult(op_l(x1, x1, x2), i(x1))}, mult(x3, \overbrace{op_l(i(x1), x1, x3)}))$

$=$ by Lemma 178 LR with $\{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x1\}$

$rd(\overbrace{rd(\underbrace{op_l(x1, x1, x2)}, x1)}, mult(x3, op_l(i(x1), x1, x3)))$

Lemma 1577: $rd(op_l(x1, x1, x2), x3) = rd(asoc(x1, x2, x1), rd(x3, x1))$

$\underbrace{rd(op_l(x1, x1, x2), x3)}$

$=$ by Lemma 1576 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{rd(\underbrace{rd(op_l(x1, x1, x2), x1)}, mult(x3, op_l(i(x1), x1, x3)))}$

$=$ by Lemma 172 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$rd(\overbrace{asoc(x1, x2, x1)}, \underbrace{mult(x3, op_l(i(x1), x1, x3))})$

$=$ by Lemma 232 LR with $\{x2 \leftarrow x1, x1 \leftarrow x3\}$

$rd(asoc(x1, x2, x1), \overbrace{rd(x3, x1)})$

Lemma 1578: $op_r(x1, asoc(x2, x3, x3), x3) = x1$

$op_r(x1, \underbrace{asoc(x2, x3, x3)}, x3)$

$=$ by Lemma 765 RL with $\{x2 \leftarrow x2, x1 \leftarrow x3\}$

$\underbrace{op_r(x1, \overbrace{rd(op_l(x3, x2, x3), x3)}, x3)}$

$=$ by Lemma 693 RL with $\{x3 \leftarrow x3, x2 \leftarrow op_l(x3, x2, x3), x1 \leftarrow x1\}$

$\overbrace{\underbrace{rd(mult(mult(x1, rd(op_l(x3, x2, x3), x3)), x3), op_l(x3, x2, x3))}}$

$=$ by Lemma 1567 LR with $\{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow mult(mult(x1, rd(op_l(x3, x2, x3), x3)), x3)\}$

$\overbrace{rd(\underbrace{rd(mult(mult(x1, rd(op_l(x3, x2, x3), x3)), x3), x3)}, asoc(x2, x3, x3))}$

$=$ by Axiom 5 RL with $\{x2 \leftarrow x3, x1 \leftarrow mult(x1, rd(op_l(x3, x2, x3), x3))\}$

$rd(\overbrace{mult(x1, \underbrace{rd(op_l(x3, x2, x3), x3)})}, asoc(x2, x3, x3))$

$=$ by Lemma 765 LR with $\{x2 \leftarrow x2, x1 \leftarrow x3\}$

$rd(\underbrace{mult(x1, \overbrace{asoc(x2, x3, x3)})}, asoc(x2, x3, x3))$

$=$ by Lemma 1300 RL with $\{x2 \leftarrow x2, x3 \leftarrow x3, x1 \leftarrow x1\}$

$\underbrace{rd(\overbrace{rd(x1, asoc(x3, x2, x3))}, asoc(x2, x3, x3))}$

$=$ by Lemma 1295 RL with $\{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow rd(x1, asoc(x3, x2, x3))\}$

$\overbrace{\underbrace{mult(rd(x1, asoc(x3, x2, x3)), asoc(x3, x2, x3))}}$

$=$ by Axiom 4 RL with $\{x2 \leftarrow asoc(x3, x2, x3), x1 \leftarrow x1\}$

$\overbrace{x1}$

Lemma 1579: $rd(op_l(x2,x2,x3),x1) = rd(rd(x2,x1),asoc(x3,i(x2),i(x2)))$

$$
\begin{aligned}
& rd(\underbrace{op_l(x2,x2,x3)}, x1) \\
= & \quad \text{by Lemma 606 RL with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& \underbrace{rd(\overbrace{op_l(x2,x3,i(x2))}, x1)} \\
= & \quad \text{by Lemma 1074 RL with } \{x4 \leftarrow i(x2), x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{rd(i(x1),op_l(i(x2),x3,i(x2)))}} \\
= & \quad \text{by Lemma 1567 LR with } \{x3 \leftarrow x3, x2 \leftarrow i(x2), x1 \leftarrow i(x1)\} \\
& \overbrace{rd(\underbrace{rd(i(x1),i(x2))}, asoc(x3,i(x2),i(x2)))} \\
= & \quad \text{by Lemma 23 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& rd(\overbrace{rd(x2,x1)}, asoc(x3,i(x2),i(x2)))
\end{aligned}
$$

Lemma 1580: $rd(rd(x1,x2),asoc(x3,x1,x1)) = rd(op_l(x1,x1,x3),x2)$

$$
\begin{aligned}
& rd(rd(x1,x2),\underbrace{asoc(x3,x1,x1)}) \\
= & \quad \text{by Lemma 772 RL with } \{x2 \leftarrow x1, x1 \leftarrow x3\} \\
& \underbrace{rd(rd(x1,x2),\overbrace{asoc(x3,i(x1),i(x1))})} \\
= & \quad \text{by Lemma 1579 RL with } \{x1 \leftarrow x2, x3 \leftarrow x3, x2 \leftarrow x1\} \\
& \overbrace{rd(op_l(x1,x1,x3),x2)}
\end{aligned}
$$

Lemma 1581: $rd(x3,op_l(x1,x2,i(x1))) = rd(asoc(x2,i(x1),i(x1)),rd(x1,x3))$

$$
\begin{aligned}
& \underbrace{rd(x3,op_l(x1,x2,i(x1)))} \\
= & \quad \text{by Lemma 1073 RL with } \{x4 \leftarrow x3, x3 \leftarrow i(x1), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{rd(op_l(i(x1),x2,i(x1)),i(x3))}} \\
= & \quad \text{by Lemma 1569 RL with } \{x3 \leftarrow i(x3), x2 \leftarrow i(x1), x1 \leftarrow x2\} \\
& \overbrace{rd(asoc(x2,i(x1),i(x1)),\underbrace{rd(i(x3),i(x1))})} \\
= & \quad \text{by Lemma 23 LR with } \{x2 \leftarrow x1, x1 \leftarrow x3\} \\
& rd(asoc(x2,i(x1),i(x1)),\overbrace{rd(x1,x3)})
\end{aligned}
$$

Lemma 1582: $rd(x3,op_l(x1,x1,x2)) = rd(asoc(x2,x1,x1),rd(x1,x3))$

$$
\begin{aligned}
& rd(x3,\underbrace{op_l(x1,x1,x2)}) \\
= & \quad \text{by Lemma 606 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{rd(x3,\overbrace{op_l(x1,x2,i(x1))})} \\
= & \quad \text{by Lemma 1581 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1, x3 \leftarrow x3\} \\
& \overbrace{rd(\underbrace{asoc(x2,i(x1),i(x1))},rd(x1,x3))} \\
= & \quad \text{by Lemma 772 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& rd(\overbrace{asoc(x2,x1,x1)},rd(x1,x3))
\end{aligned}
$$

Lemma 1583: $rd(x3, op_l(x1, i(x1), x2)) = rd(asoc(i(x1), x2, i(x1)), rd(x1, x3))$

$$
\begin{aligned}
& \underbrace{rd(x3, op_l(x1, i(x1), x2))} \\
= & \quad \text{by Lemma 1073 RL with } \{x4 \leftarrow x3, x3 \leftarrow x2, x2 \leftarrow i(x1), x1 \leftarrow x1\} \\
& \overbrace{\underbrace{rd(op_l(i(x1), i(x1), x2), i(x3))}} \\
= & \quad \text{by Lemma 1577 LR with } \{x3 \leftarrow i(x3), x2 \leftarrow x2, x1 \leftarrow i(x1)\} \\
& \overbrace{rd(asoc(i(x1), x2, i(x1)), \underbrace{rd(i(x3), i(x1))})} \\
= & \quad \text{by Lemma 23 LR with } \{x2 \leftarrow x1, x1 \leftarrow x3\} \\
& rd(asoc(i(x1), x2, i(x1)), \overbrace{rd(x1, x3)})
\end{aligned}
$$

Lemma 1584: $rd(x3, op_l(x1, x2, x1)) = rd(asoc(i(x1), x2, i(x1)), rd(x1, x3))$

$$
\begin{aligned}
& rd(x3, \underbrace{op_l(x1, x2, x1)}) \\
= & \quad \text{by Lemma 608 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& rd(x3, \overbrace{\underbrace{op_l(x1, x1, i(x2))}}) \\
= & \quad \text{by Lemma 1050 RL with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x1\} \\
& \underbrace{rd(x3, \overbrace{op_l(x1, i(x1), x2)})} \\
= & \quad \text{by Lemma 1583 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1, x3 \leftarrow x3\} \\
& \overbrace{rd(asoc(i(x1), x2, i(x1)), rd(x1, x3))}
\end{aligned}
$$

Lemma 1585: $mult(mult(x1, asoc(x2, x3, x2)), x2) = mult(x1, mult(asoc(x2, x3, x2), x2))$

$$
\begin{aligned}
& \underbrace{mult(mult(x1, asoc(x2, x3, x2)), x2)} \\
= & \quad \text{by Lemma 91 RL with } \{x3 \leftarrow x2, x2 \leftarrow asoc(x2, x3, x2), x1 \leftarrow x1\} \\
& \overbrace{mult(op_r(x1, \underbrace{asoc(x2, x3, x2)}, x2), mult(asoc(x2, x3, x2), x2))} \\
= & \quad \text{by Lemma 758 RL with } \{x2 \leftarrow x2, x1 \leftarrow x3\} \\
& mult(\underbrace{op_r(x1, \overbrace{asoc(i(x3), x2, x2)}, x2)}, mult(asoc(x2, x3, x2), x2)) \\
= & \quad \text{by Lemma 1578 LR with } \{x3 \leftarrow x2, x2 \leftarrow i(x3), x1 \leftarrow x1\} \\
& mult(\overbrace{x1}, mult(asoc(x2, x3, x2), x2))
\end{aligned}
$$

Lemma 1586: $mult(rd(x1, asoc(x3, x2, x2)), x2) = mult(x1, op_l(x2, x2, x3))$

$$
\begin{aligned}
& mult(\underbrace{rd(x1, asoc(x3, x2, x2))}, x2) \\
= & \quad \text{by Lemma 1295 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{mult(\overbrace{mult(x1, asoc(x2, x3, x2))}, x2)} \\
= & \quad \text{by Lemma 1585 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{mult(x1, \underbrace{mult(asoc(x2, x3, x2), x2)})} \\
= & \quad \text{by Lemma 173 LR with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& mult(x1, \overbrace{op_l(x2, x2, x3)})
\end{aligned}
$$

Lemma 1587: $mult(asoc(x2,x1,x2),op_l(x2,x2,x1)) = op_t(op_l(x2,x2,mult(x1,x1)),asoc(x1,x2,x2))$

$mult(\underbrace{asoc(x2,x1,x2)},op_l(x2,x2,x1))$

$=$ by Lemma 759 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{mult(\overbrace{i(asoc(x1,x2,x2))},op_l(x2,x2,x1))}$

$=$ by Lemma 14 RL with $\{x2 \leftarrow asoc(x1,x2,x2), x1 \leftarrow op_l(x2,x2,x1)\}$

$\overbrace{op_t(\underbrace{rd(op_l(x2,x2,x1),asoc(x1,x2,x2))},asoc(x1,x2,x2))}$

$=$ by Lemma 1515 RL with $\{x1 \leftarrow x1, x2 \leftarrow x2\}$

$op_t(\overbrace{op_l(x2,x2,mult(x1,x1))},asoc(x1,x2,x2))$

Lemma 1588: $mult(asoc(x1,x2,x1),op_l(x1,x1,x2)) = op_l(x1,x1,mult(x2,x2))$

$\underbrace{mult(asoc(x1,x2,x1),op_l(x1,x1,x2))}$

$=$ by Lemma 1587 LR with $\{x1 \leftarrow x2, x2 \leftarrow x1\}$

$\overbrace{op_t(op_l(x1,x1,mult(x2,x2)),asoc(x2,x1,x1))}$

$=$ by Axiom 15 LR with $\{x4 \leftarrow asoc(x2,x1,x1), x3 \leftarrow mult(x2,x2), x2 \leftarrow x1, x1 \leftarrow x1\}$

$\overbrace{op_l(op_t(x1,\underbrace{asoc(x2,x1,x1)},x1,mult(x2,x2))}$

$=$ by Lemma 756 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$op_l(op_t(x1,\overbrace{asoc(x1,i(x2),x1)},x1,mult(x2,x2))$

$=$ by Lemma 172 RL with $\{x2 \leftarrow i(x2), x1 \leftarrow x1\}$

$op_l(\underbrace{op_t(x1,\overbrace{rd(op_l(x1,x1,i(x2)),x1)})},x1,mult(x2,x2))$

$=$ by Axiom 10 RL with $\{x2 \leftarrow x1, x1 \leftarrow rd(op_l(x1,x1,i(x2)),x1)\}$

$op_l(\overbrace{mult(i(rd(op_l(x1,x1,i(x2)),x1)),\underbrace{mult(x1,rd(op_l(x1,x1,i(x2)),x1))})},x1,mult(x2,x2))$

$=$ by Lemma 170 LR with $\{x3 \leftarrow i(x2), x2 \leftarrow x1, x1 \leftarrow x1\}$

$op_l(\underbrace{mult(i(rd(op_l(x1,x1,i(x2)),x1))},\overbrace{op_l(x1,x1,i(x2))}),x1,mult(x2,x2))$

$=$ by Lemma 10 LR with $\{x2 \leftarrow x1, x1 \leftarrow op_l(x1,x1,i(x2))\}$

$op_l(\underbrace{mult(\overbrace{rd(x1,op_l(x1,x1,i(x2)))},op_l(x1,x1,i(x2)))},x1,mult(x2,x2))$

$=$ by Axiom 4 RL with $\{x2 \leftarrow op_l(x1,x1,i(x2)), x1 \leftarrow x1\}$

$op_l(\overbrace{x1},x1,mult(x2,x2))$

Lemma 1589: $op_l(x1, x1, rd(x2, x3)) = op_l(x1, rd(x3, x2), x1)$

$$
\begin{aligned}
& \underbrace{op_l(x1, x1, rd(x2, x3))} \\
= & \quad \text{by Lemma 1520 LR with } \{x2 \leftarrow x3, x3 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, x1, rd(op_t(x2, i(x1)), x3))}} \\
= & \quad \text{by Lemma 173 RL with } \{x2 \leftarrow rd(op_t(x2, i(x1)), x3), x1 \leftarrow x1\} \\
& \overbrace{mult(asoc(x1, \underbrace{rd(op_t(x2, i(x1)), x3)}, x1), x1)} \\
= & \quad \text{by Lemma 1393 RL with } \{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow i(x1)\} \\
& mult(\underbrace{asoc(x1, \overbrace{rd(i(x1), mult(rd(x3, x2), op_t(i(x1), x2)))}, x1)}, x1) \\
= & \quad \text{by Lemma 262 RL with } \{x2 \leftarrow rd(i(x1), mult(rd(x3, x2), op_t(i(x1), x2))), x1 \leftarrow x1\} \\
& mult(\overbrace{asoc(x1, \underbrace{mult(rd(i(x1), mult(rd(x3, x2), op_t(i(x1), x2))), x1)}, x1)}, x1) \\
= & \quad \text{by Lemma 21 LR with } \{x2 \leftarrow mult(rd(x3, x2), op_t(i(x1), x2)), x1 \leftarrow x1\} \\
& mult(\underbrace{asoc(x1, \overbrace{i(mult(rd(x3, x2), op_t(i(x1), x2)))}, x1)}, x1) \\
= & \quad \text{by Lemma 756 LR with } \{x2 \leftarrow mult(rd(x3, x2), op_t(i(x1), x2)), x1 \leftarrow x1\} \\
& mult(\overbrace{asoc(mult(rd(x3, x2), op_t(i(x1), x2)), x1, x1)}, x1) \\
= & \quad \text{by Lemma 1041 LR with } \{x3 \leftarrow x2, x1 \leftarrow x1, x2 \leftarrow rd(x3, x2)\} \\
& mult(\overbrace{mult(x1, op_l(i(x1), x1, rd(x3, x2)))}, x1) \\
= & \quad \text{by Lemma 778 LR with } \{x2 \leftarrow rd(x3, x2), x1 \leftarrow x1\} \\
& \underbrace{mult(\overbrace{asoc(rd(x3, x2), x1, x1)}, x1)} \\
= & \quad \text{by Lemma 771 RL with } \{x2 \leftarrow rd(x3, x2), x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, rd(x3, x2), x1)}
\end{aligned}
$$

Lemma 1590: $op_l(mult(x1, x1), x1, i(x2)) = mult(op_l(x1, x1, i(x2)), op_l(x1, x1, rd(i(x1), x2)))$

$$
\begin{aligned}
& \underbrace{op_l(mult(x1, x1), x1, i(x2))} \\
= & \quad \text{by Lemma 296 RL with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow mult(x1, x1)\} \\
& \overbrace{\underbrace{op_l(mult(x1, x1), x1, rd(i(x1), x2))}} \\
= & \quad \text{by Lemma 1537 RL with } \{x2 \leftarrow rd(i(x1), x2), x1 \leftarrow x1\} \\
& \overbrace{mult(\underbrace{op_l(x1, x1, rd(i(x1), x2))}, op_l(x1, x1, rd(i(x1), x2)))} \\
= & \quad \text{by Lemma 296 LR with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x1\} \\
& mult(\overbrace{op_l(x1, x1, i(x2))}, op_l(x1, x1, rd(i(x1), x2)))
\end{aligned}
$$

Lemma 1591: $op_l(mult(x1,x1),x2,x1) = mult(op_l(x1,x2,x1),op_l(x1,x1,i(x2)))$

$$
\begin{aligned}
& \underbrace{op_l(mult(x1,x1),x2,x1)} \\
= & \quad \text{by Lemma 1463 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(mult(x1,x1),x1,i(x2))}} \\
= & \quad \text{by Lemma 1590 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{mult(\underbrace{op_l(x1,x1,i(x2))},op_l(x1,x1,rd(i(x1),x2)))} \\
= & \quad \text{by Lemma 608 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& mult(\overbrace{op_l(x1,x2,x1)},\underbrace{op_l(x1,x1,rd(i(x1),x2))}) \\
= & \quad \text{by Lemma 296 LR with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x1\} \\
& mult(op_l(x1,x2,x1),\overbrace{op_l(x1,x1,i(x2))})
\end{aligned}
$$

Lemma 1592: $mult(i(rd(op_t(x1,i(x2)),x2)),x1) = op_r(rd(x1,mult(i(x2),x1)),x1,rd(op_t(x1,i(x2)),x2))$

$$
\begin{aligned}
& \underbrace{mult(i(rd(op_t(x1,i(x2)),x2)),x1)} \\
= & \quad \text{by Lemma 1551 LR with } \{x2 \leftarrow x1, x1 \leftarrow rd(op_t(x1,i(x2)),x2)\} \\
& \overbrace{op_r(rd(x1,op_t(rd(op_t(x1,i(x2)),x2),x1)),x1,\underbrace{i(i(rd(op_t(x1,i(x2)),x2))))}} \\
= & \quad \text{by Lemma 3 LR with } \{x1 \leftarrow rd(op_t(x1,i(x2)),x2)\} \\
& op_r(rd(x1,\underbrace{op_t(rd(op_t(x1,i(x2)),x2),x1)}),x1,\overbrace{rd(op_t(x1,i(x2)),x2)}) \\
= & \quad \text{by Lemma 910 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_r(rd(x1,\overbrace{mult(i(x2),x1)}),x1,rd(op_t(x1,i(x2)),x2))
\end{aligned}
$$

Lemma 1593: $rd(x1, mult(x1, i(x2))) = op_r(i(i(x2)), x1, rd(op_t(x1, i(x2)), x2))$

$rd(x1, \underbrace{mult(x1, i(x2))}$

$=$ by Lemma 16 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{rd(x1, \overbrace{i(mult(i(x1), x2))})$

$=$ by Lemma 359 RL with $\{x2 \leftarrow x1, x1 \leftarrow mult(i(x1), x2)\}$

$\overbrace{op_r(\underbrace{mult(mult(i(x1), x2), x1)}, x1, x1)}$

$=$ by Lemma 71 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$op_r(\overbrace{mult(i(x1), rd(x1, i(x2)))}, x1, x1)$

$=$ by Lemma 14 RL with $\{x2 \leftarrow x1, x1 \leftarrow rd(x1, i(x2))\}$

$\underbrace{op_r(\overbrace{op_t(rd(rd(x1, i(x2)), x1), x1)}, x1, x1)}$

$=$ by Axiom 14 RL with $\{x4 \leftarrow x1, x3 \leftarrow x1, x2 \leftarrow x1, x1 \leftarrow rd(rd(x1, i(x2)), x1)\}$

$\overbrace{op_t(\underbrace{op_r(rd(rd(x1, i(x2)), x1), x1, x1)}, x1)}$

$=$ by Lemma 370 LR with $\{x1 \leftarrow x1, x2 \leftarrow rd(x1, i(x2))\}$

$\underbrace{op_t(\overbrace{rd(op_r(rd(x1, i(x2)), x1, x1), x1)}, x1)}$

$=$ by Lemma 14 LR with $\{x2 \leftarrow x1, x1 \leftarrow op_r(rd(x1, i(x2)), x1, x1)\}$

$\underbrace{\overbrace{mult(i(x1), op_r(rd(x1, i(x2)), x1, x1))}$

$=$ by Lemma 956 RL with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$\overbrace{mult(\underbrace{rd(x2, op_t(x1, i(x2)))}, x1)}$

$=$ by Lemma 10 RL with $\{x2 \leftarrow x2, x1 \leftarrow op_t(x1, i(x2))\}$

$\underbrace{mult(\overbrace{i(rd(op_t(x1, i(x2)), x2))}, x1)}$

$=$ by Lemma 1592 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_r(\underbrace{rd(x1, mult(i(x2), x1))}, x1, rd(op_t(x1, i(x2)), x2))}$

$=$ by Lemma 9 LR with $\{x2 \leftarrow i(x2), x1 \leftarrow x1\}$

$op_r(\overbrace{i(i(x2))}, x1, rd(op_t(x1, i(x2)), x2))$

Lemma 1594: $op_r(x1, x2, rd(x2, x1)) = op_r(op_r(op_t(x1, i(x2)), x2, rd(x2, x1)), x2, rd(op_t(x2, i(x1)), x1))$

$\overbrace{op_r(x1, x2, rd(x2, x1))}$

$=$ by Lemma 1548 RL with $\{x2 \leftarrow x2, x1 \leftarrow op_r(x1, x2, rd(x2, x1))\}$

$\overbrace{op_r(op_t(op_r(x1, x2, rd(x2, x1)), i(x2)), x2, rd(x2, \underbrace{op_r(x1, x2, rd(x2, x1))}))}$

$=$ by Lemma 3 RL with $\{x1 \leftarrow op_r(x1, x2, rd(x2, x1))\}$

$op_r(op_t(op_r(x1, x2, rd(x2, x1)), i(x2)), x2, \underbrace{rd(x2, \overbrace{i(i(op_r(x1, x2, rd(x2, x1))))})})$

$=$ by Lemma 359 RL with $\{x2 \leftarrow x2, x1 \leftarrow i(op_r(x1, x2, rd(x2, x1)))\}$

$op_r(op_t(op_r(x1, x2, rd(x2, x1)), i(x2)), x2, \overbrace{\underbrace{op_r(mult(\overbrace{i(op_r(x1, x2, rd(x2, x1)))}, x2), x2, x2)}})$

$=$ by Lemma 641 RL with $\{x2 \leftarrow x2, x1 \leftarrow mult(i(op_r(x1, x2, rd(x2, x1))), x2)\}$

$op_r(op_t(op_r(x1, x2, rd(x2, x1)), i(x2)), x2, \overbrace{rd(mult(i(op_r(x1, x2, rd(x2, x1))), x2), \underbrace{asoc(x2, mult(i(op_r(x1, x2, rd(x2, x1))),$

$=$ by Lemma 262 LR with $\{x2 \leftarrow i(op_r(x1, x2, rd(x2, x1))), x1 \leftarrow x2\}$

$op_r(op_t(op_r(x1, x2, rd(x2, x1)), i(x2)), x2, rd(mult(i(op_r(x1, x2, rd(x2, x1))), x2), \overbrace{\underbrace{asoc(x2, i(op_r(x1, x2, rd(x2, x1))), x2)}}))$

$=$ by Lemma 756 LR with $\{x2 \leftarrow op_r(x1, x2, rd(x2, x1)), x1 \leftarrow x2\}$

$op_r(op_t(op_r(x1, x2, rd(x2, x1)), i(x2)), x2, rd(mult(i(op_r(x1, x2, rd(x2, x1))), x2), \overbrace{\underbrace{asoc(op_r(x1, x2, rd(x2, x1)), x2, x2)}}))$

$=$ by Lemma 1552 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$op_r(op_t(op_r(x1, x2, rd(x2, x1)), i(x2)), x2, rd(mult(\underbrace{i(op_r(x1, x2, rd(x2, x1)))}, x2), \overbrace{asoc(x1, x2, x2)}))$

$=$ by Lemma 1543 RL with $\{x1 \leftarrow x1, x2 \leftarrow x2\}$

$op_r(op_t(op_r(x1, x2, rd(x2, x1)), i(x2)), x2, rd(\underbrace{mult(\overbrace{rd(mult(x2, i(x1)), x2)}, x2)}, asoc(x1, x2, x2)))$

$=$ by Axiom 4 RL with $\{x2 \leftarrow x2, x1 \leftarrow mult(x2, i(x1))\}$

$op_r(op_t(op_r(x1, x2, rd(x2, x1)), i(x2)), x2, \underbrace{rd(\overbrace{mult(x2, i(x1))}, asoc(x1, x2, x2))})$

$=$ by Lemma 1494 LR with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$op_r(\underbrace{op_t(op_r(x1, x2, rd(x2, x1)), i(x2))}, x2, \overbrace{rd(op_t(x2, i(x1)), x1)})$

$=$ by Axiom 14 LR with $\{x4 \leftarrow i(x2), x3 \leftarrow rd(x2, x1), x2 \leftarrow x2, x1 \leftarrow x1\}$

$op_r(\overbrace{op_r(op_t(x1, i(x2)), x2, rd(x2, x1))}, x2, rd(op_t(x2, i(x1)), x1))$

Lemma 1595: $op_r(x1, x2, rd(x2, x1)) = rd(x2, mult(x2, i(x1)))$

$\overbrace{op_r(x1, x2, rd(x2, x1))}$

$=$ by Lemma 1594 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_r(\underbrace{op_r(op_t(x1, i(x2)), x2, rd(x2, x1))}, x2, rd(op_t(x2, i(x1)), x1))}$

$=$ by Lemma 1548 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$op_r(\overbrace{\underbrace{x1}}, x2, rd(op_t(x2, i(x1)), x1))$

$=$ by Lemma 3 RL with $\{x1 \leftarrow x1\}$

$\underbrace{op_r(\overbrace{i(i(x1))}, x2, rd(op_t(x2, i(x1)), x1))}$

$=$ by Lemma 1593 RL with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$\overbrace{rd(x2, \overbrace{mult(x2, i(x1))})}$

Lemma 1596: $mult(asoc(x1,x2,x1),op_l(x1,x1,x2)) = op_t(rd(mult(op_l(x1,x1,x2),op_l(x1,x1,x2)),x1),op_l(x1,x1,x2))$

$mult(\underbrace{asoc(x1,x2,x1)},op_l(x1,x1,x2))$

= by Lemma 172 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$mult(\overbrace{rd(op_l(x1,x1,x2),x1)},op_l(x1,x1,x2))$

= by Lemma 10 RL with $\{x2 \leftarrow op_l(x1,x1,x2), x1 \leftarrow x1\}$

$\underbrace{mult(\overbrace{i(rd(x1,op_l(x1,x1,x2)))},op_l(x1,x1,x2))}$

= by Lemma 1474 LR with $\{x2 \leftarrow op_l(x1,x1,x2), x1 \leftarrow rd(x1,op_l(x1,x1,x2))\}$

$\overbrace{rd(\underbrace{op_t(op_l(x1,x1,x2),mult(rd(x1,op_l(x1,x1,x2)),i(op_l(x1,x1,x2))))},op_t(rd(x1,op_l(x1,x1,x2)),i(op_l(x1,x1,x2))))}$

= by Lemma 551 LR with $\{x2 \leftarrow rd(x1,op_l(x1,x1,x2)), x1 \leftarrow op_l(x1,x1,x2)\}$

$rd(\overbrace{op_t(op_l(x1,x1,x2),rd(rd(x1,op_l(x1,x1,x2)),\underbrace{op_l(x1,x1,x2)}))},op_t(rd(x1,op_l(x1,x1,x2)),i(op_l(x1,x1,x2))))$

= by Lemma 773 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$rd(op_t(op_l(x1,x1,x2),rd(rd(x1,op_l(x1,x1,x2)),\overbrace{rd(x1,asoc(x2,x1,x1))})),op_t(rd(x1,op_l(x1,x1,x2)),i(op_l(x1,x1,x2))))$

= by Lemma 1031 RL with $\{x3 \leftarrow x2, x2 \leftarrow u, x1 \leftarrow x1\}$

$rd(\underbrace{op_t(op_l(x1,x1,x2),rd(rd(x1,op_l(x1,x1,x2)),\overbrace{op_l(x1,rd(op_t(x1,u),x2),x1)}))},op_t(rd(x1,op_l(x1,x1,x2)),i(op_l(x1,x1,x$

= by Lemma 1573 LR with $\{x3 \leftarrow rd(op_t(x1,u),x2), x2 \leftarrow x1, x1 \leftarrow op_l(x1,x1,x2)\}$

$\underbrace{rd(\overbrace{op_l(x1,x1,x2)},op_t(rd(x1,op_l(x1,x1,x2)),i(op_l(x1,x1,x2))))}$

= by Lemma 923 RL with $\{x2 \leftarrow rd(x1,op_l(x1,x1,x2)), x1 \leftarrow op_l(x1,x1,x2)\}$

$\overbrace{\underbrace{mult(i(op_l(x1,x1,x2)),rd(op_l(x1,x1,x2),rd(rd(x1,op_l(x1,x1,x2)),op_l(x1,x1,x2))))}}$

= by Lemma 45 RL with $\{x3 \leftarrow op_l(x1,x1,x2), x2 \leftarrow rd(rd(x1,op_l(x1,x1,x2)),op_l(x1,x1,x2)), x1 \leftarrow op_l(x1,x1,x2)\}$

$\overbrace{\underbrace{i(mult(op_l(x1,x1,x2),rd(rd(rd(x1,op_l(x1,x1,x2)),op_l(x1,x1,x2)),op_l(x1,x1,x2))))}}$

= by Axiom 11 RL with $\{x2 \leftarrow rd(rd(rd(x1,op_l(x1,x1,x2)),op_l(x1,x1,x2)),op_l(x1,x1,x2)), x1 \leftarrow op_l(x1,x1,x2)\}$

$\overbrace{\underbrace{mult(i(op_l(x1,x1,x2)),i(rd(rd(rd(x1,op_l(x1,x1,x2)),op_l(x1,x1,x2)),op_l(x1,x1,x2))))}}$

= by Lemma 1145 RL with $\{x1 \leftarrow x1, x2 \leftarrow op_l(x1,x1,x2)\}$

$\overbrace{op_t(rd(mult(op_l(x1,x1,x2),op_l(x1,x1,x2)),x1),\underbrace{i(i(op_l(x1,x1,x2)))})}$

= by Lemma 3 LR with $\{x1 \leftarrow op_l(x1,x1,x2)\}$

$op_t(rd(mult(op_l(x1,x1,x2),op_l(x1,x1,x2)),x1),\overbrace{op_l(x1,x1,x2)})$

Lemma 1597: $mult(op_l(x1, mult(x1, x1), x2), x2) = mult(x2, op_t(x1, i(x2)))$

$mult(op_l(\underbrace{x1}, mult(x1, x1), x2), x2)$

$=$ by Lemma 169 RL with $\{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x1\}$

$mult(\overbrace{op_l(\overbrace{op_t(x1, op_l(x1, x1, x2))}, mult(x1, x1), x2)}, x2)$

$=$ by Axiom 15 RL with $\{x4 \leftarrow op_l(x1, x1, x2), x3 \leftarrow x2, x2 \leftarrow mult(x1, x1), x1 \leftarrow x1\}$

$mult(\overbrace{op_t(\underbrace{op_l(x1, mult(x1, x1), x2)}, op_l(x1, x1, x2))}, x2)$

$=$ by Lemma 1528 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$mult(op_t(\overbrace{rd(\underbrace{op_l(mult(x1, x1), x1, x2)}, x1)}, op_l(x1, x1, x2)), x2)$

$=$ by Lemma 1537 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$mult(\underbrace{op_t(rd(\overbrace{mult(op_l(x1, x1, x2), op_l(x1, x1, x2))}, x1), op_l(x1, x1, x2))}, x2)$

$=$ by Lemma 1596 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$mult(\overbrace{mult(asoc(x1, x2, x1), op_l(x1, x1, x2))}, x2)$

$=$ by Lemma 1588 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{mult(\overbrace{op_l(x1, x1, mult(x2, x2))}, x2)}$

$=$ by Lemma 167 RL with $\{x4 \leftarrow mult(x2, x2), x3 \leftarrow x1, x2 \leftarrow x1, x1 \leftarrow x2\}$

$\overbrace{mult(x2, \underbrace{op_l(op_t(x1, x2), x1, mult(x2, x2))})}$

$=$ by Lemma 1130 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$mult(x2, \overbrace{op_t(x1, i(x2))})$

Lemma 1598: $asoc(op_l(x1, x2, mult(x1, x1)), x2, x2) = asoc(mult(i(x2), op_t(x1, i(i(x2)))), x2, x2)$

$asoc(\underbrace{op_l(x1, x2, mult(x1, x1))}, x2, x2)$

$=$ by Lemma 1529 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{asoc(\overbrace{op_l(x1, mult(x1, x1), i(x2))}, x2, x2)}$

$=$ by Lemma 779 RL with $\{x2 \leftarrow x2, x1 \leftarrow op_l(x1, mult(x1, x1), i(x2))\}$

$\overbrace{asoc(\underbrace{mult(op_l(x1, mult(x1, x1), i(x2)), i(x2))}, x2, x2)}$

$=$ by Lemma 1597 LR with $\{x2 \leftarrow i(x2), x1 \leftarrow x1\}$

$asoc(\overbrace{mult(i(x2), op_t(x1, i(i(x2))))}, x2, x2)$

Lemma 1599: $asoc(op_l(x1, x2, mult(x1, x1)), x2, x2) = asoc(op_t(x1, i(i(x2))), x2, x2)$

$\underbrace{asoc(op_l(x1, x2, mult(x1, x1)), x2, x2)}$

$=$ by Lemma 1598 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{\overbrace{asoc(mult(i(x2), op_t(x1, i(i(x2)))), x2, x2)}}$

$=$ by Lemma 780 LR with $\{x2 \leftarrow op_t(x1, i(i(x2))), x1 \leftarrow x2\}$

$\underbrace{\overbrace{i(asoc(x2, op_t(x1, i(i(x2))), x2))}}$

$=$ by Lemma 757 LR with $\{x2 \leftarrow op_t(x1, i(i(x2))), x1 \leftarrow x2\}$

$\overbrace{asoc(op_t(x1, i(i(x2))), x2, x2)}$

Lemma 1600: $asoc(op_l(x1, x2, mult(x1, x1)), x2, x2) = asoc(x1, x2, x2)$

$$
\begin{aligned}
& \underbrace{asoc(op_l(x1, x2, mult(x1, x1)), x2, x2)} \\
= & \quad \text{by Lemma 1599 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{asoc(op_t(x1, \underbrace{i(i(x2))}), x2, x2)} \\
= & \quad \text{by Lemma 3 LR with } \{x1 \leftarrow x2\} \\
& \underbrace{asoc(op_t(x1, \overbrace{x2}), x2, x2)} \\
= & \quad \text{by Lemma 769 LR with } \{x1 \leftarrow x2, x2 \leftarrow x1\} \\
& \overbrace{asoc(x1, x2, x2)}
\end{aligned}
$$

Lemma 1601: $op_l(x1, op_l(x2, x3, mult(x2, x2)), x3) = op_l(x1, mult(i(x3), op_t(x2, i(i(x3)))), x3)$

$$
\begin{aligned}
& op_l(x1, \underbrace{op_l(x2, x3, mult(x2, x2))}, x3) \\
= & \quad \text{by Lemma 1529 RL with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& \underbrace{op_l(x1, \overbrace{op_l(x2, mult(x2, x2), i(x3))}, x3)} \\
= & \quad \text{by Lemma 1323 RL with } \{x3 \leftarrow x3, x2 \leftarrow op_l(x2, mult(x2, x2), i(x3)), x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, \underbrace{mult(op_l(x2, mult(x2, x2), i(x3)), i(x3))}, x3)} \\
= & \quad \text{by Lemma 1597 LR with } \{x2 \leftarrow i(x3), x1 \leftarrow x2\} \\
& op_l(x1, \overbrace{mult(i(x3), op_t(x2, i(i(x3))))}, x3)
\end{aligned}
$$

Lemma 1602: $op_l(x1, op_l(x2, x3, mult(x2, x2)), x3) = op_l(x1, x3, op_t(i(x2), i(x3)))$

$$
\begin{aligned}
& \underbrace{op_l(x1, op_l(x2, x3, mult(x2, x2)), x3)} \\
= & \quad \text{by Lemma 1601 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, mult(i(x3), op_t(x2, i(i(x3)))), x3)}} \\
= & \quad \text{by Lemma 1054 LR with } \{x3 \leftarrow op_t(x2, i(i(x3))), x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x3, \underbrace{i(op_t(x2, i(i(x3))))})} \\
= & \quad \text{by Lemma 19 LR with } \{x2 \leftarrow i(x3), x1 \leftarrow x2\} \\
& op_l(x1, x3, \overbrace{op_t(i(x2), i(x3))})
\end{aligned}
$$

Lemma 1603: $op_l(x1, rd(x2, i(mult(x1, x2))), x1) = mult(asoc(x1, rd(i(x1), x2), x1), op_l(x1, x1, i(x2)))$

$op_l(x1, rd(x2, \underbrace{i(mult(x1, x2))}), x1)$

$=$ by Axiom 11 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$op_l(x1, \underbrace{rd(x2, \overbrace{mult(i(x1), i(x2))})}, x1)$

$=$ by Lemma 61 RL with $\{x3 \leftarrow x2, x2 \leftarrow i(x2), x1 \leftarrow x1\}$

$op_l(x1, \overbrace{rd(mult(x1, \underbrace{i(i(x2))}), i(x2))}, x1)$

$=$ by Lemma 332 RL with $\{x2 \leftarrow i(x2), x1 \leftarrow x1\}$

$op_l(x1, \overbrace{mult(\underbrace{rd(x1, i(x2))}, i(i(x2)))}, x1)$

$=$ by Lemma 38 LR with $\{x1 \leftarrow x2, x2 \leftarrow x1\}$

$op_l(x1, mult(\overbrace{rd(x2, i(x1))}, \underbrace{i(i(x2))}), x1)$

$=$ by Lemma 3 LR with $\{x1 \leftarrow x2\}$

$\underbrace{op_l(x1, mult(rd(x2, i(x1)), \overbrace{x2}), x1)}$

$=$ by Lemma 608 RL with $\{x2 \leftarrow mult(rd(x2, i(x1)), x2), x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_l(x1, x1, i(mult(rd(x2, i(x1)), x2)))}}$

$=$ by Lemma 296 RL with $\{x3 \leftarrow mult(rd(x2, i(x1)), x2), x2 \leftarrow x1, x1 \leftarrow x1\}$

$\overbrace{op_l(x1, x1, rd(i(x1), mult(\underbrace{rd(x2, i(x1))}, x2)))}$

$=$ by Lemma 10 RL with $\{x2 \leftarrow x2, x1 \leftarrow i(x1)\}$

$op_l(x1, x1, rd(\underbrace{i(x1)}), mult(\overbrace{i(rd(i(x1), x2))}, x2)))$

$=$ by Axiom 4 LR with $\{x2 \leftarrow x2, x1 \leftarrow i(x1)\}$

$op_l(x1, x1, rd(\overbrace{mult(rd(i(x1), x2), \underbrace{x2})}), mult(i(rd(i(x1), x2)), x2)))$

$=$ by Lemma 12 RL with $\{x2 \leftarrow x2, x1 \leftarrow rd(i(x1), x2)\}$

$op_l(x1, x1, \underbrace{rd(mult(rd(i(x1), x2), \overbrace{mult(rd(i(x1), x2), mult(i(rd(i(x1), x2)), x2))}), mult(i(rd(i(x1), x2)), x2))})$

$=$ by Lemma 41 LR with $\{x2 \leftarrow mult(i(rd(i(x1), x2)), x2), x1 \leftarrow rd(i(x1), x2)\}$

$\underbrace{op_l(x1, x1, \overbrace{mult(rd(i(x1), x2), rd(i(x1), x2))})}$

$=$ by Lemma 1588 RL with $\{x2 \leftarrow rd(i(x1), x2), x1 \leftarrow x1\}$

$\overbrace{mult(asoc(x1, rd(i(x1), x2), x1), \underbrace{op_l(x1, x1, rd(i(x1), x2))})}$

$=$ by Lemma 296 LR with $\{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x1\}$

$mult(asoc(x1, rd(i(x1), x2), x1), \overbrace{op_l(x1, x1, i(x2))})$

Lemma 1604: $mult(asoc(x1, x2, x2), op_l(x2, x1, x2)) = mult(i(x2), op_l(mult(x2, x2), x1, x2))$

$mult(\underbrace{asoc(x1, x2, x2)}, op_l(x2, x1, x2))$

$=$ by Lemma 765 RL with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$mult(\overbrace{rd(op_l(x2, x1, x2), x2)}, op_l(x2, x1, x2))$

$=$ by Lemma 171 RL with $\{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x2\}$

$\underbrace{mult(\overbrace{mult(i(x2), op_l(x2, x1, x2))}, op_l(x2, x1, x2))}$

$=$ by Lemma 91 RL with $\{x3 \leftarrow op_l(x2, x1, x2), x2 \leftarrow op_l(x2, x1, x2), x1 \leftarrow i(x2)\}$

$\overbrace{mult(op_r(i(x2), \underbrace{op_l(x2, x1, x2)}, op_l(x2, x1, x2)), mult(op_l(x2, x1, x2), op_l(x2, x1, x2)))}$

$=$ by Lemma 609 RL with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$mult(op_r(i(x2), \overbrace{op_l(\underbrace{x2}, i(x2), x1)}, op_l(x2, x1, x2)), mult(op_l(x2, x1, x2), op_l(x2, x1, x2)))$

$=$ by Lemma 3 RL with $\{x1 \leftarrow x2\}$

$mult(op_r(i(x2), \underbrace{op_l(\overbrace{i(i(x2))}, i(x2), x1)}, op_l(x2, x1, x2)), mult(op_l(x2, x1, x2), op_l(x2, x1, x2)))$

$=$ by Lemma 633 RL with $\{x2 \leftarrow x1, x1 \leftarrow i(x2)\}$

$mult(op_r(i(x2), \overbrace{\underbrace{op_l(i(i(x2)), x1, i(i(x2)))}}, op_l(x2, x1, x2)), mult(op_l(x2, x1, x2), op_l(x2, x1, x2)))$

$=$ by Axiom 12 RL with $\{x3 \leftarrow i(i(x2)), x2 \leftarrow x1, x1 \leftarrow i(i(x2))\}$

$mult(op_r(i(x2), \overbrace{mult(i(mult(i(i(x2)), x1)), \underbrace{mult(i(i(x2)), mult(x1, i(i(x2))))})}, op_l(x2, x1, x2)), mult(op_l(x2, x1, x2), op_l(x2$

$=$ by Lemma 58 RL with $\{x2 \leftarrow x1, x1 \leftarrow i(i(x2))\}$

$mult(\underbrace{op_r(i(x2), mult(i(mult(i(i(x2)), x1)), \overbrace{rd(i(i(x2)), i(mult(i(i(x2)), x1)))})}, op_l(x2, x1, x2)), mult(op_l(x2, x1, x2), op_l(x2$

$=$ by Lemma 844 LR with $\{x3 \leftarrow op_l(x2, x1, x2), x2 \leftarrow i(mult(i(i(x2)), x1)), x1 \leftarrow i(x2)\}$

$mult(\overbrace{i(x2)}, mult(op_l(x2, x1, x2), \underbrace{op_l(x2, x1, x2)}))$

$=$ by Lemma 608 RL with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$mult(i(x2), \underbrace{mult(op_l(x2, x1, x2), \overbrace{op_l(x2, x2, i(x1))})})$

$=$ by Lemma 1591 RL with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$mult(i(x2), \overbrace{op_l(mult(x2, x2), x1, x2)})$

Lemma 1605: $rd(x1, i(op_t(x2, mult(x2, x1)))) = mult(op_l(x2, x1, mult(x2, x2)), x1)$

$\overbrace{rd(x1, i(op_t(x2, mult(x2, x1))))}$

$=$ by Lemma 357 RL with $\{x2 \leftarrow x1, x1 \leftarrow op_t(x2, mult(x2, x1))\}$

$\overbrace{mult(\underbrace{op_r(op_t(x2, mult(x2, x1)), x1, x1)}, x1)}$

$=$ by Lemma 1170 LR with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$mult(\overbrace{\underbrace{op_l(x2, mult(x1, x1), x2)}}, x1)$

$=$ by Lemma 1514 RL with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$mult(\overbrace{\underbrace{op_l(x2, rd(x1, i(mult(x2, x1))), x2)}}, x1)$

$=$ by Lemma 1603 LR with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$mult(\overbrace{mult(\underbrace{asoc(x2, rd(i(x2), x1), x2)}, op_l(x2, x2, i(x1)))}, x1)$

$=$ by Lemma 262 RL with $\{x2 \leftarrow rd(i(x2), x1), x1 \leftarrow x2\}$

$mult(mult(\overbrace{asoc(x2, \underbrace{mult(rd(i(x2), x1), x2)}, x2)}, op_l(x2, x2, i(x1))), x1)$

$=$ by Lemma 21 LR with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$mult(mult(\underbrace{asoc(x2, \overbrace{i(x1)}, x2)}, op_l(x2, x2, i(x1))), x1)$

$=$ by Lemma 756 LR with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$mult(mult(\overbrace{asoc(x1, x2, x2)}, \underbrace{op_l(x2, x2, i(x1))}), x1)$

$=$ by Lemma 608 LR with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$mult(\underbrace{mult(asoc(x1, x2, x2), \overbrace{op_l(x2, x1, x2)})}, x1)$

$=$ by Lemma 1604 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$mult(\overbrace{mult(i(x2), \underbrace{op_l(mult(x2, x2), x1, x2)})}, x1)$

$=$ by Lemma 1463 RL with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$mult(mult(i(x2), \overbrace{\underbrace{op_l(mult(x2, x2), x2, i(x1))}}), x1)$

$=$ by Lemma 1531 LR with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$mult(\underbrace{mult(i(x2), \overbrace{mult(x2, op_l(x2, x1, mult(x2, x2)))})}, x1)$

$=$ by Axiom 2 RL with $\{x2 \leftarrow x2, x1 \leftarrow op_l(x2, x1, mult(x2, x2))\}$

$mult(\overbrace{op_l(x2, x1, \underbrace{mult(x2, x2)})}, x1)$

Lemma 1606: $mult(op_l(x1, mult(i(x1), i(x1)), x2), i(x2)) = i(mult(x2, op_t(i(x1), i(x2))))$

$\underbrace{mult(op_l(x1, mult(i(x1), i(x1)), x2), i(x2))}$

$=$ by Lemma 16 RL with $\{x2 \leftarrow x2, x1 \leftarrow op_l(x1, mult(i(x1), i(x1)), x2)\}$

$\overbrace{i(mult(\underbrace{i(op_l(x1, mult(i(x1), i(x1)), x2))}, x2))}$

$=$ by Lemma 1049 LR with $\{x3 \leftarrow x2, x2 \leftarrow mult(i(x1), i(x1)), x1 \leftarrow x1\}$

$i(\underbrace{mult(\overbrace{op_l(i(x1), mult(i(x1), i(x1)), x2)}, x2)})$

$=$ by Lemma 1597 LR with $\{x2 \leftarrow x2, x1 \leftarrow i(x1)\}$

$i(\overbrace{mult(x2, op_t(i(x1), i(x2)))})$

Lemma 1607: $op_l(op_l(x1, x2, mult(x1, x1)), op_l(x1, x2, mult(x1, x1)), x2) = mult(op_l(i(i(x2)), i(x2), op_l(x1, x2, mult(x1, x1))), m$

$\underbrace{op_l(op_l(x1, x2, mult(x1, x1)), op_l(x1, x2, mult(x1, x1)), x2)}$

$=$ by Lemma 606 RL with $\{x2 \leftarrow x2, x1 \leftarrow op_l(x1, x2, mult(x1, x1))\}$

$\overbrace{\underbrace{op_l(op_l(x1, x2, mult(x1, x1)), x2, i(op_l(x1, x2, mult(x1, x1))))}}$

$=$ by Lemma 1050 RL with $\{x3 \leftarrow op_l(x1, x2, mult(x1, x1)), x2 \leftarrow x2, x1 \leftarrow op_l(x1, x2, mult(x1, x1))\}$

$\overbrace{\underbrace{op_l(op_l(x1, x2, mult(x1, x1)), i(x2), op_l(x1, x2, mult(x1, x1)))}}$

$=$ by Lemma 960 LR with $\{x2 \leftarrow i(x2), x1 \leftarrow op_l(x1, x2, mult(x1, x1))\}$

$\overbrace{mult(op_l(i(i(x2)), i(x2), op_l(x1, x2, mult(x1, x1))), mult(\underbrace{op_l(x1, x2, mult(x1, x1))}, i(x2)))}$

$=$ by Lemma 1529 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$mult(op_l(i(i(x2)), i(x2), op_l(x1, x2, mult(x1, x1))), mult(\overbrace{op_l(x1, mult(x1, x1), i(x2))}, i(x2)))$

$=$ by Lemma 1066 RL with $\{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x1\}$

$mult(op_l(i(i(x2)), i(x2), op_l(x1, x2, mult(x1, x1))), mult(\overbrace{op_l(x1, \underbrace{rd(i(x1), x1)}, x2)}, i(x2)))$

$=$ by Lemma 43 RL with $\{x1 \leftarrow x1\}$

$mult(op_l(i(i(x2)), i(x2), op_l(x1, x2, mult(x1, x1))), mult(op_l(x1, \overbrace{i(mult(x1, x1))}, x2), i(x2)))$

$=$ by Axiom 11 RL with $\{x2 \leftarrow x1, x1 \leftarrow x1\}$

$mult(op_l(i(i(x2)), i(x2), op_l(x1, x2, mult(x1, x1))), \underbrace{mult(op_l(x1, \overbrace{mult(i(x1), i(x1))}, x2), i(x2)))}$

$=$ by Lemma 1606 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$mult(op_l(i(i(x2)), i(x2), op_l(x1, x2, mult(x1, x1))), \overbrace{i(mult(x2, op_t(i(x1), i(x2))))})$

$=$ by Axiom 11 RL with $\{x2 \leftarrow op_t(i(x1), i(x2)), x1 \leftarrow x2\}$

$mult(op_l(i(i(x2)), i(x2), op_l(x1, x2, mult(x1, x1))), \overbrace{mult(i(x2), \underbrace{i(op_t(i(x1), i(x2)))})})$

$=$ by Lemma 18 LR with $\{x2 \leftarrow i(x2), x1 \leftarrow x1\}$

$mult(op_l(i(i(x2)), i(x2), op_l(x1, x2, mult(x1, x1))), mult(i(x2), \overbrace{op_t(x1, \underbrace{i(i(x2))})}))$

$=$ by Lemma 3 LR with $\{x1 \leftarrow x2\}$

$mult(op_l(i(i(x2)), i(x2), op_l(x1, x2, mult(x1, x1))), mult(i(x2), op_t(x1, \overbrace{x2})))$

Lemma 1608: $op_l(op_l(x1, x2, mult(x1, x1)), x1, x2) = mult(asoc(op_l(x1, x2, mult(x1, x1)), i(x2), i(x2)), op_t(x1, x2))$

$op_l(op_l(x1, x2, mult(x1, x1)), x1, x2)$

$=$ by Lemma 1329 RL with $\{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow op_l(x1, x2, mult(x1, x1))\}$

$op_l(op_l(x1, x2, mult(x1, x1)), x2, i(op_t(x1, x2)))$

$=$ by Lemma 15 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$op_l(op_l(x1, x2, mult(x1, x1)), x2, op_t(i(x1), i(x2)))$

$=$ by Lemma 1602 RL with $\{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow op_l(x1, x2, mult(x1, x1))\}$

$op_l(op_l(x1, x2, mult(x1, x1)), op_l(x1, x2, mult(x1, x1)), x2)$

$=$ by Lemma 1607 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$mult(op_l(i(i(x2)), i(x2), op_l(x1, x2, mult(x1, x1))), mult(i(x2), op_t(x1, x2)))$

$=$ by Lemma 633 RL with $\{x2 \leftarrow op_l(x1, x2, mult(x1, x1)), x1 \leftarrow i(x2)\}$

$mult(op_l(i(i(x2)), op_l(x1, x2, mult(x1, x1)), i(i(x2))), mult(i(x2), op_t(x1, x2)))$

$=$ by Lemma 771 LR with $\{x2 \leftarrow op_l(x1, x2, mult(x1, x1)), x1 \leftarrow i(i(x2))\}$

$mult(mult(asoc(op_l(x1, x2, mult(x1, x1)), i(i(x2)), i(i(x2))), i(i(x2))), mult(i(x2), op_t(x1, x2)))$

$=$ by Axiom 8 LR with $\{x3 \leftarrow mult(i(x2), op_t(x1, x2)), x2 \leftarrow i(i(x2)), x1 \leftarrow asoc(op_l(x1, x2, mult(x1, x1)), i(i(x2)), i(i(x2)))\}$

$mult(mult(asoc(op_l(x1, x2, mult(x1, x1)), i(i(x2)), i(i(x2))), mult(i(i(x2)), mult(i(x2), op_t(x1, x2)))), asoc(asoc(op_l(x1, x2$

$=$ by Axiom 2 RL with $\{x2 \leftarrow i(x2), x1 \leftarrow op_t(x1, x2)\}$

$mult(mult(asoc(op_l(x1, x2, mult(x1, x1)), i(i(x2)), i(i(x2))), op_t(x1, x2)), asoc(asoc(op_l(x1, x2, mult(x1, x1)), i(i(x2)), i(i($

$=$ by Lemma 958 LR with $\{x3 \leftarrow mult(i(x2), op_t(x1, x2)), x2 \leftarrow i(i(x2)), x1 \leftarrow op_l(x1, x2, mult(x1, x1))\}$

$mult(mult(asoc(op_l(x1, x2, mult(x1, x1)), i(i(x2)), i(i(x2))), op_t(x1, x2)), unit())$

$=$ by Axiom 1 RL with $\{x1 \leftarrow mult(asoc(op_l(x1, x2, mult(x1, x1)), i(i(x2)), i(i(x2))), op_t(x1, x2))\}$

$mult(asoc(op_l(x1, x2, mult(x1, x1)), i(i(x2)), i(i(x2))), op_t(x1, x2))$

$=$ by Lemma 772 LR with $\{x2 \leftarrow i(x2), x1 \leftarrow op_l(x1, x2, mult(x1, x1))\}$

$mult(asoc(op_l(x1, x2, mult(x1, x1)), i(x2), i(x2)), op_t(x1, x2))$

Lemma 1609: $op_l(op_l(x1, x2, mult(x1, x1)), x1, x2) = mult(asoc(x1, x2, x2), op_t(x1, x2))$

$op_l(op_l(x1, x2, mult(x1, x1)), x1, x2)$

$=$ by Lemma 1608 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$mult(asoc(op_l(x1, x2, mult(x1, x1)), i(x2), i(x2)), op_t(x1, x2))$

$=$ by Lemma 772 LR with $\{x2 \leftarrow x2, x1 \leftarrow op_l(x1, x2, mult(x1, x1))\}$

$mult(asoc(op_l(x1, x2, mult(x1, x1)), x2, x2), op_t(x1, x2))$

$=$ by Lemma 1600 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$mult(asoc(x1, x2, x2), op_t(x1, x2))$

Lemma 1610: $op_l(mult(x1,x2),x1,i(mult(x1,x2))) = op_l(op_l(mult(x1,x2),x1,x2),i(x1),mult(mult(x1,x2),mult(x1,x2)))$

$\overbrace{op_l(mult(x1,x2),x1,i(mult(x1,x2)))}$

$=$ by Lemma 1050 RL with $\{x3 \leftarrow mult(x1,x2), x2 \leftarrow x1, x1 \leftarrow mult(x1,x2)\}$

$\overbrace{op_l(mult(x1,x2),i(x1),mult(x1,x2))}$

$=$ by Lemma 735 LR with $\{x1 \leftarrow i(x1), x2 \leftarrow mult(x1,x2)\}$

$\overbrace{rd(op_t(\underbrace{mult(x1,x2)}, i(x1)), asoc(i(x1),mult(x1,x2),i(x1)))}$

$=$ by Lemma 535 RL with $\{x2 \leftarrow i(x1), x1 \leftarrow mult(x1,x2)\}$

$rd(\underbrace{op_t(\overbrace{op_t(mult(x1,x2),asoc(i(x1),mult(x1,x2),i(x1)))}, i(x1))}, asoc(i(x1),mult(x1,x2),i(x1)))$

$=$ by Axiom 13 LR with $\{x3 \leftarrow i(x1), x2 \leftarrow asoc(i(x1),mult(x1,x2),i(x1)), x1 \leftarrow mult(x1,x2)\}$

$\underbrace{rd(\overbrace{op_t(op_t(mult(x1,x2),i(x1)),asoc(i(x1),mult(x1,x2),i(x1)))}, asoc(i(x1),mult(x1,x2),i(x1)))}$

$=$ by Lemma 139 RL with $\{x2 \leftarrow op_t(mult(x1,x2),i(x1)), x1 \leftarrow asoc(i(x1),mult(x1,x2),i(x1))\}$

$\overbrace{mult(i(asoc(i(x1),mult(x1,x2),i(x1))),\underbrace{op_r(op_t(mult(x1,x2),i(x1)),asoc(i(x1),mult(x1,x2),i(x1)),i(asoc(i(x1),mult(x$

$=$ by Axiom 14 RL with $\{x4 \leftarrow i(x1), x3 \leftarrow i(asoc(i(x1),mult(x1,x2),i(x1))), x2 \leftarrow asoc(i(x1),mult(x1,x2),i(x1)), x1 \leftarrow mul$

$mult(i(asoc(i(x1),mult(x1,x2),i(x1))),\overbrace{op_t(\underbrace{op_r(mult(x1,x2),asoc(i(x1),mult(x1,x2),i(x1)),i(asoc(i(x1),mult(x1,x2),i$

$=$ by Lemma 659 RL with $\{x3 \leftarrow i(asoc(i(x1),mult(x1,x2),i(x1))), x2 \leftarrow i(x1), x1 \leftarrow mult(x1,x2)\}$

$mult(\underbrace{i(asoc(i(x1),mult(x1,x2),i(x1)))}, op_t(\overbrace{mult(x1,x2)}, i(x1)))$

$=$ by Lemma 757 LR with $\{x2 \leftarrow mult(x1,x2), x1 \leftarrow i(x1)\}$

$\underbrace{mult(\overbrace{asoc(mult(x1,x2),i(x1),i(x1))}, op_t(mult(x1,x2),i(x1)))}$

$=$ by Lemma 1609 RL with $\{x2 \leftarrow i(x1), x1 \leftarrow mult(x1,x2)\}$

$\overbrace{\underbrace{op_l(op_l(mult(x1,x2),i(x1),mult(mult(x1,x2),mult(x1,x2))), mult(x1,x2),i(x1))}$

$=$ by Axiom 19 LR with $\{x5 \leftarrow i(x1), x4 \leftarrow mult(x1,x2), x3 \leftarrow mult(mult(x1,x2),mult(x1,x2)), x2 \leftarrow i(x1), x1 \leftarrow mult(x1,x$

$\overbrace{op_l(\underbrace{op_l(mult(x1,x2),mult(x1,x2),i(x1))}, i(x1), mult(mult(x1,x2),mult(x1,x2)))}$

$=$ by Lemma 242 LR with $\{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow mult(x1,x2)\}$

$op_l(\overbrace{op_l(mult(x1,x2),x1,x2)}, i(x1), mult(mult(x1,x2),mult(x1,x2)))$

Lemma 1611: $mult(op_t(x1,x2),x2) = op_l(op_l(mult(x1,x2),x1,x2),i(x1),mult(mult(x1,x2),mult(x1,x2)))$

$\underbrace{mult(op_t(x1,x2),x2)}$

$=$ by Lemma 636 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_l(\underbrace{mult(x1,x2)}, x2, x1)}$

$=$ by Lemma 1051 RL with $\{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow mult(x1,x2)\}$

$\overbrace{op_l(mult(x1,x2),x1,i(mult(x1,x2)))}$

$=$ by Lemma 1610 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_l(op_l(mult(x1,x2),x1,x2),i(x1),mult(mult(x1,x2),mult(x1,x2)))}$

Lemma 1612: $mult(op_t(x1,x2),x2) = op_l(mult(x2,x1),x1,i(mult(mult(x1,x2),mult(x1,x2))))$

$\overbrace{mult(op_t(x1,x2),x2)}$

$=$ by Lemma 1611 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_l(\underbrace{op_l(mult(x1,x2),x1,x2)}, i(x1), mult(mult(x1,x2),mult(x1,x2)))}$

$=$ by Lemma 1050 LR with $\{x3 \leftarrow mult(mult(x1,x2),mult(x1,x2)), x2 \leftarrow x1, x1 \leftarrow op_l(mult(x1,x2),x1,x2)\}$

$\overbrace{op_l(\underbrace{op_l(mult(x1,x2),x1,x2)}, x1, i(mult(mult(x1,x2),mult(x1,x2))))}$

$=$ by Lemma 435 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$op_l(\overbrace{mult(x2,x1)}, x1, i(mult(mult(x1,x2),mult(x1,x2))))$

Lemma 1613: $mult(x1,x2) = op_l(mult(op_t(x2,x1),x1),x2,mult(x1,x1))$

$\overbrace{mult(x1,x2)}$

$=$ by Lemma 138 RL with $\{x3 \leftarrow x2, x2 \leftarrow mult(x1,x1), x1 \leftarrow mult(x1,x2)\}$

$\overbrace{op_l(\underbrace{op_l(mult(x1,x2),mult(x1,x1),x2)}, x2, mult(x1,x1))}$

$=$ by Lemma 1051 RL with $\{x3 \leftarrow mult(x1,x1), x2 \leftarrow x2, x1 \leftarrow mult(x1,x2)\}$

$op_l(\overbrace{op_l(mult(x1,x2),x2,i(\underbrace{mult(x2,mult(x1,x1))}))}, x2, mult(x1,x1))$

$=$ by Lemma 296 RL with $\{x3 \leftarrow mult(x2,mult(x1,x1)), x2 \leftarrow x2, x1 \leftarrow mult(x1,x2)\}$

$op_l(\overbrace{op_l(mult(x1,x2),x2,rd(i(x2),\underbrace{mult(x2,mult(x1,x1))}))}, x2, mult(x1,x1))$

$=$ by Lemma 132 RL with $\{x3 \leftarrow x1, x2 \leftarrow x1, x1 \leftarrow x2\}$

$op_l(op_l(mult(x1,x2),x2,rd(\underbrace{i(x2)},\overbrace{mult(mult(x2,x1),op_l(x1,x1,x2))})),x2,mult(x1,x1))$

$=$ by Lemma 529 RL with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$op_l(op_l(mult(x1,x2),x2,rd(\overbrace{mult(i(mult(x2,x1)),\underbrace{op_l(x1,x1,x2)})},mult(mult(x2,x1),op_l(x1,x1,x2)))),x2,mult(x1,x1))$

$=$ by Axiom 2 LR with $\{x2 \leftarrow mult(x2,x1), x1 \leftarrow op_l(x1,x1,x2)\}$

$op_l(op_l(mult(x1,x2),x2,\underbrace{rd(mult(i(mult(x2,x1)),\overbrace{mult(i(mult(x2,x1)),mult(mult(x2,x1),op_l(x1,x1,x2))))},mult(mult($

$=$ by Lemma 41 LR with $\{x2 \leftarrow mult(mult(x2,x1),op_l(x1,x1,x2)), x1 \leftarrow i(mult(x2,x1))\}$

$op_l(op_l(mult(x1,x2),x2,\underbrace{\overbrace{mult(i(mult(x2,x1)),i(mult(x2,x1))))}},x2,mult(x1,x1))$

$=$ by Axiom 11 LR with $\{x2 \leftarrow mult(x2,x1), x1 \leftarrow mult(x2,x1)\}$

$op_l(\underbrace{op_l(mult(x1,x2),x2,\overbrace{i(mult(mult(x2,x1),mult(x2,x1))))}},x2,mult(x1,x1))$

$=$ by Lemma 1612 RL with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$op_l(\overbrace{mult(op_t(x2,x1),x1)},x2,mult(x1,x1))$

Lemma 1614: $op_l(mult(x2,x1),x2,mult(x1,x1)) = mult(op_t(x1,x2),x2)$

$op_l(\underbrace{mult(x2,x1)},x2,mult(x1,x1))$

$=$ by Lemma 13 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$op_l(\overbrace{mult(x1,op_t(x2,x1))},x2,mult(x1,x1))$

$=$ by Lemma 435 RL with $\{x2 \leftarrow x1, x1 \leftarrow op_t(x2,x1)\}$

$\underbrace{op_l(\overbrace{op_l(mult(op_t(x2,x1),x1),op_t(x2,x1),x1)},x2,mult(x1,x1))}$

$=$ by Axiom 19 LR with $\{x5 \leftarrow mult(x1,x1), x4 \leftarrow x2, x3 \leftarrow x1, x2 \leftarrow op_t(x2,x1), x1 \leftarrow mult(op_t(x2,x1),x1)\}$

$\overbrace{op_l(\underbrace{op_l(mult(op_t(x2,x1),x1),x2,mult(x1,x1))},op_t(x2,x1),x1)}$

$=$ by Lemma 1613 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{op_l(\overbrace{mult(x1,x2)},op_t(x2,x1),x1)}$

$=$ by Lemma 1319 LR with $\{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow mult(x1,x2)\}$

$\overbrace{\underbrace{op_l(mult(x1,x2),x1,i(x2))}}$

$=$ by Lemma 477 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_t(rd(x1,i(x2)),x2)}}$

$=$ by Lemma 337 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{mult(op_t(x1,x2),x2)}$

Lemma 1615: $op_l(op_l(op_l(x1,x2,x3),x4,x5),x3,x2) = op_l(x1,x4,x5)$

$\underbrace{op_l(op_l(op_l(x1,x2,x3),x4,x5),x3,x2)}$

$=$ by Axiom 19 RL with $\{x5 \leftarrow x5, x4 \leftarrow x4, x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow op_l(x1,x2,x3)\}$

$\overbrace{op_l(\underbrace{op_l(op_l(x1,x2,x3),x3,x2)},x4,x5)}$

$=$ by Lemma 138 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$op_l(\overbrace{x1},x4,x5)$

Lemma 1616: $op_l(x1,op_l(x1,x2,x3),i(x4)) = op_l(x1,x4,op_l(x1,x2,x3))$

$\underbrace{op_l(x1,op_l(x1,x2,x3),i(x4))}$

$=$ by Lemma 1615 RL with $\{x5 \leftarrow i(x4), x4 \leftarrow op_l(x1,x2,x3), x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_l(\underbrace{op_l(op_l(x1,x2,x3),op_l(x1,x2,x3),i(x4))},x3,x2)}$

$=$ by Lemma 608 LR with $\{x2 \leftarrow x4, x1 \leftarrow op_l(x1,x2,x3)\}$

$\underbrace{op_l(\overbrace{op_l(op_l(x1,x2,x3),x4,op_l(x1,x2,x3))},x3,x2)}$

$=$ by Lemma 1615 LR with $\{x5 \leftarrow op_l(x1,x2,x3), x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_l(x1,x4,op_l(x1,x2,x3))}$

Lemma 1617: $i(mult(x1, x2)) = rd(i(op_t(x1, x2)), op_t(x2, rd(x1, x2)))$

$$
\begin{aligned}
& \overbrace{i(mult(x1, x2))} \\
= \quad & \text{by Lemma 29 RL with } \{x2 \leftarrow i(mult(x1, x2)), x1 \leftarrow x2\} \\
& \overbrace{rd(mult(x2, i(mult(x1, x2))), \underbrace{op_t(x2, i(mult(x1, x2)))}_{})}^{} \\
= \quad & \text{by Lemma 709 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& rd(\underbrace{mult(x2, i(mult(x1, x2)))}_{}, \overbrace{op_t(x2, rd(x1, x2))}^{}) \\
= \quad & \text{by Lemma 31 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& rd(\overbrace{i(op_t(x1, x2))}^{}, op_t(x2, rd(x1, x2)))
\end{aligned}
$$

Lemma 1618: $op_l(x1, x2, x1) = mult(rd(x1, op_t(x2, x1)), op_l(x2, x1, x2))$

$$
\begin{aligned}
& \underbrace{op_l(x1, x2, x1)} \\
= \quad & \text{by Lemma 623 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{\overbrace{op_l(x1, op_t(x2, x1), x1)}^{}} \\
= \quad & \text{by Lemma 720 RL with } \{x2 \leftarrow op_t(x2, x1), x1 \leftarrow x1\} \\
& \overbrace{mult(rd(x1, op_t(x2, x1)), \underbrace{op_r(op_t(x2, x1), x1, x1)}_{})}^{} \\
= \quad & \text{by Lemma 745 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& mult(rd(x1, op_t(x2, x1)), \overbrace{op_l(x2, x1, x2)}^{})
\end{aligned}
$$

Lemma 1619: $i(op_t(x1, x2)) = rd(rd(op_t(x2, x1), x1), op_l(x2, x2, x1))$

$$
\begin{aligned}
& \underbrace{i(op_t(x1, x2))} \\
= \quad & \text{by Lemma 449 RL with } \{x2 \leftarrow op_t(x1, x2), x1 \leftarrow x2\} \\
& \overbrace{rd(\underbrace{rd(x2, op_t(x1, x2))}_{}, op_l(x2, x2, op_t(x1, x2)))}^{} \\
= \quad & \text{by Lemma 748 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& rd(\overbrace{rd(op_t(x2, x1), x1)}^{}, \underbrace{op_l(x2, x2, op_t(x1, x2))}_{}) \\
= \quad & \text{by Lemma 621 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& rd(rd(op_t(x2, x1), x1), \overbrace{op_l(x2, x2, x1)}^{})
\end{aligned}
$$

Lemma 1620: $mult(x1, mult(x2, mult(i(mult(x2, x1)), x3))) = op_l(x3, x2, x1)$

$mult(\underbrace{x1}, mult(x2, mult(i(mult(x2, x1)), x3)))$

$=$ by Lemma 3 RL with $\{x1 \leftarrow x1\}$

$mult(\overbrace{i(\underbrace{i(x1)})}, mult(x2, mult(i(mult(x2, x1)), x3)))$

$=$ by Lemma 20 RL with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$\underbrace{mult(i(\overbrace{mult(x2, i(mult(x2, x1)))}), mult(x2, mult(i(mult(x2, x1)), x3)))}$

$=$ by Axiom 12 LR with $\{x3 \leftarrow x3, x2 \leftarrow i(mult(x2, x1)), x1 \leftarrow x2\}$

$\underbrace{\overbrace{op_l(x3, i(mult(x2, x1)), x2)}}$

$=$ by Lemma 247 LR with $\{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x3\}$

$\underbrace{\overbrace{op_l(x3, i(x2), i(x1))}}$

$=$ by Lemma 1048 LR with $\{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x3\}$

$\overbrace{op_l(x3, x2, x1)}$

Lemma 1621: $op_t(mult(x1, rd(x2, x1)), rd(i(x2), x2)) = mult(mult(x2, x2), rd(mult(x1, rd(x2, x1)), mult(x2, x2)))$

$op_t(mult(x1, rd(x2, x1)), \underbrace{rd(i(x2), x2)})$

$=$ by Lemma 43 RL with $\{x1 \leftarrow x2\}$

$\underbrace{op_t(mult(x1, rd(x2, x1)), \overbrace{i(mult(x2, x2))})}$

$=$ by Lemma 688 RL with $\{x2 \leftarrow mult(x1, rd(x2, x1)), x1 \leftarrow mult(x2, x2)\}$

$\overbrace{mult(\underbrace{op_l(mult(x2, x2), mult(x2, x2), mult(x1, rd(x2, x1)))}, rd(mult(x1, rd(x2, x1)), mult(x2, x2)))}$

$=$ by Lemma 1072 RL with $\{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow mult(x2, x2)\}$

$mult(\overbrace{mult(x2, x2)}, rd(mult(x1, rd(x2, x1)), mult(x2, x2)))$

Lemma 1622: $op_t(mult(x1, rd(x2, x1)), rd(i(x2), x2)) = mult(x2, rd(mult(x2, rd(mult(x1, rd(x2, x1)), x2)), x2))$

$\underbrace{op_t(mult(x1, rd(x2, x1)), rd(i(x2), x2))}$

$=$ by Lemma 1621 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{\overbrace{mult(mult(x2, x2), rd(mult(x1, rd(x2, x1)), mult(x2, x2)))}}$

$=$ by Lemma 24 LR with $\{x2 \leftarrow rd(mult(x1, rd(x2, x1)), mult(x2, x2)), x1 \leftarrow x2\}$

$\overbrace{mult(x2, \underbrace{mult(x2, rd(mult(x1, rd(x2, x1)), mult(x2, x2)))})}$

$=$ by Lemma 841 RL with $\{x2 \leftarrow mult(x1, rd(x2, x1)), x1 \leftarrow x2\}$

$mult(x2, \overbrace{rd(mult(x2, rd(mult(x1, rd(x2, x1)), x2)), x2)})$

Lemma 1623: $op_t(mult(x1, rd(x2, x1)), rd(i(x2), x2)) = mult(x2, rd(op_l(x2, x2, x1), x2))$

$$
\begin{aligned}
& \underbrace{op_t(mult(x1, rd(x2, x1)), rd(i(x2), x2))} \\
= \quad & \text{by Lemma 1622 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{mult(x2, rd(mult(x2, \underbrace{rd(mult(x1, rd(x2, x1)), x2)}), x2))} \\
= \quad & \text{by Lemma 448 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& mult(x2, rd(\underbrace{mult(x2, \overbrace{asoc(x2, x1, x2)})}, x2)) \\
= \quad & \text{by Lemma 158 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& mult(x2, rd(\overbrace{op_l(x2, x2, x1)}, x2))
\end{aligned}
$$

Lemma 1624: $op_t(mult(x1, rd(x2, x1)), rd(i(x2), x2)) = op_l(x2, x2, x1)$

$$
\begin{aligned}
& \underbrace{op_t(mult(x1, rd(x2, x1)), rd(i(x2), x2))} \\
= \quad & \text{by Lemma 1623 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{mult(x2, \underbrace{rd(op_l(x2, x2, x1), x2)})} \\
= \quad & \text{by Lemma 172 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \underbrace{mult(x2, \overbrace{asoc(x2, x1, x2)})} \\
= \quad & \text{by Lemma 158 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \overbrace{op_l(x2, x2, x1)}
\end{aligned}
$$

Lemma 1625: $asoc(x2, x1, x2) = mult(rd(x3, op_r(x1, x2, x2)), i(rd(x3, x1)))$

$$
\begin{aligned}
& \underbrace{asoc(x2, x1, x2)} \\
= \quad & \text{by Lemma 759 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{i(asoc(x1, x2, x2))}} \\
= \quad & \text{by Lemma 22 RL with } \{x2 \leftarrow asoc(x1, x2, x2), x1 \leftarrow rd(x3, x1)\} \\
& \overbrace{mult(\underbrace{rd(rd(x3, x1), asoc(x1, x2, x2))}, i(rd(x3, x1)))} \\
= \quad & \text{by Lemma 1243 LR with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x3\} \\
& mult(\overbrace{rd(x3, op_r(x1, x2, x2))}, i(rd(x3, x1)))
\end{aligned}
$$

Lemma 1626: $mult(rd(x1, op_r(x2, x3, x3)), rd(x2, x1)) = asoc(x3, x2, x3)$

$$
\begin{aligned}
& mult(rd(x1, op_r(x2, x3, x3)), \underbrace{rd(x2, x1)}) \\
= \quad & \text{by Lemma 10 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{mult(rd(x1, op_r(x2, x3, x3)), \overbrace{i(rd(x1, x2))})} \\
= \quad & \text{by Lemma 1625 RL with } \{x3 \leftarrow x1, x1 \leftarrow x2, x2 \leftarrow x3\} \\
& \overbrace{asoc(x3, x2, x3)}
\end{aligned}
$$

Lemma 1627: $op_l(x1, rd(x2, i(op_t(x1, x3))), x1) = rd(mult(x1, rd(rd(x2, i(op_t(x1, x3))), op_t(x1, x3))), x2)$

$\overbrace{op_l(x1, rd(x2, i(op_t(x1, x3))), x1)}$

$=$ by Lemma 608 RL with $\{x2 \leftarrow rd(x2, i(op_t(x1, x3))), x1 \leftarrow x1\}$

$\overbrace{op_l(x1, x1, i(rd(x2, i(op_t(x1, x3)))))}$

$=$ by Lemma 1038 LR with $\{x3 \leftarrow x3, x2 \leftarrow rd(x2, i(op_t(x1, x3))), x1 \leftarrow x1\}$

$\overbrace{op_r(x1, rd(x2, i(op_t(x1, x3))), i(op_t(x1, x3)))}$

$=$ by Lemma 693 RL with $\{x3 \leftarrow i(op_t(x1, x3)), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{rd(\underbrace{mult(mult(x1, rd(x2, i(op_t(x1, x3)))), i(op_t(x1, x3)))}, x2)}$

$=$ by Lemma 1253 LR with $\{x3 \leftarrow x3, x2 \leftarrow rd(x2, i(op_t(x1, x3))), x1 \leftarrow x1\}$

$\overbrace{rd(mult(x1, rd(rd(x2, i(op_t(x1, x3))), op_t(x1, x3))), x2)}$

Lemma 1628: $rd(x1, asoc(op_t(x1, x2), x3, op_t(x1, x2))) = op_l(x1, x3, x1)$

$\underbrace{rd(x1, asoc(op_t(x1, x2), x3, op_t(x1, x2)))}$

$=$ by Lemma 1300 LR with $\{x2 \leftarrow x3, x3 \leftarrow op_t(x1, x2), x1 \leftarrow x1\}$

$\underbrace{mult(x1, asoc(x3, op_t(x1, x2), op_t(x1, x2)))}$

$=$ by Lemma 1246 RL with $\{x3 \leftarrow op_t(x1, x2), x2 \leftarrow x3, x1 \leftarrow x1\}$

$\overbrace{rd(mult(x1, \underbrace{op_r(x3, op_t(x1, x2), op_t(x1, x2))}), x3)}$

$=$ by Lemma 358 RL with $\{x2 \leftarrow op_t(x1, x2), x1 \leftarrow x3\}$

$\underbrace{rd(mult(x1, \overbrace{rd(rd(x3, i(op_t(x1, x2))), op_t(x1, x2))}), x3)}$

$=$ by Lemma 1627 RL with $\{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x1\}$

$\overbrace{op_l(x1, \underbrace{rd(x3, i(op_t(x1, x2)))}, x1)}$

$=$ by Lemma 38 RL with $\{x1 \leftarrow x3, x2 \leftarrow op_t(x1, x2)\}$

$\underbrace{op_l(x1, \overbrace{rd(op_t(x1, x2), i(x3))}, x1)}$

$=$ by Lemma 1031 LR with $\{x3 \leftarrow i(x3), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{\overbrace{rd(x1, asoc(i(x3), x1, x1))}}$

$=$ by Lemma 773 RL with $\{x2 \leftarrow i(x3), x1 \leftarrow x1\}$

$\underbrace{\overbrace{op_l(x1, x1, i(x3))}}$

$=$ by Lemma 608 LR with $\{x2 \leftarrow x3, x1 \leftarrow x1\}$

$\overbrace{op_l(x1, x3, x1)}$

Lemma 1629: $asoc(x3, op_t(x1, x2), op_t(x1, x2)) = mult(op_l(x1, x3, x1), i(x1))$

$$
\begin{aligned}
& \overbrace{asoc(x3, op_t(x1, x2), op_t(x1, x2))} \\
= & \quad \text{by Lemma 757 RL with } \{x2 \leftarrow x3, x1 \leftarrow op_t(x1, x2)\} \\
& \overbrace{\underbrace{i(asoc(op_t(x1, x2), x3, op_t(x1, x2)))}} \\
= & \quad \text{by Lemma 22 RL with } \{x2 \leftarrow asoc(op_t(x1, x2), x3, op_t(x1, x2)), x1 \leftarrow x1\} \\
& \overbrace{mult(\underbrace{rd(x1, asoc(op_t(x1, x2), x3, op_t(x1, x2)))}, i(x1))} \\
= & \quad \text{by Lemma 1628 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& mult(\overbrace{op_l(x1, x3, x1)}, i(x1))
\end{aligned}
$$

Lemma 1630: $asoc(x3, op_t(x1, x2), op_t(x1, x2)) = asoc(x3, x1, x1)$

$$
\begin{aligned}
& \overbrace{asoc(x3, op_t(x1, x2), op_t(x1, x2))} \\
= & \quad \text{by Lemma 1629 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1, x3 \leftarrow x3\} \\
& \overbrace{\underbrace{mult(op_l(x1, x3, x1), i(x1))}} \\
= & \quad \text{by Lemma 178 LR with } \{x3 \leftarrow x1, x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{rd(op_l(x1, x3, x1), x1)}} \\
= & \quad \text{by Lemma 765 LR with } \{x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{asoc(x3, x1, x1)}
\end{aligned}
$$

Lemma 1631: $asoc(op_t(x1,x2),x3,op_t(x1,x2)) = asoc(x1,x3,x1)$

$$
\begin{aligned}
& \underbrace{asoc(op_t(x1,x2),x3,op_t(x1,x2))} \\
= \quad & \text{by Lemma 441 RL with } \{x2 \leftarrow asoc(op_t(x1,x2),x3,op_t(x1,x2)), x1 \leftarrow x1\} \\
& \overbrace{rd(op_l(x1,x1,asoc(op_t(x1,x2),x3,op_t(x1,x2))), \underbrace{rd(x1,asoc(op_t(x1,x2),x3,op_t(x1,x2)))})} \\
= \quad & \text{by Lemma 1628 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& rd(\underbrace{op_l(x1,x1,asoc(op_t(x1,x2),x3,op_t(x1,x2)))}, \overbrace{op_l(x1,x3,x1)}) \\
= \quad & \text{by Lemma 286 RL with } \{x3 \leftarrow asoc(op_t(x1,x2),x3,op_t(x1,x2)), x2 \leftarrow x1, x1 \leftarrow x1\} \\
& rd(\overbrace{op_l(\underbrace{x1},x1,rd(asoc(op_t(x1,x2),x3,op_t(x1,x2)),x1))}, op_l(x1,x3,x1)) \\
= \quad & \text{by Lemma 1296 RL with } \{x3 \leftarrow x3, x2 \leftarrow op_t(x1,x2), x1 \leftarrow x1\} \\
& rd(\underbrace{op_l(\overbrace{op_t(x1,rd(asoc(op_t(x1,x2),x3,op_t(x1,x2)),x1))},x1,rd(asoc(op_t(x1,x2),x3,op_t(x1,x2)),x1))}, op_l(x1,x3,x1)) \\
= \quad & \text{by Lemma 421 RL with } \{x2 \leftarrow rd(asoc(op_t(x1,x2),x3,op_t(x1,x2)),x1), x1 \leftarrow x1\} \\
& rd(\overbrace{op_r(x1,rd(asoc(op_t(x1,x2),x3,op_t(x1,x2)),x1)}, \underbrace{i(rd(asoc(op_t(x1,x2),x3,op_t(x1,x2)),x1))}), op_l(x1,x3,x1)) \\
= \quad & \text{by Lemma 10 LR with } \{x2 \leftarrow x1, x1 \leftarrow asoc(op_t(x1,x2),x3,op_t(x1,x2))\} \\
& rd(\underbrace{op_r(x1,rd(asoc(op_t(x1,x2),x3,op_t(x1,x2)),x1)}, \overbrace{rd(x1,asoc(op_t(x1,x2),x3,op_t(x1,x2)))}), op_l(x1,x3,x1)) \\
= \quad & \text{by Lemma 586 LR with } \{x2 \leftarrow asoc(op_t(x1,x2),x3,op_t(x1,x2)), x1 \leftarrow x1\} \\
& rd(\overbrace{op_t(x1,\underbrace{i(asoc(op_t(x1,x2),x3,op_t(x1,x2)))})}, op_l(x1,x3,x1)) \\
= \quad & \text{by Lemma 757 LR with } \{x2 \leftarrow x3, x1 \leftarrow op_t(x1,x2)\} \\
& rd(\underbrace{op_t(x1,\overbrace{asoc(x3,op_t(x1,x2),op_t(x1,x2))})}, op_l(x1,x3,x1)) \\
= \quad & \text{by Lemma 1124 LR with } \{x2 \leftarrow x2, x3 \leftarrow x3, x1 \leftarrow x1\} \\
& \underbrace{rd(\overbrace{x1}, op_l(x1,x3,x1))} \\
= \quad & \text{by Lemma 617 LR with } \{x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{asoc(x1,x3,x1)}
\end{aligned}
$$

Lemma 1632: $op_r(x1,op_t(x2,x3),op_t(x2,x3)) = op_r(x1,x2,x2)$

$$
\begin{aligned}
& \underbrace{op_r(x1,op_t(x2,x3),op_t(x2,x3))} \\
= \quad & \text{by Lemma 767 RL with } \{x1 \leftarrow op_t(x2,x3), x2 \leftarrow x1\} \\
& \overbrace{mult(\underbrace{asoc(x1,op_t(x2,x3),op_t(x2,x3))},x1)} \\
= \quad & \text{by Lemma 1630 LR with } \{x2 \leftarrow x3, x1 \leftarrow x2, x3 \leftarrow x1\} \\
& \underbrace{mult(\overbrace{asoc(x1,x2,x2)},x1)} \\
= \quad & \text{by Lemma 767 LR with } \{x1 \leftarrow x2, x2 \leftarrow x1\} \\
& \overbrace{op_r(x1,x2,x2)}
\end{aligned}
$$

Lemma 1633: $op_t(x1, rd(op_t(x2, x3), x1)) = op_r(x1, op_t(x2, x1), i(x2))$

$\overbrace{op_t(x1, rd(op_t(x2, x3), x1))}$

$=$ by Lemma 368 RL with $\{x2 \leftarrow x2, x1 \leftarrow op_t(x1, rd(op_t(x2, x3), x1))\}$

$\overbrace{op_r(\underbrace{op_t(op_t(x1, rd(op_t(x2, x3), x1)), rd(x2, op_t(x1, rd(op_t(x2, x3), x1))))}, x2, x2)}$

$=$ by Lemma 1244 RL with $\{x3 \leftarrow x2, x2 \leftarrow x2, x1 \leftarrow op_t(x1, rd(op_t(x2, x3), x1))\}$

$\underbrace{op_r(\overbrace{op_t(op_t(x1, rd(op_t(x2, x3), x1)), rd(x2, op_r(op_t(x1, rd(op_t(x2, x3), x1)), x2, x2)))}, x2, x2)}$

$=$ by Axiom 14 RL with $\{x4 \leftarrow rd(x2, op_r(op_t(x1, rd(op_t(x2, x3), x1)), x2, x2)), x3 \leftarrow x2, x2 \leftarrow x2, x1 \leftarrow op_t(x1, rd(op_t(x2, x3),$

$\overbrace{op_t(op_r(op_t(x1, rd(op_t(x2, x3), x1)), x2, x2), \underbrace{rd(x2, op_r(op_t(x1, rd(op_t(x2, x3), x1)), x2, x2))})}$

$=$ by Lemma 709 RL with $\{x2 \leftarrow x2, x1 \leftarrow op_r(op_t(x1, rd(op_t(x2, x3), x1)), x2, x2)\}$

$\overbrace{op_t(op_r(op_t(x1, rd(op_t(x2, x3), x1)), x2, x2), i(\underbrace{mult(x2, op_r(op_t(x1, rd(op_t(x2, x3), x1)), x2, x2))}))}$

$=$ by Lemma 1415 RL with $\{x2 \leftarrow mult(x2, op_r(op_t(x1, rd(op_t(x2, x3), x1)), x2, x2)), x1 \leftarrow op_r(op_t(x1, rd(op_t(x2, x3), x1)), x2,$

$\overbrace{op_r(op_r(op_t(x1, rd(op_t(x2, x3), x1)), x2, x2), i(\underbrace{mult(x2, op_r(op_t(x1, rd(op_t(x2, x3), x1)), x2, x2))}), \underbrace{rd(mult(x2, op_r(op_t(x1,}$

$=$ by Axiom 5 RL with $\{x2 \leftarrow op_r(op_t(x1, rd(op_t(x2, x3), x1)), x2, x2), x1 \leftarrow x2\}$

$\underbrace{op_r(op_r(op_t(x1, rd(op_t(x2, x3), x1)), x2, x2), i(\underbrace{mult(x2, op_r(op_t(x1, rd(op_t(x2, x3), x1)), x2, x2))}), \overbrace{x2})}$

$=$ by Lemma 820 LR with $\{x3 \leftarrow x2, x2 \leftarrow x2, x1 \leftarrow op_r(op_t(x1, rd(op_t(x2, x3), x1)), x2, x2)\}$

$\overbrace{op_r(op_r(op_t(x1, rd(op_t(x2, x3), x1)), x2, x2), \underbrace{op_t(x2, op_r(op_t(x1, rd(op_t(x2, x3), x1)), x2, x2))}, i(x2))}$

$=$ by Lemma 550 LR with $\{x2 \leftarrow op_t(x1, rd(op_t(x2, x3), x1)), x1 \leftarrow x2\}$

$op_r(\underbrace{op_r(op_t(x1, rd(op_t(x2, x3), x1)), x2, x2)}, \overbrace{op_t(x2, op_t(x1, rd(op_t(x2, x3), x1)))}, i(x2))$

$=$ by Lemma 1632 RL with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow op_t(x1, rd(op_t(x2, x3), x1))\}$

$op_r(\overbrace{op_r(op_t(x1, rd(op_t(x2, x3), x1)), op_t(x2, x3), op_t(x2, x3))}, op_t(x2, op_t(x1, rd(op_t(x2, x3), x1))), i(x2))$

$=$ by Lemma 368 LR with $\{x2 \leftarrow op_t(x2, x3), x1 \leftarrow x1\}$

$op_r(\overbrace{x1}, op_t(x2, op_t(x1, \underbrace{rd(op_t(x2, x3), x1)})), i(x2))$

$=$ by Lemma 10 RL with $\{x2 \leftarrow op_t(x2, x3), x1 \leftarrow x1\}$

$op_r(x1, op_t(x2, \underbrace{op_t(x1, \overbrace{i(\overbrace{rd(x1, op_t(x2, x3))})})}), i(x2))$

$=$ by Lemma 18 RL with $\{x2 \leftarrow rd(x1, op_t(x2, x3)), x1 \leftarrow x1\}$

$op_r(x1, \underbrace{op_t(x2, \overbrace{i(\overbrace{op_t(i(x1), rd(x1, op_t(x2, x3)))})})}, i(x2))$

$=$ by Lemma 1119 RL with $\{x3 \leftarrow op_t(i(x1), rd(x1, op_t(x2, x3))), x2 \leftarrow x3, x1 \leftarrow x2\}$

$op_r(x1, \overbrace{op_t(x2, rd(x2, \underbrace{mult(op_t(x2, x3), op_t(i(x1), rd(x1, op_t(x2, x3))))}))}, i(x2))$

$=$ by Lemma 1167 LR with $\{x3 \leftarrow op_t(i(x1), rd(x1, op_t(x2, x3))), x2 \leftarrow x3, x1 \leftarrow x2\}$

$op_r(x1, \overbrace{op_t(x2, \underbrace{mult(i(op_t(x2, x3)), rd(op_t(x2, x3), op_t(i(x1), rd(x1, op_t(x2, x3)))))})}, i(x2))$

$=$ by Lemma 1121 RL with $\{x3 \leftarrow x3, x2 \leftarrow rd(op_t(x2, x3), op_t(i(x1), rd(x1, op_t(x2, x3)))), x1 \leftarrow x2\}$

$op_r(x1, \overbrace{op_t(x2, rd(\underbrace{rd(op_t(x2, x3), op_t(i(x1), rd(x1, op_t(x2, x3))))}, x2))}, i(x2))$

$=$ by Lemma 386 LR with $\{x2 \leftarrow x1, x1 \leftarrow op_t(x2, x3)\}$

$op_r(x1, \underbrace{op_t(x2, rd(\overbrace{mult(x1, op_t(x2, x3))}, x2))}, i(x2))$

$=$ by Lemma 1162 LR with $\{x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow x2\}$

$op_r(x1, \overbrace{op_t(x2, x1)}, i(x2))$

Lemma 1634: $op_t(x1, rd(op_t(x2, x3), x1)) = op_t(x1, rd(x2, x1))$

$$
\begin{aligned}
& \underbrace{op_t(x1, rd(op_t(x2, x3), x1))} \\
= \quad & \text{by Lemma 1633 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_r(x1, op_t(x2, x1), i(x2))}} \\
= \quad & \text{by Lemma 820 RL with } \{x3 \leftarrow x2, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, i(mult(x2, x1)), \underbrace{x2})} \\
= \quad & \text{by Axiom 5 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \underbrace{op_r(x1, i(mult(x2, x1)), \overbrace{rd(mult(x2, x1), x1)})} \\
= \quad & \text{by Lemma 1415 LR with } \{x2 \leftarrow mult(x2, x1), x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_t(x1, i(mult(x2, x1)))}} \\
= \quad & \text{by Lemma 709 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, rd(x2, x1))}
\end{aligned}
$$

Lemma 1635: $op_r(mult(x1, op_t(x2, x3)), x2, x2) = rd(op_t(x2, x3), i(x1))$

$$
\begin{aligned}
& \underbrace{op_r(mult(x1, op_t(x2, x3)), x2, x2)} \\
= \quad & \text{by Lemma 767 RL with } \{x1 \leftarrow x2, x2 \leftarrow mult(x1, op_t(x2, x3))\} \\
& \overbrace{mult(\underbrace{asoc(mult(x1, op_t(x2, x3)), x2, x2)}, mult(x1, op_t(x2, x3)))} \\
= \quad & \text{by Lemma 1028 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{mult(\overbrace{asoc(x1, x2, x2)}, mult(x1, op_t(x2, x3)))} \\
= \quad & \text{by Lemma 941 LR with } \{x3 \leftarrow op_t(x2, x3), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{mult(\underbrace{op_r(x1, x2, x2)}, op_t(x2, x3))} \\
= \quad & \text{by Lemma 1632 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{mult(\overbrace{op_r(x1, op_t(x2, x3), op_t(x2, x3))}, op_t(x2, x3))} \\
= \quad & \text{by Lemma 357 LR with } \{x2 \leftarrow op_t(x2, x3), x1 \leftarrow x1\} \\
& \overbrace{rd(op_t(x2, x3), i(x1))}
\end{aligned}
$$

Lemma 1636: $mult(op_r(x1, x2, x2), op_t(x2, x3)) = rd(op_t(x2, x3), i(x1))$

$$
\begin{aligned}
& \underbrace{mult(op_r(x1, x2, x2), op_t(x2, x3))} \\
= \quad & \text{by Lemma 941 RL with } \{x3 \leftarrow op_t(x2, x3), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{mult(\underbrace{asoc(x1, x2, x2)}, mult(x1, op_t(x2, x3)))} \\
= \quad & \text{by Lemma 1028 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{mult(\overbrace{asoc(mult(x1, op_t(x2, x3)), x2, x2)}, mult(x1, op_t(x2, x3)))} \\
= \quad & \text{by Lemma 767 LR with } \{x1 \leftarrow x2, x2 \leftarrow mult(x1, op_t(x2, x3))\} \\
& \overbrace{\underbrace{op_r(mult(x1, op_t(x2, x3)), x2, x2)}} \\
= \quad & \text{by Lemma 1635 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{rd(op_t(x2, x3), i(x1))}
\end{aligned}
$$

Lemma 1637: $rd(op_t(x1,x2),mult(i(x3),x1)) = mult(rd(x3,x1),op_t(x1,x2))$

$$
\begin{aligned}
& rd(op_t(x1,x2),\underbrace{mult(i(x3),x1)}) \\
= & \quad \text{by Lemma 17 RL with } \{x2 \leftarrow x1, x1 \leftarrow x3\} \\
& \underbrace{rd(op_t(x1,x2),\overbrace{i(mult(x3,i(x1)))})} \\
= & \quad \text{by Lemma 1636 RL with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow mult(x3,i(x1))\} \\
& \overbrace{mult(\underbrace{op_r(mult(x3,i(x1)),x1,x1)},op_t(x1,x2))} \\
= & \quad \text{by Lemma 364 LR with } \{x2 \leftarrow x1, x1 \leftarrow x3\} \\
& mult(\overbrace{rd(x3,x1)},op_t(x1,x2))
\end{aligned}
$$

Lemma 1638: $mult(rd(x1,x2),op_t(x2,x3)) = rd(x1,rd(x2,op_t(x2,x3)))$

$$
\begin{aligned}
& \underbrace{mult(rd(x1,x2),op_t(x2,x3))} \\
= & \quad \text{by Lemma 1637 RL with } \{x3 \leftarrow x1, x2 \leftarrow x3, x1 \leftarrow x2\} \\
& \underbrace{\overbrace{rd(op_t(x2,x3),mult(i(x1),x2))}} \\
= & \quad \text{by Lemma 1291 RL with } \{x3 \leftarrow i(x1), x2 \leftarrow x3, x1 \leftarrow x2\} \\
& \underbrace{\overbrace{rd(rd(op_t(x2,x3),x2),i(x1))}} \\
= & \quad \text{by Lemma 38 LR with } \{x1 \leftarrow x1, x2 \leftarrow rd(op_t(x2,x3),x2)\} \\
& \overbrace{rd(x1,\underbrace{i(rd(op_t(x2,x3),x2))})} \\
= & \quad \text{by Lemma 10 LR with } \{x2 \leftarrow x2, x1 \leftarrow op_t(x2,x3)\} \\
& rd(x1,\overbrace{rd(x2,op_t(x2,x3))})
\end{aligned}
$$

Lemma 1639: $op_t(i(x1),rd(x1,op_t(x2,x3))) = op_t(i(x1),rd(x1,x2))$

$$
\begin{aligned}
& \underbrace{op_t(i(x1),rd(x1,op_t(x2,x3)))} \\
= & \quad \text{by Lemma 717 RL with } \{x2 \leftarrow op_t(x2,x3), x1 \leftarrow x1\} \\
& \overbrace{op_t(\underbrace{i(x1)},rd(op_t(x2,x3),i(x1)))} \\
= & \quad \text{by Lemma 7 RL with } \{x1 \leftarrow x1\} \\
& \underbrace{op_t(\overbrace{op_t(i(x1),x1)},rd(op_t(x2,x3),i(x1)))} \\
= & \quad \text{by Axiom 13 LR with } \{x3 \leftarrow rd(op_t(x2,x3),i(x1)), x2 \leftarrow x1, x1 \leftarrow i(x1)\} \\
& \overbrace{op_t(\underbrace{op_t(i(x1),rd(op_t(x2,x3),i(x1)))},x1)} \\
= & \quad \text{by Lemma 1634 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow i(x1)\} \\
& \underbrace{op_t(\overbrace{op_t(i(x1),rd(x2,i(x1)))},x1)} \\
= & \quad \text{by Axiom 13 RL with } \{x3 \leftarrow rd(x2,i(x1)), x2 \leftarrow x1, x1 \leftarrow i(x1)\} \\
& \overbrace{op_t(\underbrace{op_t(i(x1),x1)},rd(x2,i(x1)))} \\
= & \quad \text{by Lemma 7 LR with } \{x1 \leftarrow x1\} \\
& \underbrace{op_t(\overbrace{i(x1)},rd(x2,i(x1)))} \\
= & \quad \text{by Lemma 717 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_t(i(x1),rd(x1,x2))}
\end{aligned}
$$

Lemma 1640: $op_t(x1, rd(rd(x2, x3), x1)) = op_t(x1, rd(mult(x3, i(x2)), x1))$

$$
\begin{aligned}
& \underbrace{op_t(x1, rd(rd(x2, x3), x1))} \\
= & \quad \text{by Lemma 1634 RL with } \{x3 \leftarrow x3, x2 \leftarrow rd(x2, x3), x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, rd(\underbrace{op_t(rd(x2, x3), x3)}, x1))} \\
= & \quad \text{by Lemma 14 LR with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& \underbrace{op_t(x1, rd(\overbrace{mult(i(x3), x2)}, x1))} \\
= & \quad \text{by Lemma 1410 LR with } \{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, rd(mult(x3, i(x2)), x1))}
\end{aligned}
$$

Lemma 1641: $op_t(x1, rd(rd(x2, rd(x2, x3)), x1)) = op_t(x1, rd(i(x3), x1))$

$$
\begin{aligned}
& op_t(x1, rd(rd(x2, rd(\underbrace{x2}, x3)), x1)) \\
= & \quad \text{by Lemma 3 RL with } \{x1 \leftarrow x2\} \\
& \underbrace{op_t(x1, rd(rd(x2, rd(\overbrace{i(i(x2))}, x3)), x1))} \\
= & \quad \text{by Lemma 1640 LR with } \{x3 \leftarrow rd(i(i(x2)), x3), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, rd(\underbrace{mult(rd(i(i(x2)), x3), i(x2))}, x1))} \\
= & \quad \text{by Lemma 21 LR with } \{x2 \leftarrow x3, x1 \leftarrow i(x2)\} \\
& op_t(x1, rd(\overbrace{i(x3)}, x1))
\end{aligned}
$$

Lemma 1642: $op_t(x1, rd(op_r(x3, x2, x2), x1)) = op_t(x1, rd(x3, x1))$

$$
\begin{aligned}
& op_t(x1, rd(\underbrace{op_r(x3, x2, x2)}, x1)) \\
= & \quad \text{by Lemma 353 RL with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& \underbrace{op_t(x1, rd(\overbrace{rd(x2, rd(x2, x3))}, x1))} \\
= & \quad \text{by Lemma 1641 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_t(x1, rd(i(x3), x1))}} \\
= & \quad \text{by Lemma 712 LR with } \{x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, rd(x3, x1))}
\end{aligned}
$$

Lemma 1643: $op_t(x1, rd(x2, x1)) = op_t(x1, rd(op_l(x2, x3, x2), x1))$

$$
\begin{aligned}
& \underbrace{op_t(x1, rd(x2, x1))} \\
= & \quad \text{by Lemma 1634 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{\overbrace{op_t(x1, rd(op_t(x2, x3), x1))}} \\
= & \quad \text{by Lemma 1642 RL with } \{x2 \leftarrow x3, x3 \leftarrow op_t(x2, x3), x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, rd(\underbrace{op_r(op_t(x2, x3), x3, x3)}, x1))} \\
= & \quad \text{by Lemma 745 RL with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& op_t(x1, rd(\overbrace{op_l(x2, x3, x2)}, x1))
\end{aligned}
$$

Lemma 1644: $rd(rd(op_t(x2, x3), x2), x1) = mult(rd(i(x2), x1), op_t(x2, x3))$

$$
\begin{aligned}
& \underbrace{rd(rd(op_t(x2, x3), x2), x1)} \\
= \quad & \text{by Lemma 40 RL with } \{x3 \leftarrow op_t(x2, x3), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{\overbrace{rd(i(x1), rd(x2, op_t(x2, x3)))}} \\
= \quad & \text{by Lemma 1638 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow i(x1)\} \\
& \overbrace{mult(\underbrace{rd(i(x1), x2)}, op_t(x2, x3))} \\
= \quad & \text{by Lemma 37 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& mult(\overbrace{rd(i(x2), x1)}, op_t(x2, x3))
\end{aligned}
$$

Lemma 1645: $mult(rd(x1, x2), i(op_t(x1, x3))) = rd(rd(x1, op_t(x1, x3)), x2)$

$$
\begin{aligned}
& \underbrace{mult(rd(x1, x2), i(op_t(x1, x3)))} \\
= \quad & \text{by Lemma 44 RL with } \{x3 \leftarrow op_t(x1, x3), x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \overbrace{i(\underbrace{mult(rd(x2, x1), op_t(x1, x3))})} \\
= \quad & \text{by Lemma 1638 LR with } \{x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \underbrace{i(\overbrace{rd(x2, rd(x1, op_t(x1, x3)))})} \\
= \quad & \text{by Lemma 10 LR with } \{x2 \leftarrow rd(x1, op_t(x1, x3)), x1 \leftarrow x2\} \\
& \overbrace{rd(rd(x1, op_t(x1, x3)), x2)}
\end{aligned}
$$

Lemma 1646: $op_l(asoc(x1,x2,x2),x3,x1) = asoc(x1,x2,x2)$

$op_l(\underbrace{asoc(x1,x2,x2)},x3,x1)$

$=$ by Lemma 777 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$op_l(\overbrace{rd(x1,\underbrace{op_r(x1,x2,i(x2))})},x3,x1)$

$=$ by Lemma 829 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$op_l(\underbrace{rd(x1,\overbrace{op_t(x1,i(\underbrace{op_t(mult(x1,x2),x2)}))})},x3,x1)$

$=$ by Lemma 1375 RL with $\{x3 \leftarrow i(op_t(mult(x1,x2),x2)), x2 \leftarrow x1, x1 \leftarrow x3\}$

$op_l(\overbrace{mult(x3,\underbrace{rd(x1,mult(x3,op_t(x1,i(\underbrace{op_t(mult(x1,x2),x2)}))))})},x3,x1)$

$=$ by Lemma 1236 LR with $\{x3 \leftarrow i(op_t(mult(x1,x2),x2)), x2 \leftarrow x3, x1 \leftarrow x1\}$

$op_l(mult(x3,\overbrace{\underbrace{rd(rd(x1,op_t(x1,i(\underbrace{op_t(mult(x1,x2),x2)}))),x3)}}),x3,x1)$

$=$ by Lemma 1645 RL with $\{x3 \leftarrow i(op_t(mult(x1,x2),x2)), x2 \leftarrow x3, x1 \leftarrow x1\}$

$\underbrace{op_l(mult(x3,\overbrace{mult(rd(x1,x3),i(op_t(x1,i(\underbrace{op_t(mult(x1,x2),x2)}))))}),x3,x1)}$

$=$ by Lemma 1471 LR with $\{x3 \leftarrow i(op_t(x1,i(op_t(mult(x1,x2),x2)))), x2 \leftarrow x1, x1 \leftarrow x3\}$

$\overbrace{\underbrace{mult(x1,i(op_t(x1,i(\underbrace{op_t(mult(x1,x2),x2)}))))}}$

$=$ by Lemma 98 RL with $\{x2 \leftarrow i(op_t(mult(x1,x2),x2)), x1 \leftarrow x1\}$

$\overbrace{rd(x1,\underbrace{op_t(x1,i(\underbrace{op_t(mult(x1,x2),x2)}))})}$

$=$ by Lemma 829 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{rd(x1,\overbrace{op_r(x1,x2,i(x2))})}$

$=$ by Lemma 777 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{asoc(x1,x2,x2)}$

Lemma 1647: $asoc(x2,x1,x2) = op_l(asoc(i(x1),x2,x2),x1,x3)$

$\underbrace{asoc(x2,x1,x2)}$

$=$ by Lemma 758 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{\underbrace{asoc(i(x1),x2,x2)}}$

$=$ by Lemma 1646 RL with $\{x3 \leftarrow mult(x1,x3), x2 \leftarrow x2, x1 \leftarrow i(x1)\}$

$\overbrace{\underbrace{op_l(asoc(i(x1),x2,x2),mult(x1,x3),i(x1))}}$

$=$ by Lemma 242 LR with $\{x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow asoc(i(x1),x2,x2)\}$

$\overbrace{op_l(asoc(i(x1),x2,x2),x1,x3)}$

Lemma 1648: $asoc(mult(x1,x2),x2,mult(x1,x2)) = op_l(rd(x1,op_t(x1,x2)),x2,x3)$

$$
\begin{aligned}
& \underbrace{asoc(mult(x1,x2),x2,mult(x1,x2))} \\
= & \quad \text{by Lemma 1647 LR with } \{x3 \leftarrow x3, x1 \leftarrow x2, x2 \leftarrow mult(x1,x2)\} \\
& \overbrace{op_l(\underbrace{asoc(i(x2),mult(x1,x2),mult(x1,x2))},x2,x3)} \\
= & \quad \text{by Lemma 758 LR with } \{x2 \leftarrow mult(x1,x2), x1 \leftarrow x2\} \\
& op_l(\overbrace{\underbrace{asoc(mult(x1,x2),x2,mult(x1,x2))}},x2,x3) \\
= & \quad \text{by Lemma 838 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_l(\overbrace{rd(x1,\underbrace{op_t(x1,x2)})},x2,x3)
\end{aligned}
$$

Lemma 1649: $rd(mult(x1,x2),asoc(x3,x2,x3)) = mult(x1,op_r(x2,x3,x3))$

$$
\begin{aligned}
& \underbrace{rd(mult(x1,x2),asoc(x3,x2,x3))} \\
= & \quad \text{by Lemma 1300 LR with } \{x2 \leftarrow x2, x3 \leftarrow x3, x1 \leftarrow mult(x1,x2)\} \\
& \overbrace{\underbrace{mult(mult(x1,x2),asoc(x2,x3,x3))}} \\
= & \quad \text{by Lemma 137 RL with } \{x3 \leftarrow asoc(x2,x3,x3), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{mult(x1,mult(x2,\underbrace{op_l(asoc(x2,x3,x3),x1,x2)}))} \\
= & \quad \text{by Lemma 1646 LR with } \{x3 \leftarrow x1, x2 \leftarrow x3, x1 \leftarrow x2\} \\
& mult(x1,\underbrace{mult(x2,\overbrace{asoc(x2,x3,x3)})}) \\
= & \quad \text{by Lemma 1300 RL with } \{x2 \leftarrow x2, x3 \leftarrow x3, x1 \leftarrow x2\} \\
& mult(x1,\overbrace{\underbrace{rd(x2,asoc(x3,x2,x3))}}) \\
= & \quad \text{by Lemma 641 LR with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& mult(x1,\overbrace{op_r(x2,x3,x3)})
\end{aligned}
$$

Lemma 1650: $mult(rd(x1, x2), op_r(x2, x3, x3)) = rd(x1, asoc(x3, x2, x3))$

$mult(rd(x1, x2), \underbrace{op_r(x2, x3, x3)})$

$=$ by Lemma 641 RL with $\{x2 \leftarrow x3, x1 \leftarrow x2\}$

$mult(rd(x1, x2), \overbrace{\underbrace{rd(x2, asoc(x3, x2, x3))}})$

$=$ by Lemma 1300 LR with $\{x2 \leftarrow x2, x3 \leftarrow x3, x1 \leftarrow x2\}$

$mult(rd(x1, x2), \overbrace{mult(x2, \underbrace{asoc(x2, x3, x3)})})$

$=$ by Lemma 1646 RL with $\{x3 \leftarrow x1, x2 \leftarrow x3, x1 \leftarrow x2\}$

$mult(rd(x1, x2), mult(x2, \overbrace{\underbrace{op_l(asoc(x2, x3, x3), x1, x2)}}))$

$=$ by Lemma 292 RL with $\{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow asoc(x2, x3, x3)\}$

$\underbrace{mult(rd(x1, x2), mult(x2, \overbrace{op_l(asoc(x2, x3, x3), rd(x1, x2), x2)}))}$

$=$ by Lemma 137 LR with $\{x3 \leftarrow asoc(x2, x3, x3), x2 \leftarrow x2, x1 \leftarrow rd(x1, x2)\}$

$\overbrace{mult(\underbrace{mult(rd(x1, x2), x2)}, asoc(x2, x3, x3))}$

$=$ by Axiom 4 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{mult(\overbrace{x1}, asoc(x2, x3, x3))}$

$=$ by Lemma 1300 RL with $\{x2 \leftarrow x2, x3 \leftarrow x3, x1 \leftarrow x1\}$

$\overbrace{rd(x1, asoc(x3, x2, x3))}$

Lemma 1651: $rd(x1, rd(op_t(x1, x2), i(x3))) = rd(rd(x1, x3), op_t(x1, x2))$

$rd(x1, \underbrace{rd(op_t(x1, x2), i(x3))})$

$=$ by Lemma 1636 RL with $\{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x3\}$

$\underbrace{rd(x1, \overbrace{mult(op_r(x3, x1, x1), op_t(x1, x2))})}$

$=$ by Lemma 1236 LR with $\{x3 \leftarrow x2, x2 \leftarrow op_r(x3, x1, x1), x1 \leftarrow x1\}$

$\overbrace{\underbrace{rd(rd(x1, op_t(x1, x2)), op_r(x3, x1, x1))}}$

$=$ by Lemma 1251 RL with $\{x3 \leftarrow rd(x1, op_t(x1, x2)), x2 \leftarrow x3, x1 \leftarrow x1\}$

$\overbrace{rd(\underbrace{asoc(x1, x3, x1)}, \overbrace{rd(x3, rd(x1, op_t(x1, x2)))})}$

$=$ by Lemma 263 RL with $\{x2 \leftarrow x3, x1 \leftarrow x1\}$

$rd(\overbrace{\underbrace{asoc(x1, rd(x3, x1), x1)}}, rd(x3, rd(x1, op_t(x1, x2))))$

$=$ by Lemma 1631 RL with $\{x3 \leftarrow rd(x3, x1), x2 \leftarrow x2, x1 \leftarrow x1\}$

$rd(\overbrace{asoc(op_t(x1, x2), rd(x3, x1), op_t(x1, x2))}, \underbrace{rd(x3, rd(x1, op_t(x1, x2)))})$

$=$ by Lemma 1638 RL with $\{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x3\}$

$rd(\underbrace{asoc(op_t(x1, x2), rd(x3, x1), op_t(x1, x2))}, \overbrace{mult(rd(x3, x1), op_t(x1, x2))})$

$=$ by Lemma 262 RL with $\{x2 \leftarrow rd(x3, x1), x1 \leftarrow op_t(x1, x2)\}$

$\underbrace{rd(\overbrace{asoc(op_t(x1, x2), mult(rd(x3, x1), op_t(x1, x2)), op_t(x1, x2))}, mult(rd(x3, x1), op_t(x1, x2)))}$

$=$ by Lemma 642 LR with $\{x2 \leftarrow mult(rd(x3, x1), op_t(x1, x2)), x1 \leftarrow op_t(x1, x2)\}$

$\overbrace{\underbrace{op_r(i(mult(rd(x3, x1), op_t(x1, x2))), op_t(x1, x2), op_t(x1, x2))}}$

$=$ by Lemma 371 LR with $\{x2 \leftarrow op_t(x1, x2), x1 \leftarrow rd(x3, x1)\}$

$\overbrace{rd(\underbrace{i(rd(x3, x1))}, op_t(x1, x2))}$

$=$ by Lemma 10 LR with $\{x2 \leftarrow x1, x1 \leftarrow x3\}$

$rd(\overbrace{rd(x1, x3)}, op_t(x1, x2))$

Lemma 1652: $rd(rd(x1, asoc(x2, x3, x3)), x2) = rd(x1, op_r(x2, x3, x3))$

$rd(\underbrace{rd(x1, asoc(x2, x3, x3))}, x2)$

$=$ by Lemma 1295 RL with $\{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x1\}$

$\underbrace{rd(\overbrace{mult(x1, asoc(x3, x2, x3))}, x2)}$

$=$ by Lemma 1152 RL with $\{x3 \leftarrow x2, x2 \leftarrow asoc(x3, x2, x3), x1 \leftarrow x1\}$

$\overbrace{mult(x1, mult(rd(\underbrace{asoc(x3, x2, x3)}, mult(x1, x2)), x1))}$

$=$ by Lemma 758 RL with $\{x2 \leftarrow x3, x1 \leftarrow x2\}$

$mult(x1, mult(\underbrace{rd(\overbrace{asoc(i(x2), x3, x3)}, mult(x1, x2))}, x1))$

$=$ by Lemma 785 RL with $\{x3 \leftarrow i(x2), x2 \leftarrow x3, x1 \leftarrow mult(x1, x2)\}$

$mult(x1, mult(\overbrace{rd(\underbrace{i(mult(x1, x2))}, asoc(x3, i(x2), x3))}, x1))$

$=$ by Axiom 11 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$mult(x1, mult(\underbrace{rd(\overbrace{mult(i(x1), i(x2))}, asoc(x3, i(x2), x3))}, x1))$

$=$ by Lemma 1649 LR with $\{x3 \leftarrow x3, x2 \leftarrow i(x2), x1 \leftarrow i(x1)\}$

$mult(x1, mult(\overbrace{mult(i(x1), op_r(i(x2), x3, x3))}, x1))$

$=$ by Lemma 1099 LR with $\{x2 \leftarrow x3, x3 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{mult(x1, mult(\overbrace{i(mult(x1, op_r(x2, x3, x3)))}, x1))}$

$=$ by Lemma 60 LR with $\{x2 \leftarrow mult(x1, op_r(x2, x3, x3)), x1 \leftarrow x1\}$

$\overbrace{rd(x1, \underbrace{mult(i(x1), mult(x1, op_r(x2, x3, x3)))})}$

$=$ by Axiom 2 RL with $\{x2 \leftarrow x1, x1 \leftarrow op_r(x2, x3, x3)\}$

$rd(x1, \overbrace{op_r(x2, x3, x3)})$

Lemma 1653: $rd(x1, rd(x2, asoc(x1, x3, x3))) = rd(op_r(x1, x3, x3), x2)$

$\underbrace{rd(x1, rd(x2, asoc(x1, x3, x3)))}$

$=$ by Lemma 10 RL with $\{x2 \leftarrow x1, x1 \leftarrow rd(x2, asoc(x1, x3, x3))\}$

$\overbrace{i(\underbrace{rd(rd(x2, asoc(x1, x3, x3)), x1)})}$

$=$ by Lemma 1652 LR with $\{x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow x2\}$

$\underbrace{i(\overbrace{rd(x2, op_r(x1, x3, x3))})}$

$=$ by Lemma 10 LR with $\{x2 \leftarrow op_r(x1, x3, x3), x1 \leftarrow x2\}$

$\overbrace{rd(op_r(x1, x3, x3), x2)}$

Lemma 1654: $rd(i(x1), op_r(x2, x3, x3)) = rd(rd(asoc(x3, x2, x3), x1), x2)$

$$
\begin{aligned}
& \underbrace{rd(i(x1), op_r(x2, x3, x3))} \\
= & \quad \text{by Lemma 1652 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow i(x1)\} \\
& \overbrace{rd(\underbrace{rd(i(x1), asoc(x2, x3, x3))}, x2)} \\
= & \quad \text{by Lemma 37 LR with } \{x2 \leftarrow asoc(x2, x3, x3), x1 \leftarrow x1\} \\
& rd(\overbrace{rd(\underbrace{i(asoc(x2, x3, x3))}, x1)}, x2) \\
= & \quad \text{by Lemma 759 LR with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& rd(rd(\overbrace{asoc(x3, x2, x3)}, x1), x2)
\end{aligned}
$$

Lemma 1655: $op_r(x1, op_t(x2, x3), i(op_t(x2, x3))) = op_r(x1, x2, i(x2))$

$$
\begin{aligned}
& \underbrace{op_r(x1, op_t(x2, x3), i(op_t(x2, x3)))} \\
= & \quad \text{by Lemma 775 RL with } \{x2 \leftarrow op_t(x2, x3), x1 \leftarrow x1\} \\
& \overbrace{rd(x1, \underbrace{asoc(x1, op_t(x2, x3), op_t(x2, x3))})} \\
= & \quad \text{by Lemma 1630 LR with } \{x2 \leftarrow x3, x1 \leftarrow x2, x3 \leftarrow x1\} \\
& \underbrace{rd(x1, \overbrace{asoc(x1, x2, x2)})} \\
= & \quad \text{by Lemma 775 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, x2, i(x2))}
\end{aligned}
$$

Lemma 1656: $rd(x1, i(mult(x2, i(op_t(x1, x3))))) = mult(rd(x2, op_t(x1, x3)), x1)$

$$
\begin{aligned}
& \underbrace{rd(x1, i(mult(x2, i(op_t(x1, x3)))))} \\
= & \quad \text{by Lemma 357 RL with } \{x2 \leftarrow x1, x1 \leftarrow mult(x2, i(op_t(x1, x3)))\} \\
& \overbrace{mult(\underbrace{op_r(mult(x2, i(op_t(x1, x3))), x1, x1)}, x1)} \\
= & \quad \text{by Lemma 1632 RL with } \{x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow mult(x2, i(op_t(x1, x3)))\} \\
& mult(\overbrace{op_r(mult(x2, i(op_t(x1, x3))), op_t(x1, x3), op_t(x1, x3))}, x1) \\
= & \quad \text{by Lemma 364 LR with } \{x2 \leftarrow op_t(x1, x3), x1 \leftarrow x2\} \\
& mult(\overbrace{rd(x2, op_t(x1, x3))}, x1)
\end{aligned}
$$

Lemma 1657: $rd(x2, rd(op_t(x1, x3), x1)) = mult(rd(x2, op_t(x1, x3)), x1)$

$$
\begin{aligned}
& \underbrace{rd(x2, rd(op_t(x1, x3), x1))} \\
= & \quad \text{by Lemma 1257 LR with } \{x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \overbrace{rd(x1, \underbrace{mult(i(x2), op_t(x1, x3))})} \\
= & \quad \text{by Lemma 17 RL with } \{x2 \leftarrow op_t(x1, x3), x1 \leftarrow x2\} \\
& \underbrace{rd(x1, \overbrace{i(mult(x2, i(op_t(x1, x3))))})} \\
= & \quad \text{by Lemma 1656 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{mult(rd(x2, op_t(x1, x3)), x1)}
\end{aligned}
$$

Lemma 1658: $mult(rd(op_t(x1,x2),x3),i(x1)) = rd(rd(op_t(x1,x2),x1),x3)$

$$
\begin{aligned}
& \overbrace{mult(rd(op_t(x1,x2),x3),i(x1))} \\
= \quad & \text{by Lemma 44 RL with } \{x3 \leftarrow x1, x2 \leftarrow op_t(x1,x2), x1 \leftarrow x3\} \\
& \overbrace{i(\underbrace{mult(rd(x3,op_t(x1,x2)),x1)})} \\
= \quad & \text{by Lemma 1657 RL with } \{x3 \leftarrow x2, x1 \leftarrow x1, x2 \leftarrow x3\} \\
& \underbrace{i(\overbrace{rd(x3,rd(op_t(x1,x2),x1))})} \\
= \quad & \text{by Lemma 10 LR with } \{x2 \leftarrow rd(op_t(x1,x2),x1), x1 \leftarrow x3\} \\
& \overbrace{rd(rd(op_t(x1,x2),x1),x3)}
\end{aligned}
$$

Lemma 1659: $rd(op_t(x1,x2),rd(x3,op_t(x2,x1))) = op_r(mult(rd(x2,x3),x1),x1,x1)$

$$
\begin{aligned}
& rd(op_t(x1,x2),\underbrace{rd(x3,op_t(x2,x1))}) \\
= \quad & \text{by Lemma 10 RL with } \{x2 \leftarrow x3, x1 \leftarrow op_t(x2,x1)\} \\
& \underbrace{rd(op_t(x1,x2),\overbrace{i(rd(op_t(x2,x1),x3))})} \\
= \quad & \text{by Lemma 1635 RL with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow rd(op_t(x2,x1),x3)\} \\
& \overbrace{op_r(\underbrace{mult(rd(op_t(x2,x1),x3),op_t(x1,x2))},x1,x1)} \\
= \quad & \text{by Lemma 1518 LR with } \{x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow x2\} \\
& op_r(\overbrace{mult(rd(x2,x3),x1)},x1,x1)
\end{aligned}
$$

Lemma 1660: $rd(op_t(x1,x2),rd(x3,op_t(x2,x1))) = rd(x1,rd(x3,x2))$

$$
\begin{aligned}
& \underbrace{rd(op_t(x1,x2),rd(x3,op_t(x2,x1)))} \\
= \quad & \text{by Lemma 1659 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_r(mult(rd(x2,x3),x1),x1,x1)}} \\
= \quad & \text{by Lemma 359 LR with } \{x2 \leftarrow x1, x1 \leftarrow rd(x2,x3)\} \\
& \overbrace{rd(x1,\underbrace{i(rd(x2,x3))})} \\
= \quad & \text{by Lemma 10 LR with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& rd(x1,\overbrace{rd(x3,x2)})
\end{aligned}
$$

Lemma 1661: $op_l(x1, x1, mult(x2, op_t(x3, x1))) = op_l(x1, x1, mult(x2, x3))$

$op_l(x1, x1, mult(\underbrace{x2}, op_t(x3, x1)))$

$=$ by Axiom 5 LR with $\{x2 \leftarrow x3, x1 \leftarrow x2\}$

$op_l(x1, x1, \underbrace{mult(\overbrace{rd(mult(x2, x3), x3)}, op_t(x3, x1))})$

$=$ by Lemma 1638 LR with $\{x3 \leftarrow x1, x2 \leftarrow x3, x1 \leftarrow mult(x2, x3)\}$

$op_l(x1, x1, \overbrace{rd(mult(x2, x3), \underbrace{rd(x3, op_t(x3, x1))})})$

$=$ by Lemma 838 RL with $\{x2 \leftarrow x1, x1 \leftarrow x3\}$

$op_l(x1, x1, rd(mult(x2, x3), \underbrace{\overbrace{asoc(mult(x3, x1), x1, mult(x3, x1))}}))$

$=$ by Lemma 759 RL with $\{x2 \leftarrow mult(x3, x1), x1 \leftarrow x1\}$

$op_l(x1, x1, \underbrace{rd(mult(x2, x3), \overbrace{i(asoc(x1, mult(x3, x1), mult(x3, x1)))})})$

$=$ by Lemma 38 RL with $\{x1 \leftarrow mult(x2, x3), x2 \leftarrow asoc(x1, mult(x3, x1), mult(x3, x1))\}$

$\underbrace{op_l(x1, x1, \overbrace{rd(asoc(x1, mult(x3, x1), mult(x3, x1)), i(mult(x2, x3)))})}$

$=$ by Lemma 1252 RL with $\{x3 \leftarrow mult(x3, x1), x2 \leftarrow i(mult(x2, x3)), x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_l(x1, i(mult(x2, x3)), x1)}}$

$=$ by Lemma 1050 LR with $\{x3 \leftarrow x1, x2 \leftarrow mult(x2, x3), x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_l(x1, mult(x2, x3), i(x1))}}$

$=$ by Lemma 606 LR with $\{x2 \leftarrow mult(x2, x3), x1 \leftarrow x1\}$

$\overbrace{op_l(x1, x1, mult(x2, x3))}$

Lemma 1662: $mult(x1, mult(x2, rd(x3, op_t(x3, x2)))) = mult(mult(x1, x2), rd(x3, op_t(x3, x2)))$

$\underbrace{mult(x1, mult(x2, rd(x3, op_t(x3, x2))))}$

$=$ by Lemma 132 RL with $\{x3 \leftarrow rd(x3, op_t(x3, x2)), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{mult(mult(x1, x2), \underbrace{op_l(rd(x3, op_t(x3, x2)), x2, x1)})}$

$=$ by Lemma 1648 RL with $\{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x3\}$

$mult(mult(x1, x2), \overbrace{asoc(mult(x3, x2), x2, mult(x3, x2))})$

$=$ by Lemma 838 LR with $\{x2 \leftarrow x2, x1 \leftarrow x3\}$

$mult(mult(x1, x2), \overbrace{rd(x3, op_t(x3, x2))})$

Lemma 1663: $mult(x1, op_t(x2, x3)) = mult(mult(x1, x2), rd(x3, op_t(x3, x2)))$

$mult(x1, \underbrace{op_t(x2, x3)})$

$=$ by Lemma 390 RL with $\{x2 \leftarrow x3, x1 \leftarrow x2\}$

$mult(x1, \underbrace{\overbrace{rd(x2, rd(op_t(x3, x2), x3))})$

$=$ by Lemma 1293 RL with $\{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x2\}$

$\underbrace{mult(x1, \overbrace{mult(x2, rd(x3, op_t(x3, x2)))})}$

$=$ by Lemma 1662 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{mult(mult(x1, x2), rd(x3, op_t(x3, x2)))}$

Lemma 1664: $rd(mult(x1, x2), rd(op_t(x3, x2), x3)) = mult(x1, op_t(x2, x3))$

$$
\begin{aligned}
& \underbrace{rd(mult(x1, x2), rd(op_t(x3, x2), x3))} \\
= \quad & \text{by Lemma 1293 RL with } \{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow mult(x1, x2)\} \\
& \overbrace{\underbrace{mult(mult(x1, x2), rd(x3, op_t(x3, x2)))}} \\
= \quad & \text{by Lemma 1663 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{mult(x1, op_t(x2, x3))}
\end{aligned}
$$

Lemma 1665: $op_t(x1, mult(x2, mult(x1, rd(x3, x1)))) = op_t(x1, mult(x2, rd(mult(x1, x3), x1)))$

$$
\begin{aligned}
& \underbrace{op_t(x1, mult(x2, mult(x1, rd(x3, x1))))} \\
= \quad & \text{by Lemma 1281 RL with } \{x3 \leftarrow mult(x1, rd(x3, x1)), x2 \leftarrow mult(x2, mult(x1, rd(x3, x1))), x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, \underbrace{rd(mult(x2, mult(x1, rd(x3, x1))), asoc(x1, mult(x1, rd(x3, x1)), x1))})} \\
= \quad & \text{by Lemma 1649 LR with } \{x3 \leftarrow x1, x2 \leftarrow mult(x1, rd(x3, x1)), x1 \leftarrow x2\} \\
& op_t(x1, \overbrace{mult(x2, \underbrace{op_r(mult(x1, rd(x3, x1)), x1, x1)})}) \\
= \quad & \text{by Lemma 802 LR with } \{x2 \leftarrow x3, x1 \leftarrow x1\} \\
& op_t(x1, mult(x2, \overbrace{rd(mult(x1, x3), x1)}))
\end{aligned}
$$

Lemma 1666: $rd(x1, op_r(x2, i(mult(x2, x3)), i(i(mult(x2, x3))))) = rd(rd(x1, asoc(i(x2), rd(x2, x3), i(mult(x2, x3)))), x2)$

$rd(x1, \underbrace{op_r(x2, i(mult(x2, x3)), i(i(mult(x2, x3))))}$

$=$ by Lemma 628 RL with $\{x2 \leftarrow x2, x1 \leftarrow i(mult(x2, x3))\}$

$\underbrace{rd(x1, \overbrace{mult(asoc(i(mult(x2, x3)), x2, i(mult(x2, x3))), x2)})}$

$=$ by Lemma 839 RL with $\{x3 \leftarrow asoc(i(mult(x2, x3)), x2, i(mult(x2, x3))), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_r(rd(rd(x1, x2), asoc(i(mult(x2, x3)), x2, i(mult(x2, x3)))), \underbrace{asoc(i(mult(x2, x3)), x2, i(mult(x2, x3)))}, x2)}$

$=$ by Lemma 639 RL with $\{x2 \leftarrow i(mult(x2, x3)), x1 \leftarrow x2\}$

$op_r(rd(rd(x1, x2), asoc(i(mult(x2, x3)), x2, i(mult(x2, x3)))), \overbrace{rd(\underbrace{op_r(x2, i(mult(x2, x3)), i(i(mult(x2, x3))))}, x2)}, x2)$

$=$ by Lemma 547 RL with $\{x2 \leftarrow i(mult(x2, x3)), x1 \leftarrow x2\}$

$\underbrace{op_r(rd(rd(x1, x2), asoc(i(mult(x2, x3)), x2, i(mult(x2, x3)))), rd(\overbrace{op_t(x2, rd(i(mult(x2, x3)), x2))}, x2), x2)}$

$=$ by Lemma 693 RL with $\{x3 \leftarrow x2, x2 \leftarrow op_t(x2, rd(i(mult(x2, x3)), x2)), x1 \leftarrow rd(rd(x1, x2), asoc(i(mult(x2, x3)), x2, i(mul$

$\overbrace{rd(mult(mult(rd(rd(x1, x2), asoc(i(mult(x2, x3)), x2, i(mult(x2, x3)))), rd(op_t(x2, rd(i(mult(x2, x3)), x2)), x2)), x2), op_t($

$=$ by Lemma 1292 RL with $\{x3 \leftarrow rd(i(mult(x2, x3)), x2), x2 \leftarrow x2, x1 \leftarrow mult(rd(rd(x1, x2), asoc(i(mult(x2, x3)), x2, i(mult($

$\overbrace{rd(mult(rd(rd(x1, x2), asoc(i(mult(x2, x3)), x2, i(mult(x2, x3)))), rd(op_t(x2, rd(i(mult(x2, x3)), x2)), x2)), rd(op_t(x2, rd(i$

$=$ by Axiom 5 RL with $\{x2 \leftarrow rd(op_t(x2, rd(i(mult(x2, x3)), x2)), x2), x1 \leftarrow rd(rd(x1, x2), asoc(i(mult(x2, x3)), x2, i(mult(x2,$

$\overbrace{rd(rd(x1, x2), asoc(i(mult(x2, x3)), x2, i(mult(x2, x3))))}$

$=$ by Lemma 1300 LR with $\{x2 \leftarrow x2, x3 \leftarrow i(mult(x2, x3)), x1 \leftarrow rd(x1, x2)\}$

$\overbrace{mult(rd(x1, x2), asoc(x2, i(mult(x2, x3)), i(mult(x2, x3))))}$

$=$ by Lemma 1246 RL with $\{x3 \leftarrow i(mult(x2, x3)), x2 \leftarrow x2, x1 \leftarrow rd(x1, x2)\}$

$\overbrace{rd(\underbrace{mult(rd(x1, x2), op_r(x2, i(mult(x2, x3)), i(mult(x2, x3))))}, x2)}$

$=$ by Lemma 1650 LR with $\{x3 \leftarrow i(mult(x2, x3)), x2 \leftarrow x2, x1 \leftarrow x1\}$

$rd(\overbrace{rd(x1, asoc(i(mult(x2, x3)), \underbrace{x2}, i(mult(x2, x3))))}, x2)$

$=$ by Lemma 3 RL with $\{x1 \leftarrow x2\}$

$rd(rd(x1, asoc(\underbrace{i(mult(x2, x3))}, \overbrace{i(i(x2))}, i(mult(x2, x3)))), x2)$

$=$ by Axiom 11 RL with $\{x2 \leftarrow x3, x1 \leftarrow x2\}$

$rd(rd(x1, \underbrace{asoc(\overbrace{mult(i(x2), i(x3))}, i(i(x2)), i(mult(x2, x3))))}, x2)$

$=$ by Lemma 1088 LR with $\{x3 \leftarrow i(mult(x2, x3)), x2 \leftarrow i(x3), x1 \leftarrow i(x2)\}$

$rd(rd(x1, \overbrace{asoc(i(x2), \underbrace{rd(i(x3), i(x2))}, i(mult(x2, x3)))}), x2)$

$=$ by Lemma 23 LR with $\{x2 \leftarrow x2, x1 \leftarrow x3\}$

$rd(rd(x1, asoc(i(x2), \overbrace{rd(x2, x3)}, i(mult(x2, x3)))), x2)$

Lemma 1667: $rd(x1, op_r(x2, x3, i(mult(x2, x3)))) = rd(rd(x1, asoc(i(x2), rd(x2, x3), i(mult(x2, x3)))), x2)$

$$
\begin{aligned}
& rd(x1, op_r(x2, x3, \underbrace{i(mult(x2, x3))})) \\
= \quad & \text{by Lemma 3 RL with } \{x1 \leftarrow i(mult(x2, x3))\} \\
& rd(x1, \underbrace{op_r(x2, x3, \overbrace{i(i(i(mult(x2, x3))))})}) \\
= \quad & \text{by Lemma 818 RL with } \{x3 \leftarrow i(i(mult(x2, x3))), x2 \leftarrow x3, x1 \leftarrow x2\} \\
& \underbrace{rd(x1, \overbrace{op_r(x2, i(mult(x2, x3)), i(i(mult(x2, x3))))})} \\
= \quad & \text{by Lemma 1666 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{rd(rd(x1, asoc(i(x2), rd(x2, x3), i(mult(x2, x3)))), x2)}
\end{aligned}
$$

Lemma 1668: $rd(x1, op_t(x2, x3)) = rd(rd(x1, i(asoc(x2, rd(x3, x2), mult(x2, x3)))), x2)$

$$
\begin{aligned}
& rd(x1, \underbrace{op_t(x2, x3)}) \\
= \quad & \text{by Lemma 79 RL with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& \underbrace{rd(x1, \overbrace{op_r(x2, x3, i(mult(x2, x3)))})} \\
= \quad & \text{by Lemma 1667 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{rd(rd(x1, asoc(i(x2), \underbrace{rd(x2, x3)}, i(mult(x2, x3)))), x2)} \\
= \quad & \text{by Lemma 10 RL with } \{x2 \leftarrow x2, x1 \leftarrow x3\} \\
& rd(rd(x1, \underbrace{asoc(i(x2), \overbrace{i(rd(x3, x2))}, i(mult(x2, x3)))}), x2) \\
= \quad & \text{by Lemma 1480 RL with } \{x3 \leftarrow mult(x2, x3), x2 \leftarrow rd(x3, x2), x1 \leftarrow x2\} \\
& rd(rd(x1, \overbrace{i(asoc(x2, rd(x3, x2), mult(x2, x3)))}), x2)
\end{aligned}
$$

Lemma 1669: $rd(x1, op_t(x2, x3)) = rd(rd(x1, rd(op_t(x2, x3), x2)), x2)$

$$
\begin{aligned}
& \underbrace{rd(x1, op_t(x2, x3))} \\
= \quad & \text{by Lemma 1668 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{rd(rd(x1, i(\underbrace{asoc(x2, rd(x3, x2), mult(x2, x3))})), x2)} \\
= \quad & \text{by Lemma 1089 LR with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& rd(rd(x1, \underbrace{i(\overbrace{rd(x2, op_t(x2, x3))})}), x2) \\
= \quad & \text{by Lemma 10 LR with } \{x2 \leftarrow op_t(x2, x3), x1 \leftarrow x2\} \\
& rd(rd(x1, \overbrace{rd(op_t(x2, x3), x2)}), x2)
\end{aligned}
$$

Lemma 1670: $rd(rd(x1, x2), op_t(x2, x3)) = rd(rd(x1, op_t(x2, x3)), x2)$

$$
\begin{aligned}
& \underbrace{rd(rd(x1, x2), op_t(x2, x3))} \\
= \quad & \text{by Lemma 1669 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow rd(x1, x2)\} \\
& \overbrace{rd(rd(rd(x1, x2), \underbrace{rd(op_t(x2, x3), x2)}), x2)} \\
= \quad & \text{by Lemma 1223 RL with } \{x3 \leftarrow x1, x2 \leftarrow x3, x1 \leftarrow x2\} \\
& rd(\underbrace{rd(rd(x1, x2), \overbrace{mult(rd(op_t(x2, x3), x1), rd(x1, x2))})}, x2) \\
= \quad & \text{by Lemma 9 LR with } \{x2 \leftarrow rd(op_t(x2, x3), x1), x1 \leftarrow rd(x1, x2)\} \\
& rd(\overbrace{\underbrace{i(rd(op_t(x2, x3), x1))}}, x2) \\
= \quad & \text{by Lemma 10 LR with } \{x2 \leftarrow x1, x1 \leftarrow op_t(x2, x3)\} \\
& rd(\overbrace{rd(x1, op_t(x2, x3))}, x2)
\end{aligned}
$$

Lemma 1671: $rd(x1, rd(x2, op_t(x1, x3))) = rd(op_t(x1, x3), rd(x2, x1))$

$$
\begin{aligned}
& \underbrace{rd(x1, rd(x2, op_t(x1, x3)))} \\
= \quad & \text{by Lemma 10 RL with } \{x2 \leftarrow x1, x1 \leftarrow rd(x2, op_t(x1, x3))\} \\
& \overbrace{i(\underbrace{rd(rd(x2, op_t(x1, x3)), x1)})} \\
= \quad & \text{by Lemma 1670 RL with } \{x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \underbrace{i(\overbrace{rd(rd(x2, x1), op_t(x1, x3))})} \\
= \quad & \text{by Lemma 10 LR with } \{x2 \leftarrow op_t(x1, x3), x1 \leftarrow rd(x2, x1)\} \\
& \overbrace{rd(op_t(x1, x3), rd(x2, x1))}
\end{aligned}
$$

Lemma 1672: $rd(op_r(x1, x2, x2), op_t(x2, x3)) = op_r(rd(x1, op_t(x2, x3)), x2, x2)$

$$
\begin{aligned}
& rd(\underbrace{op_r(x1, x2, x2)}, op_t(x2, x3)) \\
= \quad & \text{by Lemma 1632 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{rd(\overbrace{op_r(x1, op_t(x2, x3), op_t(x2, x3))}, op_t(x2, x3))} \\
= \quad & \text{by Lemma 370 RL with } \{x1 \leftarrow op_t(x2, x3), x2 \leftarrow x1\} \\
& \underbrace{\overbrace{op_r(rd(x1, op_t(x2, x3)), op_t(x2, x3), op_t(x2, x3))}} \\
= \quad & \text{by Lemma 1632 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow rd(x1, op_t(x2, x3))\} \\
& \overbrace{op_r(rd(x1, op_t(x2, x3)), x2, x2)}
\end{aligned}
$$

Lemma 1673: $mult(rd(x1, i(x2)), op_t(x1, x3)) = rd(x1, i(mult(x2, op_t(x1, x3))))$

$$
\begin{aligned}
& \underbrace{mult(rd(x1, i(x2)), op_t(x1, x3))} \\
= \quad & \text{by Lemma 1261 RL with } \{x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow rd(x1, i(x2))\} \\
& \overbrace{mult(\underbrace{rd(rd(x1, i(x2)), rd(x1, op_t(x1, x3)))}, x1)} \\
= \quad & \text{by Lemma 1390 RL with } \{x1 \leftarrow i(x2), x3 \leftarrow x3, x2 \leftarrow x1\} \\
& mult(\overbrace{\underbrace{rd(op_t(x1, x3), i(x2))}}, x1) \\
= \quad & \text{by Lemma 1635 RL with } \{x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \underbrace{mult(\overbrace{op_r(\underbrace{mult(x2, op_t(x1, x3))}, x1, x1)}, x1)} \\
= \quad & \text{by Lemma 357 LR with } \{x2 \leftarrow x1, x1 \leftarrow mult(x2, op_t(x1, x3))\} \\
& \overbrace{rd(x1, i(mult(x2, op_t(x1, x3))))}
\end{aligned}
$$

Lemma 1674: $rd(x1, mult(x2, i(op_t(x1, x3)))) = mult(rd(x1, x2), op_t(x1, x3))$

$$
\begin{aligned}
& rd(x1, \underbrace{mult(x2, i(op_t(x1, x3)))}) \\
= \quad & \text{by Lemma 16 RL with } \{x2 \leftarrow op_t(x1, x3), x1 \leftarrow x2\} \\
& \underbrace{rd(x1, \overbrace{i(mult(i(x2), op_t(x1, x3)))})} \\
= \quad & \text{by Lemma 1673 RL with } \{x3 \leftarrow x3, x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& \overbrace{mult(rd(x1, \underbrace{i(i(x2))}), op_t(x1, x3))} \\
= \quad & \text{by Lemma 3 LR with } \{x1 \leftarrow x2\} \\
& mult(rd(x1, \overbrace{x2}), op_t(x1, x3))
\end{aligned}
$$

Lemma 1675: $rd(op_t(x1, x2), i(mult(x3, x1))) = rd(x1, i(mult(x3, op_t(x1, x2))))$

$$
\begin{aligned}
& \underbrace{rd(op_t(x1, x2), i(mult(x3, x1)))} \\
= \quad & \text{by Lemma 1636 RL with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow mult(x3, x1)\} \\
& \overbrace{mult(\underbrace{op_r(\overbrace{mult(x3, x1)}, x1, x1)}, op_t(x1, x2))} \\
= \quad & \text{by Lemma 359 LR with } \{x2 \leftarrow x1, x1 \leftarrow x3\} \\
& \underbrace{mult(\overbrace{rd(x1, i(x3))}, op_t(x1, x2))} \\
= \quad & \text{by Lemma 1673 LR with } \{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{rd(x1, i(mult(x3, op_t(x1, x2))))}
\end{aligned}
$$

Lemma 1676: $rd(x1, mult(x2, i(op_t(x1, x3)))) = rd(op_t(x1, x3), mult(x2, i(x1)))$

$$
\begin{aligned}
& rd(x1, \underbrace{mult(x2, i(op_t(x1, x3)))}) \\
= \quad & \text{by Lemma 16 RL with } \{x2 \leftarrow op_t(x1, x3), x1 \leftarrow x2\} \\
& \underbrace{rd(x1, \overbrace{i(mult(i(x2), op_t(x1, x3)))})} \\
= \quad & \text{by Lemma 1675 RL with } \{x3 \leftarrow i(x2), x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{rd(op_t(x1, x3), \underbrace{i(mult(i(x2), x1))})} \\
= \quad & \text{by Lemma 16 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& rd(op_t(x1, x3), \overbrace{mult(x2, i(x1))})
\end{aligned}
$$

Lemma 1677: $rd(op_t(x1, x2), mult(x3, i(x1))) = mult(rd(x1, x3), op_t(x1, x2))$

$$
\begin{aligned}
& \overbrace{rd(op_t(x1, x2), mult(x3, i(x1)))} \\
= \quad & \text{by Lemma 1676 RL with } \{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{rd(x1, \overbrace{mult(x3, i(op_t(x1, x2)))})} \\
= \quad & \text{by Lemma 1674 LR with } \{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{mult(rd(x1, x3), op_t(x1, x2))}
\end{aligned}
$$

Lemma 1678: $rd(op_t(x1, x2), mult(mult(x3, i(x1)), x1)) = mult(i(x3), op_t(x1, x2))$

$$
\begin{aligned}
& \overbrace{rd(op_t(x1, x2), mult(mult(x3, i(x1)), x1))} \\
= \quad & \text{by Lemma 1291 RL with } \{x3 \leftarrow mult(x3, i(x1)), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{rd(\overbrace{rd(op_t(x1, x2), x1)}, mult(x3, i(x1)))} \\
= \quad & \text{by Lemma 1644 LR with } \{x1 \leftarrow mult(x3, i(x1)), x3 \leftarrow x2, x2 \leftarrow x1\} \\
& \overbrace{mult(\underbrace{rd(i(x1), mult(x3, i(x1)))}, op_t(x1, x2))} \\
= \quad & \text{by Lemma 9 LR with } \{x2 \leftarrow x3, x1 \leftarrow i(x1)\} \\
& mult(\overbrace{i(x3)}, op_t(x1, x2))
\end{aligned}
$$

Lemma 1679: $rd(x1, asoc(op_l(x2, x2, x1), x3, x3)) = rd(x1, asoc(x2, x3, x3))$

$$
\begin{aligned}
& rd(x1, \underbrace{asoc(op_l(x2, x2, x1), x3, x3)}) \\
= \quad & \text{by Lemma 756 RL with } \{x2 \leftarrow op_l(x2, x2, x1), x1 \leftarrow x3\} \\
& rd(x1, \overbrace{asoc(x3, \underbrace{i(op_l(x2, x2, x1))}, x3)}) \\
= \quad & \text{by Lemma 1049 LR with } \{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x2\} \\
& \underbrace{rd(x1, asoc(x3, \overbrace{op_l(i(x2), x2, x1)}, x3))} \\
= \quad & \text{by Lemma 1650 RL with } \{x3 \leftarrow x3, x2 \leftarrow op_l(i(x2), x2, x1), x1 \leftarrow x1\} \\
& \overbrace{mult(\underbrace{rd(x1, op_l(i(x2), x2, x1))}, op_r(op_l(i(x2), x2, x1), x3, x3))} \\
= \quad & \text{by Lemma 231 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& mult(\overbrace{mult(x1, x2)}, \underbrace{op_r(op_l(i(x2), x2, x1), x3, x3)}) \\
= \quad & \text{by Axiom 18 LR with } \{x5 \leftarrow x3, x4 \leftarrow x3, x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow i(x2)\} \\
& \underbrace{mult(mult(x1, x2), \overbrace{op_l(op_r(i(x2), x3, x3), x2, x1)})} \\
= \quad & \text{by Lemma 132 LR with } \{x3 \leftarrow op_r(i(x2), x3, x3), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{mult(x1, \underbrace{mult(x2, op_r(i(x2), x3, x3))})} \\
= \quad & \text{by Lemma 643 LR with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& \underbrace{mult(x1, \overbrace{asoc(x3, x2, x3)})} \\
= \quad & \text{by Lemma 1295 LR with } \{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{rd(x1, asoc(x2, x3, x3))}
\end{aligned}
$$

Lemma 1680: $asoc(x3, op_l(x1, x1, i(x2)), x3) = mult(rd(i(x2), asoc(x1, x3, x3)), x2)$

$$
\begin{aligned}
& \underbrace{asoc(x3, op_l(x1, x1, i(x2)), x3)} \\
= \quad & \text{by Lemma 759 RL with } \{x2 \leftarrow x3, x1 \leftarrow op_l(x1, x1, i(x2))\} \\
& \overbrace{\underbrace{i(asoc(op_l(x1, x1, i(x2)), x3, x3))}} \\
= \quad & \text{by Lemma 21 RL with } \{x2 \leftarrow asoc(op_l(x1, x1, i(x2)), x3, x3), x1 \leftarrow x2\} \\
& \overbrace{mult(\underbrace{rd(i(x2), asoc(op_l(x1, x1, i(x2)), x3, x3))}, x2)} \\
= \quad & \text{by Lemma 1679 LR with } \{x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow i(x2)\} \\
& mult(\overbrace{rd(i(x2), asoc(x1, x3, x3))}, x2)
\end{aligned}
$$

Lemma 1681: $asoc(x3, op_l(x1, x2, x1), x3) = i(asoc(x1, x3, x3))$

$$
\begin{aligned}
& asoc(x3, \underbrace{op_l(x1, x2, x1)}, x3) \\
= \quad & \text{by Lemma 608 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{asoc(x3, \overbrace{op_l(x1, x1, i(x2))}, x3)} \\
= \quad & \text{by Lemma 1680 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1, x3 \leftarrow x3\} \\
& \overbrace{\underbrace{mult(rd(i(x2), asoc(x1, x3, x3)), x2)}} \\
= \quad & \text{by Lemma 21 LR with } \{x2 \leftarrow asoc(x1, x3, x3), x1 \leftarrow x2\} \\
& \overbrace{i(asoc(x1, x3, x3))}
\end{aligned}
$$

Lemma 1682: $asoc(x1, op_l(x2, x3, x2), x1) = asoc(x1, x2, x1)$

$$
\begin{aligned}
& \underbrace{asoc(x1, op_l(x2, x3, x2), x1)} \\
= \quad & \text{by Lemma 1681 LR with } \{x2 \leftarrow x3, x1 \leftarrow x2, x3 \leftarrow x1\} \\
& \underbrace{\overbrace{i(asoc(x2, x1, x1))}} \\
= \quad & \text{by Lemma 759 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \overbrace{asoc(x1, x2, x1)}
\end{aligned}
$$

Lemma 1683: $asoc(x3, op_l(x1, x1, x2), x3) = mult(rd(x2, asoc(x1, x3, x3)), i(x2))$

$$
\begin{aligned}
& \underbrace{asoc(x3, op_l(x1, x1, x2), x3)} \\
= \quad & \text{by Lemma 759 RL with } \{x2 \leftarrow x3, x1 \leftarrow op_l(x1, x1, x2)\} \\
& \overbrace{\underbrace{i(asoc(op_l(x1, x1, x2), x3, x3))}} \\
= \quad & \text{by Lemma 22 RL with } \{x2 \leftarrow asoc(op_l(x1, x1, x2), x3, x3), x1 \leftarrow x2\} \\
& \overbrace{mult(\underbrace{rd(x2, asoc(op_l(x1, x1, x2), x3, x3))}, i(x2))} \\
= \quad & \text{by Lemma 1679 LR with } \{x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow x2\} \\
& mult(\overbrace{rd(x2, asoc(x1, x3, x3))}, i(x2))
\end{aligned}
$$

Lemma 1684: $asoc(x3, op_l(x1, x1, x2), x3) = asoc(x3, x1, x3)$

$\overbrace{asoc(x3, op_l(x1, x1, x2), x3)}$

$=$ by Lemma 1683 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1, x3 \leftarrow x3\}$

$\overbrace{mult(rd(x2, asoc(x1, x3, x3)), i(x2))}$

$=$ by Lemma 22 LR with $\{x2 \leftarrow asoc(x1, x3, x3), x1 \leftarrow x2\}$

$\overbrace{i(asoc(x1, x3, x3))}$

$=$ by Lemma 759 LR with $\{x2 \leftarrow x3, x1 \leftarrow x1\}$

$\overbrace{asoc(x3, x1, x3)}$

Lemma 1685: $op_l(x1, x1, op_l(x2, x3, x2)) = op_l(x1, x1, x2)$

$\overbrace{op_l(x1, x1, op_l(x2, x3, x2))}$

$=$ by Lemma 173 RL with $\{x2 \leftarrow op_l(x2, x3, x2), x1 \leftarrow x1\}$

$\overbrace{mult(\underbrace{asoc(x1, op_l(x2, x3, x2), x1)}, x1)}$

$=$ by Lemma 1682 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{mult(\overbrace{asoc(x1, x2, x1)}, x1)}$

$=$ by Lemma 173 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_l(x1, x1, x2)}$

Lemma 1686: $op_l(x1, op_l(x2, x3, x2), x1) = op_l(x1, x2, x1)$

$\underbrace{op_l(x1, op_l(x2, x3, x2), x1)}$

$=$ by Lemma 620 RL with $\{x2 \leftarrow op_l(x2, x3, x2), x1 \leftarrow x1\}$

$\overbrace{rd(x1, \underbrace{asoc(x1, op_l(x2, x3, x2), x1)})}$

$=$ by Lemma 1682 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{rd(x1, \overbrace{asoc(x1, x2, x1)})}$

$=$ by Lemma 620 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_l(x1, x2, x1)}$

Lemma 1687: $op_t(x1, op_l(x2, x3, x2)) = op_t(x1, x2)$

$$
\begin{aligned}
& \overbrace{op_t(x1, op_l(x2, x3, x2))} \\
= & \quad \text{by Lemma 455 RL with } \{x2 \leftarrow op_l(x2, x3, x2), x1 \leftarrow x1\} \\
& \overbrace{op_l(op_t(x1, rd(op_l(x2, x3, x2), x1)), op_l(x2, x3, x2), x1)} \\
= & \quad \text{by Lemma 1341 RL with } \{x3 \leftarrow x1, x2 \leftarrow op_l(x2, x3, x2), x1 \leftarrow op_t(x1, rd(op_l(x2, x3, x2), x1))\} \\
& \overbrace{op_l(op_t(x1, rd(op_l(x2, x3, x2), x1)), op_l(x2, x3, x2), op_t(x1, rd(op_l(x2, x3, x2), x1)))} \\
= & \quad \text{by Lemma 1686 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow op_t(x1, rd(op_l(x2, x3, x2), x1))\} \\
& \overbrace{op_l(op_t(x1, rd(op_l(x2, x3, x2), x1)), x2, op_t(x1, rd(op_l(x2, x3, x2), x1)))} \\
= & \quad \text{by Axiom 15 RL with } \{x4 \leftarrow rd(op_l(x2, x3, x2), x1), x3 \leftarrow op_t(x1, rd(op_l(x2, x3, x2), x1)), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_t(\underbrace{op_l(x1, x2, op_t(x1, rd(op_l(x2, x3, x2), x1)))}, rd(op_l(x2, x3, x2), x1))} \\
= & \quad \text{by Lemma 1024 LR with } \{x3 \leftarrow rd(op_l(x2, x3, x2), x1), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{op_t(\overbrace{op_l(x1, x2, x1)}, rd(op_l(x2, x3, x2), x1))} \\
= & \quad \text{by Axiom 15 LR with } \{x4 \leftarrow rd(op_l(x2, x3, x2), x1), x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(\underbrace{op_t(x1, rd(op_l(x2, x3, x2), x1))}, x2, x1)} \\
= & \quad \text{by Lemma 1643 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{op_l(\overbrace{op_t(x1, rd(x2, x1))}, x2, x1)} \\
= & \quad \text{by Lemma 455 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, x2)}
\end{aligned}
$$

Lemma 1688: $op_t(x1, x2) = op_t(x1, op_l(x2, x2, x3))$

$$
\begin{aligned}
& \underbrace{op_t(x1, x2)} \\
= & \quad \text{by Lemma 1687 RL with } \{x3 \leftarrow i(x3), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, \underbrace{op_l(x2, i(x3), x2)})} \\
= & \quad \text{by Lemma 1050 LR with } \{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x2\} \\
& op_t(x1, \overbrace{\underbrace{op_l(x2, x3, i(x2))}}) \\
= & \quad \text{by Lemma 606 LR with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& op_t(x1, \overbrace{op_l(x2, x2, x3)})
\end{aligned}
$$

Lemma 1689: $op_t(x1, mult(x2, rd(x3, x2))) = op_t(x1, x3)$

$$
\begin{aligned}
& op_t(x1, \underbrace{mult(x2, rd(x3, x2))}) \\
= & \quad \text{by Lemma 427 RL with } \{x2 \leftarrow x2, x1 \leftarrow x3\} \\
& \underbrace{op_t(x1, \overbrace{op_l(x3, x3, x2)})} \\
= & \quad \text{by Lemma 1688 RL with } \{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, x3)}
\end{aligned}
$$

Lemma 1690: $op_t(x1, op_t(x2, x3)) = op_t(x1, mult(x3, mult(x3, rd(mult(i(x3), x2), x3))))$

$$
\begin{aligned}
& \overbrace{op_t(x1, op_t(x2, x3))} \\
= & \quad \text{by Lemma 1689 RL with } \{x3 \leftarrow op_t(x2, x3), x2 \leftarrow mult(x3, x3), x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, mult(mult(x3, x3), \underbrace{rd(op_t(x2, x3), mult(x3, x3))}))} \\
= & \quad \text{by Lemma 205 LR with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& op_t(x1, \underbrace{mult(mult(x3, x3), \overbrace{rd(mult(i(x3), x2), x3)})}) \\
= & \quad \text{by Lemma 24 LR with } \{x2 \leftarrow rd(mult(i(x3), x2), x3), x1 \leftarrow x3\} \\
& op_t(x1, \overbrace{mult(x3, mult(x3, \underbrace{rd(mult(i(x3), x2), x3)}))})
\end{aligned}
$$

Lemma 1691: $op_t(x1, op_t(x2, x3)) = op_t(x1, op_t(x2, i(x3)))$

$$
\begin{aligned}
& \overbrace{op_t(x1, op_t(x2, x3))} \\
= & \quad \text{by Lemma 1690 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, \underbrace{mult(x3, mult(x3, rd(mult(i(x3), x2), x3)))})} \\
= & \quad \text{by Lemma 70 LR with } \{x2 \leftarrow mult(i(x3), x2), x1 \leftarrow x3\} \\
& op_t(x1, \overbrace{op_t(\underbrace{mult(x3, mult(i(x3), x2))}, i(x3))}) \\
= & \quad \text{by Lemma 12 LR with } \{x2 \leftarrow x2, x1 \leftarrow x3\} \\
& op_t(x1, op_t(\overbrace{x2}, i(x3)))
\end{aligned}
$$

Lemma 1692: $op_t(x1, mult(x2, x3)) = op_t(x1, mult(op_t(x2, x3), x3))$

$$
\begin{aligned}
& \overbrace{op_t(x1, mult(x2, x3))} \\
= & \quad \text{by Lemma 1687 RL with } \{x3 \leftarrow x3, x2 \leftarrow mult(x2, x3), x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, \underbrace{op_l(mult(x2, x3), x3, mult(x2, x3))})} \\
= & \quad \text{by Lemma 291 LR with } \{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow mult(x2, x3)\} \\
& op_t(x1, \overbrace{\underbrace{op_l(mult(x2, x3), x3, x2)}}) \\
= & \quad \text{by Lemma 636 LR with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& op_t(x1, \overbrace{mult(op_t(x2, x3), x3)})
\end{aligned}
$$

Lemma 1693: $op_t(x1, mult(op_t(x2, x3), x3)) = op_t(x1, mult(i(i(x3)), x2))$

$$
\begin{aligned}
& op_t(x1, \underbrace{mult(op_t(x2, x3), x3)}) \\
= & \quad \text{by Lemma 337 RL with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& \underbrace{op_t(x1, \overbrace{op_t(rd(x2, i(x3)), x3)})} \\
= & \quad \text{by Lemma 1691 LR with } \{x3 \leftarrow x3, x2 \leftarrow rd(x2, i(x3)), x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, \underbrace{op_t(rd(x2, i(x3)), i(x3))})} \\
= & \quad \text{by Lemma 14 LR with } \{x2 \leftarrow i(x3), x1 \leftarrow x2\} \\
& op_t(x1, \overbrace{mult(i(i(x3)), x2)})
\end{aligned}
$$

Lemma 1694: $op_t(x1, mult(x2, x3)) = op_t(x1, mult(x3, x2))$

$$
\begin{aligned}
& \overbrace{op_t(x1, mult(x2, x3))} \\
= \quad & \text{by Lemma 1692 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, mult(\underbrace{op_t(x2, x3)}, x3))} \\
= \quad & \text{by Lemma 1693 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, mult(\underbrace{i(i(x3))}, x2))} \\
= \quad & \text{by Lemma 3 LR with } \{x1 \leftarrow x3\} \\
& op_t(x1, mult(\overbrace{x3}, x2))
\end{aligned}
$$

Lemma 1695: $op_t(x1, mult(i(x2), x3)) = op_t(x1, mult(op_t(i(x2), x3), x3))$

$$
\begin{aligned}
& \overbrace{op_t(x1, mult(i(x2), x3))} \\
= \quad & \text{by Lemma 1687 RL with } \{x3 \leftarrow x2, x2 \leftarrow mult(i(x2), x3), x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, \underbrace{op_l(mult(i(x2), x3), x2, mult(i(x2), x3))})} \\
= \quad & \text{by Lemma 243 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow mult(i(x2), x3)\} \\
& op_t(x1, \overbrace{\underbrace{op_l(mult(i(x2), x3), x3, i(x2))}}) \\
= \quad & \text{by Lemma 636 LR with } \{x2 \leftarrow x3, x1 \leftarrow i(x2)\} \\
& op_t(x1, \overbrace{mult(op_t(i(x2), x3), x3)})
\end{aligned}
$$

Lemma 1696: $op_t(x1, op_r(x2, x3, x3)) = op_t(x1, rd(mult(x3, x2), x3))$

$$
\begin{aligned}
& \overbrace{op_t(x1, op_r(x2, x3, x3))} \\
= \quad & \text{by Lemma 1687 RL with } \{x3 \leftarrow rd(x2, x3), x2 \leftarrow op_r(x2, x3, x3), x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, \underbrace{op_l(op_r(x2, x3, x3), rd(x2, x3), op_r(x2, x3, x3))})} \\
= \quad & \text{by Lemma 1362 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow op_r(x2, x3, x3)\} \\
& op_t(x1, \overbrace{\underbrace{op_l(op_r(x2, x3, x3), x2, x3)}}) \\
= \quad & \text{by Lemma 453 LR with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& op_t(x1, \overbrace{rd(mult(x3, x2), x3)})
\end{aligned}
$$

Lemma 1697: $asoc(x1, op_t(x2, x3), x1) = asoc(x1, mult(x3, mult(x3, rd(mult(i(x3), x2), x3))), x1)$

$$
\begin{aligned}
& \overbrace{asoc(x1, op_t(x2, x3), x1)} \\
= \quad & \text{by Lemma 1684 RL with } \{x2 \leftarrow mult(x3, x3), x1 \leftarrow op_t(x2, x3), x3 \leftarrow x1\} \\
& \overbrace{asoc(x1, \underbrace{op_l(op_t(x2, x3), op_t(x2, x3), mult(x3, x3))}, x1)} \\
= \quad & \text{by Lemma 427 LR with } \{x2 \leftarrow mult(x3, x3), x1 \leftarrow op_t(x2, x3)\} \\
& asoc(x1, \overbrace{mult(mult(x3, x3), \underbrace{rd(op_t(x2, x3), mult(x3, x3))})}, x1) \\
= \quad & \text{by Lemma 205 LR with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& asoc(x1, \underbrace{mult(mult(x3, x3), \overbrace{rd(mult(i(x3), x2), x3)})}, x1) \\
= \quad & \text{by Lemma 24 LR with } \{x2 \leftarrow rd(mult(i(x3), x2), x3), x1 \leftarrow x3\} \\
& asoc(x1, \overbrace{mult(x3, mult(x3, rd(mult(i(x3), x2), x3)))}, x1)
\end{aligned}
$$

Lemma 1698: $asoc(x1, op_t(x2, x3), x1) = asoc(x1, op_t(x2, i(x3)), x1)$

$$
\begin{aligned}
& \underbrace{asoc(x1, op_t(x2, x3), x1)} \\
= \quad & \text{by Lemma 1697 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{asoc(x1, \underbrace{mult(x3, mult(x3, rd(mult(i(x3), x2), x3)))}, x1)} \\
= \quad & \text{by Lemma 70 LR with } \{x2 \leftarrow mult(i(x3), x2), x1 \leftarrow x3\} \\
& asoc(x1, \overbrace{op_t(\underbrace{mult(x3, mult(i(x3), x2))}, i(x3))}, x1) \\
= \quad & \text{by Lemma 12 LR with } \{x2 \leftarrow x2, x1 \leftarrow x3\} \\
& asoc(x1, op_t(\overbrace{x2}, i(x3)), x1)
\end{aligned}
$$

Lemma 1699: $asoc(x1, op_t(i(x2), x3), x1) = asoc(op_t(x2, x3), x1, x1)$

$$
\begin{aligned}
& \underbrace{asoc(x1, op_t(i(x2), x3), x1)} \\
= \quad & \text{by Lemma 1698 LR with } \{x3 \leftarrow x3, x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& \overbrace{asoc(x1, \underbrace{op_t(i(x2), i(x3))}, x1)} \\
= \quad & \text{by Lemma 15 LR with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& \underbrace{asoc(x1, \overbrace{i(op_t(x2, x3))}, x1)} \\
= \quad & \text{by Lemma 756 LR with } \{x2 \leftarrow op_t(x2, x3), x1 \leftarrow x1\} \\
& \overbrace{asoc(op_t(x2, x3), x1, x1)}
\end{aligned}
$$

Lemma 1700: $op_l(x1, x1, op_t(x2, i(x3))) = op_l(x1, x1, op_t(x2, x3))$

$$
\begin{aligned}
& \underbrace{op_l(x1, x1, op_t(x2, i(x3)))} \\
= \quad & \text{by Lemma 173 RL with } \{x2 \leftarrow op_t(x2, i(x3)), x1 \leftarrow x1\} \\
& \overbrace{mult(\underbrace{asoc(x1, op_t(x2, i(x3)), x1)}, x1)} \\
= \quad & \text{by Lemma 1698 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{mult(\overbrace{asoc(x1, op_t(x2, x3), x1)}, x1)} \\
= \quad & \text{by Lemma 173 LR with } \{x2 \leftarrow op_t(x2, x3), x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x1, op_t(x2, x3))}
\end{aligned}
$$

Lemma 1701: $op_l(x1, op_t(i(x2), x3), x1) = op_l(x1, x1, op_t(x2, x3))$

$$
\begin{aligned}
& \underbrace{op_l(x1, op_t(i(x2), x3), x1)} \\
= \quad & \text{by Lemma 1105 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{\overbrace{op_l(x1, op_t(x2, i(x3)), i(x1))}} \\
= \quad & \text{by Lemma 606 LR with } \{x2 \leftarrow op_t(x2, i(x3)), x1 \leftarrow x1\} \\
& \underbrace{\overbrace{op_l(x1, x1, op_t(x2, i(x3)))}} \\
= \quad & \text{by Lemma 1700 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x1, op_t(x2, x3))}
\end{aligned}
$$

Lemma 1702: $asoc(x1, mult(x2, x3), x1) = asoc(x1, mult(op_t(x2, x3), x3), x1)$

$$
\begin{aligned}
& \overbrace{asoc(x1, mult(x2, x3), x1)} \\
= \quad & \text{by Lemma 1682 RL with } \{x3 \leftarrow x3, x2 \leftarrow mult(x2, x3), x1 \leftarrow x1\} \\
& \overbrace{asoc(x1, \underbrace{op_l(mult(x2, x3), x3, mult(x2, x3))}, x1)} \\
= \quad & \text{by Lemma 291 LR with } \{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow mult(x2, x3)\} \\
& asoc(x1, \overbrace{\underbrace{op_l(mult(x2, x3), x3, x2)}, x1}) \\
= \quad & \text{by Lemma 636 LR with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& asoc(x1, \overbrace{mult(op_t(x2, x3), x3)}, x1)
\end{aligned}
$$

Lemma 1703: $op_l(x1, x1, mult(x2, x3)) = op_l(x1, x1, mult(op_t(x2, x3), x3))$

$$
\begin{aligned}
& \overbrace{op_l(x1, x1, mult(x2, x3))} \\
= \quad & \text{by Lemma 1685 RL with } \{x3 \leftarrow x3, x2 \leftarrow mult(x2, x3), x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x1, \underbrace{op_l(mult(x2, x3), x3, mult(x2, x3))})} \\
= \quad & \text{by Lemma 291 LR with } \{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow mult(x2, x3)\} \\
& op_l(x1, x1, \overbrace{\underbrace{op_l(mult(x2, x3), x3, x2)}}) \\
= \quad & \text{by Lemma 636 LR with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& op_l(x1, x1, \overbrace{mult(op_t(x2, x3), x3)}))
\end{aligned}
$$

Lemma 1704: $op_l(x1, mult(x2, x3), x1) = op_l(x1, x1, op_l(i(mult(x2, x3)), x3, x2))$

$$
\begin{aligned}
& \overbrace{op_l(x1, mult(x2, x3), x1)} \\
= \quad & \text{by Lemma 608 RL with } \{x2 \leftarrow mult(x2, x3), x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x1, i(mult(x2, x3)))} \\
= \quad & \text{by Lemma 1685 RL with } \{x3 \leftarrow x2, x2 \leftarrow i(mult(x2, x3)), x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x1, \underbrace{op_l(i(mult(x2, x3)), x2, i(mult(x2, x3)))})} \\
= \quad & \text{by Lemma 1051 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow i(mult(x2, x3))\} \\
& op_l(x1, x1, \overbrace{op_l(i(mult(x2, x3)), x3, x2)})
\end{aligned}
$$

Lemma 1705: $op_l(x1, mult(x2, x3), x1) = op_l(x1, mult(op_t(x2, x3), x3), x1)$

$$
\begin{aligned}
& \overbrace{op_l(x1, mult(x2, x3), x1)} \\
= \quad & \text{by Lemma 1704 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x1, op_l(i(mult(x2, x3)), x3, x2))} \\
= \quad & \text{by Lemma 1134 LR with } \{x4 \leftarrow x2, x3 \leftarrow x3, x2 \leftarrow mult(x2, x3), x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, \underbrace{op_l(mult(x2, x3), x3, x2)}, x1)} \\
= \quad & \text{by Lemma 636 LR with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& op_l(x1, \overbrace{mult(op_t(x2, x3), x3)}, x1)
\end{aligned}
$$

Lemma 1706: $mult(asoc(x1,x2,x2),mult(x3,x2)) = mult(x3,op_l(x2,x1,x2))$

$\underbrace{mult(asoc(x1,x2,x2),mult(x3,x2))}$

$=$ by Axiom 1 LR with $\{x1 \leftarrow mult(asoc(x1,x2,x2),mult(x3,x2))\}$

$\overbrace{mult(mult(asoc(x1,x2,x2),mult(x3,x2)),\underbrace{unit()})}$

$=$ by Lemma 958 RL with $\{x3 \leftarrow op_t(x3,x2), x2 \leftarrow x2, x1 \leftarrow x1\}$

$mult(mult(asoc(x1,x2,x2),\underbrace{mult(x3,x2)}),\overbrace{asoc(asoc(x1,x2,x2),x2,op_t(x3,x2))})$

$=$ by Lemma 13 RL with $\{x2 \leftarrow x3, x1 \leftarrow x2\}$

$\underbrace{mult(mult(asoc(x1,x2,x2),\overbrace{mult(x2,op_t(x3,x2))}),asoc(asoc(x1,x2,x2),x2,op_t(x3,x2)))}$

$=$ by Axiom 8 RL with $\{x3 \leftarrow op_t(x3,x2), x2 \leftarrow x2, x1 \leftarrow asoc(x1,x2,x2)\}$

$\overbrace{mult(\underbrace{mult(asoc(x1,x2,x2),x2)},op_t(x3,x2))}$

$=$ by Lemma 771 RL with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$mult(\overbrace{op_l(x2,x1,x2)},\underbrace{op_t(x3,x2)})$

$=$ by Lemma 1687 RL with $\{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x3\}$

$\underbrace{mult(op_l(x2,x1,x2),\overbrace{op_t(x3,op_l(x2,x1,x2))})}$

$=$ by Lemma 13 LR with $\{x2 \leftarrow x3, x1 \leftarrow op_l(x2,x1,x2)\}$

$\overbrace{mult(x3,op_l(x2,x1,x2))}$

Lemma 1707: $mult(op_l(x1,x2,x1),op_t(x3,x1)) = mult(x3,op_l(x1,x2,x1))$

$mult(\underbrace{op_l(x1,x2,x1)},op_t(x3,x1))$

$=$ by Lemma 771 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{mult(\overbrace{mult(asoc(x2,x1,x1),x1)},op_t(x3,x1))}$

$=$ by Axiom 8 LR with $\{x3 \leftarrow op_t(x3,x1), x2 \leftarrow x1, x1 \leftarrow asoc(x2,x1,x1)\}$

$\overbrace{mult(mult(asoc(x2,x1,x1),\underbrace{mult(x1,op_t(x3,x1))}),asoc(asoc(x2,x1,x1),x1,op_t(x3,x1)))}$

$=$ by Lemma 13 LR with $\{x2 \leftarrow x3, x1 \leftarrow x1\}$

$mult(mult(asoc(x2,x1,x1),\overbrace{mult(x3,x1)}),\underbrace{asoc(asoc(x2,x1,x1),x1,op_t(x3,x1))})$

$=$ by Lemma 958 LR with $\{x3 \leftarrow op_t(x3,x1), x2 \leftarrow x1, x1 \leftarrow x2\}$

$\underbrace{mult(mult(asoc(x2,x1,x1),mult(x3,x1)),\overbrace{unit()})}$

$=$ by Axiom 1 RL with $\{x1 \leftarrow mult(asoc(x2,x1,x1),mult(x3,x1))\}$

$\overbrace{\underbrace{mult(asoc(x2,x1,x1),mult(x3,x1))}}$

$=$ by Lemma 1706 LR with $\{x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow x2\}$

$\overbrace{mult(x3,op_l(x1,x2,x1))}$

Lemma 1708: $mult(asoc(x1, x2, x2), x3) = rd(x3, asoc(x2, x1, x2))$

$\overbrace{mult(asoc(x1, x2, x2), x3)}$

$=$ by Axiom 5 LR with $\{x2 \leftarrow x2, x1 \leftarrow mult(asoc(x1, x2, x2), x3)\}$

$\overbrace{rd(mult(\underbrace{mult(asoc(x1, x2, x2), x3)}, x2), x2)}$

$=$ by Lemma 1305 LR with $\{x2 \leftarrow x2, x3 \leftarrow x1, x1 \leftarrow mult(asoc(x1, x2, x2), x3)\}$

$rd(mult(\overbrace{op_r(mult(asoc(x1, x2, x2), x3), asoc(x1, x2, x2), asoc(x1, x2, x2))}, x2), x2)$

$=$ by Lemma 412 LR with $\{x2 \leftarrow x3, x1 \leftarrow asoc(x1, x2, x2)\}$

$rd(\underbrace{mult(\overbrace{rd(mult(asoc(x1, x2, x2), mult(x3, asoc(x1, x2, x2))), asoc(x1, x2, x2))}, x2)}, x2)$

$=$ by Lemma 1586 LR with $\{x2 \leftarrow x2, x3 \leftarrow x1, x1 \leftarrow mult(asoc(x1, x2, x2), mult(x3, asoc(x1, x2, x2)))\}$

$rd(\overbrace{mult(mult(asoc(x1, x2, x2), mult(x3, asoc(x1, x2, x2))), op_l(x2, x2, x1))}, x2)$

$=$ by Axiom 6 RL with $\{x3 \leftarrow op_l(x2, x2, x1), x2 \leftarrow x3, x1 \leftarrow asoc(x1, x2, x2)\}$

$rd(\overbrace{mult(asoc(x1, x2, x2), mult(x3, mult(asoc(x1, x2, x2), \underbrace{op_l(x2, x2, x1)})))}, x2)$

$=$ by Lemma 607 RL with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$rd(mult(asoc(x1, x2, x2), mult(x3, mult(\underbrace{asoc(x1, x2, x2)}, \overbrace{op_l(x2, i(x1), x2)}))), x2)$

$=$ by Lemma 756 RL with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$rd(mult(asoc(x1, x2, x2), mult(x3, mult(\overbrace{asoc(x2, i(x1), x2)}, op_l(x2, i(x1), x2)))), x2)$

$=$ by Lemma 617 RL with $\{x2 \leftarrow i(x1), x1 \leftarrow x2\}$

$rd(mult(asoc(x1, x2, x2), mult(x3, \underbrace{mult(\overbrace{rd(x2, op_l(x2, i(x1), x2))}, op_l(x2, i(x1), x2))})), x2)$

$=$ by Axiom 4 RL with $\{x2 \leftarrow op_l(x2, i(x1), x2), x1 \leftarrow x2\}$

$rd(\underbrace{mult(asoc(x1, x2, x2), mult(x3, \overbrace{x2}))}, x2)$

$=$ by Lemma 1706 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{\overbrace{rd(mult(x3, op_l(x2, x1, x2)), x2)}}$

$=$ by Lemma 1152 RL with $\{x3 \leftarrow x2, x2 \leftarrow op_l(x2, x1, x2), x1 \leftarrow x3\}$

$\overbrace{mult(x3, mult(\underbrace{rd(op_l(x2, x1, x2), mult(x3, x2))}, x3))}$

$=$ by Lemma 1569 RL with $\{x3 \leftarrow mult(x3, x2), x2 \leftarrow x2, x1 \leftarrow x1\}$

$mult(x3, mult(\overbrace{rd(asoc(x1, x2, x2), \underbrace{rd(mult(x3, x2), x2)})}, x3))$

$=$ by Axiom 5 RL with $\{x2 \leftarrow x2, x1 \leftarrow x3\}$

$mult(x3, \underbrace{mult(rd(asoc(x1, x2, x2), \overbrace{x3}), x3)})$

$=$ by Axiom 4 RL with $\{x2 \leftarrow x3, x1 \leftarrow asoc(x1, x2, x2)\}$

$\underbrace{mult(x3, \overbrace{asoc(x1, x2, x2)})}$

$=$ by Lemma 1300 RL with $\{x2 \leftarrow x1, x3 \leftarrow x2, x1 \leftarrow x3\}$

$\overbrace{rd(x3, asoc(x2, x1, x2))}$

Lemma 1709: $op_t(x1, asoc(x2, x3, x3)) = x1$

$$
\begin{aligned}
& \overbrace{op_t(x1, asoc(x2, x3, x3))} \\
= \quad & \text{by Lemma 1302 LR with } \{x2 \leftarrow x3, x3 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{mult(rd(x1, asoc(x2, x3, x3)), op_r(asoc(x2, x3, x3), x1, x1))} \\
= \quad & \text{by Lemma 720 LR with } \{x2 \leftarrow asoc(x2, x3, x3), x1 \leftarrow x1\} \\
& \overbrace{op_l(\underbrace{x1}, asoc(x2, x3, x3), x1)} \\
= \quad & \text{by Axiom 4 LR with } \{x2 \leftarrow asoc(x3, x2, x3), x1 \leftarrow x1\} \\
& op_l(\overbrace{mult(rd(x1, asoc(x3, x2, x3)), asoc(x3, x2, x3))}, asoc(x2, x3, x3), x1) \\
= \quad & \text{by Lemma 1295 LR with } \{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow rd(x1, asoc(x3, x2, x3))\} \\
& op_l(\overbrace{rd(\underbrace{rd(x1, asoc(x3, x2, x3))}, asoc(x2, x3, x3))}, asoc(x2, x3, x3), x1) \\
= \quad & \text{by Lemma 1708 RL with } \{x3 \leftarrow x1, x2 \leftarrow x3, x1 \leftarrow x2\} \\
& \underbrace{op_l(rd(\overbrace{mult(asoc(x2, x3, x3), x1)}, asoc(x2, x3, x3)), asoc(x2, x3, x3), x1)} \\
= \quad & \text{by Lemma 485 LR with } \{x2 \leftarrow x1, x1 \leftarrow asoc(x2, x3, x3)\} \\
& \overbrace{\underbrace{op_r(x1, asoc(x2, x3, x3), asoc(x2, x3, x3))}} \\
= \quad & \text{by Lemma 1305 RL with } \{x2 \leftarrow x3, x3 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{x1}
\end{aligned}
$$

Lemma 1710: $op_t(x1, rd(op_t(x2, x3), x2)) = x1$

$$
\begin{aligned}
& op_t(x1, \underbrace{rd(op_t(x2, x3), x2)}) \\
= \quad & \text{by Lemma 858 RL with } \{x2 \leftarrow x2, x1 \leftarrow x3\} \\
& \underbrace{op_t(x1, \overbrace{asoc(x3, mult(x2, x3), mult(x2, x3))})} \\
= \quad & \text{by Lemma 1709 LR with } \{x3 \leftarrow mult(x2, x3), x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{x1}
\end{aligned}
$$

Lemma 1711: $op_t(x1, mult(x1, asoc(x2, x3, x2))) = rd(asoc(x2, x3, x2), rd(op_t(asoc(x2, x3, x2), x1), x1))$

$\overbrace{op_t(x1, mult(x1, asoc(x2, x3, x2)))}$

$=$ by Axiom 5 LR with $\{x2 \leftarrow i(asoc(x2, x3, x2)), x1 \leftarrow op_t(x1, mult(x1, asoc(x2, x3, x2)))\}$

$\overbrace{rd(mult(\overbrace{op_t(x1, mult(x1, asoc(x2, x3, x2)))}, i(asoc(x2, x3, x2))), i(asoc(x2, x3, x2)))}$

$=$ by Lemma 251 RL with $\{x2 \leftarrow asoc(x2, x3, x2), x1 \leftarrow x1\}$

$rd(mult(\overbrace{op_l(op_t(x1, asoc(x2, x3, x2)), asoc(x2, x3, x2), x1)}, i(asoc(x2, x3, x2))), i(asoc(x2, x3, x2)))$

$=$ by Axiom 15 RL with $\{x4 \leftarrow asoc(x2, x3, x2), x3 \leftarrow x1, x2 \leftarrow asoc(x2, x3, x2), x1 \leftarrow x1\}$

$rd(mult(\overbrace{op_t(\overbrace{op_l(x1, asoc(x2, x3, x2)}, x1), asoc(x2, x3, x2))}, i(asoc(x2, x3, x2))), i(asoc(x2, x3, x2)))$

$=$ by Lemma 138 RL with $\{x3 \leftarrow asoc(x2, x3, x2), x2 \leftarrow op_t(x1, asoc(x2, x3, x2)), x1 \leftarrow op_l(x1, asoc(x2, x3, x2), x1)\}$

$rd(mult(op_t(\overbrace{op_l(\overbrace{op_l(op_l(x1, asoc(x2, x3, x2)}, x1), op_t(x1, asoc(x2, x3, x2)), asoc(x2, x3, x2))}, asoc(x2, x3, x2), op_t(x1, asoc($

$=$ by Lemma 746 RL with $\{x1 \leftarrow asoc(x2, x3, x2), x2 \leftarrow x1\}$

$rd(mult(op_t(op_l(\overbrace{rd(mult(x1, asoc(x2, x3, x2)), asoc(x2, x3, x2))}, asoc(x2, x3, x2), op_t(x1, asoc(x2, x3, x2))), asoc(x2, x3, x2$

$=$ by Axiom 5 RL with $\{x2 \leftarrow asoc(x2, x3, x2), x1 \leftarrow x1\}$

$rd(mult(\overbrace{op_t(op_l(\overbrace{x1}, asoc(x2, x3, x2), op_t(x1, asoc(x2, x3, x2))), asoc(x2, x3, x2))}, i(asoc(x2, x3, x2))), i(asoc(x2, x3, x2)))$

$=$ by Axiom 15 LR with $\{x4 \leftarrow asoc(x2, x3, x2), x3 \leftarrow op_t(x1, asoc(x2, x3, x2)), x2 \leftarrow asoc(x2, x3, x2), x1 \leftarrow x1\}$

$rd(\overbrace{mult(\overbrace{op_l(op_t(x1, asoc(x2, x3, x2)), asoc(x2, x3, x2), op_t(x1, asoc(x2, x3, x2)))}, i(asoc(x2, x3, x2)))}, i(asoc(x2, x3, x2)))$

$=$ by Lemma 1127 RL with $\{x2 \leftarrow asoc(x2, x3, x2), x1 \leftarrow x1\}$

$\overbrace{rd(\overbrace{rd(op_t(x1, asoc(x2, x3, x2)), op_t(asoc(x2, x3, x2), x1))}, i(asoc(x2, x3, x2)))}$

$=$ by Lemma 39 LR with $\{x3 \leftarrow asoc(x2, x3, x2), x2 \leftarrow op_t(asoc(x2, x3, x2), x1), x1 \leftarrow op_t(x1, asoc(x2, x3, x2))\}$

$\overbrace{rd(asoc(x2, x3, x2), rd(op_t(asoc(x2, x3, x2), x1), op_t(\underbrace{x1}, asoc(x2, x3, x2))))}$

$=$ by Lemma 1709 RL with $\{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x1\}$

$rd(asoc(x2, x3, x2), rd(op_t(asoc(x2, x3, x2), x1), op_t(\overbrace{op_t(x1, \underbrace{asoc(x3, x2, x2)})}, asoc(x2, x3, x2))))$

$=$ by Lemma 757 RL with $\{x2 \leftarrow x3, x1 \leftarrow x2\}$

$rd(asoc(x2, x3, x2), rd(op_t(asoc(x2, x3, x2), x1), op_t(\underbrace{op_t(x1, \overbrace{i(asoc(x2, x3, x2))})}, asoc(x2, x3, x2))))$

$=$ by Lemma 550 RL with $\{x2 \leftarrow i(asoc(x2, x3, x2)), x1 \leftarrow x1\}$

$rd(asoc(x2, x3, x2), rd(op_t(asoc(x2, x3, x2), x1), op_t(\overbrace{op_t(x1, \underbrace{op_r(i(asoc(x2, x3, x2)), x1, x1)})}, asoc(x2, x3, x2))))$

$=$ by Lemma 361 RL with $\{x2 \leftarrow x1, x1 \leftarrow i(asoc(x2, x3, x2))\}$

$rd(asoc(x2, x3, x2), rd(op_t(asoc(x2, x3, x2), x1), op_t(op_t(x1, \overbrace{op_r(i(asoc(x2, x3, x2)), i(x1), i(x1))}), asoc(x2, x3, x2))))$

$=$ by Lemma 362 RL with $\{x2 \leftarrow asoc(x2, x3, x2), x1 \leftarrow i(x1)\}$

$rd(asoc(x2, x3, x2), rd(op_t(asoc(x2, x3, x2), x1), op_t(\underbrace{op_t(x1, \overbrace{rd(i(x1), rd(asoc(x2, x3, x2), i(i(x1)))))}}, asoc(x2, x3, x2))))$

$=$ by Lemma 707 LR with $\{x2 \leftarrow rd(asoc(x2, x3, x2), i(i(x1))), x1 \leftarrow x1\}$

$rd(asoc(x2, x3, x2), rd(op_t(asoc(x2, x3, x2), x1), op_t(\overbrace{op_t(x1, rd(rd(asoc(x2, x3, x2), \underbrace{i(i(x1))}), x1))}, asoc(x2, x3, x2))))$

$=$ by Lemma 3 LR with $\{x1 \leftarrow x1\}$

$rd(asoc(x2, x3, x2), rd(op_t(asoc(x2, x3, x2), x1), \underbrace{op_t(op_t(x1, rd(rd(asoc(x2, x3, x2), \overbrace{x1}), x1)), asoc(x2, x3, x2))}))$

$=$ by Lemma 1481 RL with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$rd(asoc(x2, x3, x2), rd(op_t(asoc(x2, x3, x2), x1), \overbrace{op_t(x1, rd(asoc(x2, x3, x2), x1))}))$

$=$ by Lemma 1296 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$rd(asoc(x2, x3, x2), rd(op_t(asoc(x2, x3, x2), x1), \overbrace{x1}))$

Lemma 1712: $op_t(x1, rd(x1, asoc(x3, x2, x2))) = mult(mult(i(asoc(x2, x3, x2)), x1), asoc(x2, x3, x2))$

$$
\begin{aligned}
& op_t(x1, \underbrace{rd(x1, asoc(x3, x2, x2))}) \\
= & \quad \text{by Lemma 1295 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{op_t(x1, \overbrace{mult(x1, asoc(x2, x3, x2))})} \\
= & \quad \text{by Lemma 1711 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{rd(asoc(x2, x3, x2), rd(op_t(asoc(x2, x3, x2), x1), x1))}} \\
= & \quad \text{by Lemma 595 LR with } \{x2 \leftarrow x1, x1 \leftarrow asoc(x2, x3, x2)\} \\
& \overbrace{mult(mult(i(asoc(x2, x3, x2)), x1), asoc(x2, x3, x2))}
\end{aligned}
$$

Lemma 1713: $op_t(x1, rd(x1, asoc(x3, x2, x2))) = rd(mult(asoc(x3, x2, x2), x1), asoc(x3, x2, x2))$

$$
\begin{aligned}
& \underbrace{op_t(x1, rd(x1, asoc(x3, x2, x2)))} \\
= & \quad \text{by Lemma 1712 LR with } \{x2 \leftarrow x2, x3 \leftarrow x3, x1 \leftarrow x1\} \\
& \underbrace{\overbrace{mult(mult(i(asoc(x2, x3, x2)), x1), asoc(x2, x3, x2))}} \\
= & \quad \text{by Lemma 1295 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow mult(i(asoc(x2, x3, x2)), x1)\} \\
& \overbrace{rd(mult(\underbrace{i(asoc(x2, x3, x2))}, x1), asoc(x3, x2, x2))} \\
= & \quad \text{by Lemma 757 LR with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& rd(mult(\overbrace{asoc(x3, x2, x2)}, x1), asoc(x3, x2, x2))
\end{aligned}
$$

Lemma 1714: $mult(asoc(x1, x2, x1), x3) = rd(x3, asoc(x2, x1, x1))$

$mult(\underbrace{asoc(x1, x2, x1)}, x3)$

$=$ by Lemma 759 RL with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$\underbrace{mult(\overbrace{i(asoc(x2, x1, x1))}, x3)}$

$=$ by Lemma 330 RL with $\{x2 \leftarrow asoc(x2, x1, x1), x1 \leftarrow x3\}$

$\overbrace{op_r(\underbrace{rd(x3, asoc(x2, x1, x1))}, x3, i(x3))}$

$=$ by Lemma 1295 RL with $\{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x3\}$

$\underbrace{op_r(\overbrace{mult(x3, asoc(x1, x2, x1))}, x3, i(x3))}$

$=$ by Lemma 405 LR with $\{x2 \leftarrow asoc(x1, x2, x1), x1 \leftarrow x3\}$

$\overbrace{rd(x3, op_t(\underbrace{i(asoc(x1, x2, x1))}, x3))}$

$=$ by Lemma 757 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$rd(x3, \underbrace{op_t(\overbrace{asoc(x2, x1, x1)}, x3)})$

$=$ by Axiom 10 RL with $\{x2 \leftarrow asoc(x2, x1, x1), x1 \leftarrow x3\}$

$rd(x3, \overbrace{mult(i(x3), \underbrace{mult(asoc(x2, x1, x1), x3)})})$

$=$ by Lemma 1708 LR with $\{x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow x2\}$

$rd(x3, \underbrace{mult(i(x3), \overbrace{rd(x3, asoc(x1, x2, x1))})})$

$=$ by Lemma 1310 LR with $\{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x3\}$

$rd(x3, \overbrace{\underbrace{mult(rd(asoc(x2, x1, x1), x3), x3)}})$

$=$ by Axiom 4 RL with $\{x2 \leftarrow x3, x1 \leftarrow asoc(x2, x1, x1)\}$

$rd(x3, \overbrace{asoc(x2, x1, x1)})$

Lemma 1715: $x1 = op_l(op_t(x1, rd(op_t(x2, x3), x2)), x1, i(rd(op_t(x2, x3), x2)))$

$$
\begin{aligned}
& \overbrace{x1} \\
= \quad & \text{by Lemma 1710 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, rd(op_t(x2, x3), x2))} \\
= \quad & \text{by Lemma 1710 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow op_t(x1, rd(op_t(x2, x3), x2))\} \\
& \overbrace{op_t(op_t(x1, rd(op_t(x2, x3), x2)), rd(op_t(x2, x3), x2))} \\
= \quad & \text{by Lemma 376 RL with } \{x2 \leftarrow rd(op_t(x2, x3), x2), x1 \leftarrow op_t(x1, rd(op_t(x2, x3), x2))\} \\
& \overbrace{rd(\underbrace{mult(op_t(x1, rd(op_t(x2, x3), x2)), rd(op_t(x2, x3), x2))}, op_t(rd(op_t(x2, x3), x2), op_t(x1, rd(op_t(x2, x3), x2))))} \\
= \quad & \text{by Lemma 1255 RL with } \{x1 \leftarrow op_t(x1, rd(op_t(x2, x3), x2)), x3 \leftarrow x3, x2 \leftarrow x2\} \\
& rd(\overbrace{rd(op_t(x2, x3), \underbrace{mult(i(op_t(x1, rd(op_t(x2, x3), x2))), x2)})}, op_t(rd(op_t(x2, x3), x2), op_t(x1, rd(op_t(x2, x3), x2)))) \\
= \quad & \text{by Lemma 1291 RL with } \{x3 \leftarrow i(op_t(x1, rd(op_t(x2, x3), x2))), x2 \leftarrow x3, x1 \leftarrow x2\} \\
& \underbrace{rd(\overbrace{rd(rd(op_t(x2, x3), x2), i(op_t(x1, rd(op_t(x2, x3), x2))))}, op_t(rd(op_t(x2, x3), x2), op_t(x1, rd(op_t(x2, x3), x2))))} \\
= \quad & \text{by Lemma 696 RL with } \{x2 \leftarrow rd(op_t(x2, x3), x2), x1 \leftarrow op_t(x1, rd(op_t(x2, x3), x2))\} \\
& \overbrace{op_l(op_t(x1, rd(op_t(x2, x3), x2)), rd(op_t(x2, x3), x2), op_t(x1, rd(op_t(x2, x3), x2)))} \\
= \quad & \text{by Lemma 1315 LR with } \{x3 \leftarrow x1, x2 \leftarrow rd(op_t(x2, x3), x2), x1 \leftarrow op_t(x1, rd(op_t(x2, x3), x2))\} \\
& \overbrace{op_l(op_t(x1, rd(op_t(x2, x3), x2)), x1, i(rd(op_t(x2, x3), x2)))}
\end{aligned}
$$

Lemma 1716: $x1 = op_t(x1, mult(x1, rd(op_t(x2, x3), x2)))$

$$
\begin{aligned}
& \overbrace{x1} \\
= \quad & \text{by Lemma 1715 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(op_t(x1, rd(op_t(x2, x3), x2)), x1, i(rd(op_t(x2, x3), x2)))} \\
= \quad & \text{by Lemma 672 LR with } \{x3 \leftarrow rd(op_t(x2, x3), x2), x2 \leftarrow rd(op_t(x2, x3), x2), x1 \leftarrow x1\} \\
& \overbrace{op_l(op_t(x1, rd(op_t(x2, x3), x2)), rd(op_t(x2, x3), x2), x1)} \\
= \quad & \text{by Lemma 251 LR with } \{x2 \leftarrow rd(op_t(x2, x3), x2), x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, mult(x1, rd(op_t(x2, x3), x2)))}
\end{aligned}
$$

Lemma 1717: $mult(rd(x1, op_t(x1, x2)), x3) = rd(x3, rd(op_t(x1, x2), x1))$

$mult(\underbrace{rd(x1, op_t(x1, x2))}, x3)$

$=$ by Lemma 860 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{mult(\overbrace{asoc(x1, mult(x1, x2), mult(x1, x2))}, x3)}$

$=$ by Lemma 1708 LR with $\{x3 \leftarrow x3, x2 \leftarrow mult(x1, x2), x1 \leftarrow x1\}$

$\overbrace{rd(x3, \underbrace{asoc(mult(x1, x2), x1, mult(x1, x2))})}$

$=$ by Lemma 854 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$rd(x3, \overbrace{rd(\underbrace{rd(mult(x1, x2), x1)}, x2)})$

$=$ by Lemma 394 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$rd(x3, \underbrace{rd(\overbrace{mult(rd(op_t(x1, x2), x1), x2)}, x2)})$

$=$ by Axiom 5 RL with $\{x2 \leftarrow x2, x1 \leftarrow rd(op_t(x1, x2), x1)\}$

$rd(x3, \overbrace{rd(op_t(x1, x2), x1)})$

Lemma 1718: $rd(x1, op_t(x1, x2)) = rd(x3, mult(x3, rd(op_t(x1, x2), x1)))$

$\underbrace{rd(x1, op_t(x1, x2))}$

$=$ by Lemma 10 RL with $\{x2 \leftarrow x1, x1 \leftarrow op_t(x1, x2)\}$

$\overbrace{\underbrace{i(rd(op_t(x1, x2), x1))}}$

$=$ by Lemma 36 RL with $\{x2 \leftarrow rd(op_t(x1, x2), x1), x1 \leftarrow x3\}$

$\overbrace{rd(\underbrace{op_t(x3, rd(op_t(x1, x2), x1))}, mult(x3, rd(op_t(x1, x2), x1)))}$

$=$ by Lemma 1710 LR with $\{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x3\}$

$rd(\overbrace{x3}, mult(x3, rd(op_t(x1, x2), x1)))$

Lemma 1719: $rd(mult(x1, x2), asoc(x3, x1, x3)) = mult(op_r(x1, x3, x3), x2)$

$\underbrace{rd(mult(x1, x2), asoc(x3, x1, x3))}$

$=$ by Lemma 1300 LR with $\{x2 \leftarrow x1, x3 \leftarrow x3, x1 \leftarrow mult(x1, x2)\}$

$\overbrace{mult(mult(x1, x2), \underbrace{asoc(x1, x3, x3)})}$

$=$ by Lemma 1646 RL with $\{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x1\}$

$\underbrace{mult(mult(x1, x2), \overbrace{op_l(asoc(x1, x3, x3), x2, x1)})}$

$=$ by Lemma 132 LR with $\{x3 \leftarrow asoc(x1, x3, x3), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{mult(x1, \underbrace{mult(x2, asoc(x1, x3, x3))})}$

$=$ by Lemma 1300 RL with $\{x2 \leftarrow x1, x3 \leftarrow x3, x1 \leftarrow x2\}$

$mult(x1, \overbrace{\underbrace{rd(x2, asoc(x3, x1, x3))}})$

$=$ by Lemma 1708 RL with $\{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x1\}$

$\underbrace{mult(x1, \overbrace{mult(asoc(x1, x3, x3), x2)})}$

$=$ by Lemma 940 LR with $\{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x1\}$

$\overbrace{mult(op_r(x1, x3, x3), x2)}$

Lemma 1720: $mult(x1, rd(x3, asoc(x2, x1, x1))) = mult(op_l(x1, x1, x2), x3)$

$$
\begin{aligned}
& mult(x1, \underbrace{rd(x3, asoc(x2, x1, x1))}) \\
= & \quad \text{by Lemma 1714 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{mult(x1, \overbrace{mult(asoc(x1, x2, x1), x3)})} \\
= & \quad \text{by Lemma 951 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{mult(\underbrace{mult(x1, asoc(x1, x2, x1))}, x3)} \\
= & \quad \text{by Lemma 158 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& mult(\overbrace{op_l(x1, x1, x2)}, x3)
\end{aligned}
$$

Lemma 1721: $op_t(x1, op_r(x2, x3, x3)) = op_t(x1, x2)$

$$
\begin{aligned}
& \underbrace{op_t(x1, op_r(x2, x3, x3))} \\
= & \quad \text{by Axiom 10 RL with } \{x2 \leftarrow x1, x1 \leftarrow op_r(x2, x3, x3)\} \\
& \overbrace{mult(i(op_r(x2, x3, x3)), \underbrace{mult(x1, op_r(x2, x3, x3))})} \\
= & \quad \text{by Lemma 1649 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& mult(i(op_r(x2, x3, x3)), \overbrace{rd(mult(x1, x2), asoc(x3, x2, x3))}) \\
= & \quad \text{by Lemma 1708 RL with } \{x3 \leftarrow mult(x1, x2), x2 \leftarrow x3, x1 \leftarrow x2\} \\
& mult(i(op_r(x2, x3, x3)), \overbrace{mult(asoc(x2, x3, x3), \underbrace{mult(x1, x2)})}) \\
= & \quad \text{by Lemma 13 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& mult(i(op_r(x2, x3, x3)), \underbrace{mult(asoc(x2, x3, x3), \overbrace{mult(x2, op_t(x1, x2))})}) \\
= & \quad \text{by Lemma 941 LR with } \{x3 \leftarrow op_t(x1, x2), x2 \leftarrow x3, x1 \leftarrow x2\} \\
& \underbrace{mult(i(op_r(x2, x3, x3)), \overbrace{mult(op_r(x2, x3, x3), op_t(x1, x2))})} \\
= & \quad \text{by Axiom 2 RL with } \{x2 \leftarrow op_r(x2, x3, x3), x1 \leftarrow op_t(x1, x2)\} \\
& \overbrace{op_t(x1, x2)}
\end{aligned}
$$

Lemma 1722: $op_t(x1, op_t(x2, x3)) = op_t(x1, x2)$

$$
\begin{aligned}
& \underbrace{op_t(x1, op_t(x2, x3))} \\
= & \quad \text{by Lemma 1721 RL with } \{x3 \leftarrow x3, x2 \leftarrow op_t(x2, x3), x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, \underbrace{op_r(op_t(x2, x3), x3, x3)})} \\
= & \quad \text{by Lemma 745 RL with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& \underbrace{op_t(x1, \overbrace{op_l(x2, x3, x2)})} \\
= & \quad \text{by Lemma 1687 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, x2)}
\end{aligned}
$$

Lemma 1723: $op_l(x1, op_t(x2, x3), x1) = op_l(x1, x2, x1)$

$$
\begin{aligned}
& \underbrace{op_l(x1, op_t(x2, x3), x1)} \\
= \quad & \text{by Lemma 735 LR with } \{x1 \leftarrow op_t(x2, x3), x2 \leftarrow x1\} \\
& \overbrace{rd(\underbrace{op_t(x1, op_t(x2, x3))}, asoc(op_t(x2, x3), x1, op_t(x2, x3)))} \\
= \quad & \text{by Lemma 1722 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& rd(\overbrace{op_t(x1, x2)}, \underbrace{asoc(op_t(x2, x3), x1, op_t(x2, x3))}) \\
= \quad & \text{by Lemma 1631 LR with } \{x3 \leftarrow x1, x2 \leftarrow x3, x1 \leftarrow x2\} \\
& \underbrace{rd(op_t(x1, x2), \overbrace{asoc(x2, x1, x2)})} \\
= \quad & \text{by Lemma 735 RL with } \{x1 \leftarrow x2, x2 \leftarrow x1\} \\
& \overbrace{op_l(x1, x2, x1)}
\end{aligned}
$$

Lemma 1724: $op_l(x1, x2, i(x1)) = op_l(x1, x1, op_t(x2, x3))$

$$
\begin{aligned}
& \underbrace{op_l(x1, x2, i(x1))} \\
= \quad & \text{by Lemma 1050 RL with } \{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{\overbrace{op_l(x1, i(x2), x1)}} \\
= \quad & \text{by Lemma 1723 RL with } \{x3 \leftarrow x3, x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& \underbrace{\overbrace{op_l(x1, op_t(i(x2), x3), x1)}} \\
= \quad & \text{by Lemma 1701 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x1, op_t(x2, x3))}
\end{aligned}
$$

Lemma 1725: $op_l(x1, x1, op_t(x2, x3)) = op_l(x1, x1, x2)$

$$
\begin{aligned}
& \underbrace{op_l(x1, x1, op_t(x2, x3))} \\
= \quad & \text{by Lemma 1724 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{\overbrace{op_l(x1, x2, i(x1))}} \\
= \quad & \text{by Lemma 606 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x1, x2)}
\end{aligned}
$$

Lemma 1726: $asoc(x1, op_t(x2, x3), x1) = asoc(x1, x2, x1)$

$$
\begin{aligned}
& \underbrace{asoc(x1, op_t(x2, x3), x1)} \\
= \quad & \text{by Lemma 747 RL with } \{x2 \leftarrow op_t(x2, x3), x1 \leftarrow x1\} \\
& \overbrace{rd(x1, op_r(\underbrace{op_t(x1, op_t(x2, x3))}, op_t(x2, x3), op_t(x2, x3)))} \\
= \quad & \text{by Lemma 1722 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& rd(x1, \underbrace{op_r(\overbrace{op_t(x1, x2)}, op_t(x2, x3), op_t(x2, x3)))} \\
= \quad & \text{by Lemma 1632 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow op_t(x1, x2)\} \\
& \underbrace{rd(x1, \overbrace{op_r(op_t(x1, x2), x2, x2)})} \\
= \quad & \text{by Lemma 747 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{asoc(x1, x2, x1)}
\end{aligned}
$$

Lemma 1727: $asoc(x2, x1, x1) = asoc(op_t(x2, x3), x1, x1)$

$$
\begin{aligned}
& \underbrace{asoc(x2, x1, x1)} \\
= \quad & \text{by Lemma 756 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{asoc(x1, i(x2), x1)}} \\
= \quad & \text{by Lemma 1726 RL with } \{x3 \leftarrow x3, x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& \overbrace{\underbrace{asoc(x1, op_t(i(x2), x3), x1)}} \\
= \quad & \text{by Lemma 1699 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{asoc(op_t(x2, x3), x1, x1)}
\end{aligned}
$$

Lemma 1728: $op_t(x1, mult(i(x2), x3)) = op_t(x1, rd(x3, x2))$

$$
\begin{aligned}
& \underbrace{op_t(x1, mult(i(x2), x3))} \\
= \quad & \text{by Lemma 1695 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, \underbrace{mult(op_t(i(x2), x3), x3)})} \\
= \quad & \text{by Lemma 340 LR with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& \underbrace{op_t(x1, \overbrace{op_t(rd(x3, x2), x3)})} \\
= \quad & \text{by Lemma 1722 LR with } \{x3 \leftarrow x3, x2 \leftarrow rd(x3, x2), x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, rd(x3, x2))}
\end{aligned}
$$

Lemma 1729: $op_t(x1, rd(x2, i(x3))) = op_t(x1, mult(x3, x2))$

$$
\begin{aligned}
& op_t(x1, \underbrace{rd(x2, i(x3))}) \\
= \quad & \text{by Lemma 359 RL with } \{x2 \leftarrow x2, x1 \leftarrow x3\} \\
& \underbrace{op_t(x1, \overbrace{op_r(mult(x3, x2), x2, x2)})} \\
= \quad & \text{by Lemma 1721 LR with } \{x3 \leftarrow x2, x2 \leftarrow mult(x3, x2), x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, mult(x3, x2))}
\end{aligned}
$$

Lemma 1730: $op_t(x1, mult(x2, i(x3))) = op_t(x1, rd(x2, x3))$

$$
\begin{aligned}
& \underbrace{op_t(x1, mult(x2, i(x3)))} \\
= \quad & \text{by Lemma 1721 RL with } \{x3 \leftarrow x3, x2 \leftarrow mult(x2, i(x3)), x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, \underbrace{op_r(mult(x2, i(x3)), x3, x3)})} \\
= \quad & \text{by Lemma 364 LR with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& op_t(x1, \overbrace{rd(x2, x3)})
\end{aligned}
$$

Lemma 1731: $op_t(x1, rd(mult(x2, x3), x2)) = op_t(x1, x3)$

$\underbrace{op_t(x1, rd(mult(x2, x3), x2))}$

$=$ by Lemma 1696 RL with $\{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_t(x1, op_r(x3, x2, x2))}}$

$=$ by Lemma 1721 LR with $\{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x1\}$

$\overbrace{op_t(x1, x3)}$

Lemma 1732: $op_t(x1, i(mult(x2, x3))) = op_t(x1, rd(i(x2), x3))$

$\underbrace{op_t(x1, i(mult(x2, x3)))}$

$=$ by Lemma 1721 RL with $\{x3 \leftarrow x3, x2 \leftarrow i(mult(x2, x3)), x1 \leftarrow x1\}$

$\overbrace{op_t(x1, \underbrace{op_r(i(mult(x2, x3)), x3, x3)})}$

$=$ by Lemma 371 LR with $\{x2 \leftarrow x3, x1 \leftarrow x2\}$

$op_t(x1, \overbrace{rd(i(x2), x3)})$

Lemma 1733: $op_t(x1, rd(x2, mult(x2, x3))) = op_t(x1, i(x3))$

$\underbrace{op_t(x1, rd(x2, mult(x2, x3)))}$

$=$ by Lemma 1689 RL with $\{x3 \leftarrow rd(x2, mult(x2, x3)), x2 \leftarrow op_l(x2, x2, x3), x1 \leftarrow x1\}$

$\overbrace{op_t(x1, mult(op_l(x2, x2, x3), rd(rd(x2, mult(x2, x3)), \underbrace{op_l(x2, x2, x3)})))}$

$=$ by Lemma 158 RL with $\{x2 \leftarrow x3, x1 \leftarrow x2\}$

$op_t(x1, mult(op_l(x2, x2, x3), rd(rd(x2, mult(x2, x3)), \overbrace{mult(x2, \underbrace{asoc(x2, x3, x2)})})))$

$=$ by Lemma 643 RL with $\{x2 \leftarrow x2, x1 \leftarrow x3\}$

$op_t(x1, mult(op_l(x2, x2, x3), rd(rd(x2, \underbrace{mult(x2, x3)}), mult(x2, \overbrace{mult(x3, op_r(i(x3), x2, x2))}))))$

$=$ by Lemma 231 LR with $\{x2 \leftarrow x3, x1 \leftarrow x2\}$

$op_t(x1, mult(op_l(x2, x2, x3), rd(\underbrace{rd(x2, \overbrace{rd(x2, op_l(i(x3), x3, x2))})}, mult(x2, mult(x3, op_r(i(x3), x2, x2))))))$

$=$ by Lemma 353 LR with $\{x2 \leftarrow op_l(i(x3), x3, x2), x1 \leftarrow x2\}$

$op_t(x1, mult(op_l(x2, x2, x3), rd(\overbrace{\underbrace{op_r(op_l(i(x3), x3, x2), x2, x2)}}, mult(x2, mult(x3, op_r(i(x3), x2, x2))))))$

$=$ by Axiom 18 LR with $\{x5 \leftarrow x2, x4 \leftarrow x2, x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow i(x3)\}$

$op_t(x1, mult(op_l(x2, x2, x3), rd(\overbrace{\underbrace{op_l(op_r(i(x3), x2, x2), x3, x2)}}, mult(x2, mult(x3, op_r(i(x3), x2, x2))))))$

$=$ by Axiom 12 RL with $\{x3 \leftarrow op_r(i(x3), x2, x2), x2 \leftarrow x3, x1 \leftarrow x2\}$

$op_t(x1, mult(op_l(x2, x2, x3), \underbrace{rd(\overbrace{mult(i(mult(x2, x3)), mult(x2, mult(x3, op_r(i(x3), x2, x2))))}, mult(x2, mult(x3, op_r(i(x3),}$

$=$ by Axiom 5 RL with $\{x2 \leftarrow mult(x2, mult(x3, op_r(i(x3), x2, x2))), x1 \leftarrow i(mult(x2, x3))\}$

$op_t(x1, \underbrace{mult(op_l(x2, x2, x3), \overbrace{i(mult(x2, x3))})})$

$=$ by Lemma 1183 RL with $\{x2 \leftarrow x2, x1 \leftarrow x3\}$

$\underbrace{op_t(x1, \overbrace{op_r(i(x3), x2, x2)})}$

$=$ by Lemma 1721 LR with $\{x3 \leftarrow x2, x2 \leftarrow i(x3), x1 \leftarrow x1\}$

$\overbrace{op_t(x1, i(x3))}$

Lemma 1734: $asoc(x1, op_r(x2, x3, x3), x1) = asoc(x1, x2, x1)$

$$
\begin{aligned}
& \underbrace{asoc(x1, op_r(x2, x3, x3), x1)} \\
= & \quad \text{by Lemma 1009 LR with } \{x2 \leftarrow op_r(x2, x3, x3), x1 \leftarrow x1\} \\
& \overbrace{rd(op_t(x1, rd(op_r(x2, x3, x3), x1)), \underbrace{op_t(x1, op_r(x2, x3, x3))})} \\
= & \quad \text{by Lemma 1721 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& rd(\underbrace{op_t(x1, rd(op_r(x2, x3, x3), x1))}, \overbrace{op_t(x1, x2)}) \\
= & \quad \text{by Lemma 1642 LR with } \{x2 \leftarrow x3, x3 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{rd(\overbrace{op_t(x1, rd(x2, x1))}, op_t(x1, x2))} \\
= & \quad \text{by Lemma 1009 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{asoc(x1, x2, x1)}
\end{aligned}
$$

Lemma 1735: $asoc(x1, i(x2), x1) = asoc(op_r(x2, x3, x3), x1, x1)$

$$
\begin{aligned}
& \underbrace{asoc(x1, i(x2), x1)} \\
= & \quad \text{by Lemma 1734 RL with } \{x3 \leftarrow x3, x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& \overbrace{asoc(x1, \underbrace{op_r(i(x2), x3, x3)}, x1)} \\
= & \quad \text{by Lemma 356 RL with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& \underbrace{asoc(x1, \overbrace{i(op_r(x2, x3, x3))}, x1)} \\
= & \quad \text{by Lemma 756 LR with } \{x2 \leftarrow op_r(x2, x3, x3), x1 \leftarrow x1\} \\
& \overbrace{asoc(op_r(x2, x3, x3), x1, x1)}
\end{aligned}
$$

Lemma 1736: $asoc(op_r(x1, x2, x2), x3, x3) = asoc(x1, x3, x3)$

$$
\begin{aligned}
& \underbrace{asoc(op_r(x1, x2, x2), x3, x3)} \\
= & \quad \text{by Lemma 1735 RL with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x3\} \\
& \overbrace{\underbrace{asoc(x3, i(x1), x3)}} \\
= & \quad \text{by Lemma 756 LR with } \{x2 \leftarrow x1, x1 \leftarrow x3\} \\
& \overbrace{asoc(x1, x3, x3)}
\end{aligned}
$$

Lemma 1737: $op_l(x1, op_r(x2, x3, x3), x1) = op_l(x1, x2, x1)$

$$
\begin{aligned}
& \underbrace{op_l(x1, op_r(x2, x3, x3), x1)} \\
= & \quad \text{by Lemma 620 RL with } \{x2 \leftarrow op_r(x2, x3, x3), x1 \leftarrow x1\} \\
& \overbrace{rd(x1, \underbrace{asoc(x1, op_r(x2, x3, x3), x1)})} \\
= & \quad \text{by Lemma 1734 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{rd(x1, \overbrace{asoc(x1, x2, x1)})} \\
= & \quad \text{by Lemma 620 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x2, x1)}
\end{aligned}
$$

Lemma 1738: $mult(op_t(x1,x2),op_t(x3,x1)) = mult(x3,op_t(x1,x2))$

$$
\begin{aligned}
& mult(op_t(x1,x2),\underbrace{op_t(x3,x1)}) \\
= & \quad \text{by Lemma 1722 RL with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x3\} \\
& \underbrace{mult(op_t(x1,x2),\overbrace{op_t(x3,op_t(x1,x2))})} \\
= & \quad \text{by Lemma 13 LR with } \{x2 \leftarrow x3, x1 \leftarrow op_t(x1,x2)\} \\
& \overbrace{mult(x3,op_t(x1,x2))}
\end{aligned}
$$

Lemma 1739: $op_t(x1,rd(x2,op_t(x3,x2))) = op_t(x1,rd(x2,x3))$

$$
\begin{aligned}
& op_t(x1,rd(x2,op_t(\underbrace{x3},x2))) \\
= & \quad \text{by Lemma 3 RL with } \{x1 \leftarrow x3\} \\
& op_t(x1,\underbrace{rd(x2,op_t(\overbrace{i(i(x3))},x2))}) \\
= & \quad \text{by Lemma 405 RL with } \{x2 \leftarrow i(x3), x1 \leftarrow x2\} \\
& op_t(x1,\overbrace{op_r(mult(x2,i(x3)),x2,i(x2))}) \\
= & \quad \text{by Lemma 317 RL with } \{x2 \leftarrow mult(x2,i(x3)), x1 \leftarrow x2\} \\
& op_t(x1,\overbrace{rd(op_t(\underbrace{mult(x2,mult(x2,i(x3)))},x2),x2)}) \\
= & \quad \text{by Lemma 464 RL with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& op_t(x1,\underbrace{rd(op_t(\overbrace{rd(x2,op_t(rd(x3,x2),x3))},x2),x2)}) \\
= & \quad \text{by Lemma 345 LR with } \{x2 \leftarrow op_t(rd(x3,x2),x3), x1 \leftarrow x2\} \\
& op_t(x1,\overbrace{op_t(\underbrace{i(op_t(rd(x3,x2),x3))},x2)}) \\
= & \quad \text{by Lemma 46 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x3\} \\
& \underbrace{op_t(x1,op_t(\overbrace{op_t(rd(x2,x3),i(x3))},x2))} \\
= & \quad \text{by Lemma 1722 LR with } \{x3 \leftarrow x2, x2 \leftarrow op_t(rd(x2,x3),i(x3)), x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_t(x1,op_t(rd(x2,x3),i(x3)))}} \\
= & \quad \text{by Lemma 1722 LR with } \{x3 \leftarrow i(x3), x2 \leftarrow rd(x2,x3), x1 \leftarrow x1\} \\
& \overbrace{op_t(x1,rd(x2,x3))}
\end{aligned}
$$

Lemma 1740: $op_l(i(x1),x1,op_t(x2,x3)) = op_l(i(x1),x1,x2)$

$$
\begin{aligned}
& \underbrace{op_l(i(x1),x1,op_t(x2,x3))} \\
= & \quad \text{by Lemma 1185 RL with } \{x2 \leftarrow op_t(x2,x3), x1 \leftarrow x1\} \\
& \overbrace{rd(\underbrace{op_t(i(x1),op_t(x2,x3))},asoc(x1,op_t(x2,x3),op_t(x2,x3)))} \\
= & \quad \text{by Lemma 1722 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow i(x1)\} \\
& rd(\overbrace{op_t(i(x1),x2)},\underbrace{asoc(x1,op_t(x2,x3),op_t(x2,x3))}) \\
= & \quad \text{by Lemma 1630 LR with } \{x2 \leftarrow x3, x1 \leftarrow x2, x3 \leftarrow x1\} \\
& \underbrace{rd(op_t(i(x1),x2),\overbrace{asoc(x1,x2,x2)})} \\
= & \quad \text{by Lemma 1185 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(i(x1),x1,x2)}
\end{aligned}
$$

Lemma 1741: $op_l(x1, mult(i(x2), x3), x1) = op_l(x1, rd(x3, x2), x1)$

$$
\begin{aligned}
& op_l(x1, \underbrace{mult(i(x2), x3)}, x1) \\
= & \quad \text{by Lemma 14 RL with } \{x2 \leftarrow x2, x1 \leftarrow x3\} \\
& \underbrace{op_l(x1, \overbrace{op_t(rd(x3, x2), x2)}, x1)} \\
= & \quad \text{by Lemma 1723 LR with } \{x3 \leftarrow x2, x2 \leftarrow rd(x3, x2), x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, rd(x3, x2), x1)}
\end{aligned}
$$

Lemma 1742: $op_l(x1, rd(x3, x2), x1) = op_l(x1, x1, mult(x2, i(x3)))$

$$
\begin{aligned}
& \underbrace{op_l(x1, rd(x3, x2), x1)} \\
= & \quad \text{by Lemma 1741 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, mult(i(x2), x3), x1)}} \\
= & \quad \text{by Lemma 1104 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, mult(x2, i(x3)), i(x1))}} \\
= & \quad \text{by Lemma 606 LR with } \{x2 \leftarrow mult(x2, i(x3)), x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x1, mult(x2, i(x3)))}
\end{aligned}
$$

Lemma 1743: $op_l(x1, rd(i(x2), x3), x1) = op_l(x1, mult(x2, x3), i(x1))$

$$
\begin{aligned}
& \underbrace{op_l(x1, rd(i(x2), x3), x1)} \\
= & \quad \text{by Lemma 1723 RL with } \{x3 \leftarrow x2, x2 \leftarrow rd(i(x2), x3), x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, \underbrace{op_t(rd(i(x2), x3), x2)}, x1)} \\
= & \quad \text{by Lemma 33 LR with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& \underbrace{op_l(x1, \overbrace{i(mult(x2, x3))}, x1)} \\
= & \quad \text{by Lemma 1050 LR with } \{x3 \leftarrow x1, x2 \leftarrow mult(x2, x3), x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, mult(x2, x3), i(x1))}
\end{aligned}
$$

Lemma 1744: $op_l(x1, mult(x2, i(x3)), x1) = op_l(x1, rd(x2, x3), x1)$

$$
\begin{aligned}
& op_l(x1, \underbrace{mult(x2, i(x3))}, x1) \\
= & \quad \text{by Lemma 34 RL with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& \underbrace{op_l(x1, \overbrace{op_t(rd(x2, x3), i(x2))}, x1)} \\
= & \quad \text{by Lemma 1723 LR with } \{x3 \leftarrow i(x2), x2 \leftarrow rd(x2, x3), x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, rd(x2, x3), x1)}
\end{aligned}
$$

Lemma 1745: $op_l(x1, rd(x2, x3), x1) = op_l(x1, x1, mult(i(x2), x3))$

$$
\begin{aligned}
& \underbrace{op_l(x1, rd(x2, x3), x1)} \\
= & \quad \text{by Lemma 1744 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, mult(x2, i(x3)), x1)} \\
= & \quad \text{by Lemma 1103 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, mult(i(x2), x3), i(x1))} \\
= & \quad \text{by Lemma 606 LR with } \{x2 \leftarrow mult(i(x2), x3), x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x1, mult(i(x2), x3))}
\end{aligned}
$$

Lemma 1746: $op_l(x1, rd(x2, i(x3)), x1) = op_l(x1, mult(x2, x3), x1)$

$$
\begin{aligned}
& \underbrace{op_l(x1, rd(x2, i(x3)), x1)} \\
= & \quad \text{by Lemma 1723 RL with } \{x3 \leftarrow x3, x2 \leftarrow rd(x2, i(x3)), x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, \underbrace{op_t(rd(x2, i(x3)), x3)}, x1)} \\
= & \quad \text{by Lemma 337 LR with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& \underbrace{op_l(x1, \overbrace{mult(op_t(x2, x3), x3)}, x1)} \\
= & \quad \text{by Lemma 1705 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, mult(x2, x3), x1)}
\end{aligned}
$$

Lemma 1747: $op_l(x1, x1, rd(i(x2), x3)) = op_l(x1, mult(x2, x3), x1)$

$$
\begin{aligned}
& \underbrace{op_l(x1, x1, rd(i(x2), x3))} \\
= & \quad \text{by Lemma 1725 RL with } \{x3 \leftarrow x2, x2 \leftarrow rd(i(x2), x3), x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x1, \underbrace{op_t(rd(i(x2), x3), x2)})} \\
= & \quad \text{by Lemma 33 LR with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& \underbrace{op_l(x1, x1, \overbrace{i(mult(x2, x3))})} \\
= & \quad \text{by Lemma 608 LR with } \{x2 \leftarrow mult(x2, x3), x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, mult(x2, x3), x1)}
\end{aligned}
$$

Lemma 1748: $op_l(x1, x1, rd(x2, i(x3))) = op_l(x1, x1, mult(x2, x3))$

$$
\begin{aligned}
& \underbrace{op_l(x1, x1, rd(x2, i(x3)))} \\
= & \quad \text{by Lemma 1725 RL with } \{x3 \leftarrow x3, x2 \leftarrow rd(x2, i(x3)), x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x1, \underbrace{op_t(rd(x2, i(x3)), x3)})} \\
= & \quad \text{by Lemma 337 LR with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& \underbrace{op_l(x1, x1, \overbrace{mult(op_t(x2, x3), x3)})} \\
= & \quad \text{by Lemma 1703 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x1, mult(x2, x3))}
\end{aligned}
$$

Lemma 1749: $asoc(x1, mult(x2, i(x3)), x1) = asoc(x1, rd(x2, x3), x1)$

$$
\begin{aligned}
& asoc(x1, \underbrace{mult(x2, i(x3))}, x1) \\
= & \quad \text{by Lemma 34 RL with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& \underbrace{asoc(x1, \overbrace{op_t(rd(x2, x3), i(x2))}, x1)} \\
= & \quad \text{by Lemma 1726 LR with } \{x3 \leftarrow i(x2), x2 \leftarrow rd(x2, x3), x1 \leftarrow x1\} \\
& \overbrace{asoc(x1, rd(x2, x3), x1)}
\end{aligned}
$$

Lemma 1750: $asoc(x1, rd(x2, i(x3)), x1) = asoc(x1, mult(x2, x3), x1)$

$$
\begin{aligned}
& \underbrace{asoc(x1, rd(x2, i(x3)), x1)} \\
= & \quad \text{by Lemma 1726 RL with } \{x3 \leftarrow x3, x2 \leftarrow rd(x2, i(x3)), x1 \leftarrow x1\} \\
& \overbrace{asoc(x1, \underbrace{op_t(rd(x2, i(x3)), x3)}, x1)} \\
= & \quad \text{by Lemma 337 LR with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& \underbrace{asoc(x1, \overbrace{mult(op_t(x2, x3), x3)}, x1)} \\
= & \quad \text{by Lemma 1702 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{asoc(x1, mult(x2, x3), x1)}
\end{aligned}
$$

Lemma 1751: $asoc(mult(x1, i(x2)), x3, x3) = asoc(rd(x1, x2), x3, x3)$

$$
\begin{aligned}
& asoc(\underbrace{mult(x1, i(x2))}, x3, x3) \\
= & \quad \text{by Lemma 34 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{asoc(\overbrace{op_t(rd(x1, x2), i(x1))}, x3, x3)} \\
= & \quad \text{by Lemma 1727 RL with } \{x3 \leftarrow i(x1), x1 \leftarrow x3, x2 \leftarrow rd(x1, x2)\} \\
& \overbrace{asoc(rd(x1, x2), x3, x3)}
\end{aligned}
$$

Lemma 1752: $op_t(x1, mult(rd(x2, x3), x4)) = op_t(x1, rd(x4, rd(x3, x2)))$

$op_t(x1, mult(rd(x2, x3), \underbrace{x4}))$

$=$ by Lemma 3 RL with $\{x1 \leftarrow x4\}$

$op_t(x1, \underbrace{mult(rd(x2, x3), \overbrace{i(i(x4))})})$

$=$ by Lemma 44 RL with $\{x3 \leftarrow i(x4), x2 \leftarrow x2, x1 \leftarrow x3\}$

$op_t(x1, \overbrace{i(\underbrace{mult(rd(x3, x2), i(x4))})})$

$=$ by Lemma 1426 LR with $\{x3 \leftarrow x4, x1 \leftarrow x2, x2 \leftarrow x3\}$

$op_t(x1, \underbrace{i(\overbrace{op_t(rd(rd(x3, x2), x4), rd(x2, x3))})})$

$=$ by Lemma 15 RL with $\{x2 \leftarrow rd(x2, x3), x1 \leftarrow rd(rd(x3, x2), x4)\}$

$\underbrace{op_t(x1, \overbrace{op_t(i(rd(rd(x3, x2), x4)), i(rd(x2, x3)))})}$

$=$ by Lemma 1691 RL with $\{x3 \leftarrow rd(x2, x3), x2 \leftarrow i(rd(rd(x3, x2), x4)), x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_t(x1, op_t(i(rd(rd(x3, x2), x4)), rd(x2, x3)))}}$

$=$ by Lemma 1722 LR with $\{x3 \leftarrow rd(x2, x3), x2 \leftarrow i(rd(rd(x3, x2), x4)), x1 \leftarrow x1\}$

$\overbrace{op_t(x1, \underbrace{i(rd(rd(x3, x2), x4))})}$

$=$ by Lemma 10 LR with $\{x2 \leftarrow x4, x1 \leftarrow rd(x3, x2)\}$

$op_t(x1, \overbrace{rd(x4, rd(x3, x2))})$

Lemma 1753: $op_t(x1, mult(x2, rd(x3, x4))) = op_t(x1, rd(x2, rd(x4, x3)))$

$\underbrace{op_t(x1, mult(x2, rd(x3, x4)))}$

$=$ by Lemma 1729 RL with $\{x3 \leftarrow x2, x2 \leftarrow rd(x3, x4), x1 \leftarrow x1\}$

$\overbrace{op_t(x1, \underbrace{rd(rd(x3, x4), i(x2))})}$

$=$ by Lemma 39 LR with $\{x3 \leftarrow x2, x2 \leftarrow x4, x1 \leftarrow x3\}$

$op_t(x1, \overbrace{rd(x2, rd(x4, x3))})$

Lemma 1754: $mult(rd(x3, asoc(x2, x1, x1)), x1) = mult(asoc(x1, x2, x1), mult(x3, x1))$

$mult(\underbrace{rd(x3, asoc(x2, x1, x1))}, x1)$

$=$ by Lemma 1714 RL with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$mult(\overbrace{\underbrace{mult(asoc(x1, x2, x1), x3)}}, x1)$

$=$ by Lemma 368 RL with $\{x2 \leftarrow asoc(x1, x2, x1), x1 \leftarrow mult(asoc(x1, x2, x1), x3)\}$

$mult(\overbrace{op_r(\underbrace{op_t(mult(asoc(x1, x2, x1), x3), rd(asoc(x1, x2, x1), mult(asoc(x1, x2, x1), x3)))}, asoc(x1, x2, x1), asoc(x1, x2, x1)}}, x1)$

$=$ by Lemma 1296 LR with $\{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow mult(asoc(x1, x2, x1), x3)\}$

$mult(\underbrace{op_r(\overbrace{mult(asoc(x1, x2, x1), x3)}, asoc(x1, x2, x1), asoc(x1, x2, x1))}, x1)$

$=$ by Lemma 412 LR with $\{x2 \leftarrow x3, x1 \leftarrow asoc(x1, x2, x1)\}$

$mult(\overbrace{rd(mult(asoc(x1, x2, x1), mult(x3, asoc(x1, x2, x1))), asoc(x1, x2, x1))}, x1)$

$=$ by Lemma 1300 LR with $\{x2 \leftarrow x2, x3 \leftarrow x1, x1 \leftarrow mult(asoc(x1, x2, x1), mult(x3, asoc(x1, x2, x1)))\}$

$\underbrace{mult(\overbrace{mult(mult(asoc(x1, x2, x1), mult(x3, asoc(x1, x2, x1))), asoc(x2, x1, x1))}, x1)}$

$=$ by Lemma 91 RL with $\{x3 \leftarrow x1, x2 \leftarrow asoc(x2, x1, x1), x1 \leftarrow mult(asoc(x1, x2, x1), mult(x3, asoc(x1, x2, x1)))\}$

$\overbrace{mult(\underbrace{op_r(mult(asoc(x1, x2, x1), mult(x3, asoc(x1, x2, x1))), asoc(x2, x1, x1), x1)}, mult(asoc(x2, x1, x1), x1))}$

$=$ by Lemma 1578 LR with $\{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow mult(asoc(x1, x2, x1), mult(x3, asoc(x1, x2, x1)))\}$

$mult(\overbrace{mult(asoc(x1, x2, x1), mult(x3, asoc(x1, x2, x1)))}, \underbrace{mult(asoc(x2, x1, x1), x1)})$

$=$ by Lemma 771 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{mult(mult(asoc(x1, x2, x1), mult(x3, asoc(x1, x2, x1))), \overbrace{op_l(x1, x2, x1)})}$

$=$ by Axiom 6 RL with $\{x3 \leftarrow op_l(x1, x2, x1), x2 \leftarrow x3, x1 \leftarrow asoc(x1, x2, x1)\}$

$\overbrace{mult(asoc(x1, x2, x1), mult(x3, \underbrace{mult(asoc(x1, x2, x1), op_l(x1, x2, x1))}))}$

$=$ by Lemma 617 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$mult(asoc(x1, x2, x1), mult(x3, \underbrace{mult(\overbrace{rd(x1, op_l(x1, x2, x1))}, op_l(x1, x2, x1))}))$

$=$ by Axiom 4 RL with $\{x2 \leftarrow op_l(x1, x2, x1), x1 \leftarrow x1\}$

$mult(asoc(x1, x2, x1), mult(x3, \overbrace{x1}))$

Lemma 1755: $mult(rd(x1,x2),op_l(x2,x3,x2)) = rd(x1,asoc(x2,x3,x2))$

$\underbrace{mult(rd(x1,x2),op_l(x2,x3,x2))}$

$=$ by Lemma 1707 RL with $\{x3 \leftarrow rd(x1,x2), x2 \leftarrow x3, x1 \leftarrow x2\}$

$\overbrace{mult(op_l(x2,x3,x2),\underbrace{op_t(rd(x1,x2),x2)})}$

$=$ by Lemma 14 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$mult(\underbrace{op_l(x2,x3,x2)},\overbrace{mult(i(x2),x1)})$

$=$ by Lemma 771 LR with $\{x2 \leftarrow x3, x1 \leftarrow x2\}$

$\underbrace{mult(\overbrace{mult(asoc(x3,x2,x2),x2)},mult(i(x2),x1))}$

$=$ by Axiom 8 LR with $\{x3 \leftarrow mult(i(x2),x1), x2 \leftarrow x2, x1 \leftarrow asoc(x3,x2,x2)\}$

$\overbrace{mult(mult(asoc(x3,x2,x2),\underbrace{mult(x2,mult(i(x2),x1))}),asoc(asoc(x3,x2,x2),x2,mult(i(x2),x1)))}$

$=$ by Lemma 12 LR with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$mult(mult(asoc(x3,x2,x2),\overbrace{x1}),\underbrace{asoc(asoc(x3,x2,x2),x2,mult(i(x2),x1))})$

$=$ by Lemma 958 LR with $\{x3 \leftarrow mult(i(x2),x1), x2 \leftarrow x2, x1 \leftarrow x3\}$

$\underbrace{mult(mult(asoc(x3,x2,x2),x1),\overbrace{unit()})}$

$=$ by Axiom 1 RL with $\{x1 \leftarrow mult(asoc(x3,x2,x2),x1)\}$

$\overbrace{\underbrace{mult(asoc(x3,x2,x2),x1)}}$

$=$ by Lemma 1708 LR with $\{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x3\}$

$\overbrace{rd(x1,asoc(x2,x3,x2))}$

Lemma 1756: $op_t(rd(op_t(x1,x2),x3),i(x1)) = mult(op_t(x1,x2),i(x3))$

$op_t(rd(\underbrace{op_t(x1,x2)},x3),i(x1))$

$=$ by Lemma 3 RL with $\{x1 \leftarrow op_t(x1,x2)\}$

$\underbrace{op_t(rd(\overbrace{i(i(op_t(x1,x2)))},x3),i(x1))}$

$=$ by Lemma 1722 RL with $\{x3 \leftarrow x2, x2 \leftarrow i(x1), x1 \leftarrow rd(i(i(op_t(x1,x2))),x3)\}$

$\overbrace{\underbrace{op_t(rd(i(i(op_t(x1,x2))),x3),op_t(i(x1),x2))}}$

$=$ by Lemma 1691 LR with $\{x3 \leftarrow x2, x2 \leftarrow i(x1), x1 \leftarrow rd(i(i(op_t(x1,x2))),x3)\}$

$\overbrace{op_t(rd(i(i(op_t(x1,x2))),x3),\underbrace{op_t(i(x1),i(x2))})}$

$=$ by Lemma 15 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{op_t(rd(i(i(op_t(x1,x2))),x3),\overbrace{i(op_t(x1,x2))})}$

$=$ by Lemma 33 LR with $\{x2 \leftarrow x3, x1 \leftarrow i(op_t(x1,x2))\}$

$\underbrace{\overbrace{i(mult(i(op_t(x1,x2)),x3))}}$

$=$ by Lemma 16 LR with $\{x2 \leftarrow x3, x1 \leftarrow op_t(x1,x2)\}$

$\overbrace{mult(op_t(x1,x2),i(x3))}$

Lemma 1757: $mult(op_t(x1,x2),rd(x1,x3)) = op_t(rd(x1,rd(x3,op_t(x1,x2))),i(x1))$

$$
\begin{aligned}
& mult(op_t(x1,x2),\underbrace{rd(x1,x3)}) \\
= & \quad \text{by Lemma 10 RL with } \{x2 \leftarrow x1, x1 \leftarrow x3\} \\
& \underbrace{mult(op_t(x1,x2),\overbrace{i(rd(x3,x1))})} \\
= & \quad \text{by Lemma 1756 RL with } \{x3 \leftarrow rd(x3,x1), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_t(\underbrace{rd(op_t(x1,x2),rd(x3,x1))},i(x1))} \\
= & \quad \text{by Lemma 1671 RL with } \{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x1\} \\
& op_t(\overbrace{rd(x1,rd(x3,op_t(x1,x2)))},i(x1))
\end{aligned}
$$

Lemma 1758: $mult(op_t(x1,x2),rd(x1,x3)) = mult(x1,rd(op_t(x1,x2),x3))$

$$
\begin{aligned}
& \underbrace{mult(op_t(x1,x2),rd(x1,x3))} \\
= & \quad \text{by Lemma 1757 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{\overbrace{op_t(rd(x1,rd(x3,op_t(x1,x2))),i(x1))}} \\
= & \quad \text{by Lemma 34 LR with } \{x2 \leftarrow rd(x3,op_t(x1,x2)), x1 \leftarrow x1\} \\
& \overbrace{mult(x1,\underbrace{i(rd(x3,op_t(x1,x2)))})} \\
= & \quad \text{by Lemma 10 LR with } \{x2 \leftarrow op_t(x1,x2), x1 \leftarrow x3\} \\
& mult(x1,\overbrace{rd(op_t(x1,x2),x3)})
\end{aligned}
$$

Lemma 1759: $mult(op_t(x1,x2),mult(x3,x1)) = op_t(rd(x1,i(mult(x3,op_t(x1,x2)))),i(x1))$

$$
\begin{aligned}
& mult(op_t(x1,x2),\underbrace{mult(x3,x1)}) \\
= & \quad \text{by Lemma 3 RL with } \{x1 \leftarrow mult(x3,x1)\} \\
& \underbrace{mult(op_t(x1,x2),\overbrace{i(i(mult(x3,x1)))})} \\
= & \quad \text{by Lemma 1756 RL with } \{x3 \leftarrow i(mult(x3,x1)), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_t(\underbrace{rd(op_t(x1,x2),i(mult(x3,x1)))},i(x1))} \\
= & \quad \text{by Lemma 1675 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_t(\overbrace{rd(x1,i(mult(x3,op_t(x1,x2))))},i(x1))
\end{aligned}
$$

Lemma 1760: $mult(op_t(x1,x2),mult(x3,x1)) = mult(x1,mult(x3,op_t(x1,x2)))$

$$
\begin{aligned}
& \underbrace{mult(op_t(x1,x2),mult(x3,x1))} \\
= & \quad \text{by Lemma 1759 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{\overbrace{op_t(rd(x1,i(mult(x3,op_t(x1,x2)))),i(x1))}} \\
= & \quad \text{by Lemma 34 LR with } \{x2 \leftarrow i(mult(x3,op_t(x1,x2))), x1 \leftarrow x1\} \\
& \overbrace{mult(x1,\underbrace{i(i(mult(x3,op_t(x1,x2))))})} \\
= & \quad \text{by Lemma 3 LR with } \{x1 \leftarrow mult(x3,op_t(x1,x2))\} \\
& mult(x1,\overbrace{mult(x3,op_t(x1,x2))})
\end{aligned}
$$

Lemma 1761: $mult(x2, op_t(x1, x2)) = op_l(mult(x1, op_t(x2, op_t(x1, x3))), op_t(x1, x3), i(x2))$

$\overbrace{mult(x2, op_t(x1, x2))}$

$=$ by Lemma 1738 RL with $\{x3 \leftarrow x2, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{mult(op_t(x1, x2), \underbrace{op_t(x2, x1)})}$

$=$ by Lemma 1722 RL with $\{x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow x2\}$

$mult(\underbrace{op_t(x1, x2)}, \overbrace{op_t(x2, op_t(x1, x3))})$

$=$ by Lemma 1722 RL with $\{x3 \leftarrow op_t(x1, x3), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{mult(\overbrace{op_t(x1, op_t(x2, op_t(x1, x3)))}, op_t(x2, op_t(x1, x3)))}$

$=$ by Lemma 636 RL with $\{x2 \leftarrow op_t(x2, op_t(x1, x3)), x1 \leftarrow x1\}$

$\overbrace{op_l(\underbrace{mult(x1, op_t(x2, op_t(x1, x3)))}, op_t(x2, op_t(x1, x3)), x1)}$

$=$ by Lemma 138 RL with $\{x3 \leftarrow op_t(x2, op_t(x1, x3)), x2 \leftarrow op_t(x1, x3), x1 \leftarrow mult(x1, op_t(x2, op_t(x1, x3)))\}$

$op_l(\overbrace{op_l(op_l(mult(x1, \underbrace{op_t(x2, op_t(x1, x3))}), op_t(x1, x3), op_t(x2, op_t(x1, x3))), op_t(x2, op_t(x1, x3)), op_t(x1, x3))}, op_t(x2, op_t(x$

$=$ by Axiom 4 LR with $\{x2 \leftarrow op_t(x1, x3), x1 \leftarrow op_t(x2, op_t(x1, x3))\}$

$op_l(op_l(op_l(\underbrace{mult(x1, \overbrace{mult(rd(op_t(x2, op_t(x1, x3)), op_t(x1, x3)), op_t(x1, x3))})}, op_t(x1, x3), op_t(x2, op_t(x1, x3))), op_t(x2, op_t($

$=$ by Lemma 1760 RL with $\{x3 \leftarrow rd(op_t(x2, op_t(x1, x3)), op_t(x1, x3)), x2 \leftarrow x3, x1 \leftarrow x1\}$

$op_l(op_l(\underbrace{op_l(\overbrace{mult(op_t(x1, x3), mult(rd(\overbrace{op_t(x2, op_t(x1, x3))}, op_t(x1, x3)), x1))}, op_t(x1, x3), op_t(x2, op_t(x1, x3)))}, op_t(x2, op_t($

$=$ by Lemma 1471 LR with $\{x3 \leftarrow x1, x2 \leftarrow op_t(x2, op_t(x1, x3)), x1 \leftarrow op_t(x1, x3)\}$

$\underbrace{op_l(op_l(\overbrace{mult(op_t(x2, op_t(x1, x3)), x1)}, op_t(x2, op_t(x1, x3)), op_t(x1, x3)), op_t(x2, op_t(x1, x3)), x1)}$

$=$ by Axiom 19 RL with $\{x5 \leftarrow op_t(x1, x3), x4 \leftarrow op_t(x2, op_t(x1, x3)), x3 \leftarrow x1, x2 \leftarrow op_t(x2, op_t(x1, x3)), x1 \leftarrow mult(op_t(x2, o$

$\overbrace{op_l(\underbrace{op_l(mult(op_t(x2, op_t(x1, x3)), x1)}, op_t(x2, op_t(x1, x3)), x1), op_t(x2, op_t(x1, x3)), op_t(x1, x3))}$

$=$ by Lemma 435 LR with $\{x2 \leftarrow x1, x1 \leftarrow op_t(x2, op_t(x1, x3))\}$

$\underbrace{op_l(\overbrace{mult(x1, op_t(x2, op_t(x1, x3)))}, op_t(x2, op_t(x1, x3)), op_t(x1, x3))}$

$=$ by Lemma 1319 LR with $\{x3 \leftarrow op_t(x1, x3), x2 \leftarrow x2, x1 \leftarrow mult(x1, op_t(x2, op_t(x1, x3)))\}$

$\overbrace{op_l(mult(x1, op_t(x2, op_t(x1, x3))), op_t(x1, x3), i(x2))}$

Lemma 1762: $mult(x1, op_l(x3, x3, x2)) = mult(op_l(x3, x3, x2), op_t(x1, mult(x2, rd(x3, x2))))$

$mult(x1, \underbrace{op_l(x3, x3, x2)})$

$=$ by Lemma 1624 RL with $\{x2 \leftarrow x3, x1 \leftarrow x2\}$

$\underbrace{mult(x1, \overbrace{op_t(mult(x2, rd(x3, x2)), rd(i(x3), x3))})}$

$=$ by Lemma 1738 RL with $\{x3 \leftarrow x1, x2 \leftarrow rd(i(x3), x3), x1 \leftarrow mult(x2, rd(x3, x2))\}$

$\overbrace{mult(\underbrace{op_t(mult(x2, rd(x3, x2)), rd(i(x3), x3))}, op_t(x1, mult(x2, rd(x3, x2))))}$

$=$ by Lemma 1624 LR with $\{x2 \leftarrow x3, x1 \leftarrow x2\}$

$mult(\overbrace{op_l(x3, x3, x2)}, op_t(x1, mult(x2, rd(x3, x2))))$

Lemma 1763: $op_t(x1, rd(x3, mult(x2, x1))) = op_t(x1, rd(x3, mult(x1, x2)))$

$$
\begin{aligned}
& \underbrace{op_t(x1, rd(x3, mult(x2, x1)))} \\
= \quad & \text{by Lemma 1728 RL with } \{x3 \leftarrow x3, x2 \leftarrow mult(x2, x1), x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_t(x1, mult(i(mult(x2, x1)), x3))}} \\
= \quad & \text{by Lemma 1148 RL with } \{x3 \leftarrow x3, x2 \leftarrow i(mult(x2, x1)), x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, mult(\underbrace{op_r(i(mult(x2, x1)), x1, x1)}, x3))} \\
= \quad & \text{by Lemma 371 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \underbrace{op_t(x1, mult(\overbrace{rd(i(x2), x1)}, x3))} \\
= \quad & \text{by Lemma 1752 LR with } \{x4 \leftarrow x3, x3 \leftarrow x1, x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_t(x1, rd(x3, rd(x1, i(x2))))}} \\
= \quad & \text{by Lemma 1219 LR with } \{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, rd(x3, mult(x1, x2)))}
\end{aligned}
$$

Lemma 1764: $op_t(rd(x1, op_l(x2, x3, x2)), x2) = mult(op_l(i(x2), x3, x2), x1)$

$$
\begin{aligned}
& \underbrace{op_t(rd(x1, op_l(x2, x3, x2)), x2)} \\
= \quad & \text{by Lemma 1687 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow rd(x1, op_l(x2, x3, x2))\} \\
& \overbrace{\underbrace{op_t(rd(x1, op_l(x2, x3, x2)), op_l(x2, x3, x2))}} \\
= \quad & \text{by Lemma 14 LR with } \{x2 \leftarrow op_l(x2, x3, x2), x1 \leftarrow x1\} \\
& \overbrace{mult(\underbrace{i(op_l(x2, x3, x2))}, x1)} \\
= \quad & \text{by Lemma 1049 LR with } \{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x2\} \\
& mult(\overbrace{op_l(i(x2), x3, x2)}, x1)
\end{aligned}
$$

Lemma 1765: $op_t(rd(x1, op_r(x2, x3, x3)), x2) = mult(op_r(i(x2), x3, x3), x1)$

$$
\begin{aligned}
& \underbrace{op_t(rd(x1, op_r(x2, x3, x3)), x2)} \\
= \quad & \text{by Lemma 1721 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow rd(x1, op_r(x2, x3, x3))\} \\
& \overbrace{\underbrace{op_t(rd(x1, op_r(x2, x3, x3)), op_r(x2, x3, x3))}} \\
= \quad & \text{by Lemma 14 LR with } \{x2 \leftarrow op_r(x2, x3, x3), x1 \leftarrow x1\} \\
& \overbrace{mult(\underbrace{i(op_r(x2, x3, x3))}, x1)} \\
= \quad & \text{by Lemma 356 LR with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& mult(\overbrace{op_r(i(x2), x3, x3)}, x1)
\end{aligned}
$$

Lemma 1766: $op_t(x1, mult(x2, rd(i(x1), x3))) = op_t(x1, rd(x3, mult(x1, x2)))$

$\underbrace{op_t(x1, mult(x2, rd(i(x1), x3)))}$

$=$ by Lemma 1207 RL with $\{x3 \leftarrow rd(i(x1), x3), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_t(x1, mult(x2, \underbrace{op_t(rd(i(x1), x3), x1)}))}$

$=$ by Lemma 33 LR with $\{x2 \leftarrow x3, x1 \leftarrow x1\}$

$\underbrace{op_t(x1, mult(x2, \overbrace{i(mult(x1, x3))}))}$

$=$ by Lemma 1729 RL with $\{x3 \leftarrow x2, x2 \leftarrow i(mult(x1, x3)), x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_t(x1, rd(i(mult(x1, x3)), i(x2)))}}$

$=$ by Lemma 1279 LR with $\{x3 \leftarrow i(x2), x2 \leftarrow x3, x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_t(x1, rd(x3, rd(x1, i(x2))))}}$

$=$ by Lemma 1219 LR with $\{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x1\}$

$\overbrace{op_t(x1, rd(x3, mult(x1, x2)))}$

Lemma 1767: $op_t(x1, rd(x2, mult(x1, x3))) = op_t(x1, rd(x3, mult(x1, x2)))$

$\underbrace{op_t(x1, rd(x2, mult(x1, x3)))}$

$=$ by Lemma 1216 RL with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_t(x1, rd(x2, rd(x3, i(x1))))}}$

$=$ by Lemma 1753 RL with $\{x4 \leftarrow x3, x3 \leftarrow i(x1), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_t(x1, mult(x2, rd(i(x1), x3)))}}$

$=$ by Lemma 1766 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_t(x1, rd(x3, mult(x1, x2)))}$

Lemma 1768: $op_t(x1, rd(x1, rd(x3, x2))) = op_t(x1, mult(x1, mult(i(x3), x2)))$

$$
\begin{aligned}
& \overbrace{op_t(x1, rd(x1, rd(x3, x2)))} \\
= & \quad \text{by Lemma 738 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, \underbrace{mult(x1, rd(x2, x3))})} \\
= & \quad \text{by Lemma 251 RL with } \{x2 \leftarrow rd(x2, x3), x1 \leftarrow x1\} \\
& \overbrace{op_l(\underbrace{op_t(x1, rd(x2, x3))}, rd(x2, x3), x1)} \\
= & \quad \text{by Lemma 1722 RL with } \{x3 \leftarrow x3, x2 \leftarrow rd(x2, x3), x1 \leftarrow x1\} \\
& \underbrace{op_l(\overbrace{op_t(x1, op_t(rd(x2, x3), x3))}, rd(x2, x3), x1)} \\
= & \quad \text{by Axiom 15 RL with } \{x4 \leftarrow op_t(rd(x2, x3), x3), x3 \leftarrow x1, x2 \leftarrow rd(x2, x3), x1 \leftarrow x1\} \\
& \overbrace{op_t(\underbrace{op_l(x1, rd(x2, x3), x1)}, op_t(rd(x2, x3), x3))} \\
= & \quad \text{by Lemma 1024 RL with } \{x3 \leftarrow op_t(rd(x2, x3), x3), x2 \leftarrow rd(x2, x3), x1 \leftarrow x1\} \\
& \underbrace{op_t(\overbrace{op_l(x1, rd(x2, x3), op_t(x1, op_t(rd(x2, x3), x3)))}, op_t(rd(x2, x3), x3))} \\
= & \quad \text{by Axiom 15 LR with } \{x4 \leftarrow op_t(rd(x2, x3), x3), x3 \leftarrow op_t(x1, op_t(rd(x2, x3), x3)), x2 \leftarrow rd(x2, x3), x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(op_t(x1, op_t(rd(x2, x3), x3)), rd(x2, x3), op_t(x1, op_t(rd(x2, x3), x3)))}} \\
= & \quad \text{by Lemma 1723 RL with } \{x3 \leftarrow x3, x2 \leftarrow rd(x2, x3), x1 \leftarrow op_t(x1, op_t(rd(x2, x3), x3))\} \\
& \overbrace{\underbrace{op_l(op_t(x1, op_t(rd(x2, x3), x3)), op_t(rd(x2, x3), x3), op_t(x1, op_t(rd(x2, x3), x3)))}} \\
= & \quad \text{by Lemma 1315 LR with } \{x3 \leftarrow x1, x2 \leftarrow op_t(rd(x2, x3), x3), x1 \leftarrow op_t(x1, op_t(rd(x2, x3), x3))\} \\
& \overbrace{\underbrace{op_l(op_t(x1, op_t(rd(x2, x3), x3)), x1, i(op_t(rd(x2, x3), x3)))}} \\
= & \quad \text{by Lemma 672 LR with } \{x3 \leftarrow op_t(rd(x2, x3), x3), x2 \leftarrow op_t(rd(x2, x3), x3), x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(op_t(x1, op_t(rd(x2, x3), x3)), op_t(rd(x2, x3), x3), x1)}} \\
= & \quad \text{by Lemma 251 LR with } \{x2 \leftarrow op_t(rd(x2, x3), x3), x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, mult(x1, \underbrace{op_t(rd(x2, x3), x3)}))} \\
= & \quad \text{by Lemma 14 LR with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& op_t(x1, mult(x1, \overbrace{mult(i(x3), x2)}))
\end{aligned}
$$

Lemma 1769: $op_t(x1, rd(x1, rd(x3, x2))) = op_t(x1, rd(x1, mult(x3, i(x2))))$

$$
\begin{aligned}
& \overbrace{op_t(x1, rd(x1, rd(x3, x2)))} \\
= & \quad \text{by Lemma 1768 LR with } \{x2 \leftarrow x2, x3 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, mult(x1, \underbrace{mult(i(x3), x2)}))} \\
= & \quad \text{by Lemma 17 RL with } \{x2 \leftarrow x2, x1 \leftarrow x3\} \\
& \underbrace{op_t(x1, mult(x1, \overbrace{i(mult(x3, i(x2)))}))} \\
= & \quad \text{by Lemma 564 LR with } \{x2 \leftarrow mult(x3, i(x2)), x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, rd(x1, mult(x3, i(x2))))}
\end{aligned}
$$

Lemma 1770: $rd(op_t(x1,x2),op_t(op_t(x1,x2),x3)) = rd(op_l(x3,x1,x3),op_r(x3,x1,x1))$

$$
\begin{aligned}
& \overbrace{rd(op_t(x1,x2),op_t(op_t(x1,x2),x3))} \\
= \quad & \text{by Lemma 1182 RL with } \{x2 \leftarrow op_t(x1,x2), x1 \leftarrow x3\} \\
& \overbrace{rd(\underbrace{op_l(x3,op_t(x1,x2),x3)},op_r(x3,op_t(x1,x2),op_t(x1,x2)))} \\
= \quad & \text{by Lemma 1723 LR with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x3\} \\
& rd(\overbrace{op_l(x3,x1,x3)},\underbrace{op_r(x3,op_t(x1,x2),op_t(x1,x2))}) \\
= \quad & \text{by Lemma 1632 LR with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x3\} \\
& rd(op_l(x3,x1,x3),\overbrace{op_r(x3,x1,x1)})
\end{aligned}
$$

Lemma 1771: $mult(x1,mult(x3,x2)) = mult(op_t(x1,x2),mult(x3,op_t(x2,x1)))$

$$
\begin{aligned}
& mult(x1,\underbrace{mult(x3,x2)}) \\
= \quad & \text{by Lemma 13 RL with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& \underbrace{mult(x1,\overbrace{mult(x2,op_t(x3,x2))})} \\
= \quad & \text{by Lemma 1263 RL with } \{x3 \leftarrow op_t(x3,x2), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{mult(op_t(x1,x2),\underbrace{mult(op_t(x2,x1),op_t(x3,x2))})} \\
= \quad & \text{by Lemma 1738 LR with } \{x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow x2\} \\
& mult(op_t(x1,x2),\overbrace{mult(x3,op_t(x2,x1))})
\end{aligned}
$$

Lemma 1772: $mult(mult(x1,op_t(x2,x3)),op_t(x3,x2)) = op_r(op_t(mult(x3,mult(x1,x2)),op_t(x3,x2)),x3,x3)$

$$
\begin{aligned}
& \underbrace{mult(mult(x1,op_t(x2,x3)),op_t(x3,x2))} \\
= \quad & \text{by Lemma 385 RL with } \{x2 \leftarrow mult(x1,op_t(x2,x3)), x1 \leftarrow op_t(x3,x2)\} \\
& \overbrace{op_r(op_t(\underbrace{mult(op_t(x3,x2),mult(x1,op_t(x2,x3)))},op_t(x3,x2)),op_t(x3,x2),op_t(x3,x2))} \\
= \quad & \text{by Lemma 1771 RL with } \{x2 \leftarrow x2, x3 \leftarrow x1, x1 \leftarrow x3\} \\
& \underbrace{op_r(op_t(\overbrace{mult(x3,mult(x1,x2))},op_t(x3,x2)),op_t(x3,x2),op_t(x3,x2))} \\
= \quad & \text{by Lemma 1632 LR with } \{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow op_t(mult(x3,mult(x1,x2)),op_t(x3,x2))\} \\
& \overbrace{op_r(op_t(mult(x3,mult(x1,x2)),op_t(x3,x2)),x3,x3)}
\end{aligned}
$$

Lemma 1773: $op_t(x1,mult(mult(i(x2),x3),x4)) = op_t(x1,rd(x4,mult(x2,i(x3))))$

$$
\begin{aligned}
& op_t(x1,mult(\underbrace{mult(i(x2),x3)},x4)) \\
= \quad & \text{by Lemma 17 RL with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& \underbrace{op_t(x1,mult(\overbrace{i(\overbrace{mult(x2,i(x3))})},x4))} \\
= \quad & \text{by Lemma 1728 LR with } \{x3 \leftarrow x4, x2 \leftarrow mult(x2,i(x3)), x1 \leftarrow x1\} \\
& \overbrace{op_t(x1,rd(x4,mult(x2,i(x3))))}
\end{aligned}
$$

Lemma 1774: $op_t(x1, mult(x2, mult(x3, i(x4)))) = op_t(x1, rd(x2, mult(i(x3), x4)))$

$$
\begin{aligned}
& \underbrace{op_t(x1, mult(x2, mult(x3, i(x4))))} \\
= \quad & \text{by Lemma 1729 RL with } \{x3 \leftarrow x2, x2 \leftarrow mult(x3, i(x4)), x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, \underbrace{rd(mult(x3, i(x4)), i(x2))})} \\
= \quad & \text{by Lemma 61 LR with } \{x3 \leftarrow x2, x2 \leftarrow x4, x1 \leftarrow x3\} \\
& op_t(x1, \overbrace{rd(x2, mult(i(x3), x4))})
\end{aligned}
$$

Lemma 1775: $mult(i(op_t(x1, x2)), rd(x3, x1)) = mult(i(x1), rd(x3, op_t(x1, x2)))$

$$
\begin{aligned}
& \underbrace{mult(i(op_t(x1, x2)), rd(x3, x1))} \\
= \quad & \text{by Lemma 45 RL with } \{x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow op_t(x1, x2)\} \\
& \overbrace{i(\underbrace{mult(op_t(x1, x2), rd(x1, x3))})} \\
= \quad & \text{by Lemma 1758 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{i(\overbrace{mult(x1, rd(op_t(x1, x2), x3))})} \\
= \quad & \text{by Lemma 45 LR with } \{x3 \leftarrow x3, x2 \leftarrow op_t(x1, x2), x1 \leftarrow x1\} \\
& \overbrace{mult(i(x1), rd(x3, op_t(x1, x2)))}
\end{aligned}
$$

Lemma 1776: $mult(x1, mult(i(x2), op_t(x1, x3))) = rd(x1, mult(i(op_t(x1, x3)), x2))$

$\underbrace{mult(x1, mult(i(x2), op_t(x1, x3)))}$

$=$ by Lemma 1132 RL with $\{x3 \leftarrow x3, x2 \leftarrow i(x2), x1 \leftarrow x1\}$

$\overbrace{rd(mult(x1, i(x2)), i(op_t(x1, x3)))}$

$=$ by Lemma 61 LR with $\{x3 \leftarrow op_t(x1, x3), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{rd(op_t(x1, x3), \underbrace{mult(i(x1), x2)})}$

$=$ by Lemma 14 RL with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$rd(op_t(x1, x3), \overbrace{op_t(\underbrace{rd(x2, x1)}, x1)})$

$=$ by Lemma 10 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$rd(op_t(x1, x3), \underbrace{op_t(\overbrace{i(rd(x1, x2))}, x1)})$

$=$ by Lemma 1722 RL with $\{x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow i(rd(x1, x2))\}$

$\underbrace{rd(op_t(x1, x3), \overbrace{op_t(i(rd(x1, x2)), op_t(x1, x3))})}$

$=$ by Lemma 405 RL with $\{x2 \leftarrow rd(x1, x2), x1 \leftarrow op_t(x1, x3)\}$

$\overbrace{op_r(\underbrace{mult(op_t(x1, x3), rd(x1, x2))}, op_t(x1, x3), i(op_t(x1, x3)))}$

$=$ by Lemma 1758 LR with $\{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x1\}$

$\underbrace{op_r(\overbrace{mult(x1, rd(op_t(x1, x3), x2))}, op_t(x1, x3), i(op_t(x1, x3)))}$

$=$ by Lemma 1655 LR with $\{x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow mult(x1, rd(op_t(x1, x3), x2))\}$

$\underbrace{\overbrace{op_r(mult(x1, rd(op_t(x1, x3), x2)), x1, i(x1))}}$

$=$ by Lemma 405 LR with $\{x2 \leftarrow rd(op_t(x1, x3), x2), x1 \leftarrow x1\}$

$\overbrace{rd(x1, op_t(\underbrace{i(rd(op_t(x1, x3), x2))}, x1))}$

$=$ by Lemma 10 LR with $\{x2 \leftarrow x2, x1 \leftarrow op_t(x1, x3)\}$

$rd(x1, \underbrace{op_t(\overbrace{rd(x2, op_t(x1, x3))}, x1)})$

$=$ by Lemma 1722 RL with $\{x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow rd(x2, op_t(x1, x3))\}$

$rd(x1, \underbrace{\overbrace{op_t(rd(x2, op_t(x1, x3)), op_t(x1, x3))}})$

$=$ by Lemma 14 LR with $\{x2 \leftarrow op_t(x1, x3), x1 \leftarrow x2\}$

$rd(x1, \overbrace{mult(i(op_t(x1, x3)), x2)})$

Lemma 1777: $mult(x1, op_l(x2, x2, x3)) = mult(mult(x3, rd(x2, x3)), op_t(x1, x2))$

$\underbrace{mult(x1, op_l(x2, x2, x3))}$

$=$ by Lemma 1762 LR with $\{x2 \leftarrow x3, x3 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{mult(op_l(x2, x2, x3), \underbrace{op_t(x1, mult(x3, rd(x2, x3)))})}$

$=$ by Lemma 1689 LR with $\{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x1\}$

$mult(\underbrace{op_l(x2, x2, x3)}, \overbrace{op_t(x1, x2)})$

$=$ by Lemma 427 LR with $\{x2 \leftarrow x3, x1 \leftarrow x2\}$

$mult(\overbrace{mult(x3, rd(x2, x3))}, op_t(x1, x2))$

Lemma 1778: $mult(op_l(x3, x2, x1), x1) = op_l(mult(x3, op_l(x1, x1, x2)), x2, rd(x1, x2))$

$$
\begin{aligned}
& \underbrace{mult(op_l(x3, x2, x1), x1)} \\
= & \quad \text{by Lemma 1498 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{mult(rd(x1, x2), mult(x2, op_t(x3, x1)))}} \\
= & \quad \text{by Lemma 1068 RL with } \{x3 \leftarrow op_t(x3, x1), x2 \leftarrow rd(x1, x2), x1 \leftarrow x2\} \\
& \overbrace{op_l(\underbrace{mult(mult(x2, rd(x1, x2)), op_t(x3, x1))}, x2, rd(x1, x2))} \\
= & \quad \text{by Lemma 1777 RL with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x3\} \\
& op_l(\overbrace{mult(x3, op_l(x1, x1, x2))}, x2, rd(x1, x2))
\end{aligned}
$$

Lemma 1779: $op_l(mult(x1, op_l(x2, x2, x3)), x3, x2) = mult(op_l(x1, x3, x2), x2)$

$$
\begin{aligned}
& \underbrace{op_l(mult(x1, op_l(x2, x2, x3)), x3, x2)} \\
= & \quad \text{by Lemma 286 RL with } \{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow mult(x1, op_l(x2, x2, x3))\} \\
& \overbrace{\underbrace{op_l(mult(x1, op_l(x2, x2, x3)), x3, rd(x2, x3))}} \\
= & \quad \text{by Lemma 1778 RL with } \{x1 \leftarrow x2, x2 \leftarrow x3, x3 \leftarrow x1\} \\
& \overbrace{mult(op_l(x1, x3, x2), x2)}
\end{aligned}
$$

Lemma 1780: $mult(op_l(x1, x2, i(mult(x2, x3))), mult(x2, x3)) = op_l(mult(x1, op_l(mult(x2, x3), x2, x3)), i(x2), mult(x2, x3))$

$$
\begin{aligned}
& mult(\underbrace{op_l(x1, x2, i(mult(x2, x3)))}, mult(x2, x3)) \\
= & \quad \text{by Lemma 1050 RL with } \{x3 \leftarrow mult(x2, x3), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{mult(\overbrace{op_l(x1, i(x2), mult(x2, x3))}, mult(x2, x3))} \\
= & \quad \text{by Lemma 1779 RL with } \{x3 \leftarrow i(x2), x2 \leftarrow mult(x2, x3), x1 \leftarrow x1\} \\
& \overbrace{op_l(mult(x1, \underbrace{op_l(mult(x2, x3), mult(x2, x3), i(x2))}), i(x2), mult(x2, x3))} \\
= & \quad \text{by Lemma 242 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow mult(x2, x3)\} \\
& op_l(mult(x1, \overbrace{op_l(mult(x2, x3), x2, x3)}), i(x2), mult(x2, x3))
\end{aligned}
$$

Lemma 1781: $mult(op_l(x1, x3, x2), mult(x2, x3)) = op_l(mult(x1, op_l(mult(x2, x3), x2, x3)), x2, i(mult(x2, x3)))$

$$
\begin{aligned}
& mult(\underbrace{op_l(x1, x3, x2)}, mult(x2, x3)) \\
= & \quad \text{by Lemma 1051 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{mult(\overbrace{op_l(x1, x2, i(mult(x2, x3)))}, mult(x2, x3))} \\
= & \quad \text{by Lemma 1780 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(mult(x1, op_l(mult(x2, x3), x2, x3)), i(x2), mult(x2, x3))}} \\
= & \quad \text{by Lemma 1050 LR with } \{x3 \leftarrow mult(x2, x3), x2 \leftarrow x2, x1 \leftarrow mult(x1, op_l(mult(x2, x3), x2, x3))\} \\
& \overbrace{op_l(mult(x1, op_l(mult(x2, x3), x2, x3)), x2, i(mult(x2, x3)))}
\end{aligned}
$$

Lemma 1782: $mult(op_l(x1, x3, x2), mult(x2, x3)) = op_l(mult(x1, mult(x3, x2)), x3, x2)$

$$
\begin{aligned}
& \overbrace{mult(op_l(x1, x3, x2), mult(x2, x3))} \\
= & \quad \text{by Lemma 1781 LR with } \{x2 \leftarrow x2, x3 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{op_l(mult(x1, op_l(mult(x2, x3), x2, x3)), x2, i(mult(x2, x3)))} \\
= & \quad \text{by Lemma 1051 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow mult(x1, op_l(mult(x2, x3), x2, x3))\} \\
& \overbrace{op_l(mult(x1, \underbrace{op_l(mult(x2, x3), x2, x3)}), x3, x2)} \\
= & \quad \text{by Lemma 435 LR with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& op_l(mult(x1, \overbrace{mult(x3, x2)}), x3, x2)
\end{aligned}
$$

Lemma 1783: $op_t(x1, op_l(op_l(x2, x2, x3), x4, x5)) = op_t(x1, op_l(x2, x4, x5))$

$$
\begin{aligned}
& op_t(x1, op_l(\underbrace{op_l(x2, x2, x3)}, x4, x5)) \\
= & \quad \text{by Lemma 606 RL with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& op_t(x1, op_l(\overbrace{op_l(x2, x3, i(x2))}, x4, x5)) \\
= & \quad \text{by Lemma 1050 RL with } \{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x2\} \\
& op_t(x1, op_l(\overbrace{op_l(x2, i(x3), x2)}, x4, x5)) \\
= & \quad \text{by Lemma 745 LR with } \{x2 \leftarrow i(x3), x1 \leftarrow x2\} \\
& op_t(x1, \underbrace{op_l(\overbrace{op_r(op_t(x2, i(x3)), i(x3), i(x3))}, x4, x5)}) \\
= & \quad \text{by Axiom 18 RL with } \{x5 \leftarrow i(x3), x4 \leftarrow i(x3), x3 \leftarrow x5, x2 \leftarrow x4, x1 \leftarrow op_t(x2, i(x3))\} \\
& \underbrace{op_t(x1, \overbrace{op_r(op_l(op_t(x2, i(x3)), x4, x5), i(x3), i(x3))})} \\
= & \quad \text{by Lemma 1721 LR with } \{x3 \leftarrow i(x3), x2 \leftarrow op_l(op_t(x2, i(x3)), x4, x5), x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, \underbrace{op_l(op_t(x2, i(x3)), x4, x5)})} \\
= & \quad \text{by Axiom 15 RL with } \{x4 \leftarrow i(x3), x3 \leftarrow x5, x2 \leftarrow x4, x1 \leftarrow x2\} \\
& \underbrace{op_t(x1, \overbrace{op_t(op_l(x2, x4, x5), i(x3))})} \\
= & \quad \text{by Lemma 1722 LR with } \{x3 \leftarrow i(x3), x2 \leftarrow op_l(x2, x4, x5), x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, op_l(x2, x4, x5))}
\end{aligned}
$$

Lemma 1784: $mult(op_l(x1,x1,x2),op_l(x2,x2,rd(x2,x1))) = op_l(mult(x1,x2),x2,op_t(rd(x2,x1),x3))$

$mult(op_l(x1,x1,x2),\underbrace{op_l(x2,x2,rd(x2,x1))})$

$=$ by Lemma 1725 RL with $\{x3 \leftarrow x3, x2 \leftarrow rd(x2,x1), x1 \leftarrow x2\}$

$\underbrace{mult(op_l(x1,x1,x2),\overbrace{op_l(x2,x2,op_t(rd(x2,x1),x3))})}$

$=$ by Lemma 138 RL with $\{x3 \leftarrow x2, x2 \leftarrow op_t(rd(x2,x1),x3), x1 \leftarrow mult(op_l(x1,x1,x2),op_l(x2,x2,op_t(rd(x2,x1),x3)))\}$

$\overbrace{op_l(\underbrace{op_l(mult(op_l(x1,x1,x2),op_l(x2,x2,\overbrace{op_t(rd(x2,x1),x3)})),op_t(rd(x2,x1),x3),x2)},x2,op_t(rd(x2,x1),x3))}$

$=$ by Lemma 1779 LR with $\{x3 \leftarrow op_t(rd(x2,x1),x3), x2 \leftarrow x2, x1 \leftarrow op_l(x1,x1,x2)\}$

$op_l(\overbrace{mult(op_l(op_l(x1,x1,x2),op_t(\underbrace{rd(x2,x1)},x3),x2),x2)},x2,op_t(rd(x2,x1),x3))$

$=$ by Lemma 10 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$op_l(mult(op_l(\underbrace{op_l(x1,x1,x2)},op_t(\overbrace{i(rd(x1,x2))},x3),x2),x2),x2,op_t(rd(x2,x1),x3))$

$=$ by Lemma 427 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$op_l(mult(\underbrace{op_l(\overbrace{mult(x2,rd(x1,x2))},op_t(i(rd(x1,x2)),x3),x2)},x2),x2,op_t(rd(x2,x1),x3))$

$=$ by Lemma 1137 RL with $\{x4 \leftarrow x2, x3 \leftarrow x3, x2 \leftarrow rd(x1,x2), x1 \leftarrow mult(x2,rd(x1,x2))\}$

$op_l(mult(\overbrace{op_l(\underbrace{mult(x2,rd(x1,x2))},op_t(rd(x1,x2),i(x3)),i(x2))},x2),x2,op_t(rd(x2,x1),x3))$

$=$ by Lemma 13 RL with $\{x2 \leftarrow x2, x1 \leftarrow rd(x1,x2)\}$

$op_l(mult(op_l(\overbrace{mult(rd(x1,x2),\underbrace{op_t(x2,rd(x1,x2))})},op_t(rd(x1,x2),i(x3)),i(x2)),x2),x2,op_t(rd(x2,x1),x3))$

$=$ by Lemma 1722 RL with $\{x3 \leftarrow i(x3), x2 \leftarrow rd(x1,x2), x1 \leftarrow x2\}$

$op_l(mult(\underbrace{op_l(mult(rd(x1,x2),\overbrace{op_t(x2,op_t(rd(x1,x2),i(x3)))}),op_t(rd(x1,x2),i(x3)),i(x2))},x2),x2,op_t(rd(x2,x1),x3))$

$=$ by Lemma 1761 RL with $\{x3 \leftarrow i(x3), x1 \leftarrow rd(x1,x2), x2 \leftarrow x2\}$

$op_l(mult(\underbrace{mult(x2,op_t(rd(x1,x2),x2))},x2),x2,op_t(rd(x2,x1),x3))$

$=$ by Lemma 13 LR with $\{x2 \leftarrow rd(x1,x2), x1 \leftarrow x2\}$

$op_l(mult(\underbrace{mult(rd(x1,x2),x2)},x2),x2,op_t(rd(x2,x1),x3))$

$=$ by Axiom 4 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$op_l(mult(\overbrace{x1},x2),x2,op_t(rd(x2,x1),x3))$

Lemma 1785: $mult(x2, x1) = op_l(mult(x1, x2), x2, op_t(rd(x2, x1), x3))$

$\overbrace{mult(x2, x1)}$

$=$ by Lemma 231 LR with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$\overbrace{rd(x2, op_l(i(x1), x1, x2))}$

$=$ by Lemma 440 RL with $\{x2 \leftarrow x2, x1 \leftarrow op_l(i(x1), x1, x2)\}$

$\overbrace{mult(i(op_l(i(x1), x1, x2)), \underbrace{op_l(x2, x2, op_l(i(x1), x1, x2))}}$

$=$ by Lemma 286 RL with $\{x3 \leftarrow op_l(i(x1), x1, x2), x2 \leftarrow x2, x1 \leftarrow x2\}$

$mult(i(op_l(i(x1), x1, x2)), \overbrace{op_l(x2, x2, \underbrace{rd(op_l(i(x1), x1, x2), x2)})})$

$=$ by Lemma 10 RL with $\{x2 \leftarrow op_l(i(x1), x1, x2), x1 \leftarrow x2\}$

$mult(i(op_l(i(x1), x1, x2)), op_l(x2, x2, \overbrace{i(\underbrace{rd(x2, op_l(i(x1), x1, x2))})}))$

$=$ by Lemma 231 RL with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$mult(i(op_l(i(x1), x1, x2)), \underbrace{op_l(x2, x2, i(\overbrace{mult(x2, x1)}))})$

$=$ by Lemma 245 LR with $\{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x2\}$

$mult(\underbrace{i(op_l(i(x1), x1, x2))}, \overbrace{op_l(x2, i(x1), i(x2))})$

$=$ by Lemma 436 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$mult(\overbrace{op_l(x1, x1, x2)}, \underbrace{op_l(x2, i(x1), i(x2))})$

$=$ by Lemma 438 LR with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$mult(op_l(x1, x1, x2), \underbrace{\overbrace{op_l(x2, x1, x2)}})$

$=$ by Lemma 608 RL with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$mult(op_l(x1, x1, x2), \underbrace{\overbrace{op_l(x2, x2, i(x1))}})$

$=$ by Lemma 1321 RL with $\{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x2\}$

$\underbrace{mult(op_l(x1, x1, x2), \overbrace{op_l(x2, x2, rd(x2, x1))})}$

$=$ by Lemma 1784 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_l(mult(x1, x2), x2, op_t(rd(x2, x1), x3))}$

Lemma 1786: $mult(op_l(x1, x3, x2), op_l(x3, x3, x2)) = op_l(mult(x1, op_l(op_l(x3, x3, x2), x2, x3)), i(x2), op_l(x3, x3, x2))$

$mult(\underbrace{op_l(x1, x3, x2)}, op_l(x3, x3, x2))$

$=$ by Lemma 1061 RL with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$mult(\overbrace{op_l(x1, x2, \underbrace{op_l(i(x3), x3, x2)})}, op_l(x3, x3, x2))$

$=$ by Lemma 1049 RL with $\{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x3\}$

$mult(\underbrace{op_l(x1, x2, \overbrace{i(op_l(x3, x3, x2))})}, op_l(x3, x3, x2))$

$=$ by Lemma 1050 RL with $\{x3 \leftarrow op_l(x3, x3, x2), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{mult(\overbrace{op_l(x1, i(x2), op_l(x3, x3, x2))}, op_l(x3, x3, x2))}$

$=$ by Lemma 1779 RL with $\{x3 \leftarrow i(x2), x2 \leftarrow op_l(x3, x3, x2), x1 \leftarrow x1\}$

$\overbrace{op_l(mult(x1, \underbrace{op_l(op_l(x3, x3, x2), op_l(x3, x3, x2), i(x2))}), i(x2), op_l(x3, x3, x2))}$

$=$ by Lemma 484 RL with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow op_l(x3, x3, x2)\}$

$op_l(mult(x1, \overbrace{op_l(op_l(x3, x3, x2), x2, x3)}), i(x2), op_l(x3, x3, x2))$

Lemma 1787: $mult(op_l(x1, x3, x2), op_l(x3, x3, x2)) = op_l(mult(x1, x3), x2, i(op_l(x3, x3, x2)))$

$\underbrace{mult(op_l(x1, x3, x2), op_l(x3, x3, x2))}$

$=$ by Lemma 1786 LR with $\{x2 \leftarrow x2, x3 \leftarrow x3, x1 \leftarrow x1\}$

$\underbrace{\overbrace{op_l(mult(x1, op_l(op_l(x3, x3, x2), x2, x3)), i(x2), op_l(x3, x3, x2))}}$

$=$ by Lemma 1050 LR with $\{x3 \leftarrow op_l(x3, x3, x2), x2 \leftarrow x2, x1 \leftarrow mult(x1, op_l(op_l(x3, x3, x2), x2, x3))\}$

$\overbrace{op_l(mult(x1, \underbrace{op_l(op_l(x3, x3, x2), x2, x3)}), x2, i(op_l(x3, x3, x2)))}$

$=$ by Lemma 138 LR with $\{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x3\}$

$op_l(mult(x1, \overbrace{x3}), x2, i(op_l(x3, x3, x2)))$

Lemma 1788: $mult(op_l(x1, x3, x2), op_l(x3, x3, x2)) = op_l(mult(x1, x3), x3, x2)$

$\underbrace{mult(op_l(x1, x3, x2), op_l(x3, x3, x2))}$

$=$ by Lemma 1787 LR with $\{x2 \leftarrow x2, x3 \leftarrow x3, x1 \leftarrow x1\}$

$\overbrace{op_l(mult(x1, x3), x2, \underbrace{i(op_l(x3, x3, x2))})}$

$=$ by Lemma 1049 LR with $\{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x3\}$

$\underbrace{op_l(mult(x1, x3), x2, \overbrace{op_l(i(x3), x3, x2)})}$

$=$ by Lemma 1061 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow mult(x1, x3)\}$

$\overbrace{op_l(mult(x1, x3), x3, x2)}$

Lemma 1789: $mult(op_l(x1, x3, x2), op_l(x2, x3, x2)) = op_l(mult(x1, x2), x3, x2)$

$mult(\underbrace{op_l(x1, x3, x2)}, op_l(x2, x3, x2))$

$=$ by Lemma 1083 RL with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{mult(\overbrace{op_l(x1, mult(x2, x3), op_l(x2, x3, x2))}, op_l(x2, x3, x2))}$

$=$ by Lemma 1779 RL with $\{x3 \leftarrow mult(x2, x3), x2 \leftarrow op_l(x2, x3, x2), x1 \leftarrow x1\}$

$\overbrace{op_l(mult(x1, \underbrace{op_l(op_l(x2, x3, x2), op_l(x2, x3, x2), mult(x2, x3))}), mult(x2, x3), op_l(x2, x3, x2))}$

$=$ by Lemma 1077 RL with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow op_l(x2, x3, x2)\}$

$\underbrace{op_l(mult(x1, \overbrace{op_l(op_l(x2, x3, x2), x2, x3)}), mult(x2, x3), op_l(x2, x3, x2))}$

$=$ by Lemma 1083 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow mult(x1, op_l(op_l(x2, x3, x2), x2, x3))\}$

$\overbrace{op_l(mult(x1, \underbrace{op_l(op_l(x2, x3, x2), x2, x3)}), x3, x2)}$

$=$ by Lemma 138 LR with $\{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x2\}$

$op_l(mult(x1, \overbrace{x2}), x3, x2)$

Lemma 1790: $op_l(x1, op_r(x2, x3, x3), mult(i(x2), x3)) = op_l(x1, x2, rd(rd(x3, x2), x2))$

$\underbrace{op_l(x1, op_r(x2, x3, x3), mult(i(x2), x3))}$

$=$ by Lemma 1358 LR with $\{x3 \leftarrow mult(i(x2), x3), x2 \leftarrow op_r(x2, x3, x3), x1 \leftarrow x1\}$

$\underbrace{\overbrace{op_l(x1, i(op_t(op_r(x2, x3, x3), op_l(mult(i(x2), x3), mult(i(x2), x3), op_r(x2, x3, x3)))), rd(op_r(x2, x3, x3), mult(i(x2), x3)))}}$

$=$ by Lemma 1050 LR with $\{x3 \leftarrow rd(op_r(x2, x3, x3), mult(i(x2), x3)), x2 \leftarrow op_t(op_r(x2, x3, x3), op_l(mult(i(x2), x3), mult(i(x2$

$\overbrace{op_l(x1, \underbrace{op_t(op_r(x2, x3, x3), op_l(mult(i(x2), x3), mult(i(x2), x3), op_r(x2, x3, x3)))}, i(rd(op_r(x2, x3, x3), mult(i(x2), x3))))}$

$=$ by Lemma 553 RL with $\{x2 \leftarrow mult(i(x2), x3), x1 \leftarrow op_r(x2, x3, x3)\}$

$op_l(x1, \overbrace{op_t(op_r(x2, x3, x3), mult(i(x2), x3))}, \underbrace{i(rd(op_r(x2, x3, x3), mult(i(x2), x3)))})$

$=$ by Lemma 10 LR with $\{x2 \leftarrow mult(i(x2), x3), x1 \leftarrow op_r(x2, x3, x3)\}$

$op_l(x1, op_t(op_r(x2, x3, x3), mult(i(x2), x3)), \overbrace{rd(\underbrace{mult(i(x2), x3)}, op_r(x2, x3, x3))})$

$=$ by Lemma 470 RL with $\{x2 \leftarrow x2, x1 \leftarrow x3\}$

$op_l(x1, op_t(op_r(x2, x3, x3), mult(i(x2), x3)), \underbrace{rd(\overbrace{op_t(rd(x3, x2), op_r(x2, x3, x3))}, op_r(x2, x3, x3))})$

$=$ by Lemma 748 RL with $\{x2 \leftarrow op_r(x2, x3, x3), x1 \leftarrow rd(x3, x2)\}$

$op_l(x1, op_t(op_r(x2, x3, x3), mult(i(x2), x3)), \overbrace{rd(rd(x3, x2), \underbrace{op_t(op_r(x2, x3, x3), rd(x3, x2))})})$

$=$ by Axiom 14 LR with $\{x4 \leftarrow rd(x3, x2), x3 \leftarrow x3, x2 \leftarrow x3, x1 \leftarrow x2\}$

$op_l(x1, op_t(op_r(x2, x3, x3), mult(i(x2), x3)), rd(rd(x3, x2), \underbrace{\overbrace{op_r(op_t(x2, rd(x3, x2)), x3, x3)}}))$

$=$ by Lemma 368 LR with $\{x2 \leftarrow x3, x1 \leftarrow x2\}$

$op_l(x1, \underbrace{op_t(op_r(x2, x3, x3), mult(i(x2), x3))}, rd(rd(x3, x2), \overbrace{x2}))$

$=$ by Axiom 14 LR with $\{x4 \leftarrow mult(i(x2), x3), x3 \leftarrow x3, x2 \leftarrow x3, x1 \leftarrow x2\}$

$op_l(x1, \overbrace{op_r(\underbrace{op_t(x2, mult(i(x2), x3))}, x3, x3)}, rd(rd(x3, x2), x2))$

$=$ by Lemma 375 LR with $\{x2 \leftarrow x3, x1 \leftarrow x2\}$

$op_l(x1, \underbrace{op_r(\overbrace{op_t(x2, rd(x3, x2))}, x3, x3)}, rd(rd(x3, x2), x2))$

$=$ by Lemma 368 LR with $\{x2 \leftarrow x3, x1 \leftarrow x2\}$

$op_l(x1, \overbrace{x2}, rd(rd(x3, x2), x2))$

Lemma 1791: $op_l(x1, op_r(x2, x3, x3), mult(i(x2), x3)) = op_l(x1, x2, x3)$

$\underbrace{op_l(x1, op_r(x2, x3, x3), mult(i(x2), x3))}$

$=$ by Lemma 1790 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{\overbrace{op_l(x1, x2, rd(rd(x3, x2), x2))}}$

$=$ by Lemma 286 LR with $\{x3 \leftarrow rd(x3, x2), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{\overbrace{op_l(x1, x2, rd(x3, x2))}}$

$=$ by Lemma 286 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_l(x1, x2, x3)}$

Lemma 1792: $op_l(x1, x2, x3) = op_l(x1, op_t(i(x2), rd(x2, mult(i(x2), x3))), mult(i(x3), x2))$

$$
\begin{aligned}
& op_l(x1, x2, \underbrace{x3}) \\
= \quad & \text{by Lemma 3 RL with } \{x1 \leftarrow x3\} \\
& \underbrace{op_l(x1, x2, \overbrace{i(i(x3))})} \\
= \quad & \text{by Lemma 1331 RL with } \{x3 \leftarrow i(x3), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, mult(x2, i(x3)), x2)}} \\
= \quad & \text{by Lemma 1135 RL with } \{x4 \leftarrow x2, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, mult(i(x2), x3), i(x2))}} \\
= \quad & \text{by Lemma 1317 RL with } \{x3 \leftarrow mult(i(x2), x3), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, rd(x2, mult(i(x2), x3)), x2)}} \\
= \quad & \text{by Lemma 1060 RL with } \{x4 \leftarrow x2, x3 \leftarrow x2, x2 \leftarrow mult(i(x2), x3), x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, rd(mult(i(x2), x3), x2), i(x2))}} \\
= \quad & \text{by Lemma 1060 RL with } \{x4 \leftarrow i(x2), x3 \leftarrow mult(i(x2), x3), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, rd(x2, mult(i(x2), x3)), i(i(x2)))}} \\
= \quad & \text{by Lemma 1319 RL with } \{x3 \leftarrow rd(x2, mult(i(x2), x3)), x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, op_t(i(x2), rd(x2, mult(i(x2), x3))), rd(x2, mult(i(x2), x3)))}} \\
= \quad & \text{by Lemma 246 RL with } \{x3 \leftarrow i(x2), x2 \leftarrow rd(x2, mult(i(x2), x3)), x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, i(rd(x2, mult(i(x2), x3))), mult(i(x2), rd(x2, mult(i(x2), x3))))}} \\
= \quad & \text{by Lemma 285 RL with } \{x3 \leftarrow mult(i(x2), rd(x2, mult(i(x2), x3))), x2 \leftarrow i(rd(x2, mult(i(x2), x3))), x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, \underbrace{mult(i(rd(x2, mult(i(x2), x3))), mult(i(x2), rd(x2, mult(i(x2), x3))))}, mult(i(x2), rd(x2, mult(i(x2), x3))))} \\
= \quad & \text{by Axiom 10 LR with } \{x2 \leftarrow i(x2), x1 \leftarrow rd(x2, mult(i(x2), x3))\} \\
& op_l(x1, \overbrace{op_t(i(x2), rd(x2, mult(i(x2), x3)))}, \underbrace{mult(i(x2), rd(x2, mult(i(x2), x3)))}) \\
= \quad & \text{by Lemma 217 LR with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& op_l(x1, op_t(i(x2), rd(x2, mult(i(x2), x3))), \overbrace{mult(i(x3), x2)})
\end{aligned}
$$

Lemma 1793: $op_l(x1, x2, x3) = op_l(x1, op_t(i(x2), x3), mult(i(x3), x2))$

$$
\begin{aligned}
& \underbrace{op_l(x1, x2, x3)} \\
= \quad & \text{by Lemma 1792 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, \underbrace{op_t(i(x2), rd(x2, mult(i(x2), x3)))}, mult(i(x3), x2))} \\
= \quad & \text{by Lemma 715 RL with } \{x2 \leftarrow mult(i(x2), x3), x1 \leftarrow x2\} \\
& op_l(x1, \overbrace{op_t(i(x2), mult(mult(i(x2), x3), x2))}, mult(i(x3), x2)) \\
= \quad & \text{by Lemma 971 RL with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& op_l(x1, \overbrace{op_t(i(x2), x3)}, mult(i(x3), x2))
\end{aligned}
$$

Lemma 1794: $op_l(x1, x2, x3) = op_l(x1, op_l(mult(x2, x3), x2, x3), op_t(x3, mult(x2, x3)))$

$\overbrace{op_l(x1, x2, x3)}$

$=$ by Lemma 285 RL with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_l(x1, \overbrace{mult(x2, x3)}, x3)}$

$=$ by Lemma 1359 RL with $\{x3 \leftarrow x3, x2 \leftarrow mult(x2, x3), x1 \leftarrow x1\}$

$\overbrace{op_l(x1, \underbrace{op_l(mult(x2, x3), \overbrace{mult(x2, x3)}, x3)}, op_t(x3, mult(x2, x3)))}$

$=$ by Lemma 285 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow mult(x2, x3)\}$

$op_l(x1, \overbrace{op_l(mult(x2, x3), x2, x3)}, op_t(x3, mult(x2, x3)))$

Lemma 1795: $op_l(x1, x2, op_r(rd(i(x3), x2), x3, x3)) = op_l(x1, x3, x2)$

$\overbrace{op_l(x1, x2, op_r(rd(i(x3), x2), x3, x3))}$

$=$ by Lemma 1360 LR with $\{x3 \leftarrow op_r(rd(i(x3), x2), x3, x3), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_l(x1, x2, op_t(mult(\overbrace{op_r(rd(i(x3), x2), x3, x3)}, \underbrace{x2}), i(op_r(rd(i(x3), x2), x3, x3))))}$

$=$ by Lemma 12 RL with $\{x2 \leftarrow x2, x1 \leftarrow x3\}$

$op_l(x1, x2, op_t(mult(\underbrace{op_r(rd(i(x3), x2), x3, x3)}, \overbrace{mult(x3, mult(i(x3), x2))}), i(op_r(rd(i(x3), x2), x3, x3))))$

$=$ by Lemma 361 RL with $\{x2 \leftarrow x3, x1 \leftarrow rd(i(x3), x2)\}$

$op_l(x1, x2, op_t(\underbrace{mult(\overbrace{op_r(rd(i(x3), x2), i(x3), i(x3))}, mult(x3, mult(i(x3), x2)))}, i(op_r(rd(i(x3), x2), x3, x3))))$

$=$ by Lemma 1187 RL with $\{x2 \leftarrow x2, x1 \leftarrow x3\}$

$op_l(x1, x2, \underbrace{op_t(\overbrace{op_l(i(x3), x2, x3)}, i(op_r(rd(i(x3), x2), x3, x3))))}$

$=$ by Axiom 15 LR with $\{x4 \leftarrow i(op_r(rd(i(x3), x2), x3, x3)), x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow i(x3)\}$

$op_l(x1, x2, \overbrace{op_l(\underbrace{op_t(i(x3), i(op_r(rd(i(x3), x2), x3, x3)))}, x2, x3)})$

$=$ by Lemma 15 LR with $\{x2 \leftarrow op_r(rd(i(x3), x2), x3, x3), x1 \leftarrow x3\}$

$op_l(x1, x2, op_l(\overbrace{i(\underbrace{op_t(x3, op_r(rd(i(x3), x2), x3, x3)))}}, x2, x3))$

$=$ by Lemma 550 LR with $\{x2 \leftarrow rd(i(x3), x2), x1 \leftarrow x3\}$

$op_l(x1, x2, op_l(i(\overbrace{op_t(x3, rd(i(x3), x2))}), x2, x3))$

$=$ by Lemma 707 LR with $\{x2 \leftarrow x2, x1 \leftarrow x3\}$

$op_l(x1, x2, op_l(\underbrace{i(\overbrace{op_t(x3, rd(x2, x3)))}}, x2, x3))$

$=$ by Lemma 47 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x3\}$

$op_l(x1, x2, \underbrace{op_l(\overbrace{op_t(i(x3), rd(x3, x2))}, x2, x3)})$

$=$ by Lemma 966 LR with $\{x2 \leftarrow x2, x1 \leftarrow x3\}$

$\underbrace{op_l(x1, x2, \overbrace{i(op_t(x3, x2))})}$

$=$ by Lemma 1329 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_l(x1, x3, x2)}$

Lemma 1796: $op_l(x1, op_r(rd(i(x2), x3), x2, x2), x3) = op_l(x1, op_l(i(op_t(x2, op_r(rd(i(x2), x3), x2, x2))), x3, x2), x3)$

$\overbrace{op_l(x1, op_r(rd(i(x2), x3), x2, x2), x3)}$

= by Lemma 285 RL with $\{x3 \leftarrow x3, x2 \leftarrow op_r(rd(i(x2), x3), x2, x2), x1 \leftarrow x1\}$

$\overbrace{op_l(x1, \underbrace{mult(op_r(rd(i(x2), x3), x2, x2), x3)}, x3)}$

= by Lemma 285 RL with $\{x3 \leftarrow x3, x2 \leftarrow mult(op_r(rd(i(x2), x3), x2, x2), x3), x1 \leftarrow x1\}$

$\overbrace{op_l(x1, \underbrace{mult(mult(op_r(rd(i(x2), x3), x2, x2), x3), x3)}, x3)}$

= by Lemma 386 RL with $\{x2 \leftarrow mult(op_r(rd(i(x2), x3), x2, x2), x3), x1 \leftarrow x3\}$

$op_l(x1, \overbrace{rd(x3, op_t(i(\underbrace{mult(op_r(rd(i(x2), x3), x2, x2), x3)}), \underbrace{rd(mult(op_r(rd(i(x2), x3), x2, x2), x3), x3)}))}, x3)$

= by Axiom 5 RL with $\{x2 \leftarrow x3, x1 \leftarrow op_r(rd(i(x2), x3), x2, x2)\}$

$\underbrace{op_l(x1, rd(x3, op_t(i(\underbrace{mult(op_r(rd(i(x2), x3), x2, x2), x3)}), \overbrace{op_r(rd(i(x2), x3), x2, x2)})), x3)}$

= by Lemma 1317 LR with $\{x3 \leftarrow op_t(i(mult(op_r(rd(i(x2), x3), x2, x2), x3)), op_r(rd(i(x2), x3), x2, x2)), x2 \leftarrow x3, x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_l(x1, op_t(i(\underbrace{mult(op_r(rd(i(x2), x3), x2, x2), x3)}), \underbrace{op_r(rd(i(x2), x3), x2, x2)}), \underbrace{i(x3)})}}$

= by Lemma 1136 LR with $\{x4 \leftarrow x3, x3 \leftarrow op_r(rd(i(x2), x3), x2, x2), x2 \leftarrow mult(op_r(rd(i(x2), x3), x2, x2), x3), x1 \leftarrow x1\}$

$\overbrace{op_l(x1, op_t(mult(op_r(rd(i(x2), x3), x2, x2), \underbrace{x3}), i(op_r(rd(i(x2), x3), x2, x2))), x3)}$

= by Lemma 12 RL with $\{x2 \leftarrow x3, x1 \leftarrow x2\}$

$op_l(x1, op_t(mult(\underbrace{op_r(rd(i(x2), x3), x2, x2)}, \overbrace{mult(x2, mult(i(x2), x3))}), i(op_r(rd(i(x2), x3), x2, x2))), x3)$

= by Lemma 361 RL with $\{x2 \leftarrow x2, x1 \leftarrow rd(i(x2), x3)\}$

$op_l(x1, op_t(\underbrace{mult(\overbrace{op_r(rd(i(x2), x3), i(x2), i(x2))}, mult(x2, mult(i(x2), x3)))}, i(op_r(rd(i(x2), x3), x2, x2))), x3)$

= by Lemma 1187 RL with $\{x2 \leftarrow x3, x1 \leftarrow x2\}$

$op_l(x1, \underbrace{op_t(\overbrace{op_l(i(x2), x3, x2)}, i(op_r(rd(i(x2), x3), x2, x2)))}, x3)$

= by Axiom 15 LR with $\{x4 \leftarrow i(op_r(rd(i(x2), x3), x2, x2)), x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow i(x2)\}$

$op_l(x1, \overbrace{op_l(\underbrace{op_t(i(x2), i(op_r(rd(i(x2), x3), x2, x2)))}, x3, x2)}, x3)$

= by Lemma 15 LR with $\{x2 \leftarrow op_r(rd(i(x2), x3), x2, x2), x1 \leftarrow x2\}$

$op_l(x1, op_l(\overbrace{i(op_t(x2, op_r(rd(i(x2), x3), x2, x2)))}, x3, x2), x3)$

Lemma 1797: $op_l(x1, op_r(rd(i(x2), x3), x2, x2), x3) = op_l(x1, op_l(i(op_t(x2, rd(x3, x2))), x3, x2), x3)$

$\underbrace{op_l(x1, op_r(rd(i(x2), x3), x2, x2), x3)}$

= by Lemma 1796 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_l(x1, op_l(i(\underbrace{op_t(x2, op_r(rd(i(x2), x3), x2, x2))}), x3, x2), x3)}$

= by Lemma 550 LR with $\{x2 \leftarrow rd(i(x2), x3), x1 \leftarrow x2\}$

$op_l(x1, op_l(i(\overbrace{op_t(x2, rd(i(x2), x3))}), x3, x2), x3)$

= by Lemma 707 LR with $\{x2 \leftarrow x3, x1 \leftarrow x2\}$

$op_l(x1, op_l(i(\overbrace{op_t(x2, rd(x3, x2))}), x3, x2), x3)$

Lemma 1798: $op_l(x1, op_r(rd(i(x2), x3), x2, x2), x3) = op_l(x1, i(op_t(x2, x3)), x3)$

$$
\begin{aligned}
& \underbrace{op_l(x1, op_r(rd(i(x2), x3), x2, x2), x3)} \\
= & \quad \text{by Lemma 1797 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, op_l(\underbrace{i(op_t(x2, rd(x3, x2)))}, x3, x2), x3)} \\
= & \quad \text{by Lemma 47 LR with } \{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x2\} \\
& op_l(x1, \underbrace{op_l(\overbrace{op_t(i(x2), rd(x2, x3))}, x3, x2)}, x3) \\
= & \quad \text{by Lemma 966 LR with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& op_l(x1, \overbrace{i(op_t(x2, x3))}, x3)
\end{aligned}
$$

Lemma 1799: $op_l(x1, op_r(rd(i(x2), x3), x2, x2), x3) = op_l(x1, x3, x2)$

$$
\begin{aligned}
& \underbrace{op_l(x1, op_r(rd(i(x2), x3), x2, x2), x3)} \\
= & \quad \text{by Lemma 1798 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, i(op_t(x2, x3)), x3)}} \\
= & \quad \text{by Lemma 1050 LR with } \{x3 \leftarrow x3, x2 \leftarrow op_t(x2, x3), x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, op_t(x2, x3), i(x3))}} \\
= & \quad \text{by Lemma 1332 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x3, x2)}
\end{aligned}
$$

Lemma 1800: $op_l(x1, op_t(x2, rd(x2, rd(x3, i(x3)))), x3) = op_l(x1, rd(rd(i(x3), x3), i(op_t(x2, rd(i(x3), x3)))), x3)$

$$
\begin{aligned}
& op_l(x1, \underbrace{op_t(x2, rd(x2, rd(x3, i(x3))))}, x3) \\
= & \quad \text{by Lemma 738 RL with } \{x3 \leftarrow x3, x2 \leftarrow i(x3), x1 \leftarrow x2\} \\
& \underbrace{op_l(x1, \overbrace{op_t(x2, mult(x2, rd(i(x3), x3)))}, x3)} \\
= & \quad \text{by Lemma 1373 RL with } \{x3 \leftarrow op_t(x2, mult(x2, rd(i(x3), x3))), x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, \underbrace{mult(rd(i(x3), x3), op_t(x2, mult(x2, rd(i(x3), x3))))}, x3)} \\
= & \quad \text{by Lemma 461 LR with } \{x2 \leftarrow x2, x1 \leftarrow rd(i(x3), x3)\} \\
& op_l(x1, \overbrace{rd(rd(i(x3), x3), i(op_t(x2, rd(i(x3), x3))))}, x3)
\end{aligned}
$$

Lemma 1801: $op_l(x1, op_t(x2, rd(x2, mult(x3, x3))), x3) = op_l(x1, rd(op_t(x2, rd(i(x3), x3)), rd(x3, i(x3))), x3)$

$$
\begin{aligned}
& op_l(x1, op_t(x2, rd(x2, \underbrace{mult(x3, x3)})), x3) \\
= & \quad \text{by Lemma 42 RL with } \{x1 \leftarrow x3\} \\
& \underbrace{op_l(x1, op_t(x2, rd(x2, \overbrace{rd(x3, i(x3))})), x3)} \\
= & \quad \text{by Lemma 1800 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, \underbrace{rd(rd(i(x3), x3), i(op_t(x2, rd(i(x3), x3))))}, x3)} \\
= & \quad \text{by Lemma 39 LR with } \{x3 \leftarrow op_t(x2, rd(i(x3), x3)), x2 \leftarrow x3, x1 \leftarrow i(x3)\} \\
& op_l(x1, \overbrace{rd(op_t(x2, rd(i(x3), x3)), rd(x3, i(x3)))}, x3)
\end{aligned}
$$

Lemma 1802: $op_l(x1, op_t(x2, rd(x2, mult(x3, x3))), x3) = op_l(x1, op_t(x2, rd(i(x3), x3)), x3)$

$$
\begin{aligned}
& \underbrace{op_l(x1, op_t(x2, rd(x2, mult(x3, x3))), x3)} \\
= & \quad \text{by Lemma 1801 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, rd(op_t(x2, rd(i(x3), x3)), \underbrace{rd(x3, i(x3))}), x3)} \\
= & \quad \text{by Lemma 42 LR with } \{x1 \leftarrow x3\} \\
& \underbrace{op_l(x1, rd(op_t(x2, rd(i(x3), x3)), \overbrace{mult(x3, x3)}), x3)} \\
= & \quad \text{by Lemma 1349 RL with } \{x2 \leftarrow x3, x3 \leftarrow op_t(x2, rd(i(x3), x3)), x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, op_t(x2, \overbrace{rd(i(x3), x3)}), x3)}
\end{aligned}
$$

Lemma 1803: $op_l(x1, x2, x1) = rd(mult(x1, x3), mult(asoc(x1, x2, x1), x3))$

$$
\begin{aligned}
& \underbrace{op_l(x1, x2, x1)} \\
= & \quad \text{by Lemma 620 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{rd(x1, asoc(x1, x2, x1))}} \\
= & \quad \text{by Lemma 1472 RL with } \{x3 \leftarrow asoc(x1, x2, x1), x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{rd(mult(x1, \underbrace{op_l(x3, asoc(x1, x2, x1), x1)}), mult(asoc(x1, x2, x1), x3))} \\
= & \quad \text{by Lemma 799 RL with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x3\} \\
& rd(mult(x1, \overbrace{op_l(x3, x1, asoc(x2, x1, x1))}), mult(asoc(x1, x2, x1), x3)) \\
= & \quad \text{by Lemma 947 LR with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x3\} \\
& rd(mult(x1, \overbrace{x3}), mult(asoc(x1, x2, x1), x3))
\end{aligned}
$$

Lemma 1804: $rd(mult(x1, x2), rd(x2, asoc(x3, x1, x1))) = op_l(x1, x3, x1)$

$$
\begin{aligned}
& rd(mult(x1, x2), \underbrace{rd(x2, asoc(x3, x1, x1))}) \\
= & \quad \text{by Lemma 1714 RL with } \{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \underbrace{rd(mult(x1, x2), \overbrace{mult(asoc(x1, x3, x1), x2)})} \\
= & \quad \text{by Lemma 1803 RL with } \{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x3, x1)}
\end{aligned}
$$

Lemma 1805: $op_l(i(i(x1)), i(x1), x2) = op_t(op_r(x1, x2, rd(x2, x1)), rd(i(x1), x2))$

$$
\begin{aligned}
& \underbrace{op_l(i(i(x1)), i(x1), x2)} \\
= & \quad \text{by Lemma 827 LR with } \{x2 \leftarrow x2, x1 \leftarrow i(x1)\} \\
& \overbrace{\underbrace{rd(rd(x2, i(x1)), op_t(x2, i(i(x1))))}} \\
= & \quad \text{by Lemma 1206 RL with } \{x2 \leftarrow i(x1), x1 \leftarrow x2\} \\
& \overbrace{op_t(\underbrace{rd(x2, mult(x2, i(x1)))}, rd(i(x1), x2))} \\
= & \quad \text{by Lemma 1595 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_t(\overbrace{op_r(x1, x2, rd(x2, x1))}, rd(i(x1), x2))
\end{aligned}
$$

Lemma 1806: $op_l(x1, x1, i(x2)) = op_t(op_r(x1, x2, rd(x2, x1)), rd(i(x1), x2))$

$op_l(\underbrace{x1}, x1, i(x2))$

$=$ by Lemma 3 RL with $\{x1 \leftarrow x1\}$

$\underbrace{op_l(\overbrace{i(i(x1))}, x1, i(x2))}$

$=$ by Lemma 1050 RL with $\{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow i(i(x1))\}$

$\underbrace{\overbrace{op_l(i(i(x1)), i(x1), x2)}}$

$=$ by Lemma 1805 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_t(op_r(x1, x2, rd(x2, x1)), rd(i(x1), x2))}$

Lemma 1807: $asoc(x1, x2, x1) = rd(rd(op_r(x1, x3, x3), x2), mult(i(x2), op_r(x1, x3, x3)))$

$\overbrace{asoc(x1, x2, x1)}$

$=$ by Lemma 262 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{asoc(x1, mult(x2, x1), x1)}$

$=$ by Lemma 617 RL with $\{x2 \leftarrow mult(x2, x1), x1 \leftarrow x1\}$

$\overbrace{rd(x1, op_l(x1, mult(x2, x1), x1))}$

$=$ by Lemma 176 LR with $\{x3 \leftarrow x1, x2 \leftarrow mult(x2, x1), x1 \leftarrow x1\}$

$\overbrace{mult(x1, i(\overbrace{op_l(x1, mult(x2, x1), x1)}))}$

$=$ by Lemma 1039 RL with $\{x3 \leftarrow x3, x2 \leftarrow mult(x2, x1), x1 \leftarrow x1\}$

$\overbrace{mult(x1, i(\overbrace{op_l(x1, rd(mult(x2, x1), op_r(x1, x3, x3)), x1)}))}$

$=$ by Lemma 176 RL with $\{x3 \leftarrow x1, x2 \leftarrow rd(mult(x2, x1), op_r(x1, x3, x3)), x1 \leftarrow x1\}$

$\overbrace{rd(x1, op_l(x1, rd(mult(x2, x1), op_r(x1, x3, x3)), x1))}$

$=$ by Lemma 617 LR with $\{x2 \leftarrow rd(mult(x2, x1), op_r(x1, x3, x3)), x1 \leftarrow x1\}$

$\overbrace{asoc(x1, \underbrace{rd(mult(x2, x1), op_r(x1, x3, x3))}, x1)}$

$=$ by Lemma 1242 LR with $\{x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow x2\}$

$\underbrace{asoc(x1, \overbrace{rd(x2, asoc(x1, x3, x3))}, x1)}$

$=$ by Lemma 704 RL with $\{x2 \leftarrow rd(x2, asoc(x1, x3, x3)), x1 \leftarrow x1\}$

$\overbrace{rd(\underbrace{rd(x1, rd(x2, asoc(x1, x3, x3)))}, mult(i(rd(x2, asoc(x1, x3, x3))), x1))}$

$=$ by Lemma 1653 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$rd(\overbrace{rd(op_r(x1, x3, x3), x2)}, mult(\underbrace{i(rd(x2, asoc(x1, x3, x3)))}, x1))$

$=$ by Lemma 10 LR with $\{x2 \leftarrow asoc(x1, x3, x3), x1 \leftarrow x2\}$

$rd(rd(op_r(x1, x3, x3), x2), mult(\overbrace{rd(\underbrace{asoc(x1, x3, x3)}, x2)}, x1))$

$=$ by Lemma 756 RL with $\{x2 \leftarrow x1, x1 \leftarrow x3\}$

$rd(rd(op_r(x1, x3, x3), x2), mult(\underbrace{rd(\overbrace{asoc(x3, i(x1), x3)}, x2)}, x1))$

$=$ by Lemma 10 RL with $\{x2 \leftarrow asoc(x3, i(x1), x3), x1 \leftarrow x2\}$

$rd(rd(op_r(x1, x3, x3), x2), \underbrace{mult(\overbrace{i(rd(x2, asoc(x3, i(x1), x3)))}, x1)})$

$=$ by Lemma 17 RL with $\{x2 \leftarrow x1, x1 \leftarrow rd(x2, asoc(x3, i(x1), x3))\}$

$rd(rd(op_r(x1, x3, x3), x2), \overbrace{i(mult(\underbrace{rd(x2, asoc(x3, i(x1), x3))}, i(x1)))})$

$=$ by Lemma 1300 LR with $\{x2 \leftarrow i(x1), x3 \leftarrow x3, x1 \leftarrow x2\}$

$rd(rd(op_r(x1, x3, x3), x2), \underbrace{i(mult(\overbrace{mult(x2, asoc(i(x1), x3, x3))}, i(x1)))})$

$=$ by Lemma 91 RL with $\{x3 \leftarrow i(x1), x2 \leftarrow asoc(i(x1), x3, x3), x1 \leftarrow x2\}$

$rd(rd(op_r(x1, x3, x3), x2), \overbrace{i(mult(\underbrace{op_r(x2, asoc(i(x1), x3, x3), i(x1))}, mult(asoc(i(x1), x3, x3), i(x1))))})$

$=$ by Lemma 1231 RL with $\{x3 \leftarrow x3, x2 \leftarrow i(x1), x1 \leftarrow x2\}$

$rd(rd(op_r(x1, x3, x3), x2), i(mult(\overbrace{x2}, \underbrace{mult(asoc(i(x1), x3, x3), i(x1))})))$

$=$ by Lemma 767 LR with $\{x1 \leftarrow x3, x2 \leftarrow i(x1)\}$

$rd(rd(op_r(x1, x3, x3), x2), \underbrace{i(mult(x2, \overbrace{op_r(i(x1), x3, x3)}))})$

$=$ by Lemma 1100 RL with $\{x2 \leftarrow x3, x3 \leftarrow x1, x1 \leftarrow x2\}$

$rd(rd(op_r(x1, x3, x3), x2), \overbrace{mult(i(x2), op_r(x1, x3, x3))})$

Lemma 1808: $asoc(op_r(x1,x2,x2),x3,op_r(x1,x2,x2)) = asoc(x1,x3,x1)$

$\underbrace{asoc(op_r(x1,x2,x2),x3,op_r(x1,x2,x2))}$

$=$ by Lemma 704 RL with $\{x2 \leftarrow x3, x1 \leftarrow op_r(x1,x2,x2)\}$

$\overbrace{rd(rd(op_r(x1,x2,x2),x3),mult(i(x3),op_r(x1,x2,x2)))}$

$=$ by Lemma 1807 RL with $\{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x1\}$

$\overbrace{asoc(x1,x3,x1)}$

Lemma 1809: $x1 = op_t(x1,mult(mult(i(x2),rd(x1,x3)),mult(x2,x3)))$

$\underbrace{x1}$

$=$ by Lemma 442 RL with $\{x2 \leftarrow x1, x1 \leftarrow mult(i(rd(rd(x1,x3),i(x2))),mult(rd(rd(x1,x3),i(x2)),x3))\}$

$\overbrace{op_l(mult(mult(i(rd(rd(x1,x3),i(x2))),mult(rd(rd(x1,x3),i(x2)),x3)),rd(x1,mult(i(rd(rd(x1,x3),i(x2))),mult(rd(rd(x1,x3),i(x2)),x3))),x1)}$

$=$ by Lemma 169 RL with $\{x3 \leftarrow mult(rd(rd(x1,x3),i(x2)),x3), x2 \leftarrow i(rd(rd(x1,x3),i(x2))), x1 \leftarrow mult(mult(i(rd(rd(x1,x3),i(x2))),mult(rd(rd(x1,x3),i(x2)),x3)),rd(x1,mult(i(rd(rd(x1,x3),i(x2))),mult(rd(rd(x1,x3),i(x2)),x3))),x1)\}$

$\underbrace{op_l(\overbrace{op_t(mult(mult(i(rd(rd(x1,x3),i(x2))),mult(rd(rd(x1,x3),i(x2)),x3)),rd(x1,mult(i(rd(rd(x1,x3),i(x2))),mult(rd(rd(x1,x3),i(x2)),x3))),x1)}, rd(x1,mult(i(rd(rd(x1,x3),i(x2))),mult(rd(rd(x1,x3),i(x2)),x3))),x1)}$

$=$ by Axiom 15 RL with $\{x4 \leftarrow op_l(mult(mult(i(rd(rd(x1,x3),i(x2))),mult(rd(rd(x1,x3),i(x2)),x3)),rd(x1,mult(i(rd(rd(x1,x3),i(x2))),mult(rd(rd(x1,x3),i(x2)),x3))),x1)\}$

$\overbrace{op_t(\underbrace{op_l(mult(mult(i(rd(rd(x1,x3),i(x2))),mult(rd(rd(x1,x3),i(x2)),x3)),rd(x1,mult(i(rd(rd(x1,x3),i(x2))),mult(rd(rd(x1,x3),i(x2)),x3))),x1)}, rd(x1,mult(i(rd(rd(x1,x3),i(x2))),mult(rd(rd(x1,x3),i(x2)),x3))),x1)}$

$=$ by Lemma 442 LR with $\{x2 \leftarrow x1, x1 \leftarrow mult(i(rd(rd(x1,x3),i(x2))),mult(rd(rd(x1,x3),i(x2)),x3))\}$

$op_t(\overbrace{x1}, op_l(mult(mult(i(rd(rd(x1,x3),i(x2))),mult(rd(rd(x1,x3),i(x2)),x3)),rd(x1,mult(i(rd(rd(x1,x3),i(x2))),mult$

$=$ by Lemma 1068 LR with $\{x3 \leftarrow rd(x1,mult(i(rd(rd(x1,x3),i(x2))),mult(rd(rd(x1,x3),i(x2)),x3))), x2 \leftarrow mult(rd(rd(x1,x$

$op_t(x1,\overbrace{mult(mult(rd(rd(x1,x3),i(x2)),x3),mult(i(rd(rd(x1,x3),i(x2))),rd(x1,\underbrace{mult(i(rd(rd(x1,x3),i(x2))),mult(rd(rd$

$=$ by Axiom 2 RL with $\{x2 \leftarrow rd(rd(x1,x3),i(x2)), x1 \leftarrow x3\}$

$op_t(x1,\underbrace{mult(mult(rd(rd(x1,x3),i(x2)),x3),mult(i(rd(rd(x1,x3),i(x2))),rd(x1,\overbrace{x3}))))}$

$=$ by Lemma 1086 LR with $\{x3 \leftarrow rd(x1,x3), x2 \leftarrow x3, x1 \leftarrow rd(rd(x1,x3),i(x2))\}$

$op_t(x1,\overbrace{mult(rd(rd(x1,x3),i(x2)),mult(rd(x3,rd(rd(x1,x3),i(x2))),\underbrace{rd(x1,x3)}))})$

$=$ by Axiom 4 LR with $\{x2 \leftarrow i(x2), x1 \leftarrow rd(x1,x3)\}$

$op_t(x1,\underbrace{mult(rd(rd(x1,x3),i(x2)),mult(rd(x3,rd(rd(x1,x3),i(x2))),\overbrace{mult(rd(rd(x1,x3),i(x2)),i(x2))})))}$

$=$ by Lemma 54 LR with $\{x3 \leftarrow i(x2), x2 \leftarrow x3, x1 \leftarrow rd(rd(x1,x3),i(x2))\}$

$\underbrace{op_t(x1,\overbrace{mult(mult(rd(rd(x1,x3),i(x2)),x3),i(x2))})}$

$=$ by Lemma 1694 LR with $\{x3 \leftarrow i(x2), x2 \leftarrow mult(rd(rd(x1,x3),i(x2)),x3), x1 \leftarrow x1\}$

$\overbrace{op_t(x1,mult(i(x2),mult(rd(rd(x1,x3),i(x2)),\underbrace{x3})))}$

$=$ by Axiom 2 LR with $\{x2 \leftarrow x2, x1 \leftarrow x3\}$

$op_t(x1,\underbrace{mult(i(x2),mult(rd(rd(x1,x3),i(x2)),\overbrace{mult(i(x2),mult(x2,x3))}))})$

$=$ by Lemma 54 LR with $\{x3 \leftarrow mult(x2,x3), x2 \leftarrow rd(x1,x3), x1 \leftarrow i(x2)\}$

$op_t(x1,\overbrace{mult(mult(i(x2),rd(x1,x3)),mult(x2,x3))})$

Lemma 1810: $x1 = op_t(x1, rd(mult(x2, x3), mult(x2, rd(x3, x1))))$

$$\overbrace{x1}$$

$=$ by Lemma 1809 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$$\overbrace{op_t(x1, mult(\overbrace{mult(i(x2), rd(x1, x3))}, mult(x2, x3)))}$$

$=$ by Lemma 1773 LR with $\{x4 \leftarrow mult(x2, x3), x3 \leftarrow rd(x1, x3), x2 \leftarrow x2, x1 \leftarrow x1\}$

$$\overbrace{op_t(x1, rd(mult(x2, x3), mult(x2, \underbrace{i(rd(x1, x3))})))}$$

$=$ by Lemma 10 LR with $\{x2 \leftarrow x3, x1 \leftarrow x1\}$

$$op_t(x1, rd(mult(x2, x3), mult(x2, \overbrace{rd(x3, x1)})))$$

Lemma 1811: $asoc(x3, x2, x3) = rd(rd(x1, op_r(x2, x3, x3)), op_t(rd(x1, x2), i(rd(x1, op_r(x2, x3, x3)))))$

$\overbrace{asoc(x3, x2, x3)}$

$=$ by Lemma 759 RL with $\{x2 \leftarrow x3, x1 \leftarrow x2\}$

$\overbrace{i(\underbrace{asoc(x2, x3, x3)})}$

$=$ by Axiom 4 LR with $\{x2 \leftarrow rd(x1, op_r(x2, x3, x3)), x1 \leftarrow asoc(x2, x3, x3)\}$

$i(\overbrace{mult(rd(asoc(x2, x3, x3), rd(x1, op_r(x2, x3, x3))), rd(x1, op_r(x2, x3, x3)))})$

$=$ by Lemma 1310 RL with $\{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow rd(x1, op_r(x2, x3, x3))\}$

$i(\overbrace{mult(i(rd(x1, op_r(x2, x3, x3))), \underbrace{rd(rd(x1, op_r(x2, x3, x3)), asoc(x3, x2, x3))})})$

$=$ by Lemma 1708 RL with $\{x3 \leftarrow rd(x1, op_r(x2, x3, x3)), x2 \leftarrow x3, x1 \leftarrow x2\}$

$i(\underbrace{mult(i(rd(x1, op_r(x2, x3, x3))), \overbrace{mult(asoc(x2, x3, x3), rd(x1, op_r(x2, x3, x3)))})})$

$=$ by Axiom 10 LR with $\{x2 \leftarrow asoc(x2, x3, x3), x1 \leftarrow rd(x1, op_r(x2, x3, x3))\}$

$\underbrace{i(\overbrace{op_t(asoc(x2, x3, x3), rd(x1, op_r(x2, x3, x3)))})}$

$=$ by Lemma 1309 LR with $\{x3 \leftarrow rd(x1, op_r(x2, x3, x3)), x2 \leftarrow x3, x1 \leftarrow x2\}$

$\overbrace{op_t(\underbrace{asoc(x3, x2, x3)}, rd(x1, op_r(x2, x3, x3)))}$

$=$ by Lemma 1626 RL with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$op_t(\overbrace{mult(rd(x1, op_r(x2, x3, x3)), \underbrace{rd(x2, x1)})}, rd(x1, op_r(x2, x3, x3)))$

$=$ by Lemma 10 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{op_t(mult(rd(x1, op_r(x2, x3, x3)), \overbrace{i(rd(x1, x2))}), rd(x1, op_r(x2, x3, x3)))}$

$=$ by Lemma 194 RL with $\{x3 \leftarrow rd(x1, op_r(x2, x3, x3)), x2 \leftarrow rd(x1, x2), x1 \leftarrow rd(x1, op_r(x2, x3, x3))\}$

$\overbrace{\underbrace{op_t(op_t(rd(rd(x1, op_r(x2, x3, x3)), rd(x1, x2)), rd(x1, op_r(x2, x3, x3))), i(rd(x1, op_r(x2, x3, x3))))}}$

$=$ by Lemma 903 RL with $\{x2 \leftarrow rd(x1, x2), x1 \leftarrow rd(x1, op_r(x2, x3, x3))\}$

$\overbrace{rd(rd(x1, op_r(x2, x3, x3)), \underbrace{op_t(op_t(rd(x1, x2), i(rd(x1, op_r(x2, x3, x3)))), rd(x1, op_r(x2, x3, x3)))})}$

$=$ by Axiom 13 LR with $\{x3 \leftarrow rd(x1, op_r(x2, x3, x3)), x2 \leftarrow i(rd(x1, op_r(x2, x3, x3))), x1 \leftarrow rd(x1, x2)\}$

$rd(rd(x1, op_r(x2, x3, x3)), \overbrace{op_t(op_t(rd(x1, x2), \underbrace{rd(x1, op_r(x2, x3, x3))}), i(rd(x1, op_r(x2, x3, x3))))})$

$=$ by Lemma 1243 RL with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$rd(rd(x1, op_r(x2, x3, x3)), op_t(\underbrace{op_t(rd(x1, x2), \overbrace{rd(rd(x1, x2), asoc(x2, x3, x3))})}, i(rd(x1, op_r(x2, x3, x3)))))$

$=$ by Lemma 1713 LR with $\{x2 \leftarrow x3, x3 \leftarrow x2, x1 \leftarrow rd(x1, x2)\}$

$rd(rd(x1, op_r(x2, x3, x3)), op_t(\overbrace{rd(\underbrace{mult(asoc(x2, x3, x3), rd(x1, x2))}, asoc(x2, x3, x3))}, i(rd(x1, op_r(x2, x3, x3)))))$

$=$ by Lemma 1708 LR with $\{x3 \leftarrow rd(x1, x2), x2 \leftarrow x3, x1 \leftarrow x2\}$

$rd(rd(x1, op_r(x2, x3, x3)), op_t(\underbrace{rd(\overbrace{rd(rd(x1, x2), asoc(x3, x2, x3))}, asoc(x2, x3, x3))}, i(rd(x1, op_r(x2, x3, x3)))))$

$=$ by Lemma 1295 RL with $\{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow rd(rd(x1, x2), asoc(x3, x2, x3))\}$

$rd(rd(x1, op_r(x2, x3, x3)), op_t(\overbrace{mult(rd(rd(x1, x2), asoc(x3, x2, x3)), asoc(x3, x2, x3))}, i(rd(x1, op_r(x2, x3, x3)))))$

$=$ by Axiom 4 RL with $\{x2 \leftarrow asoc(x3, x2, x3), x1 \leftarrow rd(x1, x2)\}$

$rd(rd(x1, op_r(x2, x3, x3)), op_t(\overbrace{rd(x1, x2)}, i(rd(x1, op_r(x2, x3, x3)))))$

Lemma 1812: $asoc(x3, x2, x3) = rd(op_t(rd(x2, x1), rd(x1, op_r(x2, x3, x3))), rd(op_r(x2, x3, x3), x1))$

$\overbrace{asoc(x3, x2, x3)}$

$=$ by Lemma 1811 LR with $\{x1 \leftarrow x1, x2 \leftarrow x2, x3 \leftarrow x3\}$

$\overbrace{rd(rd(x1, op_r(x2, x3, x3)), \underbrace{op_t(rd(x1, x2), i(rd(x1, op_r(x2, x3, x3))))}_{})}$

$=$ by Lemma 46 RL with $\{x3 \leftarrow rd(x1, op_r(x2, x3, x3)), x2 \leftarrow x1, x1 \leftarrow x2\}$

$\underbrace{rd(rd(x1, op_r(x2, x3, x3)), \overbrace{i(op_t(rd(x2, x1), rd(x1, op_r(x2, x3, x3))))}^{})}$

$=$ by Lemma 39 LR with $\{x3 \leftarrow op_t(rd(x2, x1), rd(x1, op_r(x2, x3, x3))), x2 \leftarrow op_r(x2, x3, x3), x1 \leftarrow x1\}$

$\overbrace{rd(op_t(rd(x2, x1), rd(x1, op_r(x2, x3, x3))), rd(op_r(x2, x3, x3), x1))}$

Lemma 1813: $asoc(x3, x2, x3) = rd(rd(x2, x1), rd(op_r(x2, x3, x3), x1))$

$\overbrace{asoc(x3, x2, x3)}$

$=$ by Lemma 1812 LR with $\{x1 \leftarrow x1, x2 \leftarrow x2, x3 \leftarrow x3\}$

$\overbrace{rd(op_t(rd(x2, x1), \underbrace{rd(x1, op_r(x2, x3, x3))}_{}), rd(op_r(x2, x3, x3), x1))}$

$=$ by Lemma 1251 RL with $\{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x3\}$

$rd(\underbrace{op_t(rd(x2, x1), \overbrace{rd(asoc(x3, x2, x3), rd(x2, x1))}^{})}_{}, rd(op_r(x2, x3, x3), x1))$

$=$ by Lemma 1296 LR with $\{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow rd(x2, x1)\}$

$rd(\overbrace{rd(x2, x1)}^{}, rd(op_r(x2, x3, x3), x1))$

Lemma 1814: $op_l(x1, mult(x2, mult(x2, x3)), x2) = op_l(x1, op_t(mult(mult(x2, x2), x3), rd(x3, rd(op_t(x3, mult(x2, x2)), x3))), x2$

$op_l(x1, \underbrace{mult(x2, mult(x2, x3))}_{}, x2)$

$=$ by Lemma 24 RL with $\{x2 \leftarrow x3, x1 \leftarrow x2\}$

$\underbrace{op_l(x1, \overbrace{mult(mult(x2, x2), x3)}^{}, x2)}$

$=$ by Lemma 1370 LR with $\{x2 \leftarrow x2, x3 \leftarrow mult(mult(x2, x2), x3), x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_l(x1, op_t(mult(mult(x2, x2), x3), rd(i(x2), x2)), x2)}}$

$=$ by Lemma 1802 RL with $\{x3 \leftarrow x2, x2 \leftarrow mult(mult(x2, x2), x3), x1 \leftarrow x1\}$

$\overbrace{op_l(x1, op_t(mult(mult(x2, x2), x3), \underbrace{rd(mult(mult(x2, x2), x3), mult(x2, x2))}_{}), x2)}$

$=$ by Lemma 1108 LR with $\{x1 \leftarrow x3, x2 \leftarrow mult(x2, x2)\}$

$op_l(x1, op_t(mult(mult(x2, x2), x3), \overbrace{rd(x3, rd(op_t(x3, mult(x2, x2)), x3))}^{}), x2)$

Lemma 1815: $op_l(x1, x3, x2) = op_l(x1, op_t(mult(mult(x2, x2), x3), rd(x3, rd(op_t(x3, mult(x2, x2)), x3))), x2)$

$\underbrace{op_l(x1, x3, x2)}$

$=$ by Lemma 1051 RL with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_l(x1, x2, i(mult(x2, x3)))}}$

$=$ by Lemma 1331 RL with $\{x3 \leftarrow mult(x2, x3), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_l(x1, mult(x2, mult(x2, x3)), x2)}}$

$=$ by Lemma 1814 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_l(x1, op_t(mult(mult(x2, x2), x3), rd(x3, rd(op_t(x3, mult(x2, x2)), x3))), x2)}$

Lemma 1816: $op_l(x1, op_l(x1, x2, x3), x4) = mult(x4, rd(x1, x4))$

$\overbrace{op_l(x1, op_l(x1, x2, x3), x4)}$

$=$ by Lemma 1615 RL with $\{x5 \leftarrow x4, x4 \leftarrow op_l(x1, x2, x3), x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_l(\underbrace{op_l(op_l(x1, x2, x3), op_l(x1, x2, x3), x4)}, x3, x2)}$

$=$ by Lemma 427 LR with $\{x2 \leftarrow x4, x1 \leftarrow op_l(x1, x2, x3)\}$

$op_l(\overbrace{\underbrace{mult(x4, rd(op_l(x1, x2, x3), x4))}}, x3, x2)$

$=$ by Lemma 381 RL with $\{x2 \leftarrow x4, x1 \leftarrow op_l(x1, x2, x3)\}$

$op_l(\overbrace{op_r(\underbrace{op_t(op_l(x1, x2, x3), i(x4))}, x4, x4)}, x3, x2)$

$=$ by Axiom 15 LR with $\{x4 \leftarrow i(x4), x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$op_l(\underbrace{op_r(\overbrace{op_l(op_t(x1, i(x4)), x2, x3)}, x4, x4)}, x3, x2)$

$=$ by Axiom 18 LR with $\{x5 \leftarrow x4, x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow op_t(x1, i(x4))\}$

$op_l(\overbrace{op_l(\underbrace{op_r(op_t(x1, i(x4)), x4, x4)}, x2, x3)}, x3, x2)$

$=$ by Lemma 381 LR with $\{x2 \leftarrow x4, x1 \leftarrow x1\}$

$\underbrace{op_l(op_l(\overbrace{mult(x4, rd(x1, x4))}, x2, x3), x3, x2)}$

$=$ by Lemma 138 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow mult(x4, rd(x1, x4))\}$

$\overbrace{mult(x4, rd(x1, x4))}$

Lemma 1817: $op_l(x1, x1, i(x2)) = mult(op_l(x1, op_l(x1, x3, x4), i(x2)), mult(x2, i(x2)))$

$\underbrace{op_l(x1, x1, i(x2))}$

$=$ by Lemma 1055 RL with $\{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_l(x1, rd(i(x2), x1), x2)}}$

$=$ by Lemma 280 LR with $\{x3 \leftarrow x2, x2 \leftarrow i(x2), x1 \leftarrow x1\}$

$\overbrace{mult(\underbrace{mult(i(x2), rd(x1, i(x2)))}, mult(x2, i(x2)))}$

$=$ by Lemma 1816 RL with $\{x4 \leftarrow i(x2), x3 \leftarrow x4, x2 \leftarrow x3, x1 \leftarrow x1\}$

$mult(\overbrace{op_l(x1, op_l(x1, x3, x4), i(x2))}, mult(x2, i(x2)))$

Lemma 1818: $op_l(x1, x2, x1) = mult(op_l(x1, x2, op_l(x1, x3, x4)), mult(x2, i(x2)))$

$\underbrace{op_l(x1, x2, x1)}$

$=$ by Lemma 608 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_l(x1, x1, i(x2))}}$

$=$ by Lemma 1817 LR with $\{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{mult(\underbrace{op_l(x1, op_l(x1, x3, x4), i(x2))}, mult(x2, i(x2)))}$

$=$ by Lemma 1616 LR with $\{x4 \leftarrow x2, x3 \leftarrow x4, x2 \leftarrow x3, x1 \leftarrow x1\}$

$mult(\overbrace{op_l(x1, x2, op_l(x1, x3, x4))}, mult(x2, i(x2)))$

Lemma 1819: $op_l(x1, x2, op_l(x1, x3, x4)) = op_l(x1, x2, x1)$

$$
\begin{aligned}
& \overbrace{op_l(x1, x2, op_l(x1, x3, x4))} \\
= & \quad \text{by Axiom 1 LR with } \{x1 \leftarrow op_l(x1, x2, op_l(x1, x3, x4))\} \\
& \overbrace{mult(op_l(x1, x2, op_l(x1, x3, x4)), \underbrace{unit()})} \\
= & \quad \text{by Axiom 7 RL with } \{x1 \leftarrow x2\} \\
& \underbrace{mult(op_l(x1, x2, op_l(x1, x3, x4)), \overbrace{mult(x2, i(x2))})} \\
= & \quad \text{by Lemma 1818 RL with } \{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x2, x1)}
\end{aligned}
$$

Lemma 1820: $op_l(x1, op_l(x1, x2, x3), x4) = op_l(x1, x1, x4)$

$$
\begin{aligned}
& \underbrace{op_l(x1, op_l(x1, x2, x3), x4)} \\
= & \quad \text{by Lemma 1816 LR with } \{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{\overbrace{mult(x4, rd(x1, x4))}} \\
= & \quad \text{by Lemma 427 RL with } \{x2 \leftarrow x4, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x1, x4)}
\end{aligned}
$$

Lemma 1821: $op_l(i(x1), op_l(x1, x2, x3), x4) = op_l(i(x1), x1, x4)$

$$
\begin{aligned}
& \underbrace{op_l(i(x1), op_l(x1, x2, x3), x4)} \\
= & \quad \text{by Lemma 1049 RL with } \{x3 \leftarrow x4, x2 \leftarrow op_l(x1, x2, x3), x1 \leftarrow x1\} \\
& \overbrace{i(\underbrace{op_l(x1, op_l(x1, x2, x3), x4)})} \\
= & \quad \text{by Lemma 1820 LR with } \{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{i(\overbrace{op_l(x1, x1, x4)})} \\
= & \quad \text{by Lemma 1049 LR with } \{x3 \leftarrow x4, x2 \leftarrow x1, x1 \leftarrow x1\} \\
& \overbrace{op_l(i(x1), x1, x4)}
\end{aligned}
$$

Lemma 1822: $op_l(x1, mult(x2, op_l(x1, x3, x4)), x1) = op_l(x1, x2, x1)$

$$
\begin{aligned}
& \underbrace{op_l(x1, mult(x2, op_l(x1, x3, x4)), x1)} \\
= & \quad \text{by Lemma 1819 RL with } \{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow mult(x2, op_l(x1, x3, x4)), x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, mult(x2, op_l(x1, x3, x4)), op_l(x1, x3, x4))}} \\
= & \quad \text{by Lemma 285 LR with } \{x3 \leftarrow op_l(x1, x3, x4), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, x2, op_l(x1, x3, x4))}} \\
= & \quad \text{by Lemma 1819 LR with } \{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x2, x1)}
\end{aligned}
$$

Lemma 1823: $op_l(x1, mult(x2, mult(x3, x1)), x1) = op_l(x1, mult(x2, x3), x1)$

$$
\begin{aligned}
& op_l(x1, \underbrace{mult(x2, mult(x3, x1))}, x1) \\
= & \quad \text{by Lemma 132 RL with } \{x3 \leftarrow x1, x2 \leftarrow x3, x1 \leftarrow x2\} \\
& \underbrace{op_l(x1, \overbrace{mult(mult(x2, x3), op_l(x1, x3, x2))}, x1)} \\
= & \quad \text{by Lemma 1822 LR with } \{x4 \leftarrow x2, x3 \leftarrow x3, x2 \leftarrow mult(x2, x3), x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, mult(x2, x3), x1)}
\end{aligned}
$$

Lemma 1824: $op_l(x1, x1, mult(x2, mult(x3, x1))) = op_l(x1, x1, mult(x2, x3))$

$$
\begin{aligned}
& \underbrace{op_l(x1, x1, mult(x2, mult(x3, x1)))} \\
= & \quad \text{by Lemma 606 RL with } \{x2 \leftarrow mult(x2, mult(x3, x1)), x1 \leftarrow x1\} \\
& \underbrace{\overbrace{op_l(x1, mult(x2, mult(x3, x1)), i(x1))}} \\
= & \quad \text{by Lemma 1050 RL with } \{x3 \leftarrow x1, x2 \leftarrow mult(x2, mult(x3, x1)), x1 \leftarrow x1\} \\
& \underbrace{\overbrace{op_l(x1, i(mult(x2, mult(x3, x1))), x1)}} \\
= & \quad \text{by Lemma 1822 RL with } \{x4 \leftarrow x2, x3 \leftarrow x3, x2 \leftarrow i(mult(x2, mult(x3, x1))), x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, mult(i(mult(x2, mult(x3, x1))), \underbrace{op_l(x1, x3, x2)}), x1)} \\
= & \quad \text{by Axiom 12 RL with } \{x3 \leftarrow x1, x2 \leftarrow x3, x1 \leftarrow x2\} \\
& op_l(x1, \underbrace{mult(i(mult(x2, mult(x3, x1))), \overbrace{mult(i(mult(x2, x3)), mult(x2, mult(x3, x1)))})}, x1) \\
= & \quad \text{by Axiom 10 LR with } \{x2 \leftarrow i(mult(x2, x3)), x1 \leftarrow mult(x2, mult(x3, x1))\} \\
& \underbrace{op_l(x1, \overbrace{op_t(i(mult(x2, x3)), mult(x2, mult(x3, x1)))}, x1)} \\
= & \quad \text{by Lemma 1723 LR with } \{x3 \leftarrow mult(x2, mult(x3, x1)), x2 \leftarrow i(mult(x2, x3)), x1 \leftarrow x1\} \\
& \underbrace{\overbrace{op_l(x1, i(mult(x2, x3)), x1)}} \\
= & \quad \text{by Lemma 1050 LR with } \{x3 \leftarrow x1, x2 \leftarrow mult(x2, x3), x1 \leftarrow x1\} \\
& \underbrace{\overbrace{op_l(x1, mult(x2, x3), i(x1))}} \\
= & \quad \text{by Lemma 606 LR with } \{x2 \leftarrow mult(x2, x3), x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x1, mult(x2, x3))}
\end{aligned}
$$

Lemma 1825: $asoc(mult(x2, mult(x3, x1)), x1, x1) = mult(i(x1), op_l(x1, mult(x2, x3), x1))$

$$
\begin{aligned}
& \underbrace{asoc(mult(x2, mult(x3, x1)), x1, x1)} \\
= & \quad \text{by Lemma 765 RL with } \{x2 \leftarrow mult(x2, mult(x3, x1)), x1 \leftarrow x1\} \\
& \underbrace{\overbrace{rd(op_l(x1, mult(x2, mult(x3, x1)), x1), x1)}} \\
= & \quad \text{by Lemma 171 RL with } \{x3 \leftarrow x1, x2 \leftarrow mult(x2, mult(x3, x1)), x1 \leftarrow x1\} \\
& \overbrace{mult(i(x1), \underbrace{op_l(x1, mult(x2, mult(x3, x1)), x1)})} \\
= & \quad \text{by Lemma 1823 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& mult(i(x1), \overbrace{op_l(x1, mult(x2, x3), x1)})
\end{aligned}
$$

Lemma 1826: $asoc(mult(x2, mult(x3, x1)), x1, x1) = asoc(mult(x2, x3), x1, x1)$

$$
\begin{aligned}
& \overbrace{asoc(mult(x2, mult(x3, x1)), x1, x1)} \\
= & \quad \text{by Lemma 1825 LR with } \{x1 \leftarrow x1, x3 \leftarrow x3, x2 \leftarrow x2\} \\
& \overbrace{mult(i(x1), op_l(x1, mult(x2, x3), x1))} \\
= & \quad \text{by Lemma 171 LR with } \{x3 \leftarrow x1, x2 \leftarrow mult(x2, x3), x1 \leftarrow x1\} \\
& \overbrace{rd(op_l(x1, mult(x2, x3), x1), x1)} \\
= & \quad \text{by Lemma 765 LR with } \{x2 \leftarrow mult(x2, x3), x1 \leftarrow x1\} \\
& \overbrace{asoc(mult(x2, x3), x1, x1)}
\end{aligned}
$$

Lemma 1827: $asoc(x1, mult(x2, mult(x3, x1)), x1) = op_t(rd(x1, op_l(x1, mult(x2, x3), x1)), x1)$

$$
\begin{aligned}
& \overbrace{asoc(x1, mult(x2, mult(x3, x1)), x1)} \\
= & \quad \text{by Lemma 617 RL with } \{x2 \leftarrow mult(x2, mult(x3, x1)), x1 \leftarrow x1\} \\
& \overbrace{rd(x1, op_l(x1, mult(x2, mult(x3, x1)), x1))} \\
= & \quad \text{by Lemma 311 RL with } \{x3 \leftarrow x1, x2 \leftarrow mult(x2, mult(x3, x1)), x1 \leftarrow x1\} \\
& \overbrace{op_t(rd(x1, \underbrace{op_l(x1, mult(x2, mult(x3, x1)), x1)}), x1)} \\
= & \quad \text{by Lemma 1823 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_t(rd(x1, \overbrace{op_l(x1, mult(x2, x3), x1)}), x1)
\end{aligned}
$$

Lemma 1828: $asoc(x1, mult(x2, mult(x3, x1)), x1) = rd(x1, op_l(x1, mult(x2, x3), x1))$

$$
\begin{aligned}
& \overbrace{asoc(x1, mult(x2, mult(x3, x1)), x1)} \\
= & \quad \text{by Lemma 1827 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_t(rd(x1, op_l(x1, mult(x2, x3), x1)), x1)} \\
= & \quad \text{by Lemma 1764 LR with } \{x3 \leftarrow mult(x2, x3), x2 \leftarrow x1, x1 \leftarrow x1\} \\
& \overbrace{mult(op_l(i(x1), mult(x2, x3), x1), x1)} \\
= & \quad \text{by Lemma 1058 LR with } \{x3 \leftarrow x1, x2 \leftarrow mult(x2, x3), x1 \leftarrow x1\} \\
& \overbrace{rd(x1, op_l(x1, mult(x2, x3), x1))}
\end{aligned}
$$

Lemma 1829: $op_l(x1, mult(x1, rd(x2, i(x3))), x1) = op_l(x1, rd(mult(x1, x2), i(mult(x1, x3))), x1)$

$$
\begin{aligned}
& \overbrace{op_l(x1, mult(x1, rd(x2, i(x3))), x1)} \\
= & \quad \text{by Lemma 1823 RL with } \{x3 \leftarrow rd(x2, i(x3)), x2 \leftarrow x1, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, mult(x1, mult(rd(x2, \underbrace{i(x3)}), x1)), x1)} \\
= & \quad \text{by Lemma 20 RL with } \{x2 \leftarrow x3, x1 \leftarrow x1\} \\
& op_l(x1, \underbrace{mult(x1, mult(rd(x2, \overbrace{mult(x1, i(mult(x1, x3)))}), x1))}, x1) \\
= & \quad \text{by Lemma 1152 LR with } \{x3 \leftarrow i(mult(x1, x3)), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_l(x1, \overbrace{rd(mult(x1, x2), i(mult(x1, x3)))}, x1)
\end{aligned}
$$

Lemma 1830: $op_l(x1, rd(x1, rd(i(x3), x2)), x1) = op_l(x1, rd(mult(x1, x2), i(mult(x1, x3))), x1)$

$\underbrace{op_l(x1, rd(x1, rd(i(x3), x2)), x1)}$

$=$ by Lemma 1744 RL with $\{x3 \leftarrow rd(i(x3), x2), x2 \leftarrow x1, x1 \leftarrow x1\}$

$\overbrace{op_l(x1, mult(x1, \underbrace{i(rd(i(x3), x2))}), x1)}$

$=$ by Lemma 10 LR with $\{x2 \leftarrow x2, x1 \leftarrow i(x3)\}$

$\underbrace{op_l(x1, mult(x1, \overbrace{rd(x2, i(x3))}), x1)}$

$=$ by Lemma 1829 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_l(x1, rd(mult(x1, x2), i(mult(x1, x3))), x1)}$

Lemma 1831: $op_l(x1, rd(x2, i(x3)), x1) = op_l(x1, rd(mult(x1, x2), i(mult(x1, x3))), x1)$

$\underbrace{op_l(x1, rd(x2, i(x3)), x1)}$

$=$ by Lemma 1060 RL with $\{x4 \leftarrow x1, x3 \leftarrow x2, x2 \leftarrow i(x3), x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_l(x1, rd(i(x3), x2), i(x1))}}$

$=$ by Lemma 1317 RL with $\{x3 \leftarrow rd(i(x3), x2), x2 \leftarrow x1, x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_l(x1, rd(x1, rd(i(x3), x2)), x1)}}$

$=$ by Lemma 1830 LR with $\{x2 \leftarrow x2, x3 \leftarrow x3, x1 \leftarrow x1\}$

$\overbrace{op_l(x1, rd(mult(x1, x2), i(mult(x1, x3))), x1)}$

Lemma 1832: $op_l(x1, mult(x2, x3), x1) = op_l(x1, x1, rd(mult(x1, i(mult(x1, x3))), x2))$

$\overbrace{op_l(x1, mult(x2, x3), x1)}$

$=$ by Lemma 1746 RL with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_l(x1, rd(x2, i(x3)), x1)}$

$=$ by Lemma 1831 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_l(x1, rd(mult(x1, x2), i(mult(x1, x3))), x1)}$

$=$ by Lemma 1060 RL with $\{x4 \leftarrow x1, x3 \leftarrow mult(x1, x2), x2 \leftarrow i(mult(x1, x3)), x1 \leftarrow x1\}$

$\overbrace{op_l(x1, rd(\underbrace{i(mult(x1, x3))}, mult(x1, x2)), i(x1))}$

$=$ by Lemma 1060 RL with $\{x4 \leftarrow i(x1), x3 \leftarrow i(mult(x1, x3)), x2 \leftarrow mult(x1, x2), x1 \leftarrow x1\}$

$\overbrace{op_l(x1, rd(mult(x1, x2), i(mult(x1, x3))), i(\overbrace{i(x1)}))}$

$=$ by Lemma 1315 RL with $\{x3 \leftarrow rd(mult(x1, x2), i(mult(x1, x3))), x2 \leftarrow i(x1), x1 \leftarrow x1\}$

$\overbrace{op_l(x1, \underbrace{i(x1)}, op_t(rd(mult(x1, x2), i(mult(x1, x3))), i(x1)))}$

$=$ by Lemma 3 RL with $\{x1 \leftarrow i(x1)\}$

$op_l(x1, \overbrace{i(\overbrace{i(i(x1))})}, \underbrace{op_t(rd(mult(x1, x2), i(mult(x1, x3))), i(x1))})$

$=$ by Axiom 10 RL with $\{x2 \leftarrow rd(mult(x1, x2), i(mult(x1, x3))), x1 \leftarrow i(x1)\}$

$\underbrace{op_l(x1, i(i(i(x1))), \overbrace{mult(i(i(x1)), mult(rd(mult(x1, x2), i(mult(x1, x3))), i(x1)))})}$

$=$ by Lemma 241 LR with $\{x3 \leftarrow mult(rd(mult(x1, x2), i(mult(x1, x3))), i(x1)), x2 \leftarrow i(i(x1)), x1 \leftarrow x1\}$

$\overbrace{op_l(x1, \underbrace{mult(rd(mult(x1, x2), i(mult(x1, x3))), i(x1))}, \underbrace{i(i(x1))})}$

$=$ by Lemma 1155 LR with $\{x3 \leftarrow i(mult(x1, x3)), x2 \leftarrow x2, x1 \leftarrow x1\}$

$op_l(x1, \overbrace{mult(x1, rd(x2, mult(x1, i(mult(x1, x3)))))}, \underbrace{i(i(x1))})$

$=$ by Lemma 3 LR with $\{x1 \leftarrow x1\}$

$\underbrace{op_l(x1, mult(x1, rd(x2, mult(x1, i(mult(x1, x3))))), \overbrace{x1})}$

$=$ by Lemma 1331 LR with $\{x3 \leftarrow rd(x2, mult(x1, i(mult(x1, x3)))), x2 \leftarrow x1, x1 \leftarrow x1\}$

$\overbrace{op_l(x1, x1, \underbrace{i(rd(x2, mult(x1, i(mult(x1, x3)))))})}$

$=$ by Lemma 10 LR with $\{x2 \leftarrow mult(x1, i(mult(x1, x3))), x1 \leftarrow x2\}$

$op_l(x1, x1, \overbrace{rd(mult(x1, i(mult(x1, x3))), x2)})$

Lemma 1833: $op_l(x1, mult(x2, x3), x1) = op_l(x1, mult(mult(x1, x3), mult(x2, x1)), x1)$

$\overbrace{op_l(x1, mult(x2, x3), x1)}$

$=$ by Lemma 1832 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_l(x1, x1, rd(mult(x1, i(mult(x1, x3))), x2))}$

$=$ by Lemma 1451 LR with $\{x3 \leftarrow x2, x2 \leftarrow i(mult(x1, x3)), x1 \leftarrow x1\}$

$\overbrace{op_l(x1, x1, rd(i(mult(x1, x3)), mult(x2, x1)))}$

$=$ by Lemma 1747 LR with $\{x3 \leftarrow mult(x2, x1), x2 \leftarrow mult(x1, x3), x1 \leftarrow x1\}$

$\overbrace{op_l(x1, mult(mult(x1, x3), mult(x2, x1)), x1)}$

Lemma 1834: $op_l(x1, mult(mult(x1, x2), x3), x1) = op_l(x1, mult(x3, x2), x1)$

$$
\begin{aligned}
& \overbrace{op_l(x1, mult(mult(x1, x2), x3), x1)} \\
= \quad & \text{by Lemma 1823 RL with } \{x3 \leftarrow x3, x2 \leftarrow mult(x1, x2), x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, \underbrace{mult(mult(x1, x2), mult(x3, x1))}, x1)} \\
= \quad & \text{by Lemma 1833 RL with } \{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, mult(x3, x2), x1)}
\end{aligned}
$$

Lemma 1835: $op_l(x1, rd(mult(x2, x3), mult(mult(x1, x2), x3)), x1) = op_l(x1, i(asoc(x1, x2, x3)), x1)$

$$
\begin{aligned}
& \overbrace{op_l(x1, rd(mult(x2, x3), mult(mult(x1, x2), x3)), x1)} \\
= \quad & \text{by Lemma 1741 RL with } \{x3 \leftarrow mult(x2, x3), x2 \leftarrow mult(mult(x1, x2), x3), x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, \underbrace{mult(i(mult(mult(x1, x2), x3)), mult(x2, x3))}, x1)} \\
= \quad & \text{by Lemma 1834 RL with } \{x3 \leftarrow i(mult(mult(x1, x2), x3)), x2 \leftarrow mult(x2, x3), x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, \underbrace{mult(mult(x1, mult(x2, x3)), i(mult(mult(x1, x2), x3)))}, x1)} \\
= \quad & \text{by Lemma 964 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_l(x1, \overbrace{i(asoc(x1, x2, x3))}, x1)
\end{aligned}
$$

Lemma 1836: $op_l(x1, op_r(x1, x2, x3), i(x1)) = op_l(x1, i(asoc(x1, x2, x3)), x1)$

$$
\begin{aligned}
& \overbrace{op_l(x1, op_r(x1, x2, x3), i(x1))} \\
= \quad & \text{by Lemma 1050 RL with } \{x3 \leftarrow x1, x2 \leftarrow op_r(x1, x2, x3), x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, \underbrace{i(op_r(x1, x2, x3))}, x1)} \\
= \quad & \text{by Lemma 891 RL with } \{x3 \leftarrow x1, x2 \leftarrow x3, x1 \leftarrow x2\} \\
& \overbrace{op_l(x1, \overbrace{rd(mult(x2, x3), mult(mult(x1, x2), x3))}, x1)} \\
= \quad & \text{by Lemma 1835 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, i(asoc(x1, x2, x3)), x1)}
\end{aligned}
$$

Lemma 1837: $x1 = op_l(x1, asoc(x1, x2, x3), i(x1))$

$$
\begin{aligned}
& \underbrace{x1} \\
= \quad & \text{by Lemma 1445 RL with } \{x4 \leftarrow x3, x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, op_r(x1, x2, x3), i(x1))} \\
= \quad & \text{by Lemma 1836 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, i(asoc(x1, x2, x3)), x1)} \\
= \quad & \text{by Lemma 1050 LR with } \{x3 \leftarrow x1, x2 \leftarrow asoc(x1, x2, x3), x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, asoc(x1, x2, x3), i(x1))}
\end{aligned}
$$

Lemma 1838: $op_l(x1, x1, asoc(x1, x2, x3)) = x1$

$$
\begin{aligned}
& \underbrace{op_l(x1, x1, asoc(x1, x2, x3))} \\
= & \quad \text{by Lemma 606 RL with } \{x2 \leftarrow asoc(x1, x2, x3), x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, asoc(x1, x2, x3), i(x1))}} \\
= & \quad \text{by Lemma 1837 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{x1}
\end{aligned}
$$

Lemma 1839: $rd(x1, op_l(x1, x1, i(asoc(i(x1), x2, x3)))) = mult(x1, i(x1))$

$$
\begin{aligned}
& rd(x1, \underbrace{op_l(x1, x1, i(asoc(i(x1), x2, x3)))}) \\
= & \quad \text{by Lemma 1050 RL with } \{x3 \leftarrow asoc(i(x1), x2, x3), x2 \leftarrow x1, x1 \leftarrow x1\} \\
& \underbrace{rd(x1, \overbrace{op_l(x1, i(x1), asoc(i(x1), x2, x3))})} \\
= & \quad \text{by Lemma 1056 RL with } \{x3 \leftarrow asoc(i(x1), x2, x3), x2 \leftarrow i(x1), x1 \leftarrow x1\} \\
& \overbrace{mult(x1, \underbrace{op_l(i(x1), i(x1), asoc(i(x1), x2, x3))})} \\
= & \quad \text{by Lemma 1838 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow i(x1)\} \\
& mult(x1, \overbrace{i(x1)})
\end{aligned}
$$

Lemma 1840: $asoc(x1, asoc(i(x1), x2, x3), x1) = mult(x1, i(x1))$

$$
\begin{aligned}
& \underbrace{asoc(x1, asoc(i(x1), x2, x3), x1)} \\
= & \quad \text{by Lemma 617 RL with } \{x2 \leftarrow asoc(i(x1), x2, x3), x1 \leftarrow x1\} \\
& \overbrace{rd(x1, \underbrace{op_l(x1, asoc(i(x1), x2, x3), x1)})} \\
= & \quad \text{by Lemma 608 RL with } \{x2 \leftarrow asoc(i(x1), x2, x3), x1 \leftarrow x1\} \\
& \underbrace{rd(x1, \overbrace{op_l(x1, x1, i(asoc(i(x1), x2, x3)))})} \\
= & \quad \text{by Lemma 1839 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{mult(x1, i(x1))}
\end{aligned}
$$

Lemma 1841: $asoc(x1, x2, x3) = op_r(asoc(x1, x2, x3), x1, x1)$

$$
\begin{aligned}
& \underbrace{asoc(x1, x2, x3)} \\
= & \quad \text{by Lemma 1476 LR with } \{x2 \leftarrow x1, x1 \leftarrow asoc(x1, x2, x3)\} \\
& \overbrace{mult(\underbrace{op_l(x1, x1, asoc(x1, x2, x3))}, rd(asoc(x1, x2, x3), op_t(x1, i(asoc(x1, x2, x3)))))} \\
= & \quad \text{by Lemma 1838 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{mult(\overbrace{x1}, rd(asoc(x1, x2, x3), op_t(x1, i(asoc(x1, x2, x3)))))} \\
= & \quad \text{by Lemma 912 LR with } \{x2 \leftarrow asoc(x1, x2, x3), x1 \leftarrow x1\} \\
& \overbrace{op_r(asoc(x1, x2, x3), x1, x1)}
\end{aligned}
$$

Lemma 1842: $mult(i(asoc(x1, x2, x3)), x1) = rd(x1, asoc(x1, x2, x3))$

$$
\begin{aligned}
& mult(i(asoc(x1, x2, x3)), \underbrace{x1}) \\
= \quad & \text{by Lemma 1838 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{mult(i(asoc(x1, x2, x3)), \overbrace{op_l(x1, x1, asoc(x1, x2, x3))})} \\
= \quad & \text{by Lemma 440 LR with } \{x2 \leftarrow x1, x1 \leftarrow asoc(x1, x2, x3)\} \\
& \overbrace{rd(x1, asoc(x1, x2, x3))}
\end{aligned}
$$

Lemma 1843: $rd(asoc(x1, x2, x3), x1) = mult(asoc(x1, x2, x3), i(x1))$

$$
\begin{aligned}
& \underbrace{rd(asoc(x1, x2, x3), x1)} \\
= \quad & \text{by Lemma 1464 LR with } \{x2 \leftarrow x1, x1 \leftarrow asoc(x1, x2, x3)\} \\
& \overbrace{rd(op_r(asoc(x1, x2, x3), x1, i(x1)), \underbrace{op_l(x1, x1, asoc(x1, x2, x3))})} \\
= \quad & \text{by Lemma 1838 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{rd(op_r(asoc(x1, x2, x3), x1, i(x1)), \overbrace{x1})} \\
= \quad & \text{by Lemma 342 LR with } \{x2 \leftarrow x1, x1 \leftarrow asoc(x1, x2, x3)\} \\
& \overbrace{mult(asoc(x1, x2, x3), i(x1))}
\end{aligned}
$$

Lemma 1844: $op_t(i(i(x1)), asoc(x1,x2,x3)) = rd(op_r(asoc(x1,x2,x3), i(x1), i(x1)), op_t(rd(asoc(x1,x2,x3), x1), asoc(x1,x2,x3)$

$\underbrace{op_t(i(i(x1)), asoc(x1,x2,x3))}$

$=$ by Lemma 29 RL with $\{x2 \leftarrow op_t(i(i(x1)), asoc(x1,x2,x3)), x1 \leftarrow mult(i(x1), asoc(x1,x2,x3))\}$

$\overbrace{rd(mult(mult(i(x1), asoc(x1,x2,x3)), op_t(i(i(x1)), asoc(x1,x2,x3))), op_t(mult(i(x1), asoc(x1,x2,x3)), \underbrace{op_t(i(i(x1)), asoc(x$

$=$ by Lemma 19 RL with $\{x2 \leftarrow asoc(x1,x2,x3), x1 \leftarrow i(x1)\}$

$rd(mult(mult(i(x1), asoc(x1,x2,x3)), op_t(i(i(x1)), asoc(x1,x2,x3))), op_t(mult(i(x1), asoc(x1,x2,x3)), \overbrace{i(\underbrace{op_t(i(x1), i(asoc($

$=$ by Lemma 572 RL with $\{x2 \leftarrow asoc(x1,x2,x3), x1 \leftarrow i(x1)\}$

$rd(mult(mult(i(x1), asoc(x1,x2,x3)), op_t(i(i(x1)), asoc(x1,x2,x3))), op_t(\underbrace{mult(i(x1), asoc(x1,x2,x3))}, \overbrace{i(op_t(i(x1), op_l(i(a$

$=$ by Lemma 231 LR with $\{x2 \leftarrow asoc(x1,x2,x3), x1 \leftarrow i(x1)\}$

$rd(mult(mult(i(x1), asoc(x1,x2,x3)), op_t(i(i(x1)), asoc(x1,x2,x3))), \underbrace{op_t(\overbrace{rd(i(x1), op_l(i(asoc(x1,x2,x3)), asoc(x1,x2,x3)$

$=$ by Lemma 591 LR with $\{x2 \leftarrow op_l(i(asoc(x1,x2,x3)), asoc(x1,x2,x3), i(x1)), x1 \leftarrow i(x1)\}$

$rd(mult(mult(i(x1), asoc(x1,x2,x3)), op_t(i(i(x1)), asoc(x1,x2,x3))), \overbrace{mult(i(x1), \underbrace{i(op_l(i(asoc(x1,x2,x3)), asoc(x1,x2,x3)$

$=$ by Lemma 518 LR with $\{x3 \leftarrow i(x1), x2 \leftarrow asoc(x1,x2,x3), x1 \leftarrow asoc(x1,x2,x3)\}$

$rd(mult(mult(i(x1), asoc(x1,x2,x3)), op_t(i(i(x1)), asoc(x1,x2,x3))), mult(i(x1), \overbrace{op_l(asoc(x1,x2,x3), i(asoc(x1,x2,x3)), i$

$=$ by Lemma 429 LR with $\{x2 \leftarrow i(i(x1)), x1 \leftarrow asoc(x1,x2,x3)\}$

$rd(mult(mult(i(x1), asoc(x1,x2,x3)), op_t(i(i(x1)), asoc(x1,x2,x3))), mult(i(x1), \overbrace{op_l(asoc(x1,x2,x3), asoc(x1,x2,x3), \underbrace{i(i($

$=$ by Lemma 3 LR with $\{x1 \leftarrow i(x1)\}$

$rd(mult(mult(i(x1), asoc(x1,x2,x3)), op_t(i(i(x1)), asoc(x1,x2,x3))), \underbrace{mult(i(x1), op_l(asoc(x1,x2,x3), asoc(x1,x2,x3), \overbrace{i(x$

$=$ by Lemma 264 LR with $\{x2 \leftarrow asoc(x1,x2,x3), x1 \leftarrow i(x1)\}$

$rd(\underbrace{mult(mult(i(x1), asoc(x1,x2,x3)), op_t(i(i(x1)), asoc(x1,x2,x3)))}, \overbrace{op_t(mult(asoc(x1,x2,x3), i(x1)), asoc(x1,x2,x3)))}$

$=$ by Lemma 695 LR with $\{x2 \leftarrow asoc(x1,x2,x3), x1 \leftarrow i(x1)\}$

$rd(\overbrace{op_r(asoc(x1,x2,x3), i(x1), i(x1))}, op_t(\underbrace{mult(asoc(x1,x2,x3), i(x1))}, asoc(x1,x2,x3)))$

$=$ by Lemma 1843 RL with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$rd(op_r(asoc(x1,x2,x3), i(x1), i(x1)), op_t(\overbrace{rd(asoc(x1,x2,x3), x1)}, asoc(x1,x2,x3)))$

Lemma 1845: $i(mult(asoc(x1,x2,x3), x1)) = rd(i(x1), asoc(x1,x2,x3))$

$\underbrace{i(mult(asoc(x1,x2,x3), x1))}$

$=$ by Lemma 447 RL with $\{x2 \leftarrow x1, x1 \leftarrow asoc(x1,x2,x3)\}$

$\overbrace{rd(i(asoc(x1,x2,x3)), \underbrace{op_l(x1, x1, asoc(x1,x2,x3)))}}$

$=$ by Lemma 1838 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{rd(i(asoc(x1,x2,x3)), \overbrace{x1})}$

$=$ by Lemma 37 LR with $\{x2 \leftarrow x1, x1 \leftarrow asoc(x1,x2,x3)\}$

$\overbrace{rd(i(x1), asoc(x1,x2,x3))}$

Lemma 1846: $i(mult(asoc(x1,x2,x3),x2)) = rd(i(x2),asoc(x1,x2,x3))$

$\underbrace{i(mult(asoc(x1,x2,x3),x2))}$

$=$ by Lemma 447 RL with $\{x2 \leftarrow x2, x1 \leftarrow asoc(x1,x2,x3)\}$

$\overbrace{rd(i(asoc(x1,x2,x3)),\underbrace{op_l(x2,x2,asoc(x1,x2,x3))})}$

$=$ by Lemma 1 RL with $\{x1 \leftarrow op_l(x2,x2,asoc(x1,x2,x3))\}$

$rd(i(asoc(x1,x2,x3)),\overbrace{rd(op_l(x2,x2,asoc(x1,x2,x3)),\underbrace{unit()})})$

$=$ by Axiom 7 RL with $\{x1 \leftarrow x2\}$

$rd(i(asoc(x1,x2,x3)),rd(op_l(x2,x2,asoc(x1,x2,x3)),\overbrace{mult(x2,i(x2))}))$

$=$ by Lemma 1840 RL with $\{x3 \leftarrow x3, x2 \leftarrow rd(mult(i(i(x2)),x1),i(x2)), x1 \leftarrow x2\}$

$rd(i(asoc(x1,x2,x3)),rd(op_l(x2,x2,asoc(x1,x2,x3)),\overbrace{asoc(x2,\underbrace{asoc(i(x2),rd(mult(i(i(x2)),x1),i(x2)),x3)},x2)}))$

$=$ by Lemma 1220 LR with $\{x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow i(x2)\}$

$rd(i(asoc(x1,x2,x3)),rd(op_l(x2,x2,asoc(x1,x2,x3)),asoc(x2,\overbrace{asoc(x1,\underbrace{i(i(x2))},x3)},x2)))$

$=$ by Lemma 3 LR with $\{x1 \leftarrow x2\}$

$rd(i(asoc(x1,x2,x3)),rd(\underbrace{op_l(x2,x2,asoc(x1,x2,x3))},asoc(x2,asoc(x1,\overbrace{x2},x3),x2)))$

$=$ by Lemma 158 RL with $\{x2 \leftarrow asoc(x1,x2,x3), x1 \leftarrow x2\}$

$rd(i(asoc(x1,x2,x3)),\underbrace{rd(\overbrace{mult(x2,asoc(x2,asoc(x1,x2,x3),x2)),asoc(x2,asoc(x1,x2,x3),x2)})})$

$=$ by Axiom 5 RL with $\{x2 \leftarrow asoc(x2,asoc(x1,x2,x3),x2), x1 \leftarrow x2\}$

$\underbrace{rd(i(asoc(x1,x2,x3)),\overbrace{x2})}$

$=$ by Lemma 37 LR with $\{x2 \leftarrow x2, x1 \leftarrow asoc(x1,x2,x3)\}$

$\overbrace{rd(i(x2),asoc(x1,x2,x3))}$

Lemma 1847: $op_l(x1,rd(x3,x2),x1) = op_l(x1,rd(mult(x3,x1),x2),x1)$

$\underbrace{op_l(x1,rd(x3,x2),x1)}$

$=$ by Lemma 1741 RL with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{\overbrace{op_l(x1,mult(i(x2),x3),x1)}}$

$=$ by Lemma 1823 RL with $\{x3 \leftarrow x3, x2 \leftarrow i(x2), x1 \leftarrow x1\}$

$\underbrace{\overbrace{op_l(x1,mult(i(x2),mult(x3,x1)),x1)}}$

$=$ by Lemma 1741 LR with $\{x3 \leftarrow mult(x3,x1), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_l(x1,rd(mult(x3,x1),x2),x1)}$

Lemma 1848: $op_l(x1,x1,rd(x2,mult(x3,x1))) = op_l(x1,rd(x3,x2),x1)$

$\underbrace{op_l(x1,x1,rd(x2,mult(x3,x1)))}$

$=$ by Lemma 1589 LR with $\{x3 \leftarrow mult(x3,x1), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{\overbrace{op_l(x1,rd(mult(x3,x1),x2),x1)}}$

$=$ by Lemma 1847 RL with $\{x2 \leftarrow x2, x3 \leftarrow x3, x1 \leftarrow x1\}$

$\overbrace{op_l(x1,rd(x3,x2),x1)}$

Lemma 1849: $op_l(x1, x1, x2) = op_l(x1, x1, op_r(x2, x3, i(x1)))$

$\overbrace{op_l(x1, x1, x2)}$

$=$ by Lemma 606 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_l(x1, x2, i(x1))}$

$=$ by Lemma 1050 RL with $\{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_l(x1, \underbrace{i(x2)}, x1)}$

$=$ by Lemma 9 RL with $\{x2 \leftarrow x2, x1 \leftarrow x3\}$

$\overbrace{op_l(x1, \overbrace{rd(x3, mult(x2, x3))}, x1)}$

$=$ by Lemma 1742 LR with $\{x2 \leftarrow mult(x2, x3), x3 \leftarrow x3, x1 \leftarrow x1\}$

$\overbrace{op_l(x1, x1, \overbrace{mult(mult(x2, x3), i(x3))})}$

$=$ by Lemma 1824 RL with $\{x3 \leftarrow i(x3), x2 \leftarrow mult(x2, x3), x1 \leftarrow x1\}$

$\overbrace{op_l(x1, x1, \overbrace{mult(mult(x2, x3), mult(i(x3), x1))})}$

$=$ by Lemma 606 RL with $\{x2 \leftarrow mult(mult(x2, x3), mult(i(x3), x1)), x1 \leftarrow x1\}$

$\overbrace{op_l(x1, \overbrace{mult(mult(x2, x3), mult(i(x3), x1))}, i(x1))}$

$=$ by Lemma 1743 RL with $\{x3 \leftarrow mult(i(x3), x1), x2 \leftarrow mult(x2, x3), x1 \leftarrow x1\}$

$\overbrace{op_l(x1, \overbrace{rd(i(mult(x2, x3)), mult(i(x3), x1))}, x1)}$

$=$ by Lemma 1848 RL with $\{x3 \leftarrow i(mult(x2, x3)), x2 \leftarrow mult(i(x3), x1), x1 \leftarrow x1\}$

$\overbrace{op_l(x1, x1, rd(mult(\underbrace{i(x3)}, x1), \overbrace{mult(i(mult(x2, x3)), x1)}))}$

$=$ by Lemma 20 RL with $\{x2 \leftarrow x3, x1 \leftarrow x2\}$

$op_l(x1, x1, \underbrace{rd(mult(\overbrace{mult(x2, i(mult(x2, x3)))}, x1), mult(i(mult(x2, x3)), x1))})$

$=$ by Axiom 16 RL with $\{x3 \leftarrow x1, x2 \leftarrow i(mult(x2, x3)), x1 \leftarrow x2\}$

$op_l(x1, x1, \overbrace{\underbrace{op_r(x2, i(mult(x2, x3)), x1)})$

$=$ by Lemma 818 LR with $\{x3 \leftarrow x1, x2 \leftarrow x3, x1 \leftarrow x2\}$

$op_l(x1, x1, \overbrace{op_r(x2, x3, i(x1))})$

Lemma 1850: $op_l(x1, x1, x2) = op_l(x1, x1, op_r(x2, x3, x1))$

$\overbrace{op_l(x1, x1, x2)}$

$=$ by Lemma 1849 LR with $\{x3 \leftarrow i(mult(x2, x3)), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_l(x1, x1, \underbrace{op_r(x2, i(mult(x2, x3)), i(x1))})}$

$=$ by Lemma 818 LR with $\{x3 \leftarrow i(x1), x2 \leftarrow x3, x1 \leftarrow x2\}$

$op_l(x1, x1, \overbrace{op_r(x2, x3, \underbrace{i(i(x1))})})$

$=$ by Lemma 3 LR with $\{x1 \leftarrow x1\}$

$op_l(x1, x1, op_r(x2, x3, \overbrace{x1}))$

Lemma 1851: $op_l(x1, x1, rd(mult(x1, x2), x3)) = op_l(x1, rd(x3, x2), x1)$

$$
\begin{aligned}
& \underbrace{op_l(x1, x1, rd(mult(x1, x2), x3))} \\
= \quad & \text{by Lemma 1451 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, x1, rd(x2, mult(x3, x1)))}} \\
= \quad & \text{by Lemma 1848 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, rd(x3, x2), x1)}
\end{aligned}
$$

Lemma 1852: $op_l(x1, x1, op_r(x2, x1, x3)) = op_l(x1, x1, x2)$

$$
\begin{aligned}
& \underbrace{op_l(x1, x1, op_r(x2, x1, x3))} \\
= \quad & \text{by Lemma 773 LR with } \{x2 \leftarrow op_r(x2, x1, x3), x1 \leftarrow x1\} \\
& \overbrace{rd(x1, \underbrace{asoc(op_r(x2, x1, x3), x1, x1)})} \\
= \quad & \text{by Lemma 765 RL with } \{x2 \leftarrow op_r(x2, x1, x3), x1 \leftarrow x1\} \\
& \underbrace{rd(x1, \overbrace{rd(op_l(x1, \underbrace{op_r(x2, x1, x3)}, x1), x1)})} \\
= \quad & \text{by Lemma 1406 LR with } \{x3 \leftarrow x1, x2 \leftarrow op_r(x2, x1, x3), x1 \leftarrow x1\} \\
& \overbrace{mult(x1, rd(x1, \underbrace{op_l(x1, op_r(x2, x1, x3), x1)}))} \\
= \quad & \text{by Lemma 608 RL with } \{x2 \leftarrow op_r(x2, x1, x3), x1 \leftarrow x1\} \\
& mult(x1, rd(x1, \overbrace{op_l(x1, x1, \underbrace{i(op_r(x2, x1, x3))})})) \\
= \quad & \text{by Lemma 891 RL with } \{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x1\} \\
& mult(x1, rd(x1, \underbrace{op_l(x1, x1, \overbrace{rd(mult(x1, x3), mult(mult(x2, x1), x3))})})) \\
= \quad & \text{by Lemma 1851 LR with } \{x3 \leftarrow mult(mult(x2, x1), x3), x2 \leftarrow x3, x1 \leftarrow x1\} \\
& mult(x1, rd(x1, \overbrace{op_l(x1, \underbrace{rd(mult(mult(x2, x1), x3), x3)}, x1)})) \\
= \quad & \text{by Axiom 5 RL with } \{x2 \leftarrow x3, x1 \leftarrow mult(x2, x1)\} \\
& mult(x1, rd(x1, \underbrace{op_l(x1, \overbrace{mult(x2, x1)}, x1)})) \\
= \quad & \text{by Lemma 285 LR with } \{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{mult(x1, rd(x1, \overbrace{op_l(x1, x2, x1)}))} \\
= \quad & \text{by Lemma 1406 RL with } \{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{rd(x1, \underbrace{rd(op_l(x1, x2, x1), x1)})} \\
= \quad & \text{by Lemma 765 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{rd(x1, \overbrace{asoc(x2, x1, x1)})} \\
= \quad & \text{by Lemma 773 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x1, x2)}
\end{aligned}
$$

Lemma 1853: $op_l(x1, x1, mult(x2, i(x3))) = op_l(x1, x1, mult(x2, op_t(rd(x1, x3), x1)))$

$$
\begin{aligned}
& \underbrace{op_l(x1, x1, mult(x2, i(x3)))} \\
= \quad & \text{by Lemma 1661 RL with } \{x3 \leftarrow i(x3), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, x1, mult(x2, op_t(i(x3), x1)))}} \\
= \quad & \text{by Lemma 1824 RL with } \{x3 \leftarrow op_t(i(x3), x1), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x1, mult(x2, \underbrace{mult(op_t(i(x3), x1), x1)}))} \\
= \quad & \text{by Lemma 340 LR with } \{x2 \leftarrow x1, x1 \leftarrow x3\} \\
& op_l(x1, x1, mult(x2, \overbrace{op_t(rd(x1, x3), x1)}))
\end{aligned}
$$

Lemma 1854: $op_l(x1, rd(x3, x2), x1) = op_l(x1, x1, mult(x2, rd(x1, x3)))$

$$
\begin{aligned}
& \underbrace{op_l(x1, rd(x3, x2), x1)} \\
= \quad & \text{by Lemma 1742 LR with } \{x2 \leftarrow x2, x3 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, x1, mult(x2, i(x3)))}} \\
= \quad & \text{by Lemma 1853 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, x1, mult(x2, op_t(rd(x1, x3), x1)))}} \\
= \quad & \text{by Lemma 1661 LR with } \{x3 \leftarrow rd(x1, x3), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x1, mult(x2, rd(x1, x3)))}
\end{aligned}
$$

Lemma 1855: $op_l(x1, rd(x3, x2), x1) = op_l(x1, x1, rd(x2, rd(x3, x1)))$

$$
\begin{aligned}
& \underbrace{op_l(x1, rd(x3, x2), x1)} \\
= \quad & \text{by Lemma 1854 LR with } \{x2 \leftarrow x2, x3 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, x1, mult(x2, rd(x1, x3)))}} \\
= \quad & \text{by Lemma 1748 RL with } \{x3 \leftarrow rd(x1, x3), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x1, rd(x2, \underbrace{i(rd(x1, x3))}))} \\
= \quad & \text{by Lemma 10 LR with } \{x2 \leftarrow x3, x1 \leftarrow x1\} \\
& op_l(x1, x1, rd(x2, \overbrace{rd(x3, x1)}))
\end{aligned}
$$

Lemma 1856: $asoc(x3, rd(x2, x1), x3) = asoc(x3, rd(x2, mult(x3, x1)), x3)$

$\overbrace{asoc(x3, rd(x2, x1), x3)}$

$=$ by Lemma 759 RL with $\{x2 \leftarrow x3, x1 \leftarrow rd(x2, x1)\}$

$\overbrace{i(asoc(rd(x2, x1), x3, x3))}$

$=$ by Lemma 1478 RL with $\{x4 \leftarrow x3, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{asoc(rd(x1, x2), i(x3), i(x3))}$

$=$ by Lemma 1751 RL with $\{x3 \leftarrow i(x3), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{asoc(mult(x1, i(x2)), i(x3), i(x3))}$

$=$ by Lemma 1826 RL with $\{x1 \leftarrow i(x3), x3 \leftarrow i(x2), x2 \leftarrow x1\}$

$\overbrace{asoc(mult(x1, \underbrace{mult(i(x2), i(x3))}), i(x3), i(x3))}$

$=$ by Axiom 11 LR with $\{x2 \leftarrow x3, x1 \leftarrow x2\}$

$\underbrace{asoc(mult(x1, \overbrace{i(mult(x2, x3))}), i(x3), i(x3))}$

$=$ by Lemma 1751 LR with $\{x3 \leftarrow i(x3), x2 \leftarrow mult(x2, x3), x1 \leftarrow x1\}$

$\overbrace{asoc(rd(x1, mult(x2, x3)), i(x3), i(x3))}$

$=$ by Lemma 1478 LR with $\{x4 \leftarrow x3, x3 \leftarrow x3, x2 \leftarrow mult(x2, x3), x1 \leftarrow x1\}$

$\overbrace{i(asoc(rd(\underbrace{mult(x2, x3)}, x1), x3, x3))}$

$=$ by Lemma 759 LR with $\{x2 \leftarrow x3, x1 \leftarrow rd(mult(x2, x3), x1)\}$

$\overbrace{asoc(x3, rd(\underbrace{mult(x2, x3)}, x1), x3)}$

$=$ by Lemma 13 RL with $\{x2 \leftarrow x2, x1 \leftarrow x3\}$

$asoc(x3, \underbrace{rd(\overbrace{mult(x3, op_t(x2, x3))}, x1)}, x3)$

$=$ by Lemma 1152 RL with $\{x3 \leftarrow x1, x2 \leftarrow op_t(x2, x3), x1 \leftarrow x3\}$

$\underbrace{asoc(x3, \overbrace{mult(x3, mult(rd(op_t(x2, x3), mult(x3, x1)), x3))}, x3)}$

$=$ by Lemma 618 LR with $\{x2 \leftarrow mult(rd(op_t(x2, x3), mult(x3, x1)), x3), x1 \leftarrow x3\}$

$\overbrace{asoc(x3, mult(rd(op_t(x2, x3), mult(x3, x1)), x3), x3)}$

$=$ by Lemma 262 LR with $\{x2 \leftarrow rd(op_t(x2, x3), mult(x3, x1)), x1 \leftarrow x3\}$

$\overbrace{asoc(x3, \underbrace{rd(op_t(x2, x3), mult(x3, x1))}, x3)}$

$=$ by Lemma 1395 RL with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow mult(x3, x1)\}$

$\overbrace{asoc(rd(mult(x3, x1), x2), x3, x3)}$

$=$ by Lemma 938 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow mult(x3, x1)\}$

$\overbrace{asoc(x3, rd(x2, mult(x3, x1)), x3)}$

Lemma 1857: $asoc(i(x1), mult(x2, rd(x1, x3)), i(x1)) = asoc(i(x1), mult(x2, i(x3)), i(x1))$

$$
\begin{aligned}
& \overbrace{asoc(i(x1), mult(x2, rd(x1, x3)), i(x1))} \\
= & \quad \text{by Lemma 617 RL with } \{x2 \leftarrow mult(x2, rd(x1, x3)), x1 \leftarrow i(x1)\} \\
& \overbrace{rd(i(x1), op_l(i(x1), mult(x2, rd(x1, x3)), i(x1)))} \\
= & \quad \text{by Lemma 1828 RL with } \{x3 \leftarrow rd(x1, x3), x2 \leftarrow x2, x1 \leftarrow i(x1)\} \\
& \overbrace{asoc(i(x1), mult(x2, \underbrace{mult(rd(x1, x3), i(x1))}), i(x1))} \\
= & \quad \text{by Lemma 22 LR with } \{x2 \leftarrow x3, x1 \leftarrow x1\} \\
& asoc(i(x1), mult(x2, \overbrace{i(x3)}), i(x1))
\end{aligned}
$$

Lemma 1858: $asoc(mult(mult(x1, x2), x3), x1, x1) = mult(i(x1), op_l(x1, mult(x3, x2), x1))$

$$
\begin{aligned}
& \overbrace{asoc(mult(mult(x1, x2), x3), x1, x1)} \\
= & \quad \text{by Lemma 765 RL with } \{x2 \leftarrow mult(mult(x1, x2), x3), x1 \leftarrow x1\} \\
& \overbrace{rd(op_l(x1, mult(mult(x1, x2), x3), x1), x1)} \\
= & \quad \text{by Lemma 171 RL with } \{x3 \leftarrow x1, x2 \leftarrow mult(mult(x1, x2), x3), x1 \leftarrow x1\} \\
& \overbrace{mult(i(x1), \underbrace{op_l(x1, mult(mult(x1, x2), x3), x1)})} \\
= & \quad \text{by Lemma 1834 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& mult(i(x1), \overbrace{op_l(x1, mult(x3, x2), x1)})
\end{aligned}
$$

Lemma 1859: $asoc(mult(mult(x1, x2), x3), x1, x1) = asoc(mult(x3, x2), x1, x1)$

$$
\begin{aligned}
& \overbrace{asoc(mult(mult(x1, x2), x3), x1, x1)} \\
= & \quad \text{by Lemma 1858 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{mult(i(x1), op_l(x1, mult(x3, x2), x1))} \\
= & \quad \text{by Lemma 171 LR with } \{x3 \leftarrow x1, x2 \leftarrow mult(x3, x2), x1 \leftarrow x1\} \\
& \overbrace{rd(op_l(x1, mult(x3, x2), x1), x1)} \\
= & \quad \text{by Lemma 765 LR with } \{x2 \leftarrow mult(x3, x2), x1 \leftarrow x1\} \\
& \overbrace{asoc(mult(x3, x2), x1, x1)}
\end{aligned}
$$

Lemma 1860: $asoc(mult(x2, op_l(x1, x3, x4)), x1, x1) = rd(op_l(x1, x2, x1), x1)$

$$
\begin{aligned}
& \overbrace{asoc(mult(x2, op_l(x1, x3, x4)), x1, x1)} \\
= & \quad \text{by Lemma 765 RL with } \{x2 \leftarrow mult(x2, op_l(x1, x3, x4)), x1 \leftarrow x1\} \\
& \overbrace{rd(op_l(x1, mult(x2, op_l(x1, x3, x4)), x1), x1)} \\
= & \quad \text{by Lemma 171 RL with } \{x3 \leftarrow x1, x2 \leftarrow mult(x2, op_l(x1, x3, x4)), x1 \leftarrow x1\} \\
& \overbrace{mult(i(x1), \underbrace{op_l(x1, mult(x2, op_l(x1, x3, x4)), x1)})} \\
= & \quad \text{by Lemma 1822 LR with } \{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{mult(i(x1), \overbrace{op_l(x1, x2, x1)})} \\
= & \quad \text{by Lemma 171 LR with } \{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{rd(op_l(x1, x2, x1), x1)}
\end{aligned}
$$

Lemma 1861: $asoc(i(rd(x3,x2)),x2,x2) = asoc(op_l(i(x3),rd(x3,x2),x1),i(x2),i(x2))$

$\underbrace{asoc(i(rd(x3,x2)),x2,x2)}$

$=$ by Lemma 1736 RL with $\{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow i(rd(x3,x2))\}$

$\overbrace{asoc(\underbrace{op_r(i(rd(x3,x2)),x1,x1)},x2,x2)}$

$=$ by Lemma 354 RL with $\{x2 \leftarrow rd(x3,x2), x1 \leftarrow x1\}$

$\underbrace{asoc(\overbrace{rd(rd(x1,rd(x3,x2)),x1)},x2,x2)}$

$=$ by Lemma 1727 LR with $\{x3 \leftarrow x1, x1 \leftarrow x2, x2 \leftarrow rd(rd(x1,rd(x3,x2)),x1)\}$

$\overbrace{asoc(\underbrace{op_t(rd(rd(x1,rd(x3,x2)),x1),x1)},x2,x2)}$

$=$ by Lemma 14 LR with $\{x2 \leftarrow x1, x1 \leftarrow rd(x1,rd(x3,x2))\}$

$asoc(\overbrace{mult(i(x1),rd(x1,rd(x3,x2)))},x2,x2)$

$=$ by Lemma 56 RL with $\{x2 \leftarrow rd(x3,x2), x1 \leftarrow x1\}$

$asoc(\overbrace{mult(\underbrace{i(mult(x1,rd(x3,x2)))},x1)},x2,x2)$

$=$ by Lemma 45 LR with $\{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x1\}$

$\underbrace{asoc(mult(\overbrace{mult(i(x1),rd(x2,x3))},x1),x2,x2)}$

$=$ by Lemma 772 RL with $\{x2 \leftarrow x2, x1 \leftarrow mult(mult(i(x1),rd(x2,x3)),x1)\}$

$\overbrace{\underbrace{asoc(mult(mult(i(x1),rd(x2,x3)),x1),i(x2),i(x2))}}$

$=$ by Lemma 1826 RL with $\{x1 \leftarrow i(x2), x3 \leftarrow x1, x2 \leftarrow mult(i(x1),rd(x2,x3))\}$

$\overbrace{asoc(mult(\underbrace{mult(i(x1),rd(x2,x3))},mult(x1,i(x2))),i(x2),i(x2))}$

$=$ by Lemma 45 RL with $\{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x1\}$

$asoc(mult(\overbrace{i(mult(x1,rd(x3,x2)))},mult(x1,\underbrace{i(x2)})),i(x2),i(x2))$

$=$ by Lemma 22 RL with $\{x2 \leftarrow x2, x1 \leftarrow x3\}$

$asoc(\underbrace{mult(i(mult(x1,rd(x3,x2))),mult(x1,\overbrace{mult(rd(x3,x2),i(x3))}))},i(x2),i(x2))$

$=$ by Axiom 12 LR with $\{x3 \leftarrow i(x3), x2 \leftarrow rd(x3,x2), x1 \leftarrow x1\}$

$asoc(\overbrace{op_l(i(x3),rd(x3,x2),x1)},i(x2),i(x2))$

Lemma 1862: $asoc(x2,x3,x2) = asoc(op_l(i(x3),rd(x3,x2),x1),i(x2),i(x2))$

$\underbrace{asoc(x2,x3,x2)}$

$=$ by Lemma 263 RL with $\{x2 \leftarrow x3, x1 \leftarrow x2\}$

$\overbrace{\underbrace{asoc(x2,rd(x3,x2),x2)}}$

$=$ by Lemma 758 RL with $\{x2 \leftarrow x2, x1 \leftarrow rd(x3,x2)\}$

$\overbrace{\underbrace{asoc(i(rd(x3,x2)),x2,x2)}}$

$=$ by Lemma 1861 LR with $\{x1 \leftarrow x1, x2 \leftarrow x2, x3 \leftarrow x3\}$

$\overbrace{asoc(op_l(i(x3),rd(x3,x2),x1),i(x2),i(x2))}$

Lemma 1863: $asoc(x2, x3, x2) = asoc(i(x2), op_l(x3, rd(x3, x2), x1), i(x2))$

$$
\begin{aligned}
& \underbrace{asoc(x2, x3, x2)} \\
= \quad & \text{by Lemma 1862 LR with } \{x1 \leftarrow x1, x3 \leftarrow x3, x2 \leftarrow x2\} \\
& \overbrace{asoc(\underbrace{op_l(i(x3), rd(x3, x2), x1)}, i(x2), i(x2))} \\
= \quad & \text{by Lemma 1049 RL with } \{x3 \leftarrow x1, x2 \leftarrow rd(x3, x2), x1 \leftarrow x3\} \\
& \underbrace{asoc(\overbrace{i(op_l(x3, rd(x3, x2), x1))}, i(x2), i(x2))} \\
= \quad & \text{by Lemma 758 LR with } \{x2 \leftarrow i(x2), x1 \leftarrow op_l(x3, rd(x3, x2), x1)\} \\
& \overbrace{asoc(i(x2), op_l(x3, rd(x3, x2), x1), i(x2))}
\end{aligned}
$$

Lemma 1864: $asoc(rd(mult(x1, x2), rd(x2, x3)), i(x1), i(x1)) = asoc(op_l(x3, rd(x2, x3), x1), i(x1), i(x1))$

$$
\begin{aligned}
& \underbrace{asoc(rd(mult(x1, x2), rd(x2, x3)), i(x1), i(x1))} \\
= \quad & \text{by Lemma 1751 RL with } \{x3 \leftarrow i(x1), x2 \leftarrow rd(x2, x3), x1 \leftarrow mult(x1, x2)\} \\
& \overbrace{asoc(mult(mult(x1, x2), \underbrace{i(rd(x2, x3))}), i(x1), i(x1))} \\
= \quad & \text{by Lemma 10 LR with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& \underbrace{asoc(mult(mult(x1, x2), \overbrace{rd(x3, x2)}), i(x1), i(x1))} \\
= \quad & \text{by Lemma 1859 RL with } \{x3 \leftarrow mult(x1, x2), x2 \leftarrow rd(x3, x2), x1 \leftarrow i(x1)\} \\
& \overbrace{asoc(\underbrace{mult(mult(i(x1), rd(x3, x2)), mult(x1, x2))}, i(x1), i(x1))} \\
= \quad & \text{by Lemma 280 RL with } \{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x3\} \\
& asoc(\overbrace{op_l(x3, rd(x2, x3), x1)}, i(x1), i(x1))
\end{aligned}
$$

Lemma 1865: $asoc(x1, rd(rd(x2, x3), mult(x1, x2)), x1) = asoc(op_l(x3, rd(x2, x3), x1), i(x1), i(x1))$

$$
\begin{aligned}
& \underbrace{asoc(x1, rd(rd(x2, x3), mult(x1, x2)), x1)} \\
= \quad & \text{by Lemma 759 RL with } \{x2 \leftarrow x1, x1 \leftarrow rd(rd(x2, x3), mult(x1, x2))\} \\
& \underbrace{\overbrace{i(asoc(rd(rd(x2, x3), mult(x1, x2)), x1, x1))}} \\
= \quad & \text{by Lemma 1478 RL with } \{x4 \leftarrow x1, x3 \leftarrow x1, x2 \leftarrow rd(x2, x3), x1 \leftarrow mult(x1, x2)\} \\
& \underbrace{\overbrace{asoc(rd(mult(x1, x2), rd(x2, x3)), i(x1), i(x1))}} \\
= \quad & \text{by Lemma 1864 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{asoc(op_l(x3, rd(x2, x3), x1), i(x1), i(x1))}
\end{aligned}
$$

Lemma 1866: $asoc(x1, op_r(i(x3), x2, x2), x1) = asoc(op_l(x3, rd(x2, x3), x1), i(x1), i(x1))$

$$
\begin{aligned}
& asoc(x1, \underbrace{op_r(i(x3), x2, x2)}, x1) \\
= \quad & \text{by Lemma 354 RL with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& \underbrace{asoc(x1, \overbrace{rd(rd(x2, x3), x2)}, x1)} \\
= \quad & \text{by Lemma 1856 LR with } \{x1 \leftarrow x2, x2 \leftarrow rd(x2, x3), x3 \leftarrow x1\} \\
& \underbrace{\overbrace{asoc(x1, rd(rd(x2, x3), mult(x1, x2)), x1)}} \\
= \quad & \text{by Lemma 1865 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{asoc(op_l(x3, rd(x2, x3), x1), i(x1), i(x1))}
\end{aligned}
$$

Lemma 1867: $asoc(x3, x1, x1) = asoc(op_l(x3, rd(x2, x3), x1), i(x1), i(x1))$

$$
\begin{aligned}
& \underbrace{asoc(x3, x1, x1)} \\
= \quad & \text{by Lemma 756 RL with } \{x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{asoc(x1, i(x3), x1)} \\
= \quad & \text{by Lemma 1734 RL with } \{x3 \leftarrow x2, x2 \leftarrow i(x3), x1 \leftarrow x1\} \\
& \overbrace{asoc(x1, op_r(i(x3), x2, x2), x1)} \\
= \quad & \text{by Lemma 1866 LR with } \{x2 \leftarrow x2, x3 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{asoc(op_l(x3, \overbrace{rd(x2, x3)}, x1), i(x1), i(x1))}
\end{aligned}
$$

Lemma 1868: $asoc(x1, x2, x1) = asoc(x1, op_l(rd(x2, x1), x2, x3), x1)$

$$
\begin{aligned}
& \underbrace{asoc(x1, x2, x1)} \\
= \quad & \text{by Lemma 263 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{asoc(x1, rd(x2, x1), x1)} \\
= \quad & \text{by Lemma 1808 RL with } \{x3 \leftarrow rd(x2, x1), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{asoc(op_r(x1, x2, x2), rd(x2, x1), op_r(x1, x2, x2))} \\
= \quad & \text{by Lemma 613 LR with } \{x2 \leftarrow rd(x2, x1), x1 \leftarrow op_r(x1, x2, x2)\} \\
& \overbrace{asoc(i(op_r(x1, x2, x2)), rd(x2, x1), i(op_r(x1, x2, x2)))} \\
= \quad & \text{by Lemma 1631 RL with } \{x3 \leftarrow rd(x2, x1), x2 \leftarrow rd(op_r(x1, x2, x2), rd(x2, x1)), x1 \leftarrow i(op_r(x1, x2, x2))\} \\
& \overbrace{asoc(op_t(i(op_r(x1, x2, x2)), rd(op_r(x1, x2, x2), rd(x2, x1))), rd(x2, x1), op_t(i(op_r(x1, x2, x2)), rd(op_r(x1, x2, x2), rd(x2, x1)}} \\
= \quad & \text{by Lemma 1863 LR with } \{x1 \leftarrow x3, x3 \leftarrow rd(x2, x1), x2 \leftarrow op_t(i(op_r(x1, x2, x2)), rd(op_r(x1, x2, x2), rd(x2, x1)))\} \\
& \overbrace{asoc(i(op_t(i(op_r(x1, x2, x2)), rd(op_r(x1, x2, x2), rd(x2, x1)))), op_l(rd(x2, x1), rd(rd(x2, x1), op_t(i(op_r(x1, x2, x2)), rd(op_r(x} \\
= \quad & \text{by Lemma 613 RL with } \{x2 \leftarrow op_l(rd(x2, x1), rd(rd(x2, x1), op_t(i(op_r(x1, x2, x2)), rd(op_r(x1, x2, x2), rd(x2, x1)))), x3), x1 \leftarrow \\
& \overbrace{asoc(op_t(i(op_r(x1, x2, x2)), rd(op_r(x1, x2, x2), rd(x2, x1))), op_l(rd(x2, x1), \underbrace{rd(rd(x2, x1), op_t(i(op_r(x1, x2, x2)), rd(op_r(x1,} \\
= \quad & \text{by Lemma 386 LR with } \{x2 \leftarrow op_r(x1, x2, x2), x1 \leftarrow rd(x2, x1)\} \\
& \underbrace{asoc(op_t(i(op_r(x1, x2, x2)), rd(op_r(x1, x2, x2), rd(x2, x1))), op_l(rd(x2, x1), \overbrace{mult(op_r(x1, x2, x2), rd(x2, x1))}, x3), op_t(i(op_r(} \\
= \quad & \text{by Lemma 1631 LR with } \{x3 \leftarrow op_l(rd(x2, x1), mult(op_r(x1, x2, x2), rd(x2, x1)), x3), x2 \leftarrow rd(op_r(x1, x2, x2), rd(x2, x1)), x1 \\
& \overbrace{asoc(i(op_r(x1, x2, x2)), op_l(rd(x2, x1), mult(op_r(x1, x2, x2), rd(x2, x1)), x3), i(op_r(x1, x2, x2)))} \\
= \quad & \text{by Lemma 613 RL with } \{x2 \leftarrow op_l(rd(x2, x1), mult(op_r(x1, x2, x2), rd(x2, x1)), x3), x1 \leftarrow op_r(x1, x2, x2)\} \\
& \overbrace{asoc(op_r(x1, x2, x2), op_l(rd(x2, x1), \underbrace{mult(op_r(x1, x2, x2), rd(x2, x1))}, x3), op_r(x1, x2, x2))} \\
= \quad & \text{by Lemma 366 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{asoc(op_r(x1, x2, x2), op_l(rd(x2, x1), \overbrace{x2}, x3), op_r(x1, x2, x2))} \\
= \quad & \text{by Lemma 1808 LR with } \{x3 \leftarrow op_l(rd(x2, x1), x2, x3), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{asoc(x1, op_l(rd(x2, x1), x2, x3), x1)}
\end{aligned}
$$

Lemma 1869: $rd(x1, asoc(x1, i(x2), i(x3))) = mult(asoc(i(x1), x2, x3), x1)$

$rd(x1, \underbrace{asoc(x1, i(x2), i(x3))})$

$=$ by Lemma 1479 RL with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{rd(x1, \overbrace{i(asoc(i(x1), x2, x3))})}$

$=$ by Lemma 357 RL with $\{x2 \leftarrow x1, x1 \leftarrow asoc(i(x1), x2, x3)\}$

$\overbrace{mult(op_r(\overbrace{asoc(i(x1), x2, x3)}, x1, x1), \underbrace{x1})}$

$=$ by Axiom 5 LR with $\{x2 \leftarrow asoc(x1, asoc(i(x1), x2, x3), x1), x1 \leftarrow x1\}$

$mult(op_r(asoc(i(x1), x2, x3), x1, x1), \overbrace{rd(\underbrace{mult(x1, asoc(x1, asoc(i(x1), x2, x3), x1))}, asoc(x1, asoc(i(x1), x2, x3), x1))})$

$=$ by Lemma 158 LR with $\{x2 \leftarrow asoc(i(x1), x2, x3), x1 \leftarrow x1\}$

$mult(op_r(asoc(i(x1), x2, x3), x1, x1), rd(\overbrace{op_l(x1, x1, asoc(i(x1), x2, x3))}, \underbrace{asoc(x1, asoc(i(x1), x2, x3), x1)}))$

$=$ by Lemma 1840 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$mult(op_r(asoc(i(x1), x2, x3), x1, x1), rd(op_l(x1, x1, asoc(i(x1), x2, x3)), \overbrace{\underbrace{mult(x1, i(x1))}}))$

$=$ by Axiom 7 LR with $\{x1 \leftarrow x1\}$

$mult(op_r(asoc(i(x1), x2, x3), x1, x1), \underbrace{rd(op_l(x1, x1, asoc(i(x1), x2, x3)), \overbrace{unit()})})$

$=$ by Lemma 1 LR with $\{x1 \leftarrow op_l(x1, x1, asoc(i(x1), x2, x3))\}$

$\underbrace{mult(op_r(asoc(i(x1), x2, x3), x1, x1), \overbrace{op_l(x1, x1, asoc(i(x1), x2, x3))})}$

$=$ by Lemma 873 RL with $\{x2 \leftarrow asoc(i(x1), x2, x3), x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_t(op_r(mult(x1, \overbrace{asoc(i(x1), x2, x3)}), x1, x1), x1)}}$

$=$ by Axiom 14 LR with $\{x4 \leftarrow x1, x3 \leftarrow x1, x2 \leftarrow x1, x1 \leftarrow mult(x1, asoc(i(x1), x2, x3))\}$

$\overbrace{\underbrace{op_r(op_t(mult(x1, \overbrace{asoc(i(x1), x2, x3)}), x1), x1, x1)}}$

$=$ by Lemma 385 LR with $\{x2 \leftarrow asoc(i(x1), x2, x3), x1 \leftarrow x1\}$

$\overbrace{mult(\overbrace{asoc(i(x1), x2, x3)}, x1)}$

Lemma 1870: $rd(x1, asoc(i(x2), i(x1), x3)) = mult(i(asoc(i(x2), i(x1), x3)), x1)$

$rd(x1, \underbrace{asoc(i(x2), i(x1), x3)})$

$=$ by Lemma 1497 RL with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{rd(x1, \overbrace{asoc(x1, rd(i(x1), mult(x1, x2)), x3)})}$

$=$ by Lemma 1842 RL with $\{x3 \leftarrow x3, x2 \leftarrow rd(i(x1), mult(x1, x2)), x1 \leftarrow x1\}$

$\overbrace{mult(i(\underbrace{asoc(x1, rd(i(x1), mult(x1, x2)), x3)}), x1)}$

$=$ by Lemma 1497 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$mult(i(\overbrace{asoc(i(x2), i(x1), x3)}), x1)$

Lemma 1871: $rd(x1, asoc(i(x2), i(x1), x3)) = mult(asoc(x2, x1, i(x3)), x1)$

$$
\begin{aligned}
& \underbrace{rd(x1, asoc(i(x2), i(x1), x3))} \\
= & \quad \text{by Lemma 1870 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{mult(\underbrace{i(asoc(i(x2), i(x1), x3))}, x1)} \\
= & \quad \text{by Lemma 1479 LR with } \{x3 \leftarrow x3, x2 \leftarrow i(x1), x1 \leftarrow x2\} \\
& mult(\overbrace{asoc(x2, \underbrace{i(i(x1))}, i(x3))}, x1) \\
= & \quad \text{by Lemma 3 LR with } \{x1 \leftarrow x1\} \\
& mult(asoc(x2, \overbrace{x1}, i(x3)), x1)
\end{aligned}
$$

Lemma 1872: $op_l(x1, x1, rd(mult(i(x1), x3), x2)) = op_l(x1, x1, rd(x3, x2))$

$$
\begin{aligned}
& \underbrace{op_l(x1, x1, rd(mult(i(x1), x3), x2))} \\
= & \quad \text{by Lemma 1589 LR with } \{x3 \leftarrow x2, x2 \leftarrow mult(i(x1), x3), x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, rd(x2, mult(i(x1), x3)), x1)}} \\
= & \quad \text{by Lemma 1851 RL with } \{x3 \leftarrow x2, x2 \leftarrow mult(i(x1), x3), x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x1, rd(\underbrace{mult(x1, mult(i(x1), x3))}, x2))} \\
= & \quad \text{by Lemma 12 LR with } \{x2 \leftarrow x3, x1 \leftarrow x1\} \\
& op_l(x1, x1, rd(\overbrace{x3}, x2))
\end{aligned}
$$

Lemma 1873: $op_l(x1, x2, op_l(op_l(x1, x3, x4), x5, x6)) = op_l(x1, x2, op_l(x1, x3, x4))$

$$
\begin{aligned}
& \underbrace{op_l(x1, x2, op_l(op_l(x1, x3, x4), x5, x6))} \\
= & \quad \text{by Lemma 1615 RL with } \{x5 \leftarrow op_l(op_l(x1, x3, x4), x5, x6), x4 \leftarrow x2, x3 \leftarrow x4, x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{op_l(\underbrace{op_l(op_l(x1, x3, x4), x2, op_l(op_l(x1, x3, x4), x5, x6))}, x4, x3)} \\
= & \quad \text{by Lemma 1819 LR with } \{x4 \leftarrow x6, x3 \leftarrow x5, x2 \leftarrow x2, x1 \leftarrow op_l(x1, x3, x4)\} \\
& \underbrace{op_l(\overbrace{op_l(op_l(x1, x3, x4), x2, op_l(x1, x3, x4))}, x4, x3)} \\
= & \quad \text{by Lemma 1615 LR with } \{x5 \leftarrow op_l(x1, x3, x4), x4 \leftarrow x2, x3 \leftarrow x4, x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x2, op_l(x1, x3, x4))}
\end{aligned}
$$

Lemma 1874: $mult(x1, mult(rd(x2, x1), x3)) = mult(op_l(x2, x2, x1), op_l(x3, x2, x1))$

$$
\begin{aligned}
& \underbrace{mult(x1, mult(rd(x2, x1), x3))} \\
= & \quad \text{by Lemma 132 RL with } \{x3 \leftarrow x3, x2 \leftarrow rd(x2, x1), x1 \leftarrow x1\} \\
& \overbrace{mult(\underbrace{mult(x1, rd(x2, x1))}, op_l(x3, rd(x2, x1), x1))} \\
= & \quad \text{by Lemma 427 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& mult(\overbrace{op_l(x2, x2, x1)}, \underbrace{op_l(x3, rd(x2, x1), x1)}) \\
= & \quad \text{by Lemma 292 LR with } \{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x3\} \\
& mult(op_l(x2, x2, x1), \overbrace{op_l(x3, x2, x1)})
\end{aligned}
$$

Lemma 1875: $rd(x1, rd(mult(x1, x2), x1)) = mult(x1, rd(x1, op_t(mult(x2, x1), x2)))$

$$
\begin{aligned}
& rd(x1, rd(mult(\underbrace{x1}, x2), x1)) \\
= & \quad \text{by Lemma 3 RL with } \{x1 \leftarrow x1\} \\
& \underbrace{rd(x1, rd(mult(\overbrace{i(i(x1))}, x2), x1))} \\
= & \quad \text{by Lemma 805 RL with } \{x2 \leftarrow mult(i(i(x1)), x2), x1 \leftarrow x1\} \\
& \overbrace{mult(x1, rd(x1, \underbrace{op_t(mult(i(i(x1)), x2), i(x1))}))} \\
= & \quad \text{by Lemma 673 LR with } \{x2 \leftarrow x2, x1 \leftarrow i(x1)\} \\
& mult(x1, rd(x1, \overbrace{op_t(mult(x2, \underbrace{i(i(x1))}), x2)})) \\
= & \quad \text{by Lemma 3 LR with } \{x1 \leftarrow x1\} \\
& mult(x1, rd(x1, op_t(mult(x2, \overbrace{x1}), x2)))
\end{aligned}
$$

Lemma 1876: $op_r(i(rd(x1, op_t(x2, i(x1)))), x2, x2) = rd(mult(x2, rd(x2, x1)), x2)$

$$
\begin{aligned}
& \underbrace{op_r(i(rd(x1, op_t(x2, i(x1)))), x2, x2)} \\
= & \quad \text{by Lemma 354 RL with } \{x2 \leftarrow rd(x1, op_t(x2, i(x1))), x1 \leftarrow x2\} \\
& \overbrace{rd(\underbrace{rd(x2, rd(x1, op_t(x2, i(x1))))}, x2)} \\
= & \quad \text{by Lemma 916 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& rd(\overbrace{mult(x2, rd(x2, \underbrace{i(i(x1))}))}, x2) \\
= & \quad \text{by Lemma 3 LR with } \{x1 \leftarrow x1\} \\
& rd(mult(x2, rd(x2, \overbrace{x1})), x2)
\end{aligned}
$$

Lemma 1877: $op_l(x1, x2, op_l(op_t(x3, x4), x2, x3)) = op_l(x1, i(i(op_t(x3, x4))), i(x2))$

$$
\begin{aligned}
& \underbrace{op_l(x1, x2, op_l(op_t(x3, x4), x2, x3))} \\
= & \quad \text{by Lemma 242 RL with } \{x3 \leftarrow op_l(op_t(x3, x4), x2, x3), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, \underbrace{mult(x2, op_l(op_t(x3, x4), x2, x3))}, i(x2))} \\
= & \quad \text{by Lemma 1016 LR with } \{x3 \leftarrow x4, x2 \leftarrow x3, x1 \leftarrow x2\} \\
& \underbrace{op_l(x1, \overbrace{rd(x2, i(op_t(x3, x4)))}, i(x2))} \\
= & \quad \text{by Lemma 299 LR with } \{x3 \leftarrow i(op_t(x3, x4)), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, i(i(op_t(x3, x4))), i(x2))}
\end{aligned}
$$

Lemma 1878: $x1 = op_t(x1, rd(mult(rd(x2, mult(x3, x1)), x3), x2))$

$\overbrace{x1}$

$=$ by Lemma 442 RL with $\{x2 \leftarrow x1, x1 \leftarrow mult(x3, x2)\}$

$\overbrace{op_l(\underbrace{mult(mult(x3, x2), rd(x1, mult(x3, x2)))}, mult(x3, x2), x1)}$

$=$ by Lemma 189 RL with $\{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow mult(mult(x3, x2), rd(x1, mult(x3, x2)))\}$

$\underbrace{op_l(\overbrace{op_t(mult(mult(x3, x2), rd(x1, mult(x3, x2))), op_l(\underbrace{i(mult(mult(x3, x2), rd(x1, mult(x3, x2))))}, x3, x2))}, mult(x3, x2), x}$

$=$ by Axiom 15 RL with $\{x4 \leftarrow op_l(i(mult(mult(x3, x2), rd(x1, mult(x3, x2)))), x3, x2), x3 \leftarrow x1, x2 \leftarrow mult(x3, x2), x1 \leftarrow mu$

$\overbrace{op_t(\underbrace{op_l(mult(mult(x3, x2), rd(x1, mult(x3, x2))), mult(x3, x2), x1)}, op_l(i(mult(mult(x3, x2), rd(x1, mult(x3, x2)))), x3, x2}$

$=$ by Lemma 442 LR with $\{x2 \leftarrow x1, x1 \leftarrow mult(x3, x2)\}$

$op_t(\overbrace{x1}, op_l(\underbrace{i(mult(mult(x3, x2), rd(x1, mult(x3, x2))))}, x3, x2))$

$=$ by Lemma 45 LR with $\{x3 \leftarrow mult(x3, x2), x2 \leftarrow x1, x1 \leftarrow mult(x3, x2)\}$

$op_t(x1, op_l(\overbrace{mult(i(mult(x3, x2)), rd(mult(x3, x2), x1))}, x3, \underbrace{x2}))$

$=$ by Lemma 3 RL with $\{x1 \leftarrow x2\}$

$op_t(x1, \underbrace{op_l(mult(i(mult(x3, x2)), rd(mult(x3, x2), x1)), x3, \overbrace{i(i(x2))})})$

$=$ by Lemma 1050 RL with $\{x3 \leftarrow i(x2), x2 \leftarrow x3, x1 \leftarrow mult(i(mult(x3, x2)), rd(mult(x3, x2), x1))\}$

$op_t(x1, \overbrace{op_l(mult(\underbrace{i(mult(x3, x2))}, rd(mult(x3, x2), x1)), i(x3), i(x2)))})$

$=$ by Axiom 11 RL with $\{x2 \leftarrow x2, x1 \leftarrow x3\}$

$op_t(x1, \underbrace{op_l(mult(\overbrace{mult(i(x3), i(x2))}, rd(mult(x3, x2), x1)), i(x3), i(x2)))})$

$=$ by Lemma 1068 LR with $\{x3 \leftarrow rd(mult(x3, x2), x1), x2 \leftarrow i(x2), x1 \leftarrow i(x3)\}$

$\underbrace{op_t(x1, \overbrace{mult(i(x2), mult(i(x3), rd(mult(x3, x2), x1)))})}$

$=$ by Lemma 1728 LR with $\{x3 \leftarrow mult(i(x3), rd(mult(x3, x2), x1)), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_t(x1, rd(\underbrace{mult(i(x3), rd(mult(x3, x2), x1))}, x2))}$

$=$ by Lemma 1153 LR with $\{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x3\}$

$op_t(x1, rd(\overbrace{mult(rd(x2, mult(x3, x1)), x3)}, x2))$

Lemma 1879: $mult(mult(x1, rd(x2, x1)), x3) = op_l(mult(x2, op_l(x3, x1, x2)), x2, x1)$

$\underbrace{mult(mult(x1, rd(x2, x1)), x3)}$

$=$ by Lemma 1047 RL with $\{x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow rd(x2, x1)\}$

$\overbrace{op_l(\underbrace{mult(rd(x2, x1), mult(x1, x3))}, rd(x2, x1), x1)}$

$=$ by Lemma 1091 LR with $\{x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow x2\}$

$\underbrace{op_l(\overbrace{mult(x2, op_l(x3, x1, x2))}, rd(x2, x1), x1)}$

$=$ by Lemma 292 LR with $\{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow mult(x2, op_l(x3, x1, x2))\}$

$\overbrace{op_l(mult(x2, op_l(x3, x1, x2)), x2, x1)}$

Lemma 1880: $op_l(mult(x1, op_l(x2, x1, x3)), x3, x1) = mult(op_l(x1, x3, x1), x2)$

$op_l(mult(x1, \underbrace{op_l(x2, x1, x3)}), x3, x1)$

$=$ by Lemma 484 LR with $\{x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow x2\}$

$op_l(mult(x1, \overbrace{op_l(x2, op_l(x3, x3, x1), i(x1))}), x3, x1)$

$=$ by Lemma 1419 RL with $\{x5 \leftarrow x1, x4 \leftarrow x1, x3 \leftarrow x3, x2 \leftarrow x3, x1 \leftarrow x2\}$

$\underbrace{op_l(mult(x1, \overbrace{op_l(x2, op_l(i(x3), x3, x1), x1)}), x3, x1)}$

$=$ by Lemma 1061 RL with $\{x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow mult(x1, op_l(x2, op_l(i(x3), x3, x1), x1))\}$

$\overbrace{\underbrace{op_l(mult(x1, op_l(x2, \overbrace{op_l(i(x3), x3, x1)}, x1)), x1, op_l(i(x3), x3, x1))}}$

$=$ by Lemma 1879 RL with $\{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow op_l(i(x3), x3, x1)\}$

$\overbrace{mult(mult(op_l(i(x3), x3, x1), \underbrace{rd(x1, op_l(i(x3), x3, x1))}), x2)}$

$=$ by Lemma 231 RL with $\{x2 \leftarrow x3, x1 \leftarrow x1\}$

$mult(\underbrace{mult(op_l(i(x3), x3, x1), \overbrace{mult(x1, x3)})}, x2)$

$=$ by Lemma 1782 LR with $\{x2 \leftarrow x1, x3 \leftarrow x3, x1 \leftarrow i(x3)\}$

$mult(\overbrace{op_l(\underbrace{mult(i(x3), mult(x3, x1))}, x3, x1)}, x2)$

$=$ by Axiom 2 RL with $\{x2 \leftarrow x3, x1 \leftarrow x1\}$

$mult(op_l(\overbrace{x1}, x3, x1), x2)$

Lemma 1881: $mult(rd(x1, op_t(x1, mult(x1, x2))), x2) = op_t(op_t(i(i(x2)), x1), i(mult(x1, x2)))$

$\overbrace{mult(rd(x1, op_t(x1, mult(x1, x2))), x2)}$

$=$ by Lemma 1095 RL with $\{x3 \leftarrow x2, x2 \leftarrow mult(x1, x2), x1 \leftarrow x1\}$

$\overbrace{mult(i(op_t(x1, mult(x1, x2))), mult(x1, x2))}$

$=$ by Lemma 17 RL with $\{x2 \leftarrow mult(x1, x2), x1 \leftarrow op_t(x1, mult(x1, x2))\}$

$\overbrace{i(mult(op_t(x1, mult(x1, x2)), i(mult(x1, x2))))}$

$=$ by Axiom 11 RL with $\{x2 \leftarrow i(mult(x1, x2)), x1 \leftarrow op_t(x1, mult(x1, x2))\}$

$\overbrace{mult(\underbrace{i(op_t(x1, mult(x1, x2)))}, i(i(mult(x1, x2))))}$

$=$ by Lemma 15 RL with $\{x2 \leftarrow mult(x1, x2), x1 \leftarrow x1\}$

$\underbrace{mult(\overbrace{op_t(i(x1), i(mult(x1, x2)))}, i(i(mult(x1, x2))))}$

$=$ by Lemma 342 RL with $\{x2 \leftarrow i(mult(x1, x2)), x1 \leftarrow op_t(i(x1), i(mult(x1, x2)))\}$

$\overbrace{rd(\underbrace{op_r(op_t(i(x1), i(mult(x1, x2))), i(mult(x1, x2)), i(i(mult(x1, x2))))}, i(mult(x1, x2)))}$

$=$ by Axiom 14 RL with $\{x4 \leftarrow i(mult(x1, x2)), x3 \leftarrow i(i(mult(x1, x2))), x2 \leftarrow i(mult(x1, x2)), x1 \leftarrow i(x1)\}$

$rd(\overbrace{op_t(\underbrace{op_r(i(x1), i(mult(x1, x2)), i(i(mult(x1, x2))))}, i(mult(x1, x2)))}, i(mult(x1, x2)))$

$=$ by Lemma 404 RL with $\{x2 \leftarrow x1, x1 \leftarrow i(mult(x1, x2))\}$

$\underbrace{rd(op_t(\overbrace{rd(i(mult(x1, x2)), op_t(mult(i(\underbrace{mult(x1, x2)}), x1), i(mult(x1, x2))))}, i(mult(x1, x2))), i(mult(x1, x2)))}$

$=$ by Lemma 345 LR with $\{x2 \leftarrow op_t(mult(i(mult(x1, x2)), x1), i(mult(x1, x2))), x1 \leftarrow i(mult(x1, x2))\}$

$\overbrace{op_t(i(\underbrace{op_t(mult(i(mult(x1, x2)), x1), i(mult(x1, x2))))}, i(mult(x1, x2)))}$

$=$ by Lemma 919 RL with $\{x2 \leftarrow i(mult(x1, x2)), x1 \leftarrow x1\}$

$op_t(i(\overbrace{op_t(mult(x1, i(\underbrace{mult(x1, x2)})), i(x1))}), i(mult(x1, x2)))$

$=$ by Lemma 19 LR with $\{x2 \leftarrow x1, x1 \leftarrow mult(x1, i(mult(x1, x2)))\}$

$op_t(\overbrace{op_t(i(\underbrace{mult(x1, i(\underbrace{mult(x1, x2)}))}), x1)}, i(mult(x1, x2)))$

$=$ by Lemma 20 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$op_t(op_t(i(\overbrace{i(x2)}), x1), i(mult(x1, x2)))$

Lemma 1882: $op_t(x1, rd(x3, rd(x1, op_t(x2, x1)))) = op_t(x1, rd(x3, rd(x1, x2)))$

$$
\begin{aligned}
& \overbrace{op_t(x1, rd(x3, rd(x1, op_t(x2, x1))))} \\
= & \quad \text{by Lemma 1752 RL with } \{x4 \leftarrow x3, x3 \leftarrow x1, x2 \leftarrow op_t(x2, x1), x1 \leftarrow x1\} \\
& \overbrace{\overbrace{op_t(x1, mult(rd(op_t(x2, x1), x1), x3))}} \\
= & \quad \text{by Lemma 1148 RL with } \{x3 \leftarrow x3, x2 \leftarrow rd(op_t(x2, x1), x1), x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, mult(\overbrace{op_r(rd(op_t(x2, x1), x1), x1, x1)}, x3))} \\
= & \quad \text{by Lemma 467 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \overbrace{op_t(x1, mult(\overbrace{mult(i(x1), x2)}, x3))} \\
= & \quad \text{by Lemma 1146 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\overbrace{op_t(x1, mult(rd(x2, x1), x3))}} \\
= & \quad \text{by Lemma 1752 LR with } \{x4 \leftarrow x3, x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, rd(x3, rd(x1, x2)))}
\end{aligned}
$$

Lemma 1883: $op_t(x1, rd(mult(op_t(x2, x1), x1), x3)) = op_t(x1, rd(rd(x3, x1), mult(x1, mult(x2, x1))))$

$$
\begin{aligned}
& \overbrace{op_t(x1, rd(mult(op_t(x2, x1), x1), x3))} \\
= & \quad \text{by Lemma 1269 RL with } \{x3 \leftarrow mult(op_t(x2, x1), x1), x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, rd(rd(x3, x1), \underbrace{mult(x1, mult(op_t(x2, x1), x1))}))} \\
= & \quad \text{by Lemma 72 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{op_t(x1, rd(rd(x3, x1), \overbrace{rd(x1, i(mult(x2, x1)))}))} \\
= & \quad \text{by Lemma 1219 LR with } \{x3 \leftarrow mult(x2, x1), x2 \leftarrow rd(x3, x1), x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, rd(rd(x3, x1), mult(x1, mult(x2, x1))))}
\end{aligned}
$$

Lemma 1884: $rd(mult(x1, x2), op_t(op_t(x1, x2), x3)) = rd(x2, rd(op_t(x1, x3), x1))$

$rd(mult(x1, x2), op_t(\underbrace{op_t(x1, x2)}, x3))$

$=$ by Axiom 10 RL with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$\underbrace{rd(mult(x1, x2), op_t(\overbrace{mult(i(x2), mult(x1, x2))}, x3))}$

$=$ by Lemma 1425 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow mult(x1, x2)\}$

$\overbrace{mult(op_t(\underbrace{rd(x2, mult(x1, x2))}, i(x3)), mult(x1, x2))}$

$=$ by Lemma 9 LR with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$\underbrace{mult(op_t(\overbrace{i(x1)}, i(x3)), mult(x1, x2))}$

$=$ by Lemma 1403 LR with $\{x3 \leftarrow x2, x2 \leftarrow i(x3), x1 \leftarrow x1\}$

$\overbrace{mult(x1, mult(\underbrace{op_t(i(x1), i(x3))}, x2))}$

$=$ by Lemma 15 LR with $\{x2 \leftarrow x3, x1 \leftarrow x1\}$

$mult(\underbrace{x1}, mult(\overbrace{i(op_t(x1, x3))}, x2))$

$=$ by Axiom 5 LR with $\{x2 \leftarrow x3, x1 \leftarrow x1\}$

$mult(\overbrace{rd(\underbrace{mult(x1, x3)}, x3)}, mult(i(op_t(x1, x3)), x2))$

$=$ by Lemma 166 RL with $\{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x1\}$

$mult(\underbrace{rd(\overbrace{mult(op_r(x1, i(op_t(x1, x3))), x2)}, x3)}, mult(i(op_t(x1, x3)), x2))$

$=$ by Axiom 5 RL with $\{x2 \leftarrow x3, x1 \leftarrow op_r(x1, i(op_t(x1, x3))), x2)\}$

$\underbrace{mult(\overbrace{op_r(x1, i(op_t(x1, x3))), x2)}, mult(i(op_t(x1, x3)), x2))}$

$=$ by Lemma 91 LR with $\{x3 \leftarrow x2, x2 \leftarrow i(op_t(x1, x3)), x1 \leftarrow x1\}$

$\overbrace{mult(\underbrace{mult(x1, i(op_t(x1, x3)))}, x2)}$

$=$ by Lemma 98 RL with $\{x2 \leftarrow x3, x1 \leftarrow x1\}$

$\underbrace{mult(\overbrace{rd(x1, op_t(x1, x3))}, x2)}$

$=$ by Lemma 1717 LR with $\{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x1\}$

$\overbrace{rd(x2, rd(op_t(x1, x3), x1))}$

Lemma 1885: $mult(op_t(asoc(x1,x2,x1),i(rd(i(x2),x1))),rd(i(x2),x1)) = rd(asoc(x1,x2,x1),op_t(i(rd(i(x2),x1)),asoc(x1,x2,x$

$mult(\underbrace{op_t(asoc(x1,x2,x1),i(rd(i(x2),x1)))},rd(i(x2),x1))$

$=$ by Lemma 18 RL with $\{x2 \leftarrow rd(i(x2),x1), x1 \leftarrow asoc(x1,x2,x1)\}$

$\underbrace{mult(\overbrace{i(op_t(i(asoc(x1,x2,x1)),rd(i(x2),x1)))},rd(i(x2),x1))}$

$=$ by Lemma 343 RL with $\{x2 \leftarrow op_t(i(asoc(x1,x2,x1)),rd(i(x2),x1)),x1 \leftarrow rd(i(x2),x1)\}$

$\overbrace{rd(rd(i(x2),x1),\underbrace{op_r(op_t(i(asoc(x1,x2,x1)),rd(i(x2),x1)),rd(i(x2),x1),i(rd(i(x2),x1)))})}$

$=$ by Axiom 14 RL with $\{x4 \leftarrow rd(i(x2),x1), x3 \leftarrow i(rd(i(x2),x1)), x2 \leftarrow rd(i(x2),x1), x1 \leftarrow i(asoc(x1,x2,x1))\}$

$rd(rd(i(x2),x1),\overbrace{op_t(\underbrace{op_r(i(asoc(x1,x2,x1)),rd(i(x2),x1),i(rd(i(x2),x1)))},rd(i(x2),x1))})$

$=$ by Lemma 404 RL with $\{x2 \leftarrow asoc(x1,x2,x1), x1 \leftarrow rd(i(x2),x1)\}$

$\underbrace{rd(rd(i(x2),x1),op_t(\overbrace{rd(rd(i(x2),x1),op_t(mult(rd(i(x2),x1),asoc(x1,x2,x1)),rd(i(x2),x1)))},rd(i(x2),x1)))}$

$=$ by Lemma 344 LR with $\{x2 \leftarrow op_t(mult(rd(i(x2),x1),asoc(x1,x2,x1)),rd(i(x2),x1)),x1 \leftarrow rd(i(x2),x1)\}$

$\overbrace{op_t(\underbrace{op_t(mult(rd(i(x2),x1),asoc(x1,x2,x1)),rd(i(x2),x1))},i(rd(i(x2),x1)))}$

$=$ by Lemma 991 LR with $\{x1 \leftarrow asoc(x1,x2,x1), x2 \leftarrow rd(i(x2),x1)\}$

$\underbrace{op_t(\overbrace{op_r(mult(asoc(x1,x2,x1),rd(i(x2),x1)),asoc(x1,x2,x1),rd(i(x2),x1))},i(rd(i(x2),x1)))}$

$=$ by Axiom 14 LR with $\{x4 \leftarrow i(rd(i(x2),x1)), x3 \leftarrow rd(i(x2),x1), x2 \leftarrow asoc(x1,x2,x1), x1 \leftarrow mult(asoc(x1,x2,x1),rd(i(x2)$

$\overbrace{op_r(\underbrace{op_t(mult(asoc(x1,x2,x1),rd(i(x2),x1)),i(rd(i(x2),x1)))},asoc(x1,x2,x1),rd(i(x2),x1))}$

$=$ by Lemma 326 LR with $\{x2 \leftarrow rd(i(x2),x1), x1 \leftarrow asoc(x1,x2,x1)\}$

$op_r(\overbrace{rd(rd(i(x2),x1),op_t(i(asoc(x1,x2,x1)),rd(i(x2),x1)))},asoc(x1,x2,x1),rd(i(x2),x1))$

$=$ by Lemma 748 LR with $\{x2 \leftarrow i(asoc(x1,x2,x1)), x1 \leftarrow rd(i(x2),x1)\}$

$op_r(\overbrace{rd(op_t(rd(i(x2),x1),i(asoc(x1,x2,x1))),i(asoc(x1,x2,x1)))},asoc(x1,x2,x1),rd(i(x2),x1))$

$=$ by Lemma 65 LR with $\{x3 \leftarrow asoc(x1,x2,x1), x2 \leftarrow asoc(x1,x2,x1), x1 \leftarrow rd(i(x2),x1)\}$

$op_r(\overbrace{rd(asoc(x1,x2,x1),op_t(i(rd(i(x2),x1)),asoc(x1,x2,x1)))},\underbrace{asoc(x1,x2,x1)},rd(i(x2),x1))$

$=$ by Lemma 262 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$op_r(rd(asoc(x1,x2,x1),op_t(i(rd(i(x2),x1)),asoc(x1,x2,x1))),\overbrace{\underbrace{asoc(x1,mult(x2,x1),x1)}},rd(i(x2),x1))$

$=$ by Lemma 640 LR with $\{x2 \leftarrow mult(x2,x1), x1 \leftarrow x1\}$

$op_r(rd(asoc(x1,x2,x1),op_t(i(rd(i(x2),x1)),asoc(x1,x2,x1))),\overbrace{rd(mult(x2,x1),\underbrace{op_r(mult(x2,x1),x1,x1)})},rd(i(x2),x1))$

$=$ by Lemma 359 LR with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$\underbrace{op_r(rd(asoc(x1,x2,x1),op_t(i(rd(i(x2),x1)),asoc(x1,x2,x1))),rd(mult(x2,x1),\overbrace{rd(x1,i(x2))}),rd(i(x2),x1))}$

$=$ by Lemma 1241 RL with $\{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow rd(asoc(x1,x2,x1),op_t(i(rd(i(x2),x1)),asoc(x1,x2,x1)))\}$

$\overbrace{rd(asoc(x1,x2,x1),op_t(i(rd(i(x2),x1)),asoc(x1,x2,x1)))}$

Lemma 1886: $mult(asoc(x1,x2,x1),rd(i(x2),x1)) = rd(asoc(x1,x2,x1),op_t(i(rd(i(x2),x1)),asoc(x1,x2,x1)))$

$mult(\underbrace{asoc(x1,x2,x1)},rd(i(x2),x1))$

$=$ by Lemma 981 LR with $\{x1 \leftarrow x2, x2 \leftarrow x1\}$

$mult(\overbrace{op_t(asoc(x1,x2,x1),rd(i(x2),x1))},rd(i(x2),x1))$

$=$ by Lemma 1308 RL with $\{x3 \leftarrow rd(i(x2),x1), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{mult(\overbrace{op_t(asoc(x1,x2,x1),i(rd(i(x2),x1)))},rd(i(x2),x1))}$

$=$ by Lemma 1885 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{rd(asoc(x1,x2,x1),op_t(i(rd(i(x2),x1)),asoc(x1,x2,x1)))}$

Lemma 1887: $rd(i(x1),op_r(x2,x1,x1)) = rd(i(x2),op_l(x1,x2,x1))$

$rd(\underbrace{i(x1)},op_r(x2,x1,x1))$

$=$ by Lemma 21 RL with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$\underbrace{rd(\overbrace{mult(rd(i(x2),x1),x2)},op_r(x2,x1,x1))}$

$=$ by Lemma 1242 LR with $\{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow rd(i(x2),x1)\}$

$\underbrace{\overbrace{rd(rd(i(x2),x1),asoc(x2,x1,x1))}}$

$=$ by Lemma 1714 RL with $\{x3 \leftarrow rd(i(x2),x1), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{\overbrace{mult(asoc(x1,x2,x1),rd(i(x2),x1))}}$

$=$ by Lemma 1886 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{rd(asoc(x1,x2,x1),op_t(\underbrace{i(rd(i(x2),x1))},asoc(x1,x2,x1)))}$

$=$ by Lemma 10 LR with $\{x2 \leftarrow x1, x1 \leftarrow i(x2)\}$

$rd(asoc(x1,x2,x1),op_t(\overbrace{rd(x1,i(x2))},\underbrace{asoc(x1,x2,x1)}))$

$=$ by Lemma 502 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$rd(asoc(x1,x2,x1),op_t(\underbrace{rd(x1,i(x2))},\overbrace{asoc(x1,op_r(x2,x1,x1),x1)}))$

$=$ by Lemma 357 RL with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$rd(asoc(x1,x2,x1),op_t(\overbrace{mult(op_r(x2,x1,x1),x1)},\underbrace{asoc(x1,op_r(x2,x1,x1),x1)}))$

$=$ by Lemma 262 RL with $\{x2 \leftarrow op_r(x2,x1,x1), x1 \leftarrow x1\}$

$rd(asoc(x1,x2,x1),\underbrace{op_t(mult(op_r(x2,x1,x1),x1)},\overbrace{asoc(x1,mult(op_r(x2,x1,x1),x1),x1)}))$

$=$ by Lemma 535 LR with $\{x2 \leftarrow x1, x1 \leftarrow mult(op_r(x2,x1,x1),x1)\}$

$rd(asoc(x1,x2,x1),\underbrace{\overbrace{mult(op_r(x2,x1,x1),x1)}})$

$=$ by Lemma 357 LR with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$rd(\underbrace{asoc(x1,x2,x1)},\overbrace{rd(x1,i(x2))})$

$=$ by Lemma 613 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{rd(\overbrace{asoc(i(x1),x2,i(x1))},rd(x1,i(x2)))}$

$=$ by Lemma 1584 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1, x3 \leftarrow i(x2)\}$

$\overbrace{rd(i(x2),op_l(x1,x2,x1))}$

Lemma 1888: $rd(i(x1), rd(mult(x3, x2), x3)) = rd(rd(rd(x3, op_t(x3, x2)), x1), x2)$

$$
\begin{aligned}
& rd(i(x1), \underbrace{rd(mult(x3, x2), x3)}) \\
= \quad & \text{by Lemma 800 RL with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& \underbrace{rd(i(x1), \overbrace{op_r(x2, mult(x3, x2), mult(x3, x2))})} \\
= \quad & \text{by Lemma 1654 LR with } \{x3 \leftarrow mult(x3, x2), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{rd(rd(\underbrace{asoc(mult(x3, x2), x2, mult(x3, x2))}, x1), x2)} \\
= \quad & \text{by Lemma 838 LR with } \{x2 \leftarrow x2, x1 \leftarrow x3\} \\
& rd(rd(\overbrace{rd(x3, op_t(x3, x2))}, x1), x2)
\end{aligned}
$$

Lemma 1889: $mult(op_r(x1, x2, x3), mult(x3, x2)) = mult(mult(x1, x2), op_t(x3, x2))$

$$
\begin{aligned}
& mult(\underbrace{op_r(x1, x2, x3)}, mult(x3, x2)) \\
= \quad & \text{by Axiom 16 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& mult(\overbrace{rd(mult(mult(x1, x2), x3), mult(x2, x3))}, \underbrace{mult(x3, x2)}) \\
= \quad & \text{by Lemma 435 RL with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& mult(rd(mult(mult(x1, x2), x3), mult(x2, x3)), \overbrace{op_l(mult(x2, x3), x2, x3)}) \\
= \quad & \text{by Lemma 1077 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow mult(x2, x3)\} \\
& \underbrace{mult(rd(mult(mult(x1, x2), x3), mult(x2, x3)), \overbrace{op_l(mult(x2, x3), op_l(x2, x3, x2), mult(x2, x3))})} \\
= \quad & \text{by Lemma 1755 LR with } \{x3 \leftarrow op_l(x2, x3, x2), x2 \leftarrow mult(x2, x3), x1 \leftarrow mult(mult(x1, x2), x3)\} \\
& \overbrace{rd(mult(mult(x1, x2), x3), \underbrace{asoc(mult(x2, x3), op_l(x2, x3, x2), mult(x2, x3))})} \\
= \quad & \text{by Lemma 1682 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow mult(x2, x3)\} \\
& rd(mult(mult(x1, x2), x3), \overbrace{\underbrace{asoc(mult(x2, x3), x2, mult(x2, x3))}}) \\
= \quad & \text{by Lemma 854 LR with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& rd(mult(mult(x1, x2), x3), \overbrace{rd(\underbrace{rd(mult(x2, x3), x2)}, x3)}) \\
= \quad & \text{by Lemma 394 RL with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& rd(mult(mult(x1, x2), x3), \underbrace{rd(\overbrace{mult(rd(op_t(x2, x3), x2), x3)}, x3)}) \\
= \quad & \text{by Axiom 5 RL with } \{x2 \leftarrow x3, x1 \leftarrow rd(op_t(x2, x3), x2)\} \\
& \underbrace{rd(mult(mult(x1, x2), x3), \overbrace{rd(op_t(x2, x3), x2)})} \\
= \quad & \text{by Lemma 1664 LR with } \{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow mult(x1, x2)\} \\
& \overbrace{mult(mult(x1, x2), op_t(x3, x2))}
\end{aligned}
$$

Lemma 1890: $op_r(x1, x2, op_t(x3, x2)) = op_r(x1, x2, x3)$

$\underbrace{op_r(x1, x2, op_t(x3, x2))}$
$=$ by Axiom 16 LR with $\{x3 \leftarrow op_t(x3, x2), x2 \leftarrow x2, x1 \leftarrow x1\}$
$\overbrace{rd(mult(mult(x1, x2), op_t(x3, x2)), \underbrace{mult(x2, op_t(x3, x2))})}$
$=$ by Lemma 13 LR with $\{x2 \leftarrow x3, x1 \leftarrow x2\}$
$rd(\underbrace{mult(mult(x1, x2), op_t(x3, x2))}, \overbrace{mult(x3, x2)})$
$=$ by Lemma 1889 RL with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$
$\underbrace{rd(\overbrace{mult(op_r(x1, x2, x3), mult(x3, x2))}, mult(x3, x2))}$
$=$ by Axiom 5 RL with $\{x2 \leftarrow mult(x3, x2), x1 \leftarrow op_r(x1, x2, x3)\}$
$\overbrace{op_r(x1, x2, x3)}$

Lemma 1891: $op_r(x1, x2, mult(i(x2), x3)) = op_r(x1, x2, rd(x3, x2))$

$op_r(x1, x2, \underbrace{mult(i(x2), x3)})$
$=$ by Lemma 14 RL with $\{x2 \leftarrow x2, x1 \leftarrow x3\}$
$\underbrace{op_r(x1, x2, \overbrace{op_t(rd(x3, x2), x2)})}$
$=$ by Lemma 1890 LR with $\{x3 \leftarrow rd(x3, x2), x2 \leftarrow x2, x1 \leftarrow x1\}$
$\overbrace{op_r(x1, x2, rd(x3, x2))}$

Lemma 1892: $op_r(x1, x2, i(mult(x2, x3))) = op_r(x1, x2, rd(i(x2), x3))$

$op_r(x1, x2, \underbrace{i(mult(x2, x3))})$
$=$ by Lemma 33 RL with $\{x2 \leftarrow x3, x1 \leftarrow x2\}$
$\underbrace{op_r(x1, x2, \overbrace{op_t(rd(i(x2), x3), x2)})}$
$=$ by Lemma 1890 LR with $\{x3 \leftarrow rd(i(x2), x3), x2 \leftarrow x2, x1 \leftarrow x1\}$
$\overbrace{op_r(x1, x2, rd(i(x2), x3))}$

Lemma 1893: $op_r(x1, op_t(x2, x3), op_t(x4, x2)) = op_r(x1, op_t(x2, x3), x4)$

$op_r(x1, op_t(x2, x3), \underbrace{op_t(x4, x2)})$
$=$ by Lemma 1722 RL with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x4\}$
$\underbrace{op_r(x1, op_t(x2, x3), \overbrace{op_t(x4, op_t(x2, x3))})}$
$=$ by Lemma 1890 LR with $\{x3 \leftarrow x4, x2 \leftarrow op_t(x2, x3), x1 \leftarrow x1\}$
$\overbrace{op_r(x1, op_t(x2, x3), x4)}$

Lemma 1894: $op_r(x1, op_t(x2, x3), x3) = op_r(x1, x2, x3)$

$\underbrace{op_r(x1, op_t(x2, x3), x3)}$

$=$ by Lemma 1893 RL with $\{x4 \leftarrow x3, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_r(x1, op_t(x2, x3), op_t(x3, x2))}}$

$=$ by Axiom 16 LR with $\{x3 \leftarrow op_t(x3, x2), x2 \leftarrow op_t(x2, x3), x1 \leftarrow x1\}$

$\overbrace{rd(\underbrace{mult(mult(x1, op_t(x2, x3)), op_t(x3, x2))}, mult(op_t(x2, x3), op_t(x3, x2)))}$

$=$ by Lemma 1772 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$rd(\overbrace{op_r(\underbrace{op_t(mult(x3, mult(x1, x2)), op_t(x3, x2))}, x3, x3)}, mult(op_t(x2, x3), op_t(x3, x2)))$

$=$ by Lemma 1722 LR with $\{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow mult(x3, mult(x1, x2))\}$

$rd(\underbrace{op_r(\overbrace{op_t(mult(x3, mult(x1, x2)), x3)}, x3, x3)}, mult(op_t(x2, x3), op_t(x3, x2)))$

$=$ by Lemma 385 LR with $\{x2 \leftarrow mult(x1, x2), x1 \leftarrow x3\}$

$rd(\overbrace{mult(mult(x1, x2), x3)}, \underbrace{mult(op_t(x2, x3), op_t(x3, x2))})$

$=$ by Lemma 1738 LR with $\{x3 \leftarrow x3, x2 \leftarrow x3, x1 \leftarrow x2\}$

$rd(mult(mult(x1, x2), x3), \overbrace{\underbrace{mult(x3, op_t(x2, x3))}})$

$=$ by Lemma 13 LR with $\{x2 \leftarrow x2, x1 \leftarrow x3\}$

$\underbrace{rd(mult(mult(x1, x2), x3), \overbrace{mult(x2, x3)})}$

$=$ by Axiom 16 RL with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_r(x1, x2, x3)}$

Lemma 1895: $op_r(x1, mult(i(x2), x3), x2) = op_r(x1, rd(x3, x2), x2)$

$op_r(x1, \underbrace{mult(i(x2), x3)}, x2)$

$=$ by Lemma 14 RL with $\{x2 \leftarrow x2, x1 \leftarrow x3\}$

$\underbrace{op_r(x1, \overbrace{op_t(rd(x3, x2), x2)}, x2)}$

$=$ by Lemma 1894 LR with $\{x3 \leftarrow x2, x2 \leftarrow rd(x3, x2), x1 \leftarrow x1\}$

$\overbrace{op_r(x1, rd(x3, x2), x2)}$

Lemma 1896: $op_r(x1, rd(mult(x2, x3), x2), x2) = op_r(x1, x3, x2)$

$\underbrace{op_r(x1, rd(mult(x2, x3), x2), x2)}$

$=$ by Lemma 1895 RL with $\{x3 \leftarrow mult(x2, x3), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_r(x1, \underbrace{mult(i(x2), mult(x2, x3))}, x2)}$

$=$ by Axiom 2 RL with $\{x2 \leftarrow x2, x1 \leftarrow x3\}$

$op_r(x1, \overbrace{x3}, x2)$

Lemma 1897: $op_r(x1, mult(op_t(x2, x3), x3), x3) = op_r(x1, rd(x2, i(x3)), x3)$

$$
\begin{aligned}
& op_r(x1, \underbrace{mult(op_t(x2, x3), x3)}, x3) \\
= & \quad \text{by Lemma 337 RL with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& \underbrace{op_r(x1, \overbrace{op_t(rd(x2, i(x3)), x3)}, x3)} \\
= & \quad \text{by Lemma 1894 LR with } \{x3 \leftarrow x3, x2 \leftarrow rd(x2, i(x3)), x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, rd(x2, i(x3)), x3)}
\end{aligned}
$$

Lemma 1898: $op_r(x1, op_t(x2, rd(x3, x2)), x3) = op_r(x1, op_l(x2, x2, x3), x3)$

$$
\begin{aligned}
& op_r(x1, \underbrace{op_t(x2, rd(x3, x2))}, x3) \\
= & \quad \text{by Lemma 1509 LR with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& \underbrace{op_r(x1, \overbrace{mult(i(x3), op_r(mult(x2, x3), x3, x2))}, x3)} \\
= & \quad \text{by Lemma 1895 LR with } \{x3 \leftarrow op_r(mult(x2, x3), x3, x2), x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, \underbrace{rd(op_r(mult(x2, x3), x3, x2), x3)}, x3)} \\
= & \quad \text{by Lemma 1507 LR with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& op_r(x1, \overbrace{op_l(x2, x2, x3)}, x3)
\end{aligned}
$$

Lemma 1899: $op_r(x1, x3, x2) = op_r(x1, rd(mult(op_t(x2, x3), x3), x2), x2)$

$$
\begin{aligned}
& \underbrace{op_r(x1, x3, x2)} \\
= & \quad \text{by Lemma 1896 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{\overbrace{op_r(x1, rd(mult(x2, x3), x2), x2)}} \\
= & \quad \text{by Lemma 1896 RL with } \{x3 \leftarrow rd(mult(x2, x3), x2), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, rd(\underbrace{mult(x2, rd(mult(x2, x3), x2))}, x2), x2)} \\
= & \quad \text{by Lemma 865 RL with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& op_r(x1, rd(\overbrace{mult(op_t(x2, x3), x3)}, x2), x2)
\end{aligned}
$$

Lemma 1900: $op_r(x1, op_t(x2, x3), rd(x3, rd(x2, op_t(x2, x3)))) = op_r(x1, op_t(x2, x3), op_t(op_r(x3, x2, x2), i(mult(i(x2), x3))))$

$op_r(x1, op_t(x2, x3), rd(x3, \underbrace{rd(x2, op_t(x2, x3))}))$

$=$ by Lemma 838 RL with $\{x2 \leftarrow x3, x1 \leftarrow x2\}$

$op_r(x1, op_t(x2, x3), \underbrace{rd(x3, \overbrace{asoc(mult(x2, x3), x3, mult(x2, x3))})})$

$=$ by Lemma 1708 RL with $\{x3 \leftarrow x3, x2 \leftarrow mult(x2, x3), x1 \leftarrow x3\}$

$op_r(x1, op_t(x2, x3), \overbrace{mult(\underbrace{asoc(x3, mult(x2, x3), mult(x2, x3))}, x3)})$

$=$ by Lemma 858 LR with $\{x2 \leftarrow x2, x1 \leftarrow x3\}$

$op_r(x1, op_t(x2, x3), \underbrace{mult(\overbrace{rd(op_t(x2, x3), x2)}, x3)})$

$=$ by Lemma 1096 LR with $\{x3 \leftarrow x3, x2 \leftarrow x3, x1 \leftarrow x2\}$

$\underbrace{op_r(x1, op_t(x2, x3), \overbrace{mult(op_t(x2, x3), mult(i(x2), x3))})}$

$=$ by Lemma 1890 RL with $\{x3 \leftarrow mult(op_t(x2, x3), mult(i(x2), x3)), x2 \leftarrow op_t(x2, x3), x1 \leftarrow x1\}$

$\overbrace{op_r(x1, op_t(x2, x3), \underbrace{op_t(mult(op_t(x2, x3), mult(i(x2), x3)), op_t(x2, x3))})}$

$=$ by Lemma 919 RL with $\{x2 \leftarrow op_t(x2, x3), x1 \leftarrow mult(i(x2), x3)\}$

$op_r(x1, op_t(x2, x3), \overbrace{op_t(mult(\underbrace{mult(i(x2), x3)}, op_t(x2, x3)), i(mult(i(x2), x3)))})$

$=$ by Lemma 467 RL with $\{x2 \leftarrow x2, x1 \leftarrow x3\}$

$op_r(x1, op_t(x2, x3), op_t(mult(\overbrace{op_r(rd(op_t(x3, x2), x2), x2, x2)}, op_t(x2, x3)), i(mult(i(x2), x3))))$

$=$ by Lemma 370 LR with $\{x1 \leftarrow x2, x2 \leftarrow op_t(x3, x2)\}$

$op_r(x1, op_t(x2, x3), op_t(mult(\overbrace{rd(\underbrace{op_r(op_t(x3, x2), x2, x2)}, x2)}, op_t(x2, x3)), i(mult(i(x2), x3))))$

$=$ by Axiom 14 RL with $\{x4 \leftarrow x2, x3 \leftarrow x2, x2 \leftarrow x2, x1 \leftarrow x3\}$

$op_r(x1, op_t(x2, x3), op_t(mult(rd(\overbrace{op_t(op_r(x3, x2, x2), x2)}, x2), \underbrace{op_t(x2, x3)}), i(mult(i(x2), x3))))$

$=$ by Lemma 550 RL with $\{x2 \leftarrow x3, x1 \leftarrow x2\}$

$op_r(x1, op_t(x2, x3), op_t(\underbrace{mult(rd(op_t(op_r(x3, x2, x2), x2), x2), \overbrace{op_t(x2, op_r(x3, x2, x2))}}), i(mult(i(x2), x3))))$

$=$ by Lemma 605 LR with $\{x2 \leftarrow x2, x1 \leftarrow op_r(x3, x2, x2)\}$

$op_r(x1, op_t(x2, x3), op_t(\overbrace{op_r(x3, x2, x2)}, i(mult(i(x2), x3))))$

Lemma 1901: $op_l(x1, op_t(i(x2), x3), op_t(x4, x2)) = op_l(x1, x4, op_t(x2, i(x3)))$

$\overbrace{op_l(x1, op_t(i(x2), x3), op_t(x4, x2))}$

$=$ by Lemma 1137 RL with $\{x4 \leftarrow op_t(x4, x2), x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_l(x1, op_t(x2, i(x3)), i(op_t(x4, x2)))}$

$=$ by Lemma 1136 RL with $\{x4 \leftarrow i(op_t(x4, x2)), x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_l(x1, op_t(i(x2), x3), i(\overbrace{i(op_t(x4, x2))}))}$

$=$ by Lemma 15 RL with $\{x2 \leftarrow x2, x1 \leftarrow x4\}$

$op_l(x1, op_t(i(x2), x3), i(\overbrace{op_t(i(x4), i(x2))}))$

$=$ by Lemma 1722 RL with $\{x3 \leftarrow x3, x2 \leftarrow i(x2), x1 \leftarrow i(x4)\}$

$\overbrace{op_l(x1, op_t(i(x2), x3), i(\overbrace{op_t(i(x4), op_t(i(x2), x3))}))}$

$=$ by Lemma 1329 LR with $\{x3 \leftarrow i(x4), x2 \leftarrow op_t(i(x2), x3), x1 \leftarrow x1\}$

$\overbrace{op_l(x1, i(x4), op_t(i(x2), x3))}$

$=$ by Lemma 1050 LR with $\{x3 \leftarrow op_t(i(x2), x3), x2 \leftarrow x4, x1 \leftarrow x1\}$

$\overbrace{op_l(x1, x4, \underbrace{i(op_t(i(x2), x3))})}$

$=$ by Lemma 18 LR with $\{x2 \leftarrow x3, x1 \leftarrow x2\}$

$op_l(x1, x4, \overbrace{op_t(x2, i(x3))})$

Lemma 1902: $mult(asoc(op_r(x1,x2,x3),x2,x2),op_r(x1,x2,x3)) = rd(op_r(x1,x2,x3),i(asoc(x1,x2,x2)))$

$\overbrace{mult(asoc(op_r(x1,x2,x3),x2,x2),op_r(x1,x2,x3))}$

$=$ by Lemma 385 RL with $\{x2 \leftarrow asoc(op_r(x1,x2,x3),x2,x2), x1 \leftarrow op_r(x1,x2,x3)\}$

$\overbrace{op_r(op_t(mult(op_r(x1,x2,x3),asoc(op_r(x1,x2,x3),x2,x2)),op_r(x1,x2,x3)),op_r(x1,x2,x3),op_r(x1,x2,x3))}$

$=$ by Axiom 14 RL with $\{x4 \leftarrow op_r(x1,x2,x3), x3 \leftarrow op_r(x1,x2,x3), x2 \leftarrow op_r(x1,x2,x3), x1 \leftarrow mult(op_r(x1,x2,x3), asoc(op_r$

$\overbrace{op_t(op_r(mult(op_r(x1,x2,x3),asoc(op_r(x1,x2,x3),x2,x2)),op_r(x1,x2,x3),op_r(x1,x2,x3)),op_r(x1,x2,x3))}$

$=$ by Lemma 873 LR with $\{x2 \leftarrow asoc(op_r(x1,x2,x3),x2,x2), x1 \leftarrow op_r(x1,x2,x3)\}$

$\overbrace{mult(op_r(asoc(op_r(x1,x2,x3),x2,x2),op_r(x1,x2,x3),op_r(x1,x2,x3)),\underbrace{op_l(op_r(x1,x2,x3),op_r(x1,x2,x3),asoc(op_r(x1,x2}$

$=$ by Lemma 1838 LR with $\{x3 \leftarrow x2, x2 \leftarrow x2, x1 \leftarrow op_r(x1,x2,x3)\}$

$\underbrace{mult(op_r(asoc(op_r(x1,x2,x3),x2,x2),op_r(x1,x2,x3),op_r(x1,x2,x3)),\overbrace{op_r(x1,x2,x3)}}$

$=$ by Lemma 357 LR with $\{x2 \leftarrow op_r(x1,x2,x3), x1 \leftarrow asoc(op_r(x1,x2,x3),x2,x2)\}$

$\overbrace{rd(op_r(x1,x2,x3),i(\underbrace{asoc(op_r(x1,x2,x3),x2,x2)}))}$

$=$ by Lemma 765 RL with $\{x2 \leftarrow op_r(x1,x2,x3), x1 \leftarrow x2\}$

$rd(op_r(x1,x2,x3),i(\overbrace{rd(\underbrace{op_l(x2,op_r(x1,x2,x3),x2)},x2)}))$

$=$ by Lemma 171 RL with $\{x3 \leftarrow x2, x2 \leftarrow op_r(x1,x2,x3), x1 \leftarrow x2\}$

$rd(op_r(x1,x2,x3),i(\overbrace{mult(i(x2),\underbrace{op_l(x2,op_r(x1,x2,x3),x2)})}))$

$=$ by Lemma 608 RL with $\{x2 \leftarrow op_r(x1,x2,x3), x1 \leftarrow x2\}$

$rd(op_r(x1,x2,x3),i(mult(i(x2),\overbrace{op_l(x2,x2,\underbrace{i(op_r(x1,x2,x3))})})))$

$=$ by Lemma 891 RL with $\{x3 \leftarrow x1, x2 \leftarrow x3, x1 \leftarrow x2\}$

$rd(op_r(x1,x2,x3),i(mult(i(x2),\underbrace{op_l(x2,x2,\overbrace{rd(mult(x2,x3),mult(mult(x1,x2),x3))})})))$

$=$ by Lemma 1851 LR with $\{x3 \leftarrow mult(mult(x1,x2),x3), x2 \leftarrow x3, x1 \leftarrow x2\}$

$rd(op_r(x1,x2,x3),i(mult(i(x2),\overbrace{op_l(x2,\underbrace{rd(mult(mult(x1,x2),x3),x3)},x2)})))$

$=$ by Axiom 5 RL with $\{x2 \leftarrow x3, x1 \leftarrow mult(x1,x2)\}$

$rd(op_r(x1,x2,x3),i(mult(i(x2),\underbrace{op_l(x2,\overbrace{mult(x1,x2)},x2)})))$

$=$ by Lemma 285 LR with $\{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x2\}$

$rd(op_r(x1,x2,x3),i(\underbrace{mult(i(x2),\overbrace{op_l(x2,x1,x2)})}))$

$=$ by Lemma 171 LR with $\{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x2\}$

$rd(op_r(x1,x2,x3),i(\overbrace{rd(\underbrace{op_l(x2,x1,x2)},x2)}))$

$=$ by Lemma 765 LR with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$rd(op_r(x1,x2,x3),i(\overbrace{asoc(x1,x2,x2)}))$

Lemma 1903: $mult(x1, mult(op_t(x1, i(x2)), x2)) = op_r(rd(x2, rd(op_t(i(x1), x2), x1)), rd(x2, mult(x1, mult(x2, x1))), i(x2))$

$\overbrace{mult(x1, mult(op_t(x1, i(x2)), x2))}$

$=$ by Lemma 162 RL with $\{x3 \leftarrow x2, x2 \leftarrow i(x2), x1 \leftarrow x1\}$

$\overbrace{mult(\underbrace{mult(x1, op_t(x1, i(x2)))}, x2)}$

$=$ by Lemma 103 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$mult(\overbrace{rd(x1, \underbrace{op_t(i(x1), x2)})}, x2)$

$=$ by Lemma 10 RL with $\{x2 \leftarrow x1, x1 \leftarrow op_t(i(x1), x2)\}$

$\overbrace{mult(i(\overbrace{rd(\underbrace{op_t(i(x1), x2)}, x1)}), x2)}$

$=$ by Lemma 330 RL with $\{x2 \leftarrow rd(op_t(i(x1), x2), x1), x1 \leftarrow x2\}$

$\overbrace{op_r(rd(x2, rd(op_t(i(x1), x2), x1)), x2, i(x2))}$

$=$ by Lemma 626 RL with $\{x2 \leftarrow x2, x1 \leftarrow rd(x2, rd(op_t(i(x1), x2), x1))\}$

$\overbrace{mult(rd(x2, rd(op_t(i(x1), x2), x1)), \underbrace{asoc(x2, rd(x2, rd(op_t(i(x1), x2), x1)), x2)})}$

$=$ by Lemma 534 LR with $\{x2 \leftarrow x2, x1 \leftarrow rd(x2, rd(op_t(i(x1), x2), x1))\}$

$\overbrace{op_t(op_r(rd(x2, rd(op_t(i(x1), x2), x1)), \underbrace{rd(x2, rd(x2, rd(op_t(i(x1), x2), x1)))}, i(x2)), i(rd(x2, rd(op_t(i(x1), x2), x1))))}$

$=$ by Axiom 14 LR with $\{x4 \leftarrow i(rd(x2, rd(op_t(i(x1), x2), x1))), x3 \leftarrow i(x2), x2 \leftarrow rd(x2, rd(x2, rd(op_t(i(x1), x2), x1))), x1 \leftarrow r$

$\overbrace{op_r(\underbrace{op_t(rd(x2, rd(op_t(i(x1), x2), x1)), i(rd(x2, rd(op_t(i(x1), x2), x1))))}, rd(x2, rd(x2, rd(op_t(i(x1), x2), x1))), i(x2))}$

$=$ by Lemma 8 RL with $\{x1 \leftarrow rd(x2, rd(op_t(i(x1), x2), x1))\}$

$op_r(\overbrace{rd(x2, rd(op_t(i(x1), x2), x1))}, rd(x2, \underbrace{rd(x2, rd(op_t(i(x1), x2), x1))}), i(x2))$

$=$ by Lemma 140 LR with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$op_r(rd(x2, rd(op_t(i(x1), x2), x1)), rd(x2, \overbrace{mult(x1, mult(x2, x1))}), i(x2))$

Lemma 1904: $mult(op_t(mult(x1, x2), x2), x1) = op_r(rd(x2, rd(op_t(i(x1), x2), x1)), rd(x2, mult(x1, mult(x2, x1))), i(x2))$

$\overbrace{mult(op_t(mult(x1, x2), x2), x1)}$

$=$ by Lemma 50 RL with $\{x3 \leftarrow x2, x2 \leftarrow mult(x1, x2), x1 \leftarrow x1\}$

$\overbrace{mult(x1, \underbrace{op_t(op_t(mult(x1, x2), x1), x2)})}$

$=$ by Lemma 456 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{mult(x1, \overbrace{mult(op_t(x1, i(x2)), x2)})}$

$=$ by Lemma 1903 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_r(rd(x2, rd(op_t(i(x1), x2), x1)), rd(x2, mult(x1, mult(x2, x1))), i(x2))}$

Lemma 1905: $op_r(x1, rd(op_t(x2, i(x3)), x3), x3) = op_r(x1, mult(x2, i(x3)), x3)$

$\underbrace{op_r(x1, rd(op_t(x2, i(x3)), x3), x3)}$

$=$ by Lemma 1895 RL with $\{x3 \leftarrow op_t(x2, i(x3)), x2 \leftarrow x3, x1 \leftarrow x1\}$

$\overbrace{op_r(x1, \underbrace{mult(i(x3), op_t(x2, i(x3)))}, x3)}$

$=$ by Lemma 13 LR with $\{x2 \leftarrow x2, x1 \leftarrow i(x3)\}$

$op_r(x1, \overbrace{mult(x2, i(x3))}, x3)$

Lemma 1906: $op_r(x1, x2, op_l(x3, x2, x3)) = op_r(x1, op_t(x2, x3), op_r(i(i(x3)), x2, x2))$

$\overbrace{op_r(x1, x2, op_l(x3, x2, x3))}$

$=$ by Lemma 1894 RL with $\{x3 \leftarrow op_l(x3, x2, x3), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_r(x1, op_t(x2, op_l(x3, x2, x3)), \underbrace{op_l(x3, x2, x3)})}$

$=$ by Lemma 368 RL with $\{x2 \leftarrow x2, x1 \leftarrow op_l(x3, x2, x3)\}$

$op_r(x1, op_t(x2, op_l(x3, x2, x3)), \overbrace{op_r(\underbrace{op_t(op_l(x3, x2, x3), rd(x2, op_l(x3, x2, x3)))}, x2, x2)})$

$=$ by Lemma 712 RL with $\{x2 \leftarrow x2, x1 \leftarrow op_l(x3, x2, x3)\}$

$op_r(x1, op_t(x2, op_l(x3, x2, x3)), op_r(\overbrace{op_t(op_l(x3, x2, x3), rd(i(x2), op_l(x3, x2, x3)))}, x2, x2))$

$=$ by Lemma 712 RL with $\{x2 \leftarrow i(x2), x1 \leftarrow op_l(x3, x2, x3)\}$

$op_r(x1, op_t(x2, op_l(x3, x2, x3)), op_r(\overbrace{op_t(op_l(x3, x2, x3), rd(i(i(x2)), op_l(x3, x2, x3)))}, x2, x2))$

$=$ by Lemma 1732 RL with $\{x3 \leftarrow op_l(x3, x2, x3), x2 \leftarrow i(x2), x1 \leftarrow op_l(x3, x2, x3)\}$

$op_r(x1, op_t(x2, op_l(x3, x2, x3)), \underbrace{op_r(\overbrace{op_t(op_l(x3, x2, x3), i(mult(i(x2), op_l(x3, x2, x3))))}, x2, x2)})$

$=$ by Axiom 14 RL with $\{x4 \leftarrow i(mult(i(x2), op_l(x3, x2, x3))), x3 \leftarrow x2, x2 \leftarrow x2, x1 \leftarrow op_l(x3, x2, x3)\}$

$\underbrace{op_r(x1, op_t(x2, op_l(x3, x2, x3)), \overbrace{op_t(op_r(op_l(x3, x2, x3), x2, x2), i(mult(i(x2), op_l(x3, x2, x3))))})}$

$=$ by Lemma 1900 RL with $\{x3 \leftarrow op_l(x3, x2, x3), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_r(x1, op_t(x2, op_l(x3, x2, x3)), \underbrace{rd(op_l(x3, x2, x3), rd(x2, op_t(x2, op_l(x3, x2, x3))))})}$

$=$ by Lemma 398 LR with $\{x2 \leftarrow x2, x1 \leftarrow op_l(x3, x2, x3)\}$

$op_r(x1, op_t(x2, op_l(x3, x2, x3)), \overbrace{rd(\underbrace{mult(x2, op_l(x3, x2, x3))}, x2)})$

$=$ by Lemma 616 LR with $\{x2 \leftarrow x3, x1 \leftarrow x2\}$

$op_r(x1, \underbrace{op_t(x2, op_l(x3, x2, x3))}, rd(\overbrace{rd(x2, i(x3))}, x2))$

$=$ by Lemma 1687 LR with $\{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x2\}$

$op_r(x1, \overbrace{op_t(x2, x3)}, \underbrace{rd(rd(x2, i(x3)), x2)})$

$=$ by Lemma 354 LR with $\{x2 \leftarrow i(x3), x1 \leftarrow x2\}$

$op_r(x1, op_t(x2, x3), \overbrace{op_r(i(i(x3)), x2, x2)})$

Lemma 1907: $op_r(x1, x2, op_l(x3, x2, x3)) = op_r(x1, x2, op_r(x3, x2, x2))$

$\overbrace{op_r(x1, x2, op_l(x3, x2, x3))}$

$=$ by Lemma 1906 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_r(x1, op_t(x2, x3), op_r(\underbrace{i(i(x3))}, x2, x2))}$

$=$ by Lemma 3 LR with $\{x1 \leftarrow x3\}$

$op_r(x1, \underbrace{op_t(x2, x3)}, op_r(\overbrace{x3}, x2, x2))$

$=$ by Lemma 1721 RL with $\{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x2\}$

$\underbrace{op_r(x1, \overbrace{op_t(x2, op_r(x3, x2, x2))}, op_r(x3, x2, x2))}$

$=$ by Lemma 1894 LR with $\{x3 \leftarrow op_r(x3, x2, x2), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_r(x1, x2, op_r(x3, x2, x2))}$

Lemma 1908: $asoc(x2, x1, x1) = mult(i(mult(x2, mult(x1, op_t(x1, x3)))), op_l(mult(x1, mult(x2, op_t(x1, x3))), x1, mult(x1, mult$

$\overbrace{asoc(x2, x1, x1)}$

$=$ by Lemma 1028 RL with $\{x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow x2\}$

$\overbrace{asoc(mult(x2, op_t(x1, x3)), x1, x1)}$

$=$ by Lemma 768 RL with $\{x2 \leftarrow mult(x2, op_t(x1, x3)), x1 \leftarrow x1\}$

$\overbrace{asoc(mult(x1, mult(x2, op_t(x1, x3))), x1, x1)}$

$=$ by Lemma 990 RL with $\{x2 \leftarrow x1, x1 \leftarrow mult(x1, mult(x2, op_t(x1, x3)))\}$

$\overbrace{mult(i(op_t(\underbrace{mult(x1, mult(x2, op_t(x1, x3)))}, x1)), op_l(mult(x1, mult(x2, op_t(x1, x3))), x1, mult(x1, mult(x2, op_t(x1, x3)))))}$

$=$ by Lemma 1760 RL with $\{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x1\}$

$mult(i(\underbrace{op_t(\overbrace{mult(op_t(x1, x3), mult(x2, x1))}, x1)}), op_l(mult(x1, mult(x2, op_t(x1, x3))), x1, mult(x1, mult(x2, op_t(x1, x3)))))$

$=$ by Axiom 10 RL with $\{x2 \leftarrow mult(op_t(x1, x3), mult(x2, x1)), x1 \leftarrow x1\}$

$mult(i(\overbrace{mult(i(x1), mult(\underbrace{mult(op_t(x1, x3), mult(x2, x1))}, x1))}), op_l(mult(x1, mult(x2, op_t(x1, x3))), x1, mult(x1, mult(x2,$

$=$ by Lemma 1168 LR with $\{x3 \leftarrow mult(x2, x1), x1 \leftarrow x3, x2 \leftarrow x1\}$

$mult(i(mult(i(x1), mult(\overbrace{mult(rd(x3, op_t(x3, x1)), mult(x1, mult(x2, x1)))}, x1))), op_l(mult(x1, mult(x2, op_t(x1, x3))), x1, $

$=$ by Lemma 1717 LR with $\{x3 \leftarrow mult(x1, mult(x2, x1)), x2 \leftarrow x1, x1 \leftarrow x3\}$

$mult(i(mult(i(x1), mult(\overbrace{rd(mult(x1, mult(x2, x1)), \underbrace{rd(op_t(x3, x1), x3)}}, x1))), op_l(mult(x1, mult(x2, op_t(x1, x3))), x1, mu$

$=$ by Lemma 395 LR with $\{x1 \leftarrow x1, x2 \leftarrow x3\}$

$mult(i(mult(i(x1), \underbrace{mult(rd(mult(x1, mult(x2, x1)), \overbrace{rd(x1, op_t(x1, x3))}}, x1))), op_l(mult(x1, mult(x2, op_t(x1, x3))), x1, mu$

$=$ by Lemma 1261 LR with $\{x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow mult(x1, mult(x2, x1))\}$

$mult(i(\underbrace{mult(i(x1), \overbrace{mult(mult(x1, mult(x2, x1)), op_t(x1, x3))}}), op_l(mult(x1, mult(x2, op_t(x1, x3))), x1, mult(x1, mult(x2,$

$=$ by Lemma 1045 LR with $\{x3 \leftarrow op_t(x1, x3), x2 \leftarrow mult(x2, x1), x1 \leftarrow x1\}$

$mult(i(\overbrace{mult(\underbrace{rd(mult(x2, x1), x1)}, mult(x1, op_t(x1, x3))))}), op_l(mult(x1, mult(x2, op_t(x1, x3))), x1, mult(x1, mult(x2, op_t(x$

$=$ by Axiom 5 RL with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$mult(i(mult(\overbrace{x2}, mult(x1, op_t(x1, x3)))), op_l(mult(x1, mult(x2, op_t(x1, x3))), x1, mult(x1, mult(x2, op_t(x1, x3)))))$

Lemma 1909: $asoc(x2, x1, x1) = mult(i(mult(x2, mult(x1, op_t(x1, x3)))), mult(mult(x2, op_t(x1, x3)), x1))$

$\overbrace{asoc(x2, x1, x1)}$

$=$ by Lemma 1908 LR with $\{x3 \leftarrow x3, x1 \leftarrow x1, x2 \leftarrow x2\}$

$\overbrace{mult(i(mult(x2, mult(x1, op_t(x1, x3)))), \underbrace{op_l(mult(x1, mult(x2, op_t(x1, x3))), x1, mult(x1, mult(x2, op_t(x1, x3))))})$

$=$ by Lemma 1330 LR with $\{x3 \leftarrow mult(x2, op_t(x1, x3)), x2 \leftarrow x1, x1 \leftarrow mult(x1, mult(x2, op_t(x1, x3)))\}$

$mult(i(mult(x2, mult(x1, op_t(x1, x3)))), \overbrace{op_l(mult(x1, mult(x2, op_t(x1, x3))), mult(x2, op_t(x1, x3)), i(x1))})$

$=$ by Lemma 422 LR with $\{x2 \leftarrow mult(x2, op_t(x1, x3)), x1 \leftarrow x1\}$

$mult(i(mult(x2, mult(x1, op_t(x1, x3)))), \overbrace{mult(mult(x2, op_t(x1, x3)), x1)})$

Lemma 1910: $asoc(x1, op_t(x2, x3), x2) = asoc(x1, x2, x2)$

$\underbrace{asoc(x1, op_t(x2, x3), x2)}$

$=$ by Lemma 108 RL with $\{x3 \leftarrow x2, x2 \leftarrow op_t(x2, x3), x1 \leftarrow x1\}$

$\overbrace{mult(i(mult(x1, mult(op_t(x2, x3), x2))), mult(mult(\underbrace{x1}, op_t(x2, x3)), x2))}$

$=$ by Axiom 5 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$mult(i(mult(x1, mult(op_t(x2, x3), x2))), mult(\underbrace{mult(\overbrace{rd(mult(x1, x2), x2)}, op_t(x2, x3))}, x2))$

$=$ by Lemma 1638 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow mult(x1, x2)\}$

$mult(i(mult(x1, mult(op_t(x2, x3), x2))), \underbrace{mult(\overbrace{rd(mult(x1, x2), rd(x2, op_t(x2, x3)))}, x2)})$

$=$ by Lemma 1261 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow mult(x1, x2)\}$

$mult(i(mult(x1, \underbrace{mult(op_t(x2, x3), x2)})), \overbrace{mult(mult(x1, x2), op_t(x2, x3)))}$

$=$ by Lemma 27 LR with $\{x2 \leftarrow x3, x1 \leftarrow x2\}$

$mult(i(mult(x1, \overbrace{mult(x2, op_t(x2, x3))})), \underbrace{mult(mult(x1, x2), op_t(x2, x3)))}$

$=$ by Lemma 1261 RL with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow mult(x1, x2)\}$

$mult(i(mult(x1, mult(x2, op_t(x2, x3)))), \overbrace{mult(\underbrace{rd(mult(x1, x2), rd(x2, op_t(x2, x3)))}, x2)})$

$=$ by Lemma 1638 RL with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow mult(x1, x2)\}$

$mult(i(mult(x1, mult(x2, op_t(x2, x3)))), mult(\overbrace{mult(\underbrace{rd(mult(x1, x2), x2)}, op_t(x2, x3))}, x2))$

$=$ by Axiom 5 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{mult(i(mult(x1, mult(x2, op_t(x2, x3)))), mult(mult(\overbrace{x1}, op_t(x2, x3)), x2))}$

$=$ by Lemma 1909 RL with $\{x3 \leftarrow x3, x1 \leftarrow x2, x2 \leftarrow x1\}$

$\overbrace{asoc(x1, x2, x2)}$

Lemma 1911: $mult(op_t(mult(x1,x2),x2),op_t(x1,x2)) = op_r(mult(x1,mult(x2,op_t(x1,x2))),x2,i(x2))$

$mult(op_t(\underbrace{mult(x1,x2)}, x2), op_t(x1,x2))$

$=$ by Lemma 13 RL with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$mult(op_t(\overbrace{mult(x2,op_t(x1,x2))}, x2), op_t(x1,x2))$

$=$ by Lemma 1738 RL with $\{x3 \leftarrow x2, x2 \leftarrow x2, x1 \leftarrow x1\}$

$mult(\underbrace{op_t(\overbrace{mult(op_t(x1,x2),op_t(x2,x1))}, x2)}, op_t(x1,x2))$

$=$ by Lemma 1722 RL with $\{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow mult(op_t(x1,x2),op_t(x2,x1))\}$

$\underbrace{mult(\overbrace{op_t(mult(op_t(x1,x2),op_t(x2,x1)),op_t(x2,x1))}, op_t(x1,x2))}$

$=$ by Lemma 1904 LR with $\{x2 \leftarrow op_t(x2,x1), x1 \leftarrow op_t(x1,x2)\}$

$\overbrace{op_r(\underbrace{rd(op_t(x2,x1),rd(op_t(i(op_t(x1,x2)),op_t(x2,x1)),op_t(x1,x2)))}, rd(op_t(x2,x1),mult(op_t(x1,x2),mult(op_t(x2,x1),op_t$

$=$ by Lemma 140 LR with $\{x2 \leftarrow op_t(x1,x2), x1 \leftarrow op_t(x2,x1)\}$

$op_r(\overbrace{mult(op_t(x1,x2),mult(op_t(x2,x1),op_t(x1,x2)))}, rd(op_t(x2,x1),mult(op_t(x1,x2),mult(op_t(x2,x1),op_t(x1,x2)))), i(o$

$=$ by Lemma 8 LR with $\{x1 \leftarrow mult(op_t(x1,x2),mult(op_t(x2,x1),op_t(x1,x2)))\}$

$\underbrace{op_r(\overbrace{op_t(mult(op_t(x1,x2),mult(op_t(x2,x1),op_t(x1,x2))),i(mult(op_t(x1,x2),mult(op_t(x2,x1),op_t(x1,x2)))))}, rd(op_t(x2,x$

$=$ by Axiom 14 RL with $\{x4 \leftarrow i(mult(op_t(x1,x2),mult(op_t(x2,x1),op_t(x1,x2)))), x3 \leftarrow i(op_t(x2,x1)), x2 \leftarrow rd(op_t(x2,x1),m$

$\overbrace{op_t(\underbrace{op_r(mult(op_t(x1,x2),mult(op_t(x2,x1),op_t(x1,x2))),rd(op_t(x2,x1),mult(op_t(x1,x2),mult(op_t(x2,x1),op_t(x1,x2))))}$

$=$ by Lemma 534 RL with $\{x2 \leftarrow op_t(x2,x1), x1 \leftarrow mult(op_t(x1,x2),mult(op_t(x2,x1),op_t(x1,x2)))\}$

$\overbrace{mult(mult(op_t(x1,x2),mult(op_t(x2,x1),op_t(x1,x2))),asoc(op_t(x2,x1),mult(op_t(x1,x2),mult(op_t(x2,x1),op_t(x1,x2))),$

$=$ by Lemma 626 LR with $\{x2 \leftarrow op_t(x2,x1), x1 \leftarrow mult(op_t(x1,x2),mult(op_t(x2,x1),op_t(x1,x2)))\}$

$\overbrace{op_r(\underbrace{mult(op_t(x1,x2),mult(op_t(x2,x1),op_t(x1,x2)))}, op_t(x2,x1), i(op_t(x2,x1)))}$

$=$ by Lemma 1263 LR with $\{x3 \leftarrow op_t(x1,x2), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{op_r(\overbrace{mult(x1,mult(x2,op_t(x1,x2)))}, op_t(x2,x1), i(op_t(x2,x1)))}$

$=$ by Lemma 1655 LR with $\{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow mult(x1,mult(x2,op_t(x1,x2)))\}$

$\overbrace{op_r(mult(x1,mult(x2,op_t(x1,x2))),x2,i(x2))}$

Lemma 1912: $op_l(i(x1),x2,x3) = mult(mult(x3,x2),i(mult(x3,mult(x2,x1))))$

$\underbrace{op_l(i(x1),x2,x3)}$

$=$ by Lemma 1049 RL with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{i(op_l(x1,x2,x3))}$

$=$ by Lemma 20 RL with $\{x2 \leftarrow op_l(x1,x2,x3), x1 \leftarrow mult(x3,x2)\}$

$\overbrace{mult(mult(x3,x2),i(\underbrace{mult(mult(x3,x2),op_l(x1,x2,x3))}))}$

$=$ by Lemma 132 LR with $\{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x3\}$

$mult(mult(x3,x2),i(\overbrace{mult(x3,mult(x2,x1))}))$

Lemma 1913: $mult(x1, i(rd(x2, x3))) = rd(mult(x1, x3), op_l(x2, rd(x3, x2), x1))$

$$
\begin{aligned}
& \overbrace{mult(x1, i(rd(x2, x3)))} \\
= \quad & \text{by Lemma 16 RL with } \{x2 \leftarrow rd(x2, x3), x1 \leftarrow x1\} \\
& \overbrace{\underbrace{i(mult(i(x1), rd(x2, x3)))}} \\
= \quad & \text{by Lemma 9 RL with } \{x2 \leftarrow mult(i(x1), rd(x2, x3)), x1 \leftarrow mult(x1, x3)\} \\
& \overbrace{rd(mult(x1, x3), \underbrace{mult(mult(i(x1), rd(x2, x3)), mult(x1, x3))})} \\
= \quad & \text{by Lemma 280 RL with } \{x3 \leftarrow x1, x2 \leftarrow x3, x1 \leftarrow x2\} \\
& rd(mult(x1, x3), \overbrace{op_l(x2, rd(x3, x2), x1)})
\end{aligned}
$$

Lemma 1914: $rd(mult(x1, x2), op_l(x3, rd(x2, x3), x1)) = mult(x1, rd(x2, x3))$

$$
\begin{aligned}
& \overbrace{rd(mult(x1, x2), op_l(x3, rd(x2, x3), x1))} \\
= \quad & \text{by Lemma 1913 RL with } \{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{mult(x1, \underbrace{i(rd(x3, x2))})} \\
= \quad & \text{by Lemma 10 LR with } \{x2 \leftarrow x2, x1 \leftarrow x3\} \\
& mult(x1, \overbrace{rd(x2, x3)})
\end{aligned}
$$

Lemma 1915: $mult(x1, rd(x2, op_t(op_t(x1, x2), x2))) = rd(mult(x1, x2), op_t(op_t(x1, x2), mult(x1, x2)))$

$\underbrace{mult(x1, rd(x2, op_t(op_t(x1, x2), x2)))}$

$=$ by Lemma 1914 RL with $\{x3 \leftarrow op_t(op_t(x1, x2), x2), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{rd(mult(x1, x2), \underbrace{op_l(op_t(op_t(x1, x2), x2), rd(x2, op_t(op_t(x1, x2), x2)), x1)})}$

$=$ by Lemma 285 RL with $\{x3 \leftarrow x1, x2 \leftarrow rd(x2, op_t(op_t(x1, x2), x2)), x1 \leftarrow op_t(op_t(x1, x2), x2)\}$

$rd(mult(x1, x2), \overbrace{op_l(op_t(op_t(x1, x2), x2), \underbrace{mult(rd(x2, op_t(op_t(x1, x2), x2)), x1)}, x1)})$

$=$ by Lemma 1519 LR with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$rd(mult(x1, x2), \underbrace{op_l(op_t(op_t(x1, x2), x2), \overbrace{op_t(op_t(x2, x1), x1)}, x1)})$

$=$ by Lemma 1319 LR with $\{x3 \leftarrow x1, x2 \leftarrow op_t(x2, x1), x1 \leftarrow op_t(op_t(x1, x2), x2)\}$

$rd(mult(x1, x2), \overbrace{op_l(op_t(op_t(x1, x2), x2), x1, i(op_t(x2, x1)))})$

$=$ by Lemma 1329 LR with $\{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow op_t(op_t(x1, x2), x2)\}$

$rd(mult(x1, x2), \overbrace{op_l(op_t(op_t(x1, x2), x2), x2, x1)})$

$=$ by Axiom 12 RL with $\{x3 \leftarrow op_t(op_t(x1, x2), x2), x2 \leftarrow x2, x1 \leftarrow x1\}$

$rd(mult(x1, x2), \overbrace{mult(i(mult(x1, x2)), mult(x1, \underbrace{mult(x2, op_t(op_t(x1, x2), x2))}))})$

$=$ by Lemma 13 LR with $\{x2 \leftarrow op_t(x1, x2), x1 \leftarrow x2\}$

$rd(mult(x1, x2), mult(i(mult(x1, x2)), \underbrace{mult(x1, \overbrace{mult(op_t(x1, x2), x2)})}))$

$=$ by Lemma 162 RL with $\{x3 \leftarrow x2, x2 \leftarrow x2, x1 \leftarrow x1\}$

$rd(mult(x1, x2), mult(i(mult(x1, x2)), \overbrace{mult(\underbrace{mult(x1, op_t(x1, x2))}, x2)}))$

$=$ by Lemma 27 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$rd(mult(x1, x2), \underbrace{mult(i(mult(x1, x2)), mult(\overbrace{mult(op_t(x1, x2), x1)}, x2))})$

$=$ by Lemma 14 RL with $\{x2 \leftarrow mult(x1, x2), x1 \leftarrow mult(mult(op_t(x1, x2), x1), x2)\}$

$rd(mult(x1, x2), \overbrace{op_t(\underbrace{rd(mult(mult(op_t(x1, x2), x1), x2), mult(x1, x2))}, mult(x1, x2)))}$

$=$ by Axiom 16 RL with $\{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow op_t(x1, x2)\}$

$rd(mult(x1, x2), op_t(\overbrace{op_r(op_t(x1, x2), x1, x2)}, mult(x1, x2)))$

$=$ by Lemma 28 LR with $\{x3 \leftarrow x2, x2 \leftarrow x2, x1 \leftarrow x1\}$

$rd(mult(x1, x2), op_t(\overbrace{op_t(x1, x2)}, mult(x1, x2)))$

Lemma 1916: $rd(x1, i(rd(x2, x3))) = rd(mult(x1, op_l(x2, rd(x3, x2), x1)), x3)$

$\underbrace{rd(x1, i(rd(x2, x3)))}$

$=$ by Lemma 86 RL with $\{x2 \leftarrow rd(x2, x3), x1 \leftarrow x1\}$

$\overbrace{\underbrace{mult(x1, mult(mult(i(x1), rd(x2, x3)), x1))}}$

$=$ by Lemma 76 RL with $\{x3 \leftarrow x3, x2 \leftarrow mult(i(x1), rd(x2, x3)), x1 \leftarrow x1\}$

$\overbrace{rd(mult(x1, \underbrace{mult(mult(i(x1), rd(x2, x3)), mult(x1, x3))}), x3)}$

$=$ by Lemma 280 RL with $\{x3 \leftarrow x1, x2 \leftarrow x3, x1 \leftarrow x2\}$

$rd(mult(x1, \overbrace{op_l(x2, rd(x3, x2), x1)}), x3)$

Lemma 1917: $rd(mult(x1, op_l(x2, rd(x3, x2), x1)), x3) = rd(x1, rd(x3, x2))$

$\underbrace{rd(mult(x1, op_l(x2, rd(x3, x2), x1)), x3)}$

$=$ by Lemma 1916 RL with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{rd(x1, \underbrace{i(rd(x2, x3))})}$

$=$ by Lemma 10 LR with $\{x2 \leftarrow x3, x1 \leftarrow x2\}$

$rd(x1, \overbrace{rd(x3, x2)})$

Lemma 1918: $op_l(x1, mult(x2, i(x3)), op_l(x3, x2, x3)) = op_l(x1, i(i(x3)), i(x2))$

$\underbrace{op_l(x1, mult(x2, i(x3)), op_l(x3, x2, x3))}$

$=$ by Lemma 242 RL with $\{x3 \leftarrow op_l(x3, x2, x3), x2 \leftarrow mult(x2, i(x3)), x1 \leftarrow x1\}$

$\overbrace{op_l(x1, \underbrace{mult(mult(x2, i(x3)), op_l(x3, x2, x3))}, i(mult(x2, i(x3))))}$

$=$ by Lemma 663 LR with $\{x2 \leftarrow x3, x1 \leftarrow x2\}$

$\underbrace{op_l(x1, \overbrace{x2}, i(mult(x2, i(x3))))}$

$=$ by Lemma 245 LR with $\{x3 \leftarrow i(x3), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_l(x1, i(i(x3)), i(x2))}$

Lemma 1919: $op_t(x1, rd(x2, x1)) = rd(op_l(x1, x1, x2), asoc(x1, mult(x2, x1), mult(x2, x1)))$

$\underbrace{op_t(x1, rd(x2, x1))}$

$=$ by Lemma 709 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_t(x1, i(mult(x2, x1)))}}$

$=$ by Lemma 988 RL with $\{x2 \leftarrow mult(x2, x1), x1 \leftarrow x1\}$

$\overbrace{rd(\underbrace{op_l(x1, x1, mult(x2, x1))}, asoc(x1, mult(x2, x1), mult(x2, x1)))}$

$=$ by Lemma 291 LR with $\{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x1\}$

$rd(\overbrace{op_l(x1, x1, x2)}, asoc(x1, mult(x2, x1), mult(x2, x1)))$

Lemma 1920: $op_r(rd(x1, rd(x2, x3)), rd(x3, x2), x1) = mult(rd(x3, x2), x1)$

$$
\begin{aligned}
& op_r(rd(x1, \underbrace{rd(x2, x3)}), rd(x3, x2), x1) \\
= & \quad \text{by Lemma 10 RL with } \{x2 \leftarrow x2, x1 \leftarrow x3\} \\
& op_r(\underbrace{rd(x1, \overbrace{i(rd(x3, x2))})}, rd(x3, x2), x1) \\
= & \quad \text{by Lemma 359 RL with } \{x2 \leftarrow x1, x1 \leftarrow rd(x3, x2)\} \\
& op_r(\overbrace{op_r(mult(rd(x3, x2), x1), x1, x1)}, \underbrace{rd(x3, x2)}, x1) \\
= & \quad \text{by Axiom 5 LR with } \{x2 \leftarrow x1, x1 \leftarrow rd(x3, x2)\} \\
& \underbrace{op_r(op_r(mult(rd(x3, x2), x1), x1, x1), \overbrace{rd(mult(rd(x3, x2), x1), x1)}, x1)} \\
= & \quad \text{by Lemma 999 LR with } \{x2 \leftarrow x1, x1 \leftarrow mult(rd(x3, x2), x1)\} \\
& \overbrace{rd(\underbrace{mult(rd(mult(mult(rd(x3, x2), x1), mult(rd(x3, x2), x1)), x1), x1)}, mult(rd(x3, x2), x1))} \\
= & \quad \text{by Axiom 4 RL with } \{x2 \leftarrow x1, x1 \leftarrow mult(mult(rd(x3, x2), x1), mult(rd(x3, x2), x1))\} \\
& \underbrace{rd(\overbrace{mult(mult(rd(x3, x2), x1), mult(rd(x3, x2), x1))}, mult(rd(x3, x2), x1))} \\
= & \quad \text{by Axiom 5 RL with } \{x2 \leftarrow mult(rd(x3, x2), x1), x1 \leftarrow mult(rd(x3, x2), x1)\} \\
& \overbrace{mult(rd(x3, x2), x1)}
\end{aligned}
$$

Lemma 1921: $op_t(x1, rd(x2, x1)) = op_t(x1, rd(x2, mult(mult(i(x3), x1), x3)))$

$$
\begin{aligned}
& \underbrace{op_t(x1, rd(x2, x1))} \\
= & \quad \text{by Lemma 708 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_t(x1, i(mult(x1, x2)))}} \\
= & \quad \text{by Lemma 1281 RL with } \{x3 \leftarrow x3, x2 \leftarrow i(mult(x1, x2)), x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_t(x1, rd(i(mult(x1, x2)), asoc(x1, x3, x1)))}} \\
= & \quad \text{by Lemma 1279 LR with } \{x3 \leftarrow asoc(x1, x3, x1), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, rd(x2, \underbrace{rd(x1, asoc(x1, x3, x1))}))} \\
= & \quad \text{by Lemma 620 LR with } \{x2 \leftarrow x3, x1 \leftarrow x1\} \\
& op_t(x1, rd(x2, \overbrace{\underbrace{op_l(x1, x3, x1)}})) \\
= & \quad \text{by Lemma 744 LR with } \{x2 \leftarrow x3, x1 \leftarrow x1\} \\
& op_t(x1, rd(x2, \overbrace{mult(mult(i(x3), x1), x3)}))
\end{aligned}
$$

Lemma 1922: $op_t(mult(x1,x2),x1) = mult(mult(mult(x1,x2),x1),i(op_t(x1,x2)))$

$op_t(mult(x1,x2),\underbrace{x1}) =$ by Axiom 5 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{op_t(mult(x1,x2),\overbrace{rd(mult(x1,x2),x2)})} =$ by Lemma 554 RL with $\{x2 \leftarrow x2, x1 \leftarrow mult(x1,x2)\}$

$\overbrace{op_t(mult(x1,x2),\underbrace{mult(i(x2),mult(x1,x2))})} =$ by Axiom 10 LR with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$op_t(mult(x1,x2),\overbrace{\underbrace{op_t(x1,x2)}}) =$ by Lemma 3 RL with $\{x1 \leftarrow op_t(x1,x2)\}$

$\underbrace{op_t(mult(x1,x2),\overbrace{i(\overbrace{i(op_t(x1,x2))})})} =$ by Lemma 481 RL with $\{x2 \leftarrow i(op_t(x1,x2)), x1 \leftarrow mult(x1,x2)\}$

$\overbrace{mult(\underbrace{op_l(rd(mult(x1,x2),i(op_t(x1,x2))),mult(x1,x2),i(op_t(x1,x2)))},i(op_t(x1,x2)))} =$ by Lemma 1321 RL with $\{x3 \leftarrow op_t(x1,x2), x2 \leftarrow mult(x1,x2), x1 \leftarrow rd(mult(x1,x2),i(op_t(x1,x2)))\}$

$mult(\overbrace{op_l(rd(mult(x1,x2),i(op_t(x1,x2))),mult(x1,x2),\underbrace{rd(mult(x1,x2),op_t(x1,x2))})},i(op_t(x1,x2))) =$ by Lemma 29 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$mult(\underbrace{op_l(rd(mult(x1,x2),i(op_t(x1,x2))),mult(x1,x2),\overbrace{x2})},i(op_t(x1,x2))) =$ by Lemma 285 LR with $\{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow rd(mult(x1,x2),i(op_t(x1,x2)))\}$

$mult(\overbrace{op_l(\underbrace{rd(mult(x1,x2),i(op_t(x1,x2)))},x1,x2)},i(op_t(x1,x2))) =$ by Lemma 1132 LR with $\{x3 \leftarrow x2, x2 \leftarrow x2, x1 \leftarrow x1\}$

$mult(\underbrace{op_l(\overbrace{mult(x1,mult(x2,op_t(x1,x2)))},x1,x2)},i(op_t(x1,x2))) =$ by Lemma 1047 LR with $\{x3 \leftarrow op_t(x1,x2), x2 \leftarrow x2, x1 \leftarrow x1\}$

$mult(\underbrace{\overbrace{mult(mult(x2,x1),op_t(x1,x2))}},i(op_t(x1,x2))) =$ by Lemma 871 LR with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$mult(\overbrace{mult(mult(x1,x2),x1)},i(op_t(x1,x2)))$

Lemma 1923: $mult(rd(i(x1),x1),op_t(x2,rd(x2,x1))) = rd(rd(x2,x1),x1)$

$$
\begin{aligned}
& mult(rd(i(x1),x1),op_t(\underbrace{x2}_{},rd(x2,x1))) \\
= & \quad \text{by Axiom 4 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& mult(rd(i(x1),x1),op_t(\overbrace{mult(rd(x2,x1),\underbrace{x1}_{})}^{},rd(x2,x1))) \\
= & \quad \text{by Lemma 3 RL with } \{x1 \leftarrow x1\} \\
& mult(\underbrace{rd(i(x1),x1)}_{},op_t(mult(rd(x2,x1),\overbrace{i(i(x1))}^{}),rd(x2,x1))) \\
= & \quad \text{by Lemma 43 RL with } \{x1 \leftarrow x1\} \\
& mult(\overbrace{i(mult(x1,x1))}^{},op_t(mult(rd(x2,x1),i(i(x1))),rd(x2,x1))) \\
= & \quad \text{by Axiom 11 RL with } \{x2 \leftarrow x1, x1 \leftarrow x1\} \\
& \underbrace{mult(\overbrace{mult(i(x1),i(x1))}^{},op_t(mult(rd(x2,x1),i(i(x1))),rd(x2,x1)))}_{} \\
= & \quad \text{by Lemma 24 LR with } \{x2 \leftarrow op_t(mult(rd(x2,x1),i(i(x1))),rd(x2,x1)),x1 \leftarrow i(x1)\} \\
& \overbrace{mult(i(x1),\underbrace{mult(i(x1),op_t(mult(rd(x2,x1),i(i(x1))),rd(x2,x1)))}_{})}^{} \\
= & \quad \text{by Lemma 675 LR with } \{x2 \leftarrow rd(x2,x1),x1 \leftarrow i(x1)\} \\
& \underbrace{mult(i(x1),\overbrace{op_l(rd(x2,x1),i(x1),rd(x2,x1))}^{})}_{} \\
= & \quad \text{by Lemma 616 LR with } \{x2 \leftarrow rd(x2,x1),x1 \leftarrow i(x1)\} \\
& \overbrace{\underbrace{rd(i(x1),i(rd(x2,x1)))}_{}}^{} \\
= & \quad \text{by Lemma 23 LR with } \{x2 \leftarrow rd(x2,x1),x1 \leftarrow x1\} \\
& \overbrace{rd(rd(x2,x1),x1)}^{}
\end{aligned}
$$

Lemma 1924: $op_t(x1,i(mult(x1,x2))) = rd(mult(x1,i(i(mult(x1,x2)))),rd(x1,i(x2)))$

$$
\begin{aligned}
& \underbrace{op_t(x1,i(mult(x1,x2)))}_{} \\
= & \quad \text{by Lemma 1465 LR with } \{x2 \leftarrow i(mult(x1,x2)),x1 \leftarrow x1\} \\
& \overbrace{rd(\underbrace{op_t(rd(x1,i(mult(x1,x2))),i(\overbrace{op_t(x1,i(mult(x1,x2)))}^{}))}_{},rd(x1,mult(x1,i(mult(x1,x2)))))}^{} \\
= & \quad \text{by Lemma 591 LR with } \{x2 \leftarrow i(mult(x1,x2)),x1 \leftarrow x1\} \\
& rd(\overbrace{mult(x1,i(i(mult(x1,x2))))}^{},rd(x1,\underbrace{mult(x1,i(mult(x1,x2)))}_{})) \\
= & \quad \text{by Lemma 20 LR with } \{x2 \leftarrow x2,x1 \leftarrow x1\} \\
& rd(mult(x1,i(i(mult(x1,x2)))),rd(x1,\overbrace{i(x2)}^{})))
\end{aligned}
$$

Lemma 1925: $op_t(x1,rd(x2,x1)) = rd(mult(x1,mult(x1,x2)),rd(x1,i(x2)))$

$$
\begin{aligned}
& \underbrace{op_t(x1,rd(x2,x1))}_{} \\
= & \quad \text{by Lemma 708 RL with } \{x2 \leftarrow x2,x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_t(x1,i(mult(x1,x2)))}_{}}^{} \\
= & \quad \text{by Lemma 1924 LR with } \{x2 \leftarrow x2,x1 \leftarrow x1\} \\
& \overbrace{rd(mult(x1,\underbrace{i(i(mult(x1,x2)))}_{}),rd(x1,i(x2)))}^{} \\
= & \quad \text{by Lemma 3 LR with } \{x1 \leftarrow mult(x1,x2)\} \\
& rd(mult(x1,\overbrace{mult(x1,x2)}^{}),rd(x1,i(x2)))
\end{aligned}
$$

Lemma 1926: $op_l(x1, op_l(x2, x3, x2), i(i(mult(x3, x2)))) = op_l(x1, op_t(op_l(x2, x3, x2), rd(rd(i(x2), x3), x2)), i(i(mult(x3, x2))))$

$\underbrace{op_l(x1, op_l(x2, x3, x2), i(i(mult(x3, x2))))}$

$=$ by Lemma 1379 LR with $\{x3 \leftarrow i(mult(x3, x2)), x2 \leftarrow op_l(x2, x3, x2), x1 \leftarrow x1\}$

$\overbrace{op_l(x1, \underbrace{op_t(op_l(x2, x3, x2), mult(i(op_l(x2, x3, x2)), i(mult(x3, x2))))}, i(i(mult(x3, x2))))}$

$=$ by Lemma 375 LR with $\{x2 \leftarrow i(mult(x3, x2)), x1 \leftarrow op_l(x2, x3, x2)\}$

$op_l(x1, \overbrace{op_t(op_l(x2, x3, x2), \underbrace{rd(i(mult(x3, x2)), op_l(x2, x3, x2))})}, i(i(mult(x3, x2))))$

$=$ by Lemma 1492 RL with $\{x2 \leftarrow x3, x1 \leftarrow x2\}$

$op_l(x1, op_t(op_l(x2, x3, x2), \overbrace{rd(rd(i(x2), x3), x2)}), i(i(mult(x3, x2))))$

Lemma 1927: $op_l(x1, x3, i(x2)) = op_l(x1, op_t(op_l(x2, x3, x2), rd(rd(i(x2), x3), x2)), i(i(mult(x3, x2))))$

$\underbrace{op_l(x1, x3, i(x2))}$

$=$ by Lemma 1380 LR with $\{x2 \leftarrow x2, x3 \leftarrow x3, x1 \leftarrow x1\}$

$\overbrace{op_l(x1, op_l(x2, x3, x2), \underbrace{mult(x3, x2)})}$

$=$ by Lemma 3 RL with $\{x1 \leftarrow mult(x3, x2)\}$

$\underbrace{op_l(x1, op_l(x2, x3, x2), \overbrace{i(i(mult(x3, x2)))})}$

$=$ by Lemma 1926 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_l(x1, op_t(op_l(x2, x3, x2), rd(rd(i(x2), x3), x2)), i(i(mult(x3, x2))))}$

Lemma 1928: $op_l(x1, op_t(x2, x3), op_r(x3, x2, i(x2))) = op_l(x1, x3, i(x2))$

$op_l(x1, op_t(x2, x3), \underbrace{op_r(x3, x2, i(x2))})$

$=$ by Lemma 3 RL with $\{x1 \leftarrow op_r(x3, x2, i(x2))\}$

$\underbrace{op_l(x1, op_t(x2, x3), \overbrace{i(i(op_r(x3, x2, i(x2))))})}$

$=$ by Lemma 1050 RL with $\{x3 \leftarrow i(op_r(x3, x2, i(x2))), x2 \leftarrow op_t(x2, x3), x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_l(x1, i(op_t(x2, x3)), i(op_r(x3, x2, i(x2))))}}$

$=$ by Lemma 1321 RL with $\{x3 \leftarrow op_r(x3, x2, i(x2)), x2 \leftarrow i(op_t(x2, x3)), x1 \leftarrow x1\}$

$\overbrace{op_l(x1, i(op_t(x2, x3)), rd(\underbrace{i(op_t(x2, x3))}, op_r(x3, x2, i(x2))))}$

$=$ by Lemma 557 RL with $\{x2 \leftarrow x3, x1 \leftarrow x2\}$

$op_l(x1, i(op_t(x2, x3)), rd(\overbrace{i(op_t(x2, mult(x2, rd(x3, x2))))}, \underbrace{op_r(x3, x2, i(x2))}))$

$=$ by Lemma 349 RL with $\{x2 \leftarrow x3, x1 \leftarrow x2\}$

$op_l(x1, i(op_t(x2, x3)), rd(\underbrace{i(op_t(x2, mult(x2, rd(x3, x2))))}, \overbrace{op_t(mult(x2, rd(x3, x2)), x2)}))$

$=$ by Lemma 724 RL with $\{x2 \leftarrow mult(x2, rd(x3, x2)), x1 \leftarrow x2\}$

$op_l(x1, i(op_t(x2, x3)), \underbrace{rd(\overbrace{mult(rd(i(x2), mult(x2, rd(x3, x2))), op_t(mult(x2, rd(x3, x2)), x2)}), op_t(mult(x2, rd(x3, x2)), x2)}$

$=$ by Axiom 5 RL with $\{x2 \leftarrow op_t(mult(x2, rd(x3, x2)), x2), x1 \leftarrow rd(i(x2), mult(x2, rd(x3, x2)))\}$

$op_l(x1, i(op_t(x2, x3)), \overbrace{\underbrace{rd(i(x2), mult(x2, rd(x3, x2)))}})$

$=$ by Lemma 83 LR with $\{x2 \leftarrow x3, x1 \leftarrow x2\}$

$\underbrace{op_l(x1, i(op_t(x2, x3)), \overbrace{i(mult(x2, x3))})}$

$=$ by Lemma 1050 LR with $\{x3 \leftarrow i(mult(x2, x3)), x2 \leftarrow op_t(x2, x3), x1 \leftarrow x1\}$

$\overbrace{op_l(x1, op_t(x2, x3), \underbrace{i(i(mult(x2, x3)))})}$

$=$ by Lemma 3 LR with $\{x1 \leftarrow mult(x2, x3)\}$

$op_l(x1, \underbrace{op_t(x2, x3)}, \overbrace{mult(x2, x3)})$

$=$ by Axiom 10 RL with $\{x2 \leftarrow x2, x1 \leftarrow x3\}$

$\underbrace{op_l(x1, \overbrace{mult(i(x3), mult(x2, x3))}, mult(x2, x3))}$

$=$ by Lemma 285 LR with $\{x3 \leftarrow mult(x2, x3), x2 \leftarrow i(x3), x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_l(x1, i(x3), mult(x2, x3))}}$

$=$ by Lemma 246 LR with $\{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_l(x1, op_t(x2, x3), x3)}}$

$=$ by Lemma 1319 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_l(x1, x3, i(x2))}$

Lemma 1929: $op_r(i(op_t(x1,x2)), rd(op_t(x1,x2), i(op_r(x2,x1,i(x1)))), x3) = i(op_r(op_t(x1,x2), op_l(op_r(x2,x1,i(x1)),x2,i(x1)),x$

$op_r(i(op_t(x1,x2)), \underbrace{rd(op_t(x1,x2), i(op_r(x2,x1,i(x1))))}, x3)$

$=$ by Lemma 616 RL with $\{x2 \leftarrow op_r(x2,x1,i(x1)), x1 \leftarrow op_t(x1,x2)\}$

$\underbrace{op_r(i(op_t(x1,x2)), \overbrace{mult(op_t(x1,x2), op_l(op_r(x2,x1,i(x1)), op_t(x1,x2), op_r(x2,x1,i(x1))))}, x3)}$

$=$ by Lemma 654 LR with $\{x3 \leftarrow x3, x2 \leftarrow op_l(op_r(x2,x1,i(x1)), op_t(x1,x2), op_r(x2,x1,i(x1))), x1 \leftarrow op_t(x1,x2)\}$

$\overbrace{i(op_r(op_t(x1,x2), \underbrace{op_l(op_r(x2,x1,i(x1)), op_t(x1,x2), op_r(x2,x1,i(x1)))}, x3))}$

$=$ by Lemma 1928 LR with $\{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow op_r(x2,x1,i(x1))\}$

$i(op_r(op_t(x1,x2), \overbrace{op_l(op_r(x2,x1,i(x1)),x2,i(x1))}, x3))$

Lemma 1930: $op_r(i(op_t(x1,x2)), rd(op_t(x1,x2), op_r(i(x2),x1,i(x1))), x3) = i(op_r(op_t(x1,x2), op_l(op_r(x2,x1,i(x1)),x2,i(x1)),x$

$op_r(i(op_t(x1,x2)), rd(op_t(x1,x2), \underbrace{op_r(i(x2),x1,i(x1))}), x3)$

$=$ by Lemma 321 RL with $\{x2 \leftarrow x1, x1 \leftarrow i(x2)\}$

$op_r(i(op_t(x1,x2)), rd(op_t(x1,x2), \overbrace{op_r(i(x2),i(x1),x1)}), x3)$

$=$ by Lemma 814 RL with $\{x3 \leftarrow x1, x2 \leftarrow x1, x1 \leftarrow x2\}$

$\underbrace{op_r(i(op_t(x1,x2)), rd(op_t(x1,x2), \overbrace{i(op_r(x2,x1,i(x1)))}), x3)}$

$=$ by Lemma 1929 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{i(op_r(op_t(x1,x2), op_l(op_r(x2,x1,i(x1)),x2,i(x1)),x3))}$

Lemma 1931: $op_r(i(op_t(x1,x2)), mult(i(i(x2)), op_t(x1,x2)), x3) = i(op_r(op_t(x1,x2), op_l(op_r(x2,x1,i(x1)),x2,i(x1)),x3))$

$op_r(i(op_t(x1,x2)), \underbrace{mult(i(i(x2)), op_t(x1,x2))}, x3)$

$=$ by Lemma 1678 RL with $\{x3 \leftarrow i(x2), x2 \leftarrow x2, x1 \leftarrow x1\}$

$op_r(i(op_t(x1,x2)), \overbrace{rd(op_t(x1,x2), \underbrace{mult(mult(i(x2),i(x1)),x1)})}, x3)$

$=$ by Lemma 323 LR with $\{x2 \leftarrow x1, x1 \leftarrow i(x2)\}$

$\underbrace{op_r(i(op_t(x1,x2)), rd(op_t(x1,x2), \overbrace{op_r(i(x2),x1,i(x1))}), x3)}$

$=$ by Lemma 1930 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{i(op_r(op_t(x1,x2), op_l(op_r(x2,x1,i(x1)),x2,i(x1)),x3))}$

Lemma 1932: $op_r(i(op_t(x1,x2)), mult(x1,x2), x3) = i(op_r(op_t(x1,x2), op_l(op_r(x2,x1,i(x1)),x2,i(x1)),x3))$

$op_r(i(op_t(x1,x2)), \underbrace{mult(x1,x2)}, x3)$

$=$ by Lemma 13 RL with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$op_r(i(op_t(x1,x2)), \overbrace{mult(\underbrace{x2}, op_t(x1,x2))}, x3)$

$=$ by Lemma 3 RL with $\{x1 \leftarrow x2\}$

$\underbrace{op_r(i(op_t(x1,x2)), mult(\overbrace{i(i(x2))}, op_t(x1,x2)), x3)}$

$=$ by Lemma 1931 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{i(op_r(op_t(x1,x2), op_l(op_r(x2,x1,i(x1)),x2,i(x1)),x3))}$

Lemma 1933: $i(op_r(op_t(x1,x2),x2,x3)) = i(op_r(op_t(x1,x2),op_t(x2,x1),x3))$

$i(\underbrace{op_r(op_t(x1,x2),x2,x3)})$

$=$ by Axiom 14 RL with $\{x4 \leftarrow x2, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{i(\overbrace{op_t(op_r(x1,x2,x3),x2)})}$

$=$ by Lemma 15 RL with $\{x2 \leftarrow x2, x1 \leftarrow op_r(x1,x2,x3)\}$

$\overbrace{op_t(\underbrace{i(op_r(x1,x2,x3))},i(x2))}$

$=$ by Lemma 654 RL with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{op_t(\overbrace{op_r(i(x1),mult(x1,x2),x3)},i(x2))}$

$=$ by Axiom 14 LR with $\{x4 \leftarrow i(x2), x3 \leftarrow x3, x2 \leftarrow mult(x1,x2), x1 \leftarrow i(x1)\}$

$\overbrace{op_r(\underbrace{op_t(i(x1),i(x2))},mult(x1,x2),x3)}$

$=$ by Lemma 15 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{op_r(\overbrace{i(op_t(x1,x2))},mult(x1,x2),x3)}$

$=$ by Lemma 1932 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{i(op_r(op_t(x1,x2),\underbrace{op_l(op_r(x2,x1,i(x1)),x2,i(x1))},x3))}$

$=$ by Axiom 18 RL with $\{x5 \leftarrow i(x1), x4 \leftarrow x1, x3 \leftarrow i(x1), x2 \leftarrow x2, x1 \leftarrow x2\}$

$i(op_r(op_t(x1,x2),\overbrace{op_r(\underbrace{op_l(x2,x2,i(x1))},x1,i(x1))},x3))$

$=$ by Lemma 608 LR with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$i(op_r(op_t(x1,x2),\underbrace{op_r(\overbrace{op_l(x2,x1,x2)},x1,i(x1))},x3))$

$=$ by Axiom 18 LR with $\{x5 \leftarrow i(x1), x4 \leftarrow x1, x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x2\}$

$i(op_r(op_t(x1,x2),\overbrace{op_l(\underbrace{op_r(x2,x1,i(x1))},x1,x2)},x3))$

$=$ by Lemma 421 LR with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$i(op_r(op_t(x1,x2),\underbrace{op_l(\overbrace{op_l(op_t(x2,x1),x2,x1)},x1,x2)},x3))$

$=$ by Lemma 138 LR with $\{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow op_t(x2,x1)\}$

$i(op_r(op_t(x1,x2),\overbrace{op_t(x2,x1)},x3))$

Lemma 1934: $i(op_r(x1,x2,x3)) = i(op_r(x1,op_t(x2,rd(x1,rd(op_t(x1,x2),x1))),x3))$

$i(op_r(\underbrace{x1},x2,x3))$

$=$ by Lemma 401 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{i(op_r(\overbrace{op_t(rd(x1,rd(op_t(x1,x2),x1)),x2)},x2,x3))}$

$=$ by Lemma 1933 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow rd(x1,rd(op_t(x1,x2),x1))\}$

$\overbrace{i(op_r(\underbrace{op_t(rd(x1,rd(op_t(x1,x2),x1)),x2)},op_t(x2,rd(x1,rd(op_t(x1,x2),x1))),x3))}$

$=$ by Lemma 401 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$i(op_r(\overbrace{x1},op_t(x2,rd(x1,rd(op_t(x1,x2),x1))),x3))$

Lemma 1935: $op_r(i(x1), mult(x2, x1), x3) = i(op_r(x1, x2, x3))$

$$
\begin{aligned}
& \overbrace{op_r(i(x1), mult(x2, x1), x3)} \\
= \quad & \text{by Lemma 742 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{i(op_r(x1, \underbrace{op_t(x2, x1)}, x3))} \\
= \quad & \text{by Lemma 1237 RL with } \{x3 \leftarrow x1, x2 \leftarrow x1, x1 \leftarrow x2\} \\
& i(op_r(x1, \overbrace{op_t(x2, rd(x1, \underbrace{rd(x2, op_t(x2, x1))}))}, x3)) \\
= \quad & \text{by Lemma 395 RL with } \{x1 \leftarrow x2, x2 \leftarrow x1\} \\
& \underbrace{i(op_r(x1, op_t(x2, rd(x1, \overbrace{rd(op_t(x1, x2), x1)})), x3))} \\
= \quad & \text{by Lemma 1934 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{i(op_r(x1, x2, x3))}
\end{aligned}
$$

Lemma 1936: $op_r(x1, i(mult(x2, x1)), i(x3)) = rd(i(x1), mult(i(x1), i(op_r(x1, x2, x3))))$

$$
\begin{aligned}
& \underbrace{op_r(x1, i(mult(x2, x1)), i(x3))} \\
= \quad & \text{by Lemma 813 RL with } \{x3 \leftarrow x3, x2 \leftarrow mult(x2, x1), x1 \leftarrow x1\} \\
& \underbrace{\overbrace{i(op_r(i(x1), mult(x2, x1), x3))}} \\
= \quad & \text{by Lemma 146 RL with } \{x3 \leftarrow x3, x2 \leftarrow mult(x2, x1), x1 \leftarrow i(x1)\} \\
& \overbrace{rd(i(x1), mult(i(x1), \underbrace{op_r(i(x1), mult(x2, x1), x3)}))} \\
= \quad & \text{by Lemma 1935 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& rd(i(x1), mult(i(x1), \overbrace{i(op_r(x1, x2, x3))}))
\end{aligned}
$$

Lemma 1937: $op_r(x1, op_t(x2, x1), x3) = rd(i(x1), mult(i(x1), i(op_r(x1, x2, x3))))$

$$
\begin{aligned}
& op_r(x1, op_t(x2, x1), \underbrace{x3}) \\
= \quad & \text{by Lemma 3 RL with } \{x1 \leftarrow x3\} \\
& \underbrace{op_r(x1, op_t(x2, x1), \overbrace{i(i(x3))})} \\
= \quad & \text{by Lemma 820 RL with } \{x3 \leftarrow i(x3), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_r(x1, i(mult(x2, x1)), i(x3))}} \\
= \quad & \text{by Lemma 1936 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{rd(i(x1), mult(i(x1), i(op_r(x1, x2, x3))))}
\end{aligned}
$$

Lemma 1938: $op_r(x1, op_t(x2, x1), x3) = rd(mult(x1, op_r(x1, x2, x3)), x1)$

$$
\begin{aligned}
& \underbrace{op_r(x1, op_t(x2, x1), x3)} \\
= \quad & \text{by Lemma 1937 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{rd(i(x1), mult(i(x1), i(op_r(x1, x2, x3))))}} \\
= \quad & \text{by Lemma 64 LR with } \{x3 \leftarrow i(op_r(x1, x2, x3)), x2 \leftarrow x1, x1 \leftarrow x1\} \\
& \overbrace{rd(mult(x1, \underbrace{i(i(op_r(x1, x2, x3)))}), x1)} \\
= \quad & \text{by Lemma 3 LR with } \{x1 \leftarrow op_r(x1, x2, x3)\} \\
& rd(mult(x1, \overbrace{op_r(x1, x2, x3)}), x1)
\end{aligned}
$$

Lemma 1939: $op_r(x1, op_t(x2, x1), x3) = op_r(x1, x2, x3)$

$$
\begin{aligned}
& \underbrace{op_r(x1, op_t(x2, x1), x3)} \\
= \quad & \text{by Lemma 1938 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{rd(\underbrace{mult(x1, op_r(x1, x2, x3))}, x1)} \\
= \quad & \text{by Lemma 119 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{rd(\overbrace{mult(op_r(x1, x2, x3), x1)}, x1)} \\
= \quad & \text{by Axiom 5 RL with } \{x2 \leftarrow x1, x1 \leftarrow op_r(x1, x2, x3)\} \\
& \overbrace{op_r(x1, x2, x3)}
\end{aligned}
$$

Lemma 1940: $op_r(x1, rd(x2, x1), x3) = op_r(x1, i(x2), i(x3))$

$$
\begin{aligned}
& \underbrace{op_r(x1, rd(x2, x1), x3)} \\
= \quad & \text{by Lemma 1939 RL with } \{x3 \leftarrow x3, x2 \leftarrow rd(x2, x1), x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, \underbrace{op_t(rd(x2, x1), x1)}, x3)} \\
= \quad & \text{by Lemma 14 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \underbrace{op_r(x1, \overbrace{mult(i(x1), x2)}, x3)} \\
= \quad & \text{by Lemma 819 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, i(x2), i(x3))}
\end{aligned}
$$

Lemma 1941: $op_r(x1, rd(i(x1), x2), x3) = op_r(x1, x2, i(x3))$

$$
\begin{aligned}
& \underbrace{op_r(x1, rd(i(x1), x2), x3)} \\
= \quad & \text{by Lemma 1939 RL with } \{x3 \leftarrow x3, x2 \leftarrow rd(i(x1), x2), x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, \underbrace{op_t(rd(i(x1), x2), x1)}, x3)} \\
= \quad & \text{by Lemma 33 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{op_r(x1, \overbrace{i(mult(x1, x2))}, x3)} \\
= \quad & \text{by Lemma 818 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, x2, i(x3))}
\end{aligned}
$$

Lemma 1942: $op_r(x1, x2, i(i(x3))) = op_r(x1, op_r(x2, i(x1), i(x1)), x3)$

$$
\begin{aligned}
& \underbrace{op_r(x1, x2, i(i(x3)))} \\
= \quad & \text{by Lemma 1941 RL with } \{x3 \leftarrow i(x3), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{\overbrace{op_r(x1, rd(i(x1), x2), i(x3))}} \\
= \quad & \text{by Lemma 1941 RL with } \{x3 \leftarrow x3, x2 \leftarrow rd(i(x1), x2), x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, \underbrace{rd(i(x1), rd(i(x1), x2))}, x3)} \\
= \quad & \text{by Lemma 353 LR with } \{x2 \leftarrow x2, x1 \leftarrow i(x1)\} \\
& op_r(x1, \overbrace{op_r(x2, i(x1), i(x1))}, x3)
\end{aligned}
$$

Lemma 1943: $op_r(x1, op_r(x2, x1, x1), x3) = op_r(x1, x2, x3)$

$$
\begin{aligned}
& op_r(x1, \underbrace{op_r(x2, x1, x1)}, x3) \\
= & \quad \text{by Lemma 361 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \underbrace{op_r(x1, \overbrace{op_r(x2, i(x1), i(x1))}, x3)} \\
= & \quad \text{by Lemma 1942 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, x2, \underbrace{i(i(x3))})} \\
= & \quad \text{by Lemma 3 LR with } \{x1 \leftarrow x3\} \\
& op_r(x1, x2, \overbrace{x3}^{})
\end{aligned}
$$

Lemma 1944: $op_r(x1, x2, x3) = op_r(x1, op_l(x2, x1, x2), x3)$

$$
\begin{aligned}
& \underbrace{op_r(x1, x2, x3)} \\
= & \quad \text{by Lemma 1939 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_r(x1, op_t(x2, x1), x3)}} \\
= & \quad \text{by Lemma 1943 RL with } \{x3 \leftarrow x3, x2 \leftarrow op_t(x2, x1), x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, \underbrace{op_r(op_t(x2, x1), x1, x1)}, x3)} \\
= & \quad \text{by Lemma 745 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& op_r(x1, \overbrace{op_l(x2, x1, x2)}, x3)
\end{aligned}
$$

Lemma 1945: $op_r(x1, op_l(x2, x2, x1), x3) = op_r(x1, op_l(op_l(x2, x1, x2), x2, x1), x3)$

$$
\begin{aligned}
& \underbrace{op_r(x1, op_l(x2, x2, x1), x3)} \\
= & \quad \text{by Lemma 1944 LR with } \{x3 \leftarrow x3, x2 \leftarrow op_l(x2, x2, x1), x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, op_l(op_l(x2, x2, x1), x1, op_l(\underbrace{x2}, x2, x1)), x3)} \\
= & \quad \text{by Lemma 3 RL with } \{x1 \leftarrow x2\} \\
& op_r(x1, \underbrace{op_l(op_l(x2, x2, x1), x1, op_l(\overbrace{i(i(x2))}, x2, x1))}, x3) \\
= & \quad \text{by Lemma 1336 RL with } \{x2 \leftarrow x1, x3 \leftarrow x2, x1 \leftarrow op_l(x2, x2, x1)\} \\
& op_r(x1, \underbrace{\overbrace{op_l(op_l(x2, x2, x1), x2, i(x1))}}, x3) \\
= & \quad \text{by Lemma 1107 LR with } \{x4 \leftarrow x1, x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x2\} \\
& op_r(x1, \overbrace{op_l(op_l(x2, x1, x2), x2, x1)}, x3)
\end{aligned}
$$

Lemma 1946: $op_r(x1, i(x2), i(x3)) = op_r(x1, mult(x2, i(x1)), x3)$

$$
\begin{aligned}
& \underbrace{op_r(x1, i(x2), i(x3))} \\
= & \quad \text{by Lemma 1939 RL with } \{x3 \leftarrow i(x3), x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_r(x1, op_t(i(x2), x1), i(x3))}} \\
= & \quad \text{by Lemma 1110 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_r(\underbrace{i(i(x1))}, mult(x2, i(x1)), x3)} \\
= & \quad \text{by Lemma 3 LR with } \{x1 \leftarrow x1\} \\
& op_r(\overbrace{x1}^{}, mult(x2, i(x1)), x3)
\end{aligned}
$$

Lemma 1947: $op_r(i(x1), rd(x1, x2), x3) = i(op_r(x1, i(x2), x3))$

$$
\begin{aligned}
& \underbrace{op_r(i(x1), rd(x1, x2), x3)} \\
= & \quad \text{by Lemma 1939 RL with } \{x3 \leftarrow x3, x2 \leftarrow rd(x1, x2), x1 \leftarrow i(x1)\} \\
& \overbrace{op_r(i(x1), \underbrace{op_t(rd(x1, x2), i(x1))}, x3)} \\
= & \quad \text{by Lemma 34 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{op_r(i(x1), \overbrace{mult(x1, i(x2))}, x3)} \\
= & \quad \text{by Lemma 654 LR with } \{x3 \leftarrow x3, x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& \overbrace{i(op_r(x1, i(x2), x3))}
\end{aligned}
$$

Lemma 1948: $op_r(i(x1), rd(x1, x2), x3) = op_r(i(x1), x2, i(x3))$

$$
\begin{aligned}
& \underbrace{op_r(i(x1), rd(x1, x2), x3)} \\
= & \quad \text{by Lemma 1947 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{\overbrace{i(op_r(x1, i(x2), x3))}} \\
= & \quad \text{by Lemma 816 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_r(i(x1), x2, i(x3))}
\end{aligned}
$$

Lemma 1949: $op_r(x1, rd(x1, i(x2)), x3) = op_r(x1, mult(x2, x1), x3)$

$$
\begin{aligned}
& op_r(x1, \underbrace{rd(x1, i(x2))}, x3) \\
= & \quad \text{by Lemma 359 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \underbrace{op_r(x1, \overbrace{op_r(mult(x2, x1), x1, x1)}, x3)} \\
= & \quad \text{by Lemma 1943 LR with } \{x3 \leftarrow x3, x2 \leftarrow mult(x2, x1), x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, mult(x2, x1), x3)}
\end{aligned}
$$

Lemma 1950: $op_r(x1, op_t(x2, i(x1)), x3) = op_r(x1, x2, x3)$

$op_r(x1, \underbrace{op_t(x2, i(x1))}, x3)$

$=$ by Lemma 1001 RL with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$\underbrace{op_r(x1, \overbrace{mult(op_r(mult(x2, i(x1)), x2, x2), x1)}, x3)}$

$=$ by Lemma 1949 RL with $\{x3 \leftarrow x3, x2 \leftarrow op_r(mult(x2, i(x1)), x2, x2), x1 \leftarrow x1\}$

$\overbrace{op_r(x1, \underbrace{rd(x1, i(op_r(mult(x2, i(x1)), x2, x2)))}, x3)}$

$=$ by Lemma 38 LR with $\{x1 \leftarrow op_r(mult(x2, i(x1)), x2, x2), x2 \leftarrow x1\}$

$op_r(x1, \overbrace{\underbrace{rd(op_r(mult(x2, i(x1)), x2, x2), i(x1))}}, x3)$

$=$ by Lemma 727 LR with $\{x2 \leftarrow i(x1), x1 \leftarrow x2\}$

$op_r(x1, \overbrace{\underbrace{op_l(x2, i(x1), x2)}}, x3)$

$=$ by Lemma 1050 LR with $\{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x2\}$

$op_r(x1, \overbrace{\underbrace{op_l(x2, x1, i(x2))}}, x3)$

$=$ by Lemma 606 LR with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$\underbrace{op_r(x1, \overbrace{op_l(x2, x2, x1)}, x3)}$

$=$ by Lemma 1945 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_r(x1, \underbrace{op_l(op_l(x2, x1, x2), x2, x1)}, x3)}$

$=$ by Lemma 138 LR with $\{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x2\}$

$op_r(x1, \overbrace{x2}, x3)$

Lemma 1951: $op_r(x1, i(mult(x1, x2)), i(x3)) = op_r(i(i(x1)), mult(x1, rd(x2, x1)), x3)$

$\underbrace{op_r(x1, i(mult(x1, x2)), i(x3))}$

$=$ by Lemma 813 RL with $\{x3 \leftarrow x3, x2 \leftarrow mult(x1, x2), x1 \leftarrow x1\}$

$\overbrace{\underbrace{i(op_r(i(x1), mult(x1, x2), x3))}}$

$=$ by Lemma 1935 RL with $\{x3 \leftarrow x3, x2 \leftarrow mult(x1, x2), x1 \leftarrow i(x1)\}$

$\overbrace{op_r(i(i(x1)), \underbrace{mult(mult(x1, x2), i(x1))}, x3)}$

$=$ by Lemma 55 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$op_r(i(i(x1)), \overbrace{mult(x1, rd(x2, x1))}, x3)$

Lemma 1952: $op_r(x1, x2, x3) = op_r(i(i(x1)), mult(x1, rd(x2, x1)), x3)$

$op_r(x1, x2, \underbrace{x3})$

$=$ by Lemma 3 RL with $\{x1 \leftarrow x3\}$

$\underbrace{op_r(x1, x2, \overbrace{i(i(x3))})}$

$=$ by Lemma 818 RL with $\{x3 \leftarrow i(x3), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_r(x1, i(mult(x1, x2)), i(x3))}}$

$=$ by Lemma 1951 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_r(i(i(x1)), mult(x1, rd(x2, x1)), x3)}$

Lemma 1953: $op_r(x1, op_t(x2, i(x1)), x3) = op_r(x1, mult(x1, op_l(rd(x2, x1), x2, x1)), x3)$

$$
\begin{aligned}
& \underbrace{op_r(x1, op_t(x2, i(x1)), x3)} \\
= \quad & \text{by Lemma 1952 LR with } \{x3 \leftarrow x3, x2 \leftarrow op_t(x2, i(x1)), x1 \leftarrow x1\} \\
& \overbrace{op_r(\underbrace{i(i(x1))}, mult(x1, rd(op_t(x2, i(x1)), x1)), x3)} \\
= \quad & \text{by Lemma 3 LR with } \{x1 \leftarrow x1\} \\
& op_r(\overbrace{x1}, mult(x1, \underbrace{rd(op_t(x2, i(x1)), x1)}), x3) \\
= \quad & \text{by Lemma 928 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& op_r(x1, mult(x1, \overbrace{op_l(rd(x2, x1), x2, x1)}), x3)
\end{aligned}
$$

Lemma 1954: $op_r(x1, mult(x1, x2), x3) = op_r(x1, op_t(mult(x2, x1), x2), x3)$

$$
\begin{aligned}
& op_r(x1, mult(\underbrace{x1}, x2), x3) \\
= \quad & \text{by Lemma 3 RL with } \{x1 \leftarrow x1\} \\
& \underbrace{op_r(x1, mult(\overbrace{i(i(x1))}, x2), x3)} \\
= \quad & \text{by Lemma 1950 RL with } \{x3 \leftarrow x3, x2 \leftarrow mult(i(i(x1)), x2), x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, \underbrace{op_t(mult(i(i(x1)), x2), i(x1))}, x3)} \\
= \quad & \text{by Lemma 673 LR with } \{x2 \leftarrow x2, x1 \leftarrow i(x1)\} \\
& op_r(x1, \overbrace{op_t(mult(x2, \underbrace{i(i(x1))}), x2)}, x3) \\
= \quad & \text{by Lemma 3 LR with } \{x1 \leftarrow x1\} \\
& op_r(x1, op_t(mult(x2, \overbrace{x1}), x2), x3)
\end{aligned}
$$

Lemma 1955: $op_r(rd(x1,x2),op_r(x1,x2,x2),x3) = op_r(rd(x1,x2),rd(mult(x2,op_t(x1,rd(x1,mult(x1,i(x2))))),x2),x3)$

$op_r(rd(x1,x2),\underbrace{op_r(x1,x2,x2)},x3)$

$=$ by Lemma 698 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$op_r(rd(x1,x2),\overbrace{mult(rd(x1,x2),\underbrace{op_l(x2,x1,x2)})},x3)$

$=$ by Lemma 744 LR with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$\underbrace{op_r(rd(x1,x2),mult(rd(x1,x2),\overbrace{mult(mult(i(x1),x2),x1)}),x3)}$

$=$ by Lemma 1954 LR with $\{x3 \leftarrow x3, x2 \leftarrow mult(mult(i(x1),x2),x1), x1 \leftarrow rd(x1,x2)\}$

$\overbrace{op_r(rd(x1,x2),op_t(\underbrace{mult(mult(mult(i(x1),x2),x1),rd(x1,x2))},mult(mult(i(x1),x2),x1)),x3)}$

$=$ by Lemma 13 RL with $\{x2 \leftarrow mult(mult(i(x1),x2),x1), x1 \leftarrow rd(x1,x2)\}$

$op_r(rd(x1,x2),op_t(\overbrace{mult(rd(x1,x2),op_t(mult(mult(i(x1),x2),x1),\underbrace{rd(x1,x2)}))},mult(mult(i(x1),x2),x1)),x3)$

$=$ by Lemma 10 RL with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$op_r(rd(x1,x2),op_t(mult(rd(x1,x2),op_t(mult(\underbrace{mult(i(x1),x2)},x1),\overbrace{i(rd(x2,x1))})),mult(mult(i(x1),x2),x1)),x3)$

$=$ by Lemma 14 RL with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$op_r(rd(x1,x2),op_t(mult(rd(x1,x2),\underbrace{op_t(mult(\overbrace{op_t(rd(x2,x1),x1)},x1),i(rd(x2,x1))))},mult(mult(i(x1),x2),x1)),x3)$

$=$ by Lemma 751 RL with $\{x2 \leftarrow x1, x1 \leftarrow rd(x2,x1)\}$

$op_r(rd(x1,x2),op_t(mult(rd(x1,x2),\overbrace{op_t(\underbrace{mult(rd(x2,x1),x1)},x1)}),mult(mult(i(x1),x2),x1)),x3)$

$=$ by Axiom 4 RL with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$op_r(rd(x1,x2),op_t(\underbrace{mult(rd(x1,x2),op_t(\overbrace{x2},x1))},mult(mult(i(x1),x2),x1)),x3)$

$=$ by Lemma 650 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$op_r(rd(x1,x2),\underbrace{op_t(\overbrace{rd(mult(x2,x1),x2)},mult(mult(i(x1),x2),x1))},x3)$

$=$ by Lemma 1208 LR with $\{x3 \leftarrow mult(mult(i(x1),x2),x1), x2 \leftarrow x1, x1 \leftarrow x2\}$

$op_r(rd(x1,x2),\overbrace{rd(mult(x2,\underbrace{op_t(x1,mult(mult(i(x1),x2),x1))}),x2)},x3)$

$=$ by Lemma 1773 LR with $\{x4 \leftarrow x1, x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x1\}$

$op_r(rd(x1,x2),rd(mult(x2,\overbrace{op_t(x1,rd(x1,mult(x1,i(x2))))}),x2),x3)$

Lemma 1956: $op_r(rd(x1,x2),op_r(x1,x2,x2),x3) = op_r(rd(x1,x2),rd(mult(x2,op_t(x1,x2)),x2),x3)$

$\underbrace{op_r(rd(x1,x2),op_r(x1,x2,x2),x3)}$

$=$ by Lemma 1955 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_r(rd(x1,x2),rd(mult(x2,\underbrace{op_t(x1,rd(x1,mult(x1,i(x2))))}),x2),x3)}$

$=$ by Lemma 1733 LR with $\{x3 \leftarrow i(x2), x2 \leftarrow x1, x1 \leftarrow x1\}$

$op_r(rd(x1,x2),rd(mult(x2,\overbrace{op_t(x1,\underbrace{i(i(x2))})}),x2),x3)$

$=$ by Lemma 3 LR with $\{x1 \leftarrow x2\}$

$op_r(rd(x1,x2),rd(mult(x2,op_t(x1,\overbrace{x2})),x2),x3)$

Lemma 1957: $mult(mult(x1,x2),i(mult(op_t(x1,x3),x2))) = op_r(rd(x1,op_t(x1,x3)),mult(x1,x2),mult(x1,x2))$

$\overbrace{mult(mult(x1,x2),i(mult(op_t(x1,x3),x2)))}$

$=$ by Lemma 690 RL with $\{x2 \leftarrow mult(op_t(x1,x3),x2), x1 \leftarrow mult(x1,x2)\}$

$\overbrace{op_r(rd(op_t(mult(x1,x2),\underbrace{mult(op_t(x1,x3),x2)}),mult(op_t(x1,x3),x2)),mult(x1,x2),mult(x1,x2))}$

$=$ by Lemma 50 RL with $\{x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow x2\}$

$op_r(rd(op_t(\underbrace{mult(x1,x2)},\overbrace{mult(x2,op_t(op_t(x1,x2),x3))}),mult(op_t(x1,x3),x2)),mult(x1,x2),mult(x1,x2))$

$=$ by Lemma 13 RL with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$op_r(rd(op_t(\overbrace{mult(x2,op_t(x1,x2))},mult(\underbrace{x2},op_t(op_t(x1,x2),x3))),mult(op_t(x1,x3),x2)),mult(x1,x2),mult(x1,x2))$

$=$ by Axiom 5 LR with $\{x2 \leftarrow op_t(x1,x2), x1 \leftarrow x2\}$

$op_r(rd(op_t(mult(x2,op_t(x1,x2)),\underbrace{mult(\overbrace{rd(mult(x2,op_t(x1,x2)),op_t(x1,x2))},op_t(op_t(x1,x2),x3))}),mult(op_t(x1,x3),x2))$

$=$ by Lemma 1638 LR with $\{x3 \leftarrow x3, x2 \leftarrow op_t(x1,x2), x1 \leftarrow mult(x2,op_t(x1,x2))\}$

$op_r(rd(\underbrace{op_t(mult(x2,op_t(x1,x2))},\overbrace{rd(mult(x2,op_t(x1,x2)),rd(op_t(x1,x2),op_t(op_t(x1,x2),x3)))}),mult(op_t(x1,x3),x2)),m$

$=$ by Lemma 738 RL with $\{x3 \leftarrow op_t(x1,x2), x2 \leftarrow op_t(op_t(x1,x2),x3), x1 \leftarrow mult(x2,op_t(x1,x2))\}$

$op_r(rd(\overbrace{op_t(mult(x2,op_t(x1,x2)),mult(mult(x2,op_t(x1,x2)),rd(op_t(op_t(x1,x2),x3),op_t(x1,x2))))},mult(op_t(x1,x3),x2))$

$=$ by Lemma 1716 RL with $\{x3 \leftarrow x3, x2 \leftarrow op_t(x1,x2), x1 \leftarrow mult(x2,op_t(x1,x2))\}$

$op_r(rd(\overbrace{mult(x2,op_t(x1,x2))},mult(op_t(x1,x3),x2)),mult(x1,x2),mult(x1,x2))$

$=$ by Lemma 13 LR with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$op_r(rd(\overbrace{mult(x1,x2)},\underbrace{mult(op_t(x1,x3),x2)}),mult(x1,x2),mult(x1,x2))$

$=$ by Axiom 1 LR with $\{x1 \leftarrow mult(op_t(x1,x3),x2)\}$

$op_r(rd(mult(\underbrace{x1},x2),\overbrace{mult(mult(op_t(x1,x3),x2),unit())}),mult(x1,x2),mult(x1,x2))$

$=$ by Axiom 4 LR with $\{x2 \leftarrow op_t(x1,x3), x1 \leftarrow x1\}$

$op_r(rd(mult(\overbrace{mult(rd(x1,op_t(x1,x3)),op_t(x1,x3))},x2),mult(mult(op_t(x1,x3),x2),\underbrace{unit()})),mult(x1,x2),mult(x1,x2))$

$=$ by Lemma 942 RL with $\{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x1\}$

$op_r(rd(\underbrace{mult(mult(rd(x1,op_t(x1,x3)),op_t(x1,x3)),x2)},mult(mult(op_t(x1,x3),x2),\overbrace{asoc(rd(x1,op_t(x1,x3)),op_t(x1,x3),x}$

$=$ by Axiom 8 LR with $\{x3 \leftarrow x2, x2 \leftarrow op_t(x1,x3), x1 \leftarrow rd(x1,op_t(x1,x3))\}$

$op_r(\underbrace{rd(\overbrace{mult(mult(rd(x1,op_t(x1,x3)),mult(op_t(x1,x3),x2)),asoc(rd(x1,op_t(x1,x3)),op_t(x1,x3),x2))},mult(mult(op_t(x1$

$=$ by Axiom 16 RL with $\{x3 \leftarrow asoc(rd(x1,op_t(x1,x3)),op_t(x1,x3),x2), x2 \leftarrow mult(op_t(x1,x3),x2), x1 \leftarrow rd(x1,op_t(x1,x3))\}$

$op_r(\overbrace{op_r(rd(x1,op_t(x1,x3)),mult(op_t(x1,x3),x2),\underbrace{asoc(rd(x1,op_t(x1,x3)),op_t(x1,x3),x2)})},mult(x1,x2),mult(x1,x2))$

$=$ by Lemma 942 LR with $\{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x1\}$

$op_r(\underbrace{op_r(rd(x1,op_t(x1,x3)),mult(op_t(x1,x3),x2),\overbrace{unit()})},mult(x1,x2),mult(x1,x2))$

$=$ by Lemma 6 LR with $\{x2 \leftarrow mult(op_t(x1,x3),x2), x1 \leftarrow rd(x1,op_t(x1,x3))\}$

$op_r(\overbrace{rd(x1,op_t(x1,x3))},mult(x1,x2),mult(x1,x2))$

Lemma 1958: $op_r(x1, x2, i(mult(op_t(x1, x3), x2))) = rd(x1, rd(op_t(x1, x3), op_t(op_t(x1, x3), x2)))$

$\underbrace{op_r(x1, x2, i(mult(op_t(x1, x3), x2)))}$

$=$ by Axiom 16 LR with $\{x3 \leftarrow i(mult(op_t(x1, x3), x2)), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{rd(\underbrace{mult(mult(x1, x2), i(mult(op_t(x1, x3), x2)))}, mult(x2, i(mult(op_t(x1, x3), x2))))}$

$=$ by Lemma 1957 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$rd(\overbrace{\underbrace{op_r(rd(x1, op_t(x1, x3)), mult(x1, x2), mult(x1, x2))}}, mult(x2, i(mult(op_t(x1, x3), x2))))$

$=$ by Lemma 353 RL with $\{x2 \leftarrow rd(x1, op_t(x1, x3)), x1 \leftarrow mult(x1, x2)\}$

$rd(\overbrace{rd(mult(x1, x2), \underbrace{rd(mult(x1, x2), \underbrace{rd(x1, op_t(x1, x3))})})}, mult(x2, i(mult(op_t(x1, x3), x2))))$

$=$ by Lemma 10 RL with $\{x2 \leftarrow x1, x1 \leftarrow op_t(x1, x3)\}$

$rd(rd(mult(x1, x2), \underbrace{rd(mult(x1, x2), \overbrace{i(rd(op_t(x1, x3), x1))})}), mult(x2, i(mult(op_t(x1, x3), x2))))$

$=$ by Lemma 38 RL with $\{x1 \leftarrow mult(x1, x2), x2 \leftarrow rd(op_t(x1, x3), x1)\}$

$rd(rd(mult(x1, x2), \overbrace{\underbrace{rd(rd(op_t(x1, x3), x1), i(mult(x1, x2)))}}), mult(x2, i(mult(op_t(x1, x3), x2))))$

$=$ by Lemma 1291 LR with $\{x3 \leftarrow i(mult(x1, x2)), x2 \leftarrow x3, x1 \leftarrow x1\}$

$rd(rd(mult(x1, x2), \overbrace{\underbrace{rd(op_t(x1, x3), mult(i(mult(x1, x2)), x1))}}), mult(x2, i(mult(op_t(x1, x3), x2))))$

$=$ by Lemma 1255 LR with $\{x1 \leftarrow mult(x1, x2), x3 \leftarrow x3, x2 \leftarrow x1\}$

$rd(\underbrace{rd(mult(x1, x2), \overbrace{mult(mult(x1, x2), rd(op_t(x1, x3), x1))})}, mult(x2, i(mult(op_t(x1, x3), x2))))$

$=$ by Lemma 1718 RL with $\{x3 \leftarrow mult(x1, x2), x2 \leftarrow x3, x1 \leftarrow x1\}$

$\underbrace{rd(\overbrace{rd(x1, op_t(x1, x3))}, mult(x2, i(mult(op_t(x1, x3), x2))))}$

$=$ by Lemma 1236 RL with $\{x3 \leftarrow x3, x2 \leftarrow mult(x2, i(mult(op_t(x1, x3), x2))), x1 \leftarrow x1\}$

$\overbrace{rd(x1, mult(\underbrace{mult(x2, i(mult(op_t(x1, x3), x2)))}, op_t(x1, x3)))}$

$=$ by Lemma 31 LR with $\{x2 \leftarrow op_t(x1, x3), x1 \leftarrow x2\}$

$rd(x1, \underbrace{mult(\overbrace{i(op_t(op_t(x1, x3), x2))}, op_t(x1, x3)))}$

$=$ by Lemma 99 LR with $\{x2 \leftarrow x2, x1 \leftarrow op_t(x1, x3)\}$

$rd(x1, \overbrace{rd(op_t(x1, x3), op_t(op_t(x1, x3), x2)))}$

Lemma 1959: $op_r(x1, x2, i(mult(op_t(x1, x3), x2))) = rd(x1, rd(x1, op_t(x1, x2)))$

$\underbrace{op_r(x1, x2, i(mult(op_t(x1, x3), x2)))}$

$=$ by Lemma 1958 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{rd(x1, \underbrace{rd(op_t(x1, x3), op_t(op_t(x1, x3), x2))})}$

$=$ by Lemma 1770 LR with $\{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x1\}$

$rd(x1, \underbrace{\overbrace{rd(op_l(x2, x1, x2), op_r(x2, x1, x1))})}$

$=$ by Lemma 1182 LR with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$rd(x1, \overbrace{rd(x1, op_t(x1, x2))})$

Lemma 1960: $op_t(x1, x2) = op_r(x1, x2, rd(i(x2), op_t(x1, x3)))$

$\underbrace{op_t(x1, x2)}$

$=$ by Lemma 1722 RL with $\{x3 \leftarrow op_t(x1, x3), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_t(x1, op_t(x2, op_t(x1, x3)))}$

$=$ by Lemma 25 RL with $\{x2 \leftarrow x1, x1 \leftarrow op_t(x1, op_t(x2, op_t(x1, x3)))\}$

$\overbrace{op_l(op_t(x1, op_t(x2, op_t(x1, x3))), x1, x1)}$

$=$ by Axiom 15 RL with $\{x4 \leftarrow op_t(x2, op_t(x1, x3)), x3 \leftarrow x1, x2 \leftarrow x1, x1 \leftarrow x1\}$

$\overbrace{op_t(\underbrace{op_l(x1, x1, x1)}, op_t(x2, op_t(x1, x3)))}$

$=$ by Lemma 601 RL with $\{x2 \leftarrow x1, x1 \leftarrow x1\}$

$\underbrace{op_t(\overbrace{op_r(x1, x1, x1)}, op_t(x2, op_t(x1, x3)))}$

$=$ by Axiom 14 LR with $\{x4 \leftarrow op_t(x2, op_t(x1, x3)), x3 \leftarrow x1, x2 \leftarrow x1, x1 \leftarrow x1\}$

$\overbrace{op_r(op_t(x1, op_t(x2, op_t(x1, x3))), x1, x1)}$

$=$ by Lemma 353 RL with $\{x2 \leftarrow op_t(x1, op_t(x2, op_t(x1, x3))), x1 \leftarrow x1\}$

$\overbrace{rd(x1, rd(x1, op_t(x1, op_t(x2, op_t(x1, x3)))))}$

$=$ by Lemma 1959 RL with $\{x3 \leftarrow x3, x2 \leftarrow op_t(x2, op_t(x1, x3)), x1 \leftarrow x1\}$

$\overbrace{op_r(x1, op_t(x2, op_t(x1, x3)), i(\underbrace{mult(op_t(x1, x3), op_t(x2, op_t(x1, x3)))}))}$

$=$ by Lemma 13 LR with $\{x2 \leftarrow x2, x1 \leftarrow op_t(x1, x3)\}$

$op_r(x1, \underbrace{op_t(x2, op_t(x1, x3))}, i(\overbrace{mult(x2, op_t(x1, x3))}))$

$=$ by Lemma 1722 LR with $\{x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow x2\}$

$\underbrace{op_r(x1, \overbrace{op_t(x2, x1)}, i(mult(x2, op_t(x1, x3))))}$

$=$ by Lemma 1939 LR with $\{x3 \leftarrow i(mult(x2, op_t(x1, x3))), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_r(x1, x2, i(mult(x2, op_t(x1, x3))))}$

$=$ by Lemma 1892 LR with $\{x3 \leftarrow op_t(x1, x3), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_r(x1, x2, rd(i(x2), op_t(x1, x3)))}$

Lemma 1961: $op_t(x1, mult(x2, i(op_t(x1, x3)))) = op_r(x1, mult(x2, i(op_t(x1, x3))), i(x2))$

$\underbrace{op_t(x1, mult(x2, i(op_t(x1, x3))))}$

$=$ by Lemma 1960 LR with $\{x3 \leftarrow x3, x2 \leftarrow mult(x2, i(op_t(x1, x3))), x1 \leftarrow x1\}$

$\overbrace{op_r(x1, mult(x2, i(op_t(x1, x3))), \underbrace{rd(i(mult(x2, i(op_t(x1, x3)))), op_t(x1, x3))})}$

$=$ by Lemma 37 LR with $\{x2 \leftarrow op_t(x1, x3), x1 \leftarrow mult(x2, i(op_t(x1, x3)))\}$

$op_r(x1, mult(x2, i(op_t(x1, x3))), \overbrace{rd(i(op_t(x1, x3)), mult(x2, i(op_t(x1, x3))))})$

$=$ by Lemma 9 LR with $\{x2 \leftarrow x2, x1 \leftarrow i(op_t(x1, x3))\}$

$op_r(x1, mult(x2, i(op_t(x1, x3))), \overbrace{i(x2)})$

Lemma 1962: $op_t(x1, rd(x2, x1)) = op_r(x1, mult(x2, i(op_t(x1, x3))), i(x2))$

$$
\begin{aligned}
& \underbrace{op_t(x1, rd(x2, x1))} \\
= \quad & \text{by Lemma 1158 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_t(x1, rd(x2, op_t(x1, x3)))}} \\
= \quad & \text{by Lemma 1730 RL with } \{x3 \leftarrow op_t(x1, x3), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_t(x1, mult(x2, i(op_t(x1, x3))))}} \\
= \quad & \text{by Lemma 1961 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, mult(x2, i(op_t(x1, x3))), i(x2))}
\end{aligned}
$$

Lemma 1963: $op_t(x1, rd(x2, x1)) = op_r(x1, rd(x2, op_t(x1, x3)), i(x2))$

$$
\begin{aligned}
& \underbrace{op_t(x1, rd(x2, x1))} \\
= \quad & \text{by Lemma 1962 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, \underbrace{mult(x2, i(op_t(x1, x3)))}, i(x2))} \\
= \quad & \text{by Lemma 34 RL with } \{x2 \leftarrow op_t(x1, x3), x1 \leftarrow x2\} \\
& \underbrace{op_r(x1, \overbrace{op_t(rd(x2, op_t(x1, x3)), i(x2))}, i(x2))} \\
= \quad & \text{by Lemma 1894 LR with } \{x3 \leftarrow i(x2), x2 \leftarrow rd(x2, op_t(x1, x3)), x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, rd(x2, op_t(x1, x3)), i(x2))}
\end{aligned}
$$

Lemma 1964: $op_l(x1, op_t(x2, x3), i(x3)) = op_l(x1, op_t(op_t(x2, x3), rd(x1, mult(x2, i(op_t(x3, x2))))), i(x3))$

$\underbrace{op_l(x1, op_t(x2, x3), i(x3))}$

$=$ by Lemma 1620 RL with $\{x3 \leftarrow x1, x2 \leftarrow op_t(x2, x3), x1 \leftarrow i(x3)\}$

$\overbrace{mult(i(x3), \underbrace{mult(op_t(x2, x3), mult(i(\underbrace{mult(op_t(x2, x3), i(x3))}), x1))})}$

$=$ by Lemma 13 RL with $\{x2 \leftarrow op_t(x2, x3), x1 \leftarrow mult(i(\underbrace{mult(op_t(x2, x3), i(x3))}), x1)\}$

$mult(i(x3), \overbrace{\underbrace{mult(mult(i(\underbrace{mult(op_t(x2, x3), i(x3))}), x1)}, op_t(op_t(x2, x3), mult(i(\underbrace{mult(op_t(x2, x3), i(x3))}), x1))})$

$=$ by Lemma 1738 RL with $\{x3 \leftarrow mult(i(\underbrace{mult(op_t(x2, x3), i(x3))}), x1), x2 \leftarrow mult(i(\underbrace{mult(op_t(x2, x3), i(x3))}), x1), x1 \leftarrow op_t(x$

$\underbrace{mult(i(x3), \overbrace{mult(op_t(op_t(x2, x3), mult(i(\underbrace{mult(op_t(x2, x3), i(x3))}), x1)), op_t(mult(i(\underbrace{mult(op_t(x2, x3), i(x3))}), x1), op_t(x2, x}$

$=$ by Lemma 1068 RL with $\{x3 \leftarrow op_t(mult(i(\underbrace{mult(op_t(x2, x3), i(x3))}), x1), op_t(x2, x3)), x2 \leftarrow i(x3), x1 \leftarrow op_t(op_t(x2, x3), mu$

$\overbrace{op_l(mult(\underbrace{mult(op_t(op_t(x2, x3), mult(i(\underbrace{mult(op_t(x2, x3), i(x3))}), x1)), i(x3)}), op_t(mult(i(\underbrace{mult(op_t(x2, x3), i(x3))}), x1), op_t}$

$=$ by Lemma 231 LR with $\{x2 \leftarrow i(x3), x1 \leftarrow op_t(op_t(x2, x3), mult(i(\underbrace{mult(op_t(x2, x3), i(x3))}), x1))\}$

$op_l(mult(\overbrace{rd(op_t(op_t(x2, x3), mult(i(\underbrace{mult(op_t(x2, x3), i(x3))}), x1)), \underbrace{op_l(i(i(x3)), i(x3), op_t(op_t(x2, x3), mult(i(\underbrace{mult(op_t(x2}$

$=$ by Lemma 1740 LR with $\{x3 \leftarrow mult(i(\underbrace{mult(op_t(x2, x3), i(x3))}), x1), x2 \leftarrow op_t(x2, x3), x1 \leftarrow i(x3)\}$

$op_l(\underbrace{mult(rd(op_t(op_t(x2, x3), mult(i(\underbrace{mult(op_t(x2, x3), i(x3))}), x1)), \overbrace{op_l(i(i(x3)), i(x3), op_t(x2, x3))}), op_t(mult(i(mult(op_t(x}$

$=$ by Lemma 1518 LR with $\{x3 \leftarrow op_l(i(i(x3)), i(x3), op_t(x2, x3)), x2 \leftarrow mult(i(\underbrace{mult(op_t(x2, x3), i(x3))}), x1), x1 \leftarrow op_t(x2, x3)$

$op_l(\overbrace{mult(\underbrace{rd(op_t(x2, x3), op_l(i(i(x3)), i(x3), op_t(x2, x3)))}, mult(i(\underbrace{mult(op_t(x2, x3), i(x3))}), x1))}, op_t(op_t(x2, x3), mult(i(mu$

$=$ by Lemma 231 RL with $\{x2 \leftarrow i(x3), x1 \leftarrow op_t(x2, x3)\}$

$op_l(\underbrace{mult(\overbrace{mult(op_t(x2, x3), i(x3))}, mult(i(\underbrace{mult(op_t(x2, x3), i(x3))}), x1))}, op_t(op_t(x2, x3), mult(i(mult(op_t(x2, x3), i(x3))),$

$=$ by Lemma 12 LR with $\{x2 \leftarrow x1, x1 \leftarrow mult(op_t(x2, x3), i(x3))\}$

$op_l(\overbrace{x1}, \underbrace{op_t(op_t(x2, x3), mult(i(\underbrace{mult(op_t(x2, x3), i(x3))}), x1))}, i(x3))$

$=$ by Lemma 1728 LR with $\{x3 \leftarrow x1, x2 \leftarrow mult(op_t(x2, x3), i(x3)), x1 \leftarrow op_t(x2, x3)\}$

$op_l(x1, \overbrace{op_t(op_t(x2, x3), rd(x1, \underbrace{mult(op_t(x2, x3), i(x3))}))}, i(x3))$

$=$ by Lemma 933 LR with $\{x2 \leftarrow x3, x1 \leftarrow x2\}$

$op_l(x1, op_t(op_t(x2, x3), rd(x1, \overbrace{mult(x2, i(op_t(x3, x2)))})), i(x3))$

Lemma 1965: $op_r(rd(rd(x1, op_t(x2, x3)), x3), x3, x2) = rd(x1, mult(x2, x3))$

$\underbrace{op_r(rd(rd(x1, op_t(x2, x3)), x3), x3, x2)}$

$=$ by Lemma 1890 RL with $\{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow rd(rd(x1, op_t(x2, x3)), x3)\}$

$\overbrace{op_r(rd(rd(x1, op_t(x2, x3)), x3), x3, \underbrace{op_t(x2, x3)})}$

$=$ by Lemma 839 LR with $\{x3 \leftarrow x3, x2 \leftarrow op_t(x2, x3), x1 \leftarrow x1\}$

$\overbrace{rd(x1, \underbrace{mult(x3, op_t(x2, x3))})}$

$=$ by Lemma 13 LR with $\{x2 \leftarrow x2, x1 \leftarrow x3\}$

$rd(x1, \overbrace{mult(x2, x3)})$

Lemma 1966: $op_r(x1, op_r(rd(i(x2), x1), x2, x2), x3) = op_r(x1, x2, i(x3))$

$op_r(\underbrace{x1}, op_r(rd(i(x2), x1), x2, x2), x3)$

$=$ by Lemma 3 RL with $\{x1 \leftarrow x1\}$

$op_r(\overbrace{i(i(x1))}, \underbrace{op_r(rd(i(x2), x1), x2, x2)}, x3)$

$=$ by Lemma 361 RL with $\{x2 \leftarrow x2, x1 \leftarrow rd(i(x2), x1)\}$

$op_r(i(i(x1)), \overbrace{\underbrace{op_r(rd(i(x2), x1), i(x2), i(x2))}, x3)}$

$=$ by Lemma 369 RL with $\{x2 \leftarrow x1, x1 \leftarrow i(x2)\}$

$op_r(i(i(x1)), \overbrace{rd(i(x2), \underbrace{op_r(x1, i(x2), i(x2))})}, x3)$

$=$ by Lemma 361 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$op_r(i(i(x1)), \underbrace{rd(i(x2), \overbrace{op_r(x1, x2, x2)})}, x3)$

$=$ by Lemma 1887 LR with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$\underbrace{op_r(i(i(x1)), \overbrace{rd(i(x1), op_l(x2, x1, x2))}, x3)}$

$=$ by Lemma 1948 LR with $\{x3 \leftarrow x3, x2 \leftarrow op_l(x2, x1, x2), x1 \leftarrow i(x1)\}$

$\overbrace{op_r(\underbrace{i(i(x1))}, op_l(x2, x1, x2), i(x3))}$

$=$ by Lemma 3 LR with $\{x1 \leftarrow x1\}$

$\underbrace{op_r(\overbrace{x1}, op_l(x2, x1, x2), i(x3))}$

$=$ by Lemma 1944 RL with $\{x3 \leftarrow i(x3), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_r(x1, x2, i(x3))}$

Lemma 1967: $mult(x1, op_r(mult(i(x1), x2), x3, x4)) = mult(op_r(rd(x2, x1), x3, x4), x1)$

$mult(x1, op_r(\underbrace{mult(i(x1), x2)}, x3, x4))$

$=$ by Lemma 14 RL with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$\underbrace{mult(x1, op_r(\overbrace{op_t(rd(x2, x1), x1)}, x3, x4))}$

$=$ by Lemma 155 LR with $\{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow rd(x2, x1), x1 \leftarrow x1\}$

$\overbrace{mult(op_r(rd(x2, x1), x3, x4), x1)}$

Lemma 1968: $mult(op_r(op_r(x1, x2, x2), x3, x4), x2) = mult(x2, op_r(op_l(x1, x1, i(x2)), x3, x4))$

$mult(op_r(\underbrace{op_r(x1, x2, x2)}, x3, x4), x2)$

$=$ by Lemma 358 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{mult(op_r(\overbrace{rd(rd(x1, i(x2)), x2)}, x3, x4), x2)}$

$=$ by Lemma 1967 RL with $\{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow rd(x1, i(x2)), x1 \leftarrow x2\}$

$\overbrace{mult(x2, op_r(\underbrace{mult(i(x2), rd(x1, i(x2)))}, x3, x4))}$

$=$ by Lemma 427 RL with $\{x2 \leftarrow i(x2), x1 \leftarrow x1\}$

$mult(x2, op_r(\overbrace{op_l(x1, x1, i(x2))}, x3, x4))$

Lemma 1969: $mult(x1, op_l(op_r(x2, x3, x4), x1, x2)) = rd(x1, i(op_r(x2, x3, x4)))$

$mult(x1, \underbrace{op_l(op_r(x2, x3, x4), x1, x2)})$

$=$ by Axiom 18 RL with $\{x5 \leftarrow x4, x4 \leftarrow x3, x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x2\}$

$mult(x1, \overbrace{op_r(\underbrace{op_l(x2, x1, x2)}, x3, x4)})$

$=$ by Lemma 608 RL with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$\underbrace{mult(x1, op_r(\overbrace{op_l(x2, x2, i(x1))}, x3, x4))}$

$=$ by Lemma 1968 RL with $\{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow x2\}$

$\overbrace{mult(\underbrace{op_r(op_r(x2, x1, x1), x3, x4)}, x1)}$

$=$ by Axiom 17 RL with $\{x5 \leftarrow x1, x4 \leftarrow x1, x3 \leftarrow x4, x2 \leftarrow x3, x1 \leftarrow x2\}$

$\underbrace{mult(\overbrace{op_r(op_r(x2, x3, x4), x1, x1)}, x1)}$

$=$ by Lemma 357 LR with $\{x2 \leftarrow x1, x1 \leftarrow op_r(x2, x3, x4)\}$

$\overbrace{rd(x1, i(op_r(x2, x3, x4)))}$

Lemma 1970: $rd(op_l(op_r(x1,x2,x3),x4,x1),x4) = op_r(rd(op_r(x1,x2,x3),op_t(x4,op_r(x1,x2,x3))),x4,x4)$

$\overbrace{rd(op_l(op_r(x1,x2,x3),x4,x1),x4)}$

$=$ by Axiom 5 LR with $\{x2 \leftarrow op_t(x4,op_l(op_r(x1,x2,x3),x4,x1)), x1 \leftarrow rd(op_l(op_r(x1,x2,x3),x4,x1),x4)\}$

$\overbrace{rd(\underbrace{mult(rd(op_l(op_r(x1,x2,x3),x4,x1),x4),op_t(x4,op_l(op_r(x1,x2,x3),x4,x1)))}, op_t(x4,op_l(op_r(x1,x2,x3),x4,x1)))}$

$=$ by Lemma 650 LR with $\{x2 \leftarrow x4, x1 \leftarrow op_l(op_r(x1,x2,x3),x4,x1)\}$

$rd(\overbrace{rd(\underbrace{mult(x4,op_l(op_r(x1,x2,x3),x4,x1))},x4)}, op_t(x4,op_l(op_r(x1,x2,x3),x4,x1)))$

$=$ by Lemma 1969 LR with $\{x4 \leftarrow x3, x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x4\}$

$\underbrace{rd(\overbrace{rd(x4,i(\underbrace{op_r(x1,x2,x3)}))},x4)}, op_t(x4,op_l(op_r(x1,x2,x3),x4,x1)))$

$=$ by Lemma 354 LR with $\{x2 \leftarrow i(op_r(x1,x2,x3)), x1 \leftarrow x4\}$

$\underbrace{rd(\overbrace{op_r(i(i(op_r(x1,x2,x3))),x4,x4)}, op_t(x4,op_l(op_r(x1,x2,x3),x4,x1)))}$

$=$ by Lemma 1672 LR with $\{x3 \leftarrow op_l(op_r(x1,x2,x3),x4,x1), x2 \leftarrow x4, x1 \leftarrow i(i(op_r(x1,x2,x3)))\}$

$\overbrace{op_r(rd(\underbrace{i(i(op_r(x1,x2,x3)))}, op_t(x4,op_l(op_r(x1,x2,x3),x4,x1))),x4,x4)}$

$=$ by Lemma 3 LR with $\{x1 \leftarrow op_r(x1,x2,x3)\}$

$op_r(rd(\overbrace{op_r(x1,x2,x3)}, op_t(x4,\underbrace{op_l(op_r(x1,x2,x3),x4,x1)})),x4,x4)$

$=$ by Axiom 18 RL with $\{x5 \leftarrow x3, x4 \leftarrow x2, x3 \leftarrow x1, x2 \leftarrow x4, x1 \leftarrow x1\}$

$op_r(rd(op_r(x1,x2,x3), op_t(x4,\overbrace{op_r(\underbrace{op_l(x1,x4,x1)},x2,x3)})),x4,x4)$

$=$ by Lemma 745 LR with $\{x2 \leftarrow x4, x1 \leftarrow x1\}$

$op_r(rd(op_r(x1,x2,x3), op_t(x4,\underbrace{op_r(\overbrace{op_r(op_t(x1,x4),x4,x4)},x2,x3)})),x4,x4)$

$=$ by Axiom 17 RL with $\{x5 \leftarrow x4, x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow op_t(x1,x4)\}$

$op_r(rd(op_r(x1,x2,x3),\underbrace{op_t(x4,\overbrace{op_r(op_r(op_t(x1,x4),x2,x3),x4,x4)})},x4,x4)$

$=$ by Lemma 1721 LR with $\{x3 \leftarrow x4, x2 \leftarrow op_r(op_t(x1,x4),x2,x3), x1 \leftarrow x4\}$

$op_r(rd(op_r(x1,x2,x3),\overbrace{op_t(x4,\underbrace{op_r(op_t(x1,x4),x2,x3)})},x4,x4)$

$=$ by Axiom 14 RL with $\{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$op_r(rd(op_r(x1,x2,x3),\underbrace{op_t(x4,\overbrace{op_t(op_r(x1,x2,x3),x4)})},x4,x4)$

$=$ by Lemma 1722 LR with $\{x3 \leftarrow x4, x2 \leftarrow op_r(x1,x2,x3), x1 \leftarrow x4\}$

$op_r(rd(op_r(x1,x2,x3),\overbrace{op_t(x4,op_r(x1,x2,x3))}),x4,x4)$

Lemma 1971: $rd(op_l(op_r(x1,x2,x3),x4,x1),x4) = mult(i(x4),op_r(x1,x2,x3))$

$\overbrace{rd(op_l(op_r(x1,x2,x3),x4,x1),x4)}$

$=$ by Lemma 1970 LR with $\{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_r(rd(op_r(x1,x2,x3),op_t(x4,op_r(x1,x2,x3))),x4,x4)}}$

$=$ by Lemma 691 LR with $\{x1 \leftarrow x4, x2 \leftarrow op_r(x1,x2,x3)\}$

$\overbrace{\underbrace{rd(mult(mult(i(x4),op_r(x1,x2,x3)),x4),x4)}}$

$=$ by Axiom 5 RL with $\{x2 \leftarrow x4, x1 \leftarrow mult(i(x4),op_r(x1,x2,x3))\}$

$\overbrace{mult(i(x4),op_r(x1,x2,x3))}$

Lemma 1972: $rd(x1, op_l(op_r(x2, x3, x4), x1, x2)) = mult(x1, i(op_r(x2, x3, x4)))$

$$
\begin{aligned}
& \underbrace{rd(x1, op_l(op_r(x2, x3, x4), x1, x2))} \\
= \quad & \text{by Lemma 10 RL with } \{x2 \leftarrow x1, x1 \leftarrow op_l(op_r(x2, x3, x4), x1, x2)\} \\
& \overbrace{i(\underbrace{rd(op_l(op_r(x2, x3, x4), x1, x2), x1)})} \\
= \quad & \text{by Lemma 1971 LR with } \{x4 \leftarrow x1, x3 \leftarrow x4, x2 \leftarrow x3, x1 \leftarrow x2\} \\
& \underbrace{i(\overbrace{mult(i(x1), op_r(x2, x3, x4))})} \\
= \quad & \text{by Lemma 16 LR with } \{x2 \leftarrow op_r(x2, x3, x4), x1 \leftarrow x1\} \\
& \overbrace{mult(x1, i(op_r(x2, x3, x4)))}
\end{aligned}
$$

Lemma 1973: $mult(x1, op_l(mult(i(x1), x2), x3, x4)) = mult(op_l(rd(x2, x1), x3, x4), x1)$

$$
\begin{aligned}
& mult(x1, op_l(\underbrace{mult(i(x1), x2)}, x3, x4)) \\
= \quad & \text{by Lemma 14 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \underbrace{mult(x1, op_l(\overbrace{op_t(rd(x2, x1), x1)}, x3, x4))} \\
= \quad & \text{by Lemma 167 LR with } \{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow rd(x2, x1), x1 \leftarrow x1\} \\
& \overbrace{mult(op_l(rd(x2, x1), x3, x4), x1)}
\end{aligned}
$$

Lemma 1974: $mult(op_l(op_r(x1, x2, x2), x3, x4), x2) = mult(x2, op_l(op_l(x1, x1, i(x2)), x3, x4))$

$$
\begin{aligned}
& mult(op_l(\underbrace{op_r(x1, x2, x2)}, x3, x4), x2) \\
= \quad & \text{by Lemma 358 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{mult(op_l(\overbrace{rd(rd(x1, i(x2)), x2)}, x3, x4), x2)} \\
= \quad & \text{by Lemma 1973 RL with } \{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow rd(x1, i(x2)), x1 \leftarrow x2\} \\
& \overbrace{mult(x2, op_l(\underbrace{mult(i(x2), rd(x1, i(x2)))}, x3, x4))} \\
= \quad & \text{by Lemma 427 RL with } \{x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& mult(x2, op_l(\overbrace{op_l(x1, x1, i(x2))}, x3, x4))
\end{aligned}
$$

Lemma 1975: $mult(x1, op_l(op_l(x2, x1, x2), x3, x4)) = rd(x1, op_l(i(x2), x3, x4))$

$$
\begin{aligned}
& mult(x1, op_l(\underbrace{op_l(x2, x1, x2)}, x3, x4)) \\
= \quad & \text{by Lemma 608 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \underbrace{mult(x1, op_l(\overbrace{op_l(x2, x2, i(x1))}, x3, x4))} \\
= \quad & \text{by Lemma 1974 RL with } \{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \overbrace{mult(\underbrace{op_l(op_r(x2, x1, x1), x3, x4)}, x1)} \\
= \quad & \text{by Axiom 18 RL with } \{x5 \leftarrow x1, x4 \leftarrow x1, x3 \leftarrow x4, x2 \leftarrow x3, x1 \leftarrow x2\} \\
& \underbrace{mult(\overbrace{op_r(op_l(x2, x3, x4), x1, x1)}, x1)} \\
= \quad & \text{by Lemma 357 LR with } \{x2 \leftarrow x1, x1 \leftarrow op_l(x2, x3, x4)\} \\
& \overbrace{rd(x1, \underbrace{i(op_l(x2, x3, x4))})} \\
= \quad & \text{by Lemma 1049 LR with } \{x3 \leftarrow x4, x2 \leftarrow x3, x1 \leftarrow x2\} \\
& rd(x1, \overbrace{op_l(i(x2), x3, x4)})
\end{aligned}
$$

Lemma 1976: $rd(x1, op_l(op_l(i(i(x2)), x1, x2), x3, x4)) = mult(x1, op_l(op_l(op_l(i(x2), x1, x2), x2, x1), x3, x4))$

$rd(x1, op_l(\underbrace{op_l(i(i(x2)), x1, x2)}, x3, x4))$

$=$ by Lemma 1049 RL with $\{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow i(x2)\}$

$\underbrace{rd(x1, op_l(\overbrace{i(op_l(i(x2), x1, x2))}, x3, x4))}$

$=$ by Lemma 1975 RL with $\{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow op_l(i(x2), x1, x2), x1 \leftarrow x1\}$

$\overbrace{mult(x1, op_l(\underbrace{op_l(op_l(i(x2), x1, x2), x1, op_l(i(x2), x1, x2))}, x3, x4))}$

$=$ by Lemma 1356 RL with $\{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow op_l(i(x2), x1, x2)\}$

$mult(x1, op_l(\overbrace{op_l(op_l(i(x2), x1, x2), x2, x1)}, x3, x4))$

Lemma 1977: $rd(x1, op_l(op_l(x2, x1, x2), x3, x4)) = mult(x1, op_l(i(x2), x3, x4))$

$rd(x1, op_l(op_l(\underbrace{x2}, x1, x2), x3, x4))$

$=$ by Lemma 3 RL with $\{x1 \leftarrow x2\}$

$\underbrace{rd(x1, op_l(op_l(\overbrace{i(i(x2))}, x1, x2), x3, x4))}$

$=$ by Lemma 1976 LR with $\{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{mult(x1, op_l(\underbrace{op_l(op_l(i(x2), x1, x2), x2, x1)}, x3, x4))}$

$=$ by Lemma 138 LR with $\{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow i(x2)\}$

$mult(x1, op_l(\overbrace{i(x2)}, x3, x4))$

Lemma 1978: $rd(x1, i(op_t(op_t(x2, x1), x1))) = mult(op_t(x2, mult(x2, x1)), x1)$

$\underbrace{rd(x1, i(op_t(op_t(x2, x1), x1)))}$

$=$ by Lemma 461 RL with $\{x2 \leftarrow op_t(x2, x1), x1 \leftarrow x1\}$

$\overbrace{\underbrace{mult(x1, op_t(op_t(x2, x1), mult(op_t(x2, x1), x1)))}}$

$=$ by Lemma 50 LR with $\{x3 \leftarrow mult(op_t(x2, x1), x1), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{mult(\underbrace{op_t(x2, mult(op_t(x2, x1), x1))}, x1)}$

$=$ by Lemma 1114 LR with $\{x3 \leftarrow x1, x2 \leftarrow x1, x1 \leftarrow x2\}$

$mult(\overbrace{op_t(x2, mult(x2, x1))}, x1)$

Lemma 1979: $mult(i(op_r(x1,x2,x3)),mult(x1,x4)) = mult(rd(x1,op_r(x1,x2,x3)),x4)$

$$
\begin{aligned}
& \underbrace{mult(i(op_r(x1,x2,x3)),mult(x1,x4))} \\
= \quad & \text{by Lemma 654 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{mult(op_r(\underbrace{i(x1)},mult(x1,x2),x3),mult(x1,x4))} \\
= \quad & \text{by Lemma 52 RL with } \{x2 \leftarrow x4, x1 \leftarrow i(x1)\} \\
& mult(\underbrace{op_r(\overbrace{op_r(i(x1),i(i(x1)),x4)},mult(x1,x2),x3)},mult(x1,x4)) \\
= \quad & \text{by Axiom 17 LR with } \{x5 \leftarrow x3, x4 \leftarrow mult(x1,x2), x3 \leftarrow x4, x2 \leftarrow i(i(x1)), x1 \leftarrow i(x1)\} \\
& mult(\overbrace{op_r(\underbrace{op_r(i(x1),mult(x1,x2),x3)},i(i(x1)),x4)},mult(x1,x4)) \\
= \quad & \text{by Lemma 654 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& mult(op_r(\overbrace{i(op_r(x1,x2,x3))},\underbrace{i(i(x1))},x4),mult(x1,x4)) \\
= \quad & \text{by Lemma 3 LR with } \{x1 \leftarrow x1\} \\
& \underbrace{mult(op_r(i(op_r(x1,x2,x3)),\overbrace{x1},x4),mult(x1,x4))} \\
= \quad & \text{by Lemma 91 LR with } \{x3 \leftarrow x4, x2 \leftarrow x1, x1 \leftarrow i(op_r(x1,x2,x3))\} \\
& \overbrace{mult(\underbrace{mult(i(op_r(x1,x2,x3)),x1)},x4)} \\
= \quad & \text{by Lemma 113 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& mult(\overbrace{rd(x1,op_r(x1,x2,x3))},x4)
\end{aligned}
$$

Lemma 1980: $op_r(mult(op_t(x1,x2),x2),x1,rd(i(x1),x2)) = mult(x1,x2)$

$$
\begin{aligned}
& \underbrace{op_r(mult(op_t(x1,x2),x2),x1,rd(i(x1),x2))} \\
= \quad & \text{by Lemma 1892 RL with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow mult(op_t(x1,x2),x2)\} \\
& \overbrace{op_r(\underbrace{mult(op_t(x1,x2),x2)},x1,i(mult(x1,x2)))} \\
= \quad & \text{by Lemma 865 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{op_r(\overbrace{mult(x1,rd(mult(x1,x2),x1))},x1,i(mult(x1,x2)))} \\
= \quad & \text{by Lemma 843 RL with } \{x2 \leftarrow x1, x1 \leftarrow mult(x1,x2)\} \\
& \overbrace{mult(x1,x2)}
\end{aligned}
$$

Lemma 1981: $mult(rd(x1,x2),x3) = rd(i(x2),op_l(mult(i(x3),i(x1)),x3,rd(x1,x2)))$

$$
\begin{aligned}
& \underbrace{mult(rd(x1,x2),x3)} \\
= \quad & \text{by Lemma 888 RL with } \{x3 \leftarrow mult(i(x3),i(x1)), x2 \leftarrow x3, x1 \leftarrow rd(x1,x2)\} \\
& \overbrace{rd(mult(rd(x1,x2),\underbrace{mult(x3,mult(i(x3),i(x1)))}),op_l(mult(i(x3),i(x1)),x3,rd(x1,x2)))} \\
= \quad & \text{by Lemma 12 LR with } \{x2 \leftarrow i(x1), x1 \leftarrow x3\} \\
& rd(\underbrace{mult(rd(x1,x2),\overbrace{i(x1)})},op_l(mult(i(x3),i(x1)),x3,rd(x1,x2))) \\
= \quad & \text{by Lemma 22 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& rd(\overbrace{i(x2)},op_l(mult(i(x3),i(x1)),x3,rd(x1,x2)))
\end{aligned}
$$

Lemma 1982: $mult(x1, x2) = rd(mult(x1, mult(x2, mult(x1, i(x2)))), mult(i(x2), x1))$

$\overbrace{mult(x1, x2)}$

$=$ by Lemma 888 RL with $\{x3 \leftarrow mult(x1, i(x2)), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{rd(mult(x1, mult(x2, mult(x1, i(x2)))), op_l(mult(x1, i(x2)), \underbrace{x2}, x1))}$

$=$ by Lemma 3 RL with $\{x1 \leftarrow x2\}$

$rd(mult(x1, mult(x2, mult(x1, i(x2)))), \underbrace{op_l(mult(x1, i(x2)), \overbrace{i(i(x2))}, x1)})$

$=$ by Lemma 423 LR with $\{x2 \leftarrow i(x2), x1 \leftarrow x1\}$

$rd(mult(x1, mult(x2, mult(x1, i(x2)))), \overbrace{mult(i(x2), x1)})$

Lemma 1983: $mult(x1, x2) = rd(rd(x1, op_t(i(x1), x2)), mult(i(x2), x1))$

$\overbrace{mult(x1, x2)}$

$=$ by Lemma 1982 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{rd(mult(x1, \underbrace{mult(x2, mult(x1, i(x2)))}), mult(i(x2), x1))}$

$=$ by Lemma 30 LR with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$rd(\underbrace{mult(x1, \overbrace{op_t(x1, i(x2))})}, mult(i(x2), x1))$

$=$ by Lemma 103 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$rd(\overbrace{rd(x1, op_t(i(x1), x2))}, mult(i(x2), x1))$

Lemma 1984: $mult(x1, x2) = rd(mult(x2, i(x1)), rd(op_t(i(x1), x2), x1))$

$\overbrace{mult(x1, x2)}$

$=$ by Lemma 1983 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{rd(rd(x1, op_t(i(x1), x2)), \underbrace{mult(i(x2), x1)})}$

$=$ by Lemma 17 RL with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$\underbrace{rd(rd(x1, op_t(i(x1), x2)), \overbrace{i(mult(x2, i(x1)))})}$

$=$ by Lemma 39 LR with $\{x3 \leftarrow mult(x2, i(x1)), x2 \leftarrow op_t(i(x1), x2), x1 \leftarrow x1\}$

$\overbrace{rd(mult(x2, i(x1)), rd(op_t(i(x1), x2), x1))}$

Lemma 1985: $op_t(i(mult(rd(i(x1), x1), mult(x1, x2))), rd(x1, i(x1))) = op_t(i(op_t(rd(x2, x1), i(x1))), mult(x1, x2))$

$\underbrace{op_t(i(mult(rd(i(x1), x1), mult(x1, x2))), rd(x1, i(x1)))}$

$=$ by Lemma 47 RL with $\{x3 \leftarrow x1, x2 \leftarrow i(x1), x1 \leftarrow mult(rd(i(x1), x1), mult(x1, x2))\}$

$\overbrace{i(\underbrace{op_t(mult(rd(i(x1), x1), mult(x1, x2)), rd(i(x1), x1))})}$

$=$ by Lemma 937 LR with $\{x2 \leftarrow mult(x1, x2), x1 \leftarrow rd(i(x1), x1)\}$

$\overbrace{op_t(i(\underbrace{mult(mult(x1, x2), rd(i(x1), x1))}), mult(x1, x2))}$

$=$ by Lemma 199 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$op_t(i(\overbrace{op_t(rd(x2, x1), i(x1))}), mult(x1, x2))$

Lemma 1986: $op_t(mult(mult(x1,x1),i(mult(x1,x2))),rd(x1,i(x1))) = op_t(i(op_t(rd(x2,x1),i(x1))),mult(x1,x2))$

$op_t(mult(\underbrace{mult(x1,x1)}, i(mult(x1,x2))), rd(x1,i(x1)))$

$=$ by Lemma 42 RL with $\{x1 \leftarrow x1\}$

$op_t(\overbrace{mult(\overbrace{rd(x1,i(x1))}, i(mult(x1,x2)))}, rd(x1,i(x1)))$

$=$ by Lemma 44 RL with $\{x3 \leftarrow mult(x1,x2), x2 \leftarrow x1, x1 \leftarrow i(x1)\}$

$\underbrace{op_t(\overbrace{i(mult(rd(i(x1),x1),mult(x1,x2)))}, rd(x1,i(x1)))}$

$=$ by Lemma 1985 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_t(i(op_t(rd(x2,x1),i(x1))),mult(x1,x2))}$

Lemma 1987: $op_t(mult(x1,i(x2)),rd(x1,i(x1))) = op_t(i(op_t(rd(x2,x1),i(x1))),mult(x1,x2))$

$op_t(mult(x1,\underbrace{i(x2)}), rd(x1,i(x1)))$

$=$ by Lemma 20 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$op_t(\underbrace{mult(x1,\overbrace{mult(x1,i(mult(x1,x2)))})}, rd(x1,i(x1)))$

$=$ by Lemma 24 RL with $\{x2 \leftarrow i(mult(x1,x2)), x1 \leftarrow x1\}$

$\underbrace{op_t(\overbrace{mult(mult(x1,x1),i(mult(x1,x2)))}, rd(x1,i(x1)))}$

$=$ by Lemma 1986 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_t(i(op_t(rd(x2,x1),i(x1))),mult(x1,x2))}$

Lemma 1988: $op_t(mult(x1,i(x2)),mult(x1,x1)) = op_t(op_t(rd(x1,x2),i(i(x1))),mult(x1,x2))$

$op_t(mult(x1,i(x2)),\underbrace{mult(x1,x1)})$

$=$ by Lemma 42 RL with $\{x1 \leftarrow x1\}$

$\underbrace{op_t(mult(x1,i(x2)),\overbrace{rd(x1,i(x1))})}$

$=$ by Lemma 1987 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_t(\underbrace{i(op_t(rd(x2,x1),i(x1)))},mult(x1,x2))}$

$=$ by Lemma 46 LR with $\{x3 \leftarrow i(x1), x2 \leftarrow x1, x1 \leftarrow x2\}$

$op_t(\overbrace{op_t(rd(x1,x2),i(i(x1)))},mult(x1,x2))$

Lemma 1989: $rd(x1, x2) = mult(op_l(i(x2), x1, x2), op_r(x1, rd(i(x2), x1), rd(i(x2), x1)))$

$rd(x1, \underbrace{x2}_{})$

$=$ by Lemma 3 RL with $\{x1 \leftarrow x2\}$

$rd(x1, \overbrace{i(i(x2))}^{})$

$=$ by Lemma 22 RL with $\{x2 \leftarrow i(x2), x1 \leftarrow x1\}$

$rd(x1, \overbrace{mult(rd(x1, i(x2)), i(x1))}^{})$

$=$ by Lemma 44 RL with $\{x3 \leftarrow x1, x2 \leftarrow x1, x1 \leftarrow i(x2)\}$

$\underbrace{rd(x1, \overbrace{i(mult(rd(i(x2), x1), x1))}^{})}_{}$

$=$ by Lemma 336 RL with $\{x2 \leftarrow x1, x1 \leftarrow rd(i(x2), x1)\}$

$\overbrace{mult(rd(rd(i(x2), x1), i(x1)), \underbrace{x1}_{})}^{}$

$=$ by Lemma 368 RL with $\{x2 \leftarrow rd(i(x2), x1), x1 \leftarrow x1\}$

$mult(rd(rd(i(x2), x1), i(x1)), \overbrace{op_r(\underbrace{op_t(x1, rd(rd(i(x2), x1), x1))}_{}, rd(i(x2), x1), rd(i(x2), x1))}^{})$

$=$ by Lemma 707 RL with $\{x2 \leftarrow rd(i(x2), x1), x1 \leftarrow x1\}$

$mult(rd(rd(i(x2), x1), i(x1)), \underbrace{op_r(\overbrace{op_t(x1, rd(i(x1), rd(i(x2), x1)))}^{}, rd(i(x2), x1), rd(i(x2), x1))}_{})$

$=$ by Axiom 14 RL with $\{x4 \leftarrow rd(i(x1), rd(i(x2), x1)), x3 \leftarrow rd(i(x2), x1), x2 \leftarrow rd(i(x2), x1), x1 \leftarrow x1\}$

$mult(\underbrace{rd(rd(i(x2), x1), i(x1))}_{}, \overbrace{op_t(op_r(x1, rd(i(x2), x1), rd(i(x2), x1)), rd(i(x1), rd(i(x2), x1)))}^{})$

$=$ by Lemma 10 RL with $\{x2 \leftarrow rd(i(x2), x1), x1 \leftarrow i(x1)\}$

$\underbrace{mult(\overbrace{i(rd(i(x1), rd(i(x2), x1)))}^{}, op_t(op_r(x1, rd(i(x2), x1), rd(i(x2), x1)), rd(i(x1), rd(i(x2), x1))))}_{}$

$=$ by Lemma 963 LR with $\{x2 \leftarrow rd(i(x2), x1), x1 \leftarrow x1\}$

$\overbrace{mult(\underbrace{i(i(mult(x1, rd(i(x2), x1))))}_{}, op_r(x1, rd(i(x2), x1), rd(i(x2), x1)))}^{}$

$=$ by Lemma 3 LR with $\{x1 \leftarrow mult(x1, rd(i(x2), x1))\}$

$mult(\overbrace{mult(x1, rd(i(x2), x1))}^{}, op_r(x1, rd(i(x2), x1), rd(i(x2), x1)))$

$=$ by Lemma 904 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$mult(\overbrace{op_l(i(x2), x1, x2)}^{}, op_r(x1, rd(i(x2), x1), rd(i(x2), x1)))$

Lemma 1990: $op_l(mult(x1,x2),x3,mult(x1,x2)) = op_r(mult(x1,x2),x3,rd(i(x1),x2))$

$\underbrace{op_l(mult(x1,x2),x3,mult(x1,x2))}$

$=$ by Lemma 608 RL with $\{x2 \leftarrow x3, x1 \leftarrow mult(x1,x2)\}$

$\overbrace{op_l(mult(x1,x2),\underbrace{mult(x1,x2)},i(x3))}$

$=$ by Lemma 606 RL with $\{x2 \leftarrow i(x3), x1 \leftarrow mult(x1,x2)\}$

$\overbrace{op_l(mult(x1,x2),i(x3),i(mult(x1,x2)))}$

$=$ by Lemma 245 RL with $\{x3 \leftarrow x3, x2 \leftarrow mult(x1,x2), x1 \leftarrow mult(x1,x2)\}$

$\overbrace{op_l(mult(x1,x2),\underbrace{mult(x1,x2)},i(mult(mult(x1,x2),x3)))}$

$=$ by Lemma 400 RL with $\{x2 \leftarrow mult(x1,x2), x1 \leftarrow rd(x2,mult(x1,x2))\}$

$\underbrace{op_l(mult(x1,x2),\overbrace{op_t(rd(mult(rd(x2,mult(x1,x2)),mult(x1,x2)),rd(x2,mult(x1,x2))),op_t(rd(x2,mult(x1,x2)),mult(x1}$

$=$ by Lemma 1034 RL with $\{x3 \leftarrow i(mult(mult(x1,x2),x3)), x1 \leftarrow rd(x2,mult(x1,x2)), x2 \leftarrow mult(x1,x2)\}$

$\overbrace{op_l(mult(x1,x2),rd(\underbrace{mult(rd(x2,mult(x1,x2)),mult(x1,x2))},rd(x2,mult(x1,x2))),i(mult(mult(x1,x2),x3)))}$

$=$ by Axiom 4 RL with $\{x2 \leftarrow mult(x1,x2), x1 \leftarrow x2\}$

$op_l(mult(x1,x2),\underbrace{rd(\overbrace{x2},rd(x2,mult(x1,x2)))},i(mult(mult(x1,x2),x3)))$

$=$ by Lemma 353 LR with $\{x2 \leftarrow mult(x1,x2), x1 \leftarrow x2\}$

$\underbrace{op_l(mult(x1,x2),\overbrace{op_r(mult(x1,x2),x2,x2)},i(mult(mult(x1,x2),x3)))}$

$=$ by Lemma 743 LR with $\{x3 \leftarrow i(mult(mult(x1,x2),x3)), x2 \leftarrow x2, x1 \leftarrow mult(x1,x2)\}$

$\overbrace{op_r(mult(x1,x2),\underbrace{i(mult(mult(x1,x2),x3))},op_r(mult(x1,x2),x2,x2))}$

$=$ by Lemma 818 LR with $\{x3 \leftarrow op_r(mult(x1,x2),x2,x2), x2 \leftarrow x3, x1 \leftarrow mult(x1,x2)\}$

$\overbrace{op_r(mult(x1,x2),x3,\underbrace{i(op_r(mult(x1,x2),x2,x2))})}$

$=$ by Lemma 356 LR with $\{x2 \leftarrow x2, x1 \leftarrow mult(x1,x2)\}$

$op_r(mult(x1,x2),x3,\underbrace{op_r(i(mult(x1,x2)),x2,x2)})$

$=$ by Lemma 371 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$op_r(mult(x1,x2),x3,\overbrace{rd(i(x1),x2)})$

Lemma 1991: $mult(op_l(i(x1),x1,x2),mult(x2,x3)) = op_l(mult(rd(x2,x1),x3),x1,x2)$

$\underbrace{mult(op_l(i(x1),x1,x2),mult(x2,x3))}$

$=$ by Lemma 1068 RL with $\{x3 \leftarrow x3, x2 \leftarrow op_l(i(x1),x1,x2), x1 \leftarrow x2\}$

$\overbrace{op_l(mult(\underbrace{mult(x2,op_l(i(x1),x1,x2))},x3),x2,op_l(i(x1),x1,x2))}$

$=$ by Lemma 232 LR with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$\underbrace{op_l(mult(\overbrace{rd(x2,x1)},x3),x2,op_l(i(x1),x1,x2))}$

$=$ by Lemma 1061 LR with $\{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow mult(rd(x2,x1),x3)\}$

$\overbrace{op_l(mult(rd(x2,x1),x3),x1,x2)}$

Lemma 1992: $mult(op_l(x1,x1,x2),mult(i(x2),x3)) = op_l(mult(rd(x1,x2),x3),x2,i(op_l(x1,x1,x2)))$

$\underbrace{mult(op_l(x1,x1,x2),mult(i(x2),x3))}$

$=$ by Lemma 1068 RL with $\{x3 \leftarrow x3, x2 \leftarrow op_l(x1,x1,x2), x1 \leftarrow i(x2)\}$

$\overbrace{op_l(mult(\underbrace{mult(i(x2),op_l(x1,x1,x2))}, x3), i(x2), op_l(x1,x1,x2))}$

$=$ by Lemma 440 LR with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$\underbrace{op_l(mult(\overbrace{rd(x1,x2)}, x3), i(x2), op_l(x1,x1,x2))}$

$=$ by Lemma 1050 LR with $\{x3 \leftarrow op_l(x1,x1,x2), x2 \leftarrow x2, x1 \leftarrow mult(rd(x1,x2),x3)\}$

$\overbrace{op_l(mult(rd(x1,x2),x3),x2,i(op_l(x1,x1,x2)))}$

Lemma 1993: $mult(op_l(x1,x1,x2),mult(i(x2),x3)) = op_l(mult(rd(x1,x2),x3),x1,x2)$

$\underbrace{mult(op_l(x1,x1,x2),mult(i(x2),x3))}$

$=$ by Lemma 1992 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_l(mult(rd(x1,x2),x3),x2,\underbrace{i(op_l(x1,x1,x2))})}$

$=$ by Lemma 1049 LR with $\{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x1\}$

$\underbrace{op_l(mult(rd(x1,x2),x3),x2,\overbrace{op_l(i(x1),x1,x2)})}$

$=$ by Lemma 1061 LR with $\{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow mult(rd(x1,x2),x3)\}$

$\overbrace{op_l(mult(rd(x1,x2),x3),x1,x2)}$

Lemma 1994: $mult(x1,x2) = mult(asoc(x3,x1,x1),mult(op_l(x1,x1,x3),x2))$

$mult(x1,\underbrace{x2})$

$=$ by Lemma 948 RL with $\{x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow x2\}$

$\underbrace{mult(x1,\overbrace{op_l(x2,op_l(x1,x1,x3),x1)})}$

$=$ by Lemma 1091 RL with $\{x3 \leftarrow x2, x2 \leftarrow op_l(x1,x1,x3), x1 \leftarrow x1\}$

$\overbrace{mult(\underbrace{rd(x1,op_l(x1,x1,x3))},mult(op_l(x1,x1,x3),x2))}$

$=$ by Lemma 764 LR with $\{x2 \leftarrow x3, x1 \leftarrow x1\}$

$mult(\overbrace{asoc(x3,x1,x1)},mult(op_l(x1,x1,x3),x2))$

Lemma 1995: $rd(rd(x1, op_r(x2, x3, x3)), asoc(x3, x2, x3)) = rd(x1, x2)$

$\underbrace{rd(rd(x1, op_r(x2, x3, x3)), asoc(x3, x2, x3))}$

$=$ by Lemma 1300 LR with $\{x2 \leftarrow x2, x3 \leftarrow x3, x1 \leftarrow rd(x1, op_r(x2, x3, x3))\}$

$\overbrace{mult(rd(x1, op_r(x2, x3, x3)), \underbrace{asoc(x2, x3, x3)})}$

$=$ by Lemma 766 RL with $\{x2 \leftarrow x3, x1 \leftarrow x2\}$

$mult(rd(x1, op_r(x2, x3, x3)), \overbrace{rd(op_r(x2, x3, x3), x2)})$

$=$ by Lemma 112 RL with $\{x3 \leftarrow x3, x2 \leftarrow x3, x1 \leftarrow x2\}$

$\underbrace{mult(rd(x1, op_r(x2, x3, x3)), \overbrace{mult(op_r(x2, x3, x3), i(x2))})}$

$=$ by Lemma 1091 LR with $\{x3 \leftarrow i(x2), x2 \leftarrow op_r(x2, x3, x3), x1 \leftarrow x1\}$

$\overbrace{mult(x1, \underbrace{op_l(i(x2), op_r(x2, x3, x3), x1)})}$

$=$ by Lemma 1049 RL with $\{x3 \leftarrow x1, x2 \leftarrow op_r(x2, x3, x3), x1 \leftarrow x2\}$

$mult(x1, \overbrace{i(op_l(x2, \underbrace{op_r(x2, x3, x3)}, x1))})$

$=$ by Lemma 353 RL with $\{x2 \leftarrow x2, x1 \leftarrow x3\}$

$mult(x1, i(op_l(x2, \overbrace{rd(\underbrace{x3}, rd(x3, x2))}, x1)))$

$=$ by Axiom 4 LR with $\{x2 \leftarrow x2, x1 \leftarrow x3\}$

$mult(x1, i(\underbrace{op_l(x2, rd(\overbrace{mult(rd(x3, x2), x2)}, rd(x3, x2)), x1)}))$

$=$ by Lemma 1034 LR with $\{x3 \leftarrow x1, x1 \leftarrow rd(x3, x2), x2 \leftarrow x2\}$

$mult(x1, i(\overbrace{op_l(x2, \underbrace{op_t(rd(mult(rd(x3, x2), x2), rd(x3, x2)), op_t(rd(x3, x2), x2))}, x1)}))$

$=$ by Lemma 400 LR with $\{x2 \leftarrow x2, x1 \leftarrow rd(x3, x2)\}$

$mult(x1, \underbrace{i(op_l(x2, \overbrace{x2}, x1))})$

$=$ by Lemma 1049 LR with $\{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x2\}$

$\underbrace{mult(x1, \overbrace{op_l(i(x2), x2, x1)})}$

$=$ by Lemma 232 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{rd(x1, x2)}$

Lemma 1996: $rd(op_l(x1,x2,x3),i(op_t(x1,x4))) = mult(op_l(x1,x2,x3),op_t(x1,x4))$

$rd(op_l(x1,x2,x3),\underbrace{i(op_t(x1,x4))})$

$=$ by Lemma 15 RL with $\{x2 \leftarrow x4, x1 \leftarrow x1\}$

$rd(op_l(x1,x2,x3),\overbrace{op_t(\underbrace{i(x1)},i(x4))})$

$=$ by Lemma 189 RL with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow i(x1)\}$

$rd(op_l(x1,x2,x3),\underbrace{op_t(\overbrace{op_t(i(x1),op_l(i(i(x1)),x2,x3))},i(x4))})$

$=$ by Axiom 13 RL with $\{x3 \leftarrow op_l(i(i(x1)),x2,x3), x2 \leftarrow i(x4), x1 \leftarrow i(x1)\}$

$rd(op_l(x1,x2,x3),\overbrace{op_t(\underbrace{op_t(i(x1),i(x4))},op_l(i(i(x1)),x2,x3))})$

$=$ by Lemma 15 LR with $\{x2 \leftarrow x4, x1 \leftarrow x1\}$

$rd(op_l(x1,x2,x3),op_t(\overbrace{i(op_t(x1,x4))},op_l(\underbrace{i(i(x1))},x2,x3)))$

$=$ by Lemma 3 LR with $\{x1 \leftarrow x1\}$

$\underbrace{rd(op_l(x1,x2,x3),op_t(i(op_t(x1,x4)),op_l(\overbrace{x1},x2,x3)))}$

$=$ by Lemma 402 RL with $\{x2 \leftarrow op_t(x1,x4), x1 \leftarrow op_l(x1,x2,x3)\}$

$\overbrace{mult(op_l(x1,x2,x3),\underbrace{op_r(op_t(x1,x4),op_l(x1,x2,x3),i(op_l(x1,x2,x3))))})$

$=$ by Axiom 14 RL with $\{x4 \leftarrow x4, x3 \leftarrow i(op_l(x1,x2,x3)), x2 \leftarrow op_l(x1,x2,x3), x1 \leftarrow x1\}$

$mult(op_l(x1,x2,x3),\overbrace{op_t(\underbrace{op_r(x1,op_l(x1,x2,x3),i(op_l(x1,x2,x3)))},x4)})$

$=$ by Lemma 138 RL with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow op_r(x1,op_l(x1,x2,x3),i(op_l(x1,x2,x3)))\}$

$mult(op_l(x1,x2,x3),op_t(\overbrace{op_l(\underbrace{op_l(op_r(x1,op_l(x1,x2,x3),i(op_l(x1,x2,x3))),x2,x3)},x3,x2)},x4))$

$=$ by Axiom 18 RL with $\{x5 \leftarrow i(op_l(x1,x2,x3)), x4 \leftarrow op_l(x1,x2,x3), x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$mult(op_l(x1,x2,x3),op_t(op_l(\overbrace{op_r(op_l(x1,x2,x3),op_l(x1,x2,x3),i(op_l(x1,x2,x3)))},x3,x2),x4))$

$=$ by Lemma 26 LR with $\{x2 \leftarrow i(op_l(x1,x2,x3)), x1 \leftarrow op_l(x1,x2,x3)\}$

$mult(op_l(x1,x2,x3),op_t(\underbrace{op_l(\overbrace{op_l(x1,x2,x3)},x3,x2)},x4))$

$=$ by Lemma 138 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$mult(op_l(x1,x2,x3),op_t(\overbrace{x1},x4))$

Lemma 1997: $op_l(rd(i(op_t(x1,x2)),x2),x2,i(i(x1))) = i(mult(op_t(op_l(x1,x1,i(x2)),x2),x2))$

$op_l(\underbrace{rd(i(op_t(x1,x2)),x2)}, x2, i(i(x1)))$

$=$ by Lemma 10 RL with $\{x2 \leftarrow i(op_t(x1,x2)), x1 \leftarrow x2\}$

$op_l(\overbrace{i(rd(x2,\underbrace{i(op_t(x1,x2))}))}, x2, i(i(x1)))$

$=$ by Axiom 2 LR with $\{x2 \leftarrow x2, x1 \leftarrow i(op_t(x1,x2))\}$

$op_l(i(\underbrace{rd(x2,\overbrace{mult(i(x2),mult(x2,i(op_t(x1,x2))))})}), x2, i(i(x1)))$

$=$ by Lemma 460 RL with $\{x1 \leftarrow x2, x2 \leftarrow x1\}$

$op_l(i(\overbrace{mult(\underbrace{op_l(x1,x2,x1)}, x2)}), x2, i(i(x1)))$

$=$ by Lemma 608 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{op_l(i(mult(\overbrace{op_l(x1,x1,i(x2))}, x2)), x2, i(i(x1)))}$

$=$ by Lemma 1050 RL with $\{x3 \leftarrow i(x1), x2 \leftarrow x2, x1 \leftarrow i(mult(op_l(x1,x1,i(x2)),x2))\}$

$\overbrace{\underbrace{op_l(i(mult(op_l(x1,x1,i(x2)),x2)), i(x2), i(x1))}}$

$=$ by Lemma 1339 RL with $\{x3 \leftarrow i(x2), x2 \leftarrow x1, x1 \leftarrow i(mult(op_l(x1,x1,i(x2)),x2))\}$

$\overbrace{op_l(\underbrace{i(mult(op_l(x1,x1,i(x2)),x2))}, op_l(x1,x1,i(x2)), i(x2))}$

$=$ by Axiom 11 RL with $\{x2 \leftarrow x2, x1 \leftarrow op_l(x1,x1,i(x2))\}$

$op_l(\overbrace{mult(i(op_l(x1,x1,i(x2))), i(x2))}, op_l(x1,x1,i(x2)), i(x2))$

$=$ by Lemma 16 RL with $\{x2 \leftarrow x2, x1 \leftarrow i(op_l(x1,x1,i(x2)))\}$

$\underbrace{op_l(\overbrace{i(mult(i(i(op_l(x1,x1,i(x2))))), x2)}, op_l(x1,x1,i(x2)), i(x2))}$

$=$ by Lemma 927 LR with $\{x2 \leftarrow x2, x1 \leftarrow op_l(x1,x1,i(x2))\}$

$\overbrace{\underbrace{i(op_t(rd(x2,i(op_l(x1,x1,i(x2))))), x2))}}$

$=$ by Lemma 46 LR with $\{x3 \leftarrow x2, x2 \leftarrow i(op_l(x1,x1,i(x2))), x1 \leftarrow x2\}$

$\overbrace{\underbrace{op_t(rd(i(op_l(x1,x1,i(x2))),x2),i(x2))}}$

$=$ by Lemma 408 LR with $\{x2 \leftarrow x2, x1 \leftarrow op_l(x1,x1,i(x2))\}$

$\overbrace{i(mult(op_t(op_l(x1,x1,i(x2)),x2),x2))}$

Lemma 1998: $op_l(i(rd(x1,mult(x1,i(x2)))),x1,x2) = i(mult(op_t(rd(x2,x1),x2),x1))$

$op_l(i(\underbrace{rd(x1,mult(x1,i(x2)))}),x1,x2)$

$=$ by Lemma 465 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{op_l(i(\overbrace{mult(x1,op_t(rd(x2,x1),x2))}),x1,x2)}$

$=$ by Lemma 1344 RL with $\{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow i(mult(x1,op_t(rd(x2,x1),x2)))\}$

$\overbrace{\underbrace{op_l(i(mult(x1,op_t(rd(x2,x1),x2))),x1,op_t(rd(x2,x1),x2))}}$

$=$ by Lemma 196 LR with $\{x2 \leftarrow op_t(rd(x2,x1),x2), x1 \leftarrow x1\}$

$\overbrace{i(mult(op_t(rd(x2,x1),x2),x1))}$

Lemma 1999: $op_l(op_l(rd(x2,x1),x2,x1),x2,x1) = mult(op_t(x2,rd(x2,x1)),i(x1))$

$$
\begin{aligned}
& op_l(\underbrace{op_l(rd(x2,x1),x2,x1)}, x2, x1) \\
= & \quad \text{by Lemma 314 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{op_l(\overbrace{mult(i(x1),op_t(x2,rd(x2,x1)))}, x2, x1)} \\
= & \quad \text{by Lemma 1354 RL with } \{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow mult(i(x1),op_t(x2,rd(x2,x1)))\} \\
& \overbrace{\underbrace{op_l(mult(i(x1),op_t(x2,rd(x2,x1))),op_t(x2,rd(x2,x1)),x1)}} \\
= & \quad \text{by Lemma 426 LR with } \{x2 \leftarrow op_t(x2,rd(x2,x1)), x1 \leftarrow x1\} \\
& \overbrace{mult(op_t(x2,rd(x2,x1)),i(x1))}
\end{aligned}
$$

Lemma 2000: $mult(rd(x1,op_l(x1,x2,x3)),x4) = mult(x1,mult(op_l(i(x1),x2,x3),x4))$

$$
\begin{aligned}
& \underbrace{mult(rd(x1,op_l(x1,x2,x3)),x4)} \\
= & \quad \text{by Lemma 1221 RL with } \{x3 \leftarrow x4, x2 \leftarrow op_l(x1,x2,x3), x1 \leftarrow x1\} \\
& \overbrace{mult(x1,\underbrace{op_l(mult(i(op_l(x1,x2,x3)),x4),op_l(x1,x2,x3),x1)})} \\
= & \quad \text{by Lemma 1421 LR with } \{x4 \leftarrow x3, x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow mult(i(op_l(x1,x2,x3)),x4)\} \\
& mult(x1,\overbrace{mult(\underbrace{i(op_l(x1,x2,x3))},x4)}) \\
= & \quad \text{by Lemma 1049 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& mult(x1,mult(\overbrace{op_l(i(x1),x2,x3)},x4))
\end{aligned}
$$

Lemma 2001: $mult(op_l(x1,x2,x3),mult(i(x1),x4)) = mult(rd(op_l(x1,x2,x3),x1),x4)$

$$
\begin{aligned}
& mult(op_l(x1,x2,x3),\underbrace{mult(i(x1),x4)}) \\
= & \quad \text{by Lemma 1422 RL with } \{x4 \leftarrow x3, x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow mult(i(x1),x4)\} \\
& \underbrace{mult(op_l(x1,x2,x3),\overbrace{op_l(mult(i(x1),x4),x1,op_l(x1,x2,x3))})} \\
= & \quad \text{by Lemma 1221 LR with } \{x3 \leftarrow x4, x2 \leftarrow x1, x1 \leftarrow op_l(x1,x2,x3)\} \\
& \overbrace{mult(rd(op_l(x1,x2,x3),x1),x4)}
\end{aligned}
$$

Lemma 2002: $mult(op_l(i(x1),x2,x3),mult(x1,x4)) = mult(mult(op_l(i(x1),x2,x3),x1),x4)$

$$
\begin{aligned}
& \underbrace{mult(op_l(i(x1),x2,x3),mult(x1,x4))} \\
= & \quad \text{by Lemma 132 RL with } \{x3 \leftarrow x4, x2 \leftarrow x1, x1 \leftarrow op_l(i(x1),x2,x3)\} \\
& \overbrace{mult(mult(op_l(i(x1),x2,x3),x1),op_l(x4,x1,\underbrace{op_l(i(x1),x2,x3)}))} \\
= & \quad \text{by Lemma 1049 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& mult(mult(op_l(i(x1),x2,x3),x1),op_l(x4,\underbrace{x1},\overbrace{i(op_l(x1,x2,x3))})) \\
= & \quad \text{by Lemma 252 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& mult(mult(op_l(i(x1),x2,x3),x1),\underbrace{op_l(x4,\overbrace{op_l(x1,x1,op_l(x1,x2,x3))},i(op_l(x1,x2,x3)))}) \\
= & \quad \text{by Lemma 484 RL with } \{x3 \leftarrow x1, x2 \leftarrow op_l(x1,x2,x3), x1 \leftarrow x4\} \\
& mult(mult(op_l(i(x1),x2,x3),x1),\overbrace{op_l(x4,op_l(x1,x2,x3),x1)}) \\
= & \quad \text{by Lemma 1421 LR with } \{x4 \leftarrow x3, x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x4\} \\
& mult(mult(op_l(i(x1),x2,x3),x1),\overbrace{x4}\,)
\end{aligned}
$$

Lemma 2003: $mult(op_l(i(x1), x2, x3), mult(x1, x4)) = mult(rd(x1, op_l(x1, x2, x3)), x4)$

$\overbrace{mult(op_l(i(x1), x2, x3), mult(x1, x4))}$

$=$ by Lemma 2002 LR with $\{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{mult(\underbrace{mult(op_l(i(x1), x2, x3), x1)}, x4)}$

$=$ by Lemma 1058 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$mult(\overbrace{rd(x1, op_l(x1, x2, x3))}, x4)$

Lemma 2004: $mult(i(x1), mult(op_r(x1, x2, x3), x4)) = mult(rd(op_r(x1, x2, x3), x1), x4)$

$\overbrace{mult(i(x1), mult(op_r(x1, x2, x3), x4))}$

$=$ by Lemma 132 RL with $\{x3 \leftarrow x4, x2 \leftarrow op_r(x1, x2, x3), x1 \leftarrow i(x1)\}$

$\overbrace{mult(mult(i(x1), op_r(x1, x2, x3)), \underbrace{op_l(x4, op_r(x1, x2, x3), i(x1))})}$

$=$ by Lemma 1445 LR with $\{x4 \leftarrow x3, x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x4\}$

$mult(\underbrace{mult(i(x1), op_r(x1, x2, x3))}, \overbrace{x4})$

$=$ by Lemma 117 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$mult(\overbrace{rd(op_r(x1, x2, x3), x1)}, x4)$

Lemma 2005: $rd(rd(mult(x2, x3), x1), op_t(x2, x3)) = op_r(i(mult(rd(x1, mult(x3, x2)), x2)), op_t(x2, x3), op_t(x2, x3))$

$rd(\underbrace{rd(mult(x2, x3), x1)}, op_t(x2, x3))$

$=$ by Lemma 10 RL with $\{x2 \leftarrow mult(x2, x3), x1 \leftarrow x1\}$

$\underbrace{rd(\overbrace{i(rd(x1, mult(x2, x3)))}, op_t(x2, x3))}$

$=$ by Lemma 371 RL with $\{x2 \leftarrow op_t(x2, x3), x1 \leftarrow rd(x1, mult(x2, x3))\}$

$\overbrace{op_r(i(mult(rd(x1, mult(x2, x3)), \underbrace{op_t(x2, x3)})), op_t(x2, x3), op_t(x2, x3))}$

$=$ by Lemma 1237 RL with $\{x3 \leftarrow x3, x2 \leftarrow x3, x1 \leftarrow x2\}$

$op_r(i(mult(rd(x1, mult(x2, x3)), \overbrace{op_t(x2, rd(x3, \underbrace{rd(x2, op_t(x2, x3))}))})), op_t(x2, x3), op_t(x2, x3))$

$=$ by Lemma 395 RL with $\{x1 \leftarrow x2, x2 \leftarrow x3\}$

$op_r(i(mult(rd(x1, mult(x2, x3)), op_t(x2, \underbrace{rd(x3, \overbrace{rd(op_t(x3, x2), x3)})}))), op_t(x2, x3), op_t(x2, x3))$

$=$ by Lemma 1108 RL with $\{x1 \leftarrow x3, x2 \leftarrow x2\}$

$op_r(i(\underbrace{mult(rd(x1, mult(x2, x3)), op_t(x2, \overbrace{rd(mult(x2, x3), x2)}))}), op_t(x2, x3), op_t(x2, x3))$

$=$ by Lemma 1262 LR with $\{x3 \leftarrow mult(x2, x3), x2 \leftarrow x2, x1 \leftarrow rd(x1, mult(x2, x3))\}$

$op_r(i(\overbrace{mult(\underbrace{rd(rd(x1, mult(x2, x3)), asoc(x2, mult(x2, x3), mult(x2, x3)))}, x2)}), op_t(x2, x3), op_t(x2, x3))$

$=$ by Lemma 1567 RL with $\{x3 \leftarrow x2, x2 \leftarrow mult(x2, x3), x1 \leftarrow x1\}$

$op_r(i(mult(\overbrace{rd(x1, \underbrace{op_l(mult(x2, x3), x2, mult(x2, x3))})}, x2)), op_t(x2, x3), op_t(x2, x3))$

$=$ by Lemma 1330 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow mult(x2, x3)\}$

$op_r(i(mult(rd(x1, \overbrace{\underbrace{op_l(mult(x2, x3), x3, i(x2))}}), x2)), op_t(x2, x3), op_t(x2, x3))$

$=$ by Lemma 422 LR with $\{x2 \leftarrow x3, x1 \leftarrow x2\}$

$op_r(i(mult(rd(x1, \overbrace{mult(x3, x2)}), x2)), op_t(x2, x3), op_t(x2, x3))$

Lemma 2006: $rd(rd(mult(x2,x3),x1),op_t(x2,x3)) = rd(i(rd(x1,mult(x3,x2))),x2)$

$$
\begin{aligned}
& \underbrace{rd(rd(mult(x2,x3),x1),op_t(x2,x3))} \\
= \quad & \text{by Lemma 2005 LR with } \{x1 \leftarrow x1, x3 \leftarrow x3, x2 \leftarrow x2\} \\
& \overbrace{\underbrace{op_r(i(mult(rd(x1,mult(x3,x2)),x2)),op_t(x2,x3),op_t(x2,x3))}} \\
= \quad & \text{by Lemma 1632 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow i(mult(rd(x1,mult(x3,x2)),x2))\} \\
& \overbrace{\underbrace{op_r(i(mult(rd(x1,mult(x3,x2)),x2)),x2,x2)}} \\
= \quad & \text{by Lemma 371 LR with } \{x2 \leftarrow x2, x1 \leftarrow rd(x1,mult(x3,x2))\} \\
& \overbrace{rd(i(rd(x1,mult(x3,x2))),x2)}
\end{aligned}
$$

Lemma 2007: $rd(rd(mult(x1,x2),x3),op_t(x1,x2)) = rd(rd(mult(x2,x1),x3),x1)$

$$
\begin{aligned}
& \underbrace{rd(rd(mult(x1,x2),x3),op_t(x1,x2))} \\
= \quad & \text{by Lemma 2006 LR with } \{x1 \leftarrow x3, x3 \leftarrow x2, x2 \leftarrow x1\} \\
& \overbrace{rd(\underbrace{i(rd(x3,mult(x2,x1)))},x1)} \\
= \quad & \text{by Lemma 10 LR with } \{x2 \leftarrow mult(x2,x1), x1 \leftarrow x3\} \\
& rd(\overbrace{rd(mult(x2,x1),x3)},x1)
\end{aligned}
$$

Lemma 2008: $op_r(x1, op_t(x2, x3), mult(x3, x2)) = rd(mult(mult(x1, x2), mult(x2, x3)), mult(x2, mult(x2, x3)))$

$\underbrace{op_r(x1, op_t(x2, x3), mult(x3, x2))}$

$=$ by Axiom 16 LR with $\{x3 \leftarrow mult(x3, x2), x2 \leftarrow op_t(x2, x3), x1 \leftarrow x1\}$

$\overbrace{rd(mult(\underbrace{mult(x1, op_t(x2, x3))}, mult(x3, x2)), mult(op_t(x2, x3), mult(x3, x2)))}$

$=$ by Lemma 386 RL with $\{x2 \leftarrow x1, x1 \leftarrow op_t(x2, x3)\}$

$rd(mult(\overbrace{rd(op_t(x2, \underbrace{x3}), op_t(i(x1), rd(x1, op_t(x2, x3))))}, mult(x3, x2)), mult(op_t(x2, x3), mult(x3, x2)))$

$=$ by Axiom 5 LR with $\{x2 \leftarrow x2, x1 \leftarrow x3\}$

$rd(mult(\underbrace{rd(op_t(x2, \overbrace{rd(mult(x3, x2), x2)}), op_t(i(x1), rd(x1, op_t(x2, x3))))}, mult(x3, x2)), mult(op_t(x2, x3), mult(x3, x2)))$

$=$ by Lemma 1391 RL with $\{x3 \leftarrow mult(x3, x2), x2 \leftarrow op_t(i(x1), rd(x1, op_t(x2, x3))), x1 \leftarrow x2\}$

$rd(\underbrace{mult(\overbrace{rd(rd(x2, op_t(i(x1), rd(x1, op_t(x2, x3)))), asoc(x2, mult(x3, x2), mult(x3, x2)))}, mult(x3, x2))}, mult(op_t(x2, x3), n$

$=$ by Lemma 1586 LR with $\{x2 \leftarrow mult(x3, x2), x3 \leftarrow x2, x1 \leftarrow rd(x2, op_t(i(x1), rd(x1, op_t(x2, x3))))\}$

$rd(\overbrace{mult(rd(x2, op_t(i(x1), rd(x1, op_t(x2, x3)))), \underbrace{op_l(mult(x3, x2), mult(x3, x2), x2)})}, mult(op_t(x2, x3), mult(x3, x2)))$

$=$ by Lemma 285 LR with $\{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow mult(x3, x2)\}$

$rd(mult(rd(x2, op_t(i(x1), rd(x1, op_t(x2, x3)))), \overbrace{op_l(mult(x3, x2), x3, x2)}), mult(op_t(x2, x3), mult(x3, x2)))$

$=$ by Lemma 435 LR with $\{x2 \leftarrow x2, x1 \leftarrow x3\}$

$rd(mult(rd(x2, \underbrace{op_t(i(x1), rd(x1, op_t(x2, x3)))}), \overbrace{mult(x2, x3)}), mult(op_t(x2, x3), mult(x3, x2)))$

$=$ by Lemma 1639 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$rd(mult(\underbrace{rd(x2, \overbrace{op_t(i(x1), rd(x1, x2))})}, mult(x2, x3)), mult(op_t(x2, x3), mult(x3, x2)))$

$=$ by Lemma 386 LR with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$rd(mult(\overbrace{mult(x1, x2)}, mult(x2, x3)), \underbrace{mult(op_t(x2, x3), mult(x3, x2))})$

$=$ by Lemma 1760 LR with $\{x3 \leftarrow x3, x2 \leftarrow x3, x1 \leftarrow x2\}$

$rd(mult(mult(x1, x2), mult(x2, x3)), \overbrace{mult(x2, \underbrace{mult(x3, op_t(x2, x3))})})$

$=$ by Lemma 13 LR with $\{x2 \leftarrow x2, x1 \leftarrow x3\}$

$rd(mult(mult(x1, x2), mult(x2, x3)), mult(x2, \overbrace{mult(x2, x3)}))$

Lemma 2009: $op_r(x1, op_t(x2, x3), mult(x3, x2)) = op_r(x1, x2, mult(x2, x3))$

$\underbrace{op_r(x1, op_t(x2, x3), mult(x3, x2))}$

$=$ by Lemma 2008 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{\underbrace{rd(mult(mult(x1, x2), mult(x2, x3)), mult(x2, mult(x2, x3)))}}$

$=$ by Axiom 16 RL with $\{x3 \leftarrow mult(x2, x3), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_r(x1, x2, mult(x2, x3))}$

Lemma 2010: $op_r(x1, x2, op_t(op_t(x3, x2), rd(x2, x3))) = op_r(x1, op_t(x2, rd(x3, op_t(op_t(x2, x3), x3))), op_t(op_t(x3, x2), x2))$

$op_r(x1, x2, \underbrace{op_t(op_t(x3, x2), rd(x2, x3))})$

$=$ by Axiom 13 RL with $\{x3 \leftarrow x2, x2 \leftarrow rd(x2, x3), x1 \leftarrow x3\}$

$op_r(x1, x2, \overbrace{op_t(\underbrace{op_t(x3, rd(x2, x3))}, x2)})$

$=$ by Lemma 709 RL with $\{x2 \leftarrow x2, x1 \leftarrow x3\}$

$op_r(x1, x2, op_t(\overbrace{\underbrace{op_t(x3, i(mult(x2, x3)))}, x2}))$

$=$ by Lemma 18 RL with $\{x2 \leftarrow mult(x2, x3), x1 \leftarrow x3\}$

$op_r(x1, x2, op_t(\overbrace{\underbrace{i(op_t(i(x3), mult(x2, x3)))}, x2}))$

$=$ by Lemma 15 RL with $\{x2 \leftarrow mult(x2, x3), x1 \leftarrow i(x3)\}$

$op_r(x1, x2, \underbrace{op_t(\overbrace{op_t(i(i(x3)), i(mult(x2, x3)))}, x2)})$

$=$ by Axiom 13 LR with $\{x3 \leftarrow x2, x2 \leftarrow i(mult(x2, x3)), x1 \leftarrow i(i(x3))\}$

$op_r(x1, x2, \overbrace{\underbrace{op_t(op_t(i(i(x3)), x2), i(mult(x2, x3)))}})$

$=$ by Lemma 1881 RL with $\{x2 \leftarrow x3, x1 \leftarrow x2\}$

$op_r(x1, x2, \overbrace{\underbrace{mult(rd(x2, op_t(x2, mult(x2, x3))), x3)}})$

$=$ by Lemma 1717 LR with $\{x3 \leftarrow x3, x2 \leftarrow mult(x2, x3), x1 \leftarrow x2\}$

$op_r(x1, x2, \overbrace{\underbrace{rd(x3, rd(op_t(x2, mult(x2, x3)), x2))}})$

$=$ by Lemma 1884 RL with $\{x3 \leftarrow mult(x2, x3), x2 \leftarrow x3, x1 \leftarrow x2\}$

$op_r(x1, x2, \overbrace{\underbrace{rd(mult(x2, x3), op_t(op_t(x2, x3), mult(x2, x3)))}})$

$=$ by Lemma 1915 RL with $\{x2 \leftarrow x3, x1 \leftarrow x2\}$

$\underbrace{op_r(x1, x2, \overbrace{mult(x2, rd(x3, op_t(op_t(x2, x3), x3)))})}$

$=$ by Lemma 2009 RL with $\{x3 \leftarrow rd(x3, op_t(op_t(x2, x3), x3)), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_r(x1, op_t(x2, rd(x3, op_t(op_t(x2, x3), x3))), \underbrace{mult(rd(x3, op_t(op_t(x2, x3), x3)), x2)})}$

$=$ by Lemma 1519 LR with $\{x2 \leftarrow x2, x1 \leftarrow x3\}$

$op_r(x1, op_t(x2, rd(x3, op_t(op_t(x2, x3), x3))), \overbrace{op_t(op_t(x3, x2), x2)})$

Lemma 2011: $op_r(x1, x2, op_l(x3, x3, x2)) = op_r(x1, op_t(x2, rd(x3, x2)), x3)$

$$
\begin{aligned}
& op_r(x1, x2, \underbrace{op_l(x3, x3, x2)}) \\
= & \quad \text{by Lemma 1507 RL with } \{x2 \leftarrow x2, x1 \leftarrow x3\} \\
& \underbrace{op_r(x1, x2, \overbrace{rd(op_r(mult(x3, x2), x2, x3), x2)})} \\
= & \quad \text{by Lemma 1891 RL with } \{x3 \leftarrow op_r(mult(x3, x2), x2, x3), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, x2, \underbrace{mult(i(x2), op_r(mult(x3, x2), x2, x3))})} \\
= & \quad \text{by Lemma 1509 RL with } \{x2 \leftarrow x2, x1 \leftarrow x3\} \\
& \underbrace{op_r(x1, x2, \overbrace{op_t(x3, rd(x2, x3))})} \\
= & \quad \text{by Lemma 1890 RL with } \{x3 \leftarrow op_t(x3, rd(x2, x3)), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, x2, \underbrace{op_t(op_t(x3, rd(x2, x3)), x2)})} \\
= & \quad \text{by Axiom 13 LR with } \{x3 \leftarrow x2, x2 \leftarrow rd(x2, x3), x1 \leftarrow x3\} \\
& \underbrace{op_r(x1, x2, \overbrace{op_t(op_t(x3, x2), rd(x2, x3))})} \\
= & \quad \text{by Lemma 2010 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_r(x1, op_t(x2, rd(x3, op_t(op_t(x2, x3), x3))), op_t(op_t(x3, x2), x2)})} \\
= & \quad \text{by Lemma 1893 LR with } \{x4 \leftarrow op_t(x3, x2), x3 \leftarrow rd(x3, op_t(op_t(x2, x3), x3)), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_r(x1, op_t(x2, rd(x3, op_t(op_t(x2, x3), x3))), op_t(x3, x2)})} \\
= & \quad \text{by Lemma 1893 LR with } \{x4 \leftarrow x3, x3 \leftarrow rd(x3, op_t(op_t(x2, x3), x3)), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, \underbrace{op_t(x2, rd(x3, op_t(op_t(x2, x3), x3)))}, x3)} \\
= & \quad \text{by Lemma 1739 LR with } \{x3 \leftarrow op_t(x2, x3), x2 \leftarrow x3, x1 \leftarrow x2\} \\
& op_r(x1, \underbrace{\overbrace{op_t(x2, rd(x3, op_t(x2, x3)))}, x3}) \\
= & \quad \text{by Lemma 1739 LR with } \{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x2\} \\
& op_r(x1, \overbrace{op_t(x2, rd(x3, x2))}, x3)
\end{aligned}
$$

Lemma 2012: $op_r(x1, op_l(x2, x2, x3), x3) = op_r(x1, x2, op_l(x3, x3, x2))$

$$
\begin{aligned}
& \underbrace{op_r(x1, op_l(x2, x2, x3), x3)} \\
= & \quad \text{by Lemma 1898 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{\overbrace{op_r(x1, op_t(x2, rd(x3, x2)), x3)}} \\
= & \quad \text{by Lemma 2011 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, x2, op_l(x3, x3, x2))}
\end{aligned}
$$

Lemma 2013: $op_r(x1, op_l(x2, x3, x2), x3) = op_r(x1, op_t(op_l(x2, x3, x2), rd(x3, op_t(x2, x3))), op_l(x3, x2, x3))$

$$
\begin{aligned}
& op_r(x1, op_l(x2, x3, x2), \underbrace{x3}) \\
= & \quad \text{by Lemma 893 LR with } \{x2 \leftarrow x2, x1 \leftarrow x3\} \\
& \underbrace{op_r(x1, op_l(x2, x3, x2), \overbrace{mult(op_l(x2, x3, x2), rd(x3, op_t(x2, x3)))})} \\
= & \quad \text{by Lemma 2009 RL with } \{x3 \leftarrow rd(x3, op_t(x2, x3)), x2 \leftarrow op_l(x2, x3, x2), x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, op_t(op_l(x2, x3, x2), rd(x3, op_t(x2, x3))), \underbrace{mult(rd(x3, op_t(x2, x3)), op_l(x2, x3, x2))})} \\
= & \quad \text{by Lemma 1618 RL with } \{x2 \leftarrow x2, x1 \leftarrow x3\} \\
& op_r(x1, op_t(op_l(x2, x3, x2), rd(x3, op_t(x2, x3))), \overbrace{op_l(x3, x2, x3)})
\end{aligned}
$$

Lemma 2014: $op_r(x1, op_l(x2, x3, x2), x3) = op_r(x1, x2, op_l(x3, x2, x3))$

$$
\begin{aligned}
& \underbrace{op_r(x1, op_l(x2, x3, x2), x3)} \\
= & \quad \text{by Lemma 2013 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, \underbrace{op_t(op_l(x2, x3, x2), rd(x3, op_t(x2, x3)))}, op_l(x3, x2, x3))} \\
= & \quad \text{by Axiom 15 LR with } \{x4 \leftarrow rd(x3, op_t(x2, x3)), x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x2\} \\
& op_r(x1, \overbrace{\underbrace{op_l(\overbrace{op_t(x2, rd(x3, op_t(x2, x3)))}, x3, x2)}, op_l(x3, x2, x3))} \\
= & \quad \text{by Lemma 1739 LR with } \{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x2\} \\
& op_r(x1, \underbrace{op_l(\overbrace{op_t(x2, rd(x3, x2))}, x3, x2)}, op_l(x3, x2, x3)) \\
= & \quad \text{by Lemma 455 LR with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& op_r(x1, \underbrace{\overbrace{op_t(x2, x3)}}, op_l(x3, x2, x3)) \\
= & \quad \text{by Lemma 1687 RL with } \{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x2\} \\
& \underbrace{op_r(x1, \overbrace{op_t(x2, op_l(x3, x2, x3))}, op_l(x3, x2, x3))} \\
= & \quad \text{by Lemma 1894 LR with } \{x3 \leftarrow op_l(x3, x2, x3), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, x2, op_l(x3, x2, x3))}
\end{aligned}
$$

Lemma 2015: $op_r(x1, x2, op_l(op_t(mult(x2, x3), x2), op_t(mult(x2, x3), x2), x2)) = op_r(x1, op_t(x2, op_r(x3, x2, i(x2))), op_t(mult(x2$

$$
\begin{aligned}
& \underbrace{op_r(x1, x2, op_l(op_t(mult(x2, x3), x2), op_t(mult(x2, x3), x2), x2))} \\
= & \quad \text{by Lemma 2012 RL with } \{x3 \leftarrow op_t(mult(x2, x3), x2), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_r(x1, op_l(x2, x2, op_t(mult(x2, x3), x2)), op_t(mult(x2, x3), x2))}} \\
= & \quad \text{by Lemma 1898 RL with } \{x3 \leftarrow op_t(mult(x2, x3), x2), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, op_t(x2, \underbrace{rd(op_t(mult(x2, x3), x2), x2)}), op_t(mult(x2, x3), x2))} \\
= & \quad \text{by Lemma 317 LR with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& op_r(x1, op_t(x2, \overbrace{op_r(x3, x2, i(x2))}), op_t(mult(x2, x3), x2))
\end{aligned}
$$

Lemma 2016: $op_r(x1, x2, mult(op_t(x2, x3), x3)) = op_r(x1, op_t(x2, x3), mult(x2, x3))$

$op_r(x1, x2, \underbrace{mult(op_t(x2, x3), x3)})$

$=$ by Lemma 636 RL with $\{x2 \leftarrow x3, x1 \leftarrow x2\}$

$op_r(x1, x2, \overbrace{\underbrace{op_l(mult(x2, x3), x3, x2)}})$

$=$ by Lemma 1051 RL with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow mult(x2, x3)\}$

$op_r(x1, x2, \overbrace{\underbrace{op_l(mult(x2, x3), x2, i(mult(x2, x3)))}})$

$=$ by Lemma 1319 RL with $\{x3 \leftarrow x2, x2 \leftarrow mult(x2, x3), x1 \leftarrow mult(x2, x3)\}$

$\underbrace{op_r(x1, x2, \overbrace{op_l(mult(x2, x3), op_t(mult(x2, x3), x2), x2)})}$

$=$ by Lemma 1890 RL with $\{x3 \leftarrow op_l(mult(x2, x3), op_t(mult(x2, x3), x2), x2), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_r(x1, x2, \underbrace{op_t(op_l(mult(x2, x3), op_t(mult(x2, x3), x2), x2), x2)})}$

$=$ by Axiom 15 LR with $\{x4 \leftarrow x2, x3 \leftarrow x2, x2 \leftarrow op_t(mult(x2, x3), x2), x1 \leftarrow mult(x2, x3)\}$

$\underbrace{op_r(x1, x2, \overbrace{op_l(op_t(mult(x2, x3), x2), op_t(mult(x2, x3), x2), x2)})}$

$=$ by Lemma 2015 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_r(x1, op_t(x2, op_r(x3, x2, i(x2))), op_t(mult(x2, x3), x2)})}$

$=$ by Lemma 1893 LR with $\{x4 \leftarrow mult(x2, x3), x3 \leftarrow op_r(x3, x2, i(x2)), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_r(x1, \underbrace{op_t(x2, op_r(x3, x2, i(x2)))}, mult(x2, x3))}$

$=$ by Lemma 1687 RL with $\{x3 \leftarrow x2, x2 \leftarrow op_r(x3, x2, i(x2)), x1 \leftarrow x2\}$

$op_r(x1, \overbrace{op_t(x2, \underbrace{op_l(op_r(x3, x2, i(x2)), x2, op_r(x3, x2, i(x2)))})}, mult(x2, x3))$

$=$ by Lemma 1340 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow op_r(x3, x2, i(x2))\}$

$op_r(x1, op_t(x2, \overbrace{op_l(\underbrace{op_r(x3, x2, i(x2))}, x2, x3)}), mult(x2, x3))$

$=$ by Lemma 421 LR with $\{x2 \leftarrow x2, x1 \leftarrow x3\}$

$op_r(x1, op_t(x2, \underbrace{op_l(\overbrace{op_l(op_t(x3, x2), x3, x2)}, x2, x3)}), mult(x2, x3))$

$=$ by Lemma 138 LR with $\{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow op_t(x3, x2)\}$

$op_r(x1, \underbrace{op_t(x2, \overbrace{op_t(x3, x2)})}, mult(x2, x3))$

$=$ by Lemma 1722 LR with $\{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x2\}$

$op_r(x1, \overbrace{op_t(x2, x3)}, mult(x2, x3))$

Lemma 2017: $op_r(x1, rd(x2, x3), op_t(x2, i(x3))) = op_r(x1, op_t(rd(x2, x3), x3), x2)$

$op_r(x1, rd(x2, x3), \underbrace{op_t(x2, i(x3))})$

$=$ by Lemma 390 RL with $\{x2 \leftarrow i(x3), x1 \leftarrow x2\}$

$op_r(x1, rd(x2, x3), \overbrace{rd(x2, \underbrace{rd(op_t(i(x3), x2), i(x3))})})$

$=$ by Lemma 38 LR with $\{x1 \leftarrow x3, x2 \leftarrow op_t(i(x3), x2)\}$

$op_r(x1, rd(x2, x3), rd(x2, \overbrace{rd(x3, \underbrace{i(op_t(i(x3), x2))})}))$

$=$ by Lemma 18 LR with $\{x2 \leftarrow x2, x1 \leftarrow x3\}$

$op_r(x1, rd(x2, x3), \underbrace{rd(x2, rd(x3, \overbrace{op_t(x3, i(x2))}))})$

$=$ by Lemma 1638 RL with $\{x3 \leftarrow i(x2), x2 \leftarrow x3, x1 \leftarrow x2\}$

$\underbrace{op_r(x1, rd(x2, x3), \overbrace{mult(rd(x2, x3), op_t(x3, i(x2)))})}$

$=$ by Lemma 2009 RL with $\{x3 \leftarrow op_t(x3, i(x2)), x2 \leftarrow rd(x2, x3), x1 \leftarrow x1\}$

$\overbrace{op_r(x1, op_t(rd(x2, x3), op_t(x3, i(x2))), \underbrace{mult(op_t(x3, i(x2)), rd(x2, x3))})}$

$=$ by Lemma 830 LR with $\{x2 \leftarrow x2, x1 \leftarrow x3\}$

$op_r(x1, op_t(rd(x2, x3), op_t(x3, i(x2))), \overbrace{\underbrace{op_t(op_l(x2, x2, x3), op_t(i(x3), x2))})$

$=$ by Axiom 15 LR with $\{x4 \leftarrow op_t(i(x3), x2), x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x2\}$

$op_r(x1, op_t(rd(x2, x3), op_t(x3, i(x2))), \overbrace{op_l(\underbrace{op_t(x2, op_t(i(x3), x2))}, x2, x3)})$

$=$ by Lemma 374 LR with $\{x2 \leftarrow i(x3), x1 \leftarrow x2\}$

$op_r(x1, op_t(rd(x2, x3), op_t(x3, i(x2))), \underbrace{op_l(\overbrace{op_t(x2, i(x3))}, x2, x3)})$

$=$ by Lemma 580 LR with $\{x2 \leftarrow x3, x1 \leftarrow x2\}$

$\underbrace{op_r(x1, op_t(rd(x2, x3), op_t(x3, i(x2))), \overbrace{op_t(x2, rd(x2, x3))})}$

$=$ by Lemma 1893 LR with $\{x4 \leftarrow x2, x3 \leftarrow op_t(x3, i(x2)), x2 \leftarrow rd(x2, x3), x1 \leftarrow x1\}$

$\overbrace{op_r(x1, \underbrace{op_t(rd(x2, x3), op_t(x3, i(x2)))}, x2)}$

$=$ by Lemma 1722 LR with $\{x3 \leftarrow i(x2), x2 \leftarrow x3, x1 \leftarrow rd(x2, x3)\}$

$op_r(x1, \overbrace{op_t(rd(x2, x3), x3)}, x2)$

Lemma 2018: $op_r(x1, rd(x2, x3), op_l(x3, x3, x2)) = op_r(x1, op_l(rd(x2, x3), x2, x3), x3)$

$op_r(x1, rd(x2, x3), \underbrace{op_l(x3, x3, x2)})$

$=$ by Lemma 286 RL with $\{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x3\}$

$\underbrace{op_r(x1, rd(x2, x3), \overbrace{op_l(x3, x3, rd(x2, x3))})}$

$=$ by Lemma 2012 RL with $\{x3 \leftarrow x3, x2 \leftarrow rd(x2, x3), x1 \leftarrow x1\}$

$\overbrace{op_r(x1, \underbrace{op_l(rd(x2, x3), rd(x2, x3), x3)}, x3)}$

$=$ by Lemma 292 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow rd(x2, x3)\}$

$op_r(x1, \overbrace{op_l(rd(x2, x3), x2, x3)}, x3)$

Lemma 2019: $op_r(x1, rd(x2, x3), op_l(x3, x3, x2)) = op_r(x1, mult(x2, i(x3)), x3)$

$$
\begin{aligned}
& \underbrace{op_r(x1, rd(x2, x3), op_l(x3, x3, x2))} \\
= & \quad \text{by Lemma 2018 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, \underbrace{op_l(rd(x2, x3), x2, x3)}, x3)} \\
= & \quad \text{by Lemma 928 LR with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& \underbrace{op_r(x1, \overbrace{rd(op_t(x2, i(x3)), x3)}, x3)} \\
= & \quad \text{by Lemma 1905 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, mult(x2, i(x3)), x3)}
\end{aligned}
$$

Lemma 2020: $op_r(x1, mult(x2, x3), op_l(x3, x2, x3)) = op_r(x1, op_l(mult(x2, x3), x3, x2), x3)$

$$
\begin{aligned}
& op_r(x1, mult(x2, x3), \underbrace{op_l(x3, x2, x3)}) \\
= & \quad \text{by Lemma 285 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x3\} \\
& \underbrace{op_r(x1, mult(x2, x3), \overbrace{op_l(x3, mult(x2, x3), x3)})} \\
= & \quad \text{by Lemma 2014 RL with } \{x3 \leftarrow x3, x2 \leftarrow mult(x2, x3), x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, \underbrace{op_l(mult(x2, x3), x3, mult(x2, x3))}, x3)} \\
= & \quad \text{by Lemma 291 LR with } \{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow mult(x2, x3)\} \\
& op_r(x1, \overbrace{op_l(mult(x2, x3), x3, x2)}, x3)
\end{aligned}
$$

Lemma 2021: $op_r(x1, mult(x2, x3), op_l(x3, x2, x3)) = op_r(x1, rd(x2, i(x3)), x3)$

$$
\begin{aligned}
& \underbrace{op_r(x1, mult(x2, x3), op_l(x3, x2, x3))} \\
= & \quad \text{by Lemma 2020 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, \underbrace{op_l(mult(x2, x3), x3, x2)}, x3)} \\
= & \quad \text{by Lemma 636 LR with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& \underbrace{op_r(x1, \overbrace{mult(op_t(x2, x3), x3)}, x3)} \\
= & \quad \text{by Lemma 1897 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, rd(x2, i(x3)), x3)}
\end{aligned}
$$

Lemma 2022: $rd(rd(op_t(x1, x2), x2), op_t(x1, x3)) = mult(mult(x1, i(x2)), i(op_t(x1, x3)))$

$$
\begin{aligned}
& \underbrace{rd(rd(op_t(x1, x2), x2), op_t(x1, x3))} \\
= & \quad \text{by Lemma 352 RL with } \{x2 \leftarrow op_t(x1, x3), x1 \leftarrow rd(op_t(x1, x2), x2)\} \\
& \overbrace{mult(\underbrace{op_r(rd(op_t(x1, x2), x2), op_t(x1, x3), op_t(x1, x3))}, i(op_t(x1, x3)))} \\
= & \quad \text{by Lemma 1632 LR with } \{x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow rd(op_t(x1, x2), x2)\} \\
& mult(\overbrace{\underbrace{op_r(rd(op_t(x1, x2), x2), x1, x1)}}, i(op_t(x1, x3))) \\
= & \quad \text{by Lemma 690 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& mult(\overbrace{mult(x1, i(x2))}, i(op_t(x1, x3)))
\end{aligned}
$$

Lemma 2023: $rd(x1, rd(op_l(x1, x1, x2), mult(x1, x2))) = rd(op_t(x1, mult(x1, x2)), i(x2))$

$\underbrace{rd(x1, rd(op_l(x1, x1, x2), mult(x1, x2)))}$

$=$ by Lemma 1660 RL with $\{x3 \leftarrow op_l(x1, x1, x2), x2 \leftarrow mult(x1, x2), x1 \leftarrow x1\}$

$\overbrace{rd(op_t(x1, mult(x1, x2)), rd(op_l(x1, x1, x2), \underbrace{op_t(mult(x1, x2), x1)}))}$

$=$ by Lemma 264 RL with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$rd(op_t(x1, mult(x1, x2)), \underbrace{rd(op_l(x1, x1, x2), \overbrace{mult(x2, op_l(x1, x1, x2))})})$

$=$ by Lemma 9 LR with $\{x2 \leftarrow x2, x1 \leftarrow op_l(x1, x1, x2)\}$

$rd(op_t(x1, mult(x1, x2)), \overbrace{i(x2)})$

Lemma 2024: $rd(op_t(x1, x2), mult(rd(x3, op_t(x2, x1)), x1)) = mult(rd(x1, rd(x3, x2)), i(x1))$

$\underbrace{rd(op_t(x1, x2), mult(rd(x3, op_t(x2, x1)), x1))}$

$=$ by Lemma 1291 RL with $\{x3 \leftarrow rd(x3, op_t(x2, x1)), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{\underbrace{rd(rd(op_t(x1, x2), x1), rd(x3, op_t(x2, x1)))}}$

$=$ by Lemma 1658 RL with $\{x3 \leftarrow rd(x3, op_t(x2, x1)), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{mult(\underbrace{rd(op_t(x1, x2), rd(x3, op_t(x2, x1)))}, i(x1))}$

$=$ by Lemma 1660 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$mult(\overbrace{rd(x1, rd(x3, x2))}, i(x1))$

Lemma 2025: $rd(op_t(x1, x2), mult(rd(x3, op_t(x2, x1)), x1)) = rd(x2, x3)$

$\underbrace{rd(op_t(x1, x2), mult(rd(x3, op_t(x2, x1)), x1))}$

$=$ by Lemma 2024 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{\underbrace{mult(rd(x1, rd(x3, x2)), i(x1))}}$

$=$ by Lemma 22 LR with $\{x2 \leftarrow rd(x3, x2), x1 \leftarrow x1\}$

$\overbrace{\underbrace{i(rd(x3, x2))}}$

$=$ by Lemma 10 LR with $\{x2 \leftarrow x2, x1 \leftarrow x3\}$

$\overbrace{rd(x2, x3)}$

Lemma 2026: $op_l(x1, mult(i(x2), x3), op_t(x3, x4)) = op_l(x1, mult(rd(x3, x2), op_t(x3, x4)), op_t(x3, x4))$

$op_l(x1, \underbrace{mult(i(x2), x3)}, op_t(x3, x4))$

$=$ by Lemma 17 RL with $\{x2 \leftarrow x3, x1 \leftarrow x2\}$

$\underbrace{op_l(x1, \overbrace{i(mult(x2, i(x3)))}, op_t(x3, x4))}$

$=$ by Lemma 1050 LR with $\{x3 \leftarrow op_t(x3, x4), x2 \leftarrow mult(x2, i(x3)), x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_l(x1, mult(x2, i(x3)), i(op_t(x3, x4)))}}$

$=$ by Lemma 1317 RL with $\{x3 \leftarrow mult(x2, i(x3)), x2 \leftarrow op_t(x3, x4), x1 \leftarrow x1\}$

$\overbrace{op_l(x1, \underbrace{rd(op_t(x3, x4), mult(x2, i(x3)))}, op_t(x3, x4))}$

$=$ by Lemma 1677 LR with $\{x3 \leftarrow x2, x2 \leftarrow x4, x1 \leftarrow x3\}$

$op_l(x1, \overbrace{mult(rd(x3, x2), op_t(x3, x4))}, op_t(x3, x4))$

Lemma 2027: $op_l(x1, mult(x2, x3), op_t(x3, x4)) = op_l(x1, rd(x3, i(x2)), op_t(x3, x4))$

$$
\begin{aligned}
& \underbrace{op_l(x1, mult(x2, x3), op_t(x3, x4))} \\
= \quad & \text{by Lemma 1326 RL with } \{x3 \leftarrow op_t(x3, x4), x2 \leftarrow mult(x2, x3), x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, \underbrace{op_r(mult(x2, x3), op_t(x3, x4), op_t(x3, x4))}, op_t(x3, x4))} \\
= \quad & \text{by Lemma 1632 LR with } \{x3 \leftarrow x4, x2 \leftarrow x3, x1 \leftarrow mult(x2, x3)\} \\
& op_l(x1, \overbrace{\underbrace{op_r(mult(x2, x3), x3, x3)}}, op_t(x3, x4)) \\
= \quad & \text{by Lemma 359 LR with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& op_l(x1, \overbrace{rd(x3, i(x2))}, op_t(x3, x4))
\end{aligned}
$$

Lemma 2028: $op_l(x1, op_t(x2, x3), mult(x4, x2)) = op_l(x1, op_t(x2, x3), rd(x2, i(x4)))$

$$
\begin{aligned}
& \underbrace{op_l(x1, op_t(x2, x3), mult(x4, x2))} \\
= \quad & \text{by Lemma 1327 LR with } \{x3 \leftarrow mult(x4, x2), x2 \leftarrow op_t(x2, x3), x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, op_t(x2, x3), \underbrace{op_r(mult(x4, x2), op_t(x2, x3), op_t(x2, x3))})} \\
= \quad & \text{by Lemma 1632 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow mult(x4, x2)\} \\
& op_l(x1, op_t(x2, x3), \overbrace{\underbrace{op_r(mult(x4, x2), x2, x2)}}) \\
= \quad & \text{by Lemma 359 LR with } \{x2 \leftarrow x2, x1 \leftarrow x4\} \\
& op_l(x1, op_t(x2, x3), \overbrace{rd(x2, i(x4))})
\end{aligned}
$$

Lemma 2029: $mult(mult(x1,x2),mult(x1,mult(rd(i(x1),mult(op_t(x1,x2),x2)),x3))) = op_l(x3,op_t(x1,x2),mult(x2,x1))$

$mult(mult(x1,x2),mult(x1,mult(\underbrace{rd(i(x1),mult(op_t(x1,x2),x2))}_{},x3)))$

= by Lemma 37 RL with $\{x2 \leftarrow x1, x1 \leftarrow mult(op_t(x1,x2),x2)\}$

$mult(mult(x1,x2),mult(x1,mult(\overbrace{rd(i(mult(op_t(x1,x2),x2)),x1)}^{},x3)))$

= by Lemma 10 RL with $\{x2 \leftarrow i(mult(op_t(x1,x2),x2)), x1 \leftarrow x1\}$

$mult(mult(x1,x2),mult(x1,mult(\overbrace{i(\underbrace{rd(x1,i(mult(op_t(x1,x2),x2)))}_{})}^{},x3)))$

= by Lemma 86 RL with $\{x2 \leftarrow mult(op_t(x1,x2),x2), x1 \leftarrow x1\}$

$mult(mult(x1,x2),mult(x1,mult(i(\overbrace{mult(x1,mult(\underbrace{mult(i(x1),mult(op_t(x1,x2),x2))}_{},x1))}^{}),x3)))$

= by Lemma 1092 LR with $\{x3 \leftarrow x2, x2 \leftarrow x2, x1 \leftarrow x1\}$

$mult(mult(x1,x2),mult(x1,mult(i(mult(x1,mult(\overbrace{mult(\underbrace{rd(op_t(x1,x2),x1)}_{},x2)}^{},x1))),x3)))$

= by Lemma 858 RL with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$mult(mult(x1,x2),mult(x1,mult(i(mult(x1,mult(\underbrace{mult(\overbrace{asoc(x2,mult(x1,x2),mult(x1,x2))}^{},x2)}_{},x1))),x3)))$

= by Lemma 1708 LR with $\{x3 \leftarrow x2, x2 \leftarrow mult(x1,x2), x1 \leftarrow x2\}$

$mult(mult(x1,x2),mult(x1,mult(i(mult(x1,mult(\overbrace{rd(x2,\underbrace{asoc(mult(x1,x2),x2,mult(x1,x2))}_{})}^{},x1))),x3)))$

= by Lemma 838 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$mult(mult(x1,x2),mult(x1,mult(i(mult(x1,\underbrace{mult(rd(x2,\overbrace{rd(x1,op_t(x1,x2))}^{}),x1)}_{})),x3)))$

= by Lemma 1261 LR with $\{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x2\}$

$mult(mult(x1,x2),mult(x1,mult(i(\underbrace{mult(x1,\overbrace{mult(x2,op_t(x1,x2))}^{})}_{}),x3)))$

= by Lemma 1760 RL with $\{x3 \leftarrow x2, x2 \leftarrow x2, x1 \leftarrow x1\}$

$mult(mult(x1,x2),\underbrace{mult(x1,mult(i(\overbrace{mult(op_t(x1,x2),mult(x2,x1))}^{}),x3))}_{})$

= by Lemma 231 LR with $\{x2 \leftarrow mult(i(mult(op_t(x1,x2),mult(x2,x1))),x3), x1 \leftarrow x1\}$

$mult(mult(x1,x2),\overbrace{rd(x1,\underbrace{op_l(i(mult(i(mult(op_t(x1,x2),mult(x2,x1))),x3)),mult(i(mult(op_t(x1,x2),mult(x2,x1))),x3)}_{}}$

= by Lemma 1740 RL with $\{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow mult(i(mult(op_t(x1,x2),mult(x2,x1))),x3)\}$

$mult(\underbrace{mult(x1,x2)}_{},rd(x1,\overbrace{op_l(i(mult(i(mult(op_t(x1,x2),mult(x2,x1))),x3)),mult(i(mult(op_t(x1,x2),mult(x2,x1))),x3)}$

= by Lemma 435 RL with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$mult(\overbrace{op_l(mult(x2,x1),x2,x1)}^{},rd(x1,op_l(i(mult(i(mult(op_t(x1,x2),mult(x2,x1))),x3)),mult(i(mult(op_t(x1,x2),mult(x$

= by Lemma 285 RL with $\{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow mult(x2,x1)\}$

$\underbrace{mult(\overbrace{op_l(mult(x2,x1),mult(x2,x1),x1)}^{},rd(x1,op_l(i(mult(i(mult(op_t(x1,x2),mult(x2,x1))),x3)),mult(i(mult(op_t(x1,x$

= by Lemma 1720 RL with $\{x2 \leftarrow x1, x3 \leftarrow rd(x1,op_l(i(mult(i(mult(op_t(x1,x2),mult(x2,x1))),x3)),mult(i(mult(op_t(x1,x2$

$\overbrace{mult(mult(x2,x1),\underbrace{rd(rd(x1,op_l(i(mult(i(mult(op_t(x1,x2),mult(x2,x1))),x3)),mult(i(mult(op_t(x1,x2),mult(x2,x1))),$

= by Lemma 1391 LR with $\{x3 \leftarrow mult(x2,x1), x2 \leftarrow op_l(i(mult(i(mult(op_t(x1,x2),mult(x2,x1))),x3)),mult(i(mult(op_t(x1,$

$mult(mult(x2,x1),\overbrace{rd(op_t(x1,\underbrace{rd(mult(x2,x1),x1)}_{}),op_l(i(mult(i(mult(op_t(x1,x2),mult(x2,x1))),x3)),mult(i(mult(op_t(x$

= by Axiom 5 RL with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$mult(mult(x2,x1),\underbrace{rd(op_t(x1,\overbrace{x2}^{}),op_l(i(mult(i(mult(op_t(x1,x2),mult(x2,x1))),x3)),mult(i(mult(op_t(x1,x2),mult(x2,$

= by Lemma 231 RL with $\{x2 \leftarrow mult(i(mult(op_t(x1,x2),mult(x2,x1))),x3), x1 \leftarrow op_t(x1,x2)\}$

$\underbrace{mult(mult(x2,x1),\overbrace{mult(op_t(x1,x2),mult(i(mult(op_t(x1,x2),mult(x2,x1))),x3))}^{})}$

= by Lemma 1620 LR with $\{x3 \leftarrow x3, x2 \leftarrow op_t(x1,x2), x1 \leftarrow mult(x2,x1)\}$

$\overbrace{op_l(x3,op_t(x1,x2),mult(x2,x1))}^{}$

Lemma 2030: $op_l(x1, x2, i(x3)) = op_l(x1, x3, x2)$

$$
\begin{aligned}
& \underbrace{op_l(x1, x2, i(x3))} \\
= \quad & \text{by Lemma 1330 RL with } \{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, x3, mult(x3, x2))}} \\
= \quad & \text{by Lemma 1620 RL with } \{x3 \leftarrow x1, x2 \leftarrow x3, x1 \leftarrow mult(x3, x2)\} \\
& \overbrace{mult(mult(x3, x2), mult(x3, mult(\underbrace{i(mult(x3, mult(x3, x2)))}, x1)))} \\
= \quad & \text{by Axiom 11 RL with } \{x2 \leftarrow mult(x3, x2), x1 \leftarrow x3\} \\
& mult(mult(x3, x2), mult(x3, mult(\overbrace{mult(i(x3), \underbrace{i(mult(x3, x2))})}, x1))) \\
= \quad & \text{by Lemma 925 LR with } \{x2 \leftarrow x2, x1 \leftarrow x3\} \\
& mult(mult(x3, x2), mult(x3, mult(\overbrace{rd(\underbrace{op_t(rd(i(x3), x2), i(x2))}, x3)}, x1))) \\
= \quad & \text{by Lemma 408 LR with } \{x2 \leftarrow x2, x1 \leftarrow x3\} \\
& mult(mult(x3, x2), mult(x3, mult(\underbrace{rd(\overbrace{i(mult(op_t(x3, x2), x2))}, x3)}, x1))) \\
= \quad & \text{by Lemma 37 LR with } \{x2 \leftarrow x3, x1 \leftarrow mult(op_t(x3, x2), x2)\} \\
& \underbrace{mult(mult(x3, x2), mult(x3, mult(\overbrace{rd(i(x3), mult(op_t(x3, x2), x2))}, x1)))} \\
= \quad & \text{by Lemma 2029 LR with } \{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x3\} \\
& \overbrace{\underbrace{op_l(x1, op_t(x3, x2), mult(x2, x3))}} \\
= \quad & \text{by Lemma 1347 LR with } \{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x3, x2)}
\end{aligned}
$$

Lemma 2031: $op_l(x1, i(x2), x3) = op_l(x1, x3, x2)$

$$
\begin{aligned}
& \underbrace{op_l(x1, i(x2), x3)} \\
= \quad & \text{by Lemma 1050 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, x2, i(x3))}} \\
= \quad & \text{by Lemma 2030 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x3, x2)}
\end{aligned}
$$

Lemma 2032: $op_l(x1, x2, mult(x2, x3)) = op_l(x1, x2, x3)$

$$
\begin{aligned}
& \underbrace{op_l(x1, x2, mult(x2, x3))} \\
= \quad & \text{by Lemma 2030 RL with } \{x3 \leftarrow x2, x2 \leftarrow mult(x2, x3), x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, mult(x2, x3), i(x2))}} \\
= \quad & \text{by Lemma 242 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x2, x3)}
\end{aligned}
$$

Lemma 2033: $op_l(x1, mult(x2, x3), x2) = op_l(x1, x3, x2)$

$$
\begin{aligned}
& \overbrace{op_l(x1, mult(x2, x3), x2)} \\
= & \quad \text{by Lemma 2030 RL with } \{x3 \leftarrow mult(x2, x3), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\overbrace{op_l(x1, x2, i(mult(x2, x3)))}} \\
= & \quad \text{by Lemma 1051 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x3, x2)}
\end{aligned}
$$

Lemma 2034: $op_l(x1, op_t(x2, x3), x3) = op_l(x1, x2, x3)$

$$
\begin{aligned}
& \overbrace{op_l(x1, op_t(x2, x3), x3)} \\
= & \quad \text{by Lemma 1319 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\overbrace{op_l(x1, x3, i(x2))}} \\
= & \quad \text{by Lemma 2030 LR with } \{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x2, x3)}
\end{aligned}
$$

Lemma 2035: $op_l(x1, x2, rd(x2, x3)) = op_l(x1, x3, x2)$

$$
\begin{aligned}
& \overbrace{op_l(x1, x2, rd(x2, x3))} \\
= & \quad \text{by Lemma 1321 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\overbrace{op_l(x1, x2, i(x3))}} \\
= & \quad \text{by Lemma 2030 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x3, x2)}
\end{aligned}
$$

Lemma 2036: $op_l(x1, x2, mult(i(x2), x3)) = op_l(x1, x2, x3)$

$$
\begin{aligned}
& \overbrace{op_l(x1, x2, mult(i(x2), x3))} \\
= & \quad \text{by Lemma 243 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\overbrace{op_l(x1, x3, i(x2))}} \\
= & \quad \text{by Lemma 2030 LR with } \{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x2, x3)}
\end{aligned}
$$

Lemma 2037: $op_l(x1, x2, op_l(x3, x2, x3)) = op_l(x1, x2, x3)$

$$
\begin{aligned}
& \overbrace{op_l(x1, x2, op_l(x3, x2, x3))} \\
= & \quad \text{by Lemma 651 RL with } \{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{\overbrace{op_l(x1, x3, i(x2))}} \\
= & \quad \text{by Lemma 2030 LR with } \{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x2, x3)}
\end{aligned}
$$

Lemma 2038: $op_l(x1, rd(i(x2), x2), x3) = op_l(x1, x3, mult(x2, x2))$

$$
\begin{aligned}
& \underbrace{op_l(x1, rd(i(x2), x2), x3)} \\
= \quad & \text{by Lemma 1066 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, mult(x2, x2), i(x3))} \\
= \quad & \text{by Lemma 2030 LR with } \{x3 \leftarrow x3, x2 \leftarrow mult(x2, x2), x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x3, mult(x2, x2))}
\end{aligned}
$$

Lemma 2039: $op_l(x1, x2, rd(i(x3), x3)) = op_l(x1, mult(x3, x3), x2)$

$$
\begin{aligned}
& op_l(x1, x2, \underbrace{rd(i(x3), x3)}) \\
= \quad & \text{by Lemma 49 LR with } \{x2 \leftarrow x2, x1 \leftarrow x3\} \\
& \underbrace{op_l(x1, x2, \overbrace{rd(x2, mult(x3, mult(x3, x2)))})} \\
= \quad & \text{by Lemma 1321 LR with } \{x3 \leftarrow mult(x3, mult(x3, x2)), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, x2, i(mult(x3, mult(x3, x2))))}} \\
= \quad & \text{by Lemma 2030 LR with } \{x3 \leftarrow mult(x3, mult(x3, x2)), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, \underbrace{mult(x3, mult(x3, x2))}, x2)} \\
= \quad & \text{by Lemma 24 RL with } \{x2 \leftarrow x2, x1 \leftarrow x3\} \\
& \underbrace{op_l(x1, \overbrace{mult(mult(x3, x3), x2)}, x2)} \\
= \quad & \text{by Lemma 285 LR with } \{x3 \leftarrow x2, x2 \leftarrow mult(x3, x3), x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, mult(x3, x3), x2)}
\end{aligned}
$$

Lemma 2040: $op_l(x1, mult(x2, x3), op_l(x3, x3, x2)) = op_l(x1, x2, x3)$

$$
\begin{aligned}
& \underbrace{op_l(x1, mult(x2, x3), op_l(x3, x3, x2))} \\
= \quad & \text{by Lemma 1383 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, x3, i(x2))}} \\
= \quad & \text{by Lemma 2030 LR with } \{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x2, x3)}
\end{aligned}
$$

Lemma 2041: $op_l(x1, x2, op_l(i(x3), x4, x5)) = op_l(x1, op_l(x3, x4, x5), x2)$

$$
\begin{aligned}
& op_l(x1, x2, \underbrace{op_l(i(x3), x4, x5)}) \\
= \quad & \text{by Lemma 1049 RL with } \{x3 \leftarrow x5, x2 \leftarrow x4, x1 \leftarrow x3\} \\
& \underbrace{op_l(x1, x2, \overbrace{i(op_l(x3, x4, x5))})} \\
= \quad & \text{by Lemma 2030 LR with } \{x3 \leftarrow op_l(x3, x4, x5), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, op_l(x3, x4, x5), x2)}
\end{aligned}
$$

Lemma 2042: $op_l(x1, mult(mult(x2, x3), x3), x2) = op_l(x1, rd(x3, x2), mult(x2, x3))$

$\underbrace{op_l(x1, mult(mult(x2, x3), x3), x2)}$

$=$ by Lemma 2030 RL with $\{x3 \leftarrow mult(mult(x2, x3), x3), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_l(x1, x2, i(mult(mult(x2, x3), x3)))}$

$=$ by Lemma 1050 RL with $\{x3 \leftarrow mult(mult(x2, x3), x3), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_l(x1, i(x2), mult(mult(x2, x3), x3))}$

$=$ by Lemma 291 RL with $\{x3 \leftarrow mult(mult(x2, x3), x3), x2 \leftarrow i(x2), x1 \leftarrow x1\}$

$\overbrace{op_l(x1, i(x2), mult(mult(mult(x2, x3), x3), \underbrace{i(x2)}))}$

$=$ by Lemma 9 RL with $\{x2 \leftarrow x2, x1 \leftarrow x3\}$

$op_l(x1, i(x2), \underbrace{mult(mult(mult(x2, x3), x3), \overbrace{rd(x3, mult(x2, x3))})})$

$=$ by Lemma 309 LR with $\{x2 \leftarrow x3, x1 \leftarrow mult(x2, x3)\}$

$op_l(x1, i(x2), \overbrace{mult(mult(x2, x3), rd(x3, \underbrace{rd(mult(x2, x3), x3)}))})$

$=$ by Axiom 5 RL with $\{x2 \leftarrow x3, x1 \leftarrow x2\}$

$op_l(x1, i(x2), \underbrace{mult(mult(x2, x3), rd(x3, \overbrace{x2}))})$

$=$ by Lemma 309 LR with $\{x2 \leftarrow x3, x1 \leftarrow x2\}$

$\underbrace{op_l(x1, i(x2), \overbrace{mult(x2, rd(x3, rd(x2, x3)))})}$

$=$ by Lemma 1050 LR with $\{x3 \leftarrow mult(x2, rd(x3, rd(x2, x3))), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_l(x1, x2, i(mult(x2, rd(x3, rd(x2, x3)))))}}$

$=$ by Lemma 1051 LR with $\{x3 \leftarrow rd(x3, rd(x2, x3)), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_l(x1, rd(x3, rd(x2, x3)), x2)}}$

$=$ by Lemma 1060 RL with $\{x4 \leftarrow x2, x3 \leftarrow x3, x2 \leftarrow rd(x2, x3), x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_l(x1, rd(rd(x2, x3), x3), i(x2))}}$

$=$ by Lemma 1448 RL with $\{x2 \leftarrow x2, x3 \leftarrow x3, x1 \leftarrow x1\}$

$\overbrace{op_l(x1, rd(x3, x2), mult(x2, x3))}$

Lemma 2043: $op_l(x1, x2, rd(x3, mult(x2, x4))) = op_l(x1, x2, rd(mult(x2, x3), x4))$

$\underbrace{op_l(x1, x2, rd(x3, mult(x2, x4)))}$

$=$ by Lemma 291 RL with $\{x3 \leftarrow rd(x3, mult(x2, x4)), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_l(x1, x2, mult(rd(x3, mult(x2, x4)), x2))}}$

$=$ by Lemma 2032 RL with $\{x3 \leftarrow mult(rd(x3, mult(x2, x4)), x2), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_l(x1, x2, \underbrace{mult(x2, mult(rd(x3, mult(x2, x4)), x2))})}$

$=$ by Lemma 1152 LR with $\{x3 \leftarrow x4, x2 \leftarrow x3, x1 \leftarrow x2\}$

$op_l(x1, x2, \overbrace{rd(mult(x2, x3), x4)})$

Lemma 2044: $op_l(x1, rd(op_t(x2, x3), mult(x3, x4)), x3) = op_l(x1, mult(rd(mult(x2, x3), x4), i(x3)), x3)$

$$
\begin{aligned}
& \underbrace{op_l(x1, rd(op_t(x2, x3), mult(x3, x4)), x3)} \\
= \quad & \text{by Lemma 2033 RL with } \{x3 \leftarrow rd(op_t(x2, x3), mult(x3, x4)), x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, \underbrace{mult(x3, rd(op_t(x2, x3), mult(x3, x4)))}, x3)} \\
= \quad & \text{by Lemma 1155 RL with } \{x3 \leftarrow x4, x2 \leftarrow op_t(x2, x3), x1 \leftarrow x3\} \\
& op_l(x1, \overbrace{mult(rd(\underbrace{mult(x3, op_t(x2, x3))}, x4), i(x3))}, x3) \\
= \quad & \text{by Lemma 13 LR with } \{x2 \leftarrow x2, x1 \leftarrow x3\} \\
& op_l(x1, mult(rd(\overbrace{mult(x2, x3)}, x4), i(x3)), x3)
\end{aligned}
$$

Lemma 2045: $op_l(x1, x3, rd(x4, mult(x2, x3))) = op_l(x1, mult(rd(mult(x2, x3), x4), i(x3)), x3)$

$$
\begin{aligned}
& op_l(x1, x3, rd(x4, \underbrace{mult(x2, x3)})) \\
= \quad & \text{by Lemma 13 RL with } \{x2 \leftarrow x2, x1 \leftarrow x3\} \\
& \underbrace{op_l(x1, x3, rd(x4, \overbrace{mult(x3, op_t(x2, x3))}))} \\
= \quad & \text{by Lemma 1429 RL with } \{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow op_t(x2, x3), x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, rd(op_t(x2, x3), mult(x3, x4)), x3)}} \\
= \quad & \text{by Lemma 2044 LR with } \{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, mult(rd(mult(x2, x3), x4), i(x3)), x3)}
\end{aligned}
$$

Lemma 2046: $mult(x1, rd(x2, op_r(x2, x3, x4))) = rd(mult(x1, x2), op_r(x2, x3, x4))$

$$
\begin{aligned}
& mult(x1, \underbrace{rd(x2, op_r(x2, x3, x4))}) \\
= \quad & \text{by Lemma 115 RL with } \{x3 \leftarrow x4, x2 \leftarrow x3, x1 \leftarrow x2\} \\
& \underbrace{mult(x1, \overbrace{mult(x2, i(op_r(x2, x3, x4)))})} \\
= \quad & \text{by Lemma 132 RL with } \{x3 \leftarrow i(op_r(x2, x3, x4)), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{mult(mult(x1, x2), \underbrace{op_l(i(op_r(x2, x3, x4)), x2, x1)})} \\
= \quad & \text{by Lemma 1049 RL with } \{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow op_r(x2, x3, x4)\} \\
& mult(mult(x1, x2), \overbrace{i(\underbrace{op_l(op_r(x2, x3, x4), x2, x1)})}) \\
= \quad & \text{by Axiom 18 RL with } \{x5 \leftarrow x4, x4 \leftarrow x3, x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x2\} \\
& \underbrace{mult(mult(x1, x2), i(\overbrace{op_r(op_l(x2, x2, x1), x3, x4)}))} \\
= \quad & \text{by Lemma 1972 RL with } \{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow op_l(x2, x2, x1), x1 \leftarrow mult(x1, x2)\} \\
& \overbrace{rd(mult(x1, x2), \underbrace{op_l(op_r(op_l(x2, x2, x1), x3, x4), mult(x1, x2), op_l(x2, x2, x1))})} \\
= \quad & \text{by Lemma 2040 LR with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow op_r(op_l(x2, x2, x1), x3, x4)\} \\
& rd(mult(x1, x2), \overbrace{op_l(\underbrace{op_r(op_l(x2, x2, x1), x3, x4)}, x1, x2)}) \\
= \quad & \text{by Axiom 18 LR with } \{x5 \leftarrow x4, x4 \leftarrow x3, x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x2\} \\
& rd(mult(x1, x2), \underbrace{op_l(\overbrace{op_l(op_r(x2, x3, x4), x2, x1)}, x1, x2)}) \\
= \quad & \text{by Lemma 138 LR with } \{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow op_r(x2, x3, x4)\} \\
& rd(mult(x1, x2), \overbrace{op_r(x2, x3, x4)})
\end{aligned}
$$

Lemma 2047: $op_l(x1, x2, rd(rd(x2, x3), x4)) = op_l(x1, x2, rd(op_t(x2, rd(rd(i(x2), x4), x3)), rd(x4, rd(i(i(x2)), x3))))$

$op_l(x1, x2, rd(rd(\underbrace{x2}, x3), x4))$

$=$ by Lemma 3 RL with $\{x1 \leftarrow x2\}$

$\underbrace{op_l(x1, x2, rd(rd(\overbrace{i(i(x2))}, x3), x4))}$

$=$ by Lemma 2030 RL with $\{x3 \leftarrow x2, x2 \leftarrow rd(rd(i(i(x2)), x3), x4), x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_l(x1, rd(rd(i(i(x2)), x3), x4), i(x2))}}$

$=$ by Lemma 1050 RL with $\{x3 \leftarrow x2, x2 \leftarrow rd(rd(i(i(x2)), x3), x4), x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_l(x1, i(rd(rd(i(\overbrace{i(x2)}), x3), x4)), x2)}}$

$=$ by Lemma 1329 RL with $\{x3 \leftarrow i(rd(rd(i(i(x2)), x3), x4)), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_l(x1, x2, i(op_t(i(rd(rd(i(i(x2)), x3), x4)), x2)))}}$

$=$ by Lemma 1389 RL with $\{x3 \leftarrow rd(rd(i(i(x2)), x3), x4), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_l(x1, x2, \underbrace{i(rd(op_t(i(x2), rd(rd(i(i(x2)), x3), x4)), rd(rd(i(i(x2)), x3), x4)))}}$

$=$ by Lemma 10 LR with $\{x2 \leftarrow rd(rd(i(i(x2)), x3), x4), x1 \leftarrow op_t(i(x2), rd(rd(i(i(x2)), x3), x4))\}$

$op_l(x1, x2, \overbrace{rd(rd(rd(i(i(x2)), x3), x4), \underbrace{op_t(i(x2), rd(rd(i(i(x2)), x3), x4))})})$

$=$ by Lemma 1287 RL with $\{x2 \leftarrow x4, x3 \leftarrow x3, x1 \leftarrow i(x2)\}$

$op_l(x1, x2, rd(rd(rd(i(i(x2)), x3), x4), \overbrace{\underbrace{op_t(i(x2), rd(x3, rd(i(x2), x4)))}}))$

$=$ by Lemma 47 RL with $\{x3 \leftarrow x3, x2 \leftarrow rd(i(x2), x4), x1 \leftarrow x2\}$

$op_l(x1, x2, \underbrace{rd(rd(rd(i(i(x2)), x3), x4), \overbrace{i(op_t(x2, rd(rd(i(x2), x4), x3)))})})$

$=$ by Lemma 39 LR with $\{x3 \leftarrow op_t(x2, rd(rd(i(x2), x4), x3)), x2 \leftarrow x4, x1 \leftarrow rd(i(i(x2)), x3)\}$

$op_l(x1, x2, \overbrace{rd(op_t(x2, rd(rd(i(x2), x4), x3)), rd(x4, rd(i(i(x2)), x3)))})$

Lemma 2048: $op_l(x1, x2, rd(x3, rd(x2, x3))) = op_l(x1, rd(x2, x3), mult(x2, x3))$

$op_l(x1, x2, \underbrace{rd(x3, rd(x2, x3))})$

$=$ by Lemma 39 RL with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x3\}$

$op_l(x1, x2, \overbrace{rd(rd(x3, x2), i(\underbrace{x3}))})$

$=$ by Axiom 4 LR with $\{x2 \leftarrow x2, x1 \leftarrow x3\}$

$op_l(x1, x2, \underbrace{rd(rd(x3, x2), i(\overbrace{mult(rd(x3, x2), x2)}))})$

$=$ by Lemma 58 LR with $\{x2 \leftarrow x2, x1 \leftarrow rd(x3, x2)\}$

$\underbrace{op_l(x1, x2, \overbrace{mult(rd(x3, x2), mult(x2, rd(x3, x2)))})}$

$=$ by Lemma 1407 RL with $\{x3 \leftarrow rd(x3, x2), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_l(x1, mult(x2, \underbrace{mult(rd(x3, x2), x2)}), rd(x3, x2))}$

$=$ by Axiom 4 RL with $\{x2 \leftarrow x2, x1 \leftarrow x3\}$

$op_l(x1, mult(x2, \overbrace{x3}), \underbrace{rd(x3, x2)})$

$=$ by Axiom 5 LR with $\{x2 \leftarrow mult(x2, mult(x2, x3)), x1 \leftarrow rd(x3, x2)\}$

$op_l(x1, mult(x2, x3), \overbrace{rd(\underbrace{mult(rd(x3, x2), mult(x2, mult(x2, x3)))}, mult(x2, mult(x2, x3))))}$

$=$ by Lemma 1045 RL with $\{x3 \leftarrow mult(x2, x3), x2 \leftarrow x3, x1 \leftarrow x2\}$

$op_l(x1, mult(x2, x3), rd(\overbrace{mult(i(x2), \underbrace{mult(mult(x2, x3), mult(x2, x3))})}, mult(x2, mult(x2, x3))))$

$=$ by Lemma 530 LR with $\{x2 \leftarrow x3, x1 \leftarrow x2\}$

$op_l(x1, mult(x2, x3), rd(\underbrace{mult(i(x2), \overbrace{mult(x2, mult(x3, mult(x3, x2)))})}, mult(x2, mult(x2, x3))))$

$=$ by Axiom 2 RL with $\{x2 \leftarrow x2, x1 \leftarrow mult(x3, mult(x3, x2))\}$

$\underbrace{op_l(x1, mult(x2, x3), rd(\overbrace{mult(x3, mult(x3, x2))}, mult(x2, mult(x2, x3))))}$

$=$ by Lemma 2045 LR with $\{x2 \leftarrow x2, x4 \leftarrow mult(x3, mult(x3, x2)), x3 \leftarrow mult(x2, x3), x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_l(x1, mult(rd(mult(x2, mult(x2, x3)), mult(x3, mult(x3, x2))), i(mult(x2, x3))), mult(x2, x3))}}$

$=$ by Lemma 1323 LR with $\{x3 \leftarrow mult(x2, x3), x2 \leftarrow rd(mult(x2, mult(x2, x3)), mult(x3, mult(x3, x2))), x1 \leftarrow x1\}$

$\overbrace{op_l(x1, rd(\underbrace{mult(x2, mult(x2, x3))}, mult(x3, mult(x3, x2))), mult(x2, x3))}$

$=$ by Axiom 2 LR with $\{x2 \leftarrow x3, x1 \leftarrow mult(x2, mult(x2, x3))\}$

$op_l(x1, rd(\overbrace{mult(i(x3), \underbrace{mult(x3, mult(x2, mult(x2, x3)))})}, mult(x3, mult(x3, x2))), mult(x2, x3))$

$=$ by Lemma 530 RL with $\{x2 \leftarrow x2, x1 \leftarrow x3\}$

$op_l(x1, rd(\underbrace{mult(i(x3), \overbrace{mult(mult(x3, x2), mult(x3, x2))})}, mult(x3, mult(x3, x2))), mult(x2, x3))$

$=$ by Lemma 1045 LR with $\{x3 \leftarrow mult(x3, x2), x2 \leftarrow x2, x1 \leftarrow x3\}$

$op_l(x1, \underbrace{rd(\overbrace{mult(rd(x2, x3), mult(x3, mult(x3, x2)))}, mult(x3, mult(x3, x2)))}, mult(x2, x3))$

$=$ by Axiom 5 RL with $\{x2 \leftarrow mult(x3, mult(x3, x2)), x1 \leftarrow rd(x2, x3)\}$

$op_l(x1, \overbrace{rd(x2, x3)}, mult(x2, x3))$

Lemma 2049: $op_l(x1, x2, op_l(op_t(x3, x4), x2, x3)) = op_l(x1, x2, op_t(x3, x4))$

$$
\begin{aligned}
& \underbrace{op_l(x1, x2, op_l(op_t(x3, x4), x2, x3))} \\
= \quad & \text{by Lemma 1877 LR with } \{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, \underbrace{i(i(op_t(x3, x4)))}, i(x2))} \\
= \quad & \text{by Lemma 3 LR with } \{x1 \leftarrow op_t(x3, x4)\} \\
& \underbrace{op_l(x1, \overbrace{op_t(x3, x4)}, i(x2))} \\
= \quad & \text{by Lemma 2030 LR with } \{x3 \leftarrow x2, x2 \leftarrow op_t(x3, x4), x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x2, op_t(x3, x4))}
\end{aligned}
$$

Lemma 2050: $op_l(x1, x2, mult(i(x2), rd(x3, x4))) = op_l(x1, mult(x2, mult(rd(x4, x3), x2)), x2)$

$$
\begin{aligned}
& \underbrace{op_l(x1, x2, mult(i(x2), rd(x3, x4)))} \\
= \quad & \text{by Lemma 2030 RL with } \{x3 \leftarrow x2, x2 \leftarrow mult(i(x2), rd(x3, x4)), x1 \leftarrow x1\} \\
& \underbrace{\overbrace{op_l(x1, mult(i(x2), rd(x3, x4)), i(x2))}} \\
= \quad & \text{by Lemma 1317 RL with } \{x3 \leftarrow mult(i(x2), rd(x3, x4)), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, \underbrace{rd(x2, mult(i(x2), rd(x3, x4)))}, x2)} \\
= \quad & \text{by Lemma 1090 LR with } \{x3 \leftarrow x4, x2 \leftarrow x3, x1 \leftarrow x2\} \\
& op_l(x1, \overbrace{mult(x2, mult(rd(x4, x3), x2))}, x2)
\end{aligned}
$$

Lemma 2051: $op_l(x1, x2, mult(i(x2), rd(x3, x4))) = op_l(x1, rd(x4, x3), x2)$

$$
\begin{aligned}
& \underbrace{op_l(x1, x2, mult(i(x2), rd(x3, x4)))} \\
= \quad & \text{by Lemma 2050 LR with } \{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{\overbrace{op_l(x1, mult(x2, mult(rd(x4, x3), x2)), x2)}} \\
= \quad & \text{by Lemma 2033 LR with } \{x3 \leftarrow mult(rd(x4, x3), x2), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{\overbrace{op_l(x1, mult(rd(x4, x3), x2), x2)}} \\
= \quad & \text{by Lemma 285 LR with } \{x3 \leftarrow x2, x2 \leftarrow rd(x4, x3), x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, rd(x4, x3), x2)}
\end{aligned}
$$

Lemma 2052: $op_l(x1, x2, rd(x3, x4)) = op_l(x1, rd(x4, x3), x2)$

$$
\begin{aligned}
& \underbrace{op_l(x1, x2, rd(x3, x4))} \\
= \quad & \text{by Lemma 2036 RL with } \{x3 \leftarrow rd(x3, x4), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{\overbrace{op_l(x1, x2, mult(i(x2), rd(x3, x4)))}} \\
= \quad & \text{by Lemma 2051 LR with } \{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, rd(x4, x3), x2)}
\end{aligned}
$$

Lemma 2053: $op_l(x1, mult(x2, i(x3)), op_l(x3, x2, x3)) = op_l(x1, x2, x3)$

$$
\begin{aligned}
& \underbrace{op_l(x1, mult(x2, i(x3)), op_l(x3, x2, x3))} \\
= \quad & \text{by Lemma 1918 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, \underbrace{i(i(x3))}, i(x2))} \\
= \quad & \text{by Lemma 3 LR with } \{x1 \leftarrow x3\} \\
& \underbrace{op_l(x1, \overbrace{x3}, i(x2))} \\
= \quad & \text{by Lemma 2030 LR with } \{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x2, x3)}
\end{aligned}
$$

Lemma 2054: $mult(x1, mult(rd(i(x1), x2), mult(x2, x3))) = op_l(x3, x1, x2)$

$$
\begin{aligned}
& mult(x1, mult(\underbrace{rd(i(x1), x2)}, mult(x2, x3))) \\
= \quad & \text{by Lemma 37 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \underbrace{mult(x1, mult(\overbrace{rd(i(x2), x1)}, mult(x2, x3)))} \\
= \quad & \text{by Lemma 987 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x3, x2, i(x1))}} \\
= \quad & \text{by Lemma 2030 LR with } \{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x3\} \\
& \overbrace{op_l(x3, x1, x2)}
\end{aligned}
$$

Lemma 2055: $mult(i(x2), x1) = rd(mult(op_l(x1, x2, x1), op_l(x3, x2, x1)), mult(x2, x3))$

$$
\begin{aligned}
& \underbrace{mult(i(x2), x1)} \\
= \quad & \text{by Lemma 615 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{rd(op_l(x1, x2, x1), x2)}} \\
= \quad & \text{by Lemma 1472 RL with } \{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow op_l(x1, x2, x1)\} \\
& \overbrace{rd(mult(op_l(x1, x2, x1), \underbrace{op_l(x3, x2, op_l(x1, x2, x1))}), mult(x2, x3))} \\
= \quad & \text{by Lemma 2037 LR with } \{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x3\} \\
& rd(mult(op_l(x1, x2, x1), \overbrace{op_l(x3, x2, x1)}), mult(x2, x3))
\end{aligned}
$$

Lemma 2056: $mult(i(x2), x1) = rd(op_l(mult(x1, x3), x2, x1), mult(x2, x3))$

$$
\begin{aligned}
& \underbrace{mult(i(x2), x1)} \\
= \quad & \text{by Lemma 2055 LR with } \{x3 \leftarrow x3, x1 \leftarrow x1, x2 \leftarrow x2\} \\
& \overbrace{rd(\underbrace{mult(op_l(x1, x2, x1), op_l(x3, x2, x1))}, mult(x2, x3))} \\
= \quad & \text{by Lemma 1880 RL with } \{x3 \leftarrow x2, x2 \leftarrow op_l(x3, x2, x1), x1 \leftarrow x1\} \\
& rd(\overbrace{op_l(mult(x1, \underbrace{op_l(op_l(x3, x2, x1), x1, x2)}), x2, x1)}, mult(x2, x3)) \\
= \quad & \text{by Lemma 138 LR with } \{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x3\} \\
& rd(op_l(mult(x1, \overbrace{x3}), x2, x1), mult(x2, x3))
\end{aligned}
$$

Lemma 2057: $op_t(x1, mult(x2, op_l(i(x3), x4, x5))) = op_t(x1, rd(x2, op_l(x3, x4, x5)))$

$\overbrace{op_t(x1, mult(x2, op_l(i(x3), x4, x5)))}$

$=$ by Lemma 1729 RL with $\{x3 \leftarrow x2, x2 \leftarrow op_l(i(x3), x4, x5), x1 \leftarrow x1\}$

$\overbrace{op_t(x1, \underbrace{rd(op_l(i(x3), x4, x5), i(x2))})}$

$=$ by Lemma 1073 LR with $\{x4 \leftarrow x2, x3 \leftarrow x5, x2 \leftarrow x4, x1 \leftarrow x3\}$

$op_t(x1, \overbrace{rd(x2, op_l(x3, x4, x5))})$

Lemma 2058: $mult(rd(x3, mult(x1, op_t(x1, x2))), x1) = rd(x1, rd(op_t(x1, x2), i(rd(x1, x3))))$

$\overbrace{mult(rd(x3, mult(x1, op_t(x1, x2))), x1)}$

$=$ by Lemma 1153 RL with $\{x3 \leftarrow op_t(x1, x2), x2 \leftarrow x3, x1 \leftarrow x1\}$

$\overbrace{\underbrace{mult(i(x1), rd(mult(x1, x3), op_t(x1, x2)))}}$

$=$ by Lemma 1775 RL with $\{x3 \leftarrow mult(x1, x3), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{\underbrace{mult(i(op_t(x1, x2)), rd(mult(x1, x3), x1))}}$

$=$ by Lemma 45 RL with $\{x3 \leftarrow mult(x1, x3), x2 \leftarrow x1, x1 \leftarrow op_t(x1, x2)\}$

$\overbrace{\underbrace{i(mult(op_t(x1, x2), rd(x1, mult(x1, x3))))}}$

$=$ by Lemma 20 RL with $\{x2 \leftarrow mult(op_t(x1, x2), rd(x1, mult(x1, x3))), x1 \leftarrow x1\}$

$\overbrace{mult(x1, \underbrace{i(mult(x1, mult(op_t(x1, x2), rd(x1, mult(x1, x3)))))})}$

$=$ by Lemma 162 RL with $\{x3 \leftarrow rd(x1, mult(x1, x3)), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{mult(x1, i(\overbrace{mult(mult(x1, op_t(x1, x2)), rd(x1, mult(x1, x3)))}))}$

$=$ by Lemma 1399 RL with $\{x3 \leftarrow rd(x1, mult(x1, x3)), x1 \leftarrow op_t(x1, x2), x2 \leftarrow x1\}$

$\overbrace{\underbrace{mult(rd(x1, op_t(x1, x2)), i(mult(x1, rd(x1, mult(x1, x3)))))}}$

$=$ by Lemma 1717 LR with $\{x3 \leftarrow i(mult(x1, rd(x1, mult(x1, x3)))), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{\underbrace{rd(i(mult(x1, rd(x1, mult(x1, x3)))), rd(op_t(x1, x2), x1))}}$

$=$ by Lemma 40 LR with $\{x3 \leftarrow x1, x2 \leftarrow op_t(x1, x2), x1 \leftarrow mult(x1, rd(x1, mult(x1, x3)))\}$

$\overbrace{\underbrace{rd(rd(x1, op_t(x1, x2)), mult(x1, rd(x1, mult(x1, x3))))}}$

$=$ by Lemma 1236 RL with $\{x3 \leftarrow x2, x2 \leftarrow mult(x1, rd(x1, mult(x1, x3))), x1 \leftarrow x1\}$

$\overbrace{rd(x1, \underbrace{mult(\underbrace{mult(x1, rd(x1, mult(x1, x3)))}, op_t(x1, x2)))}}$

$=$ by Lemma 807 RL with $\{x2 \leftarrow x3, x1 \leftarrow x1\}$

$rd(x1, \underbrace{mult(\overbrace{op_r(rd(x1, x3), x1, x1)}, op_t(x1, x2)))}$

$=$ by Lemma 1636 LR with $\{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow rd(x1, x3)\}$

$rd(x1, \overbrace{rd(op_t(x1, x2), i(rd(x1, x3))))})$

Lemma 2059: $mult(rd(x3, mult(x1, op_t(x1, x2))), x1) = rd(op_r(x3, x1, x1), op_t(x1, x2))$

$$
\begin{aligned}
& \underbrace{mult(rd(x3, mult(x1, op_t(x1, x2))), x1)} \\
= \quad & \text{by Lemma 2058 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1, x3 \leftarrow x3\} \\
& \underbrace{\overbrace{rd(x1, rd(op_t(x1, x2), i(rd(x1, x3))))}} \\
= \quad & \text{by Lemma 1651 LR with } \{x3 \leftarrow rd(x1, x3), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{rd(\underbrace{rd(x1, rd(x1, x3))}, op_t(x1, x2))} \\
= \quad & \text{by Lemma 353 LR with } \{x2 \leftarrow x3, x1 \leftarrow x1\} \\
& rd(\overbrace{op_r(x3, x1, x1)}, op_t(x1, x2))
\end{aligned}
$$

Lemma 2060: $rd(x1, op_l(x2, x3, x1)) = op_l(rd(op_l(x1, x1, x3), x2), x3, x1)$

$$
\begin{aligned}
& rd(x1, op_l(\underbrace{x2}, x3, x1)) \\
= \quad & \text{by Lemma 3 RL with } \{x1 \leftarrow x2\} \\
& rd(x1, \underbrace{op_l(\overbrace{i(i(x2))}, x3, x1)}) \\
= \quad & \text{by Lemma 1049 RL with } \{x3 \leftarrow x1, x2 \leftarrow x3, x1 \leftarrow i(x2)\} \\
& \underbrace{rd(x1, \overbrace{i(op_l(i(x2), x3, x1))})} \\
= \quad & \text{by Lemma 357 RL with } \{x2 \leftarrow x1, x1 \leftarrow op_l(i(x2), x3, x1)\} \\
& \overbrace{mult(\underbrace{op_r(op_l(i(x2), x3, x1), x1, x1)}, x1)} \\
= \quad & \text{by Axiom 18 LR with } \{x5 \leftarrow x1, x4 \leftarrow x1, x3 \leftarrow x1, x2 \leftarrow x3, x1 \leftarrow i(x2)\} \\
& mult(\overbrace{op_l(\underbrace{op_r(i(x2), x1, x1)}, x3, x1)}, x1) \\
= \quad & \text{by Lemma 354 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{mult(op_l(\overbrace{rd(rd(x1, x2), x1)}, x3, x1), x1)} \\
= \quad & \text{by Lemma 1779 RL with } \{x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow rd(rd(x1, x2), x1)\} \\
& \overbrace{op_l(\underbrace{mult(rd(rd(x1, x2), x1), op_l(x1, x1, x3))}, x3, x1)} \\
= \quad & \text{by Lemma 1586 RL with } \{x2 \leftarrow x1, x3 \leftarrow x3, x1 \leftarrow rd(rd(x1, x2), x1)\} \\
& op_l(\overbrace{\underbrace{mult(rd(rd(rd(x1, x2), x1), asoc(x3, x1, x1)), x1)}}, x3, x1) \\
= \quad & \text{by Lemma 1754 LR with } \{x1 \leftarrow x1, x2 \leftarrow x3, x3 \leftarrow rd(rd(x1, x2), x1)\} \\
& op_l(\overbrace{mult(asoc(x1, x3, x1), \underbrace{mult(rd(rd(x1, x2), x1), x1)})}, x3, x1) \\
= \quad & \text{by Lemma 1714 LR with } \{x3 \leftarrow mult(rd(rd(x1, x2), x1), x1), x2 \leftarrow x3, x1 \leftarrow x1\} \\
& op_l(\overbrace{rd(\underbrace{mult(rd(rd(x1, x2), x1), x1)}, asoc(x3, x1, x1))}, x3, x1) \\
= \quad & \text{by Axiom 4 RL with } \{x2 \leftarrow x1, x1 \leftarrow rd(x1, x2)\} \\
& op_l(\underbrace{rd(\overbrace{rd(x1, x2)}, asoc(x3, x1, x1))}, x3, x1) \\
= \quad & \text{by Lemma 1580 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_l(\overbrace{rd(op_l(x1, x1, x3), x2)}, x3, x1)
\end{aligned}
$$

Lemma 2061: $rd(mult(x1,x2),op_l(x3,x2,x1)) = op_l(rd(op_l(mult(x1,x2),x1,x2),x3),x2,mult(x1,x2))$

$$
\begin{aligned}
& \underbrace{rd(mult(x1,x2),op_l(x3,x2,x1))} \\
= & \quad \text{by Lemma 291 RL with } \{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x3\} \\
& \underbrace{rd(mult(x1,x2),\overbrace{op_l(x3,x2,mult(x1,x2))})} \\
= & \quad \text{by Lemma 2060 LR with } \{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow mult(x1,x2)\} \\
& \overbrace{op_l(rd(\underbrace{op_l(mult(x1,x2),mult(x1,x2),x2)},x3),x2,mult(x1,x2))} \\
= & \quad \text{by Lemma 285 LR with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow mult(x1,x2)\} \\
& op_l(rd(\overbrace{op_l(mult(x1,x2),x1,x2)},x3),x2,mult(x1,x2))
\end{aligned}
$$

Lemma 2062: $rd(mult(x1,x2),op_l(x3,x2,x1)) = op_l(rd(mult(x2,x1),x3),x2,x1)$

$$
\begin{aligned}
& \underbrace{rd(mult(x1,x2),op_l(x3,x2,x1))} \\
= & \quad \text{by Lemma 2061 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{\overbrace{op_l(rd(op_l(mult(x1,x2),x1,x2),x3),x2,mult(x1,x2))}} \\
= & \quad \text{by Lemma 291 LR with } \{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow rd(op_l(mult(x1,x2),x1,x2),x3)\} \\
& \overbrace{op_l(rd(\underbrace{op_l(mult(x1,x2),x1,x2)},x3),x2,x1)} \\
= & \quad \text{by Lemma 435 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_l(rd(\overbrace{mult(x2,x1)},x3),x2,x1)
\end{aligned}
$$

Lemma 2063: $rd(op_l(x1,x2,x3),mult(x3,x2)) = op_l(i(rd(mult(x2,x3),x1)),x2,x3)$

$$
\begin{aligned}
& \underbrace{rd(op_l(x1,x2,x3),mult(x3,x2))} \\
= & \quad \text{by Lemma 10 RL with } \{x2 \leftarrow op_l(x1,x2,x3), x1 \leftarrow mult(x3,x2)\} \\
& \overbrace{i(\underbrace{rd(mult(x3,x2),op_l(x1,x2,x3))})} \\
= & \quad \text{by Lemma 2062 LR with } \{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x3\} \\
& \underbrace{i(\overbrace{op_l(rd(mult(x2,x3),x1),x2,x3)})} \\
= & \quad \text{by Lemma 1049 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow rd(mult(x2,x3),x1)\} \\
& \overbrace{op_l(i(rd(mult(x2,x3),x1)),x2,x3)}
\end{aligned}
$$

Lemma 2064: $rd(op_l(x1,x2,x3),mult(x3,x2)) = op_l(rd(x1,mult(x2,x3)),x2,x3)$

$$
\begin{aligned}
& \underbrace{rd(op_l(x1,x2,x3),mult(x3,x2))} \\
= & \quad \text{by Lemma 2063 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(\underbrace{i(rd(mult(x2,x3),x1))},x2,x3)} \\
= & \quad \text{by Lemma 10 LR with } \{x2 \leftarrow x1, x1 \leftarrow mult(x2,x3)\} \\
& op_l(\overbrace{rd(x1,mult(x2,x3))},x2,x3)
\end{aligned}
$$

Lemma 2065: $op_t(x1, rd(op_l(x2, x3, x1), x1)) = op_t(x1, op_l(rd(x2, mult(x3, mult(i(x3), x1))), x3, mult(i(x3), x1)))$

$op_t(x1, rd(\underbrace{op_l(x2, x3, x1)}, x1))$

$=$ by Lemma 2036 RL with $\{x3 \leftarrow x1, x2 \leftarrow x3, x1 \leftarrow x2\}$

$\underbrace{op_t(x1, rd(\overbrace{op_l(x2, x3, mult(i(x3), x1))}, x1))}$

$=$ by Lemma 1921 LR with $\{x3 \leftarrow x3, x2 \leftarrow op_l(x2, x3, mult(i(x3), x1)), x1 \leftarrow x1\}$

$\overbrace{op_t(x1, \underbrace{rd(op_l(x2, x3, mult(i(x3), x1)), mult(mult(i(x3), x1), x3))})}$

$=$ by Lemma 2064 LR with $\{x3 \leftarrow mult(i(x3), x1), x2 \leftarrow x3, x1 \leftarrow x2\}$

$op_t(x1, \overbrace{op_l(rd(x2, \underbrace{mult(x3, mult(i(x3), x1))}), x3, mult(i(x3), x1))})$

Lemma 2066: $op_t(x1, rd(op_l(x2, x3, x1), x1)) = op_t(x1, op_l(rd(x2, x1), x3, x1))$

$\underbrace{op_t(x1, rd(op_l(x2, x3, x1), x1))}$

$=$ by Lemma 2065 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_t(x1, \underbrace{op_l(rd(x2, mult(x3, mult(i(x3), x1))), x3, mult(i(x3), x1))})}$

$=$ by Lemma 2036 LR with $\{x3 \leftarrow x1, x2 \leftarrow x3, x1 \leftarrow rd(x2, mult(x3, mult(i(x3), x1)))\}$

$op_t(x1, \overbrace{op_l(rd(x2, \underbrace{mult(x3, mult(i(x3), x1))}), x3, x1)})$

$=$ by Lemma 12 LR with $\{x2 \leftarrow x1, x1 \leftarrow x3\}$

$op_t(x1, op_l(rd(x2, \overbrace{x1}), x3, x1))$

Lemma 2067: $rd(op_l(x1, x2, x1), op_l(x3, x2, x1)) = op_l(rd(x1, x3), x2, x1)$

$rd(op_l(x1, x2, x1), \underbrace{op_l(x3, x2, x1)})$

$=$ by Lemma 1083 RL with $\{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x3\}$

$\underbrace{rd(op_l(x1, x2, x1), \overbrace{op_l(x3, mult(x1, x2), op_l(x1, x2, x1))})}$

$=$ by Lemma 2060 LR with $\{x3 \leftarrow mult(x1, x2), x2 \leftarrow x3, x1 \leftarrow op_l(x1, x2, x1)\}$

$\overbrace{op_l(rd(\underbrace{op_l(op_l(x1, x2, x1), op_l(x1, x2, x1), mult(x1, x2))}, x3), mult(x1, x2), op_l(x1, x2, x1))}$

$=$ by Lemma 1077 RL with $\{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow op_l(x1, x2, x1)\}$

$\underbrace{op_l(rd(\overbrace{op_l(op_l(x1, x2, x1), x1, x2)}, x3), mult(x1, x2), op_l(x1, x2, x1))}$

$=$ by Lemma 1083 LR with $\{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow rd(op_l(op_l(x1, x2, x1), x1, x2), x3)\}$

$\overbrace{op_l(rd(\underbrace{op_l(op_l(x1, x2, x1), x1, x2)}, x3), x2, x1)}$

$=$ by Lemma 138 LR with $\{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x1\}$

$op_l(rd(\overbrace{x1}, x3), x2, x1)$

Lemma 2068: $op_l(rd(x1, mult(x2, unit())), x2, x3) = rd(op_l(x1, mult(x2, i(x3)), op_l(x3, x2, x3)), op_l(x2, x2, x3))$

$op_l(rd(x1, mult(x2, \underbrace{unit()}), x2, x3)$

$=$ by Axiom 9 RL with $\{x1 \leftarrow x3\}$

$op_l(rd(x1, mult(\underbrace{x2}, \overbrace{mult(i(x3), x3)})), x2, x3)$

$=$ by Lemma 3 RL with $\{x1 \leftarrow x2\}$

$op_l(rd(x1, mult(\overbrace{i(i(x2))}, \underbrace{mult(i(x3), x3)})), x2, x3)$

$=$ by Lemma 17 RL with $\{x2 \leftarrow x3, x1 \leftarrow x3\}$

$op_l(rd(x1, \underbrace{mult(i(i(x2)), \overbrace{i(mult(x3, i(x3)))})}), x2, x3)$

$=$ by Axiom 11 LR with $\{x2 \leftarrow mult(x3, i(x3)), x1 \leftarrow i(x2)\}$

$op_l(rd(x1, \overbrace{i(\underbrace{mult(i(x2), mult(x3, i(x3)))})}), x2, x3)$

$=$ by Lemma 132 RL with $\{x3 \leftarrow i(x3), x2 \leftarrow x3, x1 \leftarrow i(x2)\}$

$op_l(rd(x1, \underbrace{i(\overbrace{mult(mult(i(x2), x3), op_l(i(x3), x3, i(x2)))})}), x2, x3)$

$=$ by Lemma 1133 LR with $\{x4 \leftarrow i(x2), x3 \leftarrow x3, x2 \leftarrow x3, x1 \leftarrow mult(i(x2), x3)\}$

$op_l(rd(x1, \overbrace{mult(i(mult(i(x2), x3)), \underbrace{op_l(x3, x3, i(x2))})}), x2, x3)$

$=$ by Lemma 2030 LR with $\{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x3\}$

$op_l(rd(x1, mult(\underbrace{i(mult(i(x2), x3))}, \overbrace{op_l(x3, x2, x3)})), x2, x3)$

$=$ by Lemma 16 LR with $\{x2 \leftarrow x3, x1 \leftarrow x2\}$

$\underbrace{op_l(rd(x1, mult(\overbrace{mult(x2, i(x3))}, op_l(x3, x2, x3))), x2, x3)}$

$=$ by Lemma 2053 RL with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow rd(x1, mult(mult(x2, i(x3)), op_l(x3, x2, x3)))\}$

$\overbrace{\underbrace{op_l(rd(x1, mult(mult(x2, i(x3)), op_l(x3, x2, x3))), mult(x2, i(x3)), op_l(x3, x2, x3))}}$

$=$ by Lemma 2064 RL with $\{x3 \leftarrow op_l(x3, x2, x3), x2 \leftarrow mult(x2, i(x3)), x1 \leftarrow x1\}$

$\overbrace{rd(op_l(x1, mult(x2, i(x3)), op_l(x3, x2, x3)), \underbrace{mult(op_l(x3, x2, x3), mult(x2, i(x3)))})}$

$=$ by Lemma 979 LR with $\{x2 \leftarrow x2, x1 \leftarrow x3\}$

$rd(op_l(x1, mult(x2, i(x3)), op_l(x3, x2, x3)), \overbrace{op_l(x2, x2, x3)})$

Lemma 2069: $op_l(x1, rd(x2, op_t(i(x2), x3)), x3) = op_l(x1, op_t(mult(x2, mult(x3, x2)), i(x3)), op_t(x3, mult(rd(x2, op_t(i(x2), x3)),$

$\underbrace{op_l(x1, rd(x2, op_t(i(x2), x3)), x3)}$

$=$ by Lemma 1794 LR with $\{x3 \leftarrow x3, x2 \leftarrow rd(x2, op_t(i(x2), x3)), x1 \leftarrow x1\}$

$\overbrace{op_l(x1, \underbrace{op_l(mult(rd(x2, op_t(i(x2), x3)), x3), rd(x2, \overbrace{op_t(i(x2), x3)}), x3)}, op_t(x3, mult(rd(x2, op_t(i(x2), x3)), x3)))}$

$=$ by Lemma 435 LR with $\{x2 \leftarrow x3, x1 \leftarrow rd(x2, op_t(i(x2), x3))\}$

$op_l(x1, \overbrace{\underbrace{mult(x3, rd(x2, op_t(i(x2), x3)))}}, op_t(x3, mult(rd(x2, op_t(i(x2), x3)), x3)))$

$=$ by Lemma 1402 LR with $\{x2 \leftarrow x2, x1 \leftarrow x3\}$

$op_l(x1, \overbrace{op_t(mult(x2, mult(x3, x2)), i(x3))}, op_t(x3, mult(rd(x2, op_t(i(x2), x3)), x3)))$

Lemma 2070: $op_l(x1, rd(x2, op_t(i(x2), x3)), x3) = op_l(x1, op_t(mult(x2, mult(x3, x2)), i(x3)), op_t(x3, mult(x2, mult(x3, x2))))$

$\underbrace{op_l(x1, rd(x2, op_t(i(x2), x3)), x3)}$

$=$ by Lemma 2069 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_l(x1, op_t(mult(x2, mult(x3, x2)), i(x3)), \underbrace{op_t(x3, mult(rd(x2, op_t(i(x2), x3)), x3))})}$

$=$ by Lemma 1752 LR with $\{x4 \leftarrow x3, x3 \leftarrow op_t(i(x2), x3), x2 \leftarrow x2, x1 \leftarrow x3\}$

$op_l(x1, op_t(mult(x2, mult(x3, x2)), i(x3)), \overbrace{op_t(x3, \underbrace{rd(x3, rd(op_t(i(x2), x3), x2))})})$

$=$ by Lemma 140 LR with $\{x2 \leftarrow x2, x1 \leftarrow x3\}$

$op_l(x1, op_t(mult(x2, mult(x3, x2)), i(x3)), op_t(x3, \overbrace{mult(x2, mult(x3, x2))}))$

Lemma 2071: $op_l(x1, op_t(x2, x3), mult(x3, x3)) = op_l(x1, mult(mult(x3, x3), rd(mult(i(x3), x2), x3)), op_t(mult(x3, x3), op_t(x2, x$

$\underbrace{op_l(x1, op_t(x2, x3), mult(x3, x3))}$

$=$ by Lemma 1359 RL with $\{x3 \leftarrow mult(x3, x3), x2 \leftarrow op_t(x2, x3), x1 \leftarrow x1\}$

$\overbrace{op_l(x1, \underbrace{op_l(op_t(x2, x3), op_t(x2, x3), mult(x3, x3))}, op_t(mult(x3, x3), op_t(x2, x3)))}$

$=$ by Lemma 427 LR with $\{x2 \leftarrow mult(x3, x3), x1 \leftarrow op_t(x2, x3)\}$

$op_l(x1, \overbrace{mult(mult(x3, x3), \underbrace{rd(op_t(x2, x3), mult(x3, x3))})}, op_t(mult(x3, x3), op_t(x2, x3)))$

$=$ by Lemma 205 LR with $\{x2 \leftarrow x3, x1 \leftarrow x2\}$

$op_l(x1, mult(mult(x3, x3), \overbrace{rd(mult(i(x3), x2), x3)}), op_t(mult(x3, x3), op_t(x2, x3)))$

Lemma 2072: $op_l(x1, op_t(x2, x3), mult(x3, x3)) = op_l(x1, op_t(mult(x3, mult(i(x3), x2)), i(x3)), op_t(mult(x3, x3), op_t(x2, x3)))$

$\underbrace{op_l(x1, op_t(x2, x3), mult(x3, x3))}$

$=$ by Lemma 2071 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_l(x1, \underbrace{mult(mult(x3, x3), rd(mult(i(x3), x2), x3))}, op_t(mult(x3, x3), op_t(x2, x3)))}$

$=$ by Lemma 24 LR with $\{x2 \leftarrow rd(mult(i(x3), x2), x3), x1 \leftarrow x3\}$

$op_l(x1, \overbrace{\underbrace{mult(x3, mult(x3, rd(mult(i(x3), x2), x3)))}, op_t(mult(x3, x3), op_t(x2, x3)))}$

$=$ by Lemma 70 LR with $\{x2 \leftarrow mult(i(x3), x2), x1 \leftarrow x3\}$

$op_l(x1, \overbrace{op_t(mult(x3, mult(i(x3), x2)), i(x3))}, op_t(mult(x3, x3), op_t(x2, x3)))$

Lemma 2073: $mult(mult(op_l(x1, x2, x3), x4), i(x1)) = mult(op_l(x1, x2, x3), rd(x4, x1))$

$\underbrace{mult(mult(op_l(x1, x2, x3), x4), i(x1))}$

$=$ by Lemma 137 RL with $\{x3 \leftarrow i(x1), x2 \leftarrow x4, x1 \leftarrow op_l(x1, x2, x3)\}$

$\overbrace{mult(op_l(x1, x2, x3), mult(x4, \underbrace{op_l(i(x1), op_l(x1, x2, x3), x4)}))}$

$=$ by Lemma 1821 LR with $\{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$mult(op_l(x1, x2, x3), \underbrace{mult(x4, \overbrace{op_l(i(x1), x1, x4)})})$

$=$ by Lemma 232 LR with $\{x2 \leftarrow x1, x1 \leftarrow x4\}$

$mult(op_l(x1, x2, x3), \overbrace{rd(x4, x1)})$

Lemma 2074: $mult(op_r(x1,x2,x2),op_l(x2,x3,x4)) = mult(x1,op_l(op_l(x2,x1,x2),x3,x4))$

$\underbrace{mult(op_r(x1,x2,x2),op_l(x2,x3,x4))}$

$=$ by Lemma 1719 RL with $\{x3 \leftarrow x2, x2 \leftarrow op_l(x2,x3,x4), x1 \leftarrow x1\}$

$\overbrace{\underbrace{rd(mult(x1,op_l(x2,x3,x4)),asoc(x2,x1,x2))}}$

$=$ by Lemma 1300 LR with $\{x2 \leftarrow x1, x3 \leftarrow x2, x1 \leftarrow mult(x1,op_l(x2,x3,x4))\}$

$\overbrace{mult(mult(x1,op_l(x2,x3,x4)),\underbrace{asoc(x1,x2,x2)})}$

$=$ by Lemma 1646 RL with $\{x3 \leftarrow op_l(x2,x3,x4), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{mult(mult(x1,op_l(x2,x3,x4)),\overbrace{op_l(asoc(x1,x2,x2),op_l(x2,x3,x4),x1)})}$

$=$ by Lemma 132 LR with $\{x3 \leftarrow asoc(x1,x2,x2), x2 \leftarrow op_l(x2,x3,x4), x1 \leftarrow x1\}$

$\overbrace{mult(x1,\underbrace{mult(op_l(x2,x3,x4),asoc(x1,x2,x2))})}$

$=$ by Lemma 1246 RL with $\{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow op_l(x2,x3,x4)\}$

$mult(x1,\overbrace{rd(mult(op_l(x2,x3,x4),\underbrace{op_r(x1,x2,x2)}),x1)})$

$=$ by Lemma 358 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$mult(x1,rd(\underbrace{mult(op_l(x2,x3,x4),\overbrace{rd(rd(x1,i(x2)),x2)})},x1))$

$=$ by Lemma 2073 RL with $\{x4 \leftarrow rd(x1,i(x2)), x3 \leftarrow x4, x2 \leftarrow x3, x1 \leftarrow x2\}$

$mult(x1,\underbrace{rd(\overbrace{mult(mult(op_l(x2,x3,x4),rd(x1,i(x2))),i(x2))},x1)})$

$=$ by Lemma 693 LR with $\{x3 \leftarrow i(x2), x2 \leftarrow x1, x1 \leftarrow op_l(x2,x3,x4)\}$

$mult(x1,\overbrace{\underbrace{op_r(op_l(x2,x3,x4),rd(x1,i(x2)),i(x2))}})$

$=$ by Axiom 18 LR with $\{x5 \leftarrow i(x2), x4 \leftarrow rd(x1,i(x2)), x3 \leftarrow x4, x2 \leftarrow x3, x1 \leftarrow x2\}$

$mult(x1,\overbrace{op_l(\underbrace{op_r(x2,rd(x1,i(x2)),i(x2))},x3,x4)})$

$=$ by Lemma 1895 RL with $\{x3 \leftarrow x1, x2 \leftarrow i(x2), x1 \leftarrow x2\}$

$mult(x1,op_l(\overbrace{op_r(x2,mult(\underbrace{i(i(x2))},x1),i(x2))},x3,x4))$

$=$ by Lemma 3 LR with $\{x1 \leftarrow x2\}$

$mult(x1,op_l(\underbrace{op_r(x2,mult(\overbrace{x2},x1),i(x2))},x3,x4))$

$=$ by Lemma 658 LR with $\{x2 \leftarrow mult(x2,x1), x1 \leftarrow x2\}$

$mult(x1,op_l(\underbrace{\overbrace{op_l(x2,mult(x2,x1),x2)}},x3,x4))$

$=$ by Lemma 2033 LR with $\{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x2\}$

$mult(x1,op_l(\overbrace{op_l(x2,x1,x2)},x3,x4))$

Lemma 2075: $mult(op_r(x1,x2,x2),op_l(x2,x3,x4)) = rd(x1,op_l(i(x2),x3,x4))$

$\underbrace{mult(op_r(x1,x2,x2),op_l(x2,x3,x4))}$

$=$ by Lemma 2074 LR with $\{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{\underbrace{mult(x1,op_l(op_l(x2,x1,x2),x3,x4))}}$

$=$ by Lemma 1975 LR with $\{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{rd(x1,op_l(i(x2),x3,x4))}$

Lemma 2076: $op_r(op_t(x1, rd(op_l(x2,x3,x4),x1)),x2,x2) = rd(mult(x1, rd(op_r(x1,x2,x2), op_l(i(x2),x3,x4))), rd(op_r(x1,x2,x2$

$op_r(\underbrace{op_t(x1, rd(op_l(x2,x3,x4),x1))}, x2, x2)$

$=$ by Lemma 1244 RL with $\{x3 \leftarrow x2, x2 \leftarrow op_l(x2,x3,x4), x1 \leftarrow x1\}$

$\underbrace{op_r(\overbrace{op_t(x1, rd(op_l(x2,x3,x4), op_r(x1,x2,x2)))}, x2, x2)}$

$=$ by Axiom 14 RL with $\{x4 \leftarrow rd(op_l(x2,x3,x4), op_r(x1,x2,x2)), x3 \leftarrow x2, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_t(op_r(x1,x2,x2), rd(op_l(x2,x3,x4), op_r(x1,x2,x2)))}}$

$=$ by Lemma 1925 LR with $\{x2 \leftarrow op_l(x2,x3,x4), x1 \leftarrow op_r(x1,x2,x2)\}$

$\overbrace{rd(mult(op_r(x1,x2,x2), \underbrace{mult(op_r(x1,x2,x2), op_l(x2,x3,x4))}), rd(op_r(x1,x2,x2), i(op_l(x2,x3,x4))))}$

$=$ by Lemma 2075 LR with $\{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$rd(\underbrace{mult(op_r(x1,x2,x2), \overbrace{rd(x1, op_l(i(x2),x3,x4))})}, rd(op_r(x1,x2,x2), i(op_l(x2,x3,x4))))$

$=$ by Lemma 1719 RL with $\{x3 \leftarrow x2, x2 \leftarrow rd(x1, op_l(i(x2),x3,x4)), x1 \leftarrow x1\}$

$rd(\overbrace{\underbrace{rd(mult(x1, rd(x1, op_l(i(x2),x3,x4)))}, asoc(x2,x1,x2))}, rd(op_r(x1,x2,x2), i(op_l(x2,x3,x4))))$

$=$ by Lemma 1300 LR with $\{x2 \leftarrow x1, x3 \leftarrow x2, x1 \leftarrow mult(x1, rd(x1, op_l(i(x2),x3,x4)))\}$

$rd(\overbrace{mult(mult(x1, rd(x1, op_l(i(x2),x3,x4))), \underbrace{asoc(x1,x2,x2)})}, rd(op_r(x1,x2,x2), i(op_l(x2,x3,x4))))$

$=$ by Lemma 1646 RL with $\{x3 \leftarrow rd(x1, op_l(i(x2),x3,x4)), x2 \leftarrow x2, x1 \leftarrow x1\}$

$rd(mult(mult(x1, rd(x1, op_l(i(x2),x3,x4))), \overbrace{op_l(\underbrace{asoc(x1,x2,x2)}, rd(x1, op_l(i(x2),x3,x4)), x1)}), rd(op_r(x1,x2,x2), i(op_l(x$

$=$ by Lemma 132 LR with $\{x3 \leftarrow asoc(x1,x2,x2), x2 \leftarrow rd(x1, op_l(i(x2),x3,x4)), x1 \leftarrow x1\}$

$rd(\overbrace{mult(x1, \underbrace{mult(rd(x1, op_l(i(x2),x3,x4)), asoc(x1,x2,x2))})}, rd(op_r(x1,x2,x2), i(op_l(x2,x3,x4))))$

$=$ by Lemma 1300 RL with $\{x2 \leftarrow x1, x3 \leftarrow x2, x1 \leftarrow rd(x1, op_l(i(x2),x3,x4))\}$

$rd(mult(x1, \overbrace{\underbrace{rd(rd(x1, op_l(i(x2),x3,x4)), asoc(x2,x1,x2))}}), rd(op_r(x1,x2,x2), i(op_l(x2,x3,x4))))$

$=$ by Lemma 1248 LR with $\{x3 \leftarrow x2, x2 \leftarrow op_l(i(x2),x3,x4), x1 \leftarrow x1\}$

$rd(mult(x1, \overbrace{rd(op_r(x1,x2,x2), op_l(i(x2),x3,x4))}), rd(op_r(x1,x2,x2), \underbrace{i(op_l(x2,x3,x4))}))$

$=$ by Lemma 1049 LR with $\{x3 \leftarrow x4, x2 \leftarrow x3, x1 \leftarrow x2\}$

$rd(mult(x1, rd(op_r(x1,x2,x2), op_l(i(x2),x3,x4))), rd(op_r(x1,x2,x2), \overbrace{op_l(i(x2),x3,x4)}))$

Lemma 2077: $op_t(x1, rd(mult(x2, x3), x1)) = op_t(x1, rd(x2, rd(x1, x3)))$

$op_t(x1, rd(mult(x2, x3), x1))$

$=$ by Lemma 1963 LR with $\{x3 \leftarrow rd(op_l(mult(x2, x3), x2, rd(x3, x1)), x1), x2 \leftarrow mult(x2, x3), x1 \leftarrow x1\}$

$op_r(x1, rd(mult(x2, x3), op_t(x1, rd(op_l(mult(x2, x3), x2, rd(x3, x1)), x1))), i(mult(x2, x3)))$

$=$ by Axiom 5 LR with $\{x2 \leftarrow rd(op_r(x1, mult(x2, x3), mult(x2, x3)), op_l(i(mult(x2, x3)), x2, rd(x3, x1))), x1 \leftarrow x1\}$

$op_r(rd(mult(x1, rd(op_r(x1, mult(x2, x3), mult(x2, x3)), op_l(i(mult(x2, x3)), x2, rd(x3, x1)))), rd(op_r(x1, mult(x2, x3), mu$

$=$ by Lemma 2076 RL with $\{x4 \leftarrow rd(x3, x1), x3 \leftarrow x2, x2 \leftarrow mult(x2, x3), x1 \leftarrow x1\}$

$op_r(op_r(op_t(x1, rd(op_l(mult(x2, x3), x2, rd(x3, x1)), x1)), mult(x2, x3), mult(x2, x3)), rd(mult(x2, x3), op_t(x1, rd(op_l(mul$

$=$ by Lemma 892 RL with $\{x1 \leftarrow mult(x2, x3), x2 \leftarrow op_t(x1, rd(op_l(mult(x2, x3), x2, rd(x3, x1)), x1))\}$

$mult(rd(op_t(x1, rd(op_l(mult(x2, x3), x2, rd(x3, x1)), x1)), mult(x2, x3)), mult(x2, x3))$

$=$ by Axiom 4 RL with $\{x2 \leftarrow mult(x2, x3), x1 \leftarrow op_t(x1, rd(op_l(mult(x2, x3), x2, rd(x3, x1)), x1))\}$

$op_t(x1, rd(op_l(mult(x2, x3), x2, rd(x3, x1)), x1))$

$=$ by Lemma 1074 RL with $\{x4 \leftarrow rd(x3, x1), x3 \leftarrow x2, x2 \leftarrow mult(x2, x3), x1 \leftarrow x1\}$

$op_t(x1, rd(i(x1), op_l(i(mult(x2, x3)), x2, rd(x3, x1))))$

$=$ by Axiom 11 RL with $\{x2 \leftarrow x3, x1 \leftarrow x2\}$

$op_t(x1, rd(i(x1), op_l(mult(i(x2), i(x3)), x2, rd(x3, x1)))))$

$=$ by Lemma 1981 RL with $\{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x3\}$

$op_t(x1, mult(rd(x3, x1), x2))$

$=$ by Lemma 1752 LR with $\{x4 \leftarrow x2, x3 \leftarrow x1, x2 \leftarrow x3, x1 \leftarrow x1\}$

$op_t(x1, rd(x2, rd(x1, x3)))$

Lemma 2078: $op_t(x1, rd(x2, mult(x1, x3))) = op_t(x1, rd(rd(x3, x2), x1))$

$op_t(x1, rd(x2, mult(x1, x3)))$

$=$ by Lemma 1219 RL with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$op_t(x1, rd(x2, rd(x1, i(x3))))$

$=$ by Lemma 2077 RL with $\{x3 \leftarrow i(x3), x2 \leftarrow x2, x1 \leftarrow x1\}$

$op_t(x1, rd(mult(x2, i(x3)), x1))$

$=$ by Lemma 1640 RL with $\{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x1\}$

$op_t(x1, rd(rd(x3, x2), x1))$

Lemma 2079: $op_t(x1, rd(rd(x2, x1), x3)) = op_t(x1, rd(x2, mult(x1, x3)))$

$\overbrace{op_t(x1, rd(rd(x2, x1), x3))}$

$=$ by Lemma 1215 RL with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_t(x1, \underbrace{rd(mult(i(x1), x2), x3)})}$

$=$ by Lemma 10 RL with $\{x2 \leftarrow mult(i(x1), x2), x1 \leftarrow x3\}$

$\underbrace{op_t(x1, \overbrace{i(rd(x3, mult(i(x1), x2)))})}$

$=$ by Lemma 18 RL with $\{x2 \leftarrow rd(x3, mult(i(x1), x2)), x1 \leftarrow x1\}$

$\overbrace{i(\underbrace{op_t(i(x1), rd(x3, mult(i(x1), x2)))})}$

$=$ by Lemma 2078 LR with $\{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow i(x1)\}$

$\underbrace{i(\overbrace{op_t(i(x1), rd(rd(x2, x3), i(x1)))})}$

$=$ by Lemma 18 LR with $\{x2 \leftarrow rd(rd(x2, x3), i(x1)), x1 \leftarrow x1\}$

$\overbrace{op_t(x1, \underbrace{i(rd(rd(x2, x3), i(x1)))})}$

$=$ by Lemma 10 LR with $\{x2 \leftarrow i(x1), x1 \leftarrow rd(x2, x3)\}$

$op_t(x1, \overbrace{\underbrace{rd(i(x1), rd(x2, x3))}})$

$=$ by Lemma 40 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{op_t(x1, \overbrace{rd(rd(x3, x2), x1)})}$

$=$ by Lemma 2078 RL with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_t(x1, rd(x2, mult(x1, x3)))}$

Lemma 2080: $op_t(x1, op_l(x2, x3, x1)) = op_t(x1, x2)$

$op_t(x1, op_l(\underbrace{x2}, x3, x1))$

$=$ by Axiom 5 LR with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$\underbrace{op_t(x1, op_l(\overbrace{rd(mult(x2, x1), x1)}, x3, x1))}$

$=$ by Lemma 2066 RL with $\{x3 \leftarrow x3, x2 \leftarrow mult(x2, x1), x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_t(x1, rd(op_l(mult(x2, x1), x3, x1), x1))}}$

$=$ by Axiom 4 LR with $\{x2 \leftarrow mult(x2, x1), x1 \leftarrow op_t(x1, rd(op_l(mult(x2, x1), x3, x1), x1))\}$

$\overbrace{\underbrace{mult(rd(op_t(x1, rd(op_l(mult(x2, x1), x3, x1), x1)), mult(x2, x1)), mult(x2, x1))}}$

$=$ by Lemma 892 LR with $\{x1 \leftarrow mult(x2, x1), x2 \leftarrow op_t(x1, rd(op_l(mult(x2, x1), x3, x1), x1))\}$

$\overbrace{op_r(\underbrace{op_r(op_t(x1, rd(op_l(mult(x2, x1), x3, x1), x1)), mult(x2, x1), mult(x2, x1))}, rd(mult(x2, x1), op_t(x1, rd(op_l(mult(x2, x1$

$=$ by Lemma 2076 LR with $\{x4 \leftarrow x1, x3 \leftarrow x3, x2 \leftarrow mult(x2, x1), x1 \leftarrow x1\}$

$op_r(\overbrace{rd(mult(x1, rd(op_r(x1, mult(x2, x1), mult(x2, x1)), op_l(i(mult(x2, x1)), x3, x1))), rd(op_r(x1, mult(x2, x1), mult(x2, x1$

$=$ by Axiom 5 RL with $\{x2 \leftarrow rd(op_r(x1, mult(x2, x1), mult(x2, x1)), op_l(i(mult(x2, x1)), x3, x1)), x1 \leftarrow x1\}$

$\underbrace{op_r(\overbrace{x1}, rd(mult(x2, x1), op_t(x1, rd(op_l(mult(x2, x1), x3, x1), x1))), i(mult(x2, x1)))}$

$=$ by Lemma 1963 RL with $\{x3 \leftarrow rd(op_l(mult(x2, x1), x3, x1), x1), x2 \leftarrow mult(x2, x1), x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_t(x1, rd(mult(x2, x1), x1))}}$

$=$ by Lemma 2077 LR with $\{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_t(x1, rd(x2, \underbrace{rd(x1, x1)}))}$

$=$ by Lemma 2 LR with $\{x1 \leftarrow x1\}$

$op_t(x1, \underbrace{rd(x2, \overbrace{unit()})})$

$=$ by Lemma 1 LR with $\{x1 \leftarrow x2\}$

$op_t(x1, \overbrace{x2})$

Lemma 2081: $op_t(x1, mult(x2, mult(x1, x3))) = op_t(x1, mult(x1, mult(x2, x3)))$

$\overbrace{op_t(x1, mult(x2, mult(x1, x3)))}$

$=$ by Lemma 2080 RL with $\{x3 \leftarrow x2, x2 \leftarrow mult(x2, mult(x1, x3)), x1 \leftarrow x1\}$

$\overbrace{op_t(x1, \underbrace{op_l(mult(x2, mult(x1, x3)), x2, x1)})}$

$=$ by Lemma 1047 LR with $\{x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow x2\}$

$\underbrace{op_t(x1, \overbrace{mult(mult(x1, x2), x3)})}$

$=$ by Lemma 1151 RL with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_t(x1, mult(\underbrace{rd(x2, i(x1))}, x3))}$

$=$ by Lemma 38 RL with $\{x1 \leftarrow x2, x2 \leftarrow x1\}$

$op_t(x1, mult(\overbrace{rd(x1, i(x2))}, x3))$

$=$ by Lemma 359 RL with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$\underbrace{op_t(x1, mult(\overbrace{op_r(mult(x2, x1), x1, x1)}, x3))}$

$=$ by Lemma 1148 LR with $\{x3 \leftarrow x3, x2 \leftarrow mult(x2, x1), x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_t(x1, mult(mult(x2, x1), x3))}}$

$=$ by Lemma 2080 RL with $\{x3 \leftarrow x2, x2 \leftarrow mult(mult(x2, x1), x3), x1 \leftarrow x1\}$

$\overbrace{op_t(x1, \underbrace{op_l(mult(mult(x2, x1), x3), x2, x1)})}$

$=$ by Lemma 1068 LR with $\{x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow x2\}$

$op_t(x1, \overbrace{mult(x1, mult(x2, x3))})$

Lemma 2082: $op_t(x1, mult(mult(x1, i(x2)), x3)) = op_t(x1, mult(i(x2), mult(x1, x3)))$

$\underbrace{op_t(x1, mult(mult(x1, i(x2)), x3))}$

$=$ by Lemma 2080 RL with $\{x3 \leftarrow x2, x2 \leftarrow mult(mult(x1, i(x2)), x3), x1 \leftarrow x1\}$

$\overbrace{op_t(x1, \underbrace{op_l(mult(mult(x1, i(x2)), x3), x2, x1)})}$

$=$ by Lemma 2030 RL with $\{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow mult(mult(x1, i(x2)), x3)\}$

$op_t(x1, \overbrace{op_l(mult(mult(x1, i(x2)), x3), x1, i(x2))})$

$=$ by Lemma 1068 LR with $\{x3 \leftarrow x3, x2 \leftarrow i(x2), x1 \leftarrow x1\}$

$op_t(x1, \overbrace{mult(i(x2), mult(x1, x3))})$

Lemma 2083: $op_t(x1, rd(x3, mult(i(x1), x2))) = op_t(x1, mult(i(x2), mult(x1, x3)))$

$\underbrace{op_t(x1, rd(x3, mult(i(x1), x2)))}$

$=$ by Lemma 1728 RL with $\{x3 \leftarrow x3, x2 \leftarrow mult(i(x1), x2), x1 \leftarrow x1\}$

$\overbrace{op_t(x1, mult(\underbrace{i(mult(i(x1), x2))}, x3))}$

$=$ by Lemma 16 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{op_t(x1, mult(\overbrace{mult(x1, i(x2))}, x3))}$

$=$ by Lemma 2082 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_t(x1, mult(i(x2), mult(x1, x3)))}$

Lemma 2084: $op_t(x1, rd(x3, rd(x2, x1))) = op_t(x1, rd(mult(x1, x3), x2))$

$$
\begin{aligned}
& \overbrace{op_t(x1, rd(x3, rd(x2, x1)))} \\
= & \quad \text{by Lemma 1258 RL with } \{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, rd(x3, mult(i(x1), x2)))} \\
= & \quad \text{by Lemma 2083 LR with } \{x2 \leftarrow x2, x3 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, mult(i(x2), mult(x1, x3)))} \\
= & \quad \text{by Lemma 1728 LR with } \{x3 \leftarrow mult(x1, x3), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, rd(mult(x1, x3), x2))}
\end{aligned}
$$

Lemma 2085: $op_t(x1, rd(mult(x2, x1), x3)) = op_t(x1, rd(x2, rd(x3, x1)))$

$$
\begin{aligned}
& \overbrace{op_t(x1, rd(mult(x2, x1), x3))} \\
= & \quad \text{by Lemma 1269 RL with } \{x3 \leftarrow mult(x2, x1), x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, rd(rd(x3, x1), mult(x1, mult(x2, x1))))} \\
= & \quad \text{by Lemma 1883 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, rd(\underbrace{mult(op_t(x2, x1), x1)}, x3))} \\
= & \quad \text{by Axiom 2 LR with } \{x2 \leftarrow x1, x1 \leftarrow mult(op_t(x2, x1), x1)\} \\
& \underbrace{op_t(x1, rd(\overbrace{mult(i(x1), mult(x1, mult(op_t(x2, x1), x1)))}, x3))} \\
= & \quad \text{by Lemma 1215 LR with } \{x3 \leftarrow x3, x2 \leftarrow mult(x1, mult(op_t(x2, x1), x1)), x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, rd(rd(\underbrace{mult(x1, mult(op_t(x2, x1), x1))}, x1), x3))} \\
= & \quad \text{by Lemma 72 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_t(x1, rd(\underbrace{rd(\overbrace{rd(x1, i(mult(x2, x1)))}, x1)}, x3)) \\
= & \quad \text{by Lemma 354 LR with } \{x2 \leftarrow i(mult(x2, x1)), x1 \leftarrow x1\} \\
& op_t(x1, rd(\overbrace{op_r(\underbrace{i(i(mult(x2, x1)))}, x1, x1)}, x3)) \\
= & \quad \text{by Lemma 3 LR with } \{x1 \leftarrow mult(x2, x1)\} \\
& op_t(x1, rd(\underbrace{op_r(\overbrace{mult(x2, x1)}, x1, x1)}, x3)) \\
= & \quad \text{by Lemma 359 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \underbrace{op_t(x1, rd(\overbrace{rd(x1, i(x2))}, x3))} \\
= & \quad \text{by Lemma 1260 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_t(x1, rd(mult(x1, x2), x3))}} \\
= & \quad \text{by Lemma 2084 RL with } \{x2 \leftarrow x3, x3 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, rd(x2, rd(x3, x1)))}
\end{aligned}
$$

Lemma 2086: $op_t(x1, rd(x1, mult(x2, i(x3)))) = op_t(x1, rd(mult(x1, x3), x2))$

$$
\begin{aligned}
& op_t(x1, \underbrace{rd(x1, mult(x2, i(x3)))}) \\
= & \quad \text{by Lemma 63 RL with } \{x3 \leftarrow x1, x2 \leftarrow x3, x1 \leftarrow x2\} \\
& \underbrace{op_t(x1, \overbrace{rd(mult(i(x2), x3), i(x1))})} \\
= & \quad \text{by Lemma 1729 LR with } \{x3 \leftarrow x1, x2 \leftarrow mult(i(x2), x3), x1 \leftarrow x1\} \\
& \underbrace{\overbrace{op_t(x1, mult(x1, mult(i(x2), x3)))}} \\
= & \quad \text{by Lemma 2081 RL with } \{x3 \leftarrow x3, x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& \underbrace{\overbrace{op_t(x1, mult(i(x2), mult(x1, x3)))}} \\
= & \quad \text{by Lemma 1728 LR with } \{x3 \leftarrow mult(x1, x3), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, rd(mult(x1, x3), x2))}
\end{aligned}
$$

Lemma 2087: $op_t(x1, rd(x1, rd(x2, x3))) = op_t(x1, rd(x3, rd(x2, x1)))$

$$
\begin{aligned}
& \underbrace{op_t(x1, rd(x1, rd(x2, x3)))} \\
= & \quad \text{by Lemma 1769 LR with } \{x2 \leftarrow x3, x3 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{\overbrace{op_t(x1, rd(x1, mult(x2, i(x3))))}} \\
= & \quad \text{by Lemma 2086 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{\overbrace{op_t(x1, rd(mult(x1, x3), x2))}} \\
= & \quad \text{by Lemma 2084 RL with } \{x2 \leftarrow x2, x3 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, rd(x3, rd(x2, x1)))}
\end{aligned}
$$

Lemma 2088: $op_t(x1, rd(mult(x2, i(x3)), i(x3))) = op_t(x1, op_r(x2, i(x3), mult(x3, x1)))$

$$
\begin{aligned}
& op_t(x1, rd(mult(x2, i(x3)), \underbrace{i(x3)})) \\
= & \quad \text{by Lemma 9 RL with } \{x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \underbrace{op_t(x1, rd(mult(x2, i(x3)), \overbrace{rd(x1, mult(x3, x1))}))} \\
= & \quad \text{by Lemma 2077 RL with } \{x3 \leftarrow mult(x3, x1), x2 \leftarrow mult(x2, i(x3)), x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, \underbrace{rd(mult(mult(x2, i(x3)), mult(x3, x1)), x1)})} \\
= & \quad \text{by Lemma 665 LR with } \{x3 \leftarrow x1, x2 \leftarrow x3, x1 \leftarrow x2\} \\
& op_t(x1, \overbrace{op_r(x2, i(x3), mult(x3, x1))})
\end{aligned}
$$

Lemma 2089: $op_t(x1, i(i(x2))) = op_t(x1, op_r(x2, i(x3), mult(x3, x1)))$

$$
\begin{aligned}
& op_t(x1, \underbrace{i(i(x2))}) \\
= & \quad \text{by Lemma 9 RL with } \{x2 \leftarrow i(x2), x1 \leftarrow x3\} \\
& op_t(x1, \underbrace{rd(x3, mult(i(x2), x3))}) \\
= & \quad \text{by Lemma 61 RL with } \{x3 \leftarrow x3, x2 \leftarrow x3, x1 \leftarrow x2\} \\
& \underbrace{op_t(x1, \overbrace{rd(mult(x2, i(x3)), i(x3))})} \\
= & \quad \text{by Lemma 2088 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, op_r(x2, i(x3), mult(x3, x1)))}
\end{aligned}
$$

Lemma 2090: $op_t(x1, rd(x1, rd(x2, op_l(x2, x3, x1)))) = x1$

$$
\begin{aligned}
& \underbrace{op_t(x1, rd(x1, rd(x2, op_l(x2, x3, x1))))} \\
= & \quad \text{by Lemma 2087 LR with } \{x3 \leftarrow op_l(x2, x3, x1), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, rd(\underbrace{op_l(x2, x3, x1)}, rd(x2, x1)))} \\
= & \quad \text{by Lemma 1051 RL with } \{x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \underbrace{op_t(x1, rd(\overbrace{op_l(x2, x1, i(mult(x1, x3)))}, rd(x2, x1)))} \\
= & \quad \text{by Lemma 1484 LR with } \{x3 \leftarrow i(mult(x1, x3)), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{x1}
\end{aligned}
$$

Lemma 2091: $op_t(x1, mult(x1, mult(x2, op_l(i(x2), x3, x1)))) = x1$

$$
\begin{aligned}
& \underbrace{op_t(x1, mult(x1, mult(x2, op_l(i(x2), x3, x1))))} \\
= & \quad \text{by Lemma 2081 RL with } \{x3 \leftarrow op_l(i(x2), x3, x1), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_t(x1, mult(x2, mult(x1, op_l(i(x2), x3, x1))))}} \\
= & \quad \text{by Lemma 1729 RL with } \{x3 \leftarrow x2, x2 \leftarrow mult(x1, op_l(i(x2), x3, x1)), x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_t(x1, rd(mult(x1, op_l(i(x2), x3, x1)), i(x2)))}} \\
= & \quad \text{by Lemma 1483 LR with } \{x3 \leftarrow x3, x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& \overbrace{x1}
\end{aligned}
$$

Lemma 2092: $op_t(x1, rd(x1, rd(op_l(x2, x3, x1), x2))) = x1$

$$
\begin{aligned}
& \underbrace{op_t(x1, rd(x1, rd(op_l(x2, x3, x1), x2)))} \\
= & \quad \text{by Lemma 1753 RL with } \{x4 \leftarrow op_l(x2, x3, x1), x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, mult(x1, \underbrace{rd(x2, op_l(x2, x3, x1))}))} \\
= & \quad \text{by Lemma 1056 RL with } \{x3 \leftarrow x1, x2 \leftarrow x3, x1 \leftarrow x2\} \\
& \underbrace{op_t(x1, mult(x1, \overbrace{mult(x2, op_l(i(x2), x3, x1))}))} \\
= & \quad \text{by Lemma 2091 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{x1}
\end{aligned}
$$

Lemma 2093: $op_t(x1, mult(x1, rd(x2, op_l(x2, x1, x3)))) = x1$

$$
\begin{aligned}
& op_t(x1, mult(x1, \underbrace{rd(x2, op_l(x2, x1, x3))})) \\
= & \quad \text{by Lemma 1056 RL with } \{x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \underbrace{op_t(x1, mult(x1, \overbrace{mult(x2, op_l(i(x2), x1, x3))}))} \\
= & \quad \text{by Lemma 2081 RL with } \{x3 \leftarrow op_l(i(x2), x1, x3), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_t(x1, mult(x2, mult(x1, op_l(i(x2), x1, x3))))}} \\
= & \quad \text{by Lemma 1729 RL with } \{x3 \leftarrow x2, x2 \leftarrow mult(x1, op_l(i(x2), x1, x3)), x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, rd(mult(x1, \underbrace{op_l(i(x2), x1, x3)}), i(x2)))} \\
= & \quad \text{by Lemma 1076 RL with } \{x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow i(x2)\} \\
& \underbrace{op_t(x1, rd(mult(x1, \overbrace{op_l(i(x2), op_l(rd(x1, x3), x3, x1), x1)}), i(x2)))} \\
= & \quad \text{by Lemma 1483 LR with } \{x3 \leftarrow op_l(rd(x1, x3), x3, x1), x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& \overbrace{x1}
\end{aligned}
$$

Lemma 2094: $op_t(x1, rd(mult(i(x1), x2), x3)) = op_t(x1, rd(x2, mult(x1, x3)))$

$$
\begin{aligned}
& \underbrace{op_t(x1, rd(mult(i(x1), x2), x3))} \\
= & \quad \text{by Lemma 1215 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_t(x1, rd(rd(x2, x1), x3))}} \\
= & \quad \text{by Lemma 2079 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, rd(x2, mult(x1, x3)))}
\end{aligned}
$$

Lemma 2095: $op_t(x1, rd(x2, mult(x1, mult(x1, x3)))) = op_t(x1, rd(x3, x2))$

$$
\begin{aligned}
& \underbrace{op_t(x1, rd(x2, mult(x1, mult(x1, x3))))} \\
= & \quad \text{by Lemma 2079 RL with } \{x3 \leftarrow mult(x1, x3), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_t(x1, rd(rd(x2, x1), mult(x1, x3)))}} \\
= & \quad \text{by Lemma 1269 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, rd(x3, x2))}
\end{aligned}
$$

Lemma 2096: $op_t(x1, rd(mult(x1, rd(x2, x3)), x2)) = op_t(x1, mult(x1, op_l(i(x3), x3, x2)))$

$$
\begin{aligned}
& \underbrace{op_t(x1, rd(mult(x1, rd(x2, x3)), x2))} \\
= & \quad \text{by Lemma 1728 RL with } \{x3 \leftarrow mult(x1, rd(x2, x3)), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_t(x1, mult(i(x2), mult(x1, rd(x2, x3))))}} \\
= & \quad \text{by Lemma 2081 LR with } \{x3 \leftarrow rd(x2, x3), x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, mult(x1, \underbrace{mult(i(x2), rd(x2, x3))}))} \\
= & \quad \text{by Lemma 310 RL with } \{x2 \leftarrow x2, x1 \leftarrow x3\} \\
& op_t(x1, mult(x1, \overbrace{op_l(i(x3), x3, x2)}))
\end{aligned}
$$

Lemma 2097: $op_t(x1, rd(mult(x1, rd(x2, x3)), x2)) = op_t(x1, rd(x1, x3))$

$\overbrace{op_t(x1, rd(mult(x1, rd(x2, x3)), x2))}$

$=$ by Lemma 2096 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_t(x1, \underbrace{mult(x1, op_l(i(x3), x3, x2))}})$

$=$ by Lemma 2057 LR with $\{x5 \leftarrow x2, x4 \leftarrow x3, x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_t(x1, rd(x1, op_l(x3, x3, x2)))}}$

$=$ by Lemma 1202 RL with $\{x2 \leftarrow op_l(x3, x3, x2), x1 \leftarrow x1\}$

$\overbrace{rd(asoc(x1, op_l(x3, x3, x2), x1), \underbrace{op_t(i(x1), op_l(x3, x3, x2))})}$

$=$ by Lemma 1688 RL with $\{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow i(x1)\}$

$rd(\underbrace{asoc(x1, op_l(x3, x3, x2), x1)}, \overbrace{op_t(i(x1), x3)})$

$=$ by Lemma 1684 LR with $\{x2 \leftarrow x2, x1 \leftarrow x3, x3 \leftarrow x1\}$

$\underbrace{rd(\overbrace{asoc(x1, x3, x1)}, op_t(i(x1), x3))}$

$=$ by Lemma 1202 LR with $\{x2 \leftarrow x3, x1 \leftarrow x1\}$

$\overbrace{op_t(x1, rd(x1, x3))}$

Lemma 2098: $op_t(x1, mult(i(mult(x2, mult(x1, x3))), x2)) = op_t(x1, mult(x2, i(mult(mult(x1, x2), x3))))$

$\underbrace{op_t(x1, mult(i(mult(x2, mult(x1, x3))), x2))}$

$=$ by Lemma 2080 RL with $\{x3 \leftarrow x2, x2 \leftarrow mult(i(mult(x2, mult(x1, x3))), x2), x1 \leftarrow x1\}$

$\overbrace{op_t(x1, op_l(mult(i(mult(x2, mult(x1, x3))), \underbrace{x2}), x2, x1))}$

$=$ by Axiom 4 LR with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$op_t(x1, \underbrace{op_l(mult(i(mult(x2, mult(x1, x3))), \overbrace{mult(rd(x2, x1), x1)}), x2, x1)})$

$=$ by Lemma 292 RL with $\{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow mult(i(mult(x2, mult(x1, x3))), mult(rd(x2, x1), x1))\}$

$op_t(x1, \overbrace{\underbrace{op_l(mult(i(mult(x2, mult(x1, x3))), mult(rd(x2, x1), x1)), rd(x2, x1), x1)}})$

$=$ by Lemma 1782 RL with $\{x2 \leftarrow x1, x3 \leftarrow rd(x2, x1), x1 \leftarrow i(mult(x2, mult(x1, x3)))\}$

$\underbrace{op_t(x1, \overbrace{mult(op_l(i(mult(x2, mult(x1, x3))), rd(x2, x1), x1), mult(x1, rd(x2, x1)))})$

$=$ by Lemma 1665 LR with $\{x3 \leftarrow x2, x2 \leftarrow op_l(i(mult(x2, mult(x1, x3))), rd(x2, x1), x1), x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_t(x1, mult(op_l(i(mult(x2, mult(x1, x3))), rd(x2, x1), x1), rd(mult(x1, x2), x1)))}}$

$=$ by Lemma 1207 RL with $\{x3 \leftarrow rd(mult(x1, x2), x1), x2 \leftarrow op_l(i(mult(x2, mult(x1, x3))), rd(x2, x1), x1), x1 \leftarrow x1\}$

$\overbrace{op_t(x1, mult(op_l(i(mult(x2, mult(x1, x3))), rd(x2, x1), x1), \underbrace{op_t(rd(mult(x1, x2), x1), x1)}))}$

$=$ by Lemma 14 LR with $\{x2 \leftarrow x1, x1 \leftarrow mult(x1, x2)\}$

$op_t(x1, mult(op_l(i(mult(x2, mult(x1, x3))), rd(x2, x1), x1), \overbrace{mult(i(x1), mult(x1, x2))}))$

$=$ by Axiom 2 RL with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$op_t(x1, mult(\underbrace{op_l(i(mult(x2, mult(x1, x3))), rd(x2, x1), x1)}, \overbrace{x2}))$

$=$ by Lemma 292 LR with $\{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow i(mult(x2, mult(x1, x3)))\}$

$\underbrace{op_t(x1, mult(\overbrace{op_l(i(mult(x2, mult(x1, x3))), x2, x1)}, x2))}$

$=$ by Lemma 1694 LR with $\{x3 \leftarrow x2, x2 \leftarrow op_l(i(mult(x2, mult(x1, x3))), x2, x1), x1 \leftarrow x1\}$

$\overbrace{op_t(x1, mult(x2, \underbrace{op_l(i(mult(x2, mult(x1, x3))), x2, x1)}))}$

$=$ by Lemma 1049 RL with $\{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow mult(x2, mult(x1, x3))\}$

$op_t(x1, mult(x2, \overbrace{i(\underbrace{op_l(mult(x2, mult(x1, x3)), x2, x1)})}))$

$=$ by Lemma 1047 LR with $\{x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow x2\}$

$op_t(x1, mult(x2, i(\overbrace{mult(mult(x1, x2), x3)})))$

Lemma 2099: $op_t(x1, i(mult(x1, x3))) = op_t(x1, mult(x2, i(mult(mult(x1, x2), x3))))$

$\underbrace{op_t(x1, i(mult(x1, x3)))}$

$=$ by Lemma 1733 RL with $\{x3 \leftarrow mult(x1, x3), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_t(x1, rd(x2, mult(x2, mult(x1, x3))))}}$

$=$ by Lemma 1728 RL with $\{x3 \leftarrow x2, x2 \leftarrow mult(x2, mult(x1, x3)), x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_t(x1, mult(i(mult(x2, mult(x1, x3))), x2))}}$

$=$ by Lemma 2098 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_t(x1, mult(x2, i(mult(mult(x1, x2), x3))))}$

Lemma 2100: $op_t(x1, rd(x3, x1)) = op_t(x1, mult(x2, i(mult(mult(x1, x2), x3))))$

$$
\begin{aligned}
& \overbrace{op_t(x1, rd(x3, x1))} \\
= & \quad \text{by Lemma 707 RL with } \{x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, \overbrace{rd(i(x1), x3)})} \\
= & \quad \text{by Lemma 1732 RL with } \{x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, \overbrace{i(mult(x1, x3))})} \\
= & \quad \text{by Lemma 2099 LR with } \{x2 \leftarrow x2, x3 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, mult(x2, i(\overbrace{mult(mult(x1, x2), x3)})))}
\end{aligned}
$$

Lemma 2101: $op_t(x1, asoc(x1, x2, x3)) = x1$

$$
\begin{aligned}
& op_t(\underbrace{x1}, asoc(x1, x2, x3)) \\
= & \quad \text{by Axiom 4 LR with } \{x2 \leftarrow asoc(x1, x2, x3), x1 \leftarrow x1\} \\
& \underbrace{op_t(\overbrace{mult(rd(x1, asoc(x1, x2, x3)), asoc(x1, x2, x3))}, asoc(x1, x2, x3))} \\
= & \quad \text{by Lemma 751 LR with } \{x2 \leftarrow asoc(x1, x2, x3), x1 \leftarrow rd(x1, asoc(x1, x2, x3))\} \\
& \overbrace{op_t(mult(\underbrace{op_t(rd(x1, asoc(x1, x2, x3)), asoc(x1, x2, x3))}, asoc(x1, x2, x3)), i(rd(x1, asoc(x1, x2, x3))))} \\
= & \quad \text{by Lemma 14 LR with } \{x2 \leftarrow asoc(x1, x2, x3), x1 \leftarrow x1\} \\
& op_t(mult(\overbrace{mult(i(\overbrace{asoc(x1, x2, x3)}), x1)}, asoc(x1, x2, x3)), \underbrace{i(rd(x1, asoc(x1, x2, x3))))} \\
= & \quad \text{by Lemma 10 LR with } \{x2 \leftarrow asoc(x1, x2, x3), x1 \leftarrow x1\} \\
& op_t(mult(\underbrace{mult(i(asoc(x1, x2, x3)), x1)}, asoc(x1, x2, x3)), \overbrace{rd(asoc(x1, x2, x3), x1)}) \\
= & \quad \text{by Lemma 1842 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_t(\underbrace{mult(\overbrace{rd(x1, asoc(x1, x2, x3))}, asoc(x1, x2, x3))}, rd(asoc(x1, x2, x3), x1)) \\
= & \quad \text{by Axiom 4 RL with } \{x2 \leftarrow asoc(x1, x2, x3), x1 \leftarrow x1\} \\
& \underbrace{op_t(\overbrace{x1}, rd(asoc(x1, x2, x3), x1))} \\
= & \quad \text{by Lemma 2100 LR with } \{x2 \leftarrow mult(x2, x3), x3 \leftarrow asoc(x1, x2, x3), x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_t(x1, mult(mult(x2, x3), i(mult(mult(x1, mult(x2, x3)), asoc(x1, x2, x3)))))}} \\
= & \quad \text{by Lemma 1730 LR with } \{x3 \leftarrow mult(mult(x1, mult(x2, x3)), asoc(x1, x2, x3)), x2 \leftarrow mult(x2, x3), x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, rd(mult(x2, x3), \underbrace{mult(mult(x1, mult(x2, x3)), asoc(x1, x2, x3))}))} \\
= & \quad \text{by Axiom 8 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_t(x1, \underbrace{rd(mult(x2, x3), \overbrace{mult(mult(x1, x2), x3)})}) \\
= & \quad \text{by Lemma 891 LR with } \{x3 \leftarrow x1, x2 \leftarrow x3, x1 \leftarrow x2\} \\
& \underbrace{op_t(x1, \overbrace{i(op_r(x1, x2, x3))})} \\
= & \quad \text{by Lemma 160 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{x1}
\end{aligned}
$$

Lemma 2102: $x1 = op_t(x1, asoc(x2, i(i(x1)), x3))$

$\overbrace{x1}$

$=$ by Lemma 2101 RL with $\{x3 \leftarrow i(x3), x2 \leftarrow i(rd(mult(i(i(x1)), x2), i(x1))), x1 \leftarrow x1\}$

$\overbrace{op_t(x1, \underbrace{asoc(x1, i(rd(mult(i(i(x1)), x2), i(x1))), i(x3))}}$

$=$ by Lemma 1479 RL with $\{x3 \leftarrow x3, x2 \leftarrow rd(mult(i(i(x1)), x2), i(x1)), x1 \leftarrow x1\}$

$\underbrace{op_t(x1, \overbrace{i(asoc(i(x1), rd(mult(i(i(x1)), x2), i(x1)), x3))})}$

$=$ by Lemma 18 RL with $\{x2 \leftarrow asoc(i(x1), rd(mult(i(i(x1)), x2), i(x1)), x3), x1 \leftarrow x1\}$

$\overbrace{i(op_t(\underbrace{i(x1)}, asoc(i(x1), rd(mult(i(i(x1)), x2), i(x1)), x3)))}$

$=$ by Lemma 3 RL with $\{x1 \leftarrow i(x1)\}$

$i(\underbrace{op_t(\overbrace{i(i(i(x1)))}, asoc(i(x1), rd(mult(i(i(x1)), x2), i(x1)), x3))})$

$=$ by Lemma 1844 LR with $\{x3 \leftarrow x3, x2 \leftarrow rd(mult(i(i(x1)), x2), i(x1)), x1 \leftarrow i(x1)\}$

$i(\overbrace{rd(\underbrace{op_r(asoc(i(x1), rd(mult(i(i(x1)), x2), i(x1)), x3), i(i(x1)), i(i(x1)))}, op_t(rd(asoc(i(x1), rd(mult(i(i(x1)), x2), i(x1)), x3$

$=$ by Lemma 361 LR with $\{x2 \leftarrow i(x1), x1 \leftarrow asoc(i(x1), rd(mult(i(i(x1)), x2), i(x1)), x3)\}$

$i(rd(\underbrace{op_r(asoc(i(x1), rd(mult(i(i(x1)), x2), i(x1)), x3), i(x1), i(x1))}, op_t(rd(asoc(i(x1), rd(mult(i(i(x1)), x2), i(x1)), x3), i(x$

$=$ by Lemma 1841 RL with $\{x3 \leftarrow x3, x2 \leftarrow rd(mult(i(i(x1)), x2), i(x1)), x1 \leftarrow i(x1)\}$

$i(\underbrace{rd(\overbrace{asoc(i(x1), rd(mult(i(i(x1)), x2), i(x1)), x3)}, op_t(rd(asoc(i(x1), rd(mult(i(i(x1)), x2), i(x1)), x3), i(x1)), asoc(i(x1), rd$

$=$ by Lemma 344 LR with $\{x2 \leftarrow i(x1), x1 \leftarrow asoc(i(x1), rd(mult(i(i(x1)), x2), i(x1)), x3)\}$

$\underbrace{i(\overbrace{op_t(i(x1), i(asoc(i(x1), rd(mult(i(i(x1)), x2), i(x1)), x3)))})$

$=$ by Lemma 18 LR with $\{x2 \leftarrow i(asoc(i(x1), rd(mult(i(i(x1)), x2), i(x1)), x3)), x1 \leftarrow x1\}$

$\overbrace{op_t(x1, \underbrace{i(i(asoc(i(x1), rd(mult(i(i(x1)), x2), i(x1)), x3)))})}$

$=$ by Lemma 3 LR with $\{x1 \leftarrow asoc(i(x1), rd(mult(i(i(x1)), x2), i(x1)), x3)\}$

$op_t(x1, \overbrace{\underbrace{asoc(i(x1), rd(mult(i(i(x1)), x2), i(x1)), x3)}})$

$=$ by Lemma 1220 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow i(x1)\}$

$op_t(x1, \overbrace{asoc(x2, i(i(x1)), x3)})$

Lemma 2103: $mult(asoc(x1, x2, x3), x1) = mult(x1, asoc(x1, x2, 3))$

$mult(asoc(x1, x2, x3), \underbrace{x1})$

$=$ by Lemma 2101 RL with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{mult(asoc(x1, x2, x3), \overbrace{op_t(x1, asoc(x1, x2, x3))})}$

$=$ by Lemma 13 LR with $\{x2 \leftarrow x1, x1 \leftarrow asoc(x1, x2, x3)\}$

$\overbrace{mult(x1, asoc(x1, x2, x3))}$

Lemma 2104: $mult(x1, asoc(x2, x1, x3)) = mult(asoc(x2, x1, x3), x1)$

$$
\begin{aligned}
& \underbrace{mult(x1, asoc(x2, x1, x3))} \\
= \quad & \text{by Lemma 13 RL with } \{x2 \leftarrow x1, x1 \leftarrow asoc(x2, x1, x3)\} \\
& \overbrace{mult(asoc(x2, x1, x3), op_t(x1, asoc(x2, \underbrace{x1}, x3)))} \\
= \quad & \text{by Lemma 3 RL with } \{x1 \leftarrow x1\} \\
& mult(asoc(x2, x1, x3), \underbrace{op_t(x1, asoc(x2, \overbrace{i(i(x1))}, x3))}) \\
= \quad & \text{by Lemma 2102 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& mult(asoc(x2, x1, x3), \overbrace{x1})
\end{aligned}
$$

Lemma 2105: $mult(x1, asoc(i(x1), x2, x3)) = rd(mult(x1, x1), mult(i(asoc(i(x1), x2, x3)), x1))$

$$
\begin{aligned}
& \underbrace{mult(x1, asoc(i(x1), x2, x3))} \\
= \quad & \text{by Lemma 1984 LR with } \{x2 \leftarrow asoc(i(x1), x2, x3), x1 \leftarrow x1\} \\
& \overbrace{rd(mult(asoc(i(x1), x2, x3), i(x1)), rd(\underbrace{op_t(i(x1), asoc(i(x1), x2, x3))}, x1))} \\
= \quad & \text{by Lemma 2101 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow i(x1)\} \\
& rd(mult(asoc(i(x1), x2, x3), i(x1)), \underbrace{rd(\overbrace{i(x1)}, x1)}) \\
= \quad & \text{by Lemma 43 RL with } \{x1 \leftarrow x1\} \\
& \underbrace{rd(mult(asoc(i(x1), x2, x3), i(x1)), \overbrace{i(mult(x1, x1))})} \\
= \quad & \text{by Lemma 61 LR with } \{x3 \leftarrow mult(x1, x1), x2 \leftarrow x1, x1 \leftarrow asoc(i(x1), x2, x3)\} \\
& \overbrace{rd(mult(x1, x1), mult(i(asoc(i(x1), x2, x3)), x1))}
\end{aligned}
$$

Lemma 2106: $op_t(x1, rd(rd(x2, mult(x3, x1)), i(x3))) = op_t(x1, rd(x2, mult(i(x3), mult(x3, x1))))$

$$
\begin{aligned}
& op_t(x1, \underbrace{rd(rd(x2, mult(x3, x1)), i(x3))}) \\
= \quad & \text{by Lemma 3 RL with } \{x1 \leftarrow rd(rd(x2, mult(x3, x1)), i(x3))\} \\
& \underbrace{op_t(x1, \overbrace{i(i(rd(rd(x2, mult(x3, x1)), i(x3))))})} \\
= \quad & \text{by Lemma 2089 LR with } \{x3 \leftarrow x3, x2 \leftarrow rd(rd(x2, mult(x3, x1)), i(x3)), x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, \underbrace{op_r(rd(rd(x2, mult(x3, x1)), i(x3)), i(x3), mult(x3, x1))})} \\
= \quad & \text{by Lemma 839 LR with } \{x3 \leftarrow i(x3), x2 \leftarrow mult(x3, x1), x1 \leftarrow x2\} \\
& op_t(x1, \overbrace{rd(x2, mult(i(x3), mult(x3, x1)))})
\end{aligned}
$$

Lemma 2107: $op_t(x1, rd(x3, rd(mult(x3, x1), x2))) = op_t(x1, rd(x2, mult(i(x3), mult(x3, x1))))$

$$
\begin{aligned}
& \underbrace{op_t(x1, rd(x3, rd(mult(x3, x1), x2)))} \\
= \quad & \text{by Lemma 1753 RL with } \{x4 \leftarrow mult(x3, x1), x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_t(x1, mult(x3, rd(x2, mult(x3, x1))))}} \\
= \quad & \text{by Lemma 1729 RL with } \{x3 \leftarrow x3, x2 \leftarrow rd(x2, mult(x3, x1)), x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_t(x1, rd(rd(x2, mult(x3, x1)), i(x3)))}} \\
= \quad & \text{by Lemma 2106 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, rd(x2, mult(i(x3), mult(x3, x1))))}
\end{aligned}
$$

Lemma 2108: $op_t(x1, rd(asoc(x2, x3, x1), x1)) = x1$

$op_t(x1, rd(asoc(x2, x3, x1), \underbrace{x1}))$

$=$ by Axiom 2 LR with $\{x2 \leftarrow mult(x2, x3), x1 \leftarrow x1\}$

$\underbrace{op_t(x1, rd(asoc(x2, x3, x1), \overbrace{mult(i(mult(x2, x3)), mult(mult(x2, x3), x1))}))}$

$=$ by Lemma 2107 RL with $\{x2 \leftarrow asoc(x2, x3, x1), x3 \leftarrow mult(x2, x3), x1 \leftarrow x1\}$

$\overbrace{op_t(x1, rd(mult(x2, x3), rd(\underbrace{mult(mult(x2, x3), x1)}, asoc(x2, x3, x1))))}$

$=$ by Axiom 8 LR with $\{x3 \leftarrow x1, x2 \leftarrow x3, x1 \leftarrow x2\}$

$op_t(x1, rd(mult(x2, x3), \underbrace{rd(\overbrace{mult(mult(x2, mult(x3, x1)), asoc(x2, x3, x1))}, asoc(x2, x3, x1))}))$

$=$ by Axiom 5 RL with $\{x2 \leftarrow asoc(x2, x3, x1), x1 \leftarrow mult(x2, mult(x3, x1))\}$

$op_t(x1, rd(mult(\underbrace{x2}, x3), \overbrace{mult(x2, mult(x3, x1))}))$

$=$ by Axiom 5 LR with $\{x2 \leftarrow mult(x3, x1), x1 \leftarrow x2\}$

$\underbrace{op_t(x1, rd(mult(\overbrace{rd(mult(x2, mult(x3, x1)), mult(x3, x1))}, x3), mult(x2, mult(x3, x1))))}$

$=$ by Lemma 1878 RL with $\{x3 \leftarrow x3, x2 \leftarrow mult(x2, mult(x3, x1)), x1 \leftarrow x1\}$

$\overbrace{x1}$

Lemma 2109: $i(mult(x1, asoc(x2, x3, x1))) = rd(i(x1), asoc(x2, x3, x1))$

$\underbrace{i(mult(x1, asoc(x2, x3, x1)))}$

$=$ by Lemma 510 RL with $\{x2 \leftarrow x1, x1 \leftarrow asoc(x2, x3, x1)\}$

$\overbrace{rd(i(asoc(x2, x3, x1)), \underbrace{op_t(x1, rd(asoc(x2, x3, x1), x1))})}$

$=$ by Lemma 2108 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{rd(i(asoc(x2, x3, x1)), \overbrace{x1})}$

$=$ by Lemma 37 LR with $\{x2 \leftarrow x1, x1 \leftarrow asoc(x2, x3, x1)\}$

$\overbrace{rd(i(x1), asoc(x2, x3, x1))}$

Lemma 2110: $op_t(x1, rd(x3, x2)) = op_t(x1, rd(rd(mult(i(x1), i(x1)), i(x2)), x3))$

$\underbrace{op_t(x1, rd(x3, x2))}$

$=$ by Lemma 1278 RL with $\{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x1\}$

$\overbrace{op_t(x1, rd(x3, \underbrace{op_l(x2, x2, x1)}))}$

$=$ by Lemma 2031 RL with $\{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x2\}$

$\underbrace{op_t(x1, rd(x3, \overbrace{op_l(x2, i(x1), x2)}))}$

$=$ by Lemma 2057 RL with $\{x5 \leftarrow x2, x4 \leftarrow i(x1), x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x1\}$

$\overbrace{op_t(x1, mult(\underbrace{x3}, op_l(i(x2), i(x1), x2)))}$

$=$ by Axiom 2 LR with $\{x2 \leftarrow x1, x1 \leftarrow x3\}$

$\underbrace{op_t(x1, mult(\overbrace{mult(i(x1), mult(x1, x3))}, op_l(i(x2), i(x1), x2)))}$

$=$ by Lemma 1146 LR with $\{x3 \leftarrow op_l(i(x2), i(x1), x2), x2 \leftarrow mult(x1, x3), x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_t(x1, mult(rd(mult(x1, x3), x1), op_l(i(x2), i(x1), x2)))}}$

$=$ by Lemma 1752 LR with $\{x4 \leftarrow op_l(i(x2), i(x1), x2), x3 \leftarrow x1, x2 \leftarrow mult(x1, x3), x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_t(x1, rd(\underbrace{op_l(i(x2), i(x1), x2)}, rd(x1, mult(x1, x3))))}}$

$=$ by Lemma 1287 LR with $\{x2 \leftarrow mult(x1, x3), x3 \leftarrow op_l(i(x2), i(x1), x2), x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_t(x1, rd(rd(i(x1), op_l(i(x2), i(x1), x2)), mult(x1, x3)))}}$

$=$ by Lemma 2094 RL with $\{x3 \leftarrow x3, x2 \leftarrow rd(i(x1), op_l(i(x2), i(x1), x2)), x1 \leftarrow x1\}$

$\overbrace{op_t(x1, rd(mult(i(x1), rd(i(x1), \underbrace{op_l(i(x2), i(x1), x2)})), x3))}$

$=$ by Lemma 904 RL with $\{x2 \leftarrow x2, x1 \leftarrow i(x1)\}$

$op_t(x1, rd(\underbrace{mult(i(x1), rd(i(x1), \overbrace{mult(i(x1), rd(i(x2), i(x1)))})}), x3))$

$=$ by Lemma 807 RL with $\{x2 \leftarrow rd(i(x2), i(x1)), x1 \leftarrow i(x1)\}$

$op_t(x1, rd(\overbrace{\underbrace{op_r(rd(i(x1), rd(i(x2), i(x1))), i(x1), i(x1))}}, x3))$

$=$ by Lemma 868 LR with $\{x2 \leftarrow i(x2), x1 \leftarrow i(x1)\}$

$op_t(x1, rd(\overbrace{rd(mult(i(x1), i(x1)), i(x2))}, x3))$

Lemma 2111: $op_t(x1, rd(x3, x2)) = op_t(x1, rd(rd(x2, mult(x1, x1)), x3))$

$\underbrace{op_t(x1, rd(x3, x2))}$

$=$ by Lemma 2110 LR with $\{x2 \leftarrow x2, x3 \leftarrow x3, x1 \leftarrow x1\}$

$\overbrace{op_t(x1, rd(\underbrace{rd(mult(i(x1), i(x1)), i(x2))}, x3))}$

$=$ by Lemma 63 LR with $\{x3 \leftarrow x2, x2 \leftarrow i(x1), x1 \leftarrow x1\}$

$op_t(x1, rd(\overbrace{rd(x2, mult(x1, \underbrace{i(i(x1))}))}, x3))$

$=$ by Lemma 3 LR with $\{x1 \leftarrow x1\}$

$op_t(x1, rd(rd(x2, mult(x1, \overbrace{x1})), x3))$

Lemma 2112: $op_t(i(x1), rd(x1, mult(x2, x3))) = op_t(i(op_t(x1, rd(x2, rd(x1, x3)))), x1)$

$$
\begin{aligned}
& \underbrace{op_t(i(x1), rd(x1, mult(x2, x3)))} \\
= \quad & \text{by Lemma 47 RL with } \{x3 \leftarrow x1, x2 \leftarrow mult(x2, x3), x1 \leftarrow x1\} \\
& \overbrace{\underbrace{i(op_t(x1, rd(mult(x2, x3), x1)))}} \\
= \quad & \text{by Lemma 35 RL with } \{x2 \leftarrow rd(mult(x2, x3), x1), x1 \leftarrow x1\} \\
& \overbrace{op_t(i(\underbrace{op_t(x1, rd(mult(x2, x3), x1))}), x1)} \\
= \quad & \text{by Lemma 2077 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_t(i(\overbrace{op_t(x1, rd(x2, rd(x1, x3))))}), x1)
\end{aligned}
$$

Lemma 2113: $op_t(i(x1), rd(x1, mult(x2, x3))) = op_t(i(x1), rd(rd(x1, x3), x2))$

$$
\begin{aligned}
& \underbrace{op_t(i(x1), rd(x1, mult(x2, x3)))} \\
= \quad & \text{by Lemma 2112 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_t(i(op_t(x1, rd(x2, rd(x1, x3)))), x1)}} \\
= \quad & \text{by Lemma 35 LR with } \{x2 \leftarrow rd(x2, rd(x1, x3)), x1 \leftarrow x1\} \\
& \overbrace{\underbrace{i(op_t(x1, rd(x2, rd(x1, x3))))}} \\
= \quad & \text{by Lemma 47 LR with } \{x3 \leftarrow rd(x1, x3), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_t(i(x1), rd(rd(x1, x3), x2))}
\end{aligned}
$$

Lemma 2114: $op_t(i(x1), rd(x1, rd(x3, x1))) = op_t(i(x1), op_l(x3, rd(x1, x3), x2))$

$op_t(i(x1), rd(x1, \underbrace{rd(x3, x1)}))$

$=$ by Lemma 12 RL with $\{x2 \leftarrow rd(x3, x1), x1 \leftarrow x2\}$

$\underbrace{op_t(i(x1), rd(x1, \overbrace{mult(x2, mult(i(x2), rd(x3, x1)))}))}$

$=$ by Lemma 2113 LR with $\{x3 \leftarrow mult(i(x2), rd(x3, x1)), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_t(i(x1), rd(rd(x1, mult(i(x2), rd(x3, x1))), x2))}}$

$=$ by Lemma 47 RL with $\{x3 \leftarrow rd(x1, mult(i(x2), rd(x3, x1))), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{\underbrace{i(op_t(x1, rd(x2, rd(x1, mult(i(x2), rd(x3, x1))))))}}$

$=$ by Lemma 36 RL with $\{x2 \leftarrow op_t(x1, rd(x2, rd(x1, mult(i(x2), rd(x3, x1))))), x1 \leftarrow x4\}$

$\overbrace{rd(\underbrace{op_t(x4, op_t(x1, rd(x2, rd(x1, mult(i(x2), rd(x3, x1))))))}, mult(x4, op_t(x1, rd(x2, rd(x1, mult(i(x2), rd(x3, x1)))))))}$

$=$ by Lemma 1722 LR with $\{x3 \leftarrow rd(x2, rd(x1, mult(i(x2), rd(x3, x1)))), x2 \leftarrow x1, x1 \leftarrow x4\}$

$rd(\overbrace{op_t(x4, x1)}, mult(x4, \underbrace{op_t(x1, rd(x2, rd(x1, mult(i(x2), rd(x3, x1)))))}))$

$=$ by Lemma 1288 RL with $\{x3 \leftarrow x2, x2 \leftarrow mult(i(x2), rd(x3, x1)), x1 \leftarrow x1\}$

$rd(\underbrace{op_t(x4, x1)}, mult(x4, \overbrace{op_t(x1, rd(i(mult(i(x2), rd(x3, x1))), mult(x2, x1)))}))$

$=$ by Lemma 1722 RL with $\{x3 \leftarrow rd(i(mult(i(x2), rd(x3, x1))), mult(x2, x1)), x2 \leftarrow x1, x1 \leftarrow x4\}$

$\underbrace{rd(\overbrace{op_t(x4, op_t(x1, rd(i(mult(i(x2), rd(x3, x1))), mult(x2, x1))))}, mult(x4, op_t(x1, rd(i(mult(i(x2), rd(x3, x1))), mult(x2, x1$

$=$ by Lemma 36 LR with $\{x2 \leftarrow op_t(x1, rd(i(mult(i(x2), rd(x3, x1))), mult(x2, x1))), x1 \leftarrow x4\}$

$\overbrace{\underbrace{i(op_t(x1, rd(i(mult(i(x2), rd(x3, x1))), mult(x2, x1))))}}$

$=$ by Lemma 47 LR with $\{x3 \leftarrow mult(x2, x1), x2 \leftarrow i(mult(i(x2), rd(x3, x1))), x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_t(i(x1), rd(mult(x2, x1), i(mult(i(x2), rd(x3, x1)))))}}$

$=$ by Lemma 1729 LR with $\{x3 \leftarrow mult(i(x2), rd(x3, x1)), x2 \leftarrow mult(x2, x1), x1 \leftarrow i(x1)\}$

$\overbrace{op_t(i(x1), \underbrace{mult(mult(i(x2), rd(x3, x1)), mult(x2, x1))})}$

$=$ by Lemma 280 RL with $\{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x3\}$

$op_t(i(x1), \overbrace{op_l(x3, rd(x1, x3), x2)})$

Lemma 2115: $op_t(i(x1), op_l(x2, rd(x1, x2), x3)) = op_t(i(x1), x2)$

$\underbrace{op_t(i(x1), op_l(x2, rd(x1, x2), x3))}$

$=$ by Lemma 2114 RL with $\{x2 \leftarrow x3, x3 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_t(i(x1), rd(x1, rd(x2, x1)))}}$

$=$ by Lemma 718 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_t(i(x1), x2)}$

Lemma 2116: $op_t(x1, x2) = op_t(i(i(x1)), op_l(op_l(x2, x2, x1), i(mult(x1, x2)), x3))$

$$
\begin{aligned}
& op_t(\overbrace{x1}, x2) \\
= \quad & \text{by Lemma 3 RL with } \{x1 \leftarrow x1\} \\
& \overbrace{op_t(\overbrace{i(i(x1))}, x2)} \\
= \quad & \text{by Lemma 1688 LR with } \{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow i(i(x1))\} \\
& \overbrace{\overbrace{op_t(i(i(x1)), op_l(x2, x2, x1))}} \\
= \quad & \text{by Lemma 2115 RL with } \{x3 \leftarrow x3, x2 \leftarrow op_l(x2, x2, x1), x1 \leftarrow i(x1)\} \\
& \overbrace{op_t(i(i(x1)), op_l(op_l(x2, x2, x1), \overbrace{rd(i(x1), op_l(x2, x2, x1))}, x3))} \\
= \quad & \text{by Lemma 447 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_t(i(i(x1)), op_l(op_l(x2, x2, x1), \overbrace{i(mult(x1, x2))}, x3))
\end{aligned}
$$

Lemma 2117: $op_t(x1, x2) = op_t(x1, op_l(x2, i(mult(x1, x2)), x3))$

$$
\begin{aligned}
& \overbrace{op_t(x1, x2)} \\
= \quad & \text{by Lemma 2116 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\overbrace{op_t(i(i(x1)), op_l(op_l(x2, x2, x1), i(mult(x1, x2)), x3))}} \\
= \quad & \text{by Lemma 1783 LR with } \{x5 \leftarrow x3, x4 \leftarrow i(mult(x1, x2)), x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow i(i(x1))\} \\
& \overbrace{op_t(\overbrace{i(i(x1))}, op_l(x2, i(mult(x1, x2)), x3))} \\
= \quad & \text{by Lemma 3 LR with } \{x1 \leftarrow x1\} \\
& op_t(\overbrace{x1}, op_l(x2, i(mult(x1, x2)), x3))
\end{aligned}
$$

Lemma 2118: $op_t(rd(x2, x1), x1) = op_t(i(rd(x1, x2)), op_l(op_l(x1, x1, x2), i(x2), x3))$

$$
\begin{aligned}
& op_t(\overbrace{rd(x2, x1)}, x1) \\
= \quad & \text{by Lemma 10 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_t(\overbrace{i(rd(x1, x2))}, x1)} \\
= \quad & \text{by Lemma 1688 LR with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow i(rd(x1, x2))\} \\
& \overbrace{\overbrace{op_t(i(rd(x1, x2)), op_l(x1, x1, x2))}} \\
= \quad & \text{by Lemma 2115 RL with } \{x3 \leftarrow x3, x2 \leftarrow op_l(x1, x1, x2), x1 \leftarrow rd(x1, x2)\} \\
& \overbrace{op_t(i(rd(x1, x2)), op_l(op_l(x1, x1, x2), \overbrace{rd(rd(x1, x2), op_l(x1, x1, x2))}, x3))} \\
= \quad & \text{by Lemma 449 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_t(i(rd(x1, x2)), op_l(op_l(x1, x1, x2), \overbrace{i(x2)}, x3))
\end{aligned}
$$

Lemma 2119: $mult(i(x1), x2) = op_t(i(rd(x1, x2)), op_l(x1, i(x2), x3))$

$$
\begin{aligned}
& \underbrace{mult(i(x1), x2)} \\
= \quad & \text{by Lemma 14 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \overbrace{op_t(rd(x2, x1), x1)} \\
= \quad & \text{by Lemma 2118 LR with } \{x3 \leftarrow x3, x1 \leftarrow x1, x2 \leftarrow x2\} \\
& \overbrace{\underbrace{op_t(i(rd(x1, x2)), op_l(op_l(x1, x1, x2), i(x2), x3))}} \\
= \quad & \text{by Lemma 1783 LR with } \{x5 \leftarrow x3, x4 \leftarrow i(x2), x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow i(rd(x1, x2))\} \\
& \overbrace{op_t(i(rd(x1, x2)), op_l(x1, i(x2), x3))}
\end{aligned}
$$

Lemma 2120: $mult(i(x1), x2) = op_t(rd(x2, x1), op_l(x1, x3, x2))$

$$
\begin{aligned}
& \underbrace{mult(i(x1), x2)} \\
= \quad & \text{by Lemma 2119 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_t(\underbrace{i(rd(x1, x2))}, op_l(x1, i(x2), x3))} \\
= \quad & \text{by Lemma 10 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_t(\overbrace{rd(x2, x1)}, \underbrace{op_l(x1, i(x2), x3)}) \\
= \quad & \text{by Lemma 2031 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_t(rd(x2, x1), \overbrace{op_l(x1, x3, x2)})
\end{aligned}
$$

Lemma 2121: $mult(rd(x1, x2), op_l(x2, x3, x1)) = mult(rd(op_l(x2, x3, x1), x2), x1)$

$$
\begin{aligned}
& \underbrace{mult(rd(x1, x2), op_l(x2, x3, x1))} \\
= \quad & \text{by Lemma 13 RL with } \{x2 \leftarrow rd(x1, x2), x1 \leftarrow op_l(x2, x3, x1)\} \\
& \overbrace{mult(op_l(x2, x3, x1), \underbrace{op_t(rd(x1, x2), op_l(x2, x3, x1))})} \\
= \quad & \text{by Lemma 2120 RL with } \{x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \underbrace{mult(op_l(x2, x3, x1), \overbrace{mult(i(x2), x1)})} \\
= \quad & \text{by Lemma 2001 LR with } \{x4 \leftarrow x1, x3 \leftarrow x1, x2 \leftarrow x3, x1 \leftarrow x2\} \\
& \overbrace{mult(rd(op_l(x2, x3, x1), x2), x1)}
\end{aligned}
$$

Lemma 2122: $rd(rd(op_l(x1,x2,x3),x1),i(op_t(x3,mult(x3,rd(op_l(x1,x2,x3),x1)))))=mult(x3,rd(op_l(x1,x2,x3),x1))$

$rd(\underbrace{rd(op_l(x1,x2,x3),x1),i(op_t(x3,mult(x3,rd(op_l(x1,x2,x3),x1))))}$

$=$ by Lemma 1605 LR with $\{x2\leftarrow x3, x1\leftarrow rd(op_l(x1,x2,x3),x1)\}$

$\overbrace{mult(\underbrace{op_l(x3,rd(op_l(x1,x2,x3),x1),mult(x3,x3))},rd(op_l(x1,x2,x3),x1))}$

$=$ by Lemma 1535 LR with $\{x2\leftarrow rd(op_l(x1,x2,x3),x1), x1\leftarrow x3\}$

$mult(\overbrace{op_l(\underbrace{x3},mult(x3,x3),rd(x3,rd(op_l(x1,x2,x3),x1)))},rd(op_l(x1,x2,x3),x1))$

$=$ by Lemma 2092 RL with $\{x3\leftarrow x2, x2\leftarrow x1, x1\leftarrow x3\}$

$mult(\underbrace{op_l(\overbrace{op_t(x3,rd(x3,rd(op_l(x1,x2,x3),x1)))},mult(x3,x3),rd(x3,rd(op_l(x1,x2,x3),x1)))},rd(op_l(x1,x2,x3),x1))$

$=$ by Axiom 15 RL with $\{x4\leftarrow rd(x3,rd(op_l(x1,x2,x3),x1)), x3\leftarrow rd(x3,rd(op_l(x1,x2,x3),x1)), x2\leftarrow mult(x3,x3), x1\leftarrow x3$

$mult(\overbrace{op_t(op_l(x3,mult(x3,x3),rd(x3,rd(op_l(x1,x2,x3),x1))),rd(x3,rd(op_l(x1,x2,x3),x1)))},rd(op_l(x1,x2,x3),x1))$

$=$ by Axiom 10 RL with $\{x2\leftarrow op_l(x3,mult(x3,x3),rd(x3,rd(op_l(x1,x2,x3),x1))), x1\leftarrow rd(x3,rd(op_l(x1,x2,x3),x1))\}$

$mult(\overbrace{mult(i(rd(x3,rd(op_l(x1,x2,x3),x1))),\underbrace{mult(op_l(x3,mult(x3,x3),rd(x3,rd(op_l(x1,x2,x3),x1))),rd(x3,rd(op_l(x1,x2,x3),x1)))}},rd(op_l(x1,x2,x3),x1))$

$=$ by Lemma 1597 LR with $\{x2\leftarrow rd(x3,rd(op_l(x1,x2,x3),x1)), x1\leftarrow x3\}$

$mult(\underbrace{mult(i(rd(x3,rd(op_l(x1,x2,x3),x1))),\overbrace{mult(rd(x3,rd(op_l(x1,x2,x3),x1)),op_t(x3,i(rd(x3,rd(op_l(x1,x2,x3),x1)))))}},rd(op_l(x1,x2,x3),x1))$

$=$ by Axiom 2 RL with $\{x2\leftarrow rd(x3,rd(op_l(x1,x2,x3),x1)), x1\leftarrow op_t(x3,i(rd(x3,rd(op_l(x1,x2,x3),x1))))\}$

$mult(\overbrace{op_t(x3,\underbrace{i(rd(x3,rd(op_l(x1,x2,x3),x1)))})},rd(op_l(x1,x2,x3),x1))$

$=$ by Lemma 10 LR with $\{x2\leftarrow rd(op_l(x1,x2,x3),x1), x1\leftarrow x3\}$

$mult(\underbrace{op_t(x3,\overbrace{rd(rd(op_l(x1,x2,x3),x1),x3)})},rd(op_l(x1,x2,x3),x1))$

$=$ by Lemma 2078 RL with $\{x3\leftarrow op_l(x1,x2,x3), x2\leftarrow x1, x1\leftarrow x3\}$

$mult(\overbrace{op_t(x3,rd(x1,\underbrace{mult(x3,op_l(x1,x2,x3))}))},rd(op_l(x1,x2,x3),x1))$

$=$ by Lemma 1091 RL with $\{x3\leftarrow x1, x2\leftarrow x2, x1\leftarrow x3\}$

$mult(\underbrace{op_t(x3,rd(x1,\overbrace{mult(rd(x3,x2),mult(x2,x1))}))},rd(op_l(x1,x2,x3),x1))$

$=$ by Lemma 1485 RL with $\{x2\leftarrow x2, x3\leftarrow x1, x1\leftarrow x3\}$

$mult(\overbrace{x3},rd(op_l(x1,x2,x3),x1))$

Lemma 2123: $rd(op_t(x3,rd(x3,rd(x1,op_l(x1,x2,x3)))),rd(x1,op_l(x1,x2,x3)))=mult(x3,rd(op_l(x1,x2,x3),x1))$

$rd(\underbrace{op_t(x3,rd(x3,rd(x1,op_l(x1,x2,x3))))},rd(x1,op_l(x1,x2,x3)))$

$=$ by Lemma 1753 RL with $\{x4\leftarrow x1, x3\leftarrow op_l(x1,x2,x3), x2\leftarrow x3, x1\leftarrow x3\}$

$\underbrace{rd(\overbrace{op_t(x3,mult(x3,rd(op_l(x1,x2,x3),x1)))},rd(x1,op_l(x1,x2,x3)))}$

$=$ by Lemma 39 RL with $\{x3\leftarrow op_t(x3,mult(x3,rd(op_l(x1,x2,x3),x1))), x2\leftarrow x1, x1\leftarrow op_l(x1,x2,x3)\}$

$\overbrace{\underbrace{rd(rd(op_l(x1,x2,x3),x1),i(op_t(x3,mult(x3,rd(op_l(x1,x2,x3),x1)))))}}$

$=$ by Lemma 2122 LR with $\{x3\leftarrow x3, x2\leftarrow x2, x1\leftarrow x1\}$

$\overbrace{mult(x3,rd(op_l(x1,x2,x3),x1))}$

Lemma 2124: $mult(x1, rd(op_l(x2, x3, x1), x2)) = rd(x1, rd(x2, op_l(x2, x3, x1)))$

$$
\begin{aligned}
& \underbrace{mult(x1, rd(op_l(x2, x3, x1), x2))} \\
= \quad & \text{by Lemma 2123 RL with } \{x2 \leftarrow x3, x1 \leftarrow x2, x3 \leftarrow x1\} \\
& \overbrace{rd(\underbrace{op_t(x1, rd(x1, rd(x2, op_l(x2, x3, x1))))}, rd(x2, op_l(x2, x3, x1)))} \\
= \quad & \text{by Lemma 2090 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& rd(\overbrace{x1}, rd(x2, op_l(x2, x3, x1)))
\end{aligned}
$$

Lemma 2125: $rd(x1, rd(x2, op_l(x2, x3, i(x1)))) = mult(x1, mult(rd(i(x1), rd(x2, op_l(x2, x3, i(x1)))), x1))$

$$
\begin{aligned}
& rd(x1, \underbrace{rd(x2, op_l(x2, x3, i(x1)))}) \\
= \quad & \text{by Lemma 10 RL with } \{x2 \leftarrow x2, x1 \leftarrow op_l(x2, x3, i(x1))\} \\
& \underbrace{rd(x1, \overbrace{i(rd(op_l(x2, x3, i(x1)), x2))})} \\
= \quad & \text{by Lemma 86 RL with } \{x2 \leftarrow rd(op_l(x2, x3, i(x1)), x2), x1 \leftarrow x1\} \\
& \overbrace{mult(x1, mult(\underbrace{mult(i(x1), rd(op_l(x2, x3, i(x1)), x2))}, x1))} \\
= \quad & \text{by Lemma 2124 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow i(x1)\} \\
& mult(x1, mult(\overbrace{rd(i(x1), rd(x2, op_l(x2, x3, i(x1))))}, x1))
\end{aligned}
$$

Lemma 2126: $rd(x1, rd(x2, op_l(x2, x1, x3))) = mult(x1, i(rd(x2, op_l(x2, x3, i(x1)))))$

$$
\begin{aligned}
& rd(x1, rd(x2, \underbrace{op_l(x2, x1, x3)})) \\
= \quad & \text{by Lemma 2030 RL with } \{x3 \leftarrow x1, x2 \leftarrow x3, x1 \leftarrow x2\} \\
& \underbrace{rd(x1, rd(x2, \overbrace{op_l(x2, x3, i(x1))}))} \\
= \quad & \text{by Lemma 2125 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{mult(x1, \underbrace{mult(rd(i(x1), rd(x2, op_l(x2, x3, i(x1)))), x1)})} \\
= \quad & \text{by Lemma 21 LR with } \{x2 \leftarrow rd(x2, op_l(x2, x3, i(x1))), x1 \leftarrow x1\} \\
& mult(x1, \overbrace{i(rd(x2, op_l(x2, x3, i(x1))))})
\end{aligned}
$$

Lemma 2127: $rd(x1, rd(x2, op_l(x2, x1, x3))) = mult(x1, rd(op_l(x2, x1, x3), x2))$

$$
\begin{aligned}
& \underbrace{rd(x1, rd(x2, op_l(x2, x1, x3)))} \\
= \quad & \text{by Lemma 2126 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{mult(x1, \underbrace{i(rd(x2, op_l(x2, x3, i(x1))))})} \\
= \quad & \text{by Lemma 10 LR with } \{x2 \leftarrow op_l(x2, x3, i(x1)), x1 \leftarrow x2\} \\
& mult(x1, \overbrace{rd(\underbrace{op_l(x2, x3, i(x1))}, x2)}) \\
= \quad & \text{by Lemma 2030 LR with } \{x3 \leftarrow x1, x2 \leftarrow x3, x1 \leftarrow x2\} \\
& mult(x1, rd(\overbrace{op_l(x2, x1, x3)}, x2))
\end{aligned}
$$

Lemma 2128: $asoc(op_l(x1,x2,x3),x4,op_l(x1,x2,x3)) = rd(rd(x4,op_l(i(x1),x2,x3)),rd(op_r(x4,x1,x1),i(op_l(x1,x2,x3))))$

$$
\begin{aligned}
& \underbrace{asoc(op_l(x1,x2,x3),x4,op_l(x1,x2,x3))} \\
= & \quad \text{by Lemma 1734 RL with } \{x3 \leftarrow x1, x2 \leftarrow x4, x1 \leftarrow op_l(x1,x2,x3)\} \\
& \overbrace{\underbrace{asoc(op_l(x1,x2,x3),op_r(x4,x1,x1),op_l(x1,x2,x3))}} \\
= & \quad \text{by Lemma 702 RL with } \{x2 \leftarrow op_l(x1,x2,x3), x1 \leftarrow op_r(x4,x1,x1)\} \\
& \overbrace{rd(\underbrace{mult(op_r(x4,x1,x1),op_l(x1,x2,x3))},rd(op_r(x4,x1,x1),i(op_l(x1,x2,x3))))} \\
= & \quad \text{by Lemma 2075 LR with } \{x4 \leftarrow x3, x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x4\} \\
& rd(\overbrace{rd(x4,op_l(i(x1),x2,x3))},rd(op_r(x4,x1,x1),i(op_l(x1,x2,x3))))
\end{aligned}
$$

Lemma 2129: $asoc(op_l(x1,x2,x3),x4,op_l(x1,x2,x3)) = asoc(x1,x4,x1)$

$$
\begin{aligned}
& \underbrace{asoc(op_l(x1,x2,x3),x4,op_l(x1,x2,x3))} \\
= & \quad \text{by Lemma 2128 LR with } \{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{rd(rd(x4,op_l(i(x1),x2,x3)),rd(op_r(x4,x1,x1),\underbrace{i(op_l(x1,x2,x3))}))} \\
= & \quad \text{by Lemma 1049 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{rd(rd(x4,op_l(i(x1),x2,x3)),rd(op_r(x4,x1,x1),\overbrace{op_l(i(x1),x2,x3)}))} \\
= & \quad \text{by Lemma 1813 RL with } \{x1 \leftarrow op_l(i(x1),x2,x3), x2 \leftarrow x4, x3 \leftarrow x1\} \\
& \overbrace{asoc(x1,x4,x1)}
\end{aligned}
$$

Lemma 2130: $op_r(x1,x2,x2) = rd(op_t(rd(x1,op_l(i(x2),x3,x4)),i(op_r(x1,x2,x2))),op_l(op_l(x2,x3,x4),op_l(x2,x3,x4),op_r(x1,x2$

$$
\begin{aligned}
& \underbrace{op_r(x1,x2,x2)} \\
= & \quad \text{by Lemma 283 RL with } \{x2 \leftarrow op_r(x1,x2,x2), x1 \leftarrow op_l(x2,x3,x4)\} \\
& \overbrace{rd(\underbrace{op_t(mult(op_l(x2,x3,x4),op_r(x1,x2,x2)),op_l(x2,x3,x4))},op_l(op_l(x2,x3,x4),op_l(x2,x3,x4),op_r(x1,x2,x2)))} \\
= & \quad \text{by Lemma 919 RL with } \{x2 \leftarrow op_l(x2,x3,x4), x1 \leftarrow op_r(x1,x2,x2)\} \\
& rd(\overbrace{op_t(\underbrace{mult(op_r(x1,x2,x2),op_l(x2,x3,x4))},i(op_r(x1,x2,x2)))},op_l(op_l(x2,x3,x4),op_l(x2,x3,x4),op_r(x1,x2,x2))) \\
= & \quad \text{by Lemma 2075 LR with } \{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& rd(op_t(\overbrace{rd(x1,op_l(i(x2),x3,x4))},i(op_r(x1,x2,x2))),op_l(op_l(x2,x3,x4),op_l(x2,x3,x4),op_r(x1,x2,x2)))
\end{aligned}
$$

Lemma 2131: $op_r(x1,x2,x2) = rd(op_t(rd(x1,op_l(i(x2),x3,x4)),i(x1)),op_l(op_l(x2,x3,x4),op_l(x2,x3,x4),op_r(x1,x2,x2)))$

$$
\begin{aligned}
& \underbrace{op_r(x1,x2,x2)} \\
= & \quad \text{by Lemma 2130 LR with } \{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{rd(op_t(rd(x1,op_l(i(x2),x3,x4)),\underbrace{i(op_r(x1,x2,x2))}),op_l(op_l(x2,x3,x4),op_l(x2,x3,x4),op_r(x1,x2,x2)))} \\
= & \quad \text{by Lemma 356 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& rd(\underbrace{op_t(rd(x1,op_l(i(x2),x3,x4)),\overbrace{op_r(i(x1),x2,x2)})},op_l(op_l(x2,x3,x4),op_l(x2,x3,x4),op_r(x1,x2,x2))) \\
= & \quad \text{by Lemma 1721 LR with } \{x3 \leftarrow x2, x2 \leftarrow i(x1), x1 \leftarrow rd(x1,op_l(i(x2),x3,x4))\} \\
& rd(\overbrace{op_t(rd(x1,op_l(i(x2),x3,x4)),i(x1))},op_l(op_l(x2,x3,x4),op_l(x2,x3,x4),op_r(x1,x2,x2)))
\end{aligned}
$$

Lemma 2132: $op_r(x1, x2, x2) = rd(mult(x1, op_l(i(i(x2)), x3, x4)), op_l(op_l(x2, x3, x4), op_l(x2, x3, x4), op_r(x1, x2, x2)))$

$\overbrace{op_r(x1, x2, x2)}$

$=$ by Lemma 2131 LR with $\{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{rd(\underbrace{op_t(rd(x1, op_l(i(x2), x3, x4)), i(x1))}, op_l(op_l(x2, x3, x4), op_l(x2, x3, x4), op_r(x1, x2, x2)))}$

$=$ by Lemma 34 LR with $\{x2 \leftarrow op_l(i(x2), x3, x4), x1 \leftarrow x1\}$

$rd(\overbrace{mult(x1, \underbrace{i(op_l(i(x2), x3, x4))})}, op_l(op_l(x2, x3, x4), op_l(x2, x3, x4), op_r(x1, x2, x2)))$

$=$ by Lemma 1049 LR with $\{x3 \leftarrow x4, x2 \leftarrow x3, x1 \leftarrow i(x2)\}$

$rd(mult(x1, \overbrace{op_l(i(i(x2)), x3, x4)}), op_l(op_l(x2, x3, x4), op_l(x2, x3, x4), op_r(x1, x2, x2)))$

Lemma 2133: $op_r(x1, op_l(x2, x3, x4), op_l(x2, x3, x4)) = op_r(x1, x2, x2)$

$\overbrace{op_r(x1, op_l(x2, x3, x4), op_l(x2, x3, x4))}$

$=$ by Lemma 641 RL with $\{x2 \leftarrow op_l(x2, x3, x4), x1 \leftarrow x1\}$

$\overbrace{rd(x1, \underbrace{asoc(op_l(x2, x3, x4), x1, op_l(x2, x3, x4))})}$

$=$ by Lemma 1734 RL with $\{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow op_l(x2, x3, x4)\}$

$rd(\underbrace{x1}, \overbrace{asoc(op_l(x2, x3, x4), op_r(x1, x2, x2), op_l(x2, x3, x4))})$

$=$ by Axiom 5 LR with $\{x2 \leftarrow op_l(x2, x3, x4), x1 \leftarrow x1\}$

$\underbrace{rd(\overbrace{rd(mult(x1, op_l(x2, x3, x4)), op_l(x2, x3, x4))}, asoc(op_l(x2, x3, x4), op_r(x1, x2, x2), op_l(x2, x3, x4)))}$

$=$ by Lemma 1575 RL with $\{x3 \leftarrow op_r(x1, x2, x2), x2 \leftarrow op_l(x2, x3, x4), x1 \leftarrow mult(x1, op_l(x2, x3, x4))\}$

$\overbrace{rd(mult(x1, op_l(\underbrace{x2}, x3, x4)), op_l(op_l(x2, x3, x4), op_l(x2, x3, x4), op_r(x1, x2, x2)))}$

$=$ by Lemma 3 RL with $\{x1 \leftarrow x2\}$

$\underbrace{rd(mult(x1, op_l(\overbrace{i(i(x2))}, x3, x4)), op_l(op_l(x2, x3, x4), op_l(x2, x3, x4), op_r(x1, x2, x2)))}$

$=$ by Lemma 2132 RL with $\{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_r(x1, x2, x2)}$

Lemma 2134: $op_t(x1, rd(x2, rd(x1, x3))) = op_t(x1, rd(x2, rd(op_t(x1, x4), x3)))$

$$
\begin{aligned}
& \overbrace{op_t(x1, rd(x2, rd(x1, x3)))} \\
= & \quad \text{by Lemma 2077 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, \underbrace{rd(mult(x2, x3), x1)})} \\
= & \quad \text{by Lemma 1158 RL with } \{x3 \leftarrow x4, x2 \leftarrow mult(x2, x3), x1 \leftarrow x1\} \\
& \overbrace{op_t(\underbrace{x1}, rd(mult(x2, x3), op_t(x1, x4)))} \\
= & \quad \text{by Axiom 5 LR with } \{x2 \leftarrow x4, x1 \leftarrow x1\} \\
& op_t(\overbrace{\underbrace{rd(mult(x1, x4), x4)}}, rd(mult(x2, x3), op_t(x1, x4))) \\
= & \quad \text{by Lemma 977 LR with } \{x1 \leftarrow x4, x2 \leftarrow mult(x1, x4)\} \\
& \underbrace{op_t(\overbrace{op_r(mult(i(x4), mult(x1, x4)), x4, mult(x1, x4))}, rd(mult(x2, x3), op_t(x1, x4)))} \\
= & \quad \text{by Axiom 14 LR with } \{x4 \leftarrow rd(mult(x2, x3), op_t(x1, x4)), x3 \leftarrow mult(x1, x4), x2 \leftarrow x4, x1 \leftarrow mult(i(x4), mult(x1, x4))\} \\
& \overbrace{op_r(op_t(\underbrace{mult(i(x4), mult(x1, x4))}, rd(mult(x2, x3), op_t(x1, x4))), x4, mult(x1, x4))} \\
= & \quad \text{by Axiom 10 LR with } \{x2 \leftarrow x1, x1 \leftarrow x4\} \\
& op_r(\underbrace{op_t(\overbrace{op_t(x1, x4)}, rd(mult(x2, x3), op_t(x1, x4)))}, x4, mult(x1, x4)) \\
= & \quad \text{by Lemma 2077 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow op_t(x1, x4)\} \\
& op_r(\overbrace{op_t(\underbrace{op_t(x1, x4)}, rd(x2, rd(op_t(x1, x4), x3)))}, x4, mult(x1, x4)) \\
= & \quad \text{by Axiom 10 RL with } \{x2 \leftarrow x1, x1 \leftarrow x4\} \\
& \underbrace{op_r(op_t(\overbrace{mult(i(x4), mult(x1, x4))}, rd(x2, rd(op_t(x1, x4), x3))), x4, mult(x1, x4))} \\
= & \quad \text{by Axiom 14 RL with } \{x4 \leftarrow rd(x2, rd(op_t(x1, x4), x3)), x3 \leftarrow mult(x1, x4), x2 \leftarrow x4, x1 \leftarrow mult(i(x4), mult(x1, x4))\} \\
& \overbrace{op_t(\underbrace{op_r(mult(i(x4), mult(x1, x4)), x4, mult(x1, x4))}, rd(x2, rd(op_t(x1, x4), x3)))} \\
= & \quad \text{by Lemma 977 RL with } \{x1 \leftarrow x4, x2 \leftarrow mult(x1, x4)\} \\
& op_t(\overbrace{\underbrace{rd(mult(x1, x4), x4)}}, rd(x2, rd(op_t(x1, x4), x3))) \\
= & \quad \text{by Axiom 5 RL with } \{x2 \leftarrow x4, x1 \leftarrow x1\} \\
& op_t(\overbrace{x1}, rd(x2, rd(op_t(x1, x4), x3)))
\end{aligned}
$$

Lemma 2135: $mult(i(x1), op_l(x2, x1, x3)) = mult(op_l(x2, x1, x3), op_t(i(x1), x2))$

$$
\begin{aligned}
& \underbrace{mult(i(x1), op_l(x2, x1, x3))} \\
= & \quad \text{by Lemma 13 RL with } \{x2 \leftarrow i(x1), x1 \leftarrow op_l(x2, x1, x3)\} \\
& \overbrace{mult(op_l(x2, x1, x3), op_t(i(x1), \underbrace{op_l(x2, x1, x3)}))} \\
= & \quad \text{by Lemma 2030 RL with } \{x3 \leftarrow x1, x2 \leftarrow x3, x1 \leftarrow x2\} \\
& mult(op_l(x2, x1, x3), \underbrace{op_t(i(x1), \overbrace{op_l(x2, x3, i(x1))})}) \\
= & \quad \text{by Lemma 2080 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow i(x1)\} \\
& mult(op_l(x2, x1, x3), \overbrace{op_t(i(x1), x2)})
\end{aligned}
$$

Lemma 2136: $mult(i(op_l(x1,x2,x3)),op_l(i(x2),op_l(x1,x2,x3),x2)) = mult(op_l(i(x2),x1,x2),op_t(i(op_l(x1,x2,x3)),i(x2)))$

$\underbrace{mult(i(op_l(x1,x2,x3)),op_l(i(x2),op_l(x1,x2,x3),x2))}$

$=$ by Lemma 2135 LR with $\{x3 \leftarrow x2, x2 \leftarrow i(x2), x1 \leftarrow op_l(x1,x2,x3)\}$

$\overbrace{mult(\underbrace{op_l(i(x2),op_l(x1,x2,x3),x2)},op_t(i(op_l(x1,x2,x3)),i(x2)))}$

$=$ by Lemma 627 RL with $\{x2 \leftarrow op_l(x1,x2,x3), x1 \leftarrow x2\}$

$mult(\overbrace{\underbrace{rd(asoc(x2,op_l(x1,x2,x3),x2),x2)}},op_t(i(op_l(x1,x2,x3)),i(x2)))$

$=$ by Lemma 784 RL with $\{x3 \leftarrow x2, x2 \leftarrow op_l(x1,x2,x3), x1 \leftarrow x2\}$

$mult(\overbrace{\underbrace{rd(i(x2),asoc(op_l(x1,x2,x3),x2,x2))}},op_t(i(op_l(x1,x2,x3)),i(x2)))$

$=$ by Lemma 1846 RL with $\{x3 \leftarrow x2, x2 \leftarrow x2, x1 \leftarrow op_l(x1,x2,x3)\}$

$mult(\overbrace{i(mult(\underbrace{asoc(op_l(x1,x2,x3),x2,x2)},x2))},op_t(i(op_l(x1,x2,x3)),i(x2)))$

$=$ by Lemma 1867 LR with $\{x2 \leftarrow mult(x3,op_l(x1,x2,x3)), x1 \leftarrow x2, x3 \leftarrow op_l(x1,x2,x3)\}$

$mult(i(mult(\overbrace{\underbrace{asoc(op_l(op_l(x1,x2,x3),rd(mult(x3,op_l(x1,x2,x3)),op_l(x1,x2,x3)),x2),i(x2),i(x2))}},x2)),op_t(i(op_l(x1,x2,$

$=$ by Lemma 772 LR with $\{x2 \leftarrow x2, x1 \leftarrow op_l(op_l(x1,x2,x3),rd(mult(x3,op_l(x1,x2,x3)),op_l(x1,x2,x3)),x2)\}$

$mult(i(mult(\overbrace{asoc(op_l(op_l(x1,x2,x3),\underbrace{rd(mult(x3,op_l(x1,x2,x3)),op_l(x1,x2,x3))},x2),x2,x2)},x2)),op_t(i(op_l(x1,x2,x3)),$

$=$ by Axiom 5 RL with $\{x2 \leftarrow op_l(x1,x2,x3), x1 \leftarrow x3\}$

$mult(i(mult(asoc(\underbrace{op_l(op_l(x1,x2,x3),\overbrace{x3},x2)},x2,x2),x2)),op_t(i(op_l(x1,x2,x3)),i(x2)))$

$=$ by Lemma 138 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$mult(\underbrace{i(mult(asoc(\overbrace{x1},x2,x2),x2))},op_t(i(op_l(x1,x2,x3)),i(x2)))$

$=$ by Lemma 1846 LR with $\{x3 \leftarrow x2, x2 \leftarrow x2, x1 \leftarrow x1\}$

$mult(\overbrace{\underbrace{rd(i(x2),asoc(x1,x2,x2))}},op_t(i(op_l(x1,x2,x3)),i(x2)))$

$=$ by Lemma 784 LR with $\{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x2\}$

$mult(\overbrace{\underbrace{rd(asoc(x2,x1,x2),x2)}},op_t(i(op_l(x1,x2,x3)),i(x2)))$

$=$ by Lemma 627 LR with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$mult(\overbrace{op_l(i(x2),x1,x2)},op_t(i(op_l(x1,x2,x3)),i(x2)))$

Lemma 2137: $rd(i(x2),op_l(x1,x2,x3)) = i(rd(x2,op_l(i(x1),x2,x3)))$

$\underbrace{rd(i(x2),op_l(x1,x2,x3))}$

$=$ by Lemma 10 RL with $\{x2 \leftarrow i(x2), x1 \leftarrow op_l(x1,x2,x3)\}$

$\overbrace{i(\underbrace{rd(op_l(x1,x2,x3),i(x2))})}$

$=$ by Lemma 38 LR with $\{x1 \leftarrow x2, x2 \leftarrow op_l(x1,x2,x3)\}$

$i(\overbrace{rd(x2,\underbrace{i(op_l(x1,x2,x3))})})$

$=$ by Lemma 1049 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$i(rd(x2,\overbrace{op_l(i(x1),x2,x3)}))$

Lemma 2138: $mult(mult(x1, op_l(x2, x3, x4)), asoc(mult(x1, op_l(x2, x3, x4)), x2, x2)) = mult(asoc(x1, x2, x2), mult(x1, op_l(x2, x3$

$\underbrace{mult(mult(x1, op_l(x2, x3, x4)), asoc(mult(x1, op_l(x2, x3, x4)), x2, x2))}$

$=$ by Lemma 2103 RL with $\{x3 \leftarrow x2, x2 \leftarrow x2, x1 \leftarrow mult(x1, op_l(x2, x3, x4))\}$

$\overbrace{mult(\underbrace{asoc(mult(x1, op_l(x2, x3, x4)), x2, x2)}, mult(x1, op_l(x2, x3, x4)))}$

$=$ by Lemma 1860 LR with $\{x4 \leftarrow x4, x3 \leftarrow x3, x1 \leftarrow x2, x2 \leftarrow x1\}$

$mult(\overbrace{\underbrace{rd(op_l(x2, x1, x2), x2)}}, mult(x1, op_l(x2, x3, x4)))$

$=$ by Lemma 765 LR with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$mult(\overbrace{asoc(x1, x2, x2)}, mult(x1, op_l(x2, x3, x4)))$

Lemma 2139: $op_t(x1, rd(x2, mult(op_t(x3, x1), x1))) = op_t(x1, rd(mult(i(x1), x3), x2))$

$op_t(x1, rd(x2, \underbrace{mult(op_t(x3, x1), x1)}))$

$=$ by Lemma 636 RL with $\{x2 \leftarrow x1, x1 \leftarrow x3\}$

$\underbrace{op_t(x1, rd(x2, \overbrace{op_l(mult(x3, x1), x1, x3)}))}$

$=$ by Lemma 2111 LR with $\{x2 \leftarrow op_l(mult(x3, x1), x1, x3), x3 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_t(x1, rd(\underbrace{rd(op_l(mult(x3, x1), x1, x3), mult(x1, x1))}, x2))}$

$=$ by Lemma 2056 RL with $\{x3 \leftarrow x1, x1 \leftarrow x3, x2 \leftarrow x1\}$

$op_t(x1, rd(\overbrace{mult(i(x1), x3)}, x2))$

Lemma 2140: $op_t(x1, rd(x2, mult(x1, op_t(x3, x1)))) = op_t(x1, rd(x3, mult(x1, x2)))$

$\underbrace{op_t(x1, rd(x2, mult(x1, op_t(x3, x1))))}$

$=$ by Lemma 1763 RL with $\{x2 \leftarrow op_t(x3, x1), x3 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{\overbrace{op_t(x1, rd(x2, mult(op_t(x3, x1), x1)))}}$

$=$ by Lemma 2139 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{\overbrace{op_t(x1, rd(mult(i(x1), x3), x2))}}$

$=$ by Lemma 2094 LR with $\{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x1\}$

$\overbrace{op_t(x1, rd(x3, mult(x1, x2)))}$

Lemma 2141: $op_t(x1, rd(x2, rd(op_t(x4, x1), x3))) = op_t(x1, rd(x2, rd(x4, x3)))$

$op_t(x1, rd(x2, \underbrace{rd(op_t(x4, x1), x3)}))$

$=$ by Lemma 10 RL with $\{x2 \leftarrow op_t(x4, x1), x1 \leftarrow x3\}$

$op_t(x1, rd(x2, \underbrace{\overbrace{i(\overbrace{rd(x3, op_t(x4, x1))})}}))$

$=$ by Lemma 9 RL with $\{x2 \leftarrow rd(x3, op_t(x4, x1)), x1 \leftarrow x1\}$

$\underbrace{op_t(x1, rd(x2, \overbrace{rd(x1, mult(rd(x3, op_t(x4, x1)), x1))}))}$

$=$ by Lemma 2134 LR with $\{x4 \leftarrow x4, x3 \leftarrow mult(rd(x3, op_t(x4, x1)), x1), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_t(x1, rd(x2, \underbrace{rd(op_t(x1, x4), mult(rd(x3, op_t(x4, x1)), x1))}))}$

$=$ by Lemma 2025 LR with $\{x3 \leftarrow x3, x2 \leftarrow x4, x1 \leftarrow x1\}$

$op_t(x1, rd(x2, \overbrace{rd(x4, x3)}))$

Lemma 2142: $mult(mult(x1, x2), op_l(x3, x2, x1)) = mult(op_l(x3, x2, x1), op_t(mult(x1, x2), x3))$

$$
\begin{aligned}
& \overbrace{mult(mult(x1, x2), op_l(x3, x2, x1))} \\
= \quad & \text{by Lemma 13 RL with } \{x2 \leftarrow mult(x1, x2), x1 \leftarrow op_l(x3, x2, x1)\} \\
& \overbrace{mult(op_l(x3, x2, x1), op_t(mult(x1, x2), \underbrace{op_l(x3, x2, x1)}))} \\
= \quad & \text{by Lemma 291 RL with } \{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x3\} \\
& mult(op_l(x3, x2, x1), \underbrace{op_t(mult(x1, x2), \overbrace{op_l(x3, x2, mult(x1, x2))})}) \\
= \quad & \text{by Lemma 2080 LR with } \{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow mult(x1, x2)\} \\
& mult(op_l(x3, x2, x1), \overbrace{op_t(mult(x1, x2), x3)})
\end{aligned}
$$

Lemma 2143: $op_t(x1, rd(x2, x1)) = op_t(x1, rd(rd(mult(x2, mult(i(x1), x3)), x3), rd(x1, x1)))$

$$
\begin{aligned}
& \underbrace{op_t(x1, rd(x2, x1))} \\
= \quad & \text{by Lemma 1634 RL with } \{x3 \leftarrow mult(i(x1), x3), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, rd(\underbrace{op_t(x2, mult(i(x1), x3))}, x1))} \\
= \quad & \text{by Axiom 10 RL with } \{x2 \leftarrow x2, x1 \leftarrow mult(i(x1), x3)\} \\
& \underbrace{op_t(x1, rd(\overbrace{mult(i(mult(i(x1), x3)), mult(x2, mult(i(x1), x3)))}, x1))} \\
= \quad & \text{by Lemma 2077 LR with } \{x3 \leftarrow mult(x2, mult(i(x1), x3)), x2 \leftarrow i(mult(i(x1), x3)), x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, \underbrace{rd(i(mult(i(x1), x3)), rd(x1, mult(x2, mult(i(x1), x3))))})} \\
= \quad & \text{by Lemma 40 LR with } \{x3 \leftarrow mult(x2, mult(i(x1), x3)), x2 \leftarrow x1, x1 \leftarrow mult(i(x1), x3)\} \\
& \underbrace{op_t(x1, \overbrace{rd(rd(mult(x2, mult(i(x1), x3)), x1), mult(i(x1), x3))})} \\
= \quad & \text{by Lemma 2079 LR with } \{x3 \leftarrow mult(i(x1), x3), x2 \leftarrow mult(x2, mult(i(x1), x3)), x1 \leftarrow x1\} \\
& \underbrace{\overbrace{op_t(x1, rd(mult(x2, mult(i(x1), x3)), mult(x1, mult(i(x1), x3))))}} \\
= \quad & \text{by Lemma 1767 LR with } \{x3 \leftarrow mult(i(x1), x3), x2 \leftarrow mult(x2, mult(i(x1), x3)), x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, \underbrace{rd(mult(i(x1), x3), mult(x1, mult(x2, mult(i(x1), x3))))})} \\
= \quad & \text{by Lemma 1427 LR with } \{x3 \leftarrow mult(x2, mult(i(x1), x3)), x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \underbrace{op_t(x1, \overbrace{rd(i(x1), mult(x1, rd(mult(x2, mult(i(x1), x3)), x3)))})} \\
= \quad & \text{by Lemma 1210 RL with } \{x3 \leftarrow mult(x1, rd(mult(x2, mult(i(x1), x3)), x3)), x2 \leftarrow i(x1), x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, rd(\underbrace{op_t(i(x1), x1)}, mult(x1, rd(mult(x2, mult(i(x1), x3)), x3))))} \\
= \quad & \text{by Lemma 345 RL with } \{x2 \leftarrow x1, x1 \leftarrow x1\} \\
& \underbrace{op_t(x1, rd(\overbrace{rd(op_t(rd(x1, x1), x1), x1)}, mult(x1, rd(mult(x2, mult(i(x1), x3)), x3))))} \\
= \quad & \text{by Lemma 1269 LR with } \{x3 \leftarrow rd(mult(x2, mult(i(x1), x3)), x3), x2 \leftarrow op_t(rd(x1, x1), x1), x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, rd(\underbrace{rd(mult(x2, mult(i(x1), x3)), x3)}, op_t(rd(x1, x1), x1)))} \\
= \quad & \text{by Axiom 4 LR with } \{x2 \leftarrow rd(x1, x1), x1 \leftarrow rd(mult(x2, mult(i(x1), x3)), x3)\} \\
& \underbrace{op_t(x1, rd(\overbrace{mult(rd(rd(mult(x2, mult(i(x1), x3)), x3), rd(x1, x1)), rd(x1, x1))}, op_t(rd(x1, x1), x1)))} \\
= \quad & \text{by Lemma 1240 RL with } \{x3 \leftarrow rd(x1, x1), x2 \leftarrow rd(rd(mult(x2, mult(i(x1), x3)), x3), rd(x1, x1)), x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, rd(rd(mult(x2, mult(i(x1), x3)), x3), rd(x1, x1)))}
\end{aligned}
$$

Lemma 2144: $op_t(x1, rd(x2, x1)) = op_t(x1, rd(mult(x2, mult(i(x1), x3)), x3))$

$$
\begin{aligned}
& \overbrace{op_t(x1, rd(x2, x1))} \\
= \quad & \text{by Lemma 2143 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, rd(\underbrace{rd(mult(x2, mult(i(x1), x3)), x3)}, \underbrace{rd(x1, x1)}))} \\
= \quad & \text{by Lemma 2 LR with } \{x1 \leftarrow x1\} \\
& op_t(x1, \underbrace{rd(rd(mult(x2, mult(i(x1), x3)), x3)}, \overbrace{unit()})) \\
= \quad & \text{by Lemma 1 LR with } \{x1 \leftarrow rd(mult(x2, mult(i(x1), x3)), x3)\} \\
& op_t(x1, \overbrace{rd(mult(x2, mult(i(x1), x3)), x3)})
\end{aligned}
$$

Lemma 2145: $op_t(x1, rd(x2, x1)) = op_t(x1, op_r(mult(x2, i(x1)), i(mult(x2, mult(i(x1), x2))), i(x3)))$

$$
\begin{aligned}
& \overbrace{op_t(x1, rd(x2, x1))} \\
= \quad & \text{by Lemma 2144 LR with } \{x3 \leftarrow mult(x2, op_t(x3, mult(x2, mult(i(x1), x2)))), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, rd(\underbrace{mult(x2, mult(i(x1), mult(x2, op_t(x3, mult(x2, mult(i(x1), x2))))))}, mult(x2, op_t(x3, mult(x2, mult(i(x1), x2))))} \\
= \quad & \text{by Axiom 6 LR with } \{x3 \leftarrow op_t(x3, mult(x2, mult(i(x1), x2))), x2 \leftarrow i(x1), x1 \leftarrow x2\} \\
& op_t(x1, rd(\overbrace{mult(mult(x2, mult(i(x1), x2)), op_t(x3, mult(x2, mult(i(x1), x2))))}, mult(x2, op_t(x3, mult(x2, mult(i(x1), x2))) \\
= \quad & \text{by Lemma 13 LR with } \{x2 \leftarrow x3, x1 \leftarrow mult(x2, mult(i(x1), x2))\} \\
& op_t(x1, \underbrace{rd(\overbrace{mult(x3, mult(x2, mult(i(x1), x2)))}, mult(x2, op_t(x3, mult(x2, mult(i(x1), x2))))}) \\
= \quad & \text{by Lemma 10 RL with } \{x2 \leftarrow mult(x3, mult(x2, mult(i(x1), x2))), x1 \leftarrow mult(x2, op_t(x3, mult(x2, mult(i(x1), x2))))\} \\
& op_t(x1, \overbrace{i(\underbrace{rd(mult(x2, op_t(x3, mult(x2, mult(i(x1), x2)))), mult(x3, mult(x2, mult(i(x1), x2)))}))}) \\
= \quad & \text{by Lemma 1965 RL with } \{x3 \leftarrow mult(x2, mult(i(x1), x2)), x2 \leftarrow x3, x1 \leftarrow mult(x2, op_t(x3, mult(x2, mult(i(x1), x2))))\} \\
& op_t(x1, i(\overbrace{op_r(\underbrace{rd(\underbrace{rd(mult(x2, op_t(x3, mult(x2, mult(i(x1), x2)))), op_t(x3, mult(x2, mult(i(x1), x2))))}, mult(x2, mult(i(x1),} \\
= \quad & \text{by Axiom 5 RL with } \{x2 \leftarrow op_t(x3, mult(x2, mult(i(x1), x2))), x1 \leftarrow x2\} \\
& op_t(x1, i(op_r(\underbrace{rd(\overbrace{x2}, mult(x2, mult(i(x1), x2)))}, mult(x2, mult(i(x1), x2)), x3))) \\
= \quad & \text{by Lemma 527 LR with } \{x2 \leftarrow i(x1), x1 \leftarrow x2\} \\
& op_t(x1, i(\underbrace{op_r(\overbrace{op_r(i(mult(x2, i(x1))), x2, x2)}, mult(x2, mult(i(x1), x2)), x3)})) \\
= \quad & \text{by Axiom 17 RL with } \{x5 \leftarrow x2, x4 \leftarrow x2, x3 \leftarrow x3, x2 \leftarrow mult(x2, mult(i(x1), x2)), x1 \leftarrow i(mult(x2, i(x1)))\} \\
& op_t(x1, \underbrace{i(\overbrace{op_r(op_r(i(mult(x2, i(x1))), mult(x2, mult(i(x1), x2)), x3), x2, x2)})}) \\
= \quad & \text{by Lemma 356 LR with } \{x2 \leftarrow x2, x1 \leftarrow op_r(i(mult(x2, i(x1))), mult(x2, mult(i(x1), x2)), x3)\} \\
& \underbrace{op_t(x1, \overbrace{op_r(i(op_r(i(mult(x2, i(x1))), mult(x2, mult(i(x1), x2)), x3)), x2, x2)})} \\
= \quad & \text{by Lemma 1721 LR with } \{x3 \leftarrow x2, x2 \leftarrow i(op_r(i(mult(x2, i(x1))), mult(x2, mult(i(x1), x2)), x3)), x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, \underbrace{i(op_r(i(mult(x2, i(x1))), mult(x2, mult(i(x1), x2)), x3))})} \\
= \quad & \text{by Lemma 813 LR with } \{x3 \leftarrow x3, x2 \leftarrow mult(x2, mult(i(x1), x2)), x1 \leftarrow mult(x2, i(x1))\} \\
& op_t(x1, \overbrace{op_r(mult(x2, i(x1)), i(mult(x2, mult(i(x1), x2))), i(x3))})
\end{aligned}
$$

Lemma 2146: $op_t(x1, rd(x2, x1)) = op_t(x1, i(op_r(mult(i(x2), x1), mult(x2, mult(i(x1), x2)), x3)))$

$\overbrace{op_t(x1, rd(x2, x1))}$

$=$ by Lemma 2145 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_t(x1, \underbrace{op_r(mult(x2, i(x1)), i(mult(x2, mult(i(x1), x2)))}, i(x3)))}$

$=$ by Lemma 813 RL with $\{x3 \leftarrow x3, x2 \leftarrow mult(x2, mult(i(x1), x2)), x1 \leftarrow mult(x2, i(x1))\}$

$op_t(x1, \overbrace{i(op_r(\underbrace{i(mult(x2, i(x1)))}, mult(x2, mult(i(x1), x2)), x3)))}$

$=$ by Lemma 17 LR with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$op_t(x1, i(op_r(\overbrace{mult(i(x2), x1)}, mult(x2, mult(i(x1), x2)), x3)))$

Lemma 2147: $op_t(x1, rd(x2, x1)) = op_t(x1, op_r(mult(x2, i(x1)), x2, x3))$

$\overbrace{op_t(x1, rd(x2, x1))}$

$=$ by Lemma 2146 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_t(x1, i(op_r(mult(i(x2), x1), \underbrace{mult(x2, mult(i(x1), x2))}, x3)))}$

$=$ by Lemma 60 LR with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$op_t(x1, i(\underbrace{op_r(mult(i(x2), x1), \overbrace{rd(x2, mult(i(x2), x1))}, x3)}))$

$=$ by Lemma 1940 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow mult(i(x2), x1)\}$

$op_t(x1, i(\overbrace{op_r(mult(i(x2), x1), i(x2), i(x3))}))$

$=$ by Lemma 813 RL with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow mult(i(x2), x1)\}$

$op_t(x1, i(\overbrace{i(op_r(\underbrace{i(mult(i(x2), x1))}, x2, x3))}))$

$=$ by Lemma 16 LR with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$op_t(x1, \underbrace{i(i(op_r(\overbrace{mult(x2, i(x1))}, x2, x3)))})$

$=$ by Lemma 3 LR with $\{x1 \leftarrow op_r(mult(x2, i(x1)), x2, x3)\}$

$op_t(x1, \overbrace{op_r(mult(x2, i(x1)), x2, x3)})$

Lemma 2148: $mult(rd(x1,x2),op_r(x2,x1,x3)) = mult(op_r(x2,x1,x3),mult(i(x2),x1))$

$\underbrace{mult(rd(x1,x2),op_r(x2,x1,x3))}$

$=$ by Lemma 13 RL with $\{x2 \leftarrow rd(x1,x2), x1 \leftarrow op_r(x2,x1,x3)\}$

$\overbrace{mult(op_r(x2,x1,x3),\underbrace{op_t(rd(x1,x2),op_r(x2,x1,x3))})}$

$=$ by Lemma 1721 RL with $\{x3 \leftarrow x1, x2 \leftarrow op_r(x2,x1,x3), x1 \leftarrow rd(x1,x2)\}$

$mult(op_r(x2,x1,x3),\overbrace{op_t(rd(x1,x2),\underbrace{op_r(op_r(x2,x1,x3),x1,x1)})})$

$=$ by Axiom 17 LR with $\{x5 \leftarrow x1, x4 \leftarrow x1, x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow x2\}$

$mult(op_r(x2,x1,x3),op_t(rd(x1,x2),\overbrace{op_r(\underbrace{op_r(x2,x1,x1)},x1,x3)}))$

$=$ by Lemma 353 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$mult(op_r(x2,x1,x3),\underbrace{op_t(rd(x1,x2),op_r(\overbrace{rd(x1,rd(x1,x2))},x1,x3))})$

$=$ by Lemma 1722 RL with $\{x3 \leftarrow i(x1), x2 \leftarrow op_r(rd(x1,rd(x1,x2)),x1,x3), x1 \leftarrow rd(x1,x2)\}$

$mult(op_r(x2,x1,x3),\overbrace{op_t(rd(x1,x2),\underbrace{op_t(op_r(rd(x1,rd(x1,x2)),x1,x3),i(x1))})})$

$=$ by Axiom 14 LR with $\{x4 \leftarrow i(x1), x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow rd(x1,rd(x1,x2))\}$

$mult(op_r(x2,x1,x3),op_t(rd(x1,x2),\overbrace{op_r(\underbrace{op_t(rd(x1,rd(x1,x2)),i(x1))},x1,x3)}))$

$=$ by Lemma 34 LR with $\{x2 \leftarrow rd(x1,x2), x1 \leftarrow x1\}$

$mult(op_r(x2,x1,x3),\underbrace{op_t(rd(x1,x2),op_r(\overbrace{mult(x1,i(rd(x1,x2)))},x1,x3))})$

$=$ by Lemma 2147 RL with $\{x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow rd(x1,x2)\}$

$mult(op_r(x2,x1,x3),\overbrace{op_t(rd(x1,x2),\underbrace{rd(x1,rd(x1,x2))})})$

$=$ by Lemma 353 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$mult(op_r(x2,x1,x3),\underbrace{op_t(rd(x1,x2),\overbrace{op_r(x2,x1,x1)})})$

$=$ by Lemma 1721 LR with $\{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow rd(x1,x2)\}$

$mult(op_r(x2,x1,x3),\overbrace{\underbrace{op_t(rd(x1,x2),x2)}})$

$=$ by Lemma 14 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$mult(op_r(x2,x1,x3),\overbrace{mult(i(x2),x1)})$

Lemma 2149: $mult(rd(x1,x2),op_r(x2,x1,x3)) = mult(rd(op_r(x2,x1,x3),x2),x1)$

$\underbrace{mult(rd(x1,x2),op_r(x2,x1,x3))}$

$=$ by Lemma 2148 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{mult(op_r(x2,x1,x3),\underbrace{mult(i(x2),x1)})}$

$=$ by Lemma 1444 RL with $\{x4 \leftarrow x3, x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow mult(i(x2),x1)\}$

$\underbrace{mult(op_r(x2,x1,x3),\overbrace{op_l(mult(i(x2),x1),x2,op_r(x2,x1,x3))})}$

$=$ by Lemma 1221 LR with $\{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow op_r(x2,x1,x3)\}$

$\overbrace{mult(rd(op_r(x2,x1,x3),x2),x1)}$

Lemma 2150: $rd(mult(x1,x2),op_l(x2,x1,x3)) = mult(op_l(i(x2),x1,x3),mult(x2,x1))$

$\overbrace{rd(mult(x1,x2),op_l(x2,x1,x3))}$

$=$ by Lemma 440 RL with $\{x2 \leftarrow mult(x1,x2), x1 \leftarrow op_l(x2,x1,x3)\}$

$\overbrace{mult(i(op_l(x2,x1,x3)),op_l(mult(x1,x2),mult(x1,x2),\underbrace{op_l(x2,x1,x3)}))}$

$=$ by Lemma 286 RL with $\{x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow x2\}$

$mult(i(op_l(x2,x1,x3)),op_l(mult(x1,x2),mult(x1,x2),\overbrace{op_l(x2,x1,rd(x3,x1))}))$

$=$ by Axiom 12 RL with $\{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow rd(x3,x1)\}$

$mult(i(op_l(x2,x1,x3)),op_l(mult(x1,x2),mult(x1,x2),\overbrace{mult(i(\underbrace{mult(rd(x3,x1),x1)}),mult(rd(x3,x1),mult(x1,x2)))}))$

$=$ by Axiom 4 RL with $\{x2 \leftarrow x1, x1 \leftarrow x3\}$

$mult(i(op_l(x2,x1,x3)),\underbrace{op_l(mult(x1,x2),mult(x1,x2),mult(i(\overbrace{x3}),mult(rd(x3,x1),mult(x1,x2))))})$

$=$ by Lemma 1824 LR with $\{x3 \leftarrow rd(x3,x1), x2 \leftarrow i(x3), x1 \leftarrow mult(x1,x2)\}$

$mult(i(op_l(x2,x1,x3)),\overbrace{op_l(mult(x1,x2),mult(x1,x2),mult(i(x3),rd(x3,x1)))})$

$=$ by Lemma 1745 RL with $\{x3 \leftarrow rd(x3,x1), x2 \leftarrow x3, x1 \leftarrow mult(x1,x2)\}$

$mult(i(op_l(x2,x1,x3)),\overbrace{op_l(mult(x1,x2),\underbrace{rd(x3,rd(x3,x1))},mult(x1,x2))})$

$=$ by Lemma 353 LR with $\{x2 \leftarrow x1, x1 \leftarrow x3\}$

$mult(i(op_l(x2,x1,x3)),\underbrace{op_l(mult(x1,x2),\overbrace{op_r(x1,x3,x3)},mult(x1,x2))})$

$=$ by Lemma 1737 LR with $\{x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow mult(x1,x2)\}$

$mult(i(op_l(x2,x1,x3)),\overbrace{\underbrace{op_l(mult(x1,x2),x1,mult(x1,x2))}})$

$=$ by Lemma 1330 LR with $\{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow mult(x1,x2)\}$

$mult(i(op_l(x2,x1,x3)),\overbrace{\underbrace{op_l(mult(x1,x2),x2,i(x1))}})$

$=$ by Lemma 422 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$mult(\underbrace{i(op_l(x2,x1,x3))},\overbrace{mult(x2,x1)})$

$=$ by Lemma 1049 LR with $\{x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow x2\}$

$mult(\overbrace{op_l(i(x2),x1,x3)},mult(x2,x1))$

Lemma 2151: $mult(rd(x1,op_l(x1,x2,x3)),x2) = rd(mult(x2,x1),op_l(x1,x2,x3))$

$\underbrace{mult(rd(x1,op_l(x1,x2,x3)),x2)}$

$=$ by Lemma 2003 RL with $\{x4 \leftarrow x2, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{\underbrace{mult(op_l(i(x1),x2,x3),mult(x1,x2))}}$

$=$ by Lemma 2150 RL with $\{x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow x2\}$

$\overbrace{rd(mult(x2,x1),op_l(x1,x2,x3))}$

Lemma 2152: $rd(mult(x1, op_l(x2, x3, x1)), x2) = mult(rd(op_l(x2, x3, x1), x2), x1)$

$$
\begin{aligned}
& rd(mult(x1, op_l(x2, x3, x1)), \underbrace{x2}) \\
= & \quad \text{by Lemma 138 RL with } \{x3 \leftarrow x1, x2 \leftarrow x3, x1 \leftarrow x2\} \\
& \underbrace{rd(mult(x1, op_l(x2, x3, x1)), \overbrace{op_l(op_l(x2, x3, x1), x1, x3)})} \\
= & \quad \text{by Lemma 2151 RL with } \{x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow op_l(x2, x3, x1)\} \\
& \overbrace{mult(rd(op_l(x2, x3, x1), \underbrace{op_l(op_l(x2, x3, x1), x1, x3)}), x1)} \\
= & \quad \text{by Lemma 138 LR with } \{x3 \leftarrow x1, x2 \leftarrow x3, x1 \leftarrow x2\} \\
& mult(rd(op_l(x2, x3, x1), \overbrace{x2}), x1)
\end{aligned}
$$

Lemma 2153: $mult(rd(x1, x2), op_l(x2, x3, x1)) = rd(mult(x1, op_l(x2, x3, x1)), x2)$

$$
\begin{aligned}
& \underbrace{mult(rd(x1, x2), op_l(x2, x3, x1))} \\
= & \quad \text{by Lemma 2121 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{mult(rd(op_l(x2, x3, x1), x2), x1)}} \\
= & \quad \text{by Lemma 2152 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{rd(mult(x1, op_l(x2, x3, x1)), x2)}
\end{aligned}
$$

Lemma 2154: $rd(mult(x1, x2), op_l(x2, x3, x1)) = mult(rd(x2, op_l(x2, x3, x1)), x1)$

$$
\begin{aligned}
& rd(mult(x1, x2), \underbrace{op_l(x2, x3, x1)}) \\
= & \quad \text{by Lemma 1081 RL with } \{x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \underbrace{rd(mult(x1, x2), \overbrace{op_l(x2, x1, op_l(rd(x1, x3), x3, x1))})} \\
= & \quad \text{by Lemma 2151 RL with } \{x3 \leftarrow op_l(rd(x1, x3), x3, x1), x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \overbrace{mult(rd(x2, \underbrace{op_l(x2, x1, op_l(rd(x1, x3), x3, x1))}), x1)} \\
= & \quad \text{by Lemma 1081 LR with } \{x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow x2\} \\
& mult(rd(x2, \overbrace{op_l(x2, x3, x1)}), x1)
\end{aligned}
$$

Lemma 2155: $mult(rd(op_l(x1,x2,x3),x1),x2) = rd(mult(x2,op_l(x1,x2,x3)),x1)$

$\overbrace{mult(rd(op_l(x1,x2,x3),x1),x2)}$

$=$ by Lemma 2001 RL with $\{x4 \leftarrow x2, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{mult(op_l(x1,x2,x3),\underbrace{mult(i(x1),x2)})}$

$=$ by Lemma 14 RL with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$mult(op_l(x1,x2,x3),\overbrace{op_t(rd(x2,x1),x1)})$

$=$ by Lemma 2117 LR with $\{x3 \leftarrow op_l(rd(mult(rd(x2,x1),x1),x3),x3,mult(rd(x2,x1),x1)), x2 \leftarrow x1, x1 \leftarrow rd(x2,x1)\}$

$mult(op_l(x1,x2,x3),\overbrace{op_t(rd(x2,x1),\underbrace{op_l(x1,i(mult(rd(x2,x1),x1)),op_l(rd(mult(rd(x2,x1),x1),x3),x3,mult(rd(x2,x1),x}$

$=$ by Lemma 2031 LR with $\{x3 \leftarrow op_l(rd(mult(rd(x2,x1),x1),x3),x3,mult(rd(x2,x1),x1)), x2 \leftarrow mult(rd(x2,x1),x1), x1 \leftarrow$

$mult(op_l(x1,x2,x3),op_t(rd(x2,x1),\overbrace{op_l(x1,op_l(rd(mult(rd(x2,x1),x1),x3),x3,mult(rd(x2,x1),x1)),mult(rd(x2,x1),x1}$

$=$ by Lemma 1076 LR with $\{x3 \leftarrow x3, x2 \leftarrow mult(rd(x2,x1),x1), x1 \leftarrow x1\}$

$mult(op_l(x1,x2,x3),op_t(rd(x2,x1),\overbrace{op_l(x1,\underbrace{mult(rd(x2,x1),x1)},x3)}))$

$=$ by Axiom 4 RL with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$\underbrace{mult(op_l(x1,x2,x3),op_t(rd(x2,x1),op_l(x1,\overbrace{x2},x3)))}$

$=$ by Lemma 13 LR with $\{x2 \leftarrow rd(x2,x1), x1 \leftarrow op_l(x1,x2,x3)\}$

$\overbrace{mult(rd(x2,x1),\underbrace{op_l(x1,x2,x3)})}$

$=$ by Lemma 1076 RL with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{mult(rd(x2,x1),\overbrace{op_l(x1,op_l(rd(x2,x3),x3,x2),x2)})}$

$=$ by Lemma 2153 LR with $\{x3 \leftarrow op_l(rd(x2,x3),x3,x2), x2 \leftarrow x1, x1 \leftarrow x2\}$

$\overbrace{rd(mult(x2,\underbrace{op_l(x1,op_l(rd(x2,x3),x3,x2),x2)}),x1)}$

$=$ by Lemma 1076 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$rd(mult(x2,\overbrace{op_l(x1,x2,x3)}),x1)$

Lemma 2156: $op_l(mult(x1,x2),x1,x2) = op_l(mult(x1,x2),mult(x1,x2),i(op_r(x1,x2,x3)))$

$\underbrace{op_l(mult(x1,x2),x1,x2)}$

$=$ by Lemma 242 RL with $\{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow mult(x1,x2)\}$

$\overbrace{\underbrace{op_l(mult(x1,x2),mult(x1,x2),i(x1))}}$

$=$ by Lemma 1852 RL with $\{x3 \leftarrow x3, x2 \leftarrow i(x1), x1 \leftarrow mult(x1,x2)\}$

$\overbrace{op_l(mult(x1,x2),mult(x1,x2),\underbrace{op_r(i(x1),mult(x1,x2),x3)})}$

$=$ by Lemma 654 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$op_l(mult(x1,x2),mult(x1,x2),\overbrace{i(op_r(x1,x2,x3))})$

Lemma 2157: $mult(x2, x1) = op_l(mult(x1, x2), op_r(x1, x2, x3), mult(x1, x2))$

$$
\begin{aligned}
& \overbrace{mult(x2, x1)} \\
= \quad & \text{by Lemma 435 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(mult(x1, x2), x1, x2)} \\
= \quad & \text{by Lemma 2156 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(mult(x1, x2), mult(x1, x2), i(op_r(x1, x2, x3)))} \\
= \quad & \text{by Lemma 608 LR with } \{x2 \leftarrow op_r(x1, x2, x3), x1 \leftarrow mult(x1, x2)\} \\
& \overbrace{op_l(mult(x1, x2), op_r(x1, x2, x3), mult(x1, x2))}
\end{aligned}
$$

Lemma 2158: $mult(rd(x1, op_r(x1, x2, x3)), x2) = rd(mult(x2, x1), op_r(x1, x2, x3))$

$$
\begin{aligned}
& \overbrace{mult(rd(x1, op_r(x1, x2, x3)), x2)} \\
= \quad & \text{by Lemma 1979 RL with } \{x4 \leftarrow x2, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{mult(i(op_r(x1, x2, x3)), mult(x1, x2))} \\
= \quad & \text{by Lemma 615 RL with } \{x2 \leftarrow op_r(x1, x2, x3), x1 \leftarrow mult(x1, x2)\} \\
& \overbrace{rd(\underbrace{op_l(mult(x1, x2), op_r(x1, x2, x3), mult(x1, x2))}, op_r(x1, x2, x3))} \\
= \quad & \text{by Lemma 2157 RL with } \{x3 \leftarrow x3, x1 \leftarrow x1, x2 \leftarrow x2\} \\
& rd(\overbrace{mult(x2, x1)}, op_r(x1, x2, x3))
\end{aligned}
$$

Lemma 2159: $op_t(x1, rd(x2, op_r(x2, x1, x3))) = x1$

$$
\begin{aligned}
& \underbrace{op_t(x1, rd(x2, op_r(x2, x1, x3)))} \\
= \quad & \text{by Lemma 29 RL with } \{x2 \leftarrow op_t(x1, rd(x2, op_r(x2, x1, x3))), x1 \leftarrow mult(rd(x2, op_r(x2, x1, x3)), x1)\} \\
& \overbrace{rd(\underbrace{mult(mult(rd(x2, op_r(x2, x1, x3)), x1), op_t(x1, rd(x2, op_r(x2, x1, x3))))}, op_t(mult(rd(x2, op_r(x2, x1, x3)), x1), op_t(x1, rd} \\
= \quad & \text{by Lemma 871 RL with } \{x2 \leftarrow x1, x1 \leftarrow rd(x2, op_r(x2, x1, x3))\} \\
& rd(\overbrace{mult(mult(x1, rd(x2, op_r(x2, x1, x3))), x1)}, \underbrace{op_t(mult(rd(x2, op_r(x2, x1, x3)), x1), op_t(x1, rd(x2, op_r(x2, x1, x3))))}) \\
= \quad & \text{by Lemma 575 RL with } \{x2 \leftarrow x1, x1 \leftarrow rd(x2, op_r(x2, x1, x3))\} \\
& rd(mult(mult(x1, rd(x2, op_r(x2, x1, x3))), x1), \overbrace{op_t(\underbrace{mult(rd(x2, op_r(x2, x1, x3)), x1)}, x1)}) \\
= \quad & \text{by Lemma 2158 LR with } \{x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow x2\} \\
& rd(mult(\underbrace{mult(x1, rd(x2, op_r(x2, x1, x3)))}, x1), op_t(\overbrace{rd(mult(x1, x2), op_r(x2, x1, x3))}, x1)) \\
= \quad & \text{by Lemma 2046 LR with } \{x4 \leftarrow x3, x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{rd(mult(\overbrace{rd(mult(x1, x2), op_r(x2, x1, x3))}, x1), op_t(rd(mult(x1, x2), op_r(x2, x1, x3)), x1))} \\
= \quad & \text{by Lemma 29 LR with } \{x2 \leftarrow x1, x1 \leftarrow rd(mult(x1, x2), op_r(x2, x1, x3))\} \\
& \overbrace{x1}
\end{aligned}
$$

Lemma 2160: $op_l(mult(x1,x2),mult(x1,x2),x2) = op_l(mult(x1,x2),mult(x1,x2),op_l(i(x1),x3,x2))$

$\underbrace{op_l(mult(x1,x2),mult(x1,x2),x2)}$

$=$ by Lemma 773 LR with $\{x2 \leftarrow x2, x1 \leftarrow mult(x1,x2)\}$

$\overbrace{rd(mult(x1,x2),\underbrace{asoc(x2,mult(x1,x2),mult(x1,x2))})}$

$=$ by Lemma 1472 RL with $\{x3 \leftarrow asoc(x2,mult(x1,x2),mult(x1,x2)), x2 \leftarrow x4, x1 \leftarrow mult(x1,x2)\}$

$\overbrace{rd(mult(mult(x1,x2),\underbrace{op_l(x4,asoc(x2,mult(x1,x2),mult(x1,x2)),mult(x1,x2))}),mult(asoc(x2,mult(x1,x2),mult(x1,x$

$=$ by Lemma 946 LR with $\{x3 \leftarrow mult(x1,x2), x2 \leftarrow x2, x1 \leftarrow x4\}$

$rd(mult(mult(x1,x2),\overbrace{x4}),\underbrace{mult(asoc(x2,mult(x1,x2),mult(x1,x2)),x4)})$

$=$ by Lemma 1708 LR with $\{x3 \leftarrow x4, x2 \leftarrow mult(x1,x2), x1 \leftarrow x2\}$

$rd(mult(mult(x1,x2),x4),\overbrace{rd(x4,\underbrace{asoc(mult(x1,x2),x2,mult(x1,x2))})})$

$=$ by Lemma 1868 LR with $\{x3 \leftarrow i(mult(x2,x3)), x2 \leftarrow x2, x1 \leftarrow mult(x1,x2)\}$

$rd(mult(mult(x1,x2),x4),rd(x4,\overbrace{asoc(mult(x1,x2),\underbrace{op_l(rd(x2,mult(x1,x2)),x2,i(mult(x2,x3)))},mult(x1,x2))}))$

$=$ by Lemma 1051 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow rd(x2,mult(x1,x2))\}$

$rd(mult(mult(x1,x2),x4),\underbrace{rd(x4,asoc(mult(x1,x2),\overbrace{op_l(rd(x2,mult(x1,x2)),x3,x2)},mult(x1,x2)))})$

$=$ by Lemma 1708 RL with $\{x3 \leftarrow x4, x2 \leftarrow mult(x1,x2), x1 \leftarrow op_l(rd(x2,mult(x1,x2)),x3,x2)\}$

$rd(mult(mult(x1,x2),\underbrace{x4}),\overbrace{mult(asoc(op_l(rd(x2,mult(x1,x2)),x3,x2),mult(x1,x2),mult(x1,x2)),x4)})$

$=$ by Lemma 946 RL with $\{x3 \leftarrow mult(x1,x2), x2 \leftarrow op_l(rd(x2,mult(x1,x2)),x3,x2), x1 \leftarrow x4\}$

$\underbrace{rd(mult(mult(x1,x2),\overbrace{op_l(x4,asoc(op_l(rd(x2,mult(x1,x2)),x3,x2),mult(x1,x2),mult(x1,x2)),mult(x1,x2))}),mult(asoc}$

$=$ by Lemma 1472 LR with $\{x3 \leftarrow asoc(op_l(rd(x2,mult(x1,x2)),x3,x2),mult(x1,x2),mult(x1,x2)), x2 \leftarrow x4, x1 \leftarrow mult(x1,$

$\overbrace{rd(mult(x1,x2),\underbrace{asoc(op_l(rd(x2,mult(x1,x2)),x3,x2),mult(x1,x2),mult(x1,x2))})}$

$=$ by Lemma 773 RL with $\{x2 \leftarrow op_l(rd(x2,mult(x1,x2)),x3,x2), x1 \leftarrow mult(x1,x2)\}$

$\overbrace{op_l(mult(x1,x2),mult(x1,x2),\underbrace{op_l(rd(x2,mult(x1,x2)),x3,x2)})}$

$=$ by Lemma 9 LR with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$op_l(mult(x1,x2),mult(x1,x2),op_l(\overbrace{i(x1)},x3,x2))$

Lemma 2161: $mult(x2,x1) = op_l(mult(x1,x2),mult(x1,x2),op_l(i(x1),x3,x2))$

$\underbrace{mult(x2,x1)}$

$=$ by Lemma 435 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_l(mult(x1,x2),x1,x2)}}$

$=$ by Lemma 285 RL with $\{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow mult(x1,x2)\}$

$\overbrace{\underbrace{op_l(mult(x1,x2),mult(x1,x2),x2)}}$

$=$ by Lemma 2160 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_l(mult(x1,x2),mult(x1,x2),op_l(i(x1),x3,x2))}$

Lemma 2162: $op_r(x1, op_r(x3, x2, x2), x2) = op_r(x1, op_l(x3, x2, x3), x2)$

$op_r(x1, \underbrace{op_r(x3, x2, x2)}, x2)$

$=$ by Lemma 353 RL with $\{x2 \leftarrow x3, x1 \leftarrow x2\}$

$op_r(x1, \overbrace{rd(x2, \underbrace{rd(x2, x3)})}, x2)$

$=$ by Axiom 2 LR with $\{x2 \leftarrow x2, x1 \leftarrow rd(x2, x3)\}$

$op_r(x1, \underbrace{rd(x2, \overbrace{mult(i(x2), mult(x2, rd(x2, x3)))})}, x2)$

$=$ by Lemma 60 RL with $\{x2 \leftarrow mult(x2, rd(x2, x3)), x1 \leftarrow x2\}$

$op_r(x1, \overbrace{mult(x2, mult(\underbrace{i(mult(x2, rd(x2, x3)))}, x2))}, x2)$

$=$ by Lemma 45 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x2\}$

$op_r(x1, \underbrace{mult(x2, mult(\overbrace{mult(i(x2), rd(x3, x2))}, x2))}, x2)$

$=$ by Lemma 58 RL with $\{x2 \leftarrow mult(i(x2), rd(x3, x2)), x1 \leftarrow x2\}$

$op_r(x1, \underbrace{rd(x2, i(\overbrace{mult(x2, mult(i(x2), rd(x3, x2)))}))}, x2)$

$=$ by Lemma 38 RL with $\{x1 \leftarrow x2, x2 \leftarrow mult(x2, mult(i(x2), rd(x3, x2)))\}$

$\underbrace{op_r(x1, \overbrace{rd(mult(x2, mult(i(x2), rd(x3, x2))), i(x2))}, x2)}$

$=$ by Lemma 1897 RL with $\{x3 \leftarrow x2, x2 \leftarrow mult(x2, mult(i(x2), rd(x3, x2))), x1 \leftarrow x1\}$

$\overbrace{op_r(x1, \underbrace{mult(op_t(mult(x2, \overbrace{mult(i(x2), rd(x3, x2))}), x2), x2)}, x2)}$

$=$ by Lemma 350 LR with $\{x2 \leftarrow mult(i(x2), rd(x3, x2)), x1 \leftarrow x2\}$

$op_r(x1, \underbrace{\overbrace{mult(mult(i(x2), rd(x3, x2)), mult(x2, x2))}, x2)}$

$=$ by Lemma 280 RL with $\{x3 \leftarrow x2, x2 \leftarrow x2, x1 \leftarrow x3\}$

$op_r(x1, \underbrace{\overbrace{op_l(x3, rd(x2, x3), x2)}, x2)}$

$=$ by Lemma 2052 RL with $\{x4 \leftarrow x2, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x3\}$

$op_r(x1, \underbrace{\overbrace{op_l(x3, x2, rd(x3, x2))}, x2)}$

$=$ by Lemma 286 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x3\}$

$op_r(x1, \overbrace{op_l(x3, x2, x3)}, x2)$

Lemma 2163: $op_r(i(op_t(rd(x1, mult(x1, x2)), mult(x2, mult(x1, x2)))), x3, i(x4)) = op_r(i(i(x2)), rd(rd(x1, mult(x1, x2)), x3), x4$

$op_r(\overbrace{i(op_t(rd(x1, mult(x1, x2)), mult(x2, mult(x1, x2))))}, x3, i(x4))$

$=$ by Lemma 15 RL with $\{x2 \leftarrow mult(x2, mult(x1, x2)), x1 \leftarrow rd(x1, mult(x1, x2))\}$

$op_r(\overbrace{op_t(i(rd(x1, mult(x1, x2))), i(mult(x2, mult(x1, x2))))}, x3, i(x4))$

$=$ by Lemma 1188 RL with $\{x2 \leftarrow mult(x2, mult(x1, x2)), x1 \leftarrow i(rd(x1, mult(x1, x2)))\}$

$\underbrace{op_r(\overbrace{op_r(i(rd(x1, mult(x1, x2))), mult(x2, mult(x1, x2)), rd(i(rd(x1, mult(x1, x2))), mult(x2, mult(x1, x2))))}, x3, i(x4))}$

$=$ by Axiom 17 LR with $\{x5 \leftarrow i(x4), x4 \leftarrow x3, x3 \leftarrow rd(i(rd(x1, mult(x1, x2))), mult(x2, mult(x1, x2))), x2 \leftarrow mult(x2, mult($

$\overbrace{op_r(\underbrace{op_r(i(rd(x1, mult(x1, x2))), x3, i(x4))}, mult(x2, mult(x1, x2)), rd(i(rd(x1, mult(x1, x2))), mult(x2, mult(x1, x2))))}$

$=$ by Lemma 1948 RL with $\{x3 \leftarrow x4, x2 \leftarrow x3, x1 \leftarrow rd(x1, mult(x1, x2))\}$

$\underbrace{op_r(\overbrace{op_r(i(rd(x1, mult(x1, x2))), rd(rd(x1, mult(x1, x2)), x3), x4)}, mult(x2, mult(x1, x2)), rd(i(rd(x1, mult(x1, x2))), mult($

$=$ by Axiom 17 RL with $\{x5 \leftarrow x4, x4 \leftarrow rd(rd(x1, mult(x1, x2)), x3), x3 \leftarrow rd(i(rd(x1, mult(x1, x2))), mult(x2, mult(x1, x2)))$

$\overbrace{op_r(\underbrace{op_r(i(rd(x1, mult(x1, x2))), mult(x2, mult(x1, x2)), rd(i(rd(x1, mult(x1, x2))), mult(x2, mult(x1, x2))))}, rd(rd(x1, mu$

$=$ by Lemma 1188 LR with $\{x2 \leftarrow mult(x2, mult(x1, x2)), x1 \leftarrow i(rd(x1, mult(x1, x2)))\}$

$op_r(\overbrace{op_t(i(rd(x1, mult(x1, x2))), i(mult(x2, mult(x1, x2))))}, rd(rd(x1, mult(x1, x2)), x3), x4)$

$=$ by Lemma 15 LR with $\{x2 \leftarrow mult(x2, mult(x1, x2)), x1 \leftarrow rd(x1, mult(x1, x2))\}$

$op_r(\overbrace{i(op_t(\underbrace{rd(x1, mult(x1, x2))}, mult(x2, mult(x1, x2))))}, rd(rd(x1, mult(x1, x2)), x3), x4)$

$=$ by Lemma 10 RL with $\{x2 \leftarrow x1, x1 \leftarrow mult(x1, x2)\}$

$op_r(i(op_t(\overbrace{i(\underbrace{rd(mult(x1, x2), x1)})}, mult(x2, mult(x1, x2)))), rd(rd(x1, mult(x1, x2)), x3), x4)$

$=$ by Lemma 823 RL with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$op_r(i(op_t(i(\overbrace{op_r(x2, mult(x2, x1), mult(x2, \underbrace{x1}))})), mult(x2, mult(x1, x2)))), rd(rd(x1, mult(x1, x2)), x3), x4)$

$=$ by Axiom 1 LR with $\{x1 \leftarrow x1\}$

$op_r(i(op_t(\underbrace{i(op_r(x2, mult(x2, x1), mult(x2, \overbrace{mult(x1, unit())})))}, mult(x2, mult(x1, x2)))), rd(rd(x1, mult(x1, x2)), x3), x4)$

$=$ by Lemma 1196 RL with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$op_r(i(\underbrace{op_t(\overbrace{rd(mult(x2, x1), mult(x2, mult(x1, x2)))}, mult(x2, mult(x1, x2))))}, rd(rd(x1, mult(x1, x2)), x3), x4)$

$=$ by Lemma 14 LR with $\{x2 \leftarrow mult(x2, mult(x1, x2)), x1 \leftarrow mult(x2, x1)\}$

$op_r(i(\overbrace{mult(\underbrace{i(mult(x2, mult(x1, x2)))}, mult(x2, x1)))}, rd(rd(x1, mult(x1, x2)), x3), x4)$

$=$ by Lemma 59 LR with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$op_r(i(\underbrace{mult(\overbrace{rd(i(x2), mult(x2, x1))}, mult(x2, x1)))}, rd(rd(x1, mult(x1, x2)), x3), x4)$

$=$ by Axiom 4 RL with $\{x2 \leftarrow mult(x2, x1), x1 \leftarrow i(x2)\}$

$op_r(i(\overbrace{i(x2)}), rd(rd(x1, mult(x1, x2)), x3), x4)$

Lemma 2164: $op_r(rd(mult(x1, op_t(x2, i(mult(x2, mult(x1, x2))))), x1), x3, i(x4)) = op_r(i(i(x2)), rd(rd(x1, mult(x1, x2)), x3), x4$

$op_r(\underbrace{rd(mult(x1, op_t(x2, i(mult(x2, mult(x1, x2))))), x1)}, x3, i(x4))$

$=$ by Lemma 1208 RL with $\{x3 \leftarrow i(mult(x2, mult(x1, x2))), x2 \leftarrow x2, x1 \leftarrow x1\}$

$op_r(\overbrace{op_t(rd(mult(x1, x2), x1), i(mult(x2, mult(x1, x2))))}, x3, i(x4))$

$=$ by Lemma 46 RL with $\{x3 \leftarrow mult(x2, mult(x1, x2)), x2 \leftarrow mult(x1, x2), x1 \leftarrow x1\}$

$\underbrace{op_r(\overbrace{i(op_t(rd(x1, mult(x1, x2)), mult(x2, mult(x1, x2))))}, x3, i(x4))}$

$=$ by Lemma 2163 LR with $\{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_r(i(i(x2)), rd(rd(x1, mult(x1, x2)), x3), x4)}$

Lemma 2165: $op_r(rd(mult(x1, op_t(x2, rd(x1, rd(x2, x2)))), x1), x3, i(x4)) = op_r(i(i(x2)), rd(rd(x1, mult(x1, x2)), x3), x4)$

$op_r(rd(mult(x1, \underbrace{op_t(x2, rd(x1, rd(x2, x2)))}), x1), x3, i(x4))$

$=$ by Lemma 1288 RL with $\{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x2\}$

$op_r(rd(mult(x1, \overbrace{op_t(x2, rd(i(x2), mult(x1, x2)))}), x1), x3, i(x4))$

$=$ by Lemma 1732 RL with $\{x3 \leftarrow mult(x1, x2), x2 \leftarrow x2, x1 \leftarrow x2\}$

$\underbrace{op_r(rd(mult(x1, \overbrace{op_t(x2, i(mult(x2, mult(x1, x2))))}), x1), x3, i(x4))}$

$=$ by Lemma 2164 LR with $\{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_r(i(i(x2)), rd(rd(x1, mult(x1, x2)), x3), x4)}$

Lemma 2166: $op_r(rd(mult(x1, op_t(x2, x1)), x1), x3, i(x4)) = op_r(i(i(x2)), rd(rd(x1, mult(x1, x2)), x3), x4)$

$op_r(rd(mult(x1, op_t(x2, \underbrace{x1})), x1), x3, i(x4))$

$=$ by Lemma 1 RL with $\{x1 \leftarrow x1\}$

$op_r(rd(mult(x1, op_t(x2, \overbrace{rd(x1, \underbrace{unit()})})), x1), x3, i(x4))$

$=$ by Lemma 2 RL with $\{x1 \leftarrow x2\}$

$\underbrace{op_r(rd(mult(x1, op_t(x2, rd(x1, \overbrace{rd(x2, x2)}))), x1), x3, i(x4))}$

$=$ by Lemma 2165 LR with $\{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_r(i(i(x2)), rd(rd(x1, mult(x1, x2)), x3), x4)}$

Lemma 2167: $op_r(x1, i(x2), x3) = op_r(x1, i(rd(op_t(x1, x4), x2)), i(x3))$

$op_r(x1, i(x2), \underbrace{x3}_{})$

$=$ by Lemma 3 RL with $\{x1 \leftarrow x3\}$

$\underbrace{op_r(x1, i(x2), \overbrace{i(i(x3))}^{})}_{}$

$=$ by Lemma 1940 RL with $\{x3 \leftarrow i(x3), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_r(\underbrace{x1}_{}, rd(x2, x1), i(x3))}^{}$

$=$ by Axiom 5 LR with $\{x2 \leftarrow x4, x1 \leftarrow x1\}$

$op_r(\overbrace{rd(\underbrace{mult(x1, x4)}_{}, x4)}^{}, rd(x2, x1), i(x3))$

$=$ by Lemma 13 RL with $\{x2 \leftarrow x1, x1 \leftarrow x4\}$

$\underbrace{op_r(rd(\overbrace{mult(x4, op_t(x1, x4))}^{}, x4), rd(x2, x1), i(x3))}_{}$

$=$ by Lemma 2166 LR with $\{x4 \leftarrow x3, x3 \leftarrow rd(x2, x1), x2 \leftarrow x1, x1 \leftarrow x4\}$

$\overbrace{op_r(\underbrace{i(i(x1))}_{}, rd(rd(x4, mult(x4, x1)), rd(x2, x1)), x3)}^{}$

$=$ by Lemma 3 LR with $\{x1 \leftarrow x1\}$

$op_r(\overbrace{x1}^{}, \underbrace{rd(rd(x4, mult(x4, x1)), rd(x2, x1))}_{}, x3)$

$=$ by Lemma 40 RL with $\{x3 \leftarrow x4, x2 \leftarrow mult(x4, x1), x1 \leftarrow rd(x2, x1)\}$

$op_r(x1, \overbrace{\underbrace{rd(i(rd(x2, x1)), rd(mult(x4, x1), x4))}_{}}^{}, x3)$

$=$ by Lemma 1888 LR with $\{x2 \leftarrow x1, x3 \leftarrow x4, x1 \leftarrow rd(x2, x1)\}$

$op_r(x1, \overbrace{rd(\underbrace{rd(rd(x4, op_t(x4, x1)), rd(x2, x1))}_{}, x1)}^{}, x3)$

$=$ by Lemma 1236 RL with $\{x3 \leftarrow x1, x2 \leftarrow rd(x2, x1), x1 \leftarrow x4\}$

$op_r(x1, rd(\overbrace{\underbrace{rd(x4, mult(rd(x2, x1), op_t(x4, x1)))}_{}}^{}, x1), x3)$

$=$ by Lemma 1393 LR with $\{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x4\}$

$\underbrace{op_r(x1, rd(\overbrace{rd(op_t(x1, x4), x2)}^{}, x1), x3)}_{}$

$=$ by Lemma 1940 LR with $\{x3 \leftarrow x3, x2 \leftarrow rd(op_t(x1, x4), x2), x1 \leftarrow x1\}$

$\overbrace{op_r(x1, i(rd(op_t(x1, x4), x2)), i(x3))}^{}$

Lemma 2168: $op_r(x1, i(x2), i(x3)) = op_r(x1, rd(x2, op_t(x1, x4)), x3)$

$\underbrace{op_r(x1, i(x2), i(x3))}_{}$

$=$ by Lemma 2167 LR with $\{x4 \leftarrow x4, x3 \leftarrow i(x3), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_r(x1, \underbrace{i(rd(op_t(x1, x4), x2))}_{}, i(i(x3)))}^{}$

$=$ by Lemma 10 LR with $\{x2 \leftarrow x2, x1 \leftarrow op_t(x1, x4)\}$

$op_r(x1, \overbrace{rd(x2, op_t(x1, x4))}^{}, \underbrace{i(i(x3))}_{})$

$=$ by Lemma 3 LR with $\{x1 \leftarrow x3\}$

$op_r(x1, rd(x2, op_t(x1, x4)), \overbrace{x3}^{})$

Lemma 2169: $op_r(x1, rd(op_t(i(x1), x2), x3), x4) = op_r(x1, x3, i(x4))$

$$
\begin{aligned}
& op_r(x1, \underbrace{rd(op_t(i(x1), x2), x3)}, x4) \\
= & \quad \text{by Lemma 66 RL with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x3\} \\
& \underbrace{op_r(x1, \overbrace{rd(i(x3), op_t(x1, i(x2)))}, x4)} \\
= & \quad \text{by Lemma 2168 RL with } \{x4 \leftarrow i(x2), x3 \leftarrow x4, x2 \leftarrow i(x3), x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, \underbrace{i(i(x3))}, i(x4))} \\
= & \quad \text{by Lemma 3 LR with } \{x1 \leftarrow x3\} \\
& op_r(x1, \overbrace{\ x3\ }, i(x4))
\end{aligned}
$$

Lemma 2170: $op_r(x1, i(rd(x2, x3)), i(i(x4))) = op_r(x1, rd(x3, op_t(x2, i(x1))), x4)$

$$
\begin{aligned}
& \underbrace{op_r(x1, i(rd(x2, x3)), i(i(x4)))} \\
= & \quad \text{by Lemma 1946 LR with } \{x3 \leftarrow i(x4), x2 \leftarrow rd(x2, x3), x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_r(x1, mult(\overbrace{rd(x2, x3)}, i(x1)), i(x4))}} \\
= & \quad \text{by Lemma 2169 RL with } \{x4 \leftarrow x4, x3 \leftarrow mult(rd(x2, x3), i(x1)), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, \underbrace{rd(op_t(i(x1), x2), mult(rd(x2, x3), i(x1)))}, x4)} \\
= & \quad \text{by Lemma 1398 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow i(x1)\} \\
& op_r(x1, \overbrace{rd(x3, op_t(x2, i(x1)))}, x4)
\end{aligned}
$$

Lemma 2171: $op_r(x1, rd(x3, x2), x4) = op_r(x1, rd(x3, op_t(x2, i(x1))), x4)$

$$
\begin{aligned}
& op_r(x1, rd(x3, x2), \underbrace{\ x4\ }) \\
= & \quad \text{by Lemma 3 RL with } \{x1 \leftarrow x4\} \\
& op_r(x1, \underbrace{rd(x3, x2)}, \overbrace{i(i(x4))}) \\
= & \quad \text{by Lemma 10 RL with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& \underbrace{op_r(x1, \overbrace{i(rd(x2, x3))}, i(i(x4)))} \\
= & \quad \text{by Lemma 2170 LR with } \{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, rd(x3, op_t(x2, i(x1))), x4)}
\end{aligned}
$$

Lemma 2172: $op_r(x1, rd(x3, x2), x4) = op_r(x1, rd(op_t(x3, x1), x2), x4)$

$$
\begin{aligned}
& op_r(x1, \underbrace{rd(x3, x2)}, x4) \\
= & \quad \text{by Lemma 23 RL with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& \underbrace{op_r(x1, \overbrace{rd(i(x2), i(x3))}, x4)} \\
= & \quad \text{by Lemma 2171 LR with } \{x4 \leftarrow x4, x2 \leftarrow i(x3), x3 \leftarrow i(x2), x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, \underbrace{rd(i(x2), op_t(i(x3), i(x1)))}, x4)} \\
= & \quad \text{by Lemma 68 LR with } \{x3 \leftarrow i(x1), x2 \leftarrow x3, x1 \leftarrow x2\} \\
& op_r(x1, \overbrace{rd(op_t(x3, \underbrace{i(i(x1))}), x2)}, x4) \\
= & \quad \text{by Lemma 3 LR with } \{x1 \leftarrow x1\} \\
& op_r(x1, rd(op_t(x3, \overbrace{\ x1\ }), x2), x4)
\end{aligned}
$$

Lemma 2173: $op_r(x1, rd(op_t(x1, x2), x3), x4) = op_r(x1, rd(x1, x3), x4)$

$$
\begin{aligned}
& op_r(x1, \underbrace{rd(op_t(x1, x2), x3)}, x4) \\
= & \quad \text{by Lemma 10 RL with } \{x2 \leftarrow op_t(x1, x2), x1 \leftarrow x3\} \\
& op_r(x1, \overbrace{i(rd(x3, op_t(x1, x2)))}, x4) \\
= & \quad \text{by Lemma 9 RL with } \{x2 \leftarrow rd(x3, op_t(x1, x2)), x1 \leftarrow x2\} \\
& \underbrace{op_r(x1, \overbrace{rd(x2, mult(rd(x3, op_t(x1, x2)), x2))}, x4)} \\
= & \quad \text{by Lemma 2172 LR with } \{x4 \leftarrow x4, x2 \leftarrow mult(rd(x3, op_t(x1, x2)), x2), x3 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, \underbrace{rd(op_t(x2, x1), mult(rd(x3, op_t(x1, x2)), x2))}, x4)} \\
= & \quad \text{by Lemma 2025 LR with } \{x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow x2\} \\
& op_r(x1, \overbrace{rd(x1, x3)}, x4)
\end{aligned}
$$

Lemma 2174: $op_r(x1, rd(x1, i(x3)), x4) = op_r(x1, mult(x3, op_t(x1, x2)), x4)$

$$
\begin{aligned}
& \underbrace{op_r(x1, rd(x1, i(x3)), x4)} \\
= & \quad \text{by Lemma 2173 RL with } \{x4 \leftarrow x4, x3 \leftarrow i(x3), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, \underbrace{rd(op_t(x1, x2), i(x3))}, x4)} \\
= & \quad \text{by Lemma 1635 RL with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x3\} \\
& \underbrace{op_r(x1, \overbrace{op_r(mult(x3, op_t(x1, x2)), x1, x1)}, x4)} \\
= & \quad \text{by Lemma 1943 LR with } \{x3 \leftarrow x4, x2 \leftarrow mult(x3, op_t(x1, x2)), x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, mult(x3, op_t(x1, x2)), x4)}
\end{aligned}
$$

Lemma 2175: $op_r(x1, i(rd(x2, x3)), x4) = op_r(x1, rd(x3, op_t(x2, x1)), x4)$

$$
\begin{aligned}
& op_r(x1, \underbrace{i(rd(x2, x3))}, x4) \\
= & \quad \text{by Lemma 9 RL with } \{x2 \leftarrow rd(x2, x3), x1 \leftarrow x1\} \\
& \underbrace{op_r(x1, \overbrace{rd(x1, mult(rd(x2, x3), x1))}, x4)} \\
= & \quad \text{by Lemma 2173 RL with } \{x4 \leftarrow x4, x3 \leftarrow mult(rd(x2, x3), x1), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, \underbrace{rd(op_t(x1, x2), mult(rd(x2, x3), x1))}, x4)} \\
= & \quad \text{by Lemma 1398 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_r(x1, \overbrace{rd(x3, op_t(x2, x1))}, x4)
\end{aligned}
$$

Lemma 2176: $op_r(x1, rd(x2, op_t(x3, x1)), x4) = op_r(x1, rd(x2, x3), x4)$

$$
\begin{aligned}
& \underbrace{op_r(x1, rd(x2, op_t(x3, x1)), x4)} \\
= & \quad \text{by Lemma 2175 RL with } \{x4 \leftarrow x4, x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, \underbrace{i(rd(x3, x2))}, x4)} \\
= & \quad \text{by Lemma 10 LR with } \{x2 \leftarrow x2, x1 \leftarrow x3\} \\
& op_r(x1, \overbrace{rd(x2, x3)}, x4)
\end{aligned}
$$

Lemma 2177: $op_r(x1, rd(op_t(x3, i(x1)), x2), i(i(x4))) = op_r(x1, rd(x3, x2), x4)$

$op_r(x1, \underbrace{rd(op_t(x3, i(x1)), x2)}, i(i(x4)))$

$=$ by Lemma 10 RL with $\{x2 \leftarrow op_t(x3, i(x1)), x1 \leftarrow x2\}$

$\underbrace{op_r(x1, \overbrace{i(rd(x2, op_t(x3, i(x1))))}, i(i(x4)))}$

$=$ by Lemma 1946 LR with $\{x3 \leftarrow i(x4), x2 \leftarrow rd(x2, op_t(x3, i(x1))), x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_r(x1, mult(rd(x2, \overbrace{op_t(x3, i(x1))}), i(x1)), i(x4))}}$

$=$ by Lemma 2169 RL with $\{x4 \leftarrow x4, x3 \leftarrow mult(rd(x2, op_t(x3, i(x1))), i(x1)), x2 \leftarrow x3, x1 \leftarrow x1\}$

$\overbrace{op_r(x1, \underbrace{rd(op_t(i(x1), x3), mult(rd(x2, op_t(x3, i(x1))), i(x1)))}, x4)}$

$=$ by Lemma 2025 LR with $\{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow i(x1)\}$

$op_r(x1, \overbrace{rd(x3, x2)}, x4)$

Lemma 2178: $op_r(x1, x2, x3) = op_r(x1, rd(x2, asoc(x4, x1, x4)), x3)$

$op_r(x1, x2, \underbrace{x3}_{})$

$=$ by Lemma 3 RL with $\{x1 \leftarrow x3\}$

$\underbrace{op_r(x1, x2, \overbrace{i(i(x3))})}$

$=$ by Lemma 1939 RL with $\{x3 \leftarrow i(i(x3)), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_r(x1, op_t(x2, x1), i(i(x3)))}}$

$=$ by Lemma 820 RL with $\{x3 \leftarrow i(x3), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_r(x1, i(mult(x2, x1)), i(x3))}}$

$=$ by Lemma 2168 LR with $\{x4 \leftarrow rd(x4, x1), x3 \leftarrow x3, x2 \leftarrow mult(x2, x1), x1 \leftarrow x1\}$

$\overbrace{op_r(x1, \underbrace{rd(mult(x2, x1), op_t(x1, rd(x4, x1)))}, x3)}$

$=$ by Lemma 1292 RL with $\{x3 \leftarrow rd(x4, x1), x2 \leftarrow x1, x1 \leftarrow x2\}$

$op_r(x1, \overbrace{rd(x2, rd(\underbrace{op_t(x1, rd(x4, x1))}, x1))}, x3)$

$=$ by Lemma 547 LR with $\{x2 \leftarrow x4, x1 \leftarrow x1\}$

$op_r(x1, rd(x2, \underbrace{rd(\overbrace{op_r(x1, x4, i(x4))}, x1)}), x3)$

$=$ by Lemma 639 LR with $\{x2 \leftarrow x4, x1 \leftarrow x1\}$

$op_r(x1, rd(x2, \overbrace{asoc(x4, x1, x4)}), x3)$

Lemma 2179: $op_r(x1, i(x2), x3) = op_r(x1, rd(asoc(x4, x1, x1), x2), x3)$

$op_r(x1, \underbrace{i(x2)}, x3)$

$=$ by Lemma 9 RL with $\{x2 \leftarrow x2, x1 \leftarrow op_t(op_l(x4, x4, x1), x1)\}$

$\underbrace{op_r(x1, \overbrace{rd(op_t(op_l(x4, x4, x1), x1), mult(x2, op_t(op_l(x4, x4, x1), x1)))}, x3)}$

$=$ by Lemma 2172 RL with $\{x4 \leftarrow x3, x2 \leftarrow mult(x2, op_t(op_l(x4, x4, x1), x1)), x3 \leftarrow op_l(x4, x4, x1), x1 \leftarrow x1\}$

$\overbrace{op_r(x1, \underbrace{rd(op_l(x4, x4, x1), mult(x2, op_t(op_l(x4, x4, x1), x1)))}, x3)}$

$=$ by Lemma 1580 RL with $\{x3 \leftarrow x1, x2 \leftarrow mult(x2, op_t(op_l(x4, x4, x1), x1)), x1 \leftarrow x4\}$

$op_r(x1, \underbrace{\overbrace{rd(rd(x4, mult(x2, op_t(op_l(x4, x4, x1), x1))), asoc(x1, x4, x4))}, x3)}$

$=$ by Lemma 1295 RL with $\{x3 \leftarrow x1, x2 \leftarrow x4, x1 \leftarrow rd(x4, mult(x2, op_t(op_l(x4, x4, x1), x1)))\}$

$\underbrace{op_r(x1, \overbrace{mult(rd(x4, mult(x2, op_t(op_l(x4, x4, x1), x1))), asoc(x4, x1, x4))}, x3)}$

$=$ by Lemma 2178 LR with $\{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow mult(rd(x4, mult(x2, op_t(op_l(x4, x4, x1), x1))), asoc(x4, x1, x4)), x1 \leftarrow x1$

$\overbrace{op_r(x1, \underbrace{rd(mult(rd(x4, mult(x2, op_t(op_l(x4, x4, x1), x1))), asoc(x4, x1, x4)), asoc(x4, x1, x4))}, x3)}$

$=$ by Axiom 5 RL with $\{x2 \leftarrow asoc(x4, x1, x4), x1 \leftarrow rd(x4, mult(x2, op_t(op_l(x4, x4, x1), x1)))\}$

$op_r(x1, \overbrace{rd(x4, mult(x2, \underbrace{op_t(op_l(x4, x4, x1), x1)}))}, x3)$

$=$ by Axiom 15 LR with $\{x4 \leftarrow x1, x3 \leftarrow x1, x2 \leftarrow x4, x1 \leftarrow x4\}$

$op_r(x1, rd(x4, mult(x2, \overbrace{\underbrace{op_l(op_t(x4, x1), x4, x1)}})), x3)$

$=$ by Lemma 421 RL with $\{x2 \leftarrow x1, x1 \leftarrow x4\}$

$op_r(x1, rd(x4, mult(x2, \overbrace{\underbrace{op_r(x4, x1, i(x1))}})), x3)$

$=$ by Lemma 829 RL with $\{x2 \leftarrow x1, x1 \leftarrow x4\}$

$op_r(x1, \underbrace{rd(x4, mult(x2, \overbrace{op_t(x4, i(op_t(mult(x4, x1), x1)))}))}, x3)$

$=$ by Lemma 1236 LR with $\{x3 \leftarrow i(op_t(mult(x4, x1), x1)), x2 \leftarrow x2, x1 \leftarrow x4\}$

$op_r(x1, \overbrace{rd(rd(x4, \underbrace{op_t(x4, i(op_t(mult(x4, x1), x1)))}), x2)}, x3)$

$=$ by Lemma 829 LR with $\{x2 \leftarrow x1, x1 \leftarrow x4\}$

$op_r(x1, rd(\underbrace{rd(x4, \overbrace{op_r(x4, x1, i(x1))})}, x2), x3)$

$=$ by Lemma 777 LR with $\{x2 \leftarrow x1, x1 \leftarrow x4\}$

$op_r(x1, rd(\overbrace{asoc(x4, x1, x1)}, x2), x3)$

Lemma 2180: $op_r(x1, rd(x2, op_r(x3, x1, x1)), x4) = op_r(x1, rd(x2, x3), x4)$

$op_r(x1, \underbrace{rd(x2, op_r(x3, x1, x1))}, x4)$

$=$ by Lemma 10 RL with $\{x2 \leftarrow x2, x1 \leftarrow op_r(x3, x1, x1)\}$

$\underbrace{op_r(x1, \overbrace{i(rd(op_r(x3, x1, x1), x2))}, x4)}$

$=$ by Lemma 2179 LR with $\{x4 \leftarrow x3, x3 \leftarrow x4, x2 \leftarrow rd(op_r(x3, x1, x1), x2), x1 \leftarrow x1\}$

$\overbrace{op_r(x1, \underbrace{rd(asoc(x3, x1, x1), rd(op_r(x3, x1, x1), x2))}, x4)}$

$=$ by Lemma 10 RL with $\{x2 \leftarrow asoc(x3, x1, x1), x1 \leftarrow rd(op_r(x3, x1, x1), x2)\}$

$op_r(x1, \underbrace{\overbrace{i(rd(rd(op_r(x3, x1, x1), x2), asoc(x3, x1, x1)))}, x4)}$

$=$ by Lemma 1437 RL with $\{x3 \leftarrow x1, x4 \leftarrow x3, x2 \leftarrow op_r(x3, x1, x1), x1 \leftarrow x2\}$

$op_r(x1, \underbrace{\overbrace{mult(rd(x2, op_r(x3, x1, x1)), asoc(x3, x1, x1))}, x4)}$

$=$ by Lemma 1300 RL with $\{x2 \leftarrow x3, x3 \leftarrow x1, x1 \leftarrow rd(x2, op_r(x3, x1, x1))\}$

$op_r(x1, \underbrace{\overbrace{rd(rd(x2, op_r(x3, x1, x1)), asoc(x1, x3, x1))}, x4)}$

$=$ by Lemma 1995 LR with $\{x3 \leftarrow x1, x2 \leftarrow x3, x1 \leftarrow x2\}$

$op_r(x1, \overbrace{rd(x2, x3)}, x4)$

Lemma 2181: $mult(op_t(x1, i(x2)), i(rd(x2, i(x1)))) = op_l(op_l(op_r(i(x2), x1, x1), x1, rd(x2, i(x1))), x1, rd(x2, i(x1)))$

$mult(\underbrace{op_t(x1, i(x2))}, i(rd(x2, i(x1))))$

$=$ by Lemma 1733 RL with $\{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x1\}$

$mult(\overbrace{op_t(x1, rd(x1, mult(x1, x2)))}, i(rd(x2, i(x1))))$

$=$ by Lemma 1216 RL with $\{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x1\}$

$\underbrace{mult(\overbrace{op_t(x1, rd(x1, rd(x2, i(x1))))}, i(rd(x2, i(x1))))}$

$=$ by Lemma 1999 RL with $\{x1 \leftarrow rd(x2, i(x1)), x2 \leftarrow x1\}$

$\overbrace{op_l(op_l(\underbrace{rd(x1, rd(x2, i(x1)))}, x1, rd(x2, i(x1))), x1, rd(x2, i(x1)))}$

$=$ by Lemma 362 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$op_l(op_l(\overbrace{op_r(i(x2), x1, x1)}, x1, rd(x2, i(x1))), x1, rd(x2, i(x1)))$

Lemma 2182: $mult(op_t(x1, i(x2)), rd(i(x1), x2)) = op_l(op_l(op_r(i(x2), x1, x1), x1, rd(x2, i(x1))), x1, x2)$

$mult(op_t(x1, i(x2)), \underbrace{rd(i(x1), x2)})$

$=$ by Lemma 10 RL with $\{x2 \leftarrow i(x1), x1 \leftarrow x2\}$

$\underbrace{mult(op_t(x1, i(x2)), \overbrace{i(rd(x2, i(x1)))})}$

$=$ by Lemma 2181 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_l(op_l(op_r(i(x2), x1, x1), x1, rd(x2, i(x1))), x1, rd(x2, i(x1)))}}$

$=$ by Lemma 1328 LR with $\{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow op_l(op_r(i(x2), x1, x1), x1, rd(x2, i(x1)))\}$

$\overbrace{op_l(op_l(op_r(i(x2), x1, x1), x1, rd(x2, i(x1))), x1, x2)}$

Lemma 2183: $op_l(x1, x2, rd(i(x4), op_t(x2, x3))) = op_l(x1, mult(x4, op_t(x2, x3)), x2)$

$op_l(x1, x2, \underbrace{rd(i(x4), op_t(x2, x3))})$

$=$ by Axiom 5 LR with $\{x2 \leftarrow rd(x2, i(x4)), x1 \leftarrow rd(i(x4), op_t(x2, x3))\}$

$op_l(x1, x2, \overbrace{rd(\underbrace{mult(rd(i(x4), op_t(x2, x3)), rd(x2, i(x4)))}, rd(x2, i(x4)))})$

$=$ by Lemma 1374 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow i(x4)\}$

$\underbrace{op_l(x1, x2, rd(\overbrace{rd(x2, op_t(x2, x3))}, rd(x2, i(x4))))}$

$=$ by Lemma 2047 LR with $\{x4 \leftarrow rd(x2, i(x4)), x3 \leftarrow op_t(x2, x3), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_l(x1, x2, rd(\underbrace{op_t(x2, rd(rd(i(x2), rd(x2, i(x4))), op_t(x2, x3)))}, rd(rd(x2, i(x4)), rd(i(i(x2)), op_t(x2, x3)))))}$

$=$ by Lemma 1287 RL with $\{x2 \leftarrow op_t(x2, x3), x3 \leftarrow rd(x2, i(x4)), x1 \leftarrow x2\}$

$op_l(x1, x2, rd(\overbrace{op_t(x2, rd(rd(x2, i(x4)), rd(x2, op_t(x2, x3))))}, rd(rd(x2, i(x4)), rd(\underbrace{i(i(x2))}, op_t(x2, x3)))))$

$=$ by Lemma 3 LR with $\{x1 \leftarrow x2\}$

$\underbrace{op_l(x1, x2, rd(op_t(x2, rd(rd(x2, i(x4)), rd(x2, op_t(x2, x3)))), rd(rd(x2, i(x4)), rd(\overbrace{x2}, op_t(x2, x3)))))}$

$=$ by Lemma 1357 LR with $\{x3 \leftarrow rd(rd(x2, i(x4)), rd(x2, op_t(x2, x3))), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_l(x1, x2, i(op_t(rd(rd(x2, i(x4)), rd(x2, op_t(x2, x3))), x2)))}}$

$=$ by Lemma 1329 LR with $\{x3 \leftarrow rd(rd(x2, i(x4)), rd(x2, op_t(x2, x3))), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_l(x1, \underbrace{rd(rd(x2, i(x4)), rd(x2, op_t(x2, x3)))}, x2)}$

$=$ by Lemma 1390 RL with $\{x1 \leftarrow i(x4), x3 \leftarrow x3, x2 \leftarrow x2\}$

$op_l(x1, \underbrace{\overbrace{rd(op_t(x2, x3), i(x4))}}, x2)$

$=$ by Lemma 1635 RL with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x4\}$

$\underbrace{op_l(x1, \overbrace{op_r(mult(x4, op_t(x2, x3)), x2, x2)}, x2)}$

$=$ by Lemma 1326 LR with $\{x3 \leftarrow x2, x2 \leftarrow mult(x4, op_t(x2, x3)), x1 \leftarrow x1\}$

$\overbrace{op_l(x1, mult(x4, op_t(x2, x3)), x2)}$

Lemma 2184: $op_l(mult(rd(x1, i(x2)), x3), x2, x1) = mult(op_l(x1, x2, x1), mult(i(i(x2)), x3))$

$\underbrace{op_l(mult(rd(x1, i(x2)), x3), x2, x1)}$

$=$ by Lemma 2030 RL with $\{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow mult(rd(x1, i(x2)), x3)\}$

$\overbrace{\underbrace{op_l(mult(rd(x1, i(x2)), x3), x1, i(x2))}}$

$=$ by Lemma 1993 RL with $\{x3 \leftarrow x3, x2 \leftarrow i(x2), x1 \leftarrow x1\}$

$\overbrace{mult(\underbrace{op_l(x1, x1, i(x2))}, mult(i(i(x2)), x3))}$

$=$ by Lemma 2030 LR with $\{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x1\}$

$mult(\overbrace{op_l(x1, x2, x1)}, mult(i(i(x2)), x3))$

Lemma 2185: $op_l(mult(rd(x1, i(x2)), x3), x1, x2) = mult(op_l(i(i(x2)), x1, x2), mult(x1, x3))$

$$
\begin{aligned}
& \overbrace{op_l(mult(rd(x1, i(x2)), x3), x1, x2)} \\
= \quad & \text{by Lemma 2031 RL with } \{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow mult(rd(x1, i(x2)), x3)\} \\
& \overbrace{\underbrace{op_l(mult(rd(x1, i(x2)), x3), i(x2), x1)}} \\
= \quad & \text{by Lemma 1991 RL with } \{x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow i(x2)\} \\
& \overbrace{mult(\underbrace{op_l(i(i(x2)), i(x2), x1)}, mult(x1, x3))} \\
= \quad & \text{by Lemma 2031 LR with } \{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow i(i(x2))\} \\
& mult(\overbrace{op_l(i(i(x2)), x1, x2)}, mult(x1, x3))
\end{aligned}
$$

Lemma 2186: $mult(x1, op_l(i(mult(x1, x2)), x3, x4)) = rd(x1, op_l(mult(x2, x1), x3, x4))$

$$
\begin{aligned}
& \overbrace{mult(x1, op_l(i(mult(x1, x2)), x3, x4))} \\
= \quad & \text{by Lemma 1977 RL with } \{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow mult(x1, x2), x1 \leftarrow x1\} \\
& \overbrace{rd(x1, op_l(\underbrace{op_l(mult(x1, x2), x1, mult(x1, x2))}, x3, x4))} \\
= \quad & \text{by Lemma 2032 LR with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow mult(x1, x2)\} \\
& rd(x1, op_l(\overbrace{op_l(mult(x1, x2), x1, x2)}, x3, x4)) \\
= \quad & \text{by Lemma 435 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& rd(x1, op_l(\overbrace{mult(x2, x1)}, x3, x4))
\end{aligned}
$$

Lemma 2187: $mult(i(x1), rd(op_r(x2, x3, x4), x2)) = rd(op_r(x2, x3, x4), mult(x1, x2))$

$$
\begin{aligned}
& mult(i(x1), \underbrace{rd(op_r(x2, x3, x4), x2)}) \\
= & \quad \text{by Lemma 117 RL with } \{x3 \leftarrow x4, x2 \leftarrow x3, x1 \leftarrow x2\} \\
& mult(i(x1), \overbrace{mult(i(x2), op_r(x2, x3, x4))}) \\
= & \quad \text{by Lemma 17 RL with } \{x2 \leftarrow op_r(x2, x3, x4), x1 \leftarrow x2\} \\
& \underbrace{mult(i(x1), \overbrace{i(mult(x2, i(op_r(x2, x3, x4))))})} \\
= & \quad \text{by Axiom 11 LR with } \{x2 \leftarrow mult(x2, i(op_r(x2, x3, x4))), x1 \leftarrow x1\} \\
& \overbrace{i(\underbrace{mult(x1, mult(x2, i(op_r(x2, x3, x4))))})} \\
= & \quad \text{by Lemma 132 RL with } \{x3 \leftarrow i(op_r(x2, x3, x4)), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{i(\overbrace{mult(mult(x1, x2), op_l(i(op_r(x2, x3, x4)), x2, x1))})} \\
= & \quad \text{by Lemma 1133 LR with } \{x4 \leftarrow x1, x3 \leftarrow x2, x2 \leftarrow op_r(x2, x3, x4), x1 \leftarrow mult(x1, x2)\} \\
& \overbrace{mult(i(mult(x1, x2)), \underbrace{op_l(op_r(x2, x3, x4), x2, x1)})} \\
= & \quad \text{by Axiom 18 RL with } \{x5 \leftarrow x4, x4 \leftarrow x3, x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x2\} \\
& \underbrace{mult(i(mult(x1, x2)), \overbrace{op_r(op_l(x2, x2, x1), x3, x4)})} \\
= & \quad \text{by Lemma 1971 RL with } \{x4 \leftarrow mult(x1, x2), x3 \leftarrow x4, x2 \leftarrow x3, x1 \leftarrow op_l(x2, x2, x1)\} \\
& \overbrace{rd(\underbrace{op_l(op_r(op_l(x2, x2, x1), x3, x4), mult(x1, x2), op_l(x2, x2, x1))}, mult(x1, x2))} \\
= & \quad \text{by Lemma 2040 LR with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow op_r(op_l(x2, x2, x1), x3, x4)\} \\
& rd(\overbrace{op_l(\underbrace{op_r(op_l(x2, x2, x1), x3, x4)}, x1, x2)}, mult(x1, x2)) \\
= & \quad \text{by Axiom 18 LR with } \{x5 \leftarrow x4, x4 \leftarrow x3, x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x2\} \\
& rd(\underbrace{op_l(\overbrace{op_l(op_r(x2, x3, x4), x2, x1)}, x1, x2)}, mult(x1, x2)) \\
= & \quad \text{by Lemma 138 LR with } \{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow op_r(x2, x3, x4)\} \\
& rd(\overbrace{op_r(x2, x3, x4)}, mult(x1, x2))
\end{aligned}
$$

Lemma 2188: $rd(x1, rd(x2, op_r(x2, x3, x4))) = mult(x1, mult(rd(op_r(x2, x3, x4), mult(x1, x2)), x1))$

$$
\begin{aligned}
& rd(x1, \underbrace{rd(x2, op_r(x2, x3, x4))}) \\
= & \quad \text{by Lemma 10 RL with } \{x2 \leftarrow x2, x1 \leftarrow op_r(x2, x3, x4)\} \\
& \underbrace{rd(x1, \overbrace{i(rd(op_r(x2, x3, x4), x2))})} \\
= & \quad \text{by Lemma 86 RL with } \{x2 \leftarrow rd(op_r(x2, x3, x4), x2), x1 \leftarrow x1\} \\
& \overbrace{mult(x1, mult(\underbrace{mult(i(x1), rd(op_r(x2, x3, x4), x2))}, x1))} \\
= & \quad \text{by Lemma 2187 LR with } \{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& mult(x1, mult(\overbrace{rd(op_r(x2, x3, x4), mult(x1, x2))}, x1))
\end{aligned}
$$

Lemma 2189: $rd(mult(x1, op_r(x2, x3, x4)), x2) = rd(x1, rd(x2, op_r(x2, x3, x4)))$

$$
\begin{aligned}
& \overbrace{rd(mult(x1, op_r(x2, x3, x4)), x2)} \\
= \quad & \text{by Lemma 1152 RL with } \{x3 \leftarrow x2, x2 \leftarrow op_r(x2, x3, x4), x1 \leftarrow x1\} \\
& \overbrace{mult(x1, \overbrace{mult(rd(op_r(x2, x3, x4), mult(x1, x2)), x1)})} \\
= \quad & \text{by Lemma 2188 RL with } \{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{rd(x1, rd(x2, op_r(x2, x3, x4)))}
\end{aligned}
$$

Lemma 2190: $rd(x1, mult(x2, op_r(x1, x3, x4))) = rd(rd(x1, op_r(x1, x3, x4)), x2)$

$$
\begin{aligned}
& \overbrace{rd(x1, mult(x2, op_r(x1, x3, x4)))} \\
= \quad & \text{by Lemma 10 RL with } \{x2 \leftarrow x1, x1 \leftarrow mult(x2, op_r(x1, x3, x4))\} \\
& \overbrace{i(\overbrace{rd(mult(x2, op_r(x1, x3, x4)), x1)})} \\
= \quad & \text{by Lemma 2189 LR with } \{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \overbrace{i(\overbrace{rd(x2, rd(x1, op_r(x1, x3, x4)))})} \\
= \quad & \text{by Lemma 10 LR with } \{x2 \leftarrow rd(x1, op_r(x1, x3, x4)), x1 \leftarrow x2\} \\
& \overbrace{rd(rd(x1, op_r(x1, x3, x4)), x2)}
\end{aligned}
$$

Lemma 2191: $op_l(x1, mult(x2, x3), rd(i(x2), x2)) = op_l(x1, rd(i(x2), x2), op_t(i(op_t(rd(x3, x2), i(x2))), mult(x2, x3)))$

$$
\begin{aligned}
& \overbrace{op_l(x1, mult(x2, x3), rd(i(x2), x2))} \\
= \quad & \text{by Lemma 2030 RL with } \{x3 \leftarrow mult(x2, x3), x2 \leftarrow rd(i(x2), x2), x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, rd(i(x2), x2), i(mult(x2, x3)))} \\
= \quad & \text{by Lemma 1385 RL with } \{x3 \leftarrow mult(x2, x3), x2 \leftarrow rd(i(x2), x2), x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, rd(i(x2), x2), op_t(i(\underbrace{mult(mult(x2, x3), rd(i(x2), x2))}), mult(x2, x3)))} \\
= \quad & \text{by Lemma 199 LR with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& op_l(x1, rd(i(x2), x2), op_t(i(\overbrace{op_t(rd(x3, x2), i(x2))}), mult(x2, x3)))
\end{aligned}
$$

Lemma 2192: $op_l(x1, mult(x2, x2), mult(x2, x3)) = op_l(x1, op_t(i(op_t(rd(x3, x2), i(x2))), mult(x2, x3)), mult(x2, x2))$

$$
\begin{aligned}
& \overbrace{op_l(x1, mult(x2, x2), mult(x2, x3))} \\
= \quad & \text{by Lemma 2039 RL with } \{x3 \leftarrow x2, x2 \leftarrow mult(x2, x3), x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, mult(x2, x3), rd(i(x2), x2))} \\
= \quad & \text{by Lemma 2191 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, rd(i(x2), x2), op_t(i(op_t(rd(x3, x2), i(x2))), mult(x2, x3)))} \\
= \quad & \text{by Lemma 2038 LR with } \{x3 \leftarrow op_t(i(op_t(rd(x3, x2), i(x2))), mult(x2, x3)), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, op_t(i(op_t(rd(x3, x2), i(x2))), mult(x2, x3)), mult(x2, x2))}
\end{aligned}
$$

Lemma 2193: $op_l(x1, mult(x2, x2), mult(x2, x3)) = op_l(x1, op_t(op_t(rd(x2, x3), x2), mult(x2, x3)), mult(x2, x2))$

$\underbrace{op_l(x1, mult(x2, x2), mult(x2, x3))}$

$=$ by Lemma 2192 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_l(x1, op_t(\underbrace{i(op_t(rd(x3, x2), i(x2)))}, mult(x2, x3)), mult(x2, x2))}$

$=$ by Lemma 46 LR with $\{x3 \leftarrow i(x2), x2 \leftarrow x2, x1 \leftarrow x3\}$

$op_l(x1, op_t(\overbrace{op_t(rd(x2, x3), \underbrace{i(i(x2))})}, mult(x2, x3)), mult(x2, x2))$

$=$ by Lemma 3 LR with $\{x1 \leftarrow x2\}$

$op_l(x1, op_t(op_t(rd(x2, x3), \overbrace{x2}), mult(x2, x3)), mult(x2, x2))$

Lemma 2194: $op_l(x1, mult(x2, x2), mult(x2, x3)) = op_l(x1, op_t(mult(x2, i(x3)), mult(x2, x2)), mult(x2, x2))$

$\underbrace{op_l(x1, mult(x2, x2), mult(x2, x3))}$

$=$ by Lemma 2193 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_l(x1, op_t(op_t(rd(x2, x3), \underbrace{x2}), mult(x2, x3)), mult(x2, x2))}$

$=$ by Lemma 3 RL with $\{x1 \leftarrow x2\}$

$op_l(x1, \underbrace{op_t(op_t(rd(x2, x3), \overbrace{i(i(x2))}), mult(x2, x3))}, mult(x2, x2))$

$=$ by Lemma 1988 RL with $\{x2 \leftarrow x3, x1 \leftarrow x2\}$

$op_l(x1, \overbrace{op_t(mult(x2, i(x3)), mult(x2, x2))}, mult(x2, x2))$

Lemma 2195: $rd(mult(op_l(mult(x1,x2),x1,mult(x1,x2)),x3),x1) = rd(mult(mult(x1,x2),x3),op_t(x1,rd(x2,rd(op_t(x2,x1),x2$

$rd(mult(\underbrace{op_l(mult(x1,x2),x1,mult(x1,x2))},x3),x1)$

$=$ by Lemma 2030 RL with $\{x3 \leftarrow x1, x2 \leftarrow mult(x1,x2), x1 \leftarrow mult(x1,x2)\}$

$\underbrace{rd(mult(\overbrace{op_l(mult(x1,x2),mult(x1,x2),i(x1))},x3),x1)}$

$=$ by Lemma 364 RL with $\{x2 \leftarrow x1, x1 \leftarrow mult(op_l(mult(x1,x2),mult(x1,x2),i(x1)),x3)\}$

$\overbrace{op_r(mult(\underbrace{mult(op_l(mult(x1,x2),mult(x1,x2),i(x1)),x3)},i(x1)),x1,x1)}$

$=$ by Lemma 957 RL with $\{x3 \leftarrow x3, x2 \leftarrow i(x1), x1 \leftarrow mult(x1,x2)\}$

$op_r(mult(\overbrace{mult(asoc(mult(x1,x2),i(x1),mult(x1,x2)),mult(mult(x1,x2),x3))},i(x1)),x1,x1)$

$=$ by Lemma 1714 LR with $\{x3 \leftarrow mult(mult(x1,x2),x3), x2 \leftarrow i(x1), x1 \leftarrow mult(x1,x2)\}$

$op_r(\underbrace{mult(\overbrace{rd(mult(mult(x1,x2),x3),asoc(i(x1),mult(x1,x2),mult(x1,x2)))},i(x1))},x1,x1)$

$=$ by Lemma 1262 RL with $\{x3 \leftarrow mult(x1,x2), x2 \leftarrow i(x1), x1 \leftarrow mult(mult(x1,x2),x3)\}$

$\underbrace{op_r(\overbrace{mult(mult(mult(x1,x2),x3),op_t(i(x1),rd(mult(x1,x2),i(x1))))},x1,x1)}$

$=$ by Lemma 361 RL with $\{x2 \leftarrow x1, x1 \leftarrow mult(mult(mult(x1,x2),x3),op_t(i(x1),rd(mult(x1,x2),i(x1))))\}$

$\overbrace{\underbrace{op_r(mult(mult(mult(x1,x2),x3),op_t(i(x1),rd(mult(x1,x2),i(x1))))},i(x1),i(x1))}$

$=$ by Lemma 1635 LR with $\{x3 \leftarrow rd(mult(x1,x2),i(x1)), x2 \leftarrow i(x1), x1 \leftarrow mult(mult(x1,x2),x3)\}$

$\overbrace{\underbrace{rd(op_t(i(x1),rd(mult(x1,x2),i(x1))),i(mult(mult(x1,x2),x3)))}}$

$=$ by Lemma 67 LR with $\{x3 \leftarrow mult(mult(x1,x2),x3), x2 \leftarrow rd(mult(x1,x2),i(x1)), x1 \leftarrow x1\}$

$\overbrace{rd(mult(mult(x1,x2),x3),op_t(x1,\underbrace{i(rd(mult(x1,x2),i(x1)))}))}$

$=$ by Lemma 10 LR with $\{x2 \leftarrow i(x1), x1 \leftarrow mult(x1,x2)\}$

$rd(mult(mult(x1,x2),x3),\underbrace{op_t(x1,\overbrace{rd(i(x1),mult(x1,x2))})})$

$=$ by Lemma 707 LR with $\{x2 \leftarrow mult(x1,x2), x1 \leftarrow x1\}$

$rd(mult(mult(x1,x2),x3),\overbrace{op_t(x1,\underbrace{rd(mult(x1,x2),x1)})})$

$=$ by Lemma 1108 LR with $\{x1 \leftarrow x2, x2 \leftarrow x1\}$

$rd(mult(mult(x1,x2),x3),op_t(x1,\overbrace{rd(x2,rd(op_t(x2,x1),x2))}))$

Lemma 2196: $rd(mult(mult(x2,x1),x3),x1) = rd(mult(mult(x1,x2),x3),op_t(x1,rd(x2,rd(op_t(x2,x1),x2))))$

$rd(mult(\underbrace{mult(x2,x1)},x3),x1)$

$=$ by Lemma 435 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$rd(mult(\overbrace{op_l(mult(x1,x2),x1,x2)},x3),x1)$

$=$ by Lemma 2032 RL with $\{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow mult(x1,x2)\}$

$\underbrace{rd(mult(\overbrace{op_l(mult(x1,x2),x1,mult(x1,x2))},x3),x1)}$

$=$ by Lemma 2195 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{rd(mult(mult(x1,x2),x3),op_t(x1,rd(x2,rd(op_t(x2,x1),x2))))}$

Lemma 2197: $rd(mult(mult(x2,x1),x3),x1) = rd(mult(mult(x1,x2),x3),op_t(x1,op_r(x2,x2,x2)))$

$\overbrace{rd(mult(mult(x2,x1),x3),x1)}$

$=$ by Lemma 2196 LR with $\{x3 \leftarrow x3, x1 \leftarrow x1, x2 \leftarrow x2\}$

$\overbrace{rd(mult(mult(x1,x2),x3),\underbrace{op_t(x1,rd(x2,rd(op_t(x2,x1),x2)))}}$

$=$ by Lemma 2141 LR with $\{x3 \leftarrow x2, x4 \leftarrow x2, x2 \leftarrow x2, x1 \leftarrow x1\}$

$rd(mult(mult(x1,x2),x3),\overbrace{op_t(x1,\underbrace{rd(x2,rd(x2,x2))})})$

$=$ by Lemma 353 LR with $\{x2 \leftarrow x2, x1 \leftarrow x2\}$

$rd(mult(mult(x1,x2),x3),op_t(x1,\overbrace{op_r(x2,x2,x2)}))$

Lemma 2198: $rd(mult(mult(x2,x1),x3),x1) = rd(mult(mult(x1,x2),x3),op_t(x1,x2))$

$\overbrace{rd(mult(mult(x2,x1),x3),x1)}$

$=$ by Lemma 2197 LR with $\{x3 \leftarrow x3, x1 \leftarrow x1, x2 \leftarrow x2\}$

$\overbrace{rd(mult(mult(x1,x2),x3),op_t(x1,\underbrace{op_r(x2,x2,x2)}))}$

$=$ by Lemma 601 LR with $\{x2 \leftarrow x2, x1 \leftarrow x2\}$

$rd(mult(mult(x1,x2),x3),op_t(x1,\overbrace{op_l(x2,x2,x2)}))$

$=$ by Lemma 25 LR with $\{x2 \leftarrow x2, x1 \leftarrow x2\}$

$rd(mult(mult(x1,x2),x3),op_t(x1,\overbrace{x2}))$

Lemma 2199: $rd(mult(x3,mult(x1,x2)),x2) = rd(mult(x3,mult(x2,x1)),op_t(x2,x1))$

$rd(\underbrace{mult(x3,mult(x1,x2))},x2)$

$=$ by Lemma 13 RL with $\{x2 \leftarrow x3, x1 \leftarrow mult(x1,x2)\}$

$rd(\overbrace{mult(mult(x1,x2),\underbrace{op_t(x3,mult(x1,x2))})},x2)$

$=$ by Lemma 1694 LR with $\{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x3\}$

$\underbrace{rd(mult(mult(x1,x2),\overbrace{op_t(x3,mult(x2,x1))}),x2)}$

$=$ by Lemma 2198 LR with $\{x3 \leftarrow op_t(x3,mult(x2,x1)), x1 \leftarrow x2, x2 \leftarrow x1\}$

$\overbrace{rd(\underbrace{mult(mult(x2,x1),op_t(x3,mult(x2,x1)))},op_t(x2,x1))}$

$=$ by Lemma 13 LR with $\{x2 \leftarrow x3, x1 \leftarrow mult(x2,x1)\}$

$rd(\overbrace{mult(x3,mult(x2,x1))},op_t(x2,x1))$

Lemma 2200: $rd(op_l(x2,x3,x4),mult(x1,x2)) = mult(rd(i(x1),x2),op_l(x2,x3,x4))$

$\underbrace{rd(op_l(x2,x3,x4),mult(x1,x2))}$

$=$ by Lemma 1074 RL with $\{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow mult(x1,x2)\}$

$\underbrace{\overbrace{rd(i(mult(x1,x2)),op_l(i(x2),x3,x4))}}$

$=$ by Lemma 2075 RL with $\{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow i(mult(x1,x2))\}$

$\overbrace{mult(\underbrace{op_r(i(mult(x1,x2)),x2,x2)},op_l(x2,x3,x4))}$

$=$ by Lemma 371 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$mult(\overbrace{rd(i(x1),x2)},op_l(x2,x3,x4))$

Lemma 2201: $rd(op_t(rd(x1, rd(x2, op_l(x2, x1, x3))), x1), x1) = rd(rd(x1, rd(x2, op_l(x2, x1, x3))), x1)$

$$
\begin{aligned}
& \overbrace{rd(op_t(rd(x1, rd(x2, op_l(x2, x1, x3))), x1), x1)} \\
= \quad & \text{by Lemma 748 RL with } \{x2 \leftarrow x1, x1 \leftarrow rd(x1, rd(x2, op_l(x2, x1, x3)))\} \\
& \overbrace{rd(rd(x1, rd(x2, op_l(x2, x1, x3))), \overbrace{op_t(x1, rd(x1, rd(x2, op_l(x2, x1, x3))))})} \\
= \quad & \text{by Lemma 2087 LR with } \{x3 \leftarrow op_l(x2, x1, x3), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& rd(rd(x1, rd(x2, op_l(x2, x1, x3))), \overbrace{op_t(x1, rd(op_l(x2, x1, x3), rd(x2, x1)))}) \\
= \quad & \text{by Lemma 1484 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& rd(rd(x1, rd(x2, op_l(x2, x1, x3))), \overbrace{x1})
\end{aligned}
$$

Lemma 2202: $op_r(rd(op_l(x1, x2, x3), x1), x2, x2) = op_t(rd(op_l(x1, x2, x3), x1), x2)$

$$
\begin{aligned}
& op_r(\overbrace{rd(op_l(x1, x2, x3), x1)}, x2, x2) \\
= \quad & \text{by Lemma 10 RL with } \{x2 \leftarrow op_l(x1, x2, x3), x1 \leftarrow x1\} \\
& \overbrace{op_r(\overbrace{i(rd(x1, op_l(x1, x2, x3)))}, x2, x2)} \\
= \quad & \text{by Lemma 354 RL with } \{x2 \leftarrow rd(x1, op_l(x1, x2, x3)), x1 \leftarrow x2\} \\
& \overbrace{\overbrace{rd(rd(x2, rd(x1, op_l(x1, x2, x3))), x2)}} \\
= \quad & \text{by Lemma 2201 RL with } \{x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \overbrace{\overbrace{rd(op_t(rd(x2, rd(x1, op_l(x1, x2, x3))), x2), x2)}} \\
= \quad & \text{by Lemma 345 LR with } \{x2 \leftarrow rd(x1, op_l(x1, x2, x3)), x1 \leftarrow x2\} \\
& \overbrace{op_t(\overbrace{i(rd(x1, op_l(x1, x2, x3)))}, x2)} \\
= \quad & \text{by Lemma 10 LR with } \{x2 \leftarrow op_l(x1, x2, x3), x1 \leftarrow x1\} \\
& op_t(\overbrace{rd(op_l(x1, x2, x3), x1)}, x2)
\end{aligned}
$$

Lemma 2203: $mult(i(x1), rd(x2, op_l(x2, x3, x1))) = rd(rd(x2, op_l(x2, x3, x1)), x1)$

$$
\begin{aligned}
& \overbrace{mult(i(x1), rd(x2, op_l(x2, x3, x1)))} \\
= \quad & \text{by Lemma 45 RL with } \{x3 \leftarrow x2, x2 \leftarrow op_l(x2, x3, x1), x1 \leftarrow x1\} \\
& \overbrace{i(\overbrace{mult(x1, rd(op_l(x2, x3, x1), x2))})} \\
= \quad & \text{by Lemma 2124 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{i(\overbrace{rd(x1, rd(x2, op_l(x2, x3, x1)))})} \\
= \quad & \text{by Lemma 10 LR with } \{x2 \leftarrow rd(x2, op_l(x2, x3, x1)), x1 \leftarrow x1\} \\
& \overbrace{rd(rd(x2, op_l(x2, x3, x1)), x1)}
\end{aligned}
$$

Lemma 2204: $op_t(x1, i(rd(x2, op_l(x2, x3, x1)))) = mult(op_r(i(rd(rd(x2, op_l(x2, x3, x1)), x1)), i(x1), i(x1)), rd(x2, op_l(x2, x3, x1))$

$\overbrace{op_t(x1, i(rd(x2, op_l(x2, x3, x1))))}$

$=$ by Lemma 18 RL with $\{x2 \leftarrow rd(x2, op_l(x2, x3, x1)), x1 \leftarrow x1\}$

$\overbrace{i(\overbrace{op_t(i(x1), rd(x2, op_l(x2, x3, x1)))}^{})}$

$=$ by Lemma 1002 LR with $\{x2 \leftarrow rd(x2, op_l(x2, x3, x1)), x1 \leftarrow i(x1)\}$

$\overbrace{mult(op_r(i(\overbrace{mult(i(x1), rd(x2, op_l(x2, x3, x1)))}^{})), i(x1), i(x1)), \overbrace{i(i(rd(x2, op_l(x2, x3, x1))))}^{})}$

$=$ by Lemma 3 LR with $\{x1 \leftarrow rd(x2, op_l(x2, x3, x1))\}$

$mult(op_r(i(\overbrace{mult(i(x1), rd(x2, op_l(x2, x3, x1)))}^{})), i(x1), i(x1)), \overbrace{rd(x2, op_l(x2, x3, x1))}^{})$

$=$ by Lemma 2203 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$mult(op_r(i(\overbrace{rd(rd(x2, op_l(x2, x3, x1)), x1)}^{})), i(x1), i(x1)), rd(x2, op_l(x2, x3, x1)))$

Lemma 2205: $op_t(x1, rd(op_l(x2, x3, x1), x2)) = mult(i(op_r(rd(rd(x2, op_l(x2, x3, x1)), x1), x1, x1)), rd(x2, op_l(x2, x3, x1)))$

$op_t(x1, \overbrace{rd(op_l(x2, x3, x1), x2)}^{})$

$=$ by Lemma 10 RL with $\{x2 \leftarrow op_l(x2, x3, x1), x1 \leftarrow x2\}$

$\overbrace{op_t(x1, \overbrace{i(rd(x2, op_l(x2, x3, x1)))}^{})}$

$=$ by Lemma 2204 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{mult(\overbrace{op_r(i(rd(rd(x2, op_l(x2, x3, x1)), x1)), i(x1), i(x1))}^{}, rd(x2, op_l(x2, x3, x1)))}$

$=$ by Lemma 815 LR with $\{x3 \leftarrow x1, x2 \leftarrow x1, x1 \leftarrow rd(rd(x2, op_l(x2, x3, x1)), x1)\}$

$mult(\overbrace{i(op_r(rd(rd(x2, op_l(x2, x3, x1)), x1), x1, x1))}^{}, rd(x2, op_l(x2, x3, x1)))$

Lemma 2206: $op_t(x1, rd(op_l(x2, x3, x1), x2)) = mult(op_r(rd(x1, rd(x2, op_l(x2, x3, x1))), x1, x1), rd(x2, op_l(x2, x3, x1)))$

$\overbrace{op_t(x1, rd(op_l(x2, x3, x1), x2))}^{}$

$=$ by Lemma 2205 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{mult(\overbrace{i(op_r(rd(rd(x2, op_l(x2, x3, x1)), x1), x1, x1))}^{}, rd(x2, op_l(x2, x3, x1)))}$

$=$ by Lemma 356 LR with $\{x2 \leftarrow x1, x1 \leftarrow rd(rd(x2, op_l(x2, x3, x1)), x1)\}$

$mult(\overbrace{op_r(\overbrace{i(rd(rd(x2, op_l(x2, x3, x1)), x1))}^{}), x1, x1)}^{}, rd(x2, op_l(x2, x3, x1)))$

$=$ by Lemma 10 LR with $\{x2 \leftarrow x1, x1 \leftarrow rd(x2, op_l(x2, x3, x1))\}$

$mult(op_r(\overbrace{rd(x1, rd(x2, op_l(x2, x3, x1)))}^{}), x1, x1), rd(x2, op_l(x2, x3, x1)))$

Lemma 2207: $op_t(x1, rd(op_l(x2, x3, x1), x2)) = op_t(x1, rd(x2, op_l(x2, x3, x1)))$

$\underbrace{op_t(x1, rd(op_l(x2, x3, x1), x2))}$

$=$ by Lemma 2206 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{\underbrace{mult(op_r(rd(x1, rd(x2, op_l(x2, x3, x1))), x1, x1), rd(x2, op_l(x2, x3, x1)))}}$

$=$ by Lemma 808 LR with $\{x2 \leftarrow rd(x2, op_l(x2, x3, x1)), x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_l(x1, x1, rd(x2, op_l(x2, x3, x1)))}}$

$=$ by Lemma 2052 LR with $\{x4 \leftarrow op_l(x2, x3, x1), x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_l(x1, rd(op_l(x2, x3, x1), x2), x1)}}$

$=$ by Lemma 2035 RL with $\{x3 \leftarrow rd(op_l(x2, x3, x1), x2), x2 \leftarrow x1, x1 \leftarrow x1\}$

$\overbrace{op_l(\underbrace{x1}, x1, rd(x1, rd(op_l(x2, x3, x1), x2)))}$

$=$ by Lemma 2092 RL with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{op_l(\overbrace{op_t(x1, rd(x1, rd(op_l(x2, x3, x1), x2)))}, x1, rd(x1, rd(op_l(x2, x3, x1), x2)))}$

$=$ by Lemma 421 RL with $\{x2 \leftarrow rd(x1, rd(op_l(x2, x3, x1), x2)), x1 \leftarrow x1\}$

$\overbrace{op_r(x1, rd(x1, rd(op_l(x2, x3, x1), x2)), \underbrace{i(rd(x1, rd(op_l(x2, x3, x1), x2)))})$

$=$ by Lemma 10 LR with $\{x2 \leftarrow rd(op_l(x2, x3, x1), x2), x1 \leftarrow x1\}$

$\underbrace{op_r(x1, rd(x1, rd(op_l(x2, x3, x1), x2)), \overbrace{rd(rd(op_l(x2, x3, x1), x2), x1)})$

$=$ by Lemma 857 RL with $\{x2 \leftarrow rd(op_l(x2, x3, x1), x2), x1 \leftarrow x1\}$

$\overbrace{op_t(x1, \underbrace{i(rd(op_l(x2, x3, x1), x2))})}$

$=$ by Lemma 10 LR with $\{x2 \leftarrow x2, x1 \leftarrow op_l(x2, x3, x1)\}$

$op_t(x1, \overbrace{rd(x2, op_l(x2, x3, x1))})$

Lemma 2208: $rd(rd(op_l(x1,x2,x3),x1),x3) = mult(i(x3),rd(op_l(x1,x2,x3),x1))$

$\overbrace{rd(rd(op_l(x1,x2,x3),x1),x3)}$

$=$ by Lemma 1989 LR with $\{x2 \leftarrow x3, x1 \leftarrow rd(op_l(x1,x2,x3),x1)\}$

$\overbrace{mult(op_l(i(x3),rd(op_l(x1,x2,x3),x1),x3),\underbrace{op_r(rd(op_l(x1,x2,x3),x1),rd(i(x3),rd(op_l(x1,x2,x3),x1)),rd(i(x3),rd(op_l(x1}$

$=$ by Lemma 695 RL with $\{x2 \leftarrow rd(op_l(x1,x2,x3),x1), x1 \leftarrow rd(i(x3),rd(op_l(x1,x2,x3),x1))\}$

$mult(op_l(i(x3),rd(op_l(x1,x2,x3),x1),x3),\overbrace{mult(\underbrace{mult(rd(i(x3),rd(op_l(x1,x2,x3),x1)),rd(op_l(x1,x2,x3),x1))}, op_t(i(rd(i($

$=$ by Axiom 4 RL with $\{x2 \leftarrow rd(op_l(x1,x2,x3),x1), x1 \leftarrow i(x3)\}$

$mult(op_l(i(x3),rd(op_l(x1,x2,x3),x1),x3),mult(\overbrace{i(x3)}, op_t(\underbrace{i(rd(i(x3),rd(op_l(x1,x2,x3),x1)))}, rd(op_l(x1,x2,x3),x1))))$

$=$ by Lemma 10 LR with $\{x2 \leftarrow rd(op_l(x1,x2,x3),x1), x1 \leftarrow i(x3)\}$

$mult(op_l(i(x3),rd(op_l(x1,x2,x3),x1),x3),\underbrace{mult(i(x3),op_t(\overbrace{rd(rd(op_l(x1,x2,x3),x1),i(x3))}, rd(op_l(x1,x2,x3),x1))))}$

$=$ by Lemma 465 LR with $\{x2 \leftarrow rd(op_l(x1,x2,x3),x1), x1 \leftarrow i(x3)\}$

$mult(op_l(i(x3),rd(op_l(x1,x2,x3),x1),x3),\overbrace{rd(i(x3),mult(i(x3),i(rd(op_l(x1,x2,x3),x1)))))}$

$=$ by Lemma 64 LR with $\{x3 \leftarrow i(rd(op_l(x1,x2,x3),x1)), x2 \leftarrow x3, x1 \leftarrow x3\}$

$mult(op_l(i(x3),rd(op_l(x1,x2,x3),x1),x3),\overbrace{rd(mult(x3,\underbrace{i(i(rd(op_l(x1,x2,x3),x1))))}, x3))}$

$=$ by Lemma 3 LR with $\{x1 \leftarrow rd(op_l(x1,x2,x3),x1)\}$

$mult(op_l(i(x3),rd(op_l(x1,x2,x3),x1),x3),rd(\underbrace{mult(x3,\overbrace{rd(op_l(x1,x2,x3),x1)})}, x3))$

$=$ by Lemma 2124 LR with $\{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x3\}$

$mult(op_l(i(x3),rd(op_l(x1,x2,x3),x1),x3),\underbrace{rd(\overbrace{rd(x3,rd(x1,op_l(x1,x2,x3)))}, x3))}$

$=$ by Lemma 354 LR with $\{x2 \leftarrow rd(x1,op_l(x1,x2,x3)), x1 \leftarrow x3\}$

$mult(op_l(i(x3),rd(op_l(x1,x2,x3),x1),x3),\overbrace{op_r(\underbrace{i(rd(x1,op_l(x1,x2,x3)))}, x3, x3))}$

$=$ by Lemma 10 LR with $\{x2 \leftarrow op_l(x1,x2,x3), x1 \leftarrow x1\}$

$mult(\underbrace{op_l(i(x3),rd(op_l(x1,x2,x3),x1),x3)}, op_r(\overbrace{rd(op_l(x1,x2,x3),x1)}, x3, x3))$

$=$ by Lemma 433 RL with $\{x2 \leftarrow rd(op_l(x1,x2,x3),x1), x1 \leftarrow x3\}$

$mult(\overbrace{i(op_l(x3,rd(op_l(x1,x2,x3),x1),x3))}, \underbrace{op_r(rd(op_l(x1,x2,x3),x1),x3,x3)})$

$=$ by Lemma 698 RL with $\{x2 \leftarrow x3, x1 \leftarrow rd(op_l(x1,x2,x3),x1)\}$

$\underbrace{mult(i(op_l(x3,rd(op_l(x1,x2,x3),x1),x3)),\overbrace{mult(rd(rd(op_l(x1,x2,x3),x1),x3),op_l(x3,rd(op_l(x1,x2,x3),x1),x3))})}$

$=$ by Axiom 10 LR with $\{x2 \leftarrow rd(rd(op_l(x1,x2,x3),x1),x3), x1 \leftarrow op_l(x3,rd(op_l(x1,x2,x3),x1),x3)\}$

$\overbrace{\underbrace{op_t(rd(rd(op_l(x1,x2,x3),x1),x3),op_l(x3,rd(op_l(x1,x2,x3),x1),x3))}}$

$=$ by Lemma 587 RL with $\{x1 \leftarrow rd(op_l(x1,x2,x3),x1), x2 \leftarrow x3\}$

$\overbrace{mult(i(x3),rd(op_l(x1,x2,x3),x1))}$

Lemma 2209: $op_r(x1, i(x2), mult(x2, op_t(i(x3), rd(x3, x2)))) = rd(mult(mult(x1, i(x2)), rd(op_t(i(x3), rd(x3, i(x2))), i(x2))), mu$

$op_r(x1, i(x2), mult(x2, \underbrace{op_t(i(x3), rd(x3, x2))}))$

$=$ by Lemma 715 RL with $\{x2 \leftarrow x2, x1 \leftarrow x3\}$

$op_r(x1, i(x2), mult(x2, \overbrace{op_t(i(x3), mult(x2, x3))}))$

$=$ by Lemma 1729 RL with $\{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow i(x3)\}$

$op_r(x1, i(x2), mult(\underbrace{x2}, \overbrace{op_t(i(x3), rd(x3, i(x2)))}))$

$=$ by Lemma 3 RL with $\{x1 \leftarrow x2\}$

$\underbrace{op_r(x1, i(x2), mult(\overbrace{i(i(x2))}, op_t(i(x3), rd(x3, i(x2)))))}$

$=$ by Lemma 1891 LR with $\{x3 \leftarrow op_t(i(x3), rd(x3, i(x2))), x2 \leftarrow i(x2), x1 \leftarrow x1\}$

$\overbrace{op_r(x1, i(x2), rd(op_t(i(x3), rd(x3, i(x2))), i(x2)))}$

$=$ by Axiom 16 LR with $\{x3 \leftarrow rd(op_t(i(x3), rd(x3, i(x2))), i(x2)), x2 \leftarrow i(x2), x1 \leftarrow x1\}$

$\overbrace{rd(mult(mult(x1, i(x2)), rd(op_t(i(x3), rd(x3, i(x2))), i(x2))), \underbrace{mult(i(x2), rd(op_t(i(x3), rd(x3, i(x2))), i(x2)))}})$

$=$ by Lemma 69 LR with $\{x2 \leftarrow op_t(i(x3), rd(x3, i(x2))), x1 \leftarrow x2\}$

$rd(mult(mult(x1, i(x2)), rd(op_t(i(x3), rd(x3, i(x2))), i(x2))), \overbrace{mult(mult(i(x2), \underbrace{op_t(i(x3), rd(x3, i(x2)))}), x2)})$

$=$ by Lemma 714 RL with $\{x2 \leftarrow i(x2), x1 \leftarrow x3\}$

$rd(mult(mult(x1, i(x2)), rd(op_t(i(x3), rd(x3, i(x2))), i(x2))), mult(\underbrace{mult(i(x2), \overbrace{op_t(i(x3), mult(x3, i(x2)))})}, x2))$

$=$ by Lemma 57 RL with $\{x2 \leftarrow op_t(i(x3), mult(x3, i(x2))), x1 \leftarrow i(x2)\}$

$rd(mult(mult(x1, i(x2)), rd(op_t(i(x3), rd(x3, i(x2))), i(x2))), mult(\overbrace{rd(i(x2), mult(i(i(x2)), rd(\underbrace{i(x2)}, op_t(i(x3), mult(x3, i(x$

$=$ by Axiom 2 LR with $\{x2 \leftarrow x3, x1 \leftarrow i(x2)\}$

$rd(mult(mult(x1, i(x2)), rd(op_t(i(x3), rd(x3, i(x2))), i(x2))), mult(rd(i(x2), mult(i(i(x2)), \underbrace{rd(\overbrace{mult(i(x3), mult(x3, i(x2)))}$

$=$ by Lemma 29 LR with $\{x2 \leftarrow mult(x3, i(x2)), x1 \leftarrow i(x3)\}$

$rd(mult(mult(x1, i(x2)), rd(op_t(i(x3), rd(x3, i(x2))), i(x2))), mult(rd(i(x2), \underbrace{mult(i(i(x2)), \overbrace{mult(x3, i(x2))})}), x2))$

$=$ by Axiom 10 LR with $\{x2 \leftarrow x3, x1 \leftarrow i(x2)\}$

$rd(mult(mult(x1, i(x2)), rd(op_t(i(x3), rd(x3, i(x2))), i(x2))), mult(rd(i(x2), \overbrace{op_t(x3, i(x2))}), x2))$

Lemma 2210: $op_r(x1, i(x2), mult(i(x3), x2)) = rd(mult(mult(x1, i(x2)), rd(op_t(i(x3), rd(x3, i(x2))), i(x2))), mult(rd(i(x2), op_t($

$op_r(x1, i(x2), \underbrace{mult(i(x3), x2)})$

$=$ by Lemma 343 RL with $\{x2 \leftarrow x3, x1 \leftarrow x2\}$

$op_r(x1, i(x2), \overbrace{rd(x2, \underbrace{op_r(x3, x2, i(x2))})})$

$=$ by Lemma 321 RL with $\{x2 \leftarrow x2, x1 \leftarrow x3\}$

$op_r(x1, i(x2), \underbrace{rd(x2, \overbrace{op_r(x3, i(x2), x2)})})$

$=$ by Lemma 23 RL with $\{x2 \leftarrow x2, x1 \leftarrow op_r(x3, i(x2), x2)\}$

$op_r(x1, i(x2), \overbrace{rd(\underbrace{i(op_r(x3, i(x2), x2))}, i(x2))})$

$=$ by Lemma 816 LR with $\{x3 \leftarrow x2, x2 \leftarrow x2, x1 \leftarrow x3\}$

$op_r(x1, i(x2), rd(\overbrace{\underbrace{op_r(i(x3), x2, i(x2))}}, i(x2)))$

$=$ by Lemma 321 RL with $\{x2 \leftarrow x2, x1 \leftarrow i(x3)\}$

$op_r(x1, i(x2), rd(\overbrace{\underbrace{op_r(i(x3), i(x2), x2)}}, i(x2)))$

$=$ by Lemma 207 RL with $\{x2 \leftarrow x3, x1 \leftarrow x2\}$

$\underbrace{op_r(x1, i(x2), rd(\overbrace{mult(i(x2), rd(x2, op_t(x3, x2)))}, i(x2)))}$

$=$ by Lemma 1891 RL with $\{x3 \leftarrow mult(i(x2), rd(x2, op_t(x3, x2))), x2 \leftarrow i(x2), x1 \leftarrow x1\}$

$\overbrace{op_r(x1, i(x2), \underbrace{mult(i(i(x2)), \underbrace{mult(i(x2), rd(x2, op_t(x3, x2)))})})}$

$=$ by Axiom 2 RL with $\{x2 \leftarrow i(x2), x1 \leftarrow rd(x2, op_t(x3, x2))\}$

$op_r(x1, i(x2), \overbrace{rd(x2, \underbrace{op_t(x3, x2)})})$

$=$ by Axiom 10 RL with $\{x2 \leftarrow x3, x1 \leftarrow x2\}$

$op_r(x1, i(x2), rd(x2, \overbrace{mult(i(x2), \underbrace{mult(x3, x2)})}))$

$=$ by Lemma 29 RL with $\{x2 \leftarrow mult(x3, x2), x1 \leftarrow i(x3)\}$

$op_r(x1, i(x2), rd(x2, mult(i(x2), \overbrace{rd(\underbrace{mult(i(x3), mult(x3, x2))}, op_t(i(x3), mult(x3, x2)))})))$

$=$ by Axiom 2 RL with $\{x2 \leftarrow x3, x1 \leftarrow x2\}$

$op_r(x1, i(x2), \underbrace{rd(x2, mult(i(x2), rd(\overbrace{x2}, op_t(i(x3), mult(x3, x2)))))})$

$=$ by Lemma 57 LR with $\{x2 \leftarrow op_t(i(x3), mult(x3, x2)), x1 \leftarrow x2\}$

$op_r(x1, i(x2), \overbrace{mult(x2, \underbrace{op_t(i(x3), mult(x3, x2))})})$

$=$ by Lemma 714 LR with $\{x2 \leftarrow x2, x1 \leftarrow x3\}$

$\underbrace{op_r(x1, i(x2), mult(x2, \overbrace{op_t(i(x3), rd(x3, x2))}))}$

$=$ by Lemma 2209 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{rd(mult(mult(x1, i(x2)), rd(op_t(i(x3), rd(x3, i(x2))), i(x2))), mult(rd(i(x2), op_t(x3, i(x2))), x2))}$

Lemma 2211: $op_r(x1, i(x2), mult(i(x3), x2)) = rd(mult(mult(x1, i(x2)), rd(x2, op_t(x3, rd(i(x2), x3)))), mult(rd(i(x2), op_t(x3, i($

$\overbrace{op_r(x1, i(x2), mult(i(x3), x2))}$

$=$ by Lemma 2210 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{rd(mult(mult(x1, i(x2)), \underbrace{rd(op_t(i(x3), rd(x3, i(x2))), i(x2))}), mult(rd(i(x2), op_t(x3, i(x2))), x2))}$

$=$ by Lemma 67 LR with $\{x3 \leftarrow x2, x2 \leftarrow rd(x3, i(x2)), x1 \leftarrow x3\}$

$rd(mult(mult(x1, i(x2)), \overbrace{rd(x2, op_t(x3, \underbrace{i(rd(x3, i(x2)))})))}), mult(rd(i(x2), op_t(x3, i(x2))), x2))$

$=$ by Lemma 10 LR with $\{x2 \leftarrow i(x2), x1 \leftarrow x3\}$

$rd(mult(mult(x1, i(x2)), rd(x2, op_t(x3, \overbrace{rd(i(x2), x3)}))), mult(rd(i(x2), op_t(x3, i(x2))), x2))$

Lemma 2212: $op_r(x1, i(x2), mult(i(x3), x2)) = rd(mult(mult(x1, i(x2)), mult(i(x3), x2)), mult(rd(i(x2), op_t(x3, i(x2))), x2))$

$\overbrace{op_r(x1, i(x2), mult(i(x3), x2))}$

$=$ by Lemma 2211 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{rd(mult(mult(x1, i(x2)), rd(x2, \underbrace{op_t(x3, rd(i(x2), x3))})), mult(rd(i(x2), op_t(x3, i(x2))), x2))}$

$=$ by Lemma 712 LR with $\{x2 \leftarrow x2, x1 \leftarrow x3\}$

$rd(mult(mult(x1, i(x2)), \underbrace{rd(x2, \overbrace{op_t(x3, rd(x2, x3))})}), mult(rd(i(x2), op_t(x3, i(x2))), x2))$

$=$ by Lemma 380 LR with $\{x2 \leftarrow x3, x1 \leftarrow x2\}$

$rd(mult(mult(x1, i(x2)), \overbrace{mult(i(x3), x2)}), mult(rd(i(x2), op_t(x3, i(x2))), x2))$

Lemma 2213: $op_r(x1, i(x2), mult(i(x3), x2)) = rd(mult(mult(x1, i(x2)), mult(i(x3), x2)), op_t(i(x3), x2))$

$\overbrace{op_r(x1, i(x2), mult(i(x3), x2))}$

$=$ by Lemma 2212 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{rd(mult(mult(x1, i(x2)), mult(i(x3), x2)), \underbrace{mult(rd(i(x2), op_t(x3, i(x2))), x2)})}$

$=$ by Lemma 21 LR with $\{x2 \leftarrow op_t(x3, i(x2)), x1 \leftarrow x2\}$

$rd(mult(mult(x1, i(x2)), mult(i(x3), x2)), \overbrace{i(\underbrace{op_t(x3, i(x2))})})$

$=$ by Lemma 19 LR with $\{x2 \leftarrow x2, x1 \leftarrow x3\}$

$rd(mult(mult(x1, i(x2)), mult(i(x3), x2)), \overbrace{op_t(i(x3), x2)})$

Lemma 2214: $op_r(x1, i(x2), mult(i(x3), x2)) = op_r(x1, i(x2), mult(x2, i(x3)))$

$\overbrace{op_r(x1, i(x2), mult(i(x3), x2))}$

$=$ by Lemma 2213 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{rd(mult(mult(x1, i(x2)), mult(i(x3), x2)), op_t(i(x3), x2))}$

$=$ by Lemma 2199 RL with $\{x2 \leftarrow i(x3), x1 \leftarrow x2, x3 \leftarrow mult(x1, i(x2))\}$

$\overbrace{rd(mult(mult(x1, i(x2)), mult(x2, i(x3))), i(x3))}$

$=$ by Lemma 665 LR with $\{x3 \leftarrow i(x3), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_r(x1, i(x2), mult(x2, i(x3)))}$

Lemma 2215: $op_r(x1, i(x2), mult(i(x3), x2)) = op_r(x1, i(x2), rd(x2, x3))$

$$
\begin{aligned}
& \underbrace{op_r(x1, i(x2), mult(i(x3), x2))} \\
= & \quad \text{by Lemma 2214 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, i(x2), \underbrace{mult(x2, i(x3))})} \\
= & \quad \text{by Lemma 34 RL with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& \underbrace{op_r(x1, i(x2), \overbrace{op_t(rd(x2, x3), i(x2))})} \\
= & \quad \text{by Lemma 1890 LR with } \{x3 \leftarrow rd(x2, x3), x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, i(x2), rd(x2, x3))}
\end{aligned}
$$

Lemma 2216: $op_r(x1, x2, i(mult(i(x3), x2))) = i(op_r(i(x1), i(x2), rd(x2, x3)))$

$$
\begin{aligned}
& op_r(x1, \underbrace{x2}, i(mult(i(x3), x2))) \\
= & \quad \text{by Lemma 3 RL with } \{x1 \leftarrow x2\} \\
& \underbrace{op_r(x1, \overbrace{i(i(x2))}, i(mult(i(x3), x2)))} \\
= & \quad \text{by Lemma 813 RL with } \{x3 \leftarrow mult(i(x3), x2), x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& \overbrace{i(\underbrace{op_r(i(x1), i(x2), mult(i(x3), x2))})} \\
= & \quad \text{by Lemma 2215 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow i(x1)\} \\
& i(\overbrace{op_r(i(x1), i(x2), rd(x2, x3))})
\end{aligned}
$$

Lemma 2217: $op_r(x1, x2, mult(x3, i(x2))) = op_r(x1, i(i(x2)), i(rd(x2, x3)))$

$$
\begin{aligned}
& op_r(x1, x2, \underbrace{mult(x3, i(x2))}) \\
= & \quad \text{by Lemma 16 RL with } \{x2 \leftarrow x2, x1 \leftarrow x3\} \\
& \underbrace{op_r(x1, x2, \overbrace{i(mult(i(x3), x2))})} \\
= & \quad \text{by Lemma 2216 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{i(op_r(i(x1), i(x2), rd(x2, x3)))}} \\
= & \quad \text{by Lemma 813 LR with } \{x3 \leftarrow rd(x2, x3), x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, i(i(x2)), i(rd(x2, x3)))}
\end{aligned}
$$

Lemma 2218: $op_r(x1, x2, mult(x3, i(x2))) = op_r(x1, x2, rd(x3, x2))$

$$
\begin{aligned}
& \underbrace{op_r(x1, x2, mult(x3, i(x2)))} \\
= & \quad \text{by Lemma 2217 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, \underbrace{i(i(x2))}, i(rd(x2, x3)))} \\
= & \quad \text{by Lemma 3 LR with } \{x1 \leftarrow x2\} \\
& op_r(x1, \overbrace{x2}, \underbrace{i(rd(x2, x3))}) \\
= & \quad \text{by Lemma 10 LR with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& op_r(x1, x2, \overbrace{rd(x3, x2)})
\end{aligned}
$$

Lemma 2219: $op_r(x1, x2, op_l(x3, x2, x3)) = op_r(x1, x2, x3)$

$$
\begin{aligned}
& \underbrace{op_r(x1, x2, op_l(x3, x2, x3))} \\
= \quad & \text{by Lemma 1907 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, x2, \underbrace{op_r(x3, x2, x2)})} \\
= \quad & \text{by Lemma 358 RL with } \{x2 \leftarrow x2, x1 \leftarrow x3\} \\
& \underbrace{op_r(x1, x2, \overbrace{rd(rd(x3, i(x2)), x2)})} \\
= \quad & \text{by Lemma 2218 RL with } \{x3 \leftarrow rd(x3, i(x2)), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, x2, \underbrace{mult(rd(x3, i(x2)), i(x2))})} \\
= \quad & \text{by Axiom 4 RL with } \{x2 \leftarrow i(x2), x1 \leftarrow x3\} \\
& op_r(x1, x2, \overbrace{x3}^{})
\end{aligned}
$$

Lemma 2220: $op_r(x1, op_r(x2, x3, x3), x3) = op_r(x1, x2, x3)$

$$
\begin{aligned}
& \underbrace{op_r(x1, op_r(x2, x3, x3), x3)} \\
= \quad & \text{by Lemma 2162 LR with } \{x2 \leftarrow x3, x3 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_r(x1, op_l(x2, x3, x2), x3)}} \\
= \quad & \text{by Lemma 2014 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_r(x1, x2, op_l(x3, x2, x3))}} \\
= \quad & \text{by Lemma 2219 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, x2, x3)}
\end{aligned}
$$

Lemma 2221: $op_r(x1, op_l(x2, x2, x3), x3) = op_r(x1, x2, x3)$

$$
\begin{aligned}
& \underbrace{op_r(x1, op_l(x2, x2, x3), x3)} \\
= \quad & \text{by Lemma 2012 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, x2, \underbrace{op_l(x3, x3, x2)})} \\
= \quad & \text{by Lemma 2031 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x3\} \\
& op_r(x1, x2, \overbrace{\underbrace{op_l(x3, i(x2), x3)}}) \\
= \quad & \text{by Lemma 744 LR with } \{x2 \leftarrow i(x2), x1 \leftarrow x3\} \\
& \underbrace{op_r(x1, x2, \overbrace{mult(mult(i(i(x2)), x3), i(x2))})} \\
= \quad & \text{by Lemma 2218 LR with } \{x3 \leftarrow mult(i(i(x2)), x3), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, x2, rd(mult(\underbrace{i(i(x2))}, x3), x2))} \\
= \quad & \text{by Lemma 3 LR with } \{x1 \leftarrow x2\} \\
& \underbrace{op_r(x1, x2, rd(mult(\overbrace{x2}^{}, x3), x2))} \\
= \quad & \text{by Lemma 1891 RL with } \{x3 \leftarrow mult(x2, x3), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, x2, \underbrace{mult(i(x2), mult(x2, x3))})} \\
= \quad & \text{by Axiom 2 RL with } \{x2 \leftarrow x2, x1 \leftarrow x3\} \\
& op_r(x1, x2, \overbrace{x3}^{})
\end{aligned}
$$

Lemma 2222: $op_r(x1, op_l(x2, x2, x3), x3) = op_r(x1, op_l(op_l(x2, x2, x3), x2, x3), x3)$

$$
\begin{aligned}
& \underbrace{op_r(x1, op_l(x2, x2, x3), x3)} \\
= \quad & \text{by Lemma 2221 RL with } \{x3 \leftarrow x3, x2 \leftarrow op_l(x2, x2, x3), x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, \underbrace{op_l(op_l(x2, x2, x3), op_l(x2, x2, x3), x3)}, x3)} \\
= \quad & \text{by Lemma 1339 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow op_l(x2, x2, x3)\} \\
& op_r(x1, \overbrace{op_l(op_l(x2, x2, x3), x3, i(x2))}, x3) \\
= \quad & \text{by Lemma 2030 LR with } \{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow op_l(x2, x2, x3)\} \\
& op_r(x1, \overbrace{op_l(op_l(x2, x2, x3), x2, x3)}, x3)
\end{aligned}
$$

Lemma 2223: $op_r(x1, x2, x3) = op_r(x1, op_l(x2, x2, mult(x3, x3)), x3)$

$$
\begin{aligned}
& \underbrace{op_r(x1, x2, x3)} \\
= \quad & \text{by Lemma 2221 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_r(x1, op_l(x2, x2, x3), x3)}} \\
= \quad & \text{by Lemma 2222 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, \underbrace{op_l(op_l(x2, x2, x3), x2, x3)}, x3)} \\
= \quad & \text{by Lemma 1174 LR with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& op_r(x1, \overbrace{op_l(x2, x2, mult(x3, x3))}, x3)
\end{aligned}
$$

Lemma 2224: $op_r(x1, op_t(x2, i(x3)), x3) = op_r(x1, x2, x3)$

$$
\begin{aligned}
& \underbrace{op_r(x1, op_t(x2, i(x3)), x3)} \\
= \quad & \text{by Lemma 1896 RL with } \{x3 \leftarrow op_t(x2, i(x3)), x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, rd(\underbrace{mult(x3, op_t(x2, i(x3)))}, x3), x3)} \\
= \quad & \text{by Lemma 936 RL with } \{x1 \leftarrow x2, x2 \leftarrow x3\} \\
& op_r(x1, rd(\overbrace{mult(op_t(x3, i(x2)), x2)}, x3), x3) \\
= \quad & \text{by Lemma 456 RL with } \{x2 \leftarrow x2, x1 \leftarrow x3\} \\
& op_r(x1, rd(\overbrace{op_t(op_t(mult(x3, x2), x3), x2)}, x3), x3) \\
= \quad & \text{by Axiom 13 LR with } \{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow mult(x3, x2)\} \\
& op_r(x1, \underbrace{rd(\overbrace{op_t(op_t(mult(x3, x2), x2), x3)}, x3)}, x3) \\
= \quad & \text{by Lemma 1194 RL with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& \underbrace{op_r(x1, \overbrace{op_l(x2, x2, mult(x3, x3))}, x3)} \\
= \quad & \text{by Lemma 2223 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, x2, x3)}
\end{aligned}
$$

Lemma 2225: $op_r(x1, mult(x2, x3), x3) = op_r(x1, rd(x2, i(x3)), x3)$

$$
\begin{aligned}
& \underbrace{op_r(x1, mult(x2, x3), x3)} \\
= \quad & \text{by Lemma 2219 RL with } \{x3 \leftarrow x3, x2 \leftarrow mult(x2, x3), x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, mult(x2, x3), \underbrace{op_l(x3, mult(x2, x3), x3)})} \\
= \quad & \text{by Lemma 285 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x3\} \\
& \underbrace{op_r(x1, mult(x2, x3), \overbrace{op_l(x3, x2, x3)})} \\
= \quad & \text{by Lemma 2021 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, rd(x2, i(x3)), x3)}
\end{aligned}
$$

Lemma 2226: $op_r(x1, mult(x2, i(x3)), x3) = op_r(x1, rd(x2, x3), x3)$

$$
\begin{aligned}
& \underbrace{op_r(x1, mult(x2, i(x3)), x3)} \\
= \quad & \text{by Lemma 2019 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_r(x1, rd(x2, x3), op_l(x3, x3, x2))}} \\
= \quad & \text{by Lemma 2219 RL with } \{x3 \leftarrow op_l(x3, x3, x2), x2 \leftarrow rd(x2, x3), x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, rd(x2, x3), \underbrace{op_l(op_l(x3, x3, x2), rd(x2, x3), op_l(x3, x3, x2))})} \\
= \quad & \text{by Lemma 1312 RL with } \{x3 \leftarrow rd(x2, x3), x2 \leftarrow op_l(x3, x3, x2), x1 \leftarrow op_l(x3, x3, x2)\} \\
& op_r(x1, rd(x2, x3), \overbrace{op_l(op_l(x3, x3, x2), op_t(op_l(x3, x3, x2), rd(x2, x3)), i(\underbrace{mult(op_l(x3, x3, x2), rd(x2, x3))}))}) \\
= \quad & \text{by Lemma 688 LR with } \{x2 \leftarrow x2, x1 \leftarrow x3\} \\
& op_r(x1, rd(x2, x3), op_l(op_l(x3, x3, x2), \underbrace{op_t(op_l(x3, x3, x2), rd(x2, x3))}, i(\overbrace{op_t(x2, i(x3))}))) \\
= \quad & \text{by Axiom 15 LR with } \{x4 \leftarrow rd(x2, x3), x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x3\} \\
& op_r(x1, rd(x2, x3), op_l(op_l(x3, x3, x2), \overbrace{op_l(op_t(x3, rd(x2, x3)), x3, x2)}, i(op_t(x2, i(x3))))) \\
= \quad & \text{by Lemma 511 LR with } \{x2 \leftarrow x2, x1 \leftarrow x3\} \\
& op_r(x1, rd(x2, x3), \underbrace{op_l(op_l(x3, x3, x2), \overbrace{op_t(x3, i(x2))}, i(op_t(x2, i(x3))))}) \\
= \quad & \text{by Lemma 1137 LR with } \{x4 \leftarrow op_t(x2, i(x3)), x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow op_l(x3, x3, x2)\} \\
& op_r(x1, rd(x2, x3), \overbrace{\underbrace{op_l(op_l(x3, x3, x2), op_t(i(x3), x2), op_t(x2, i(x3)))}}) \\
= \quad & \text{by Lemma 1264 LR with } \{x3 \leftarrow x2, x2 \leftarrow i(x3), x1 \leftarrow op_l(x3, x3, x2)\} \\
& op_r(x1, rd(x2, x3), \overbrace{\underbrace{op_l(op_l(x3, x3, x2), i(x3), x2)}}) \\
= \quad & \text{by Lemma 1050 LR with } \{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow op_l(x3, x3, x2)\} \\
& op_r(x1, rd(x2, x3), \overbrace{\underbrace{op_l(op_l(x3, x3, x2), x3, i(x2))}}) \\
= \quad & \text{by Lemma 2030 LR with } \{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow op_l(x3, x3, x2)\} \\
& op_r(x1, rd(x2, x3), \overbrace{\underbrace{op_l(op_l(x3, x3, x2), x2, x3)}}) \\
= \quad & \text{by Lemma 138 LR with } \{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x3\} \\
& op_r(x1, rd(x2, x3), \overbrace{x3}^{})
\end{aligned}
$$

Lemma 2227: $op_r(x1, rd(x2, x3), x2) = op_r(x1, i(mult(i(x2), x3)), x2)$

$$
\begin{aligned}
& op_r(x1, rd(\underbrace{x2}, x3), x2) \\
= \quad & \text{by Lemma 3 RL with } \{x1 \leftarrow x2\} \\
& \underbrace{op_r(x1, rd(\overbrace{i(i(x2))}, x3), x2)} \\
= \quad & \text{by Lemma 2224 RL with } \{x3 \leftarrow x2, x2 \leftarrow rd(i(i(x2)), x3), x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, \underbrace{op_t(rd(i(i(x2)), x3), i(x2))}, x2)} \\
= \quad & \text{by Lemma 33 LR with } \{x2 \leftarrow x3, x1 \leftarrow i(x2)\} \\
& op_r(x1, \overbrace{i(mult(i(x2), x3))}, x2)
\end{aligned}
$$

Lemma 2228: $op_r(x1, op_r(i(x3), x2, x2), x2) = op_r(x1, rd(x2, mult(x2, x3)), x2)$

$$
\begin{aligned}
& op_r(x1, \underbrace{op_r(i(x3), x2, x2)}, x2) \\
= \quad & \text{by Lemma 354 RL with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& \underbrace{op_r(x1, \overbrace{rd(rd(x2, x3), x2)}, x2)} \\
= \quad & \text{by Lemma 1895 RL with } \{x3 \leftarrow rd(x2, x3), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_r(x1, mult(i(x2), rd(x2, x3)), x2)}} \\
= \quad & \text{by Lemma 2220 RL with } \{x3 \leftarrow x2, x2 \leftarrow mult(i(x2), rd(x2, x3)), x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, \underbrace{op_r(mult(i(x2), rd(x2, x3)), x2, x2)}, x2)} \\
= \quad & \text{by Lemma 1156 RL with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& op_r(x1, \overbrace{rd(x2, mult(x2, x3))}, x2)
\end{aligned}
$$

Lemma 2229: $op_r(x1, rd(op_t(mult(x3, x2), x3), x2), x2) = op_r(x1, rd(x2, rd(x2, rd(mult(x2, x3), x2))), x2)$

$$
\begin{aligned}
& op_r(x1, \underbrace{rd(op_t(mult(x3, x2), x3), x2)}, x2) \\
= \quad & \text{by Lemma 10 RL with } \{x2 \leftarrow op_t(mult(x3, x2), x3), x1 \leftarrow x2\} \\
& \underbrace{op_r(x1, \overbrace{i(rd(x2, op_t(mult(x3, x2), x3)))}, x2)} \\
= \quad & \text{by Lemma 2220 RL with } \{x3 \leftarrow x2, x2 \leftarrow i(rd(x2, op_t(mult(x3, x2), x3))), x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_r(x1, op_r(i(rd(x2, op_t(mult(x3, x2), x3))), x2, x2), x2)}} \\
= \quad & \text{by Lemma 2228 LR with } \{x2 \leftarrow x2, x3 \leftarrow rd(x2, op_t(mult(x3, x2), x3)), x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, rd(x2, \underbrace{mult(x2, rd(x2, op_t(mult(x3, x2), x3)))}), x2)} \\
= \quad & \text{by Lemma 1875 RL with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& op_r(x1, rd(x2, \overbrace{rd(x2, rd(mult(x2, x3), x2))}), x2)
\end{aligned}
$$

Lemma 2230: $op_r(x1, rd(op_t(mult(x3, x2), x3), x2), x2) = op_r(x1, x3, x2)$

$$
\begin{aligned}
& \overbrace{op_r(x1, rd(op_t(mult(x3, x2), x3), x2), x2)} \\
= \quad & \text{by Lemma 2229 LR with } \{x2 \leftarrow x2, x3 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, \underbrace{rd(x2, rd(x2, rd(mult(x2, x3), x2)))}, x2)} \\
= \quad & \text{by Lemma 353 LR with } \{x2 \leftarrow rd(mult(x2, x3), x2), x1 \leftarrow x2\} \\
& \underbrace{op_r(x1, \overbrace{op_r(rd(mult(x2, x3), x2), x2, x2)}, x2)} \\
= \quad & \text{by Lemma 2220 LR with } \{x3 \leftarrow x2, x2 \leftarrow rd(mult(x2, x3), x2), x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_r(x1, rd(mult(x2, x3), x2), x2)}} \\
= \quad & \text{by Lemma 1896 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, x3, x2)}
\end{aligned}
$$

Lemma 2231: $op_r(x1, x3, x2) = op_r(x1, rd(op_t(mult(x2, x3), x3), x2), x2)$

$$
\begin{aligned}
& \underbrace{op_r(x1, x3, x2)} \\
= \quad & \text{by Lemma 1899 LR with } \{x2 \leftarrow x2, x3 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_r(x1, rd(mult(op_t(x2, x3), x3), x2), x2)}} \\
= \quad & \text{by Lemma 2226 RL with } \{x3 \leftarrow x2, x2 \leftarrow mult(op_t(x2, x3), x3), x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_r(x1, mult(mult(op_t(x2, x3), x3), i(x2)), x2)}} \\
= \quad & \text{by Lemma 1905 RL with } \{x3 \leftarrow x2, x2 \leftarrow mult(op_t(x2, x3), x3), x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, rd(\underbrace{op_t(mult(op_t(x2, x3), x3), i(x2))}, x2), x2)} \\
= \quad & \text{by Lemma 751 RL with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& op_r(x1, rd(\overbrace{op_t(mult(x2, x3), x3)}, x2), x2)
\end{aligned}
$$

Lemma 2232: $op_l(x1, i(x2), rd(x2, op_t(i(x3), x4))) = op_l(x1, op_t(i(x2), op_t(mult(mult(i(x2), x3), x2), i(x4))), mult(rd(x2, op_t(i(x$

$\underbrace{op_l(x1, i(x2), rd(x2, op_t(i(x3), x4)))}$

$=$ by Lemma 2030 RL with $\{x3 \leftarrow i(x2), x2 \leftarrow rd(x2, op_t(i(x3), x4)), x1 \leftarrow x1\}$

$\overbrace{op_l(x1, rd(x2, op_t(i(x3), x4)), i(i(x2)))}$

$=$ by Lemma 1927 LR with $\{x2 \leftarrow i(x2), x3 \leftarrow rd(x2, op_t(i(x3), x4)), x1 \leftarrow x1\}$

$\overbrace{op_l(x1, \underbrace{op_t(op_l(i(x2), rd(x2, op_t(i(x3), x4)), i(x2)), rd(rd(i(i(x2)), rd(x2, op_t(i(x3), x4))), i(x2)))}, i(i(mult(rd(x2, op_t(i(x3),}$

$=$ by Axiom 15 LR with $\{x4 \leftarrow rd(rd(i(i(x2)), rd(x2, op_t(i(x3), x4))), i(x2)), x3 \leftarrow i(x2), x2 \leftarrow rd(x2, op_t(i(x3), x4)), x1 \leftarrow i(x2$

$op_l(x1, \overbrace{op_l(\underbrace{op_t(i(x2), rd(rd(i(i(x2)), rd(x2, op_t(i(x3), x4))), i(x2)))}, rd(x2, op_t(i(x3), x4)), i(x2))}, i(i(mult(rd(x2, op_t(i(x3),}$

$=$ by Lemma 1450 RL with $\{x3 \leftarrow rd(x2, op_t(i(x3), x4)), x2 \leftarrow i(x2), x1 \leftarrow i(x2)\}$

$op_l(x1, op_l(\overbrace{op_t(i(x2), rd(\underbrace{mult(i(x2), op_l(rd(x2, op_t(i(x3), x4)), i(x2), rd(x2, op_t(i(x3), x4))))}, i(x2)))}, rd(x2, op_t(i(x3), x4)),$

$=$ by Lemma 616 LR with $\{x2 \leftarrow rd(x2, op_t(i(x3), x4)), x1 \leftarrow i(x2)\}$

$op_l(x1, op_l(\underbrace{op_t(i(x2), rd(\overbrace{rd(i(x2), i(rd(x2, op_t(i(x3), x4))))}, i(x2)))}, rd(x2, op_t(i(x3), x4)), i(x2)), i(i(mult(rd(x2, op_t(i(x3),}$

$=$ by Lemma 1260 LR with $\{x3 \leftarrow i(x2), x2 \leftarrow rd(x2, op_t(i(x3), x4)), x1 \leftarrow i(x2)\}$

$op_l(x1, op_l(\overbrace{op_t(i(x2), rd(mult(i(x2), rd(x2, op_t(i(x3), x4))), i(x2)))}, rd(x2, op_t(i(x3), x4)), i(x2)), i(i(mult(rd(x2, op_t(i(x3),}$

$=$ by Lemma 378 LR with $\{x2 \leftarrow rd(x2, op_t(i(x3), x4)), x1 \leftarrow i(x2)\}$

$op_l(x1, \underbrace{op_l(\overbrace{op_t(i(x2), rd(x2, op_t(i(x3), x4)))}, rd(x2, op_t(i(x3), x4)), i(x2))}, i(i(mult(rd(x2, op_t(i(x3), x4)), i(x2)))))$

$=$ by Lemma 251 LR with $\{x2 \leftarrow rd(x2, op_t(i(x3), x4)), x1 \leftarrow i(x2)\}$

$op_l(x1, \overbrace{op_t(i(x2), mult(i(x2), rd(x2, op_t(i(x3), x4))))}, \underbrace{i(i(mult(rd(x2, op_t(i(x3), x4)), i(x2)))))}$

$=$ by Lemma 3 LR with $\{x1 \leftarrow mult(rd(x2, op_t(i(x3), x4)), i(x2))\}$

$op_l(x1, op_t(i(x2), mult(i(x2), \underbrace{rd(x2, op_t(i(x3), x4))})), \overbrace{mult(rd(x2, op_t(i(x3), x4)), i(x2)))}$

$=$ by Lemma 65 RL with $\{x3 \leftarrow x2, x2 \leftarrow x4, x1 \leftarrow x3\}$

$op_l(x1, op_t(i(x2), \underbrace{mult(i(x2), \overbrace{rd(op_t(x3, i(x4)), i(x2))})}), mult(rd(x2, op_t(i(x3), x4)), i(x2)))$

$=$ by Lemma 1209 LR with $\{x3 \leftarrow i(x4), x2 \leftarrow x3, x1 \leftarrow i(x2)\}$

$op_l(x1, op_t(i(x2), \overbrace{op_t(\underbrace{mult(i(x2), rd(x3, i(x2)))}, i(x4))}), mult(rd(x2, op_t(i(x3), x4)), i(x2)))$

$=$ by Lemma 69 LR with $\{x2 \leftarrow x3, x1 \leftarrow x2\}$

$op_l(x1, op_t(i(x2), op_t(\overbrace{mult(mult(i(x2), x3), x2)}, i(x4))), mult(rd(x2, op_t(i(x3), x4)), i(x2)))$

Lemma 2233: $op_l(x1, rd(x2, op_t(i(x3), x4)), x2) = op_l(x1, mult(rd(op_t(i(x3), x4), x2), x2), op_t(i(x2), op_t(mult(mult(i(x2), x3), x$

$\overbrace{op_l(x1, rd(x2, op_t(i(x3), x4)), x2)}$

$=$ by Lemma 2031 RL with $\{x3 \leftarrow rd(x2, op_t(i(x3), x4)), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_l(x1, i(x2), rd(x2, op_t(i(x3), x4)))}$

$=$ by Lemma 2232 LR with $\{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_l(x1, op_t(i(x2), op_t(mult(mult(i(x2), x3), x2), i(x4))), \underbrace{mult(rd(x2, op_t(i(x3), x4)), i(x2))}}$

$=$ by Lemma 44 RL with $\{x3 \leftarrow x2, x2 \leftarrow x2, x1 \leftarrow op_t(i(x3), x4)\}$

$\underbrace{op_l(x1, op_t(i(x2), op_t(mult(mult(i(x2), x3), x2), i(x4))), \overbrace{i(mult(rd(op_t(i(x3), x4), x2), x2))}}$

$=$ by Lemma 2030 LR with $\{x3 \leftarrow mult(rd(op_t(i(x3), x4), x2), x2), x2 \leftarrow op_t(i(x2), op_t(mult(mult(i(x2), x3), x2), i(x4))), x1 \leftarrow$

$\overbrace{op_l(x1, mult(rd(op_t(i(x3), x4), x2), x2), op_t(i(x2), op_t(mult(mult(i(x2), x3), x2), i(x4))))}$

Lemma 2234: $op_l(x1, x2, op_t(i(x3), x4)) = op_l(x1, op_t(x3, i(x4)), x2)$

$\underbrace{op_l(x1, x2, op_t(i(x3), x4))}$

$=$ by Lemma 286 RL with $\{x3 \leftarrow op_t(i(x3), x4), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_l(x1, x2, rd(op_t(i(x3), x4), x2))}$

$=$ by Lemma 2052 LR with $\{x4 \leftarrow x2, x3 \leftarrow op_t(i(x3), x4), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_l(x1, rd(x2, op_t(i(x3), x4)), x2)}$

$=$ by Lemma 2233 LR with $\{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_l(x1, \underbrace{mult(rd(op_t(i(x3), x4), x2), x2)}, op_t(i(x2), op_t(mult(mult(i(x2), x3), x2), i(x4))))}$

$=$ by Axiom 4 RL with $\{x2 \leftarrow x2, x1 \leftarrow op_t(i(x3), x4)\}$

$op_l(x1, \overbrace{op_t(i(x3), x4)}, \underbrace{op_t(i(x2), op_t(mult(mult(i(x2), x3), x2), i(x4)))})$

$=$ by Lemma 1722 LR with $\{x3 \leftarrow i(x4), x2 \leftarrow mult(mult(i(x2), x3), x2), x1 \leftarrow i(x2)\}$

$op_l(x1, op_t(i(x3), x4), \overbrace{op_t(i(x2), mult(mult(i(x2), x3), x2))})$

$=$ by Lemma 715 LR with $\{x2 \leftarrow mult(i(x2), x3), x1 \leftarrow x2\}$

$op_l(x1, op_t(i(x3), x4), \overbrace{op_t(i(x2), rd(x2, \underbrace{mult(i(x2), x3)}))})$

$=$ by Lemma 14 RL with $\{x2 \leftarrow x2, x1 \leftarrow x3\}$

$op_l(x1, op_t(i(x3), x4), \underbrace{op_t(i(x2), rd(x2, \overbrace{op_t(rd(x3, x2), x2)}))})$

$=$ by Lemma 1214 LR with $\{x2 \leftarrow rd(x3, x2), x3 \leftarrow x2, x1 \leftarrow x2\}$

$op_l(x1, op_t(i(x3), x4), \overbrace{op_t(i(x2), rd(x2, rd(x3, x2)))})$

$=$ by Lemma 718 LR with $\{x2 \leftarrow x3, x1 \leftarrow x2\}$

$\underbrace{op_l(x1, op_t(i(x3), x4), \overbrace{op_t(i(x2), x3)})}$

$=$ by Lemma 1901 LR with $\{x4 \leftarrow i(x2), x3 \leftarrow x4, x2 \leftarrow x3, x1 \leftarrow x1\}$

$\overbrace{op_l(x1, i(x2), op_t(x3, i(x4)))}$

$=$ by Lemma 2031 LR with $\{x3 \leftarrow op_t(x3, i(x4)), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_l(x1, op_t(x3, i(x4)), x2)}$

Lemma 2235: $mult(rd(x1, op_l(x1, x2, x3)), i(x3)) = i(rd(mult(x3, op_l(x1, x2, x3)), x1))$

$$
\begin{aligned}
& \underbrace{mult(rd(x1, op_l(x1, x2, x3)), i(x3))} \\
= \quad & \text{by Lemma 44 RL with } \{x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow op_l(x1, x2, x3)\} \\
& \overbrace{i(\underbrace{mult(rd(op_l(x1, x2, x3), x1), x3)})} \\
= \quad & \text{by Lemma 2121 RL with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x3\} \\
& i(\overbrace{mult(rd(x3, x1), op_l(x1, x2, x3))}) \\
= \quad & \text{by Lemma 2153 LR with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x3\} \\
& i(\overbrace{rd(mult(x3, op_l(x1, x2, x3)), x1)})
\end{aligned}
$$

Lemma 2236: $mult(rd(x1, op_l(x1, x2, x3)), i(x3)) = rd(x1, mult(x3, op_l(x1, x2, x3)))$

$$
\begin{aligned}
& \underbrace{mult(rd(x1, op_l(x1, x2, x3)), i(x3))} \\
= \quad & \text{by Lemma 2235 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{\overbrace{i(rd(mult(x3, op_l(x1, x2, x3)), x1))}} \\
= \quad & \text{by Lemma 10 LR with } \{x2 \leftarrow x1, x1 \leftarrow mult(x3, op_l(x1, x2, x3))\} \\
& \overbrace{rd(x1, mult(x3, op_l(x1, x2, x3)))}
\end{aligned}
$$

Lemma 2237: $op_t(mult(x1, op_r(op_l(x2, x1, x2), x3, x1)), x1) = mult(op_r(op_l(x2, x1, x2), x3, x1), op_l(x1, x1, x2))$

$$
\begin{aligned}
& \underbrace{op_t(mult(x1, op_r(op_l(x2, x1, x2), x3, x1)), x1)} \\
= \quad & \text{by Lemma 264 RL with } \{x2 \leftarrow x1, x1 \leftarrow op_r(op_l(x2, x1, x2), x3, x1)\} \\
& \overbrace{mult(op_r(op_l(x2, x1, x2), x3, x1), \underbrace{op_l(x1, x1, op_r(op_l(x2, x1, x2), x3, x1))})} \\
= \quad & \text{by Lemma 1850 RL with } \{x3 \leftarrow x3, x2 \leftarrow op_l(x2, x1, x2), x1 \leftarrow x1\} \\
& mult(op_r(op_l(x2, x1, x2), x3, x1), \overbrace{op_l(x1, x1, op_l(x2, x1, x2))}) \\
= \quad & \text{by Lemma 2037 LR with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x1\} \\
& mult(op_r(op_l(x2, x1, x2), x3, x1), \overbrace{op_l(x1, x1, x2)})
\end{aligned}
$$

Lemma 2238: $op_t(mult(x1, op_r(op_l(x2, x1, x2), x1, x3)), x1) = mult(op_r(op_l(x2, x1, x2), x1, x3), op_l(x1, x1, x2))$

$$
\begin{aligned}
& \underbrace{op_t(mult(x1, op_r(op_l(x2, x1, x2), x1, x3)), x1)} \\
= \quad & \text{by Lemma 264 RL with } \{x2 \leftarrow x1, x1 \leftarrow op_r(op_l(x2, x1, x2), x1, x3)\} \\
& \overbrace{mult(op_r(op_l(x2, x1, x2), x1, x3), \underbrace{op_l(x1, x1, op_r(op_l(x2, x1, x2), x1, x3))})} \\
= \quad & \text{by Lemma 1852 LR with } \{x3 \leftarrow x3, x2 \leftarrow op_l(x2, x1, x2), x1 \leftarrow x1\} \\
& mult(op_r(op_l(x2, x1, x2), x1, x3), \overbrace{op_l(x1, x1, op_l(x2, x1, x2))}) \\
= \quad & \text{by Lemma 2037 LR with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x1\} \\
& mult(op_r(op_l(x2, x1, x2), x1, x3), \overbrace{op_l(x1, x1, x2)})
\end{aligned}
$$

Lemma 2239: $mult(op_l(x1,x2,x3),i(mult(x1,x3))) = rd(op_l(x1,x2,x3),mult(x3,x1))$

$$
\begin{aligned}
& \underbrace{mult(op_l(x1,x2,x3),i(mult(x1,x3)))} \\
= & \quad \text{by Lemma 614 RL with } \{x2 \leftarrow mult(x1,x3), x1 \leftarrow op_l(x1,x2,x3)\} \\
& \overbrace{rd(op_l(x1,x2,x3),\underbrace{op_l(mult(x1,x3),op_l(x1,x2,x3),mult(x1,x3))})} \\
= & \quad \text{by Lemma 1134 RL with } \{x4 \leftarrow x3, x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow mult(x1,x3)\} \\
& rd(op_l(x1,x2,x3),\overbrace{\underbrace{op_l(mult(x1,x3),mult(x1,x3),op_l(i(x1),x2,x3))}}) \\
= & \quad \text{by Lemma 2161 RL with } \{x3 \leftarrow x2, x1 \leftarrow x1, x2 \leftarrow x3\} \\
& rd(op_l(x1,x2,x3),\overbrace{mult(x3,x1)})
\end{aligned}
$$

Lemma 2240: $op_r(x1,rd(x2,rd(x1,x3)),x4) = op_r(x1,rd(x2,mult(x1,i(x3))),x4)$

$$
\begin{aligned}
& op_r(x1,rd(x2,rd(\underbrace{x1},x3)),x4) \\
= & \quad \text{by Lemma 3 RL with } \{x1 \leftarrow x1\} \\
& \underbrace{op_r(x1,rd(x2,rd(\overbrace{i(i(x1))},x3)),x4)} \\
= & \quad \text{by Lemma 2171 LR with } \{x4 \leftarrow x4, x2 \leftarrow rd(i(i(x1)),x3), x3 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_r(x1,rd(x2,\underbrace{op_t(rd(i(i(x1)),x3),i(x1))}),x4)} \\
= & \quad \text{by Lemma 33 LR with } \{x2 \leftarrow x3, x1 \leftarrow i(x1)\} \\
& op_r(x1,rd(x2,\overbrace{\underbrace{i(mult(i(x1),x3))}}),x4) \\
= & \quad \text{by Lemma 16 LR with } \{x2 \leftarrow x3, x1 \leftarrow x1\} \\
& op_r(x1,rd(x2,\overbrace{mult(x1,i(x3))}),x4)
\end{aligned}
$$

Lemma 2241: $op_r(x1,rd(x2,mult(i(x1),x3)),x4) = op_r(x1,rd(x2,rd(x3,x1)),x4)$

$$
\begin{aligned}
& op_r(x1,rd(x2,\underbrace{mult(i(x1),x3)}),x4) \\
= & \quad \text{by Lemma 14 RL with } \{x2 \leftarrow x1, x1 \leftarrow x3\} \\
& \underbrace{op_r(x1,rd(x2,\overbrace{op_t(rd(x3,x1),x1)}),x4)} \\
= & \quad \text{by Lemma 2176 LR with } \{x4 \leftarrow x4, x3 \leftarrow rd(x3,x1), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_r(x1,rd(x2,rd(x3,x1)),x4)}
\end{aligned}
$$

Lemma 2242: $op_r(x1,rd(mult(x2,x1),x3),x4) = op_r(x1,rd(rd(x1,i(x2)),x3),x4)$

$$
\begin{aligned}
& op_r(x1,\underbrace{rd(mult(x2,x1),x3)},x4) \\
= & \quad \text{by Lemma 10 RL with } \{x2 \leftarrow mult(x2,x1), x1 \leftarrow x3\} \\
& \underbrace{op_r(x1,\overbrace{i(rd(x3,mult(x2,x1)))},x4)} \\
= & \quad \text{by Lemma 2179 LR with } \{x4 \leftarrow mult(x2,x1), x3 \leftarrow x4, x2 \leftarrow rd(x3,mult(x2,x1)), x1 \leftarrow x1\} \\
& \overbrace{op_r(x1,\underbrace{rd(asoc(mult(x2,x1),x1,x1),rd(x3,mult(x2,x1)))},x4)} \\
= & \quad \text{by Lemma 1249 LR with } \{x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow mult(x2,x1)\} \\
& op_r(x1,\overbrace{rd(\underbrace{op_r(mult(x2,x1),x1,x1)},x3)},x4) \\
= & \quad \text{by Lemma 359 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& op_r(x1,rd(\overbrace{rd(x1,i(x2))},x3),x4)
\end{aligned}
$$

Lemma 2243: $op_l(x1, mult(x2, x2), mult(x2, x3)) = op_l(x1, mult(x2, x2), mult(x3, op_t(i(x2), x3)))$

$op_l(x1, mult(x2, x2), mult(x2, \underbrace{x3}))$

$=$ by Lemma 29 RL with $\{x2 \leftarrow x3, x1 \leftarrow x2\}$

$op_l(x1, mult(x2, x2), \underbrace{mult(x2, \overbrace{rd(mult(x2, x3), op_t(x2, x3))})})$

$=$ by Lemma 1253 RL with $\{x3 \leftarrow x3, x2 \leftarrow mult(x2, x3), x1 \leftarrow x2\}$

$op_l(x1, mult(x2, x2), \overbrace{mult(\underbrace{mult(x2, mult(x2, x3))}, i(op_t(x2, x3)))})$

$=$ by Lemma 24 RL with $\{x2 \leftarrow x3, x1 \leftarrow x2\}$

$op_l(x1, mult(x2, x2), \underbrace{mult(\overbrace{mult(mult(x2, x2), x3)}, i(op_t(x2, x3)))})$

$=$ by Lemma 137 RL with $\{x3 \leftarrow i(op_t(x2, x3)), x2 \leftarrow x3, x1 \leftarrow mult(x2, x2)\}$

$\underbrace{op_l(x1, mult(x2, x2), \overbrace{mult(mult(x2, x2), mult(x3, op_l(i(op_t(x2, x3)), mult(x2, x2), x3)))})}$

$=$ by Lemma 2032 LR with $\{x3 \leftarrow mult(x3, op_l(i(op_t(x2, x3)), mult(x2, x2), x3)), x2 \leftarrow mult(x2, x2), x1 \leftarrow x1\}$

$\overbrace{op_l(x1, mult(x2, x2), mult(x3, \underbrace{op_l(i(op_t(x2, x3)), mult(x2, x2), x3)}))}$

$=$ by Lemma 1049 RL with $\{x3 \leftarrow x3, x2 \leftarrow mult(x2, x2), x1 \leftarrow op_t(x2, x3)\}$

$op_l(x1, mult(x2, x2), mult(x3, \underbrace{i(\overbrace{op_l(op_t(x2, x3), mult(x2, x2), x3)})}))$

$=$ by Axiom 15 RL with $\{x4 \leftarrow x3, x3 \leftarrow x3, x2 \leftarrow mult(x2, x2), x1 \leftarrow x2\}$

$op_l(x1, mult(x2, x2), mult(x3, \underbrace{i(\overbrace{op_t(op_l(x2, mult(x2, x2), x3), x3)})}))$

$=$ by Lemma 31 RL with $\{x2 \leftarrow op_l(x2, mult(x2, x2), x3), x1 \leftarrow x3\}$

$op_l(x1, mult(x2, x2), mult(x3, \overbrace{mult(x3, i(\underbrace{mult(op_l(x2, mult(x2, x2), x3), x3)}))}))$

$=$ by Lemma 1597 LR with $\{x2 \leftarrow x3, x1 \leftarrow x2\}$

$op_l(x1, mult(x2, x2), mult(x3, \underbrace{mult(x3, i(\overbrace{mult(x3, op_t(x2, i(x3)))}))}))$

$=$ by Lemma 20 LR with $\{x2 \leftarrow op_t(x2, i(x3)), x1 \leftarrow x3\}$

$op_l(x1, mult(x2, x2), mult(x3, \underbrace{i(\overbrace{i(op_t(x2, i(x3)))})}))$

$=$ by Lemma 19 LR with $\{x2 \leftarrow x3, x1 \leftarrow x2\}$

$op_l(x1, mult(x2, x2), mult(x3, \overbrace{op_t(i(x2), x3)}))$

Lemma 2244: $rd(x1, rd(i(x2), i(op_r(x2, x3, x4)))) = rd(mult(x1, i(op_r(x2, x3, x4))), i(x2))$

$rd(x1, rd(i(x2), \underbrace{i(op_r(x2, x3, x4))}))$

$=$ by Lemma 654 RL with $\{x3 \leftarrow x4, x2 \leftarrow x3, x1 \leftarrow x2\}$

$\underbrace{rd(x1, rd(i(x2), \overbrace{op_r(i(x2), mult(x2, x3), x4)}))}$

$=$ by Lemma 2189 RL with $\{x4 \leftarrow x4, x3 \leftarrow mult(x2, x3), x2 \leftarrow i(x2), x1 \leftarrow x1\}$

$\overbrace{rd(mult(x1, \underbrace{op_r(i(x2), mult(x2, x3), x4)}), i(x2))}$

$=$ by Lemma 654 LR with $\{x3 \leftarrow x4, x2 \leftarrow x3, x1 \leftarrow x2\}$

$rd(mult(x1, \overbrace{i(op_r(x2, x3, x4))}), i(x2))$

Lemma 2245: $rd(x1, rd(op_r(x2, x3, x4), x2)) = rd(x2, mult(i(x1), op_r(x2, x3, x4)))$

$$
\begin{aligned}
& rd(x1, \underbrace{rd(op_r(x2, x3, x4), x2)}) \\
= & \quad \text{by Lemma 23 RL with } \{x2 \leftarrow op_r(x2, x3, x4), x1 \leftarrow x2\} \\
& \underbrace{rd(x1, \overbrace{rd(i(x2), i(op_r(x2, x3, x4)))})} \\
= & \quad \text{by Lemma 2244 LR with } \{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{\overbrace{rd(mult(x1, i(op_r(x2, x3, x4))), i(x2))}} \\
= & \quad \text{by Lemma 61 LR with } \{x3 \leftarrow x2, x2 \leftarrow op_r(x2, x3, x4), x1 \leftarrow x1\} \\
& \overbrace{rd(x2, mult(i(x1), op_r(x2, x3, x4)))}
\end{aligned}
$$

Lemma 2246: $op_l(x1, mult(x2, x2), mult(x2, i(x3))) = op_l(x1, mult(x2, x3), mult(x2, x2))$

$$
\begin{aligned}
& \underbrace{op_l(x1, mult(x2, x2), mult(x2, i(x3)))} \\
= & \quad \text{by Lemma 2194 LR with } \{x3 \leftarrow i(x3), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{\overbrace{op_l(x1, op_t(mult(x2, i(i(x3))), mult(x2, x2)), mult(x2, x2))}} \\
= & \quad \text{by Lemma 2034 LR with } \{x3 \leftarrow mult(x2, x2), x2 \leftarrow mult(x2, i(i(x3))), x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, mult(x2, \underbrace{i(i(x3))}), mult(x2, x2))} \\
= & \quad \text{by Lemma 3 LR with } \{x1 \leftarrow x3\} \\
& op_l(x1, mult(x2, \overbrace{x3}), mult(x2, x2))
\end{aligned}
$$

Lemma 2247: $rd(op_l(x1, x2, x3), mult(i(x4), x1)) = mult(rd(x4, x1), op_l(x1, x2, x3))$

$$
\begin{aligned}
& \underbrace{rd(op_l(x1, x2, x3), mult(i(x4), x1))} \\
= & \quad \text{by Lemma 2200 LR with } \{x1 \leftarrow i(x4), x4 \leftarrow x3, x3 \leftarrow x2, x2 \leftarrow x1\} \\
& \overbrace{mult(rd(\underbrace{i(i(x4))}, x1), op_l(x1, x2, x3))} \\
= & \quad \text{by Lemma 3 LR with } \{x1 \leftarrow x4\} \\
& mult(rd(\overbrace{x4}, x1), op_l(x1, x2, x3))
\end{aligned}
$$

Lemma 2248: $op_r(x1, rd(rd(x1, i(x3)), x2), x4) = op_r(x1, rd(mult(i(i(x1)), x3), x2), x4)$

$$
\begin{aligned}
& op_r(x1, \underbrace{rd(rd(x1, i(x3)), x2)}, x4) \\
= & \quad \text{by Lemma 40 RL with } \{x3 \leftarrow x1, x2 \leftarrow i(x3), x1 \leftarrow x2\} \\
& \underbrace{op_r(x1, \overbrace{rd(i(x2), rd(i(x3), x1))}, x4)} \\
= & \quad \text{by Lemma 2241 RL with } \{x4 \leftarrow x4, x3 \leftarrow i(x3), x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, \underbrace{rd(i(x2), mult(i(x1), i(x3)))}, x4)} \\
= & \quad \text{by Lemma 62 LR with } \{x3 \leftarrow x3, x2 \leftarrow i(x1), x1 \leftarrow x2\} \\
& op_r(x1, \overbrace{rd(mult(i(i(x1)), x3), x2)}, x4)
\end{aligned}
$$

Lemma 2249: $op_r(x1, rd(mult(x3, x1), x2), x4) = op_r(x1, rd(mult(x1, x3), x2), x4)$

$\underbrace{op_r(x1, rd(mult(x3, x1), x2), x4)}$

$=$ by Lemma 2242 LR with $\{x4 \leftarrow x4, x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_r(x1, rd(rd(x1, i(x3)), x2), x4)}}$

$=$ by Lemma 2248 LR with $\{x4 \leftarrow x4, x2 \leftarrow x2, x3 \leftarrow x3, x1 \leftarrow x1\}$

$\overbrace{op_r(x1, rd(mult(\underbrace{i(i(x1))}, x3), x2), x4)}$

$=$ by Lemma 3 LR with $\{x1 \leftarrow x1\}$

$op_r(x1, rd(mult(\overbrace{x1}, x3), x2), x4)$

Lemma 2250: $op_r(x1, mult(mult(x1, x2), x3), x4) = op_r(x1, mult(mult(x2, x1), x3), x4)$

$op_r(x1, mult(mult(\overbrace{x1}, x2), x3), x4)$

$=$ by Lemma 3 RL with $\{x1 \leftarrow x1\}$

$op_r(x1, mult(\underbrace{mult(\overbrace{i(i(x1))}, x2)}, x3), x4)$

$=$ by Lemma 14 RL with $\{x2 \leftarrow i(x1), x1 \leftarrow x2\}$

$op_r(x1, \underbrace{mult(\overbrace{op_t(rd(x2, i(x1)), i(x1))}, x3)}, x4)$

$=$ by Lemma 231 LR with $\{x2 \leftarrow x3, x1 \leftarrow op_t(rd(x2, i(x1)), i(x1))\}$

$op_r(x1, \overbrace{rd(op_t(rd(x2, i(x1)), i(x1)), op_l(i(x3), x3, op_t(rd(x2, i(x1)), i(x1))))}, \overbrace{x4})$

$=$ by Lemma 3 RL with $\{x1 \leftarrow x4\}$

$\underbrace{op_r(x1, rd(op_t(rd(x2, i(x1)), i(x1)), op_l(i(x3), x3, op_t(rd(x2, i(x1)), i(x1)))), \overbrace{i(i(x4))})}$

$=$ by Lemma 2177 LR with $\{x4 \leftarrow x4, x2 \leftarrow op_l(i(x3), x3, op_t(rd(x2, i(x1)), i(x1))), x3 \leftarrow rd(x2, i(x1)), x1 \leftarrow x1\}$

$\overbrace{op_r(x1, rd(rd(x2, i(x1)), \underbrace{op_l(i(x3), x3, op_t(rd(x2, i(x1)), i(x1)))}), x4)}$

$=$ by Lemma 1740 LR with $\{x3 \leftarrow i(x1), x2 \leftarrow rd(x2, i(x1)), x1 \leftarrow x3\}$

$op_r(x1, \underbrace{rd(rd(x2, i(x1)), \overbrace{op_l(i(x3), x3, rd(x2, i(x1)))})}, x4)$

$=$ by Lemma 231 RL with $\{x2 \leftarrow x3, x1 \leftarrow rd(x2, i(x1))\}$

$op_r(x1, \overbrace{mult(\underbrace{rd(x2, i(x1))}, x3)}, x4)$

$=$ by Lemma 38 LR with $\{x1 \leftarrow x1, x2 \leftarrow x2\}$

$op_r(x1, mult(\overbrace{rd(x1, i(x2))}, x3), x4)$

$=$ by Lemma 359 RL with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$op_r(x1, \underbrace{mult(\overbrace{op_r(mult(x2, x1), x1, x1)}, x3)}, x4)$

$=$ by Lemma 1719 RL with $\{x3 \leftarrow x1, x2 \leftarrow x3, x1 \leftarrow mult(x2, x1)\}$

$op_r(x1, \overbrace{rd(mult(mult(x2, x1), x3), \underbrace{asoc(x1, mult(x2, x1), x1)})}, x4)$

$=$ by Lemma 759 RL with $\{x2 \leftarrow x1, x1 \leftarrow mult(x2, x1)\}$

$op_r(x1, \underbrace{rd(mult(mult(x2, x1), x3), \overbrace{i(asoc(mult(x2, x1), x1, x1))})}, x4)$

$=$ by Lemma 38 RL with $\{x1 \leftarrow mult(mult(x2, x1), x3), x2 \leftarrow asoc(mult(x2, x1), x1, x1)\}$

$\underbrace{op_r(x1, \overbrace{rd(asoc(mult(x2, x1), x1, x1), i(mult(mult(x2, x1), x3)))}, x4)}$

$=$ by Lemma 2179 RL with $\{x4 \leftarrow mult(x2, x1), x3 \leftarrow x4, x2 \leftarrow i(mult(mult(x2, x1), x3)), x1 \leftarrow x1\}$

$\overbrace{op_r(x1, \underbrace{i(i(mult(mult(x2, x1), x3)))}, x4)}$

$=$ by Lemma 3 LR with $\{x1 \leftarrow mult(mult(x2, x1), x3)\}$

$op_r(x1, \overbrace{mult(mult(x2, x1), x3)}, x4)$

Lemma 2251: $mult(rd(x1, op_l(x2, x3, x4)), x2) = rd(x2, mult(i(x1), op_l(x2, x3, x4)))$

$mult(rd(x1, op_l(x2, x3, x4)), \underbrace{x2}_{})$

$=$ by Lemma 138 RL with $\{x3 \leftarrow x4, x2 \leftarrow x3, x1 \leftarrow x2\}$

$\underbrace{mult(rd(x1, op_l(x2, x3, x4)), \overbrace{op_l(op_l(x2, x3, x4), x4, x3)}}$

$=$ by Lemma 2247 RL with $\{x4 \leftarrow x1, x3 \leftarrow x3, x2 \leftarrow x4, x1 \leftarrow op_l(x2, x3, x4)\}$

$\overbrace{rd(\underbrace{op_l(op_l(x2, x3, x4), x4, x3)}, mult(i(x1), op_l(x2, x3, x4)))}$

$=$ by Lemma 138 LR with $\{x3 \leftarrow x4, x2 \leftarrow x3, x1 \leftarrow x2\}$

$rd(\overbrace{x2}, mult(i(x1), op_l(x2, x3, x4)))$

Lemma 2252: $x1 = op_t(x1, rd(i(x2), mult(i(mult(x2, mult(x3, mult(x2, x1)))), mult(x2, x3))))$

$\underbrace{x1}$

$=$ by Lemma 400 RL with $\{x2 \leftarrow x1, x1 \leftarrow i(rd(i(x2), mult(x2, x3)))\}$

$\overbrace{op_t(\underbrace{rd(mult(i(rd(i(x2), mult(x2, x3))), x1), i(rd(i(x2), mult(x2, x3))))}, op_t(i(rd(i(x2), mult(x2, x3))), x1))}$

$=$ by Lemma 120 RL with $\{x3 \leftarrow mult(x2, x3), x2 \leftarrow mult(rd(i(x2), mult(x2, x3)), i(x1)), x1 \leftarrow rd(mult(i(rd(i(x2), mult(x2, x$

$\underbrace{op_t(\overbrace{op_t(rd(mult(i(rd(i(x2), mult(x2, x3))), x1), i(rd(i(x2), mult(x2, x3)))), op_r(rd(mult(i(rd(i(x2), mult(x2, x3))), x1), i($

$=$ by Axiom 13 LR with $\{x3 \leftarrow op_t(i(rd(i(x2), mult(x2, x3))), x1), x2 \leftarrow op_r(rd(mult(i(rd(i(x2), mult(x2, x3))), x1), i(rd(i(x2)$

$\overbrace{op_t(\underbrace{op_t(rd(mult(i(rd(i(x2), mult(x2, x3))), x1), i(rd(i(x2), mult(x2, x3))))}, op_t(i(rd(i(x2), mult(x2, x3))), x1))}, op_r(rd(mu$

$=$ by Lemma 400 LR with $\{x2 \leftarrow x1, x1 \leftarrow i(rd(i(x2), mult(x2, x3)))\}$

$op_t(\overbrace{x1}, op_r(\underbrace{rd(mult(i(rd(i(x2), mult(x2, x3))), x1), i(rd(i(x2), mult(x2, x3))))}, mult(rd(i(x2), mult(x2, x3)), i(x1)), mult$

$=$ by Lemma 38 LR with $\{x1 \leftarrow rd(i(x2), mult(x2, x3)), x2 \leftarrow mult(i(rd(i(x2), mult(x2, x3))), x1)\}$

$op_t(x1, op_r(\overbrace{rd(rd(i(x2), mult(x2, x3)), \underbrace{i(mult(i(rd(i(x2), mult(x2, x3))), x1))})}, mult(rd(i(x2), mult(x2, x3)), i(x1)), mult($

$=$ by Lemma 16 LR with $\{x2 \leftarrow x1, x1 \leftarrow rd(i(x2), mult(x2, x3))\}$

$op_t(x1, \underbrace{op_r(rd(rd(i(x2), mult(x2, x3)), \overbrace{mult(rd(i(x2), mult(x2, x3)), i(x1))}), mult(rd(i(x2), mult(x2, x3)), i(x1)), mult(x2,}$

$=$ by Lemma 839 LR with $\{x3 \leftarrow mult(rd(i(x2), mult(x2, x3)), i(x1)), x2 \leftarrow mult(x2, x3), x1 \leftarrow i(x2)\}$

$op_t(x1, \overbrace{rd(i(x2), mult(mult(\underbrace{rd(i(x2), mult(x2, x3))}, i(x1)), mult(x2, x3))))}$

$=$ by Lemma 37 RL with $\{x2 \leftarrow x2, x1 \leftarrow mult(x2, x3)\}$

$op_t(x1, rd(i(x2), mult(\underbrace{mult(\overbrace{rd(i(mult(x2, x3)), x2)}, i(x1))}, mult(x2, x3))))$

$=$ by Lemma 44 RL with $\{x3 \leftarrow x1, x2 \leftarrow i(mult(x2, x3)), x1 \leftarrow x2\}$

$op_t(x1, rd(i(x2), mult(\overbrace{i(mult(\underbrace{rd(x2, i(mult(x2, x3)))}, x1))}, mult(x2, x3))))$

$=$ by Lemma 58 LR with $\{x2 \leftarrow x3, x1 \leftarrow x2\}$

$op_t(x1, rd(i(x2), mult(i(\underbrace{mult(\overbrace{mult(x2, mult(x3, x2))}, x1)}), mult(x2, x3))))$

$=$ by Axiom 6 RL with $\{x3 \leftarrow x1, x2 \leftarrow x3, x1 \leftarrow x2\}$

$op_t(x1, rd(i(x2), mult(i(\overbrace{mult(x2, mult(x3, mult(x2, x1)))}), mult(x2, x3))))$

Lemma 2253: $x1 = op_t(x1, rd(mult(x2, mult(rd(mult(x3, mult(x2, x1)), x2), i(x3))), x2))$

$$
\begin{aligned}
& \overbrace{x1} \\
= \quad & \text{by Lemma 2252 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, \underbrace{rd(i(x2), mult(i(mult(x2, mult(x3, mult(x2, x1)))), mult(x2, x3)))}_{})}^{} \\
= \quad & \text{by Lemma 64 LR with } \{x3 \leftarrow mult(x2, x3), x2 \leftarrow mult(x2, mult(x3, mult(x2, x1))), x1 \leftarrow x2\} \\
& op_t(x1, \overbrace{rd(\underbrace{mult(mult(x2, mult(x3, mult(x2, x1))), i(mult(x2, x3)))}_{}, x2)}^{}) \\
= \quad & \text{by Lemma 1400 LR with } \{x3 \leftarrow x3, x2 \leftarrow mult(x3, mult(x2, x1)), x1 \leftarrow x2\} \\
& op_t(x1, rd(\overbrace{mult(x2, mult(rd(mult(x3, mult(x2, x1)), x2), i(x3)))}^{}, x2))
\end{aligned}
$$

Lemma 2254: $x1 = op_t(x1, rd(i(x3), rd(x2, mult(x3, mult(x2, x1)))))$

$$
\begin{aligned}
& \overbrace{x1} \\
= \quad & \text{by Lemma 2253 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_t(x1, rd(mult(x2, mult(rd(mult(x3, mult(x2, x1)), x2), i(x3))), x2))}_{}}^{} \\
= \quad & \text{by Lemma 1731 LR with } \{x3 \leftarrow mult(rd(mult(x3, mult(x2, x1)), x2), i(x3)), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_t(x1, mult(rd(mult(x3, mult(x2, x1)), x2), i(x3)))}_{}}^{} \\
= \quad & \text{by Lemma 1752 LR with } \{x4 \leftarrow i(x3), x3 \leftarrow x2, x2 \leftarrow mult(x3, mult(x2, x1)), x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, rd(i(x3), rd(x2, mult(x3, mult(x2, x1)))))}^{}
\end{aligned}
$$

Lemma 2255: $x1 = op_t(x1, rd(rd(mult(x1,x2), mult(x3,x2)), rd(op_t(x1,x2), mult(x3,x1))))$

$\overbrace{x1}$

$=$ by Lemma 2254 LR with $\{x2 \leftarrow rd(mult(x1,x2),x1), x3 \leftarrow rd(mult(x3,x2), mult(rd(mult(x1,x2),x1),x1)), x1 \leftarrow x1\}$

$\overbrace{op_t(x1, \underbrace{rd(i(rd(mult(x3,x2), mult(rd(mult(x1,x2),x1),x1))), rd(rd(mult(x1,x2),x1), mult(rd(mult(x3,x2), mult(rd(mu}$

$=$ by Lemma 40 LR with $\{x3 \leftarrow mult(rd(mult(x3,x2), mult(rd(mult(x1,x2),x1),x1)), mult(rd(mult(x1,x2),x1),x1)), x2 \leftarrow$

$op_t(x1, \overbrace{rd(rd(\underbrace{mult(rd(mult(x3,x2), mult(rd(mult(x1,x2),x1),x1)), mult(rd(mult(x1,x2),x1),x1))}, rd(mult(x1,x2),x1)}$

$=$ by Axiom 4 RL with $\{x2 \leftarrow mult(rd(mult(x1,x2),x1),x1), x1 \leftarrow mult(x3,x2)\}$

$op_t(x1, rd(rd(\overbrace{mult(x3,x2)}, rd(mult(x1,x2),x1)), rd(mult(x3,x2), \underbrace{mult(rd(mult(x1,x2),x1),x1)})))$

$=$ by Axiom 4 RL with $\{x2 \leftarrow x1, x1 \leftarrow mult(x1,x2)\}$

$op_t(x1, \underbrace{rd(rd(mult(x3,x2), rd(mult(x1,x2),x1)), rd(mult(x3,x2), \overbrace{mult(x1,x2)}))})$

$=$ by Lemma 1475 LR with $\{x4 \leftarrow mult(x1,x2), x3 \leftarrow mult(x3,x2), x2 \leftarrow rd(mult(x1,x2),x1), x1 \leftarrow mult(x3,x2)\}$

$op_t(x1, \overbrace{rd(rd(mult(x1,x2), mult(x3,x2)), \underbrace{rd(rd(mult(x1,x2),x1)}, mult(x3,x2))))}$

$=$ by Lemma 394 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$op_t(x1, rd(rd(mult(x1,x2), mult(x3,x2)), rd(\overbrace{mult(\underbrace{rd(op_t(x1,x2),x1)}, x2)}, mult(x3,x2))))$

$=$ by Lemma 10 RL with $\{x2 \leftarrow op_t(x1,x2), x1 \leftarrow x1\}$

$op_t(x1, rd(rd(mult(x1,x2), mult(x3,x2)), \underbrace{rd(mult(\overbrace{i(rd(x1, op_t(x1,x2)))}, x2), mult(x3,x2))}))$

$=$ by Lemma 62 RL with $\{x3 \leftarrow x2, x2 \leftarrow rd(x1, op_t(x1,x2)), x1 \leftarrow mult(x3,x2)\}$

$op_t(x1, rd(rd(mult(x1,x2), mult(x3,x2)), \overbrace{rd(\underbrace{i(mult(x3,x2))}, mult(rd(x1, op_t(x1,x2)), i(x2)))}))$

$=$ by Axiom 11 RL with $\{x2 \leftarrow x2, x1 \leftarrow x3\}$

$op_t(x1, rd(rd(mult(x1,x2), mult(x3,x2)), rd(\overbrace{mult(\underbrace{i(x3)}, i(x2))}, mult(rd(x1, op_t(x1,x2)), i(x2)))))$

$=$ by Lemma 21 RL with $\{x2 \leftarrow x3, x1 \leftarrow rd(x1, op_t(x1,x2))\}$

$op_t(x1, rd(rd(mult(x1,x2), mult(x3,x2)), \underbrace{rd(mult(\overbrace{mult(rd(i(rd(x1, op_t(x1,x2))), x3), rd(x1, op_t(x1,x2)))}, i(x2)), mult(rd}$

$=$ by Axiom 16 RL with $\{x3 \leftarrow i(x2), x2 \leftarrow rd(x1, op_t(x1,x2)), x1 \leftarrow rd(i(rd(x1, op_t(x1,x2))), x3)\}$

$op_t(x1, rd(rd(mult(x1,x2), mult(x3,x2)), \overbrace{op_r(rd(i(rd(x1, op_t(x1,x2))), x3), rd(x1, op_t(x1,x2)), i(x2))}))$

$=$ by Lemma 1234 RL with $\{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow rd(i(rd(x1, op_t(x1,x2))), x3)\}$

$op_t(x1, rd(rd(mult(x1,x2), mult(x3,x2)), \overbrace{rd(\underbrace{i(rd(x1, op_t(x1,x2)))}, x3)}))$

$=$ by Lemma 10 LR with $\{x2 \leftarrow op_t(x1,x2), x1 \leftarrow x1\}$

$op_t(x1, rd(rd(mult(x1,x2), mult(x3,x2)), \underbrace{rd(\overbrace{rd(op_t(x1,x2),x1)}, x3)}))$

$=$ by Lemma 1291 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$op_t(x1, rd(rd(mult(x1,x2), mult(x3,x2)), \overbrace{rd(op_t(x1,x2), mult(x3,x1))}))$

Lemma 2256: $x1 = op_t(x1, rd(rd(mult(x1, x2), mult(x3, x2)), i(x3)))$

$\overbrace{x1}$

$=$ by Lemma 2255 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_t(x1, rd(rd(mult(x1, x2), mult(x3, x2)), rd(op_t(x1, x2), mult(x3, x1))))}$

$=$ by Lemma 2134 RL with $\{x4 \leftarrow x2, x3 \leftarrow mult(x3, x1), x2 \leftarrow rd(mult(x1, x2), mult(x3, x2)), x1 \leftarrow x1\}$

$\overbrace{op_t(x1, rd(rd(mult(x1, x2), mult(x3, x2)), \underbrace{rd(x1, mult(x3, x1))}))}$

$=$ by Lemma 9 LR with $\{x2 \leftarrow x3, x1 \leftarrow x1\}$

$op_t(x1, rd(rd(mult(x1, x2), mult(x3, x2)), \overbrace{i(x3)}))$

Lemma 2257: $asoc(x1, rd(x2, op_r(x2, x1, x3)), x1) = rd(op_t(x1, rd(rd(x2, rd(op_r(x2, x1, x3), x1)), x1)), x1)$

$\underbrace{asoc(x1, rd(x2, op_r(x2, x1, x3)), x1)}$

$=$ by Lemma 1749 RL with $\{x3 \leftarrow op_r(x2, x1, x3), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{\underbrace{asoc(x1, mult(x2, \overbrace{i(op_r(x2, x1, x3))}), x1)}}$

$=$ by Lemma 613 LR with $\{x2 \leftarrow mult(x2, i(op_r(x2, x1, x3))), x1 \leftarrow x1\}$

$\overbrace{\underbrace{asoc(i(x1), mult(x2, i(op_r(x2, x1, x3))), i(x1))}}$

$=$ by Lemma 1857 RL with $\{x3 \leftarrow op_r(x2, x1, x3), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{\underbrace{asoc(i(x1), mult(x2, rd(x1, op_r(x2, x1, x3))), i(x1))}}$

$=$ by Lemma 613 RL with $\{x2 \leftarrow mult(x2, rd(x1, op_r(x2, x1, x3))), x1 \leftarrow x1\}$

$\overbrace{asoc(x1, mult(x2, \underbrace{rd(x1, op_r(x2, x1, x3))}), x1)}$

$=$ by Lemma 10 RL with $\{x2 \leftarrow x1, x1 \leftarrow op_r(x2, x1, x3)\}$

$\underbrace{asoc(x1, mult(x2, \overbrace{i(rd(op_r(x2, x1, x3), x1))}), x1)}$

$=$ by Lemma 1749 LR with $\{x3 \leftarrow rd(op_r(x2, x1, x3), x1), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{\underbrace{asoc(x1, rd(x2, rd(op_r(x2, x1, x3), x1)), x1)}}$

$=$ by Lemma 1009 LR with $\{x2 \leftarrow rd(x2, rd(op_r(x2, x1, x3), x1)), x1 \leftarrow x1\}$

$\overbrace{rd(op_t(x1, rd(rd(x2, rd(op_r(x2, x1, x3), x1)), x1)), \underbrace{op_t(x1, rd(x2, rd(op_r(x2, x1, x3), x1)))})}$

$=$ by Lemma 2085 RL with $\{x3 \leftarrow op_r(x2, x1, x3), x2 \leftarrow x2, x1 \leftarrow x1\}$

$rd(op_t(x1, rd(rd(x2, rd(op_r(x2, x1, x3), x1)), x1)), \overbrace{op_t(x1, rd(mult(x2, x1), \underbrace{op_r(x2, x1, x3)}))})$

$=$ by Axiom 16 LR with $\{x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow x2\}$

$rd(op_t(x1, rd(rd(x2, rd(op_r(x2, x1, x3), x1)), x1)), op_t(x1, \underbrace{rd(mult(x2, x1), \overbrace{rd(mult(mult(x2, x1), x3), mult(x1, x3))})}))$

$=$ by Lemma 39 RL with $\{x3 \leftarrow mult(x2, x1), x2 \leftarrow mult(mult(x2, x1), x3), x1 \leftarrow mult(x1, x3)\}$

$rd(op_t(x1, rd(rd(x2, rd(op_r(x2, x1, x3), x1)), x1)), \underbrace{op_t(x1, \overbrace{rd(rd(mult(x1, x3), mult(mult(x2, x1), x3)), i(mult(x2, x1)))})})$

$=$ by Lemma 2256 RL with $\{x3 \leftarrow mult(x2, x1), x2 \leftarrow x3, x1 \leftarrow x1\}$

$rd(op_t(x1, rd(rd(x2, rd(op_r(x2, x1, x3), x1)), x1)), \overbrace{x1})$

Lemma 2258: $asoc(x1, rd(x2, op_r(x2, x1, x3)), x1) = unit()$

$\underbrace{asoc(x1, rd(x2, op_r(x2, x1, x3)), x1)}$

$=$ by Lemma 2257 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{rd(\underbrace{op_t(x1, rd(rd(x2, rd(op_r(x2, x1, x3), x1)), x1))}, x1)}$

$=$ by Lemma 2078 RL with $\{x3 \leftarrow x2, x2 \leftarrow rd(op_r(x2, x1, x3), x1), x1 \leftarrow x1\}$

$rd(\overbrace{\underbrace{op_t(x1, rd(rd(op_r(x2, x1, x3), x1), mult(x1, x2)))}}, x1)$

$=$ by Lemma 2079 LR with $\{x3 \leftarrow mult(x1, x2), x2 \leftarrow op_r(x2, x1, x3), x1 \leftarrow x1\}$

$rd(\overbrace{\underbrace{op_t(x1, rd(op_r(x2, x1, x3), mult(x1, mult(x1, x2))))}}, x1)$

$=$ by Lemma 2095 LR with $\{x3 \leftarrow x2, x2 \leftarrow op_r(x2, x1, x3), x1 \leftarrow x1\}$

$rd(\overbrace{\underbrace{op_t(x1, rd(x2, op_r(x2, x1, x3)))}}, x1)$

$=$ by Lemma 2159 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{rd(\overbrace{x1}, x1)}$

$=$ by Lemma 2 LR with $\{x1 \leftarrow x1\}$

$\overbrace{unit()}$

Lemma 2259: $op_t(x1, rd(rd(op_r(x2, x1, x3), x1), mult(x1, x2))) = op_l(x1, x1, mult(op_t(x1, x4), rd(x2, rd(op_r(x2, x1, x3), x1))))$

$\overbrace{op_t(x1, rd(rd(op_r(x2, x1, x3), x1), mult(x1, x2)))}$

$=$ by Lemma 2078 LR with $\{x3 \leftarrow x2, x2 \leftarrow rd(op_r(x2, x1, x3), x1), x1 \leftarrow x1\}$

$\overbrace{op_t(x1, rd(rd(x2, rd(op_r(x2, x1, x3), x1)), x1))}$

$=$ by Lemma 1120 RL with $\{x3 \leftarrow rd(x2, rd(op_r(x2, x1, x3), x1)), x2 \leftarrow x4, x1 \leftarrow x1\}$

$\overbrace{op_t(x1, i(mult(op_t(x1, x4), rd(x2, rd(op_r(x2, x1, x3), x1)))))}$

$=$ by Lemma 586 RL with $\{x2 \leftarrow mult(op_t(x1, x4), rd(x2, rd(op_r(x2, x1, x3), x1))), x1 \leftarrow x1\}$

$\overbrace{op_r(x1, rd(mult(op_t(x1, x4), rd(x2, rd(op_r(x2, x1, x3), x1))), x1), \underbrace{rd(x1, mult(op_t(x1, x4), rd(x2, rd(op_r(x2, x1, x3), x1))))}}$

$=$ by Lemma 10 RL with $\{x2 \leftarrow x1, x1 \leftarrow mult(op_t(x1, x4), rd(x2, rd(op_r(x2, x1, x3), x1)))\}$

$\underbrace{op_r(x1, rd(mult(op_t(x1, x4), rd(x2, rd(op_r(x2, x1, x3), x1))), x1), \overbrace{i(rd(mult(op_t(x1, x4), rd(x2, rd(op_r(x2, x1, x3), x1))), x1}}$

$=$ by Lemma 421 LR with $\{x2 \leftarrow rd(mult(op_t(x1, x4), rd(x2, rd(op_r(x2, x1, x3), x1))), x1), x1 \leftarrow x1\}$

$\overbrace{op_l(\underbrace{op_t(x1, rd(mult(op_t(x1, x4), rd(x2, rd(op_r(x2, x1, x3), x1))), x1))}, x1, rd(mult(op_t(x1, x4), rd(x2, rd(op_r(x2, x1, x3), x1}}$

$=$ by Lemma 1123 LR with $\{x3 \leftarrow rd(x2, rd(op_r(x2, x1, x3), x1)), x2 \leftarrow x4, x1 \leftarrow x1\}$

$\underbrace{op_l(\overbrace{op_t(x1, rd(x2, rd(op_r(x2, x1, x3), x1)))}, x1, rd(mult(op_t(x1, x4), rd(x2, rd(op_r(x2, x1, x3), x1))), x1))}$

$=$ by Lemma 286 LR with $\{x3 \leftarrow mult(op_t(x1, x4), rd(x2, rd(op_r(x2, x1, x3), x1))), x2 \leftarrow x1, x1 \leftarrow op_t(x1, rd(x2, rd(op_r(x2, x1}}$

$\overbrace{op_l(\underbrace{op_t(x1, rd(x2, rd(op_r(x2, x1, x3), x1)))}, x1, mult(op_t(x1, x4), rd(x2, rd(op_r(x2, x1, x3), x1))))}$

$=$ by Lemma 2085 RL with $\{x3 \leftarrow op_r(x2, x1, x3), x2 \leftarrow x2, x1 \leftarrow x1\}$

$op_l(\overbrace{op_t(x1, rd(mult(x2, x1), \underbrace{op_r(x2, x1, x3)}))}, x1, mult(op_t(x1, x4), rd(x2, rd(op_r(x2, x1, x3), x1))))$

$=$ by Axiom 16 LR with $\{x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow x2\}$

$op_l(op_t(x1, \underbrace{rd(mult(x2, x1), \overbrace{rd(mult(mult(x2, x1), x3), mult(x1, x3))})}), x1, mult(op_t(x1, x4), rd(x2, rd(op_r(x2, x1, x3), x1}}$

$=$ by Lemma 39 RL with $\{x3 \leftarrow mult(x2, x1), x2 \leftarrow mult(mult(x2, x1), x3), x1 \leftarrow mult(x1, x3)\}$

$op_l(\underbrace{op_t(x1, \overbrace{rd(rd(mult(x1, x3), mult(mult(x2, x1), x3)), i(mult(x2, x1))))}}, x1, mult(op_t(x1, x4), rd(x2, rd(op_r(x2, x1, x3), x}$

$=$ by Lemma 2256 RL with $\{x3 \leftarrow mult(x2, x1), x2 \leftarrow x3, x1 \leftarrow x1\}$

$op_l(\overbrace{x1}, x1, mult(op_t(x1, x4), rd(x2, rd(op_r(x2, x1, x3), x1))))$

Lemma 2260: $op_t(x1, rd(x2, op_r(x2, x1, x3))) = op_l(x1, x1, mult(op_t(x1, x4), rd(x2, rd(op_r(x2, x1, x3), x1))))$

$\overbrace{op_t(x1, rd(x2, op_r(x2, x1, x3)))}$

$=$ by Lemma 2095 RL with $\{x3 \leftarrow x2, x2 \leftarrow op_r(x2, x1, x3), x1 \leftarrow x1\}$

$\overbrace{op_t(x1, rd(op_r(x2, x1, x3), mult(x1, mult(x1, x2))))}$

$=$ by Lemma 2079 RL with $\{x3 \leftarrow mult(x1, x2), x2 \leftarrow op_r(x2, x1, x3), x1 \leftarrow x1\}$

$\overbrace{op_t(x1, rd(rd(op_r(x2, x1, x3), x1), mult(x1, x2)))}$

$=$ by Lemma 2259 LR with $\{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_l(x1, x1, mult(op_t(x1, x4), rd(x2, rd(op_r(x2, x1, x3), x1))))}$

Lemma 2261: $x1 = op_l(x1, x1, rd(x2, op_r(x2, x1, x3)))$

$$
\begin{aligned}
& \overbrace{x1} \\
= \quad & \text{by Lemma 2159 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_t(x1, rd(x2, op_r(x2, x1, x3)))}} \\
= \quad & \text{by Lemma 2260 LR with } \{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, x1, mult(op_t(x1, x4), rd(x2, rd(op_r(x2, x1, x3), x1))))}} \\
= \quad & \text{by Lemma 1063 LR with } \{x3 \leftarrow rd(x2, rd(op_r(x2, x1, x3), x1)), x2 \leftarrow x4, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, x1, rd(x2, rd(op_r(x2, x1, x3), x1)))}} \\
= \quad & \text{by Lemma 1855 RL with } \{x2 \leftarrow x2, x3 \leftarrow op_r(x2, x1, x3), x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, rd(op_r(x2, x1, x3), x2), x1)}} \\
= \quad & \text{by Lemma 2052 RL with } \{x4 \leftarrow op_r(x2, x1, x3), x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x1, rd(x2, op_r(x2, x1, x3)))}
\end{aligned}
$$

Lemma 2262: $asoc(x1, rd(x2, op_r(x2, x1, x3)), x1) = rd(op_t(x1, rd(rd(x2, mult(x1, op_r(x2, x1, x3))), x1)), x1)$

$\underbrace{asoc(x1, rd(x2, op_r(x2, x1, x3)), x1)}$

$=$ by Lemma 1856 LR with $\{x1 \leftarrow op_r(x2, x1, x3), x2 \leftarrow x2, x3 \leftarrow x1\}$

$\overbrace{\underbrace{asoc(x1, rd(x2, mult(x1, op_r(x2, x1, x3))), x1)}}$

$=$ by Lemma 1009 LR with $\{x2 \leftarrow rd(x2, mult(x1, op_r(x2, x1, x3))), x1 \leftarrow x1\}$

$\overbrace{rd(op_t(x1, rd(rd(x2, mult(x1, op_r(x2, x1, x3))), x1)), \underbrace{op_t(x1, rd(x2, mult(x1, op_r(x2, x1, x3))))})}$

$=$ by Lemma 1763 RL with $\{x2 \leftarrow op_r(x2, x1, x3), x3 \leftarrow x2, x1 \leftarrow x1\}$

$rd(op_t(x1, rd(rd(x2, mult(x1, op_r(x2, x1, x3))), x1)), \overbrace{op_t(x1, rd(x2, \underbrace{mult(op_r(x2, x1, x3), x1)}))})$

$=$ by Lemma 13 RL with $\{x2 \leftarrow op_r(x2, x1, x3), x1 \leftarrow x1\}$

$rd(op_t(x1, rd(rd(x2, mult(x1, op_r(x2, x1, x3))), x1)), \underbrace{op_t(x1, rd(x2, \overbrace{mult(x1, op_t(op_r(x2, x1, x3), x1))}))})$

$=$ by Lemma 2140 LR with $\{x3 \leftarrow op_r(x2, x1, x3), x2 \leftarrow x2, x1 \leftarrow x1\}$

$rd(op_t(x1, rd(rd(x2, mult(x1, op_r(x2, x1, x3))), x1)), \overbrace{op_t(x1, rd(op_r(x2, x1, x3), mult(x1, x2)))})$

$=$ by Lemma 2078 LR with $\{x3 \leftarrow x2, x2 \leftarrow op_r(x2, x1, x3), x1 \leftarrow x1\}$

$rd(op_t(x1, rd(rd(x2, mult(x1, op_r(x2, x1, x3))), x1)), \overbrace{op_t(x1, rd(rd(x2, op_r(x2, x1, x3)), x1))})$

$=$ by Lemma 1919 LR with $\{x2 \leftarrow rd(x2, op_r(x2, x1, x3)), x1 \leftarrow x1\}$

$rd(op_t(x1, rd(rd(x2, mult(x1, op_r(x2, x1, x3))), x1)), \overbrace{rd(op_l(x1, x1, rd(x2, op_r(x2, x1, x3))), \underbrace{asoc(x1, mult(rd(x2, op_r(x2, x1$

$=$ by Lemma 858 LR with $\{x2 \leftarrow rd(x2, op_r(x2, x1, x3)), x1 \leftarrow x1\}$

$rd(op_t(x1, rd(rd(x2, mult(x1, op_r(x2, x1, x3))), x1)), rd(\underbrace{op_l(x1, x1, rd(x2, op_r(x2, x1, x3)))}, \overbrace{rd(op_t(rd(x2, op_r(x2, x1, x3)), x$

$=$ by Lemma 2261 RL with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$rd(op_t(x1, rd(rd(x2, mult(x1, op_r(x2, x1, x3))), x1)), \underbrace{rd(\overbrace{x1}, rd(op_t(rd(x2, op_r(x2, x1, x3)), x1), rd(x2, op_r(x2, x1, x3)))))}$

$=$ by Lemma 390 LR with $\{x2 \leftarrow rd(x2, op_r(x2, x1, x3)), x1 \leftarrow x1\}$

$rd(op_t(x1, rd(rd(x2, mult(x1, op_r(x2, x1, x3))), x1)), \overbrace{op_t(x1, rd(x2, op_r(x2, x1, x3)))})$

$=$ by Lemma 2159 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$rd(op_t(x1, rd(rd(x2, mult(x1, op_r(x2, x1, x3))), x1)), \overbrace{x1})$

Lemma 2263: $op_t(x1, rd(op_r(x2, x1, x3), x2)) = x1$

$$
\begin{aligned}
& \overbrace{op_t(x1, rd(op_r(x2, x1, x3), x2))} \\
= \quad & \text{by Axiom 4 LR with } \{x2 \leftarrow x1, x1 \leftarrow op_t(x1, rd(op_r(x2, x1, x3), x2))\} \\
& \overbrace{mult(rd(op_t(x1, rd(op_r(x2, x1, x3), \underbrace{x2}_{}\,)), x1), x1)} \\
= \quad & \text{by Axiom 2 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& mult(rd(\overbrace{op_t(x1, rd(op_r(x2, x1, x3), \overbrace{mult(i(x1), mult(x1, x2))}))}, x1), x1) \\
= \quad & \text{by Lemma 1258 LR with } \{x3 \leftarrow mult(x1, x2), x2 \leftarrow op_r(x2, x1, x3), x1 \leftarrow x1\} \\
& mult(rd(\overbrace{op_t(x1, rd(op_r(x2, x1, x3), rd(mult(x1, x2), x1)))}, x1), x1) \\
= \quad & \text{by Lemma 2084 LR with } \{x2 \leftarrow mult(x1, x2), x3 \leftarrow op_r(x2, x1, x3), x1 \leftarrow x1\} \\
& mult(rd(\overbrace{op_t(x1, rd(mult(x1, op_r(x2, x1, x3)), mult(x1, x2)))}, x1), x1) \\
= \quad & \text{by Lemma 2078 LR with } \{x3 \leftarrow x2, x2 \leftarrow mult(x1, op_r(x2, x1, x3)), x1 \leftarrow x1\} \\
& mult(\underbrace{rd(\overbrace{op_t(x1, rd(rd(x2, mult(x1, op_r(x2, x1, x3))), x1))}, x1)}, x1) \\
= \quad & \text{by Lemma 2262 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& mult(\overbrace{asoc(x1, rd(x2, op_r(x2, x1, x3)), x1)}, x1) \\
= \quad & \text{by Lemma 2258 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{mult(\overbrace{unit()}, x1)} \\
= \quad & \text{by Axiom 3 RL with } \{x1 \leftarrow x1\} \\
& \overbrace{x1}
\end{aligned}
$$

Lemma 2264: $op_t(i(x1), rd(x2, op_r(x2, x1, x3))) = op_t(i(x1), x1)$

$$
\begin{aligned}
& \underbrace{op_t(i(x1), rd(x2, op_r(x2, x1, x3)))} \\
= \quad & \text{by Lemma 47 RL with } \{x3 \leftarrow x2, x2 \leftarrow op_r(x2, x1, x3), x1 \leftarrow x1\} \\
& \underbrace{\overbrace{i(op_t(x1, rd(op_r(x2, x1, x3), x2)))}} \\
= \quad & \text{by Lemma 35 RL with } \{x2 \leftarrow rd(op_r(x2, x1, x3), x2), x1 \leftarrow x1\} \\
& \overbrace{op_t(i(\underbrace{op_t(x1, rd(op_r(x2, x1, x3), x2))}), x1)} \\
= \quad & \text{by Lemma 2263 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_t(i(\overbrace{x1}), x1)
\end{aligned}
$$

Lemma 2265: $i(op_t(i(x1), mult(i(x1), rd(x2, op_r(x2, x1, x3))))) = op_l(i(i(x1)), rd(x2, op_r(x2, x1, x3)), i(x1))$

$$
\begin{aligned}
& \underbrace{i(op_t(i(x1), mult(i(x1), rd(x2, op_r(x2, x1, x3)))))} \\
= \quad & \text{by Lemma 479 LR with } \{x1 \leftarrow rd(x2, op_r(x2, x1, x3)), x2 \leftarrow i(x1)\} \\
& \overbrace{op_l(i(\underbrace{op_t(i(x1), rd(x2, op_r(x2, x1, x3)))}), rd(x2, op_r(x2, x1, x3)), i(x1))} \\
= \quad & \text{by Lemma 2264 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_l(i(\overbrace{op_t(i(x1), x1)}), rd(x2, op_r(x2, x1, x3)), i(x1)) \\
= \quad & \text{by Lemma 7 LR with } \{x1 \leftarrow x1\} \\
& op_l(i(\overbrace{i(x1)}), rd(x2, op_r(x2, x1, x3)), i(x1))
\end{aligned}
$$

Lemma 2266: $op_t(x1, rd(i(i(x1)), rd(x2, op_r(x2, x1, x3)))) = op_l(i(i(x1)), rd(x2, op_r(x2, x1, x3)), i(x1))$

$$
\begin{aligned}
& \underbrace{op_t(x1, rd(i(i(x1)), rd(x2, op_r(x2, x1, x3))))} \\
= & \quad \text{by Lemma 1732 RL with } \{x3 \leftarrow rd(x2, op_r(x2, x1, x3)), x2 \leftarrow i(x1), x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, \underbrace{i(mult(i(x1), rd(x2, op_r(x2, x1, x3))))}_{})} \\
= & \quad \text{by Lemma 18 RL with } \{x2 \leftarrow mult(i(x1), rd(x2, op_r(x2, x1, x3))), x1 \leftarrow x1\} \\
& \overbrace{\underbrace{i(op_t(i(x1), mult(i(x1), rd(x2, op_r(x2, x1, x3)))))}_{}} \\
= & \quad \text{by Lemma 2265 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(i(i(x1)), rd(x2, \underbrace{op_r(x2, x1, x3)}_{}), i(x1))}
\end{aligned}
$$

Lemma 2267: $op_t(x1, rd(x1, rd(x2, op_r(x2, x1, x3)))) = op_l(i(i(x1)), rd(x2, op_r(x2, x1, x3)), i(x1))$

$$
\begin{aligned}
& op_t(x1, \underbrace{rd(x1, rd(x2, op_r(x2, x1, x3)))}) \\
= & \quad \text{by Lemma 39 RL with } \{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow op_r(x2, x1, x3)\} \\
& op_t(x1, \overbrace{\underbrace{rd(rd(op_r(x2, x1, x3), x2), i(x1))}}) \\
= & \quad \text{by Lemma 40 RL with } \{x3 \leftarrow op_r(x2, x1, x3), x2 \leftarrow x2, x1 \leftarrow i(x1)\} \\
& \underbrace{op_t(x1, \overbrace{rd(i(i(x1)), rd(x2, op_r(x2, x1, x3)))})} \\
= & \quad \text{by Lemma 2266 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(i(i(x1)), rd(x2, \underbrace{op_r(x2, x1, x3)}_{}), i(x1))}
\end{aligned}
$$

Lemma 2268: $op_t(x1, rd(x1, rd(x2, op_r(x2, x1, x3)))) = op_l(x1, x1, rd(x2, op_r(x2, x1, x3)))$

$$
\begin{aligned}
& \underbrace{op_t(x1, rd(x1, rd(x2, op_r(x2, x1, x3))))} \\
= & \quad \text{by Lemma 2267 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{\overbrace{op_l(i(i(x1)), rd(x2, op_r(x2, x1, x3)), i(x1))}} \\
= & \quad \text{by Lemma 2030 LR with } \{x3 \leftarrow x1, x2 \leftarrow rd(x2, op_r(x2, x1, x3)), x1 \leftarrow i(i(x1))\} \\
& \overbrace{op_l(\underbrace{i(i(x1))}, x1, rd(x2, op_r(x2, x1, x3)))} \\
= & \quad \text{by Lemma 3 LR with } \{x1 \leftarrow x1\} \\
& op_l(\overbrace{x1}, x1, rd(x2, op_r(x2, x1, x3)))
\end{aligned}
$$

Lemma 2269: $mult(rd(x1, x2), op_r(x2, x1, x3)) = rd(x1, rd(x2, op_r(x2, x1, x3)))$

$$
\begin{aligned}
& \underbrace{mult(rd(x1, x2), op_r(x2, x1, x3))} \\
= & \quad \text{by Lemma 2149 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{mult(\underbrace{rd(op_r(x2, x1, x3), x2)}, x1)} \\
= & \quad \text{by Lemma 10 RL with } \{x2 \leftarrow op_r(x2, x1, x3), x1 \leftarrow x2\} \\
& mult(\overbrace{i(\underbrace{rd(x2, op_r(x2, x1, x3))})}, \underbrace{x1}) \\
= & \quad \text{by Lemma 2261 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{mult(i(rd(x2, op_r(x2, x1, x3))), \overbrace{op_l(x1, x1, rd(x2, op_r(x2, x1, x3)))})} \\
= & \quad \text{by Lemma 440 LR with } \{x2 \leftarrow x1, x1 \leftarrow rd(x2, op_r(x2, x1, x3))\} \\
& \overbrace{rd(x1, rd(x2, op_r(x2, x1, x3)))}
\end{aligned}
$$

Lemma 2270: $mult(rd(x1, op_r(x1, x2, x3)), x2) = rd(x2, rd(op_r(x1, x2, x3), x1))$

$$
\begin{aligned}
& \underbrace{mult(rd(x1, op_r(x1, x2, x3)), x2)} \\
= \quad & \text{by Lemma 435 RL with } \{x2 \leftarrow rd(x1, op_r(x1, x2, x3)), x1 \leftarrow x2\} \\
& \overbrace{\underbrace{op_l(mult(x2, rd(x1, op_r(x1, x2, x3)))}, x2, rd(x1, op_r(x1, x2, x3)))} \\
= \quad & \text{by Lemma 1789 RL with } \{x2 \leftarrow rd(x1, op_r(x1, x2, x3)), x3 \leftarrow x2, x1 \leftarrow x2\} \\
& \overbrace{mult(\underbrace{op_l(x2, x2, rd(x1, op_r(x1, x2, x3)))}, op_l(rd(x1, op_r(x1, x2, x3)), x2, rd(x1, op_r(x1, x2, x3))))} \\
= \quad & \text{by Lemma 2261 RL with } \{x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \underbrace{mult(\overbrace{x2}, op_l(rd(x1, op_r(x1, x2, x3)), x2, rd(x1, op_r(x1, x2, x3))))} \\
= \quad & \text{by Lemma 616 LR with } \{x2 \leftarrow rd(x1, op_r(x1, x2, x3)), x1 \leftarrow x2\} \\
& \overbrace{rd(x2, \underbrace{i(rd(x1, op_r(x1, x2, x3)))})} \\
= \quad & \text{by Lemma 10 LR with } \{x2 \leftarrow op_r(x1, x2, x3), x1 \leftarrow x1\} \\
& rd(x2, \overbrace{rd(op_r(x1, x2, x3), x1)})
\end{aligned}
$$

Lemma 2271: $op_r(rd(op_r(x1, x2, x3), x1), x2, x2) = rd(unit(), rd(x1, op_r(x1, x2, x3)))$

$$
\begin{aligned}
& op_r(\underbrace{rd(op_r(x1, x2, x3), x1)}, x2, x2) \\
= \quad & \text{by Lemma 10 RL with } \{x2 \leftarrow op_r(x1, x2, x3), x1 \leftarrow x1\} \\
& \underbrace{op_r(\overbrace{i(rd(x1, op_r(x1, x2, x3)))}, x2, x2)} \\
= \quad & \text{by Lemma 642 RL with } \{x2 \leftarrow rd(x1, op_r(x1, x2, x3)), x1 \leftarrow x2\} \\
& \overbrace{rd(\underbrace{asoc(x2, rd(x1, op_r(x1, x2, x3)), x2)}, rd(x1, op_r(x1, x2, x3)))} \\
= \quad & \text{by Lemma 2258 LR with } \{x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow x2\} \\
& rd(\overbrace{unit()}, rd(x1, op_r(x1, x2, x3)))
\end{aligned}
$$

Lemma 2272: $op_r(rd(op_r(x1, x2, x3), x1), x2, x2) = rd(op_r(x1, x2, x3), x1)$

$$
\begin{aligned}
& \underbrace{op_r(rd(op_r(x1, x2, x3), x1), x2, x2)} \\
= \quad & \text{by Lemma 2271 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{rd(unit(), rd(x1, op_r(x1, x2, x3)))}} \\
= \quad & \text{by Lemma 5 LR with } \{x1 \leftarrow rd(x1, op_r(x1, x2, x3))\} \\
& \overbrace{\underbrace{i(rd(x1, op_r(x1, x2, x3)))}} \\
= \quad & \text{by Lemma 10 LR with } \{x2 \leftarrow op_r(x1, x2, x3), x1 \leftarrow x1\} \\
& \overbrace{rd(op_r(x1, x2, x3), x1)}
\end{aligned}
$$

Lemma 2273: $x1 = op_l(x1, rd(mult(mult(x2, x1), x3), mult(x2, x3)), x1)$

$$
\begin{aligned}
& \overbrace{x1} \\
= \quad & \text{by Lemma 1443 RL with } \{x4 \leftarrow x3, x3 \leftarrow x2, x2 \leftarrow rd(mult(x2, x1), x2), x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, op_r(rd(mult(x2, x1), x2), x2, x3), rd(mult(x2, x1), x2))} \\
= \quad & \text{by Lemma 1022 LR with } \{x3 \leftarrow x2, x2 \leftarrow op_r(rd(mult(x2, x1), x2), x2, x3), x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, \underbrace{op_r(rd(mult(x2, x1), x2), x2, x3)}, x1)} \\
= \quad & \text{by Lemma 676 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow mult(x2, x1)\} \\
& op_l(x1, \overbrace{rd(mult(mult(x2, x1), x3), mult(x2, x3))}, x1)
\end{aligned}
$$

Lemma 2274: $rd(x1, mult(i(x1), mult(mult(x1, x2), x3))) = rd(mult(x1, mult(i(mult(x1, x2)), rd(mult(x1, x2), x3))), x2)$

$$
\begin{aligned}
& \underbrace{rd(x1, mult(i(x1), mult(mult(x1, x2), x3)))} \\
= \quad & \text{by Lemma 60 RL with } \{x2 \leftarrow mult(mult(x1, x2), x3), x1 \leftarrow x1\} \\
& \overbrace{mult(x1, mult(i(\overbrace{mult(mult(x1, x2), x3)}), x1))} \\
= \quad & \text{by Lemma 76 RL with } \{x3 \leftarrow x2, x2 \leftarrow i(mult(mult(x1, x2), x3)), x1 \leftarrow x1\} \\
& \overbrace{rd(mult(x1, \underbrace{mult(i(mult(mult(x1, x2), x3)), mult(x1, x2))}), x2)} \\
= \quad & \text{by Lemma 56 LR with } \{x2 \leftarrow x3, x1 \leftarrow mult(x1, x2)\} \\
& rd(mult(x1, \overbrace{mult(i(mult(x1, x2)), rd(mult(x1, x2), x3))}), x2)
\end{aligned}
$$

Lemma 2275: $x1 = op_t(x1, rd(rd(x3, rd(op_r(x2, x3, i(x1)), x2)), rd(x3, x1)))$

$\overbrace{x1}$

$=$ by Lemma 3 RL with $\{x1 \leftarrow x1\}$

$\overbrace{i(\underbrace{i(x1)}_{})}$

$=$ by Lemma 1810 LR with $\{x3 \leftarrow mult(mult(x2, x3), mult(x3, i(x1))), x2 \leftarrow i(mult(x2, x3)), x1 \leftarrow i(x1)\}$

$i(\overbrace{op_t(i(x1), rd(\underbrace{mult(i(mult(x2, x3)), mult(mult(x2, x3), mult(x3, i(x1))))}_{}, mult(i(mult(x2, x3)), rd(mult(mult(x2, x3), m}$

$=$ by Axiom 2 RL with $\{x2 \leftarrow mult(x2, x3), x1 \leftarrow mult(x3, i(x1))\}$

$i(op_t(i(x1), rd(\overbrace{mult(x3, i(x1))}, \underbrace{mult(i(mult(x2, x3)), rd(mult(mult(x2, x3), mult(x3, i(x1))), i(x1)))}_{})))$

$=$ by Lemma 1153 LR with $\{x3 \leftarrow i(x1), x2 \leftarrow mult(x3, i(x1)), x1 \leftarrow mult(x2, x3)\}$

$\underbrace{i(op_t(i(x1), rd(mult(x3, i(x1)), \overbrace{mult(rd(mult(x3, i(x1)), mult(mult(x2, x3), i(x1))), mult(x2, x3))}))}$

$=$ by Lemma 18 LR with $\{x2 \leftarrow rd(mult(x3, i(x1)), mult(rd(mult(x3, i(x1)), mult(mult(x2, x3), i(x1))), mult(x2, x3))), x1 \leftarrow$

$\overbrace{op_t(x1, \underbrace{i(rd(mult(x3, i(x1)), mult(rd(mult(x3, i(x1)), mult(mult(x2, x3), i(x1))), mult(x2, x3))))}_{})}$

$=$ by Lemma 10 LR with $\{x2 \leftarrow mult(rd(mult(x3, i(x1)), mult(mult(x2, x3), i(x1))), mult(x2, x3)), x1 \leftarrow mult(x3, i(x1))\}$

$op_t(x1, \overbrace{rd(mult(\underbrace{rd(mult(x3, i(x1)), mult(mult(x2, x3), i(x1)))}_{}, mult(x2, x3)), mult(x3, i(x1)))})$

$=$ by Lemma 891 LR with $\{x3 \leftarrow x2, x2 \leftarrow i(x1), x1 \leftarrow x3\}$

$op_t(x1, rd(mult(\overbrace{i(op_r(x2, x3, i(x1)))}, mult(x2, x3)), \underbrace{mult(x3, i(x1))}_{}))$

$=$ by Lemma 197 RL with $\{x2 \leftarrow x3, x1 \leftarrow x1\}$

$\underbrace{op_t(x1, rd(mult(i(op_r(x2, x3, i(x1))), mult(x2, x3)), \overbrace{op_l(mult(\overbrace{i(x1)}, x3), x1, i(x3))}))}$

$=$ by Lemma 1285 RL with $\{x3 \leftarrow x3, x2 \leftarrow mult(i(op_r(x2, x3, i(x1))), mult(x2, x3)), x1 \leftarrow x1\}$

$\overbrace{op_t(x1, rd(\underbrace{mult(i(op_r(x2, x3, i(x1))), mult(x2, x3))}_{}, rd(x3, x1)))}$

$=$ by Lemma 1979 LR with $\{x4 \leftarrow x3, x3 \leftarrow i(x1), x2 \leftarrow x3, x1 \leftarrow x2\}$

$op_t(x1, rd(\overbrace{mult(rd(x2, op_r(x2, x3, i(x1))), x3)}, rd(x3, x1)))$

$=$ by Lemma 2270 LR with $\{x3 \leftarrow i(x1), x2 \leftarrow x3, x1 \leftarrow x2\}$

$op_t(x1, rd(\overbrace{rd(x3, rd(op_r(x2, x3, i(x1)), x2))}, rd(x3, x1)))$

Lemma 2276: $op_t(x1, rd(x1, rd(op_r(x2, x3, i(x1)), x2))) = x1$

$\underbrace{op_t(x1, rd(x1, rd(op_r(x2, x3, i(x1)), x2)))}$

$=$ by Lemma 2097 RL with $\{x3 \leftarrow rd(op_r(x2, x3, i(x1)), x2), x2 \leftarrow x3, x1 \leftarrow x1\}$

$\underbrace{\overbrace{op_t(x1, rd(mult(x1, rd(x3, rd(op_r(x2, x3, i(x1)), x2))), x3))}}$

$=$ by Lemma 2084 RL with $\{x2 \leftarrow x3, x3 \leftarrow rd(x3, rd(op_r(x2, x3, i(x1)), x2)), x1 \leftarrow x1\}$

$\underbrace{\overbrace{op_t(x1, rd(rd(x3, rd(op_r(x2, x3, i(x1)), x2)), rd(x3, x1)))}}$

$=$ by Lemma 2275 RL with $\{x2 \leftarrow x2, x3 \leftarrow x3, x1 \leftarrow x1\}$

$\overbrace{x1}$

Lemma 2277: $op_t(x1, rd(rd(op_r(x2, x3, i(i(x1))), x2), i(x1))) = i(i(x1))$

$$
\begin{aligned}
& op_t(x1, \underbrace{rd(rd(op_r(x2, x3, i(i(x1))), x2), i(x1))}) \\
= & \quad \text{by Lemma 10 RL with } \{x2 \leftarrow rd(op_r(x2, x3, i(i(x1))), x2), x1 \leftarrow i(x1)\} \\
& \underbrace{op_t(x1, \overbrace{i(rd(i(x1), rd(op_r(x2, x3, i(i(x1))), x2)))})} \\
= & \quad \text{by Lemma 18 RL with } \{x2 \leftarrow rd(i(x1), rd(op_r(x2, x3, i(i(x1))), x2)), x1 \leftarrow x1\} \\
& \overbrace{i(\underbrace{op_t(i(x1), rd(i(x1), rd(op_r(x2, x3, i(i(x1))), x2)))})} \\
= & \quad \text{by Lemma 2276 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow i(x1)\} \\
& i(\overbrace{i(x1)})
\end{aligned}
$$

Lemma 2278: $op_t(x1, rd(x1, rd(x2, op_r(x2, x3, x1)))) = i(i(x1))$

$$
\begin{aligned}
& op_t(x1, rd(x1, rd(x2, op_r(x2, x3, \underbrace{x1}_{})))) \\
= & \quad \text{by Lemma 3 RL with } \{x1 \leftarrow x1\} \\
& op_t(x1, \underbrace{rd(x1, rd(x2, op_r(x2, x3, \overbrace{i(i(x1))})))}) \\
= & \quad \text{by Lemma 39 RL with } \{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow op_r(x2, x3, i(i(x1)))\} \\
& \underbrace{op_t(x1, \overbrace{rd(rd(op_r(x2, x3, i(i(x1))), x2), i(x1))})} \\
= & \quad \text{by Lemma 2277 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{i(i(x1))}
\end{aligned}
$$

Lemma 2279: $op_t(x1, rd(x1, rd(x2, op_r(x2, x3, x1)))) = x1$

$$
\begin{aligned}
& \underbrace{op_t(x1, rd(x1, rd(x2, op_r(x2, x3, x1))))} \\
= & \quad \text{by Lemma 2278 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{\overbrace{i(i(x1))}} \\
= & \quad \text{by Lemma 3 LR with } \{x1 \leftarrow x1\} \\
& \overbrace{x1}
\end{aligned}
$$

Lemma 2280: $x1 = op_t(x1, rd(x1, rd(op_r(x2, x3, x1), x2)))$

$$
\begin{aligned}
& \underbrace{x1} \\
= & \quad \text{by Lemma 2276 RL with } \{x3 \leftarrow i(mult(x2, x3)), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, rd(x1, rd(\underbrace{op_r(x2, i(mult(x2, x3)), i(x1))}, x2)))} \\
= & \quad \text{by Lemma 818 LR with } \{x3 \leftarrow i(x1), x2 \leftarrow x3, x1 \leftarrow x2\} \\
& op_t(x1, rd(x1, rd(\overbrace{op_r(x2, x3, \underbrace{i(i(x1))})}, x2))) \\
= & \quad \text{by Lemma 3 LR with } \{x1 \leftarrow x1\} \\
& op_t(x1, rd(x1, rd(op_r(x2, x3, \overbrace{x1}), x2)))
\end{aligned}
$$

Lemma 2281: $rd(op_t(rd(x1, rd(x2, op_r(x2, x3, i(i(x1))))), x1), x1) = rd(i(x1), rd(i(x1), rd(op_r(x2, x3, i(i(x1))), x2)))$

$rd(op_t(\overbrace{rd(x1, rd(x2, op_r(x2, x3, i(i(x1)))))}, x1), x1)$

$=$ by Lemma 39 RL with $\{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow op_r(x2, x3, i(i(x1)))\}$

$rd(op_t(\overbrace{rd(rd(op_r(x2, x3, i(i(x1))), x2), i(x1))}, x1), x1)$

$=$ by Lemma 10 RL with $\{x2 \leftarrow rd(op_r(x2, x3, i(i(x1))), x2), x1 \leftarrow i(x1)\}$

$\overbrace{rd(op_t(\overbrace{i(rd(i(x1), rd(op_r(x2, x3, i(i(x1))), x2)))}, x1), x1)}$

$=$ by Lemma 329 RL with $\{x2 \leftarrow x1, x1 \leftarrow rd(i(x1), rd(op_r(x2, x3, i(i(x1))), x2))\}$

$\overbrace{op_t(\overbrace{i(mult(rd(i(x1), rd(op_r(x2, x3, i(i(x1))), x2)), x1))}, x1)}$

$=$ by Lemma 682 LR with $\{x2 \leftarrow x1, x1 \leftarrow rd(i(x1), rd(op_r(x2, x3, i(i(x1))), x2))\}$

$\overbrace{op_t(\overbrace{op_t(rd(i(rd(i(x1), rd(op_r(x2, x3, i(i(x1))), x2))), x1)}, op_t(rd(i(x1), rd(op_r(x2, x3, i(i(x1))), x2)), x1))}, x1)$

$=$ by Axiom 13 RL with $\{x3 \leftarrow op_t(rd(i(x1), rd(op_r(x2, x3, i(i(x1))), x2)), x1), x2 \leftarrow x1, x1 \leftarrow rd(i(rd(i(x1), rd(op_r(x2, x3, i(i($

$\overbrace{op_t(\overbrace{op_t(rd(i(rd(i(x1), rd(op_r(x2, x3, i(i(x1))), x2))), x1)}, x1), op_t(rd(i(x1), rd(op_r(x2, x3, i(i(x1))), x2)), x1))}$

$=$ by Lemma 14 LR with $\{x2 \leftarrow x1, x1 \leftarrow i(rd(i(x1), rd(op_r(x2, x3, i(i(x1))), x2)))\}$

$op_t(\overbrace{mult(i(x1), i(rd(i(x1), rd(op_r(x2, x3, i(i(x1))), x2))))}, op_t(rd(i(x1), rd(op_r(x2, x3, i(i(x1))), x2)), x1))$

$=$ by Axiom 11 LR with $\{x2 \leftarrow rd(i(x1), rd(op_r(x2, x3, i(i(x1))), x2)), x1 \leftarrow x1\}$

$op_t(\overbrace{i(mult(x1, rd(i(x1), rd(op_r(x2, x3, i(i(x1))), x2))))}, op_t(rd(i(x1), rd(op_r(x2, x3, i(i(x1))), x2)), \underbrace{x1}))$

$=$ by Axiom 5 LR with $\{x2 \leftarrow rd(i(x1), rd(op_r(x2, x3, i(i(x1))), x2)), x1 \leftarrow x1\}$

$\overbrace{op_t(i(mult(x1, rd(i(x1), rd(op_r(x2, x3, i(i(x1))), x2)))), op_t(rd(i(x1), rd(op_r(x2, x3, i(i(x1))), x2)), \overbrace{rd(mult(x1, rd(i(x1), rd}$

$=$ by Lemma 592 LR with $\{x2 \leftarrow rd(i(x1), rd(op_r(x2, x3, i(i(x1))), x2)), x1 \leftarrow mult(x1, rd(i(x1), rd(op_r(x2, x3, i(i(x1))), x2)))\}$

$\overbrace{op_t(i(mult(x1, rd(i(x1), rd(op_r(x2, x3, i(i(x1))), x2)))), rd(i(x1), rd(op_r(x2, x3, i(i(x1))), x2)))}$

$=$ by Lemma 329 LR with $\{x2 \leftarrow rd(i(x1), rd(op_r(x2, x3, i(i(x1))), x2)), x1 \leftarrow x1\}$

$\overbrace{rd(\overbrace{op_t(i(x1), rd(i(x1), rd(op_r(x2, x3, i(i(x1))), x2)))}, rd(i(x1), rd(op_r(x2, x3, i(i(x1))), x2)))}$

$=$ by Lemma 2276 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow i(x1)\}$

$rd(\overbrace{i(x1)}, rd(i(x1), rd(op_r(x2, x3, i(i(x1))), x2)))$

Lemma 2282: $op_t(rd(op_r(x2, x3, x1), x2), x1) = op_r(rd(op_r(x2, x3, i(i(x1))), x2), x1, x1)$

$op_t(rd(op_r(x2, x3, \underbrace{x1}), x2), x1)$

$=$ by Lemma 3 RL with $\{x1 \leftarrow x1\}$

$op_t(\underbrace{rd(op_r(x2, x3, \overbrace{i(i(x1))}), x2)}, x1)$

$=$ by Lemma 10 RL with $\{x2 \leftarrow op_r(x2, x3, i(i(x1))), x1 \leftarrow x2\}$

$\underbrace{op_t(\overbrace{i(rd(x2, op_r(x2, x3, i(i(x1)))))}, x1)}$

$=$ by Lemma 345 RL with $\{x2 \leftarrow rd(x2, op_r(x2, x3, i(i(x1)))), x1 \leftarrow x1\}$

$\overbrace{\underbrace{rd(op_t(rd(x1, rd(x2, op_r(x2, x3, i(i(x1))))), x1), x1)}}$

$=$ by Lemma 2281 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{\underbrace{rd(i(x1), rd(i(x1), rd(op_r(x2, x3, i(i(x1))), x2)))}}$

$=$ by Lemma 40 LR with $\{x3 \leftarrow rd(op_r(x2, x3, i(i(x1))), x2), x2 \leftarrow i(x1), x1 \leftarrow x1\}$

$\overbrace{\underbrace{rd(rd(rd(op_r(x2, x3, i(i(x1))), x2), i(x1)), x1)}}$

$=$ by Lemma 358 LR with $\{x2 \leftarrow x1, x1 \leftarrow rd(op_r(x2, x3, i(i(x1))), x2)\}$

$\overbrace{op_r(rd(op_r(x2, x3, i(i(x1))), x2), x1, x1)}$

Lemma 2283: $op_r(rd(op_r(x1, x2, x3), x1), x3, x3) = op_t(rd(op_r(x1, x2, x3), x1), x3)$

$op_r(rd(op_r(x1, x2, \underbrace{x3}), x1), x3, x3)$

$=$ by Lemma 3 RL with $\{x1 \leftarrow x3\}$

$\underbrace{op_r(rd(op_r(x1, x2, \overbrace{i(i(x3))}), x1), x3, x3)}$

$=$ by Lemma 2282 RL with $\{x1 \leftarrow x3, x3 \leftarrow x2, x2 \leftarrow x1\}$

$\overbrace{op_t(rd(op_r(x1, x2, x3), x1), x3)}$

Lemma 2284: $op_t(x1, rd(op_r(x2, x3, x1), x2)) = op_l(x1, rd(x2, op_r(x2, x3, x1)), x1)$

$op_t(x1, \underbrace{rd(op_r(x2, x3, x1), x2)})$

$=$ by Lemma 10 RL with $\{x2 \leftarrow op_r(x2, x3, x1), x1 \leftarrow x2\}$

$\underbrace{op_t(x1, \overbrace{i(rd(x2, op_r(x2, x3, x1)))})}$

$=$ by Lemma 681 RL with $\{x2 \leftarrow rd(x2, op_r(x2, x3, x1)), x1 \leftarrow x1\}$

$\overbrace{op_l(\underbrace{op_t(x1, rd(x1, rd(x2, op_r(x2, x3, x1))))}, rd(x2, op_r(x2, x3, x1)), x1)}$

$=$ by Lemma 2279 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$op_l(\overbrace{x1}, rd(x2, op_r(x2, x3, x1)), x1)$

Lemma 2285: $op_l(x1, x1, rd(x2, op_r(x2, x3, x1))) = op_t(x1, rd(x2, op_r(x2, x3, x1)))$

$\overbrace{op_l(x1, x1, rd(x2, op_r(x2, x3, x1)))}$

$=$ by Lemma 2052 LR with $\{x4 \leftarrow op_r(x2, x3, x1), x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x1\}$

$\overbrace{op_l(x1, rd(op_r(x2, x3, x1), x2), x1)}$

$=$ by Lemma 2035 RL with $\{x3 \leftarrow rd(op_r(x2, x3, x1), x2), x2 \leftarrow x1, x1 \leftarrow x1\}$

$\overbrace{op_l(\underbrace{x1}, x1, rd(x1, rd(op_r(x2, x3, x1), x2)))}$

$=$ by Lemma 2280 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_l(\overbrace{op_t(x1, rd(x1, rd(op_r(x2, x3, x1), x2)))}, x1, rd(x1, rd(op_r(x2, x3, x1), x2)))}$

$=$ by Lemma 421 RL with $\{x2 \leftarrow rd(x1, rd(op_r(x2, x3, x1), x2)), x1 \leftarrow x1\}$

$\overbrace{op_r(x1, rd(x1, rd(op_r(x2, x3, x1), x2)), \underbrace{i(rd(x1, rd(op_r(x2, x3, x1), x2)))})$

$=$ by Lemma 10 LR with $\{x2 \leftarrow rd(op_r(x2, x3, x1), x2), x1 \leftarrow x1\}$

$\overbrace{op_r(x1, rd(x1, rd(op_r(x2, x3, x1), x2)), \overbrace{rd(rd(op_r(x2, x3, x1), x2), x1)})$

$=$ by Lemma 857 RL with $\{x2 \leftarrow rd(op_r(x2, x3, x1), x2), x1 \leftarrow x1\}$

$\overbrace{op_t(x1, \underbrace{i(rd(op_r(x2, x3, x1), x2))})$

$=$ by Lemma 10 LR with $\{x2 \leftarrow x2, x1 \leftarrow op_r(x2, x3, x1)\}$

$op_t(x1, \overbrace{rd(x2, op_r(x2, x3, x1))})$

Lemma 2286: $op_t(rd(op_r(x1, x2, x3), x1), i(x3)) = rd(rd(op_t(x3, rd(x1, op_r(x1, x2, x3))), rd(x1, op_r(x1, x2, x3))), op_t(x3, rd(x1, o$

$\overbrace{op_t(rd(op_r(x1, x2, x3), x1), i(x3))}$

$=$ by Lemma 46 RL with $\{x3 \leftarrow x3, x2 \leftarrow op_r(x1, x2, x3), x1 \leftarrow x1\}$

$\overbrace{i(\overbrace{op_t(rd(x1, op_r(x1, x2, x3)), x3)})}$

$=$ by Lemma 1619 LR with $\{x2 \leftarrow x3, x1 \leftarrow rd(x1, op_r(x1, x2, x3))\}$

$\overbrace{rd(rd(op_t(x3, rd(x1, op_r(x1, x2, x3))), rd(x1, op_r(x1, x2, x3))), \underbrace{op_l(x3, x3, rd(x1, op_r(x1, x2, x3)))})$

$=$ by Lemma 2285 LR with $\{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x3\}$

$rd(rd(op_t(x3, rd(x1, op_r(x1, x2, x3))), rd(x1, op_r(x1, x2, x3))), \overbrace{op_t(x3, rd(x1, op_r(x1, x2, x3)))})$

Lemma 2287: $op_t(rd(op_r(x1, x2, x3), x1), i(x3)) = mult(x3, rd(i(rd(x1, op_r(x1, x2, x3))), op_t(x3, rd(x1, op_r(x1, x2, x3)))))$

$\overbrace{op_t(rd(op_r(x1, x2, x3), x1), i(x3))}$

$=$ by Lemma 2286 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{rd(rd(op_t(x3, rd(x1, op_r(x1, x2, x3))), rd(x1, op_r(x1, x2, x3))), op_t(x3, rd(x1, op_r(x1, x2, x3))))}$

$=$ by Lemma 2022 LR with $\{x3 \leftarrow rd(x1, op_r(x1, x2, x3)), x2 \leftarrow rd(x1, op_r(x1, x2, x3)), x1 \leftarrow x3\}$

$\overbrace{mult(mult(x3, i(rd(x1, op_r(x1, x2, x3)))), i(op_t(x3, rd(x1, op_r(x1, x2, x3)))))}$

$=$ by Lemma 1253 LR with $\{x3 \leftarrow rd(x1, op_r(x1, x2, x3)), x2 \leftarrow i(rd(x1, op_r(x1, x2, x3))), x1 \leftarrow x3\}$

$\overbrace{mult(x3, rd(i(rd(x1, op_r(x1, x2, x3))), op_t(x3, rd(x1, op_r(x1, x2, x3)))))}$

Lemma 2288: $op_t(rd(op_r(x1,x2,x3),x1),i(x3)) = op_r(rd(op_r(x1,x2,x3),x1),x3,x3)$

$\overbrace{op_t(rd(op_r(x1,x2,x3),x1),i(x3))}$

$=$ by Lemma 2287 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{mult(x3,rd(i(rd(x1,op_r(x1,x2,x3))),op_t(x3,rd(x1,op_r(x1,x2,x3)))))}$

$=$ by Lemma 913 LR with $\{x2 \leftarrow rd(x1,op_r(x1,x2,x3)), x1 \leftarrow x3\}$

$\overbrace{op_r(\underbrace{i(rd(x1,op_r(x1,x2,x3)))},x3,x3)}$

$=$ by Lemma 10 LR with $\{x2 \leftarrow op_r(x1,x2,x3), x1 \leftarrow x1\}$

$op_r(\overbrace{rd(op_r(x1,x2,x3),x1)},x3,x3)$

Lemma 2289: $x1 = op_l(x1,rd(x3,rd(x3,rd(op_r(x2,x3,x1),x2))),x1)$

$\overbrace{x1}$

$=$ by Lemma 1394 LR with $\{x3 \leftarrow mult(i(op_r(x2,x3,x1)),mult(x2,x3)), x2 \leftarrow op_l(x1,mult(x2,x3),i(op_r(x2,x3,x1))), x1 \leftarrow x$

$\overbrace{op_l(x1,op_l(x1,mult(x2,x3),i(op_r(x2,x3,x1))),\underbrace{i(rd(op_l(x1,mult(x2,x3),i(op_r(x2,x3,x1))),\overbrace{rd(mult(i(op_r(x2,x3,x1)),m}}$

$=$ by Lemma 10 LR with $\{x2 \leftarrow rd(mult(i(op_r(x2,x3,x1)),mult(x2,x3)),op_t(mult(i(op_r(x2,x3,x1)),mult(x2,x3)),x1)), x1 \leftarrow$

$op_l(x1,op_l(x1,mult(x2,x3),i(op_r(x2,x3,x1))),\overbrace{rd(rd(mult(i(op_r(x2,x3,x1)),mult(x2,x3)),op_t(mult(i(op_r(x2,x3,x1)),m}$

$=$ by Lemma 1236 RL with $\{x3 \leftarrow x1, x2 \leftarrow op_l(x1,mult(x2,x3),i(op_r(x2,x3,x1))), x1 \leftarrow mult(i(op_r(x2,x3,x1)),mult(x2,x3$

$op_l(x1,op_l(x1,mult(x2,x3),i(op_r(x2,x3,x1))),\overbrace{rd(mult(i(op_r(x2,x3,x1)),mult(x2,x3)),\underbrace{mult(op_l(x1,mult(x2,x3),i(op_r$

$=$ by Lemma 2142 RL with $\{x3 \leftarrow x1, x2 \leftarrow mult(x2,x3), x1 \leftarrow i(op_r(x2,x3,x1))\}$

$op_l(x1,op_l(x1,mult(x2,x3),i(op_r(x2,x3,x1))),rd(mult(i(op_r(x2,x3,x1)),mult(x2,x3)),\overbrace{mult(mult(i(op_r(x2,x3,x1)),m}$

$=$ by Lemma 132 LR with $\{x3 \leftarrow x1, x2 \leftarrow mult(x2,x3), x1 \leftarrow i(op_r(x2,x3,x1))\}$

$\underbrace{op_l(x1,op_l(x1,mult(x2,x3),i(op_r(x2,x3,x1))),rd(mult(i(op_r(x2,x3,x1)),mult(x2,x3)),\overbrace{mult(i(op_r(x2,x3,x1)),mult(m}}$

$=$ by Lemma 1820 LR with $\{x4 \leftarrow rd(mult(i(op_r(x2,x3,x1)),mult(x2,x3)),mult(i(op_r(x2,x3,x1)),mult(mult(x2,x3),x1))),$

$\overbrace{op_l(x1,x1,rd(mult(i(op_r(x2,x3,x1)),mult(x2,x3)),mult(i(op_r(x2,x3,x1)),\underbrace{mult(mult(x2,x3),x1)})))}$

$=$ by Lemma 91 RL with $\{x3 \leftarrow x1, x2 \leftarrow x3, x1 \leftarrow x2\}$

$op_l(x1,x1,rd(mult(i(op_r(x2,x3,x1)),mult(x2,x3)),\underbrace{mult(i(op_r(x2,x3,x1)),\overbrace{mult(op_r(x2,x3,x1),mult(x3,x1))})}))$

$=$ by Axiom 2 RL with $\{x2 \leftarrow op_r(x2,x3,x1), x1 \leftarrow mult(x3,x1)\}$

$\underbrace{op_l(x1,x1,rd(mult(i(op_r(x2,x3,x1)),mult(x2,x3)),\overbrace{mult(x3,x1)}))}$

$=$ by Lemma 1848 LR with $\{x3 \leftarrow x3, x2 \leftarrow mult(i(op_r(x2,x3,x1)),mult(x2,x3)), x1 \leftarrow x1\}$

$\overbrace{op_l(x1,rd(x3,\underbrace{mult(i(op_r(x2,x3,x1)),mult(x2,x3))}),x1)}$

$=$ by Lemma 1979 LR with $\{x4 \leftarrow x3, x3 \leftarrow x1, x2 \leftarrow x3, x1 \leftarrow x2\}$

$op_l(x1,rd(x3,\overbrace{mult(rd(x2,op_r(x2,x3,x1)),x3)}),x1)$

$=$ by Lemma 2270 LR with $\{x3 \leftarrow x1, x2 \leftarrow x3, x1 \leftarrow x2\}$

$op_l(x1,rd(x3,\overbrace{rd(x3,rd(op_r(x2,x3,x1),x2))}),x1)$

Lemma 2290: $x1 = op_l(x1, rd(op_r(x2, x3, x1), x2), x1)$

$$
\begin{aligned}
& \overbrace{x1} \\
= \quad & \text{by Lemma 2289 LR with } \{x2 \leftarrow x2, x3 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, \underbrace{rd(x3, rd(x3, rd(op_r(x2, x3, x1), x2)))}, x1)} \\
= \quad & \text{by Lemma 353 LR with } \{x2 \leftarrow rd(op_r(x2, x3, x1), x2), x1 \leftarrow x3\} \\
& op_l(x1, \overbrace{op_r(rd(op_r(x2, x3, x1), x2), x3, x3)}, x1) \\
= \quad & \text{by Lemma 2272 LR with } \{x3 \leftarrow x1, x2 \leftarrow x3, x1 \leftarrow x2\} \\
& op_l(x1, \overbrace{rd(op_r(x2, x3, x1), x2)}, x1)
\end{aligned}
$$

Lemma 2291: $x1 = op_t(x1, rd(x2, op_r(x2, x3, x1)))$

$$
\begin{aligned}
& \overbrace{x1} \\
= \quad & \text{by Lemma 2290 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, rd(op_r(x2, x3, x1), x2), x1)}} \\
= \quad & \text{by Lemma 2052 RL with } \{x4 \leftarrow op_r(x2, x3, x1), x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, x1, rd(x2, op_r(x2, x3, x1)))}} \\
= \quad & \text{by Lemma 2285 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, rd(x2, op_r(x2, x3, x1)))}
\end{aligned}
$$

Lemma 2292: $op_t(x1, rd(op_r(x2, x3, x1), x2)) = x1$

$$
\begin{aligned}
& \underbrace{op_t(x1, rd(op_r(x2, x3, x1), x2))} \\
= \quad & \text{by Lemma 1730 RL with } \{x3 \leftarrow x2, x2 \leftarrow op_r(x2, x3, x1), x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_t(x1, mult(op_r(x2, x3, x1), i(x2)))}} \\
= \quad & \text{by Lemma 1729 RL with } \{x3 \leftarrow op_r(x2, x3, x1), x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, rd(i(x2), \underbrace{i(op_r(x2, x3, x1))}))} \\
= \quad & \text{by Lemma 654 RL with } \{x3 \leftarrow x1, x2 \leftarrow x3, x1 \leftarrow x2\} \\
& \underbrace{op_t(x1, rd(i(x2), \overbrace{op_r(i(x2), mult(x2, x3), x1)}))} \\
= \quad & \text{by Lemma 2291 RL with } \{x3 \leftarrow mult(x2, x3), x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& \overbrace{x1}
\end{aligned}
$$

Lemma 2293: $op_l(x1, x1, rd(op_r(x2, x3, x1), x2)) = x1$

$$
\begin{aligned}
& \underbrace{op_l(x1, x1, rd(op_r(x2, x3, x1), x2))} \\
= \quad & \text{by Lemma 2052 LR with } \{x4 \leftarrow x2, x3 \leftarrow op_r(x2, x3, x1), x2 \leftarrow x1, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, rd(x2, op_r(x2, x3, x1)), x1)}} \\
= \quad & \text{by Lemma 2284 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_t(x1, rd(op_r(x2, x3, x1), x2))}} \\
= \quad & \text{by Lemma 2292 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{x1}
\end{aligned}
$$

Lemma 2294: $op_t(rd(op_r(x1,x2,x3),x1),x3) = i(rd(x1,op_r(x1,x2,x3)))$

$$
\begin{aligned}
& \underbrace{op_t(rd(op_r(x1,x2,x3),x1),x3)} \\
= & \quad \text{by Lemma 2283 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_r(rd(op_r(x1,x2,x3),x1),x3,x3)}} \\
= & \quad \text{by Lemma 2288 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_t(rd(op_r(x1,x2,x3),x1),i(x3))}} \\
= & \quad \text{by Lemma 46 RL with } \{x3 \leftarrow x3, x2 \leftarrow op_r(x1,x2,x3), x1 \leftarrow x1\} \\
& \overbrace{\underbrace{i(op_t(rd(x1,op_r(x1,x2,x3)),x3))}} \\
= & \quad \text{by Lemma 373 RL with } \{x2 \leftarrow rd(x1,op_r(x1,x2,x3)), x1 \leftarrow x3\} \\
& \overbrace{rd(\underbrace{op_t(x3,rd(x1,op_r(x1,x2,x3)))}, mult(rd(x1,op_r(x1,x2,x3)),x3))} \\
= & \quad \text{by Lemma 2291 RL with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x3\} \\
& \underbrace{rd(\overbrace{x3}, mult(rd(x1,op_r(x1,x2,x3)),x3))} \\
= & \quad \text{by Lemma 9 LR with } \{x2 \leftarrow rd(x1,op_r(x1,x2,x3)), x1 \leftarrow x3\} \\
& \overbrace{i(rd(x1,op_r(x1,x2,x3)))}
\end{aligned}
$$

Lemma 2295: $rd(mult(x1,x2),op_r(x2,x3,x1)) = rd(x1,rd(op_r(x2,x3,x1),x2))$

$$
\begin{aligned}
& rd(mult(\underbrace{x1}, x2), op_r(x2,x3,x1)) \\
= & \quad \text{by Lemma 2280 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& rd(mult(\overbrace{op_t(x1,rd(x1,rd(op_r(x2,x3,x1),x2)))}, x2), op_r(x2,x3,x1)) \\
= & \quad \text{by Lemma 1753 RL with } \{x4 \leftarrow op_r(x2,x3,x1), x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x1\} \\
& \underbrace{rd(mult(\overbrace{op_t(x1,mult(x1,rd(x2,op_r(x2,x3,x1))))}, x2), op_r(x2,x3,x1))} \\
= & \quad \text{by Lemma 2046 RL with } \{x4 \leftarrow x1, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow op_t(x1,mult(x1,rd(x2,op_r(x2,x3,x1))))\} \\
& \overbrace{\underbrace{mult(op_t(x1,mult(x1,rd(x2,op_r(x2,x3,x1)))), rd(x2,op_r(x2,x3,x1)))}} \\
= & \quad \text{by Lemma 1978 RL with } \{x2 \leftarrow x1, x1 \leftarrow rd(x2,op_r(x2,x3,x1))\} \\
& \overbrace{rd(rd(x2,op_r(x2,x3,x1)), i(op_t(\underbrace{op_t(x1,rd(x2,op_r(x2,x3,x1)))}, rd(x2,op_r(x2,x3,x1)))))} \\
= & \quad \text{by Lemma 2291 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{rd(rd(x2,op_r(x2,x3,x1)), i(op_t(\overbrace{x1}, rd(x2,op_r(x2,x3,x1)))))} \\
= & \quad \text{by Lemma 2190 RL with } \{x4 \leftarrow x1, x3 \leftarrow x3, x2 \leftarrow i(op_t(x1,rd(x2,op_r(x2,x3,x1)))), x1 \leftarrow x2\} \\
& \overbrace{\underbrace{rd(x2, mult(i(op_t(x1,rd(x2,op_r(x2,x3,x1))))), op_r(x2,x3,x1)))}} \\
= & \quad \text{by Lemma 2245 RL with } \{x4 \leftarrow x1, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow op_t(x1,rd(x2,op_r(x2,x3,x1)))\} \\
& \overbrace{rd(\underbrace{op_t(x1,rd(x2,op_r(x2,x3,x1)))}, rd(op_r(x2,x3,x1),x2))} \\
= & \quad \text{by Lemma 2291 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& rd(\overbrace{x1}, rd(op_r(x2,x3,x1),x2))
\end{aligned}
$$

Lemma 2296: $rd(mult(x1, x2), op_r(x2, x3, x1)) = mult(rd(x2, op_r(x2, x3, x1)), x1)$

$\underbrace{rd(mult(x1, x2), op_r(x2, x3, x1))}$

$=$ by Lemma 2046 RL with $\{x4 \leftarrow x1, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{mult(x1, rd(x2, op_r(x2, x3, x1)))}$

$=$ by Lemma 1980 RL with $\{x2 \leftarrow rd(x2, op_r(x2, x3, x1)), x1 \leftarrow x1\}$

$\overbrace{op_r(mult(\underbrace{op_t(x1, rd(x2, op_r(x2, x3, x1)))}, rd(x2, op_r(x2, x3, x1))), x1, rd(i(x1), rd(x2, op_r(x2, x3, x1))))}$

$=$ by Lemma 2291 RL with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{op_r(mult(\overbrace{x1}, rd(x2, op_r(x2, x3, x1))), x1, rd(i(x1), rd(x2, op_r(x2, x3, x1))))}$

$=$ by Lemma 1990 RL with $\{x3 \leftarrow x1, x2 \leftarrow rd(x2, op_r(x2, x3, x1)), x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_l(mult(x1, rd(x2, op_r(x2, x3, x1))), x1, mult(x1, rd(x2, op_r(x2, x3, x1))))}}$

$=$ by Lemma 2032 LR with $\{x3 \leftarrow rd(x2, op_r(x2, x3, x1)), x2 \leftarrow x1, x1 \leftarrow mult(x1, rd(x2, op_r(x2, x3, x1)))\}$

$\overbrace{\underbrace{op_l(mult(x1, rd(x2, op_r(x2, x3, x1))), x1, rd(x2, op_r(x2, x3, x1)))}}$

$=$ by Lemma 435 LR with $\{x2 \leftarrow rd(x2, op_r(x2, x3, x1)), x1 \leftarrow x1\}$

$\overbrace{mult(rd(x2, op_r(x2, x3, x1)), x1)}$

Lemma 2297: $mult(i(op_t(x1, mult(x1, rd(x2, op_r(x2, x3, x1))))), rd(op_r(x2, x3, x1), x2)) = rd(i(rd(x2, op_r(x2, x3, x1))), op_t(x1, r$

$\underbrace{mult(i(op_t(x1, mult(x1, rd(x2, op_r(x2, x3, x1))))), rd(op_r(x2, x3, x1), x2))}$

$=$ by Lemma 45 RL with $\{x3 \leftarrow op_r(x2, x3, x1), x2 \leftarrow x2, x1 \leftarrow op_t(x1, mult(x1, rd(x2, op_r(x2, x3, x1))))\}$

$\overbrace{i(mult(\underbrace{op_t(x1, mult(x1, rd(x2, op_r(x2, x3, x1))))}, rd(x2, op_r(x2, x3, x1))))}$

$=$ by Lemma 251 RL with $\{x2 \leftarrow rd(x2, op_r(x2, x3, x1)), x1 \leftarrow x1\}$

$i(mult(\overbrace{op_l(op_t(x1, rd(x2, op_r(x2, x3, x1))), rd(x2, op_r(x2, x3, x1)), x1)}, rd(x2, op_r(x2, x3, x1))))$

$=$ by Lemma 672 RL with $\{x3 \leftarrow rd(x2, op_r(x2, x3, x1)), x2 \leftarrow rd(x2, op_r(x2, x3, x1)), x1 \leftarrow x1\}$

$i(mult(\overbrace{op_l(op_t(x1, rd(x2, op_r(x2, x3, x1))), x1, i(rd(x2, op_r(x2, x3, x1))))}, rd(x2, op_r(x2, x3, x1))))$

$=$ by Axiom 15 RL with $\{x4 \leftarrow rd(x2, op_r(x2, x3, x1)), x3 \leftarrow i(rd(x2, op_r(x2, x3, x1))), x2 \leftarrow x1, x1 \leftarrow x1\}$

$\underbrace{i(mult(\overbrace{op_t(op_l(x1, x1, i(rd(x2, op_r(x2, x3, x1))))), rd(x2, op_r(x2, x3, x1)))}, rd(x2, op_r(x2, x3, x1))))}$

$=$ by Lemma 1997 RL with $\{x2 \leftarrow rd(x2, op_r(x2, x3, x1)), x1 \leftarrow x1\}$

$\overbrace{op_l(rd(i(op_t(x1, rd(x2, op_r(x2, x3, x1))))), rd(x2, op_r(x2, x3, x1))), rd(x2, op_r(x2, x3, x1)), \underbrace{i(i(x1))})}$

$=$ by Lemma 3 LR with $\{x1 \leftarrow x1\}$

$\underbrace{op_l(rd(i(op_t(x1, rd(x2, op_r(x2, x3, x1))))), rd(x2, op_r(x2, x3, x1))), rd(x2, op_r(x2, x3, x1)), \overbrace{x1})}$

$=$ by Lemma 1332 RL with $\{x3 \leftarrow rd(x2, op_r(x2, x3, x1)), x2 \leftarrow x1, x1 \leftarrow rd(i(op_t(x1, rd(x2, op_r(x2, x3, x1))))), rd(x2, op_r(x2, x$

$\underbrace{\overbrace{op_l(rd(i(op_t(x1, rd(x2, op_r(x2, x3, x1))))), rd(x2, op_r(x2, x3, x1))), op_t(x1, rd(x2, op_r(x2, x3, x1))), i(rd(x2, op_r(x2, x3, x1)))}$

$=$ by Lemma 930 LR with $\{x2 \leftarrow rd(x2, op_r(x2, x3, x1)), x1 \leftarrow op_t(x1, rd(x2, op_r(x2, x3, x1)))\}$

$\overbrace{rd(i(rd(x2, op_r(x2, x3, x1))), op_t(\underbrace{op_t(x1, rd(x2, op_r(x2, x3, x1)))}, rd(x2, op_r(x2, x3, x1))))}$

$=$ by Lemma 2291 RL with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$rd(i(rd(x2, op_r(x2, x3, x1))), op_t(\overbrace{x1}, rd(x2, op_r(x2, x3, x1))))$

Lemma 2298: $rd(op_r(x2,x3,x1), mult(op_t(x1, rd(x1, rd(op_r(x2,x3,x1),x2))),x2)) = rd(i(rd(x2,op_r(x2,x3,x1))), op_t(x1, rd(x2$

$rd(op_r(x2,x3,x1), mult(\overbrace{op_t(x1, rd(x1, rd(op_r(x2,x3,x1),x2)))}, x2))$

$=$ by Lemma 1753 RL with $\{x4 \leftarrow op_r(x2,x3,x1), x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x1\}$

$\overbrace{rd(op_r(x2,x3,x1), mult(\overbrace{op_t(x1, mult(x1, rd(x2, op_r(x2,x3,x1))))}, x2))}$

$=$ by Lemma 2187 RL with $\{x4 \leftarrow x1, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow op_t(x1, mult(x1, rd(x2, op_r(x2,x3,x1))))\}$

$\overbrace{mult(i(\overbrace{op_t(x1, mult(x1, rd(x2, op_r(x2,x3,x1))))}), rd(op_r(x2,x3,x1),x2))}$

$=$ by Lemma 2297 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{rd(i(rd(x2,op_r(x2,x3,x1))), op_t(x1, rd(x2,op_r(x2,x3,x1))))}$

Lemma 2299: $rd(rd(op_r(x1,x2,x3),x1),x3) = rd(op_r(x1,x2,x3), mult(x3,x1))$

$rd(rd(op_r(x1,x2,x3),x1), \underbrace{x3})$

$=$ by Lemma 2291 LR with $\{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x3\}$

$rd(\underbrace{rd(op_r(x1,x2,x3),x1)}, \overbrace{op_t(x3, rd(x1, op_r(x1,x2,x3)))})$

$=$ by Lemma 10 RL with $\{x2 \leftarrow op_r(x1,x2,x3), x1 \leftarrow x1\}$

$\underbrace{rd(\overbrace{i(\overbrace{rd(x1, op_r(x1,x2,x3))}), op_t(x3, rd(x1, op_r(x1,x2,x3))))}}$

$=$ by Lemma 2298 RL with $\{x1 \leftarrow x3, x3 \leftarrow x2, x2 \leftarrow x1\}$

$\overbrace{rd(op_r(x1,x2,x3), mult(\underbrace{op_t(x3, rd(x3, rd(op_r(x1,x2,x3),x1)))}, x1))}$

$=$ by Lemma 2280 RL with $\{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x3\}$

$rd(op_r(x1,x2,x3), mult(\overbrace{x3}, x1))$

Lemma 2300: $mult(op_t(x1, rd(x1, rd(x2, op_r(x2,x3,x1)))), rd(op_r(x2,x3,x1),x2)) = rd(rd(op_r(x2,x3,x1),x2), i(op_t(x1, rd(op_r$

$mult(\underbrace{op_t(x1, rd(x1, rd(x2, op_r(x2,x3,x1))))}, rd(op_r(x2,x3,x1),x2))$

$=$ by Lemma 1753 RL with $\{x4 \leftarrow x2, x3 \leftarrow op_r(x2,x3,x1), x2 \leftarrow x1, x1 \leftarrow x1\}$

$\underbrace{mult(\overbrace{op_t(x1, mult(x1, rd(op_r(x2,x3,x1),x2)))}, rd(op_r(x2,x3,x1),x2))}$

$=$ by Lemma 1978 RL with $\{x2 \leftarrow x1, x1 \leftarrow rd(op_r(x2,x3,x1),x2)\}$

$\overbrace{rd(rd(op_r(x2,x3,x1),x2), i(op_t(\underbrace{op_t(x1, rd(op_r(x2,x3,x1),x2))}, rd(op_r(x2,x3,x1),x2))))}$

$=$ by Lemma 2292 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$rd(rd(op_r(x2,x3,x1),x2), i(op_t(\overbrace{x1}, rd(op_r(x2,x3,x1),x2))))$

Lemma 2301: $mult(x1, rd(op_r(x2,x3,x1),x2)) = rd(op_t(x1, rd(op_r(x2,x3,x1),x2)), rd(x2, op_r(x2,x3,x1)))$

$mult(\underbrace{x1}, rd(op_r(x2,x3,x1),x2))$

$=$ by Lemma 2279 RL with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{mult(\overbrace{op_t(x1, rd(x1, rd(x2, op_r(x2,x3,x1))))}, rd(op_r(x2,x3,x1),x2))}$

$=$ by Lemma 2300 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{\overbrace{rd(rd(op_r(x2,x3,x1),x2), i(op_t(x1, rd(op_r(x2,x3,x1),x2))))}}$

$=$ by Lemma 39 LR with $\{x3 \leftarrow op_t(x1, rd(op_r(x2,x3,x1),x2)), x2 \leftarrow x2, x1 \leftarrow op_r(x2,x3,x1)\}$

$\overbrace{rd(op_t(x1, rd(op_r(x2,x3,x1),x2)), rd(x2, op_r(x2,x3,x1)))}$

Lemma 2302: $mult(x1, rd(op_r(x2, x3, x1), x2)) = rd(x1, rd(x2, op_r(x2, x3, x1)))$

$$
\begin{aligned}
& \underbrace{mult(x1, rd(op_r(x2, x3, x1), x2))} \\
= \quad & \text{by Lemma 2301 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{rd(\underbrace{op_t(x1, rd(op_r(x2, x3, x1), x2))}, rd(x2, op_r(x2, x3, x1)))} \\
= \quad & \text{by Lemma 2292 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& rd(\overbrace{x1}, rd(x2, op_r(x2, x3, x1)))
\end{aligned}
$$

Lemma 2303: $mult(x1, rd(op_r(x2, x3, x1), x2)) = op_l(mult(x1, rd(op_r(x2, x3, x1), x2)), x1, mult(x1, rd(op_r(x2, x3, x1), x2)))$

$$
\begin{aligned}
& \underbrace{mult(x1, rd(op_r(x2, x3, x1), x2))} \\
= \quad & \text{by Lemma 1980 RL with } \{x2 \leftarrow rd(op_r(x2, x3, x1), x2), x1 \leftarrow x1\} \\
& \overbrace{op_r(mult(\underbrace{op_t(x1, rd(op_r(x2, x3, x1), x2))}, rd(op_r(x2, x3, x1), x2)), x1, rd(i(x1), rd(op_r(x2, x3, x1), x2)))} \\
= \quad & \text{by Lemma 2292 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{op_r(mult(\overbrace{x1}, rd(op_r(x2, x3, x1), x2)), x1, rd(i(x1), rd(op_r(x2, x3, x1), x2)))} \\
= \quad & \text{by Lemma 1990 RL with } \{x3 \leftarrow x1, x2 \leftarrow rd(op_r(x2, x3, x1), x2), x1 \leftarrow x1\} \\
& \overbrace{op_l(mult(x1, rd(op_r(x2, x3, x1), x2)), x1, mult(x1, rd(op_r(x2, x3, x1), x2)))}
\end{aligned}
$$

Lemma 2304: $mult(x1, rd(op_r(x2, x3, x1), x2)) = mult(rd(op_r(x2, x3, x1), x2), x1)$

$$
\begin{aligned}
& \underbrace{mult(x1, rd(op_r(x2, x3, x1), x2))} \\
= \quad & \text{by Lemma 2303 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(mult(x1, rd(op_r(x2, x3, x1), x2)), x1, mult(x1, rd(op_r(x2, x3, x1), x2)))}} \\
= \quad & \text{by Lemma 2032 LR with } \{x3 \leftarrow rd(op_r(x2, x3, x1), x2), x2 \leftarrow x1, x1 \leftarrow mult(x1, rd(op_r(x2, x3, x1), x2))\} \\
& \underbrace{\overbrace{op_l(mult(x1, rd(op_r(x2, x3, x1), x2)), x1, rd(op_r(x2, x3, x1), x2))}} \\
= \quad & \text{by Lemma 435 LR with } \{x2 \leftarrow rd(op_r(x2, x3, x1), x2), x1 \leftarrow x1\} \\
& \overbrace{mult(rd(op_r(x2, x3, x1), x2), x1)}
\end{aligned}
$$

Lemma 2305: $mult(rd(op_r(x1, x2, x3), x1), x3) = rd(x3, rd(x1, op_r(x1, x2, x3)))$

$$
\begin{aligned}
& \underbrace{mult(rd(op_r(x1, x2, x3), x1), x3)} \\
= \quad & \text{by Lemma 2304 RL with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x3\} \\
& \overbrace{\underbrace{mult(x3, rd(op_r(x1, x2, x3), x1))}} \\
= \quad & \text{by Lemma 2302 LR with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x3\} \\
& \overbrace{rd(x3, rd(x1, op_r(x1, x2, x3)))}
\end{aligned}
$$

Lemma 2306: $op_r(rd(op_r(x1,x2,x3),x1),x3,x3) = rd(op_r(x1,x2,x3),x1)$

$$
\begin{aligned}
& \overbrace{op_r(rd(op_r(x1,x2,x3),x1),x3,x3)} \\
= & \quad \text{by Lemma 2283 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\overbrace{op_t(rd(op_r(x1,x2,x3),x1),x3)}} \\
= & \quad \text{by Lemma 2294 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\overbrace{i(rd(x1,op_r(x1,x2,x3)))}} \\
= & \quad \text{by Lemma 10 LR with } \{x2 \leftarrow op_r(x1,x2,x3), x1 \leftarrow x1\} \\
& \overbrace{rd(op_r(x1,x2,x3),x1)}
\end{aligned}
$$

Lemma 2307: $mult(rd(x1,op_r(x1,x2,x3)),i(x3)) = rd(i(rd(op_r(x1,x2,x3),x1)),x3)$

$$
\begin{aligned}
& \overbrace{mult(rd(x1,op_r(x1,x2,x3)),i(x3))} \\
= & \quad \text{by Lemma 44 RL with } \{x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow op_r(x1,x2,x3)\} \\
& \overbrace{\overbrace{i(mult(rd(op_r(x1,x2,x3),x1),x3))}} \\
= & \quad \text{by Lemma 447 RL with } \{x2 \leftarrow x3, x1 \leftarrow rd(op_r(x1,x2,x3),x1)\} \\
& \overbrace{rd(i(rd(op_r(x1,x2,x3),x1)),\overbrace{op_l(x3,x3,rd(op_r(x1,x2,x3),x1))})} \\
= & \quad \text{by Lemma 2293 LR with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x3\} \\
& rd(i(rd(op_r(x1,x2,x3),x1)),\overbrace{x3}) \\
\end{aligned}
$$

Lemma 2308: $x1 = op_l(x1,x1,rd(x2,mult(mult(x3,x1),mult(i(x3),x2))))$

$$
\begin{aligned}
& \overbrace{x1} \\
= & \quad \text{by Lemma 2273 LR with } \{x3 \leftarrow mult(i(x3),x2), x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{\overbrace{op_l(x1,rd(mult(mult(x3,x1),mult(i(x3),x2)),mult(x3,mult(i(x3),x2))),x1)}} \\
= & \quad \text{by Lemma 2052 RL with } \{x4 \leftarrow mult(mult(x3,x1),mult(i(x3),x2)), x3 \leftarrow mult(x3,mult(i(x3),x2)), x2 \leftarrow x1, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1,x1,rd(\underbrace{mult(x3,mult(i(x3),x2))},mult(mult(x3,x1),mult(i(x3),x2))))} \\
= & \quad \text{by Lemma 12 LR with } \{x2 \leftarrow x2, x1 \leftarrow x3\} \\
& op_l(x1,x1,rd(\overbrace{x2},mult(mult(x3,x1),mult(i(x3),x2)))) \\
\end{aligned}
$$

Lemma 2309: $x1 = op_t(x1, rd(x2, op_l(x2, x1, x3)))$

$\overbrace{x1}$

$=$ by Lemma 2308 LR with $\{x3 \leftarrow x3, x2 \leftarrow mult(i(x1), x2), x1 \leftarrow x1\}$

$\overbrace{op_l(x1, x1, rd(mult(i(x1), x2), \underbrace{mult(mult(x3, x1), mult(i(x3), mult(i(x1), x2)))}))}$

$=$ by Lemma 1086 LR with $\{x3 \leftarrow mult(i(x1), x2), x2 \leftarrow x1, x1 \leftarrow x3\}$

$op_l(x1, x1, rd(mult(i(x1), x2), \overbrace{mult(x3, \underbrace{mult(rd(x1, x3), mult(i(x1), x2))})}))$

$=$ by Lemma 1418 LR with $\{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x1\}$

$op_l(x1, x1, rd(mult(i(x1), x2), mult(x3, \overbrace{mult(x1, mult(\underbrace{mult(i(x1), i(x3))}, x2))})))$

$=$ by Axiom 11 LR with $\{x2 \leftarrow x3, x1 \leftarrow x1\}$

$\underbrace{op_l(x1, x1, rd(mult(i(x1), x2), mult(x3, mult(x1, mult(\overbrace{i(mult(x1, x3))}, x2)))))}$

$=$ by Lemma 1872 LR with $\{x2 \leftarrow mult(x3, mult(x1, mult(i(mult(x1, x3)), x2))), x3 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_l(x1, x1, rd(x2, \underbrace{mult(x3, mult(x1, mult(i(mult(x1, x3)), x2)))}))}$

$=$ by Lemma 1620 LR with $\{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x3\}$

$\underbrace{op_l(x1, x1, rd(x2, \overbrace{op_l(x2, x1, x3)}))}$

$=$ by Lemma 2052 LR with $\{x4 \leftarrow op_l(x2, x1, x3), x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_l(x1, rd(op_l(x2, x1, x3), x2), x1)}}$

$=$ by Lemma 2035 RL with $\{x3 \leftarrow rd(op_l(x2, x1, x3), x2), x2 \leftarrow x1, x1 \leftarrow x1\}$

$\overbrace{op_l(\underbrace{x1}, x1, rd(x1, \overbrace{rd(op_l(x2, x1, x3), x2)}))}$

$=$ by Lemma 2093 RL with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$op_l(\overbrace{\underbrace{op_t(x1, mult(x1, rd(x2, op_l(x2, x1, x3))))}}, x1, rd(x1, rd(op_l(x2, x1, x3), x2)))$

$=$ by Lemma 1753 LR with $\{x4 \leftarrow op_l(x2, x1, x3), x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x1\}$

$\underbrace{op_l(\overbrace{op_t(x1, rd(x1, rd(op_l(x2, x1, x3), x2)))}, x1, rd(x1, rd(op_l(x2, x1, x3), x2)))}$

$=$ by Lemma 421 RL with $\{x2 \leftarrow rd(x1, rd(op_l(x2, x1, x3), x2)), x1 \leftarrow x1\}$

$\overbrace{op_r(x1, rd(x1, rd(op_l(x2, x1, x3), x2)), \underbrace{i(rd(x1, rd(op_l(x2, x1, x3), x2)))})}$

$=$ by Lemma 10 LR with $\{x2 \leftarrow rd(op_l(x2, x1, x3), x2), x1 \leftarrow x1\}$

$\underbrace{op_r(x1, rd(x1, rd(op_l(x2, x1, x3), x2)), \overbrace{rd(rd(op_l(x2, x1, x3), x2), x1)})}$

$=$ by Lemma 857 RL with $\{x2 \leftarrow rd(op_l(x2, x1, x3), x2), x1 \leftarrow x1\}$

$\overbrace{op_t(x1, \underbrace{i(rd(op_l(x2, x1, x3), x2))})}$

$=$ by Lemma 10 LR with $\{x2 \leftarrow x2, x1 \leftarrow op_l(x2, x1, x3)\}$

$op_t(x1, \overbrace{rd(x2, op_l(x2, x1, x3))})$

Lemma 2310: $op_t(x1, rd(op_l(x2, x3, x1), x2)) = i(i(x1))$

$$
\begin{aligned}
& op_t(x1, rd(\overbrace{op_l(x2, x3, x1)}, x2)) \\
= & \quad \text{by Lemma 2031 RL with } \{x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow x2\} \\
& op_t(x1, \overbrace{rd(\overbrace{op_l(x2, i(x1), x3)}, x2)}) \\
= & \quad \text{by Lemma 10 RL with } \{x2 \leftarrow op_l(x2, i(x1), x3), x1 \leftarrow x2\} \\
& \overbrace{op_t(x1, \overbrace{i(rd(x2, op_l(x2, i(x1), x3)))})} \\
= & \quad \text{by Lemma 18 RL with } \{x2 \leftarrow rd(x2, op_l(x2, i(x1), x3)), x1 \leftarrow x1\} \\
& \overbrace{i(\overbrace{op_t(i(x1), rd(x2, op_l(x2, i(x1), x3)))})} \\
= & \quad \text{by Lemma 2309 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow i(x1)\} \\
& \overbrace{i(\overbrace{i(x1)})}
\end{aligned}
$$

Lemma 2311: $op_t(x1, rd(x2, op_l(x2, x3, x1))) = x1$

$$
\begin{aligned}
& \overbrace{op_t(x1, rd(x2, op_l(x2, x3, x1)))} \\
= & \quad \text{by Lemma 2207 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\overbrace{op_t(x1, rd(op_l(x2, x3, x1), x2))}} \\
= & \quad \text{by Lemma 2310 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\overbrace{i(i(x1))}} \\
= & \quad \text{by Lemma 3 LR with } \{x1 \leftarrow x1\} \\
& \overbrace{x1}
\end{aligned}
$$

Lemma 2312: $mult(x1, op_t(rd(op_l(x2, x1, x3), x2), x1)) = mult(x1, i(rd(x2, op_l(x2, x1, x3))))$

$mult(x1, \underbrace{op_t(rd(op_l(x2, x1, x3), x2), x1)})$

$=$ by Lemma 2202 RL with $\{x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow x2\}$

$mult(x1, \overbrace{op_r(\underbrace{rd(op_l(x2, x1, x3), x2)}, x1, x1)})$

$=$ by Lemma 10 RL with $\{x2 \leftarrow op_l(x2, x1, x3), x1 \leftarrow x2\}$

$mult(x1, \underbrace{op_r(\overbrace{i(rd(x2, op_l(x2, x1, x3)))}, x1, x1)})$

$=$ by Lemma 356 RL with $\{x2 \leftarrow x1, x1 \leftarrow rd(x2, op_l(x2, x1, x3))\}$

$mult(x1, \overbrace{i(op_r(\underbrace{rd(x2, op_l(x2, x1, x3))}, x1, x1))})$

$=$ by Lemma 10 RL with $\{x2 \leftarrow x2, x1 \leftarrow op_l(x2, x1, x3)\}$

$mult(x1, \underbrace{i(op_r(\overbrace{i(rd(op_l(x2, x1, x3), x2))}, x1, x1))})$

$=$ by Lemma 813 LR with $\{x3 \leftarrow x1, x2 \leftarrow x1, x1 \leftarrow rd(op_l(x2, x1, x3), x2)\}$

$mult(x1, \overbrace{op_r(rd(op_l(x2, x1, x3), x2), i(x1), i(x1))})$

$=$ by Lemma 1632 RL with $\{x3 \leftarrow x3, x2 \leftarrow i(x1), x1 \leftarrow rd(op_l(x2, x1, x3), x2)\}$

$mult(x1, \overbrace{op_r(rd(\underbrace{op_l(x2, x1, x3)}, x2), op_t(i(x1), x3), op_t(i(x1), x3))})$

$=$ by Lemma 1793 LR with $\{x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow x2\}$

$mult(x1, \underbrace{op_r(rd(\overbrace{op_l(x2, op_t(i(x1), x3), mult(i(x3), x1))}, x2), op_t(i(x1), x3), op_t(i(x1), x3))})$

$=$ by Lemma 2202 LR with $\{x3 \leftarrow mult(i(x3), x1), x2 \leftarrow op_t(i(x1), x3), x1 \leftarrow x2\}$

$mult(x1, \overbrace{op_t(rd(op_l(x2, op_t(i(x1), x3), mult(i(x3), x1)), x2), op_t(i(x1), x3))})$

$=$ by Lemma 1722 LR with $\{x3 \leftarrow x3, x2 \leftarrow i(x1), x1 \leftarrow rd(op_l(x2, op_t(i(x1), x3), mult(i(x3), x1)), x2)\}$

$mult(x1, \overbrace{op_t(rd(\underbrace{op_l(x2, op_t(i(x1), x3), mult(i(x3), x1))}, x2), i(x1)))})$

$=$ by Lemma 1793 RL with $\{x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow x2\}$

$mult(x1, \underbrace{op_t(rd(\overbrace{op_l(x2, x1, x3)}, x2), i(x1)))})$

$=$ by Lemma 46 RL with $\{x3 \leftarrow x1, x2 \leftarrow op_l(x2, x1, x3), x1 \leftarrow x2\}$

$\underbrace{mult(x1, \overbrace{i(op_t(rd(x2, op_l(x2, x1, x3)), x1)))})}$

$=$ by Lemma 933 RL with $\{x2 \leftarrow rd(x2, op_l(x2, x1, x3)), x1 \leftarrow x1\}$

$\overbrace{mult(\underbrace{op_t(x1, rd(x2, op_l(x2, x1, x3)))}, i(rd(x2, op_l(x2, x1, x3))))}$

$=$ by Lemma 2309 RL with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$mult(\overbrace{x1}, i(rd(x2, op_l(x2, x1, x3))))$

Lemma 2313: $rd(mult(x1, op_l(x2, x1, x3)), x2) = mult(x1, i(rd(x2, op_l(x2, x1, x3))))$

$$
\begin{aligned}
& \underbrace{rd(mult(x1, op_l(x2, x1, x3)), x2)} \\
= \quad & \text{by Lemma 2155 RL with } \{x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \overbrace{\underbrace{mult(rd(op_l(x2, x1, x3), x2), x1)}} \\
= \quad & \text{by Lemma 13 RL with } \{x2 \leftarrow rd(op_l(x2, x1, x3), x2), x1 \leftarrow x1\} \\
& \overbrace{\underbrace{mult(x1, op_t(rd(op_l(x2, x1, x3), x2), x1))}} \\
= \quad & \text{by Lemma 2312 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{mult(x1, i(rd(x2, op_l(x2, x1, x3))))}
\end{aligned}
$$

Lemma 2314: $rd(mult(x1, op_l(x2, x1, x3)), x2) = rd(x1, rd(x2, op_l(x2, x1, x3)))$

$$
\begin{aligned}
& \underbrace{rd(mult(x1, op_l(x2, x1, x3)), x2)} \\
= \quad & \text{by Lemma 2313 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{mult(x1, \underbrace{i(rd(x2, op_l(x2, x1, x3)))})} \\
= \quad & \text{by Lemma 10 LR with } \{x2 \leftarrow op_l(x2, x1, x3), x1 \leftarrow x2\} \\
& \underbrace{mult(x1, \overbrace{rd(op_l(x2, x1, x3), x2)})} \\
= \quad & \text{by Lemma 2127 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{rd(x1, rd(x2, op_l(x2, x1, x3)))}
\end{aligned}
$$

Lemma 2315: $rd(mult(x1, x2), op_l(x2, x3, x1)) = rd(rd(x2, op_l(x2, i(x1), x3)), i(x1))$

$\underbrace{rd(mult(x1, x2), op_l(x2, x3, x1))}$

$=$ by Lemma 2154 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{\overbrace{mult(rd(x2, op_l(x2, x3, x1)), x1)}}$

$=$ by Lemma 1920 RL with $\{x3 \leftarrow x2, x2 \leftarrow op_l(x2, x3, x1), x1 \leftarrow x1\}$

$\overbrace{op_r(rd(x1, rd(op_l(x2, x3, x1), x2)), rd(x2, \underbrace{op_l(x2, x3, x1)}), x1)}$

$=$ by Lemma 2031 RL with $\{x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow x2\}$

$op_r(rd(x1, \underbrace{rd(op_l(x2, x3, x1), x2)}), rd(x2, \overbrace{op_l(x2, i(x1), x3)}), x1)$

$=$ by Lemma 1074 RL with $\{x4 \leftarrow x1, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x2\}$

$op_r(\underbrace{rd(x1, \overbrace{rd(i(x2), op_l(i(x2), x3, x1))})}, rd(x2, op_l(x2, i(x1), x3)), x1)$

$=$ by Lemma 2124 RL with $\{x3 \leftarrow x3, x2 \leftarrow i(x2), x1 \leftarrow x1\}$

$op_r(\overbrace{mult(x1, \underbrace{rd(op_l(i(x2), x3, x1), i(x2))})}, rd(x2, op_l(x2, i(x1), x3)), x1)$

$=$ by Lemma 38 LR with $\{x1 \leftarrow x2, x2 \leftarrow op_l(i(x2), x3, x1)\}$

$op_r(mult(x1, \overbrace{rd(x2, \underbrace{i(op_l(i(x2), x3, x1))})}), rd(x2, op_l(x2, i(x1), x3)), x1)$

$=$ by Lemma 1049 LR with $\{x3 \leftarrow x1, x2 \leftarrow x3, x1 \leftarrow i(x2)\}$

$op_r(mult(x1, rd(x2, \overbrace{\underbrace{op_l(i(i(x2)), x3, x1)}})), rd(x2, op_l(x2, i(x1), x3)), x1)$

$=$ by Lemma 3 LR with $\{x1 \leftarrow x2\}$

$op_r(mult(x1, rd(x2, \underbrace{op_l(\overbrace{x2}, x3, x1)})), rd(x2, op_l(x2, i(x1), x3)), x1)$

$=$ by Lemma 2031 RL with $\{x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow x2\}$

$\underbrace{op_r(mult(x1, rd(x2, \overbrace{op_l(x2, i(x1), x3)})), rd(x2, op_l(x2, i(x1), x3)), x1)}$

$=$ by Lemma 1516 RL with $\{x2 \leftarrow x1, x1 \leftarrow rd(x2, op_l(x2, i(x1), x3))\}$

$\overbrace{rd(rd(x2, op_l(x2, i(x1), x3)), \underbrace{op_t(i(x1), rd(x2, op_l(x2, i(x1), x3)))})$

$=$ by Lemma 2309 RL with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow i(x1)\}$

$rd(rd(x2, op_l(x2, i(x1), x3)), \overbrace{i(x1)})$

Lemma 2316: $rd(mult(x1, x2), op_l(x2, x3, x1)) = rd(x1, rd(op_l(x2, x3, x1), x2))$

$\underbrace{rd(mult(x1, x2), op_l(x2, x3, x1))}$

$=$ by Lemma 2315 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{\overbrace{rd(rd(x2, op_l(x2, i(x1), x3)), i(x1))}}$

$=$ by Lemma 39 LR with $\{x3 \leftarrow x1, x2 \leftarrow op_l(x2, i(x1), x3), x1 \leftarrow x2\}$

$\overbrace{rd(x1, rd(\underbrace{op_l(x2, i(x1), x3)}, x2))}$

$=$ by Lemma 2031 LR with $\{x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow x2\}$

$rd(x1, rd(\overbrace{op_l(x2, x3, x1)}, x2))$

Lemma 2317: $rd(rd(op_l(x2, x3, x1), x2), x1) = mult(rd(op_l(x2, x3, x1), x2), i(x1))$

$$
\begin{aligned}
& \underbrace{rd(rd(op_l(x2, x3, x1), x2), x1)} \\
= \quad & \text{by Lemma 2208 LR with } \{x3 \leftarrow x1, x2 \leftarrow x3, x1 \leftarrow x2\} \\
& \overbrace{mult(i(x1), rd(op_l(x2, x3, x1), x2))} \\
= \quad & \text{by Lemma 45 RL with } \{x3 \leftarrow op_l(x2, x3, x1), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{i(mult(x1, rd(x2, op_l(x2, x3, x1))))} \\
= \quad & \text{by Lemma 1617 LR with } \{x2 \leftarrow rd(x2, op_l(x2, x3, x1)), x1 \leftarrow x1\} \\
& \overbrace{rd(i(\underbrace{op_t(x1, rd(x2, op_l(x2, x3, x1)))}), op_t(rd(x2, op_l(x2, x3, x1)), rd(x1, rd(x2, op_l(x2, x3, x1)))))} \\
= \quad & \text{by Lemma 2311 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{rd(i(\overbrace{x1}), op_t(rd(x2, op_l(x2, x3, x1)), rd(x1, rd(x2, op_l(x2, x3, x1)))))} \\
= \quad & \text{by Lemma 510 LR with } \{x2 \leftarrow rd(x2, op_l(x2, x3, x1)), x1 \leftarrow x1\} \\
& \overbrace{i(\underbrace{mult(rd(x2, op_l(x2, x3, x1)), x1)})} \\
= \quad & \text{by Lemma 44 LR with } \{x3 \leftarrow x1, x2 \leftarrow op_l(x2, x3, x1), x1 \leftarrow x2\} \\
& \overbrace{mult(rd(op_l(x2, x3, x1), x2), i(x1))}
\end{aligned}
$$

Lemma 2318: $rd(rd(op_l(x2, x3, x1), x2), x1) = rd(op_l(x2, x3, x1), mult(x1, x2))$

$$
\begin{aligned}
& \underbrace{rd(rd(op_l(x2, x3, x1), x2), x1)} \\
= \quad & \text{by Lemma 2317 LR with } \{x1 \leftarrow x1, x3 \leftarrow x3, x2 \leftarrow x2\} \\
& \overbrace{\underbrace{mult(rd(op_l(x2, x3, x1), x2), i(x1))}} \\
= \quad & \text{by Lemma 44 RL with } \{x3 \leftarrow x1, x2 \leftarrow op_l(x2, x3, x1), x1 \leftarrow x2\} \\
& \overbrace{i(\underbrace{mult(rd(x2, op_l(x2, x3, x1)), x1)})} \\
= \quad & \text{by Lemma 2154 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{i(\overbrace{rd(mult(x1, x2), op_l(x2, x3, x1))})} \\
= \quad & \text{by Lemma 10 LR with } \{x2 \leftarrow op_l(x2, x3, x1), x1 \leftarrow mult(x1, x2)\} \\
& \overbrace{rd(op_l(x2, x3, x1), mult(x1, x2))}
\end{aligned}
$$

Lemma 2319: $rd(rd(x2, op_l(x2, x3, x1)), x1) = mult(rd(x2, op_l(x2, x3, x1)), i(x1))$

$\overbrace{rd(rd(x2, op_l(x2, x3, x1)), x1)}$

$=$ by Lemma 2203 RL with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{mult(i(x1), rd(x2, op_l(x2, x3, x1)))}$

$=$ by Lemma 45 RL with $\{x3 \leftarrow x2, x2 \leftarrow op_l(x2, x3, x1), x1 \leftarrow x1\}$

$\overbrace{i(mult(x1, rd(op_l(x2, x3, x1), x2)))}$

$=$ by Lemma 1617 LR with $\{x2 \leftarrow rd(op_l(x2, x3, x1), x2), x1 \leftarrow x1\}$

$\overbrace{rd(i(\overbrace{op_t(x1, rd(op_l(x2, x3, x1), x2))}), op_t(\overbrace{rd(op_l(x2, x3, x1), x2)}, rd(x1, rd(op_l(x2, x3, x1), x2))))}$

$=$ by Lemma 2207 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$rd(i(\overbrace{op_t(x1, rd(x2, op_l(x2, x3, x1)))}), op_t(rd(op_l(x2, x3, x1), x2), rd(x1, rd(op_l(x2, x3, x1), x2))))$

$=$ by Lemma 2311 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{rd(i(\overbrace{x1}), op_t(rd(op_l(x2, x3, x1), x2), rd(x1, rd(op_l(x2, x3, x1), x2))))}$

$=$ by Lemma 510 LR with $\{x2 \leftarrow rd(op_l(x2, x3, x1), x2), x1 \leftarrow x1\}$

$\overbrace{i(mult(rd(op_l(x2, x3, x1), x2), x1))}$

$=$ by Lemma 44 LR with $\{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow op_l(x2, x3, x1)\}$

$\overbrace{mult(rd(x2, op_l(x2, x3, x1)), i(x1))}$

Lemma 2320: $op_l(x1, rd(op_l(x1, x2, x3), x1), x3) = mult(rd(x1, mult(x3, op_l(x1, x2, x3))), mult(x3, op_l(x1, x2, x3)))$

$\overbrace{op_l(x1, rd(op_l(x1, x2, x3), x1), x3)}$

$=$ by Lemma 280 LR with $\{x3 \leftarrow x3, x2 \leftarrow op_l(x1, x2, x3), x1 \leftarrow x1\}$

$\overbrace{mult(\overbrace{mult(i(x3), rd(x1, op_l(x1, x2, x3)))}, mult(x3, op_l(x1, x2, x3)))}$

$=$ by Lemma 2203 LR with $\{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x3\}$

$mult(\overbrace{rd(rd(x1, op_l(x1, x2, x3)), x3)}, mult(x3, op_l(x1, x2, x3)))$

$=$ by Lemma 2319 LR with $\{x1 \leftarrow x3, x3 \leftarrow x2, x2 \leftarrow x1\}$

$mult(\overbrace{mult(rd(x1, op_l(x1, x2, x3)), i(x3))}, mult(x3, op_l(x1, x2, x3)))$

$=$ by Lemma 2236 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$mult(\overbrace{rd(x1, mult(x3, op_l(x1, x2, x3)))}, mult(x3, op_l(x1, x2, x3)))$

Lemma 2321: $op_l(op_t(x1, x2), x2, x3) = mult(mult(i(x3), i(x2)), mult(x3, mult(x1, x2)))$

$op_l(op_t(x1, x2), \underbrace{x2}, x3)$

$=$ by Lemma 29 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_l(op_t(x1, x2), \overbrace{rd(mult(x1, x2), op_t(x1, x2))}, x3)}$

$=$ by Lemma 280 LR with $\{x3 \leftarrow x3, x2 \leftarrow mult(x1, x2), x1 \leftarrow op_t(x1, x2)\}$

$\overbrace{mult(mult(i(x3), \underbrace{rd(op_t(x1, x2), mult(x1, x2))}), mult(x3, mult(x1, x2)))}$

$=$ by Lemma 36 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$mult(mult(i(x3), \overbrace{i(x2)}), mult(x3, mult(x1, x2)))$

Lemma 2322: $op_l(op_t(x1,x2),op_t(x2,x1),x3) = mult(i(x3),mult(rd(x3,x2),mult(x1,x2)))$

$op_l(\underbrace{op_t(x1,x2)},op_t(x2,x1),x3)$

$=$ by Lemma 1722 RL with $\{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{op_l(\overbrace{op_t(x1,op_t(x2,x1))},op_t(x2,x1),x3)}$

$=$ by Lemma 2321 LR with $\{x3 \leftarrow x3, x2 \leftarrow op_t(x2,x1), x1 \leftarrow x1\}$

$\overbrace{mult(\underbrace{mult(i(x3),i(op_t(x2,x1)))},mult(x3,mult(x1,op_t(x2,x1))))}$

$=$ by Axiom 11 LR with $\{x2 \leftarrow op_t(x2,x1), x1 \leftarrow x3\}$

$\underbrace{mult(\overbrace{i(mult(x3,op_t(x2,x1)))},mult(x3,mult(x1,op_t(x2,x1))))}$

$=$ by Lemma 1417 LR with $\{x3 \leftarrow mult(x1,op_t(x2,x1)), x2 \leftarrow op_t(x2,x1), x1 \leftarrow x3\}$

$\overbrace{mult(i(x3),mult(rd(x3,op_t(x2,x1)),\underbrace{mult(x1,op_t(x2,x1))}))}$

$=$ by Lemma 13 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$mult(i(x3),mult(\underbrace{rd(x3,op_t(x2,x1))},\overbrace{mult(x2,x1)}))$

$=$ by Lemma 1398 RL with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$mult(i(x3),mult(\overbrace{rd(op_t(x1,\underbrace{x2}),mult(rd(x2,x3),x1))},mult(x2,x1)))$

$=$ by Axiom 5 LR with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$mult(i(x3),mult(\underbrace{rd(op_t(x1,\overbrace{rd(mult(x2,x1),x1)}),mult(rd(x2,x3),x1))},mult(x2,x1)))$

$=$ by Lemma 1391 RL with $\{x3 \leftarrow mult(x2,x1), x2 \leftarrow mult(rd(x2,x3),x1), x1 \leftarrow x1\}$

$mult(i(x3),\underbrace{mult(\overbrace{rd(rd(x1,mult(rd(x2,x3),x1)),asoc(x1,mult(x2,x1),mult(x2,x1)))},mult(x2,x1))})$

$=$ by Lemma 1586 LR with $\{x2 \leftarrow mult(x2,x1), x3 \leftarrow x1, x1 \leftarrow rd(x1,mult(rd(x2,x3),x1))\}$

$mult(i(x3),\overbrace{mult(rd(x1,mult(rd(x2,x3),x1)),\underbrace{op_l(mult(x2,x1),mult(x2,x1),x1)})})$

$=$ by Lemma 285 LR with $\{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow mult(x2,x1)\}$

$mult(i(x3),mult(rd(x1,mult(rd(x2,x3),x1)),\underbrace{\overbrace{op_l(mult(x2,x1),x2,x1)}}))$

$=$ by Lemma 435 LR with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$mult(i(x3),mult(\underbrace{rd(x1,mult(rd(x2,x3),x1))},\overbrace{mult(x1,x2)}))$

$=$ by Lemma 9 LR with $\{x2 \leftarrow rd(x2,x3), x1 \leftarrow x1\}$

$mult(i(x3),mult(\overbrace{i(\underbrace{rd(x2,x3)})},mult(x1,x2)))$

$=$ by Lemma 10 LR with $\{x2 \leftarrow x3, x1 \leftarrow x2\}$

$mult(i(x3),mult(\overbrace{rd(x3,x2)},mult(x1,x2)))$

Lemma 2323: $op_l(i(x1), x3, x2) = op_l(i(x1), x3, op_t(x2, rd(x1, rd(op_t(x1, x2), x1))))$

$op_l(\underbrace{i(x1)}, x3, x2)$

$=$ by Lemma 1011 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{op_l(\overbrace{op_t(rd(op_t(x1, x2), mult(x1, x1)), i(x2))}, x3, x2)}$

$=$ by Lemma 2031 RL with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow op_t(rd(op_t(x1, x2), mult(x1, x1)), i(x2))\}$

$\overbrace{op_l(op_t(rd(op_t(x1, x2), mult(x1, x1)), i(x2)), i(x2), x3)}$

$=$ by Lemma 2321 LR with $\{x3 \leftarrow x3, x2 \leftarrow i(x2), x1 \leftarrow rd(op_t(x1, x2), mult(x1, x1))\}$

$\overbrace{mult(\underbrace{mult(i(x3), i(i(x2)))}, mult(x3, mult(rd(op_t(x1, x2), mult(x1, x1)), i(x2))))}$

$=$ by Axiom 11 LR with $\{x2 \leftarrow i(x2), x1 \leftarrow x3\}$

$\underbrace{mult(\overbrace{i(mult(x3, i(x2)))}, mult(x3, mult(rd(op_t(x1, x2), mult(x1, x1)), i(x2))))}$

$=$ by Lemma 1417 LR with $\{x3 \leftarrow mult(rd(op_t(x1, x2), mult(x1, x1)), i(x2)), x2 \leftarrow i(x2), x1 \leftarrow x3\}$

$\underbrace{mult(i(x3), \overbrace{mult(rd(x3, i(x2)), mult(rd(op_t(x1, x2), mult(x1, x1)), i(x2)))})}$

$=$ by Lemma 2322 RL with $\{x3 \leftarrow x3, x2 \leftarrow i(x2), x1 \leftarrow rd(op_t(x1, x2), mult(x1, x1))\}$

$\overbrace{op_l(\underbrace{op_t(rd(op_t(x1, x2), mult(x1, x1)), i(x2))}, op_t(i(x2), rd(op_t(x1, x2), mult(x1, x1))), x3)}$

$=$ by Lemma 1011 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$op_l(\overbrace{i(x1)}, \underbrace{op_t(i(x2), rd(op_t(x1, x2), mult(x1, x1)))}, x3)$

$=$ by Lemma 47 RL with $\{x3 \leftarrow op_t(x1, x2), x2 \leftarrow mult(x1, x1), x1 \leftarrow x2\}$

$\underbrace{op_l(i(x1), \overbrace{i(op_t(x2, rd(mult(x1, x1), op_t(x1, x2))))}, x3)}$

$=$ by Lemma 2031 LR with $\{x3 \leftarrow x3, x2 \leftarrow op_t(x2, rd(mult(x1, x1), op_t(x1, x2))), x1 \leftarrow i(x1)\}$

$\overbrace{op_l(i(x1), x3, op_t(x2, \underbrace{rd(mult(x1, x1), op_t(x1, x2))}))}$

$=$ by Lemma 868 RL with $\{x2 \leftarrow op_t(x1, x2), x1 \leftarrow x1\}$

$op_l(i(x1), x3, \underbrace{op_t(x2, \overbrace{op_r(rd(x1, rd(op_t(x1, x2), x1)), x1, x1)})})$

$=$ by Lemma 1721 LR with $\{x3 \leftarrow x1, x2 \leftarrow rd(x1, rd(op_t(x1, x2), x1)), x1 \leftarrow x2\}$

$op_l(i(x1), x3, \overbrace{op_t(x2, rd(x1, \underbrace{rd(op_t(x1, x2), x1)}))})$

Lemma 2324: $op_l(i(x1), x3, x2) = op_l(i(x1), x3, op_t(x2, op_r(x1, x1, x1)))$

$\underbrace{op_l(i(x1), x3, x2)}$

$=$ by Lemma 2323 LR with $\{x2 \leftarrow x2, x3 \leftarrow x3, x1 \leftarrow x1\}$

$\overbrace{op_l(i(x1), x3, \underbrace{op_t(x2, rd(x1, rd(op_t(x1, x2), x1)))})}$

$=$ by Lemma 2141 LR with $\{x3 \leftarrow x1, x4 \leftarrow x1, x2 \leftarrow x1, x1 \leftarrow x2\}$

$op_l(i(x1), x3, \overbrace{op_t(x2, \underbrace{rd(x1, rd(x1, x1))})})$

$=$ by Lemma 353 LR with $\{x2 \leftarrow x1, x1 \leftarrow x1\}$

$op_l(i(x1), x3, op_t(x2, \overbrace{op_r(x1, x1, x1)}))$

Lemma 2325: $op_l(i(x1), x3, x2) = op_l(i(x1), x3, op_t(x2, x1))$

$$
\begin{aligned}
& \underbrace{op_l(i(x1), x3, x2)} \\
= \quad & \text{by Lemma 2324 LR with } \{x2 \leftarrow x2, x3 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{op_l(i(x1), x3, op_t(x2, \underbrace{op_r(x1, x1, x1)}))} \\
= \quad & \text{by Lemma 601 LR with } \{x2 \leftarrow x1, x1 \leftarrow x1\} \\
& op_l(i(x1), x3, op_t(x2, \overbrace{\underbrace{op_l(x1, x1, x1)}})) \\
= \quad & \text{by Lemma 25 LR with } \{x2 \leftarrow x1, x1 \leftarrow x1\} \\
& op_l(i(x1), x3, op_t(x2, \overbrace{x1})))
\end{aligned}
$$

Lemma 2326: $op_l(x1, x3, x2) = op_l(i(i(x1)), x2, i(op_t(x3, x1)))$

$$
\begin{aligned}
& op_l(\underbrace{x1}, x3, x2) \\
= \quad & \text{by Lemma 3 RL with } \{x1 \leftarrow x1\} \\
& \underbrace{op_l(\overbrace{i(i(x1))}, x3, x2)} \\
= \quad & \text{by Lemma 2030 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow i(i(x1))\} \\
& \overbrace{\underbrace{op_l(i(i(x1)), x2, i(x3))}} \\
= \quad & \text{by Lemma 2325 LR with } \{x2 \leftarrow i(x3), x3 \leftarrow x2, x1 \leftarrow i(x1)\} \\
& \overbrace{op_l(i(i(x1)), x2, \underbrace{op_t(i(x3), i(x1))})} \\
= \quad & \text{by Lemma 15 LR with } \{x2 \leftarrow x1, x1 \leftarrow x3\} \\
& op_l(i(i(x1)), x2, \overbrace{i(op_t(x3, x1))})
\end{aligned}
$$

Lemma 2327: $op_l(x1, x3, x2) = op_l(x1, op_t(x3, x1), x2)$

$$
\begin{aligned}
& \underbrace{op_l(x1, x3, x2)} \\
= \quad & \text{by Lemma 2326 LR with } \{x2 \leftarrow x2, x3 \leftarrow x3, x1 \leftarrow x1\} \\
& \underbrace{\overbrace{op_l(i(i(x1)), x2, i(op_t(x3, x1)))}} \\
= \quad & \text{by Lemma 2030 LR with } \{x3 \leftarrow op_t(x3, x1), x2 \leftarrow x2, x1 \leftarrow i(i(x1))\} \\
& \overbrace{op_l(\underbrace{i(i(x1))}, op_t(x3, x1), x2)} \\
= \quad & \text{by Lemma 3 LR with } \{x1 \leftarrow x1\} \\
& op_l(\overbrace{x1}, op_t(x3, x1), x2)
\end{aligned}
$$

Lemma 2328: $op_l(x1, x2, op_t(x3, x1)) = op_l(i(i(x1)), x2, x3)$

$$
\begin{aligned}
& op_l(\underbrace{x1}, x2, op_t(x3, x1)) \\
= \quad & \text{by Lemma 3 RL with } \{x1 \leftarrow x1\} \\
& \underbrace{op_l(\overbrace{i(i(x1))}, x2, op_t(x3, x1))} \\
= \quad & \text{by Lemma 1049 RL with } \{x3 \leftarrow op_t(x3, x1), x2 \leftarrow x2, x1 \leftarrow i(x1)\} \\
& \overbrace{i(\underbrace{op_l(i(x1), x2, op_t(x3, x1))})} \\
= \quad & \text{by Lemma 2325 RL with } \{x2 \leftarrow x3, x3 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{i(\overbrace{op_l(i(x1), x2, x3)})} \\
= \quad & \text{by Lemma 1049 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow i(x1)\} \\
& \overbrace{op_l(i(i(x1)), x2, x3)}
\end{aligned}
$$

Lemma 2329: $op_l(x1, x2, op_t(x3, x1)) = op_l(x1, x2, x3)$

$$
\begin{aligned}
& \underbrace{op_l(x1, x2, op_t(x3, x1))} \\
= \quad & \text{by Lemma 2328 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(\underbrace{i(i(x1))}, x2, x3)} \\
= \quad & \text{by Lemma 3 LR with } \{x1 \leftarrow x1\} \\
& op_l(\overbrace{x1}, x2, x3)
\end{aligned}
$$

Lemma 2330: $op_l(i(x1), op_t(x2, x1), x3) = op_l(i(x1), x2, x3)$

$$
\begin{aligned}
& \underbrace{op_l(i(x1), op_t(x2, x1), x3)} \\
= \quad & \text{by Lemma 2031 RL with } \{x3 \leftarrow op_t(x2, x1), x2 \leftarrow x3, x1 \leftarrow i(x1)\} \\
& \overbrace{\underbrace{op_l(i(x1), i(x3), op_t(x2, x1))}} \\
= \quad & \text{by Lemma 2325 RL with } \{x2 \leftarrow x2, x3 \leftarrow i(x3), x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(i(x1), i(x3), x2)}} \\
= \quad & \text{by Lemma 2031 LR with } \{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow i(x1)\} \\
& \overbrace{op_l(i(x1), x2, x3)}
\end{aligned}
$$

Lemma 2331: $op_l(x1, x2, rd(x1, x3)) = op_l(x1, x2, mult(x1, i(x3)))$

$$
\begin{aligned}
& op_l(\underbrace{x1}, x2, rd(x1, x3)) \\
= \quad & \text{by Lemma 3 RL with } \{x1 \leftarrow x1\} \\
& \underbrace{op_l(\overbrace{i(i(x1))}, x2, rd(x1, x3))} \\
= \quad & \text{by Lemma 2325 LR with } \{x2 \leftarrow rd(x1, x3), x3 \leftarrow x2, x1 \leftarrow i(x1)\} \\
& \overbrace{op_l(i(i(x1)), x2, \underbrace{op_t(rd(x1, x3), i(x1))})} \\
= \quad & \text{by Lemma 34 LR with } \{x2 \leftarrow x3, x1 \leftarrow x1\} \\
& op_l(\underbrace{i(i(x1))}, x2, \overbrace{mult(x1, i(x3))}) \\
= \quad & \text{by Lemma 3 LR with } \{x1 \leftarrow x1\} \\
& op_l(\overbrace{x1}, x2, mult(x1, i(x3)))
\end{aligned}
$$

Lemma 2332: $op_l(x1, x2, mult(i(x1), x3)) = op_l(x1, x2, rd(x3, x1))$

$$
\begin{aligned}
& op_l(x1, x2, \underbrace{mult(i(x1), x3)}) \\
= & \quad \text{by Lemma 14 RL with } \{x2 \leftarrow x1, x1 \leftarrow x3\} \\
& \underbrace{op_l(x1, x2, \overbrace{op_t(rd(x3, x1), x1)})} \\
= & \quad \text{by Lemma 2329 LR with } \{x3 \leftarrow rd(x3, x1), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x2, rd(x3, x1))}
\end{aligned}
$$

Lemma 2333: $op_l(x1, x2, rd(i(x1), x3)) = op_l(x1, mult(x1, x3), x2)$

$$
\begin{aligned}
& \underbrace{op_l(x1, x2, rd(i(x1), x3))} \\
= & \quad \text{by Lemma 2329 RL with } \{x3 \leftarrow rd(i(x1), x3), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x2, \underbrace{op_t(rd(i(x1), x3), x1)})} \\
= & \quad \text{by Lemma 33 LR with } \{x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \underbrace{op_l(x1, x2, \overbrace{i(mult(x1, x3))})} \\
= & \quad \text{by Lemma 2030 LR with } \{x3 \leftarrow mult(x1, x3), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, mult(x1, x3), x2)}
\end{aligned}
$$

Lemma 2334: $op_l(x1, x2, rd(x1, mult(x1, x3))) = op_l(x1, x3, x2)$

$$
\begin{aligned}
& \underbrace{op_l(x1, x2, rd(x1, mult(x1, x3)))} \\
= & \quad \text{by Lemma 2331 LR with } \{x3 \leftarrow mult(x1, x3), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x2, \underbrace{mult(x1, i(mult(x1, x3)))})} \\
= & \quad \text{by Lemma 20 LR with } \{x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \underbrace{op_l(x1, x2, \overbrace{i(x3)})} \\
= & \quad \text{by Lemma 2030 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x3, x2)}
\end{aligned}
$$

Lemma 2335: $op_l(x1, x2, x3) = op_l(x1, op_t(x3, op_r(mult(x1, x2), x2, x1)), op_t(i(x2), x1))$

$\overbrace{op_l(x1, x2, x3)}$

$=$ by Lemma 2030 RL with $\{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x1\}$

$\overbrace{\overbrace{op_l(x1, x3, i(x2))}}$

$=$ by Lemma 2329 RL with $\{x3 \leftarrow i(x2), x2 \leftarrow x3, x1 \leftarrow x1\}$

$\overbrace{op_l(\overbrace{x1}, x3, \overbrace{op_t(i(x2), x1)})}$

$=$ by Lemma 1511 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_l(\overbrace{mult(op_r(mult(x1, x2), x2, x1), op_t(i(x2), x1))}, x3, op_t(i(x2), x1))}$

$=$ by Lemma 1789 RL with $\{x2 \leftarrow op_t(i(x2), x1), x3 \leftarrow x3, x1 \leftarrow op_r(mult(x1, x2), x2, x1)\}$

$\overbrace{mult(op_l(op_r(mult(x1, x2), x2, x1), x3, op_t(i(x2), x1)), \underbrace{op_l(op_t(i(x2), x1), x3, op_t(i(x2), x1))})}$

$=$ by Lemma 1723 RL with $\{x3 \leftarrow op_r(mult(x1, x2), x2, x1), x2 \leftarrow x3, x1 \leftarrow op_t(i(x2), x1)\}$

$mult(\underbrace{op_l(op_r(mult(x1, x2), x2, x1), x3, op_t(i(x2), x1))}, \overbrace{op_l(op_t(i(x2), x1), op_t(x3, op_r(mult(x1, x2), x2, x1)), op_t(i(x2), x1))}$

$=$ by Lemma 2327 LR with $\{x2 \leftarrow op_t(i(x2), x1), x3 \leftarrow x3, x1 \leftarrow op_r(mult(x1, x2), x2, x1)\}$

$\underbrace{mult(\overbrace{op_l(op_r(mult(x1, x2), x2, x1), op_t(x3, op_r(mult(x1, x2), x2, x1)), op_t(i(x2), x1))}, op_l(op_t(i(x2), x1), op_t(x3, op_r(mult($

$=$ by Lemma 1789 LR with $\{x2 \leftarrow op_t(i(x2), x1), x3 \leftarrow op_t(x3, op_r(mult(x1, x2), x2, x1)), x1 \leftarrow op_r(mult(x1, x2), x2, x1)\}$

$\overbrace{op_l(\underbrace{mult(op_r(mult(x1, x2), x2, x1), op_t(i(x2), x1))}, op_t(x3, op_r(mult(x1, x2), x2, x1)), op_t(i(x2), x1))}$

$=$ by Lemma 1511 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$op_l(\overbrace{x1}, op_t(x3, op_r(mult(x1, x2), x2, x1)), op_t(i(x2), x1))$

Lemma 2336: $op_l(x1, x2, x3) = op_l(x1, x2, op_t(x3, op_r(mult(x1, x2), x2, x1)))$

$\overbrace{op_l(x1, x2, x3)}$

$=$ by Lemma 2335 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_l(x1, op_t(x3, op_r(mult(x1, x2), x2, x1)), op_t(i(x2), x1))}}$

$=$ by Lemma 2329 LR with $\{x3 \leftarrow i(x2), x2 \leftarrow op_t(x3, op_r(mult(x1, x2), x2, x1)), x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_l(x1, op_t(x3, op_r(mult(x1, x2), x2, x1)), i(x2))}}$

$=$ by Lemma 2030 LR with $\{x3 \leftarrow x2, x2 \leftarrow op_t(x3, op_r(mult(x1, x2), x2, x1)), x1 \leftarrow x1\}$

$\overbrace{op_l(x1, x2, op_t(x3, op_r(mult(x1, x2), x2, x1)))}$

Lemma 2337: $op_l(x1, x3, x2) = op_l(x1, i(x2), op_t(x3, op_t(rd(x2, x1), x2)))$

$\overbrace{op_l(x1, x3, x2)}$

$=$ by Lemma 2031 RL with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_l(x1, i(x2), x3)}$

$=$ by Lemma 2334 RL with $\{x3 \leftarrow i(x2), x2 \leftarrow x3, x1 \leftarrow x1\}$

$\overbrace{op_l(\underbrace{x1}, x3, rd(x1, mult(x1, i(x2))))}$

$=$ by Axiom 5 LR with $\{x2 \leftarrow op_t(rd(x2, x1), x2), x1 \leftarrow x1\}$

$op_l(\overbrace{rd(\underbrace{mult(x1, op_t(rd(x2, x1), x2))}, op_t(rd(x2, x1), x2))}, x3, rd(x1, mult(x1, i(x2))))$

$=$ by Lemma 465 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{op_l(rd(\overbrace{rd(x1, mult(x1, i(x2)))}, op_t(rd(x2, x1), x2)), x3, rd(x1, mult(x1, i(x2))))}$

$=$ by Lemma 2067 RL with $\{x3 \leftarrow op_t(rd(x2, x1), x2), x2 \leftarrow x3, x1 \leftarrow rd(x1, mult(x1, i(x2)))\}$

$\overbrace{rd(\underbrace{op_l(rd(x1, mult(x1, i(x2))), x3, rd(x1, mult(x1, i(x2))))}, op_l(op_t(rd(x2, x1), x2), x3, rd(x1, mult(x1, i(x2)))))}$

$=$ by Lemma 1723 RL with $\{x3 \leftarrow op_t(rd(x2, x1), x2), x2 \leftarrow x3, x1 \leftarrow rd(x1, mult(x1, i(x2)))\}$

$rd(\overbrace{op_l(rd(x1, mult(x1, i(x2))), op_t(x3, op_t(rd(x2, x1), x2)), rd(x1, mult(x1, i(x2))))}, \underbrace{op_l(op_t(rd(x2, x1), x2), x3, rd(x1, mult}$

$=$ by Lemma 2327 LR with $\{x2 \leftarrow rd(x1, mult(x1, i(x2))), x3 \leftarrow x3, x1 \leftarrow op_t(rd(x2, x1), x2)\}$

$\underbrace{rd(op_l(rd(x1, mult(x1, i(x2))), op_t(x3, op_t(rd(x2, x1), x2)), rd(x1, mult(x1, i(x2))))}, \overbrace{op_l(op_t(rd(x2, x1), x2), op_t(x3, op_t(rd($

$=$ by Lemma 2067 LR with $\{x3 \leftarrow op_t(rd(x2, x1), x2), x2 \leftarrow op_t(x3, op_t(rd(x2, x1), x2)), x1 \leftarrow rd(x1, mult(x1, i(x2)))\}$

$\overbrace{op_l(rd(\underbrace{rd(x1, mult(x1, i(x2)))}, op_t(rd(x2, x1), x2)), op_t(x3, op_t(rd(x2, x1), x2)), rd(x1, mult(x1, i(x2))))}$

$=$ by Lemma 465 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$op_l(\underbrace{rd(\overbrace{mult(x1, op_t(rd(x2, x1), x2))}, op_t(rd(x2, x1), x2))}, op_t(x3, op_t(rd(x2, x1), x2)), rd(x1, mult(x1, i(x2))))$

$=$ by Axiom 5 RL with $\{x2 \leftarrow op_t(rd(x2, x1), x2), x1 \leftarrow x1\}$

$\underbrace{op_l(\overbrace{x1}, op_t(x3, op_t(rd(x2, x1), x2)), rd(x1, mult(x1, i(x2))))}$

$=$ by Lemma 2334 LR with $\{x3 \leftarrow i(x2), x2 \leftarrow op_t(x3, op_t(rd(x2, x1), x2)), x1 \leftarrow x1\}$

$\overbrace{op_l(x1, i(x2), op_t(x3, op_t(rd(x2, x1), x2)))}$

Lemma 2338: $op_l(x1, mult(x1, x3), x2) = op_l(x1, x2, i(op_t(mult(x3, x1), x3)))$

$\overbrace{op_l(x1, mult(x1, x3), x2)}$

$=$ by Lemma 2030 RL with $\{x3 \leftarrow mult(x1, x3), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_l(x1, x2, i(mult(x1, x3)))}$

$=$ by Lemma 2329 RL with $\{x3 \leftarrow i(mult(x1, x3)), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_l(x1, x2, \underbrace{op_t(i(mult(x1, x3)), x1)})}$

$=$ by Lemma 937 RL with $\{x2 \leftarrow x1, x1 \leftarrow x3\}$

$op_l(x1, x2, \overbrace{i(op_t(mult(x3, x1), x3))})$

Lemma 2339: $op_l(x1, x2, op_t(x3, op_l(x1, x4, x5))) = op_l(x1, x2, x3)$

$$
\begin{aligned}
& \overbrace{op_l(x1, x2, op_t(x3, op_l(x1, x4, x5)))} \\
= \quad & \text{by Lemma 1615 RL with } \{x5 \leftarrow op_t(x3, op_l(x1, x4, x5)), x4 \leftarrow x2, x3 \leftarrow x5, x2 \leftarrow x4, x1 \leftarrow x1\} \\
& \overbrace{op_l(\underbrace{op_l(op_l(x1, x4, x5), x2, op_t(x3, op_l(x1, x4, x5)))}, x5, x4)} \\
= \quad & \text{by Lemma 2329 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow op_l(x1, x4, x5)\} \\
& \underbrace{op_l(\overbrace{op_l(op_l(x1, x4, x5), x2, x3)}, x5, x4)} \\
= \quad & \text{by Lemma 1615 LR with } \{x5 \leftarrow x3, x4 \leftarrow x2, x3 \leftarrow x5, x2 \leftarrow x4, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x2, x3)}
\end{aligned}
$$

Lemma 2340: $op_l(x1, x3, x2) = op_l(x1, x3, op_r(x2, x1, rd(x1, x2)))$

$$
\begin{aligned}
& \underbrace{op_l(x1, x3, x2)} \\
= \quad & \text{by Lemma 2031 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, i(x2), x3)}} \\
= \quad & \text{by Lemma 2334 RL with } \{x3 \leftarrow i(x2), x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x3, \underbrace{rd(x1, mult(x1, i(x2)))})} \\
= \quad & \text{by Lemma 1595 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& op_l(x1, x3, \overbrace{op_r(x2, x1, rd(x1, x2))})
\end{aligned}
$$

Lemma 2341: $op_l(x1, mult(x1, op_t(i(x2), x1)), x3) = op_l(x1, op_t(op_t(rd(x1, x2), x1), op_t(i(x2), x1)), x3)$

$$
\begin{aligned}
& \underbrace{op_l(x1, mult(x1, op_t(i(x2), x1)), x3)} \\
= \quad & \text{by Lemma 2338 LR with } \{x2 \leftarrow x3, x3 \leftarrow op_t(i(x2), x1), x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, x3, i(op_t(mult(op_t(i(x2), x1), x1), op_t(i(x2), x1))))}} \\
= \quad & \text{by Lemma 2030 LR with } \{x3 \leftarrow op_t(mult(op_t(i(x2), x1), x1), op_t(i(x2), x1)), x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, op_t(\underbrace{mult(op_t(i(x2), x1), x1)}, op_t(i(x2), x1)), x3)} \\
= \quad & \text{by Lemma 340 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& op_l(x1, op_t(\overbrace{op_t(rd(x1, x2), x1)}, op_t(i(x2), x1)), x3)
\end{aligned}
$$

Lemma 2342: $op_l(x1, x2, op_t(x3, rd(x1, x3))) = op_l(x1, x2, op_r(op_t(x3, rd(x1, x3)), x1, rd(x1, x3)))$

$\underbrace{op_l(x1, x2, op_t(x3, rd(x1, x3)))}$

$=$ by Lemma 2340 LR with $\{x2 \leftarrow op_t(x3, rd(x1, x3)), x3 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_l(x1, x2, op_r(op_t(x3, rd(x1, x3)), x1, \underbrace{rd(x1, op_t(x3, rd(x1, x3)))}))}$

$=$ by Lemma 380 LR with $\{x2 \leftarrow x3, x1 \leftarrow x1\}$

$op_l(x1, x2, op_r(op_t(x3, rd(x1, x3)), x1, \overbrace{mult(i(x3), x1)}))$

$=$ by Lemma 910 RL with $\{x2 \leftarrow x3, x1 \leftarrow x1\}$

$op_l(x1, x2, \underbrace{op_r(op_t(x3, rd(x1, x3)), x1, \overbrace{op_t(rd(op_t(x1, i(x3)), x3), x1)})})$

$=$ by Lemma 1890 LR with $\{x3 \leftarrow rd(op_t(x1, i(x3)), x3), x2 \leftarrow x1, x1 \leftarrow op_t(x3, rd(x1, x3))\}$

$op_l(x1, x2, \overbrace{op_r(op_t(x3, rd(x1, x3)), x1, \underbrace{rd(op_t(x1, i(x3)), x3)})})$

$=$ by Axiom 4 LR with $\{x2 \leftarrow i(x1), x1 \leftarrow rd(op_t(x1, i(x3)), x3)\}$

$op_l(x1, x2, \underbrace{op_r(op_t(x3, rd(x1, x3)), x1, \overbrace{mult(rd(rd(op_t(x1, i(x3)), x3), i(x1)), i(x1))})})$

$=$ by Lemma 2218 LR with $\{x3 \leftarrow rd(rd(op_t(x1, i(x3)), x3), i(x1)), x2 \leftarrow x1, x1 \leftarrow op_t(x3, rd(x1, x3))\}$

$op_l(x1, x2, \overbrace{op_r(op_t(x3, rd(x1, x3)), x1, \underbrace{rd(rd(rd(op_t(x1, i(x3)), x3), i(x1)), x1)})})$

$=$ by Lemma 358 LR with $\{x2 \leftarrow x1, x1 \leftarrow rd(op_t(x1, i(x3)), x3)\}$

$op_l(x1, x2, op_r(op_t(x3, rd(x1, x3)), x1, \overbrace{op_r(\underbrace{rd(op_t(x1, i(x3)), x3)}, x1, x1)}))$

$=$ by Lemma 10 RL with $\{x2 \leftarrow op_t(x1, i(x3)), x1 \leftarrow x3\}$

$op_l(x1, x2, op_r(op_t(x3, rd(x1, x3)), x1, \underbrace{op_r(\overbrace{i(rd(x3, op_t(x1, i(x3))))}, x1, x1)}))$

$=$ by Lemma 1876 LR with $\{x2 \leftarrow x1, x1 \leftarrow x3\}$

$op_l(x1, x2, \underbrace{op_r(op_t(x3, rd(x1, x3)), x1, \overbrace{rd(mult(x1, rd(x1, x3)), x1)})})$

$=$ by Lemma 1891 RL with $\{x3 \leftarrow mult(x1, rd(x1, x3)), x2 \leftarrow x1, x1 \leftarrow op_t(x3, rd(x1, x3))\}$

$op_l(x1, x2, \overbrace{op_r(op_t(x3, rd(x1, x3)), x1, \underbrace{mult(i(x1), mult(x1, rd(x1, x3)))})})$

$=$ by Axiom 2 RL with $\{x2 \leftarrow x1, x1 \leftarrow rd(x1, x3)\}$

$op_l(x1, x2, op_r(op_t(x3, rd(x1, x3)), x1, \overbrace{rd(x1, x3)}))$

Lemma 2343: $op_l(i(x1), x2, rd(x3, op_t(x2, x1))) = op_l(i(x1), x2, x3)$

$\underbrace{op_l(i(x1), x2, rd(x3, op_t(x2, x1)))}$

$=$ by Lemma 2330 RL with $\{x3 \leftarrow rd(x3, op_t(x2, x1)), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_l(i(x1), op_t(x2, x1), rd(x3, op_t(x2, x1)))}}$

$=$ by Lemma 286 LR with $\{x3 \leftarrow x3, x2 \leftarrow op_t(x2, x1), x1 \leftarrow i(x1)\}$

$\overbrace{\underbrace{op_l(i(x1), op_t(x2, x1), x3)}}$

$=$ by Lemma 2330 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_l(i(x1), x2, x3)}$

Lemma 2344: $x1 = op_l(x1, x3, rd(mult(x3, x1), op_l(x1, x2, x3)))$

$\underbrace{x1}$

$=$ by Lemma 138 RL with $\{x3 \leftarrow rd(rd(op_l(x1, x2, x3), x1), x3), x2 \leftarrow op_t(x3, x1), x1 \leftarrow x1\}$

$\overbrace{op_l(\underbrace{op_l(x1, op_t(x3, x1), rd(rd(op_l(x1, x2, x3), x1), x3))}, rd(rd(op_l(x1, x2, x3), x1), x3), op_t(x3, x1))}$

$=$ by Lemma 2327 RL with $\{x2 \leftarrow rd(rd(op_l(x1, x2, x3), x1), x3), x3 \leftarrow x3, x1 \leftarrow x1\}$

$op_l(\overbrace{op_l(x1, x3, rd(rd(op_l(x1, x2, x3), x1), x3))}, rd(rd(op_l(x1, x2, x3), x1), x3), op_t(x3, x1))$

$=$ by Lemma 286 LR with $\{x3 \leftarrow rd(op_l(x1, x2, x3), x1), x2 \leftarrow x3, x1 \leftarrow x1\}$

$op_l(\overbrace{op_l(x1, x3, rd(op_l(x1, x2, x3), x1))}, rd(rd(op_l(x1, x2, x3), x1), x3), op_t(x3, x1))$

$=$ by Lemma 2052 LR with $\{x4 \leftarrow x1, x3 \leftarrow op_l(x1, x2, x3), x2 \leftarrow x3, x1 \leftarrow x1\}$

$op_l(\overbrace{op_l(\underbrace{x1}, rd(x1, op_l(x1, x2, x3)), x3)}, rd(rd(op_l(x1, x2, x3), x1), x3), op_t(x3, x1))$

$=$ by Axiom 4 LR with $\{x2 \leftarrow mult(x3, op_l(x1, x2, x3)), x1 \leftarrow x1\}$

$op_l(op_l(\overbrace{mult(rd(x1, mult(x3, op_l(x1, x2, x3))), mult(x3, op_l(x1, x2, x3)))}, rd(x1, op_l(x1, x2, x3)), x3), rd(rd(op_l(x1, x2, x3),$

$=$ by Lemma 2320 RL with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$op_l(op_l(\overbrace{op_l(x1, rd(op_l(x1, x2, x3), x1), x3)}, rd(x1, op_l(x1, x2, x3)), x3), rd(rd(op_l(x1, x2, x3), x1), x3), op_t(x3, x1))$

$=$ by Lemma 2052 RL with $\{x4 \leftarrow op_l(x1, x2, x3), x3 \leftarrow x1, x2 \leftarrow x3, x1 \leftarrow x1\}$

$op_l(\underbrace{op_l(\overbrace{op_l(x1, x3, rd(x1, op_l(x1, x2, x3)))}, rd(x1, op_l(x1, x2, x3)), x3)}, rd(rd(op_l(x1, x2, x3), x1), x3), op_t(x3, x1))$

$=$ by Lemma 138 LR with $\{x3 \leftarrow rd(x1, op_l(x1, x2, x3)), x2 \leftarrow x3, x1 \leftarrow x1\}$

$\underbrace{op_l(\overbrace{x1}, rd(rd(op_l(x1, x2, x3), x1), x3), op_t(x3, x1))}$

$=$ by Lemma 2329 LR with $\{x3 \leftarrow x3, x2 \leftarrow rd(rd(op_l(x1, x2, x3), x1), x3), x1 \leftarrow x1\}$

$\overbrace{op_l(x1, \underbrace{rd(rd(op_l(x1, x2, x3), x1), x3)}, x3)}$

$=$ by Lemma 2318 LR with $\{x1 \leftarrow x3, x3 \leftarrow x2, x2 \leftarrow x1\}$

$\underbrace{op_l(x1, \overbrace{rd(op_l(x1, x2, x3), mult(x3, x1))}, x3)}$

$=$ by Lemma 2052 RL with $\{x4 \leftarrow op_l(x1, x2, x3), x3 \leftarrow mult(x3, x1), x2 \leftarrow x3, x1 \leftarrow x1\}$

$\overbrace{op_l(x1, x3, rd(mult(x3, x1), op_l(x1, x2, x3)))}$

Lemma 2345: $op_l(x1, x2, op_l(x3, x3, mult(x1, x2))) = op_l(x1, x2, x3)$

$op_l(x1, x2, \overbrace{op_l(x3, x3, mult(x1, x2))})$

$=$ by Lemma 1507 RL with $\{x2 \leftarrow mult(x1, x2), x1 \leftarrow x3\}$

$\overbrace{op_l(x1, x2, \overbrace{rd(op_r(mult(x3, mult(x1, x2)), mult(x1, x2), x3), mult(x1, x2))})}$

$=$ by Lemma 2336 LR with $\{x3 \leftarrow rd(op_r(mult(x3, mult(x1, x2)), mult(x1, x2), x3), mult(x1, x2)), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_l(x1, x2, \overbrace{op_t(rd(op_r(mult(x3, mult(x1, x2)), mult(x1, x2), x3), mult(x1, x2)), op_r(mult(x1, x2), x2, x1))})}$

$=$ by Lemma 1721 RL with $\{x3 \leftarrow x1, x2 \leftarrow op_r(mult(x1, x2), x2, x1), x1 \leftarrow rd(op_r(mult(x3, mult(x1, x2)), mult(x1, x2), x3), n$

$op_l(x1, x2, \overbrace{op_t(rd(op_r(mult(x3, mult(x1, x2)), mult(x1, x2), x3), mult(x1, x2)), \overbrace{op_r(op_r(mult(x1, x2), x2, x1), x1, x1)})})$

$=$ by Lemma 353 RL with $\{x2 \leftarrow op_r(mult(x1, x2), x2, x1), x1 \leftarrow x1\}$

$op_l(x1, x2, op_t(rd(op_r(mult(x3, mult(x1, x2)), mult(x1, x2), x3), mult(x1, x2)), \overbrace{rd(x1, \overbrace{rd(x1, op_r(mult(x1, x2), x2, x1))})}))$

$=$ by Lemma 1501 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$op_l(x1, x2, op_t(rd(op_r(mult(x3, mult(x1, x2)), mult(x1, x2), x3), mult(x1, x2)), \underbrace{rd(x1, \overbrace{op_l(i(x2), x2, x1)})}))$

$=$ by Lemma 231 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$op_l(x1, x2, \underbrace{op_t(rd(op_r(mult(x3, mult(x1, x2)), mult(x1, x2), x3), mult(x1, x2)), \overbrace{mult(x1, x2)})})$

$=$ by Lemma 14 LR with $\{x2 \leftarrow mult(x1, x2), x1 \leftarrow op_r(mult(x3, mult(x1, x2)), mult(x1, x2), x3)\}$

$op_l(x1, x2, \overbrace{mult(i(mult(x1, x2)), op_r(mult(x3, mult(x1, x2)), mult(x1, x2), x3))})$

$=$ by Lemma 1509 RL with $\{x2 \leftarrow mult(x1, x2), x1 \leftarrow x3\}$

$op_l(x1, x2, \overbrace{op_t(x3, rd(mult(x1, x2), x3))})$

$=$ by Lemma 2077 LR with $\{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x3\}$

$op_l(x1, x2, \overbrace{op_t(x3, rd(x1, rd(x3, x2)))})$

$=$ by Lemma 1882 RL with $\{x2 \leftarrow x2, x3 \leftarrow x1, x1 \leftarrow x3\}$

$op_l(x1, x2, \overbrace{op_t(x3, rd(x1, rd(x3, op_t(x2, x3))))})$

$=$ by Lemma 1217 LR with $\{x3 \leftarrow op_t(x2, x3), x2 \leftarrow x1, x1 \leftarrow x3\}$

$\overbrace{op_l(x1, x2, \overbrace{op_t(x3, rd(x1, mult(x3, i(op_t(x2, x3)))))})}$

$=$ by Lemma 1517 RL with $\{x4 \leftarrow rd(x1, mult(x3, i(op_t(x2, x3)))), x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x1\}$

$\overbrace{op_l(x1, op_t(op_t(x3, x2), rd(x1, mult(x3, i(op_t(x2, x3))))), i(x2))}$

$=$ by Lemma 1964 RL with $\{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x1\}$

$\overbrace{op_l(x1, op_t(x3, x2), i(x2))}$

$=$ by Lemma 1332 LR with $\{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x1\}$

$\overbrace{op_l(x1, x2, x3)}$

Lemma 2346: $op_l(x1, x2, op_l(x3, x3, x1)) = op_l(x1, x2, x3)$

$op_l(\underbrace{x1}, x2, op_l(x3, x3, x1))$

$=$ by Axiom 4 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{op_l(\overbrace{mult(rd(x1, x2), x2)}, x2, op_l(x3, x3, x1))}$

$=$ by Lemma 1788 RL with $\{x2 \leftarrow op_l(x3, x3, x1), x3 \leftarrow x2, x1 \leftarrow rd(x1, x2)\}$

$\overbrace{mult(op_l(rd(x1, x2), x2, op_l(x3, x3, \underbrace{x1})), op_l(x2, x2, op_l(x3, x3, x1)))}$

$=$ by Axiom 4 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$mult(\underbrace{op_l(rd(x1, x2), x2, op_l(x3, x3, \overbrace{mult(rd(x1, x2), x2)}))}, op_l(x2, x2, op_l(x3, x3, x1)))$

$=$ by Lemma 2345 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow rd(x1, x2)\}$

$mult(\overbrace{op_l(rd(x1, x2), x2, x3)}, \underbrace{op_l(x2, x2, op_l(x3, x3, x1))})$

$=$ by Lemma 173 RL with $\{x2 \leftarrow op_l(x3, x3, x1), x1 \leftarrow x2\}$

$mult(op_l(rd(x1, x2), x2, x3), \overbrace{mult(\underbrace{asoc(x2, op_l(x3, x3, x1), x2)}, x2)})$

$=$ by Lemma 1684 LR with $\{x2 \leftarrow x1, x1 \leftarrow x3, x3 \leftarrow x2\}$

$mult(op_l(rd(x1, x2), x2, x3), \underbrace{mult(\overbrace{asoc(x2, x3, x2)}, x2)})$

$=$ by Lemma 173 LR with $\{x2 \leftarrow x3, x1 \leftarrow x2\}$

$\underbrace{mult(op_l(rd(x1, x2), x2, x3), \overbrace{op_l(x2, x2, x3)})}$

$=$ by Lemma 1788 LR with $\{x2 \leftarrow x3, x3 \leftarrow x2, x1 \leftarrow rd(x1, x2)\}$

$\overbrace{op_l(\underbrace{mult(rd(x1, x2), x2)}, x2, x3)}$

$=$ by Axiom 4 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$op_l(\overbrace{x1}, x2, x3)$

Lemma 2347: $op_l(x1, x2, x3) = op_l(x1, x2, op_r(x3, x1, x1))$

$\overbrace{op_l(x1, x2, x3)}$

$=$ by Lemma 2346 RL with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_l(x1, x2, \underbrace{op_l(x3, x3, x1)})}$

$=$ by Lemma 1507 RL with $\{x2 \leftarrow x1, x1 \leftarrow x3\}$

$\underbrace{op_l(x1, x2, \overbrace{rd(op_r(mult(x3, x1), x1, x3), x1)})}$

$=$ by Lemma 2332 RL with $\{x3 \leftarrow op_r(mult(x3, x1), x1, x3), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_l(x1, x2, \underbrace{mult(i(x1), op_r(mult(x3, x1), x1, x3))})}$

$=$ by Lemma 1509 RL with $\{x2 \leftarrow x1, x1 \leftarrow x3\}$

$\underbrace{op_l(x1, x2, \overbrace{op_t(x3, rd(x1, x3))})}$

$=$ by Lemma 2342 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_l(x1, x2, op_r(\underbrace{op_t(x3, rd(x1, x3))}, x1, rd(x1, x3)))}$

$=$ by Lemma 707 RL with $\{x2 \leftarrow x1, x1 \leftarrow x3\}$

$op_l(x1, x2, \underbrace{op_r(\overbrace{op_t(x3, rd(i(x3), x1))}, x1, rd(x1, x3))})$

$=$ by Axiom 14 RL with $\{x4 \leftarrow rd(i(x3), x1), x3 \leftarrow rd(x1, x3), x2 \leftarrow x1, x1 \leftarrow x3\}$

$op_l(x1, x2, \underbrace{\overbrace{op_t(op_r(x3, x1, rd(x1, x3)), rd(i(x3), x1))}})$

$=$ by Lemma 1806 RL with $\{x2 \leftarrow x1, x1 \leftarrow x3\}$

$op_l(x1, x2, \underbrace{\overbrace{op_l(x3, x3, i(x1))}})$

$=$ by Lemma 608 LR with $\{x2 \leftarrow x1, x1 \leftarrow x3\}$

$op_l(x1, x2, \underbrace{\overbrace{op_l(x3, x1, x3)}})$

$=$ by Lemma 286 RL with $\{x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow x3\}$

$op_l(x1, x2, \underbrace{\overbrace{op_l(x3, x1, rd(x3, x1))}})$

$=$ by Lemma 2052 LR with $\{x4 \leftarrow x1, x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow x3\}$

$op_l(x1, x2, \underbrace{\overbrace{op_l(x3, rd(x1, x3), x1)}})$

$=$ by Axiom 2 LR with $\{x2 \leftarrow x1, x1 \leftarrow op_l(x3, rd(x1, x3), x1)\}$

$\underbrace{op_l(x1, x2, \overbrace{mult(i(x1), mult(x1, op_l(x3, rd(x1, x3), x1)))})}$

$=$ by Lemma 2332 LR with $\{x3 \leftarrow mult(x1, op_l(x3, rd(x1, x3), x1)), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_l(x1, x2, \underbrace{rd(mult(x1, op_l(x3, rd(x1, x3), x1)), x1)})}$

$=$ by Lemma 1917 LR with $\{x3 \leftarrow x1, x2 \leftarrow x3, x1 \leftarrow x1\}$

$op_l(x1, x2, \underbrace{\overbrace{rd(x1, rd(x1, x3))}})$

$=$ by Lemma 353 LR with $\{x2 \leftarrow x3, x1 \leftarrow x1\}$

$op_l(x1, x2, \overbrace{op_r(x3, x1, x1)})$

Lemma 2348: $op_l(x1, mult(i(x3), x1), x2) = op_l(x1, x2, rd(x3, x1))$

$op_l(x1, \underbrace{mult(i(x3), x1)}, x2)$

$=$ by Lemma 13 RL with $\{x2 \leftarrow i(x3), x1 \leftarrow x1\}$

$\underbrace{op_l(x1, \overbrace{mult(x1, op_t(i(x3), x1))}, x2)}$

$=$ by Lemma 2341 LR with $\{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x1\}$

$\overbrace{op_l(x1, \underbrace{op_t(op_t(rd(x1, x3), x1), op_t(i(x3), x1))}, x2)}$

$=$ by Axiom 13 RL with $\{x3 \leftarrow x1, x2 \leftarrow op_t(i(x3), x1), x1 \leftarrow rd(x1, x3)\}$

$\underbrace{op_l(x1, \overbrace{op_t(op_t(rd(x1, x3), op_t(i(x3), x1)), x1)}, x2)}$

$=$ by Lemma 2327 RL with $\{x2 \leftarrow x2, x3 \leftarrow op_t(rd(x1, x3), op_t(i(x3), x1)), x1 \leftarrow x1\}$

$\overbrace{op_l(x1, \underbrace{op_t(rd(x1, x3), op_t(i(x3), x1))}, x2)}$

$=$ by Lemma 1722 LR with $\{x3 \leftarrow x1, x2 \leftarrow i(x3), x1 \leftarrow rd(x1, x3)\}$

$op_l(x1, \underbrace{\overbrace{op_t(rd(x1, x3), i(x3))}}, x2)$

$=$ by Lemma 46 RL with $\{x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow x3\}$

$\underbrace{op_l(x1, \overbrace{i(op_t(rd(x3, x1), x3))}, x2)}$

$=$ by Lemma 2031 LR with $\{x3 \leftarrow x2, x2 \leftarrow op_t(rd(x3, x1), x3), x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_l(x1, x2, \overbrace{op_t(rd(x3, x1), x3)})}}$

$=$ by Lemma 2346 RL with $\{x3 \leftarrow op_t(rd(x3, x1), x3), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_l(x1, x2, \underbrace{op_l(op_t(rd(x3, x1), x3), op_t(rd(x3, x1), x3), x1)})}$

$=$ by Lemma 1343 LR with $\{x3 \leftarrow x1, x2 \leftarrow x3, x1 \leftarrow op_t(rd(x3, x1), x3)\}$

$op_l(x1, x2, \overbrace{\underbrace{op_l(op_t(rd(x3, x1), x3), x3, x1)}})$

$=$ by Lemma 250 LR with $\{x2 \leftarrow x1, x1 \leftarrow x3\}$

$\underbrace{op_l(x1, x2, \overbrace{mult(i(x1), x3)})}$

$=$ by Lemma 2332 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_l(x1, x2, rd(x3, x1))}$

Lemma 2349: $op_l(x1, x2, mult(mult(x1, x2), x3)) = op_l(x1, x2, rd(mult(x1, x2), i(x3)))$

$op_l(\underbrace{x1}, x2, mult(mult(x1, x2), x3))$

$=$ by Axiom 5 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$op_l(\overbrace{rd(mult(x1, x2), \underbrace{x2}}), x2, mult(mult(x1, x2), x3))$

$=$ by Axiom 1 LR with $\{x1 \leftarrow x2\}$

$\underbrace{op_l(rd(mult(x1, x2), \overbrace{mult(x2, unit())}), x2, mult(mult(x1, x2), x3))}$

$=$ by Lemma 2068 LR with $\{x3 \leftarrow mult(mult(x1, x2), x3), x2 \leftarrow x2, x1 \leftarrow mult(x1, x2)\}$

$\overbrace{rd(\underbrace{op_l(mult(x1, x2), mult(x2, i(mult(mult(x1, x2), x3))), op_l(mult(mult(x1, x2), x3), x2, mult(mult(x1, x2), x3)))}, op_l(x2,$

$=$ by Lemma 2053 LR with $\{x3 \leftarrow mult(mult(x1, x2), x3), x2 \leftarrow x2, x1 \leftarrow mult(x1, x2)\}$

$rd(\overbrace{op_l(mult(x1, x2), x2, mult(mult(x1, x2), x3))}, \underbrace{op_l(x2, x2, mult(mult(x1, x2), x3))})$

$=$ by Lemma 1748 RL with $\{x3 \leftarrow x3, x2 \leftarrow mult(x1, x2), x1 \leftarrow x2\}$

$rd(op_l(mult(x1, x2), x2, mult(mult(x1, x2), \underbrace{x3})), \overbrace{op_l(x2, x2, rd(mult(x1, x2), i(x3)))})$

$=$ by Lemma 3 RL with $\{x1 \leftarrow x3\}$

$rd(\underbrace{op_l(mult(x1, x2), x2, mult(mult(x1, x2), \overbrace{i(i(x3))}))}, op_l(x2, x2, rd(mult(x1, x2), i(x3))))$

$=$ by Lemma 2331 RL with $\{x3 \leftarrow i(x3), x2 \leftarrow x2, x1 \leftarrow mult(x1, x2)\}$

$rd(\overbrace{\underbrace{op_l(mult(x1, x2), x2, rd(mult(x1, x2), i(x3)))}}, op_l(x2, x2, rd(mult(x1, x2), i(x3))))$

$=$ by Lemma 2053 RL with $\{x3 \leftarrow rd(mult(x1, x2), i(x3)), x2 \leftarrow x2, x1 \leftarrow mult(x1, x2)\}$

$\underbrace{rd(\overbrace{op_l(mult(x1, x2), mult(x2, i(rd(mult(x1, x2), i(x3)))), op_l(rd(mult(x1, x2), i(x3)), x2, rd(mult(x1, x2), i(x3))))}, op_l(x2,$

$=$ by Lemma 2068 RL with $\{x3 \leftarrow rd(mult(x1, x2), i(x3)), x2 \leftarrow x2, x1 \leftarrow mult(x1, x2)\}$

$\overbrace{op_l(rd(mult(x1, x2), \underbrace{mult(x2, unit())}), x2, rd(mult(x1, x2), i(x3)))}$

$=$ by Axiom 1 RL with $\{x1 \leftarrow x2\}$

$op_l(\underbrace{rd(mult(x1, x2), \overbrace{x2}}), x2, rd(mult(x1, x2), i(x3)))$

$=$ by Axiom 5 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$op_l(\overbrace{x1}, x2, rd(mult(x1, x2), i(x3)))$

Lemma 2350: $op_l(x1, x2, rd(x1, mult(x2, i(x3)))) = op_l(x1, x2, mult(mult(x1, x2), x3))$

$\overbrace{op_l(x1, x2, rd(x1, mult(x2, i(x3))))}$

$=$ by Lemma 2043 LR with $\{x4 \leftarrow i(x3), x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_l(x1, x2, rd(mult(x2, x1), i(x3)))}$

$=$ by Lemma 2327 LR with $\{x2 \leftarrow rd(mult(x2, x1), i(x3)), x3 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_l(x1, op_t(x2, x1), rd(mult(x2, x1), i(x3)))}$

$=$ by Lemma 286 RL with $\{x3 \leftarrow rd(mult(x2, x1), i(x3)), x2 \leftarrow op_t(x2, x1), x1 \leftarrow x1\}$

$\overbrace{op_l(x1, op_t(x2, x1), rd(rd(mult(x2, x1), i(x3)), op_t(x2, x1)))}$

$=$ by Lemma 2327 RL with $\{x2 \leftarrow rd(rd(mult(x2, x1), i(x3)), op_t(x2, x1)), x3 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_l(x1, x2, \underbrace{rd(rd(mult(x2, x1), i(x3)), op_t(x2, x1))})}$

$=$ by Lemma 2007 LR with $\{x3 \leftarrow i(x3), x2 \leftarrow x1, x1 \leftarrow x2\}$

$\underbrace{op_l(x1, x2, \overbrace{rd(rd(mult(x1, x2), i(x3)), x2)})}$

$=$ by Lemma 286 LR with $\{x3 \leftarrow rd(mult(x1, x2), i(x3)), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_l(x1, x2, rd(mult(x1, x2), i(x3)))}$

$=$ by Lemma 2349 RL with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_l(x1, x2, mult(mult(x1, x2), x3))}$

Lemma 2351: $op_l(x1, i(mult(x2, x3)), rd(x3, x1)) = op_l(x1, rd(i(x2), x3), rd(x3, x1))$

$$
\begin{aligned}
& \overbrace{op_l(x1, i(mult(x2, x3)), rd(x3, x1))} \\
= \quad & \text{by Lemma 2348 RL with } \{x2 \leftarrow i(mult(x2, x3)), x3 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{op_l(\underbrace{x1}, mult(i(x3), x1), i(mult(x2, x3)))} \\
= \quad & \text{by Lemma 12 RL with } \{x2 \leftarrow x1, x1 \leftarrow x3\} \\
& \underbrace{op_l(\overbrace{mult(x3, mult(i(x3), x1))}, mult(i(x3), x1), i(mult(x2, x3)))} \\
= \quad & \text{by Lemma 1788 RL with } \{x2 \leftarrow i(mult(x2, x3)), x3 \leftarrow mult(i(x3), x1), x1 \leftarrow x3\} \\
& \overbrace{mult(op_l(x3, mult(i(x3), x1), i(mult(x2, x3))), \underbrace{op_l(mult(i(x3), x1), mult(i(x3), x1), i(mult(x2, x3)))})} \\
= \quad & \text{by Lemma 773 LR with } \{x2 \leftarrow i(mult(x2, x3)), x1 \leftarrow mult(i(x3), x1)\} \\
& mult(op_l(x3, mult(i(x3), x1), i(mult(x2, x3))), \overbrace{rd(mult(i(x3), x1), \underbrace{asoc(i(mult(x2, x3)), mult(i(x3), x1), mult(i(x3), x1))})} \\
= \quad & \text{by Lemma 1736 RL with } \{x3 \leftarrow mult(i(x3), x1), x2 \leftarrow x3, x1 \leftarrow i(mult(x2, x3))\} \\
& mult(op_l(x3, mult(i(x3), x1), i(mult(x2, x3))), \overbrace{rd(mult(i(x3), x1), \underbrace{asoc(op_r(i(mult(x2, x3)), x3, x3), mult(i(x3), x1), mult(} \\
= \quad & \text{by Lemma 773 RL with } \{x2 \leftarrow op_r(i(mult(x2, x3)), x3, x3), x1 \leftarrow mult(i(x3), x1)\} \\
& mult(\underbrace{op_l(x3, mult(i(x3), x1), i(mult(x2, x3)))}, \overbrace{op_l(mult(i(x3), x1), mult(i(x3), x1), op_r(i(mult(x2, x3)), x3, x3))}) \\
= \quad & \text{by Lemma 2347 LR with } \{x3 \leftarrow i(mult(x2, x3)), x2 \leftarrow mult(i(x3), x1), x1 \leftarrow x3\} \\
& \underbrace{mult(\overbrace{op_l(x3, mult(i(x3), x1), op_r(i(mult(x2, x3)), x3, x3))}, op_l(mult(i(x3), x1), mult(i(x3), x1), op_r(i(mult(x2, x3)), x3, x3} \\
= \quad & \text{by Lemma 1788 LR with } \{x2 \leftarrow op_r(i(mult(x2, x3)), x3, x3), x3 \leftarrow mult(i(x3), x1), x1 \leftarrow x3\} \\
& \overbrace{op_l(\underbrace{mult(x3, mult(i(x3), x1))}, mult(i(x3), x1), op_r(i(mult(x2, x3)), x3, x3))} \\
= \quad & \text{by Lemma 12 LR with } \{x2 \leftarrow x1, x1 \leftarrow x3\} \\
& \underbrace{op_l(\overbrace{x1}, mult(i(x3), x1), op_r(i(mult(x2, x3)), x3, x3))} \\
= \quad & \text{by Lemma 2348 LR with } \{x2 \leftarrow op_r(i(mult(x2, x3)), x3, x3), x3 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, \underbrace{op_r(i(mult(x2, x3)), x3, x3)}, rd(x3, x1))} \\
= \quad & \text{by Lemma 371 LR with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& op_l(x1, \overbrace{rd(i(x2), x3)}, rd(x3, x1))
\end{aligned}
$$

Lemma 2352: $mult(x1, mult(rd(mult(i(x1), x2), x3), x1)) = rd(x2, mult(i(x1), x3))$

$$
\begin{aligned}
& \underbrace{mult(x1, mult(rd(mult(i(x1), x2), x3), x1))} \\
= \quad & \text{by Lemma 76 RL with } \{x3 \leftarrow i(x2), x2 \leftarrow rd(mult(i(x1), x2), x3), x1 \leftarrow x1\} \\
& \overbrace{rd(mult(x1, \underbrace{mult(rd(mult(i(x1), x2), x3), mult(x1, i(x2)))}), i(x2))} \\
= \quad & \text{by Lemma 1003 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{rd(mult(x1, \overbrace{i(x3)}), i(x2))} \\
= \quad & \text{by Lemma 61 LR with } \{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{rd(x2, mult(i(x1), x3))}
\end{aligned}
$$

Lemma 2353: $rd(x1, rd(op_l(x2, x3, x4), x2)) = rd(x2, mult(i(x1), op_l(x2, x3, x4)))$

$$
\begin{aligned}
& \underbrace{rd(x1, \underbrace{rd(op_l(x2, x3, x4), x2)}_{}}_{} \\
= \quad & \text{by Lemma 171 RL with } \{x3 \leftarrow x4, x2 \leftarrow x3, x1 \leftarrow x2\} \\
& rd(x1, \overbrace{\underbrace{mult(i(x2), op_l(x2, x3, x4))}_{}}^{}) \\
= \quad & \text{by Lemma 1133 RL with } \{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x2\} \\
& rd(x1, \overbrace{\underbrace{i(mult(x2, op_l(i(x2), x3, x4)))}_{}}^{}) \\
= \quad & \text{by Axiom 11 RL with } \{x2 \leftarrow op_l(i(x2), x3, x4), x1 \leftarrow x2\} \\
& rd(x1, \overbrace{mult(i(x2), \underbrace{i(op_l(i(x2), x3, x4))}_{})}^{}) \\
= \quad & \text{by Lemma 1049 LR with } \{x3 \leftarrow x4, x2 \leftarrow x3, x1 \leftarrow i(x2)\} \\
& \underbrace{rd(x1, mult(i(x2), \overbrace{op_l(i(i(x2)), x3, x4)}^{}))}_{} \\
= \quad & \text{by Lemma 2352 RL with } \{x3 \leftarrow op_l(i(i(x2)), x3, x4), x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \overbrace{mult(x2, mult(\underbrace{rd(mult(i(x2), x1), op_l(i(i(x2)), x3, x4))}_{}, x2))}^{} \\
= \quad & \text{by Lemma 1975 RL with } \{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow i(x2), x1 \leftarrow mult(i(x2), x1)\} \\
& mult(x2, mult(\overbrace{mult(mult(i(x2), x1), \underbrace{op_l(op_l(i(x2), mult(i(x2), x1), i(x2)), x3, x4)}_{})}^{}, x2)) \\
= \quad & \text{by Axiom 19 LR with } \{x5 \leftarrow x4, x4 \leftarrow x3, x3 \leftarrow i(x2), x2 \leftarrow mult(i(x2), x1), x1 \leftarrow i(x2)\} \\
& mult(x2, mult(mult(mult(i(x2), x1), \overbrace{\underbrace{op_l(op_l(i(x2), x3, x4), mult(i(x2), x1), i(x2))}_{}}^{}), x2)) \\
= \quad & \text{by Lemma 2033 LR with } \{x3 \leftarrow x1, x2 \leftarrow i(x2), x1 \leftarrow op_l(i(x2), x3, x4)\} \\
& mult(x2, mult(\underbrace{mult(mult(i(x2), x1), \overbrace{op_l(op_l(i(x2), x3, x4), x1, i(x2))}^{})}_{}, x2)) \\
= \quad & \text{by Lemma 132 LR with } \{x3 \leftarrow op_l(i(x2), x3, x4), x2 \leftarrow x1, x1 \leftarrow i(x2)\} \\
& \underbrace{mult(x2, mult(\overbrace{mult(i(x2), mult(x1, op_l(i(x2), x3, x4)))}^{}, x2))}_{} \\
= \quad & \text{by Lemma 86 LR with } \{x2 \leftarrow mult(x1, op_l(i(x2), x3, x4)), x1 \leftarrow x2\} \\
& \overbrace{\underbrace{rd(x2, i(mult(x1, op_l(i(x2), x3, x4))))}_{}}^{} \\
= \quad & \text{by Lemma 1133 LR with } \{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& rd(x2, \overbrace{mult(i(x1), op_l(x2, x3, x4))}^{})
\end{aligned}
$$

Lemma 2354: $mult(rd(x1, op_l(x2, x3, x4)), x2) = rd(x1, rd(op_l(x2, x3, x4), x2))$

$$
\begin{aligned}
& \underbrace{mult(rd(x1, op_l(x2, x3, x4)), x2)}_{} \\
= \quad & \text{by Lemma 2251 LR with } \{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{\overbrace{rd(x2, mult(i(x1), op_l(x2, x3, x4)))}^{}}_{} \\
= \quad & \text{by Lemma 2353 RL with } \{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{rd(x1, rd(op_l(x2, x3, x4), x2))}^{}
\end{aligned}
$$

Lemma 2355: $rd(i(x1), rd(op_l(x2, x3, x4), x2)) = rd(x2, mult(x1, op_l(x2, x3, x4)))$

$$
\begin{aligned}
& \underbrace{rd(i(x1), rd(op_l(x2, x3, x4), x2))} \\
= \quad & \text{by Lemma 2354 RL with } \{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow i(x1)\} \\
& \overbrace{mult(rd(i(x1), op_l(x2, x3, x4)), \underbrace{x2})} \\
= \quad & \text{by Lemma 138 RL with } \{x3 \leftarrow x4, x2 \leftarrow x3, x1 \leftarrow x2\} \\
& \underbrace{mult(rd(i(x1), op_l(x2, x3, x4)), \overbrace{op_l(op_l(x2, x3, x4), x4, x3)})} \\
= \quad & \text{by Lemma 2200 RL with } \{x1 \leftarrow x1, x4 \leftarrow x3, x3 \leftarrow x4, x2 \leftarrow op_l(x2, x3, x4)\} \\
& \overbrace{rd(\underbrace{op_l(op_l(x2, x3, x4), x4, x3)}, mult(x1, op_l(x2, x3, x4)))} \\
= \quad & \text{by Lemma 138 LR with } \{x3 \leftarrow x4, x2 \leftarrow x3, x1 \leftarrow x2\} \\
& rd(\overbrace{x2}, mult(x1, op_l(x2, x3, x4)))
\end{aligned}
$$

Lemma 2356: $rd(rd(x1, op_l(x1, x2, x3)), x4) = rd(x1, mult(x4, op_l(x1, x2, x3)))$

$$
\begin{aligned}
& \underbrace{rd(rd(x1, op_l(x1, x2, x3)), x4)} \\
= \quad & \text{by Lemma 40 RL with } \{x3 \leftarrow x1, x2 \leftarrow op_l(x1, x2, x3), x1 \leftarrow x4\} \\
& \underbrace{\overbrace{rd(i(x4), rd(op_l(x1, x2, x3), x1))}} \\
= \quad & \text{by Lemma 2355 LR with } \{x4 \leftarrow x3, x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x4\} \\
& \overbrace{rd(x1, mult(x4, op_l(x1, x2, x3)))}
\end{aligned}
$$

Lemma 2357: $rd(x1, rd(x2, op_l(x2, x3, x4))) = mult(rd(x1, x2), op_l(x2, x3, x4))$

$$
\begin{aligned}
& rd(x1, rd(\underbrace{x2}, op_l(x2, x3, x4))) \\
= \quad & \text{by Lemma 138 RL with } \{x3 \leftarrow x4, x2 \leftarrow x3, x1 \leftarrow x2\} \\
& \underbrace{rd(x1, rd(\overbrace{op_l(op_l(x2, x3, x4), x4, x3)}, op_l(x2, x3, x4)))} \\
= \quad & \text{by Lemma 2354 RL with } \{x4 \leftarrow x3, x3 \leftarrow x4, x2 \leftarrow op_l(x2, x3, x4), x1 \leftarrow x1\} \\
& \overbrace{mult(rd(x1, \underbrace{op_l(op_l(x2, x3, x4), x4, x3)}), op_l(x2, x3, x4))} \\
= \quad & \text{by Lemma 138 LR with } \{x3 \leftarrow x4, x2 \leftarrow x3, x1 \leftarrow x2\} \\
& mult(rd(x1, \overbrace{x2}), op_l(x2, x3, x4))
\end{aligned}
$$

Lemma 2358: $rd(rd(op_l(x1, x2, x3), x1), x4) = rd(op_l(x1, x2, x3), mult(x4, x1))$

$$
\begin{aligned}
& \underbrace{rd(rd(op_l(x1, x2, x3), x1), x4)} \\
= \quad & \text{by Lemma 40 RL with } \{x3 \leftarrow op_l(x1, x2, x3), x2 \leftarrow x1, x1 \leftarrow x4\} \\
& \underbrace{\overbrace{rd(i(x4), rd(x1, op_l(x1, x2, x3)))}} \\
= \quad & \text{by Lemma 2357 LR with } \{x4 \leftarrow x3, x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow i(x4)\} \\
& \underbrace{\overbrace{mult(rd(i(x4), x1), op_l(x1, x2, x3))}} \\
= \quad & \text{by Lemma 2200 RL with } \{x1 \leftarrow x4, x4 \leftarrow x3, x3 \leftarrow x2, x2 \leftarrow x1\} \\
& \overbrace{rd(op_l(x1, x2, x3), mult(x4, x1))}
\end{aligned}
$$

Lemma 2359: $i(op_r(x4, rd(op_l(x1, x2, x3), x1), x1)) = i(x4)$

$$
\begin{aligned}
& \underbrace{i(op_r(x4, rd(op_l(x1, x2, x3), x1), x1))} \\
= & \quad \text{by Lemma 693 RL with } \{x3 \leftarrow x1, x2 \leftarrow op_l(x1, x2, x3), x1 \leftarrow x4\} \\
& \overbrace{i(\underbrace{rd(mult(mult(x4, rd(op_l(x1, x2, x3), x1)), x1), op_l(x1, x2, x3))})} \\
= & \quad \text{by Lemma 10 LR with } \{x2 \leftarrow op_l(x1, x2, x3), x1 \leftarrow mult(mult(x4, rd(op_l(x1, x2, x3), x1)), x1)\} \\
& \overbrace{\underbrace{rd(op_l(x1, x2, x3), mult(mult(x4, rd(op_l(x1, x2, x3), x1)), x1))}} \\
= & \quad \text{by Lemma 2358 RL with } \{x4 \leftarrow mult(x4, rd(op_l(x1, x2, x3), x1)), x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{rd(rd(op_l(x1, x2, x3), x1), mult(x4, rd(op_l(x1, x2, x3), x1)))} \\
= & \quad \text{by Lemma 9 LR with } \{x2 \leftarrow x4, x1 \leftarrow rd(op_l(x1, x2, x3), x1)\} \\
& \overbrace{i(x4)}
\end{aligned}
$$

Lemma 2360: $op_r(x1, rd(op_l(x2, x3, x4), x2), x2) = x1$

$$
\begin{aligned}
& \underbrace{op_r(x1, rd(op_l(x2, x3, x4), x2), x2)} \\
= & \quad \text{by Lemma 3 RL with } \{x1 \leftarrow op_r(x1, rd(op_l(x2, x3, x4), x2), x2)\} \\
& \overbrace{i(\underbrace{i(op_r(x1, rd(op_l(x2, x3, x4), x2), x2))})} \\
= & \quad \text{by Lemma 2359 LR with } \{x3 \leftarrow x4, x2 \leftarrow x3, x1 \leftarrow x2, x4 \leftarrow x1\} \\
& \underbrace{i(\overbrace{i(x1)})} \\
= & \quad \text{by Lemma 3 LR with } \{x1 \leftarrow x1\} \\
& \overbrace{x1}
\end{aligned}
$$

Lemma 2361: $rd(x1, rd(x2, op_l(x2, x3, x4))) = rd(mult(x1, op_l(x2, x3, x4)), x2)$

$$
\begin{aligned}
& \underbrace{rd(x1, rd(x2, op_l(x2, x3, x4)))} \\
= & \quad \text{by Lemma 10 RL with } \{x2 \leftarrow x1, x1 \leftarrow rd(x2, op_l(x2, x3, x4))\} \\
& \overbrace{i(\underbrace{rd(rd(x2, op_l(x2, x3, x4)), x1)})} \\
= & \quad \text{by Lemma 2356 LR with } \{x4 \leftarrow x1, x3 \leftarrow x4, x2 \leftarrow x3, x1 \leftarrow x2\} \\
& \underbrace{i(\overbrace{rd(x2, mult(x1, op_l(x2, x3, x4)))})} \\
= & \quad \text{by Lemma 10 LR with } \{x2 \leftarrow mult(x1, op_l(x2, x3, x4)), x1 \leftarrow x2\} \\
& \overbrace{rd(mult(x1, op_l(x2, x3, x4)), x2)}
\end{aligned}
$$

Lemma 2362: $rd(x1, mult(i(x2), op_l(x1, x3, x4))) = rd(x2, rd(op_l(x1, x3, x4), x1))$

$$
\begin{aligned}
& \underbrace{rd(x1, mult(i(x2), op_l(x1, x3, x4)))} \\
= & \quad \text{by Lemma 2251 RL with } \{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \overbrace{\underbrace{mult(rd(x2, op_l(x1, x3, x4)), x1)}} \\
= & \quad \text{by Lemma 2354 LR with } \{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \overbrace{rd(x2, rd(op_l(x1, x3, x4), x1))}
\end{aligned}
$$

Lemma 2363: $rd(x4, rd(x1, op_l(x1, x2, x3))) = mult(x4, rd(op_l(x1, x2, x3), x1))$

$\underbrace{rd(x4, rd(x1, op_l(x1, x2, x3)))}$

$=$ by Lemma 2357 LR with $\{x4 \leftarrow x3, x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x4\}$

$\overbrace{\underbrace{mult(rd(x4, x1), op_l(x1, x2, x3))}}$

$=$ by Lemma 2247 RL with $\{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{rd(op_l(x1, x2, x3), mult(i(x4), \underbrace{x1}))}$

$=$ by Axiom 4 LR with $\{x2 \leftarrow mult(i(x4), op_l(x1, x2, x3)), x1 \leftarrow x1\}$

$rd(op_l(x1, x2, x3), \underbrace{mult(i(x4), \overbrace{mult(rd(x1, mult(i(x4), op_l(x1, x2, x3))), mult(i(x4), op_l(x1, x2, x3)))})})$

$=$ by Axiom 6 LR with $\{x3 \leftarrow op_l(x1, x2, x3), x2 \leftarrow rd(x1, mult(i(x4), op_l(x1, x2, x3))), x1 \leftarrow i(x4)\}$

$\underbrace{rd(op_l(x1, x2, x3), \overbrace{mult(mult(i(x4), mult(rd(x1, mult(i(x4), op_l(x1, x2, x3))), i(x4))), op_l(x1, x2, x3))})}$

$=$ by Lemma 9 LR with $\{x2 \leftarrow mult(i(x4), mult(rd(x1, mult(i(x4), op_l(x1, x2, x3))), i(x4))), x1 \leftarrow op_l(x1, x2, x3)\}$

$\overbrace{\underbrace{i(mult(i(x4), mult(rd(x1, mult(i(x4), op_l(x1, x2, x3))), i(x4))))}}$

$=$ by Lemma 59 LR with $\{x2 \leftarrow rd(x1, mult(i(x4), op_l(x1, x2, x3))), x1 \leftarrow i(x4)\}$

$\overbrace{rd(i(i(x4)), mult(i(x4), \underbrace{rd(x1, mult(i(x4), op_l(x1, x2, x3)))}))}$

$=$ by Lemma 2362 LR with $\{x4 \leftarrow x3, x3 \leftarrow x2, x2 \leftarrow x4, x1 \leftarrow x1\}$

$\underbrace{rd(i(i(x4)), mult(i(x4), \overbrace{rd(x4, rd(op_l(x1, x2, x3), x1))}))}$

$=$ by Lemma 64 LR with $\{x3 \leftarrow rd(x4, rd(op_l(x1, x2, x3), x1)), x2 \leftarrow x4, x1 \leftarrow i(x4)\}$

$\overbrace{\underbrace{rd(mult(x4, i(rd(x4, rd(op_l(x1, x2, x3), x1)))), i(x4))}}$

$=$ by Lemma 61 LR with $\{x3 \leftarrow x4, x2 \leftarrow rd(x4, rd(op_l(x1, x2, x3), x1)), x1 \leftarrow x4\}$

$\overbrace{\underbrace{rd(x4, mult(i(x4), rd(x4, rd(op_l(x1, x2, x3), x1))))}}$

$=$ by Lemma 1090 LR with $\{x3 \leftarrow rd(op_l(x1, x2, x3), x1), x2 \leftarrow x4, x1 \leftarrow x4\}$

$\overbrace{mult(x4, \underbrace{mult(rd(rd(op_l(x1, x2, x3), x1), x4), x4)})}$

$=$ by Axiom 4 RL with $\{x2 \leftarrow x4, x1 \leftarrow rd(op_l(x1, x2, x3), x1)\}$

$mult(x4, \overbrace{rd(op_l(x1, x2, x3), x1)})$

Lemma 2364: $op_r(x1, i(x2), rd(x2, op_l(x2, x3, x4))) = rd(rd(mult(x1, i(x2)), rd(op_l(x2, x3, x4), x2)), op_l(i(x2), x3, x4))$

$op_r(x1, i(x2), \underbrace{rd(x2, op_l(x2, x3, x4))}_{})$

$=$ by Lemma 1056 RL with $\{x3 \leftarrow x4, x2 \leftarrow x3, x1 \leftarrow x2\}$

$op_r(x1, i(x2), \overbrace{mult(\underbrace{x2}_{}, op_l(i(x2), x3, x4))}^{})$

$=$ by Lemma 3 RL with $\{x1 \leftarrow x2\}$

$\underbrace{op_r(x1, i(x2), mult(\overbrace{i(i(x2))}^{}, op_l(i(x2), x3, x4)))}_{}$

$=$ by Lemma 1891 LR with $\{x3 \leftarrow op_l(i(x2), x3, x4), x2 \leftarrow i(x2), x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_r(x1, i(x2), rd(\overbrace{op_l(i(x2), x3, x4)}^{}, i(x2)))}_{}}^{}$

$=$ by Axiom 16 LR with $\{x3 \leftarrow rd(op_l(i(x2), x3, x4), i(x2)), x2 \leftarrow i(x2), x1 \leftarrow x1\}$

$\overbrace{rd(mult(mult(x1, i(x2)), rd(\overbrace{op_l(i(x2), x3, x4)}^{}, i(x2))), \underbrace{mult(i(x2), rd(op_l(i(x2), x3, x4), i(x2)))}_{})}^{}$

$=$ by Lemma 69 LR with $\{x2 \leftarrow op_l(i(x2), x3, x4), x1 \leftarrow x2\}$

$rd(\underbrace{mult(mult(x1, i(x2)), rd(op_l(i(x2), x3, x4), i(x2)))}_{}, \overbrace{mult(mult(i(x2), op_l(i(x2), x3, x4)), x2)}^{})$

$=$ by Lemma 2363 RL with $\{x3 \leftarrow x4, x2 \leftarrow x3, x1 \leftarrow i(x2), x4 \leftarrow mult(x1, i(x2))\}$

$rd(\overbrace{rd(mult(x1, i(x2)), \underbrace{rd(i(x2), op_l(i(x2), x3, x4))}_{})}^{}, mult(mult(i(x2), op_l(i(x2), x3, x4)), x2))$

$=$ by Lemma 1074 LR with $\{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x2\}$

$rd(rd(mult(x1, i(x2)), \overbrace{rd(op_l(x2, x3, x4), x2)}^{}), \underbrace{mult(mult(i(x2), op_l(i(x2), x3, x4)), x2)}_{})$

$=$ by Lemma 1420 LR with $\{x4 \leftarrow x2, x3 \leftarrow x4, x2 \leftarrow x3, x1 \leftarrow i(x2)\}$

$rd(rd(mult(x1, i(x2)), rd(op_l(x2, x3, x4), x2)), \overbrace{\underbrace{mult(i(x2), mult(op_l(i(x2), x3, x4), x2))}_{}}^{})$

$=$ by Axiom 10 LR with $\{x2 \leftarrow op_l(i(x2), x3, x4), x1 \leftarrow x2\}$

$rd(rd(mult(x1, i(x2)), rd(op_l(x2, x3, x4), x2)), \overbrace{\underbrace{op_t(op_l(i(x2), x3, x4), x2)}_{}}^{})$

$=$ by Axiom 15 LR with $\{x4 \leftarrow x2, x3 \leftarrow x4, x2 \leftarrow x3, x1 \leftarrow i(x2)\}$

$rd(rd(mult(x1, i(x2)), rd(op_l(x2, x3, x4), x2)), \overbrace{op_l(\underbrace{op_t(i(x2), x2)}_{}, x3, x4)}^{})$

$=$ by Lemma 7 LR with $\{x1 \leftarrow x2\}$

$rd(rd(mult(x1, i(x2)), rd(op_l(x2, x3, x4), x2)), op_l(\overbrace{i(x2)}^{}, x3, x4))$

Lemma 2365: $op_r(x1, i(x2), rd(x2, op_l(x2, x3, x4))) = rd(op_l(x2, x3, x4), rd(rd(op_l(x2, x3, x4), x2), mult(x1, i(x2))))$

$\underbrace{op_r(x1, i(x2), rd(x2, op_l(x2, x3, x4)))}_{}$

$=$ by Lemma 2364 LR with $\{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{rd(rd(mult(x1, i(x2)), rd(\overbrace{op_l(x2, x3, x4)}^{}, x2)), \underbrace{op_l(i(x2), x3, x4)}_{})}^{}$

$=$ by Lemma 1049 RL with $\{x3 \leftarrow x4, x2 \leftarrow x3, x1 \leftarrow x2\}$

$\underbrace{rd(rd(mult(x1, i(x2)), rd(op_l(x2, x3, x4), x2)), \overbrace{i(op_l(x2, x3, x4))}^{})}_{}$

$=$ by Lemma 39 LR with $\{x3 \leftarrow op_l(x2, x3, x4), x2 \leftarrow rd(op_l(x2, x3, x4), x2), x1 \leftarrow mult(x1, i(x2))\}$

$\overbrace{rd(op_l(x2, x3, x4), rd(rd(op_l(x2, x3, x4), x2), mult(x1, i(x2))))}^{}$

Lemma 2366: $op_r(x1, i(x2), rd(x2, op_l(x2, x3, x4))) = op_r(mult(mult(x1, i(x2)), x2), op_l(x2, x3, x4), op_l(x2, x3, x4))$

$\underbrace{op_r(x1, i(x2), rd(x2, op_l(x2, x3, x4)))}$

$=$ by Lemma 2365 LR with $\{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{rd(op_l(x2, x3, x4), \underbrace{rd(rd(op_l(x2, x3, x4), x2), mult(x1, i(x2)))})}$

$=$ by Lemma 2358 LR with $\{x4 \leftarrow mult(x1, i(x2)), x3 \leftarrow x4, x2 \leftarrow x3, x1 \leftarrow x2\}$

$\underbrace{rd(op_l(x2, x3, x4), \overbrace{rd(op_l(x2, x3, x4), mult(mult(x1, i(x2)), x2))})}$

$=$ by Lemma 353 LR with $\{x2 \leftarrow mult(mult(x1, i(x2)), x2), x1 \leftarrow op_l(x2, x3, x4)\}$

$\overbrace{op_r(mult(mult(x1, i(x2)), x2), op_l(x2, x3, x4), op_l(x2, x3, x4))}$

Lemma 2367: $op_r(x1, i(x2), rd(x2, op_l(x2, x3, x4))) = rd(x2, i(mult(x1, i(x2))))$

$\underbrace{op_r(x1, i(x2), rd(x2, op_l(x2, x3, x4)))}$

$=$ by Lemma 2366 LR with $\{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_r(mult(mult(x1, i(x2)), x2), op_l(x2, x3, x4), op_l(x2, x3, x4))}}$

$=$ by Lemma 2133 LR with $\{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow mult(mult(x1, i(x2)), x2)\}$

$\overbrace{\underbrace{op_r(mult(mult(x1, i(x2)), x2), x2, x2)}}$

$=$ by Lemma 359 LR with $\{x2 \leftarrow x2, x1 \leftarrow mult(x1, i(x2))\}$

$\overbrace{rd(x2, i(mult(x1, i(x2))))}$

Lemma 2368: $op_r(x1, i(x2), rd(x2, op_l(x2, x3, x4))) = i(i(x1))$

$\underbrace{op_r(x1, i(x2), rd(x2, op_l(x2, x3, x4)))}$

$=$ by Lemma 2367 LR with $\{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{rd(x2, \underbrace{i(mult(x1, i(x2)))})}$

$=$ by Lemma 17 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{rd(x2, \overbrace{mult(i(x1), x2)})}$

$=$ by Lemma 9 LR with $\{x2 \leftarrow i(x1), x1 \leftarrow x2\}$

$\overbrace{i(i(x1))}$

Lemma 2369: $x1 = op_r(x1, x2, rd(i(x2), op_l(i(x2), x3, x4)))$

$\underbrace{x1}$

$=$ by Lemma 3 RL with $\{x1 \leftarrow x1\}$

$\underbrace{\overbrace{i(i(x1))}}$

$=$ by Lemma 2368 RL with $\{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow i(x2), x1 \leftarrow x1\}$

$\overbrace{op_r(x1, \underbrace{i(i(x2))}, rd(i(x2), op_l(i(x2), x3, x4)))}$

$=$ by Lemma 3 LR with $\{x1 \leftarrow x2\}$

$op_r(x1, \overbrace{x2}, rd(i(x2), op_l(i(x2), x3, x4)))$

Lemma 2370: $mult(x1, rd(x2, op_l(x2, x3, x4))) = rd(x2, mult(i(x1), op_l(x2, x3, x4)))$

$mult(x1, \underbrace{rd(x2, op_l(x2, x3, x4))})$

$=$ by Lemma 10 RL with $\{x2 \leftarrow x2, x1 \leftarrow op_l(x2, x3, x4)\}$

$\underbrace{mult(x1, \overbrace{i(\overbrace{rd(op_l(x2, x3, x4), x2)})})}$

$=$ by Lemma 16 RL with $\{x2 \leftarrow rd(op_l(x2, x3, x4), x2), x1 \leftarrow x1\}$

$\overbrace{i(\underbrace{mult(i(x1), \overbrace{rd(op_l(x2, x3, x4), x2)})})}$

$=$ by Lemma 2363 RL with $\{x3 \leftarrow x4, x2 \leftarrow x3, x1 \leftarrow x2, x4 \leftarrow i(x1)\}$

$\underbrace{i(\overbrace{rd(i(x1), \overbrace{rd(x2, op_l(x2, x3, x4))})})}$

$=$ by Lemma 10 LR with $\{x2 \leftarrow rd(x2, op_l(x2, x3, x4)), x1 \leftarrow i(x1)\}$

$\overbrace{\underbrace{rd(rd(x2, op_l(x2, x3, x4)), i(x1))}}$

$=$ by Lemma 2356 LR with $\{x4 \leftarrow i(x1), x3 \leftarrow x4, x2 \leftarrow x3, x1 \leftarrow x2\}$

$\overbrace{rd(x2, mult(i(x1), op_l(x2, x3, x4)))}$

Lemma 2371: $op_l(x1, rd(op_l(x1, x2, x3), x1), x4) = mult(rd(i(x4), rd(op_l(x1, x2, x3), x1)), mult(x4, op_l(x1, x2, x3)))$

$\underbrace{op_l(x1, rd(op_l(x1, x2, x3), x1), x4)}$

$=$ by Lemma 280 LR with $\{x3 \leftarrow x4, x2 \leftarrow op_l(x1, x2, x3), x1 \leftarrow x1\}$

$\overbrace{mult(\underbrace{mult(i(x4), rd(x1, op_l(x1, x2, x3)))}, mult(x4, op_l(x1, x2, x3)))}$

$=$ by Lemma 2370 LR with $\{x4 \leftarrow x3, x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow i(x4)\}$

$mult(\overbrace{\underbrace{rd(x1, mult(i(i(x4)), op_l(x1, x2, x3)))}}, mult(x4, op_l(x1, x2, x3)))$

$=$ by Lemma 2362 LR with $\{x4 \leftarrow x3, x3 \leftarrow x2, x2 \leftarrow i(x4), x1 \leftarrow x1\}$

$mult(\overbrace{rd(i(x4), rd(op_l(x1, x2, x3), x1))}, mult(x4, op_l(x1, x2, x3)))$

Lemma 2372: $op_l(x1, rd(op_l(x1, x2, x3), x1), x4) = mult(rd(x1, mult(x4, op_l(x1, x2, x3))), mult(x4, op_l(x1, x2, x3)))$

$\underbrace{op_l(x1, rd(op_l(x1, x2, x3), x1), x4)}$

$=$ by Lemma 2371 LR with $\{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{mult(\underbrace{rd(i(x4), rd(op_l(x1, x2, x3), x1))}, mult(x4, op_l(x1, x2, x3)))}$

$=$ by Lemma 40 LR with $\{x3 \leftarrow x1, x2 \leftarrow op_l(x1, x2, x3), x1 \leftarrow x4\}$

$mult(\overbrace{\underbrace{rd(rd(x1, op_l(x1, x2, x3)), x4)}}, mult(x4, op_l(x1, x2, x3)))$

$=$ by Lemma 2356 LR with $\{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$mult(\overbrace{rd(x1, mult(x4, op_l(x1, x2, x3)))}, mult(x4, op_l(x1, x2, x3)))$

Lemma 2373: $mult(rd(op_l(x1,x2,x3),x4),i(x1)) = rd(rd(op_l(x1,x2,x3),x1),x4)$

$\overbrace{mult(rd(op_l(x1,x2,x3),x4),i(x1))}$

$=$ by Lemma 44 RL with $\{x3 \leftarrow x1, x2 \leftarrow op_l(x1,x2,x3), x1 \leftarrow x4\}$

$\overbrace{i(\underbrace{mult(rd(x4,op_l(x1,x2,x3)),x1)})}$

$=$ by Lemma 2354 LR with $\{x4 \leftarrow x3, x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x4\}$

$\underbrace{i(\overbrace{rd(x4,rd(op_l(x1,x2,x3),x1))})}$

$=$ by Lemma 10 LR with $\{x2 \leftarrow rd(op_l(x1,x2,x3),x1), x1 \leftarrow x4\}$

$\overbrace{rd(rd(op_l(x1,x2,x3),x1),x4)}$

Lemma 2374: $rd(op_l(x1,x2,x3),mult(op_r(x4,op_l(x1,x2,x3),op_l(x1,x2,x3)),x1)) = mult(op_r(rd(op_l(x1,x2,x3),x4),op_l(x1,x2$

$\overbrace{rd(op_l(x1,x2,x3),mult(op_r(x4,op_l(x1,x2,x3),op_l(x1,x2,x3)),x1))}$

$=$ by Lemma 2358 RL with $\{x4 \leftarrow op_r(x4,op_l(x1,x2,x3),op_l(x1,x2,x3)), x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{\underbrace{rd(rd(op_l(x1,x2,x3),x1),op_r(x4,op_l(x1,x2,x3),op_l(x1,x2,x3)))}}$

$=$ by Lemma 2373 RL with $\{x4 \leftarrow op_r(x4,op_l(x1,x2,x3),op_l(x1,x2,x3)), x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{mult(\underbrace{rd(op_l(x1,x2,x3),op_r(x4,op_l(x1,x2,x3),op_l(x1,x2,x3)))},i(x1))}$

$=$ by Lemma 369 LR with $\{x2 \leftarrow x4, x1 \leftarrow op_l(x1,x2,x3)\}$

$mult(\overbrace{op_r(rd(op_l(x1,x2,x3),x4),op_l(x1,x2,x3),op_l(x1,x2,x3))},i(x1))$

Lemma 2375: $rd(op_l(x1,x2,x3),rd(x1,i(x4))) = mult(op_r(rd(op_l(x1,x2,x3),x4),op_l(x1,x2,x3),op_l(x1,x2,x3)),i(x1))$

$rd(op_l(x1,x2,x3),\underbrace{rd(x1,i(x4))})$

$=$ by Lemma 357 RL with $\{x2 \leftarrow x1, x1 \leftarrow x4\}$

$rd(op_l(x1,x2,x3),\overbrace{mult(\underbrace{op_r(x4,x1,x1)},x1)})$

$=$ by Lemma 2133 RL with $\{x4 \leftarrow x3, x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x4\}$

$\underbrace{rd(op_l(x1,x2,x3),mult(\overbrace{op_r(x4,op_l(x1,x2,x3),op_l(x1,x2,x3))},x1))}$

$=$ by Lemma 2374 LR with $\{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{mult(op_r(rd(op_l(x1,x2,x3),x4),op_l(x1,x2,x3),op_l(x1,x2,x3)),i(x1))}$

Lemma 2376: $rd(op_l(x1,x2,x3),rd(x1,i(x4))) = rd(rd(op_l(x1,x2,x3),x4),x1)$

$\underbrace{rd(op_l(x1,x2,x3),rd(x1,i(x4)))}$

$=$ by Lemma 2375 LR with $\{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{mult(\underbrace{op_r(rd(op_l(x1,x2,x3),x4),op_l(x1,x2,x3),op_l(x1,x2,x3))},i(x1))}$

$=$ by Lemma 2133 LR with $\{x4 \leftarrow x3, x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow rd(op_l(x1,x2,x3),x4)\}$

$\underbrace{mult(\overbrace{op_r(rd(op_l(x1,x2,x3),x4),x1,x1)},i(x1))}$

$=$ by Lemma 352 LR with $\{x2 \leftarrow x1, x1 \leftarrow rd(op_l(x1,x2,x3),x4)\}$

$\overbrace{rd(rd(op_l(x1,x2,x3),x4),x1)}$

Lemma 2377: $rd(x1, rd(x4, op_l(i(x1), x2, x3))) = rd(rd(x1, x4), op_l(x1, x2, x3))$

$\underbrace{rd(x1, rd(x4, op_l(i(x1), x2, x3)))}$

$=$ by Lemma 39 RL with $\{x3 \leftarrow x1, x2 \leftarrow x4, x1 \leftarrow op_l(i(x1), x2, x3)\}$

$\overbrace{\underbrace{rd(rd(op_l(i(x1), x2, x3), x4), i(x1))}}$

$=$ by Lemma 2376 RL with $\{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow i(x1)\}$

$\overbrace{rd(op_l(i(x1), x2, x3), \underbrace{rd(i(x1), i(x4))})}$

$=$ by Lemma 23 LR with $\{x2 \leftarrow x4, x1 \leftarrow x1\}$

$rd(\underbrace{op_l(i(x1), x2, x3)}, \overbrace{rd(x4, x1)})$

$=$ by Lemma 1049 RL with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{rd(\overbrace{i(op_l(x1, x2, x3))}, rd(x4, x1))}$

$=$ by Lemma 40 LR with $\{x3 \leftarrow x1, x2 \leftarrow x4, x1 \leftarrow op_l(x1, x2, x3)\}$

$\overbrace{rd(rd(x1, x4), op_l(x1, x2, x3))}$

Lemma 2378: $rd(op_t(x2, i(x3)), x1) = rd(op_t(x3, i(x2)), mult(rd(x1, x2), x3))$

$\underbrace{rd(op_t(x2, i(x3)), x1)}$

$=$ by Lemma 68 RL with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{\underbrace{rd(i(x1), op_t(i(x2), x3))}}$

$=$ by Lemma 1398 RL with $\{x3 \leftarrow i(x1), x2 \leftarrow i(x2), x1 \leftarrow x3\}$

$\overbrace{rd(op_t(x3, i(x2)), mult(\underbrace{rd(i(x2), i(x1))}, x3))}$

$=$ by Lemma 23 LR with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$rd(op_t(x3, i(x2)), mult(\overbrace{rd(x1, x2)}, x3))$

Lemma 2379: $rd(mult(op_l(x1, x2, x3), x4), mult(x1, x4)) = rd(op_l(x1, x2, x3), x1)$

$rd(mult(op_l(x1, x2, x3), \underbrace{x4}), mult(x1, x4))$

$=$ by Lemma 1422 RL with $\{x4 \leftarrow x3, x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x4\}$

$\underbrace{rd(mult(op_l(x1, x2, x3), \overbrace{op_l(x4, x1, op_l(x1, x2, x3))}), mult(x1, x4))}$

$=$ by Lemma 1472 LR with $\{x3 \leftarrow x1, x2 \leftarrow x4, x1 \leftarrow op_l(x1, x2, x3)\}$

$\overbrace{rd(op_l(x1, x2, x3), x1)}$

Lemma 2380: $rd(x1, asoc(op_t(x2, x3), rd(x3, i(x2)), op_t(x2, x3))) = mult(rd(x1, mult(rd(x3, i(x2)), i(x3))), op_r(op_t(x2, x3), x3, x$

$rd(x1, \underbrace{asoc(op_t(x2, x3), rd(x3, i(x2)), op_t(x2, x3))}$

= by Lemma 1808 RL with $\{x3 \leftarrow rd(x3, i(x2)), x2 \leftarrow x3, x1 \leftarrow op_t(x2, x3)\}$

$\underbrace{rd(x1, \overbrace{asoc(op_r(op_t(x2, x3), x3, x3), rd(x3, i(x2)), op_r(op_t(x2, x3), x3, x3))}}$

= by Lemma 1755 RL with $\{x3 \leftarrow rd(x3, i(x2)), x2 \leftarrow op_r(op_t(x2, x3), x3, x3), x1 \leftarrow x1\}$

$\overbrace{mult(rd(x1, op_r(op_t(x2, x3), x3, x3)), \underbrace{op_l(op_r(op_t(x2, x3), x3, x3), rd(x3, i(x2)), op_r(op_t(x2, x3), x3, x3))}}$

= by Lemma 771 LR with $\{x2 \leftarrow rd(x3, i(x2)), x1 \leftarrow op_r(op_t(x2, x3), x3, x3)\}$

$mult(\underbrace{rd(x1, op_r(op_t(x2, x3), x3, x3))}, \overbrace{mult(asoc(rd(x3, i(x2)), op_r(op_t(x2, x3), x3, x3), op_r(op_t(x2, x3), x3, x3)), op_r(op_t(x2$

= by Lemma 1578 RL with $\{x3 \leftarrow op_r(op_t(x2, x3), x3, x3), x2 \leftarrow rd(x3, i(x2)), x1 \leftarrow rd(x1, op_r(op_t(x2, x3), x3, x3))\}$

$\underbrace{mult(\overbrace{op_r(rd(x1, op_r(op_t(x2, x3), x3, x3)), asoc(rd(x3, i(x2)), op_r(op_t(x2, x3), x3, x3), op_r(op_t(x2, x3), x3, x3)), op_r(op_t(x2,$

= by Lemma 91 LR with $\{x3 \leftarrow op_r(op_t(x2, x3), x3, x3), x2 \leftarrow asoc(rd(x3, i(x2)), op_r(op_t(x2, x3), x3, x3), op_r(op_t(x2, x3), x3, x$

$\overbrace{mult(\underbrace{mult(rd(x1, op_r(op_t(x2, x3), x3, x3)), asoc(rd(x3, i(x2)), op_r(op_t(x2, x3), x3, x3), op_r(op_t(x2, x3), x3, x3)))}, op_r(op_t(x$

= by Lemma 1300 RL with $\{x2 \leftarrow rd(x3, i(x2)), x3 \leftarrow op_r(op_t(x2, x3), x3, x3), x1 \leftarrow rd(x1, op_r(op_t(x2, x3), x3, x3))\}$

$mult(\overbrace{rd(rd(x1, op_r(op_t(x2, x3), x3, x3)), asoc(op_r(op_t(x2, x3), x3, x3), rd(x3, i(x2)), op_r(op_t(x2, x3), x3, x3)))}, op_r(op_t(x2,$

= by Lemma 1575 RL with $\{x3 \leftarrow rd(x3, i(x2)), x2 \leftarrow op_r(op_t(x2, x3), x3, x3), x1 \leftarrow x1\}$

$mult(\overbrace{rd(x1, \underbrace{op_l(op_r(op_t(x2, x3), x3, x3), op_r(op_t(x2, x3), x3, x3), rd(x3, i(x2))))}), op_r(op_t(x2, x3), x3, x3))$

= by Lemma 427 LR with $\{x2 \leftarrow rd(x3, i(x2)), x1 \leftarrow op_r(op_t(x2, x3), x3, x3)\}$

$mult(rd(x1, \overbrace{mult(rd(x3, i(x2)), rd(\underbrace{op_r(op_t(x2, x3), x3, x3)}, rd(x3, i(x2))))}), op_r(op_t(x2, x3), x3, x3))$

= by Axiom 14 RL with $\{x4 \leftarrow x3, x3 \leftarrow x3, x2 \leftarrow x3, x1 \leftarrow x2\}$

$mult(rd(x1, mult(rd(x3, i(x2)), rd(\overbrace{op_t(op_r(x2, x3, x3), x3)}, \underbrace{rd(x3, i(x2))}))), op_r(op_t(x2, x3), x3, x3))$

= by Lemma 357 RL with $\{x2 \leftarrow x3, x1 \leftarrow x2\}$

$mult(rd(x1, mult(rd(x3, i(x2)), \underbrace{rd(op_t(op_r(x2, x3, x3), x3), \overbrace{mult(op_r(x2, x3, x3), x3)})})), op_r(op_t(x2, x3), x3, x3))$

= by Lemma 36 LR with $\{x2 \leftarrow x3, x1 \leftarrow op_r(x2, x3, x3)\}$

$mult(rd(x1, mult(rd(x3, i(x2)), \overbrace{i(x3)})), op_r(op_t(x2, x3), x3, x3))$

Lemma 2381: $mult(rd(x1,x2),op_r(op_t(x2,x3),x3,x3)) = rd(x1,asoc(x2,x3,x2))$

$mult(rd(x1,\underbrace{x2}),op_r(op_t(x2,x3),x3,x3))$

$=$ by Lemma 3 RL with $\{x1 \leftarrow x2\}$

$mult(rd(x1,\overbrace{\underbrace{i(i(x2))}}),op_r(op_t(x2,x3),x3,x3))$

$=$ by Lemma 22 RL with $\{x2 \leftarrow i(x2), x1 \leftarrow x3\}$

$\underbrace{mult(rd(x1,\overbrace{mult(rd(x3,i(x2)),i(x3))}),op_r(op_t(x2,x3),x3,x3))}$

$=$ by Lemma 2380 RL with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{rd(x1,\underbrace{asoc(op_t(x2,x3),rd(x3,i(x2)),op_t(x2,x3))})}$

$=$ by Lemma 1631 LR with $\{x3 \leftarrow rd(x3,i(x2)), x2 \leftarrow x3, x1 \leftarrow x2\}$

$rd(x1,\underbrace{asoc(x2,rd(x3,i(x2)),x2)})$

$=$ by Lemma 1750 LR with $\{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x2\}$

$rd(x1,\underbrace{asoc(x2,mult(x3,x2),x2)})$

$=$ by Lemma 262 LR with $\{x2 \leftarrow x3, x1 \leftarrow x2\}$

$rd(x1,\overbrace{asoc(x2,x3,x2)})$

Lemma 2382: $rd(mult(x1,x2),asoc(x3,x4,x3)) = mult(mult(x1,rd(x2,x3)),op_r(op_t(op_l(x3,rd(x2,x3)),x1),x4),x4,x4))$

$rd(mult(x1,x2),\underbrace{asoc(x3,x4,x3)})$

$=$ by Lemma 2129 RL with $\{x4 \leftarrow x4, x3 \leftarrow x1, x2 \leftarrow rd(x2,x3), x1 \leftarrow x3\}$

$\underbrace{rd(mult(x1,x2),\overbrace{asoc(op_l(x3,rd(x2,x3),x1),x4,op_l(x3,rd(x2,x3),x1))})}$

$=$ by Lemma 2381 RL with $\{x3 \leftarrow x4, x2 \leftarrow op_l(x3,rd(x2,x3),x1), x1 \leftarrow mult(x1,x2)\}$

$\overbrace{mult(\underbrace{rd(mult(x1,x2),op_l(x3,rd(x2,x3),x1))},op_r(op_t(op_l(x3,rd(x2,x3),x1),x4),x4,x4))}$

$=$ by Lemma 1914 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$mult(\overbrace{mult(x1,rd(x2,x3))},op_r(op_t(op_l(x3,rd(x2,x3),x1),x4),x4,x4))$

Lemma 2383: $mult(x1,rd(x2,asoc(x3,x4,x3))) = rd(mult(x1,x2),asoc(x3,x4,x3))$

$mult(x1,\underbrace{rd(x2,asoc(x3,x4,x3))})$

$=$ by Lemma 2381 RL with $\{x3 \leftarrow x4, x2 \leftarrow x3, x1 \leftarrow x2\}$

$\underbrace{mult(x1,\overbrace{mult(rd(x2,x3),op_r(op_t(x3,x4),x4,x4))})}$

$=$ by Lemma 132 RL with $\{x3 \leftarrow op_r(op_t(x3,x4),x4,x4), x2 \leftarrow rd(x2,x3), x1 \leftarrow x1\}$

$\overbrace{mult(mult(x1,rd(x2,x3)),\underbrace{op_l(op_r(op_t(x3,x4),x4,x4),rd(x2,x3),x1)})}$

$=$ by Axiom 18 RL with $\{x5 \leftarrow x4, x4 \leftarrow x4, x3 \leftarrow x1, x2 \leftarrow rd(x2,x3), x1 \leftarrow op_t(x3,x4)\}$

$mult(mult(x1,rd(x2,x3)),\overbrace{op_r(\underbrace{op_l(op_t(x3,x4),rd(x2,x3),x1)},x4,x4)})$

$=$ by Axiom 15 RL with $\{x4 \leftarrow x4, x3 \leftarrow x1, x2 \leftarrow rd(x2,x3), x1 \leftarrow x3\}$

$\underbrace{mult(mult(x1,rd(x2,x3)),op_r(\overbrace{op_t(op_l(x3,rd(x2,x3),x1),x4)},x4,x4))}$

$=$ by Lemma 2382 RL with $\{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{rd(mult(x1,x2),asoc(x3,x4,x3))}$

Lemma 2384: $rd(mult(x1,x2),asoc(x4,x3,x3)) = mult(x1,rd(x2,asoc(x4,x3,x3)))$

$$
\begin{aligned}
& \underbrace{rd(mult(x1,x2),asoc(x4,x3,x3))} \\
= \quad & \text{by Lemma 756 RL with } \{x2 \leftarrow x4, x1 \leftarrow x3\} \\
& \underbrace{rd(mult(x1,x2),\overbrace{asoc(x3,i(x4),x3)})} \\
= \quad & \text{by Lemma 2383 RL with } \{x4 \leftarrow i(x4), x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{mult(x1,rd(x2,\underbrace{asoc(x3,i(x4),x3)}))} \\
= \quad & \text{by Lemma 756 LR with } \{x2 \leftarrow x4, x1 \leftarrow x3\} \\
& mult(x1,rd(x2,\overbrace{asoc(x4,x3,x3)}))
\end{aligned}
$$

Lemma 2385: $rd(x1,rd(x2,asoc(x3,x4,x4))) = i(rd(x2,rd(x1,asoc(x4,x3,x4))))$

$$
\begin{aligned}
& \underbrace{rd(x1,rd(x2,asoc(x3,x4,x4)))} \\
= \quad & \text{by Lemma 10 RL with } \{x2 \leftarrow x1, x1 \leftarrow rd(x2,asoc(x3,x4,x4))\} \\
& \overbrace{i(rd(rd(\underbrace{x2},asoc(x3,x4,x4)),x1))} \\
= \quad & \text{by Axiom 4 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& i(rd(rd(\overbrace{mult(rd(x2,x1),x1)},asoc(x3,x4,x4)),x1)) \\
= \quad & \text{by Lemma 12 RL with } \{x2 \leftarrow mult(rd(x2,x1),x1), x1 \leftarrow x2\} \\
& i(rd(rd(\overbrace{mult(x2,\underbrace{mult(i(x2),mult(rd(x2,x1),x1))})},asoc(x3,x4,x4)),x1)) \\
= \quad & \text{by Lemma 1417 RL with } \{x3 \leftarrow x1, x2 \leftarrow x1, x1 \leftarrow x2\} \\
& i(\underbrace{rd(rd(mult(x2,\overbrace{mult(i(mult(x2,x1)),mult(x2,x1))}),asoc(x3,x4,x4))},x1)) \\
= \quad & \text{by Lemma 2384 LR with } \{x3 \leftarrow x4, x4 \leftarrow x3, x2 \leftarrow mult(i(mult(x2,x1)),mult(x2,x1)), x1 \leftarrow x2\} \\
& i(rd(\overbrace{mult(x2,\underbrace{rd(mult(i(mult(x2,x1)),mult(x2,x1)),asoc(x3,x4,x4))})},x1)) \\
= \quad & \text{by Lemma 2384 LR with } \{x3 \leftarrow x4, x4 \leftarrow x3, x2 \leftarrow mult(x2,x1), x1 \leftarrow i(mult(x2,x1))\} \\
& i(\underbrace{rd(mult(x2,\overbrace{mult(i(mult(x2,x1)),rd(mult(x2,x1),asoc(x3,x4,x4)))}),x1)}) \\
= \quad & \text{by Lemma 2274 RL with } \{x3 \leftarrow asoc(x3,x4,x4), x2 \leftarrow x1, x1 \leftarrow x2\} \\
& i(\overbrace{rd(x2,\underbrace{mult(i(x2),mult(mult(x2,x1),asoc(x3,x4,x4)))})}) \\
= \quad & \text{by Lemma 1045 LR with } \{x3 \leftarrow asoc(x3,x4,x4), x2 \leftarrow x1, x1 \leftarrow x2\} \\
& i(rd(x2,\overbrace{mult(rd(x1,x2),\underbrace{mult(x2,asoc(x3,x4,x4))})})) \\
= \quad & \text{by Lemma 1300 RL with } \{x2 \leftarrow x3, x3 \leftarrow x4, x1 \leftarrow x2\} \\
& i(rd(x2,\underbrace{mult(rd(x1,x2),\overbrace{rd(x2,asoc(x4,x3,x4))})})) \\
= \quad & \text{by Lemma 2383 LR with } \{x4 \leftarrow x3, x3 \leftarrow x4, x2 \leftarrow x2, x1 \leftarrow rd(x1,x2)\} \\
& i(rd(x2,\overbrace{rd(\underbrace{mult(rd(x1,x2),x2)},asoc(x4,x3,x4))})) \\
= \quad & \text{by Axiom 4 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& i(rd(x2,rd(\overbrace{x1},asoc(x4,x3,x4))))
\end{aligned}
$$

Lemma 2386: $rd(rd(x1, asoc(x2, x3, x2)), x4) = rd(x1, rd(x4, asoc(x3, x2, x2)))$

$$
\begin{aligned}
& \underbrace{rd(rd(x1, asoc(x2, x3, x2)), x4)} \\
= & \quad \text{by Lemma 10 RL with } \{x2 \leftarrow rd(x1, asoc(x2, x3, x2)), x1 \leftarrow x4\} \\
& \overbrace{\underbrace{i(rd(x4, rd(x1, asoc(x2, x3, x2))))}} \\
= & \quad \text{by Lemma 2385 RL with } \{x4 \leftarrow x2, x3 \leftarrow x3, x2 \leftarrow x4, x1 \leftarrow x1\} \\
& \overbrace{rd(x1, rd(x4, asoc(x3, x2, x2)))}
\end{aligned}
$$

Lemma 2387: $mult(i(x1), rd(asoc(x2, x3, x2), x4)) = rd(asoc(x2, x3, x2), mult(x1, x4))$

$$
\begin{aligned}
& \underbrace{mult(i(x1), rd(asoc(x2, x3, x2), x4))} \\
= & \quad \text{by Lemma 45 RL with } \{x3 \leftarrow asoc(x2, x3, x2), x2 \leftarrow x4, x1 \leftarrow x1\} \\
& \overbrace{i(\underbrace{mult(x1, rd(x4, asoc(x2, x3, x2)))})} \\
= & \quad \text{by Lemma 2383 LR with } \{x4 \leftarrow x3, x3 \leftarrow x2, x2 \leftarrow x4, x1 \leftarrow x1\} \\
& \underbrace{i(\overbrace{rd(mult(x1, x4), asoc(x2, x3, x2))})} \\
= & \quad \text{by Lemma 10 LR with } \{x2 \leftarrow asoc(x2, x3, x2), x1 \leftarrow mult(x1, x4)\} \\
& \overbrace{rd(asoc(x2, x3, x2), mult(x1, x4))}
\end{aligned}
$$

Lemma 2388: $op_r(x1, x2, rd(x5, asoc(x3, x4, x3))) = rd(mult(mult(x1, x2), x5), rd(mult(x2, x5), unit()))$

$op_r(x1, x2, \underbrace{rd(x5, asoc(x3, x4, x3))})$

$=$ by Lemma 10 RL with $\{x2 \leftarrow x5, x1 \leftarrow asoc(x3, x4, x3)\}$

$\underbrace{op_r(x1, x2, \overbrace{i(rd(asoc(x3, x4, x3), x5))})}$

$=$ by Lemma 818 RL with $\{x3 \leftarrow rd(asoc(x3, x4, x3), x5), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_r(x1, i(mult(x1, x2)), rd(asoc(x3, x4, x3), x5))}}$

$=$ by Axiom 16 LR with $\{x3 \leftarrow rd(asoc(x3, x4, x3), x5), x2 \leftarrow i(mult(x1, x2)), x1 \leftarrow x1\}$

$\overbrace{rd(mult(\underbrace{mult(x1, i(mult(x1, x2)))}, \underbrace{rd(asoc(x3, x4, x3), x5)}), mult(i(mult(x1, x2)), rd(asoc(x3, x4, x3), x5)))}$

$=$ by Lemma 20 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$rd(\underbrace{mult(\overbrace{i(x2)}, rd(asoc(x3, x4, x3), x5))}, mult(i(mult(x1, x2)), rd(asoc(x3, x4, x3), x5)))$

$=$ by Lemma 2387 LR with $\{x4 \leftarrow x5, x3 \leftarrow x4, x2 \leftarrow x3, x1 \leftarrow x2\}$

$rd(\overbrace{rd(asoc(x3, x4, x3), mult(x2, x5))}, \underbrace{mult(i(mult(x1, x2)), rd(asoc(x3, x4, x3), x5))})$

$=$ by Lemma 45 RL with $\{x3 \leftarrow asoc(x3, x4, x3), x2 \leftarrow x5, x1 \leftarrow mult(x1, x2)\}$

$\underbrace{rd(rd(asoc(x3, x4, x3), mult(x2, x5)), \overbrace{i(mult(mult(x1, x2), rd(x5, asoc(x3, x4, x3))))})}$

$=$ by Lemma 39 LR with $\{x3 \leftarrow mult(mult(x1, x2), rd(x5, asoc(x3, x4, x3))), x2 \leftarrow mult(x2, x5), x1 \leftarrow asoc(x3, x4, x3)\}$

$\overbrace{rd(\underbrace{mult(mult(x1, x2), rd(x5, asoc(x3, x4, x3)))}, rd(mult(x2, x5), asoc(x3, x4, x3)))}$

$=$ by Lemma 2383 LR with $\{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x5, x1 \leftarrow mult(x1, x2)\}$

$\underbrace{rd(\overbrace{rd(mult(mult(x1, x2), x5), asoc(x3, x4, x3))}, rd(mult(x2, x5), asoc(x3, x4, x3)))}$

$=$ by Lemma 2386 LR with $\{x4 \leftarrow rd(mult(x2, x5), asoc(x3, x4, x3)), x3 \leftarrow x4, x2 \leftarrow x3, x1 \leftarrow mult(mult(x1, x2), x5)\}$

$\overbrace{rd(mult(mult(x1, x2), x5), \underbrace{rd(rd(mult(x2, x5), asoc(x3, x4, x3)), asoc(x4, x3, x3))})}$

$=$ by Lemma 2386 LR with $\{x4 \leftarrow asoc(x4, x3, x3), x3 \leftarrow x4, x2 \leftarrow x3, x1 \leftarrow mult(x2, x5)\}$

$rd(mult(mult(x1, x2), x5), \overbrace{rd(mult(x2, x5), \underbrace{rd(asoc(x4, x3, x3), asoc(x4, x3, x3))})})$

$=$ by Lemma 2 LR with $\{x1 \leftarrow asoc(x4, x3, x3)\}$

$rd(mult(mult(x1, x2), x5), rd(mult(x2, x5), \overbrace{unit()}))$

Lemma 2389: $op_r(x1, x2, rd(x3, asoc(x4, x5, x4))) = op_r(x1, x2, x3)$

$\underbrace{op_r(x1, x2, rd(x3, asoc(x4, x5, x4)))}$

$=$ by Lemma 2388 LR with $\{x4 \leftarrow x5, x3 \leftarrow x4, x5 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{rd(mult(\overbrace{mult(x1, x2)}, x3), \underbrace{rd(mult(x2, x3), unit())})}$

$=$ by Lemma 1 LR with $\{x1 \leftarrow mult(x2, x3)\}$

$\underbrace{rd(mult(mult(x1, x2), x3), \overbrace{mult(x2, x3)})}$

$=$ by Axiom 16 RL with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_r(x1, x2, x3)}$

Lemma 2390: $op_r(x1, x2, op_l(x3, x4, x3)) = op_r(x1, x2, x3)$

$op_r(x1, x2, \underbrace{op_l(x3, x4, x3)})$

$=$ by Lemma 620 RL with $\{x2 \leftarrow x4, x1 \leftarrow x3\}$

$\underbrace{op_r(x1, x2, \overbrace{rd(x3, asoc(x3, x4, x3))})}$

$=$ by Lemma 2389 LR with $\{x5 \leftarrow x4, x4 \leftarrow x3, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_r(x1, x2, x3)}$

Lemma 2391: $op_r(x1, x2, op_r(x3, x4, x4)) = op_r(x1, x2, x3)$

$op_r(x1, x2, \underbrace{op_r(x3, x4, x4)})$

$=$ by Lemma 641 RL with $\{x2 \leftarrow x4, x1 \leftarrow x3\}$

$\underbrace{op_r(x1, x2, \overbrace{rd(x3, asoc(x4, x3, x4))})}$

$=$ by Lemma 2389 LR with $\{x5 \leftarrow x3, x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_r(x1, x2, x3)}$

Lemma 2392: $op_r(x1, x2, op_t(x3, x4)) = op_r(x1, x2, x3)$

$\underbrace{op_r(x1, x2, op_t(x3, x4))}$

$=$ by Lemma 2391 RL with $\{x4 \leftarrow x4, x3 \leftarrow op_t(x3, x4), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_r(x1, x2, \underbrace{op_r(op_t(x3, x4), x4, x4)})}$

$=$ by Lemma 745 RL with $\{x2 \leftarrow x4, x1 \leftarrow x3\}$

$\underbrace{op_r(x1, x2, \overbrace{op_l(x3, x4, x3)})}$

$=$ by Lemma 2390 LR with $\{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_r(x1, x2, x3)}$

Lemma 2393: $op_r(x1, x2, x3) = op_r(x1, x2, rd(x3, asoc(x4, x3, x3)))$

$\underbrace{op_r(x1, x2, x3)}$

$=$ by Lemma 2389 RL with $\{x5 \leftarrow i(x4), x4 \leftarrow i(x3), x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_r(x1, x2, rd(x3, asoc(i(x3), i(x4), \underbrace{i(x3)})))}$

$=$ by Lemma 3 RL with $\{x1 \leftarrow i(x3)\}$

$op_r(x1, x2, rd(x3, \underbrace{asoc(i(x3), i(x4), \overbrace{i(i(i(x3)))})}))$

$=$ by Lemma 1480 RL with $\{x3 \leftarrow i(i(x3)), x2 \leftarrow x4, x1 \leftarrow x3\}$

$op_r(x1, x2, \underbrace{rd(x3, \overbrace{i(asoc(x3, x4, i(i(x3))))})})$

$=$ by Lemma 357 RL with $\{x2 \leftarrow x3, x1 \leftarrow asoc(x3, x4, i(i(x3)))\}$

$op_r(x1, x2, \overbrace{mult(op_r(asoc(x3, x4, i(i(x3))), x3, x3), \underbrace{x3})})$

$=$ by Lemma 1838 RL with $\{x3 \leftarrow i(i(x3)), x2 \leftarrow x4, x1 \leftarrow x3\}$

$op_r(x1, x2, \underbrace{mult(op_r(asoc(x3, x4, i(i(x3))), x3, x3), \overbrace{op_l(x3, x3, asoc(x3, x4, i(i(x3))))})})$

$=$ by Lemma 873 RL with $\{x2 \leftarrow asoc(x3, x4, i(i(x3))), x1 \leftarrow x3\}$

$op_r(x1, x2, \overbrace{\underbrace{op_t(op_r(mult(x3, asoc(x3, x4, i(i(x3)))), x3, x3), x3)}})$

$=$ by Axiom 14 LR with $\{x4 \leftarrow x3, x3 \leftarrow x3, x2 \leftarrow x3, x1 \leftarrow mult(x3, asoc(x3, x4, i(i(x3))))\}$

$op_r(x1, x2, \overbrace{\underbrace{op_r(op_t(mult(x3, asoc(x3, x4, i(i(x3)))), x3), x3, x3)}})$

$=$ by Lemma 385 LR with $\{x2 \leftarrow asoc(x3, x4, i(i(x3))), x1 \leftarrow x3\}$

$op_r(x1, x2, \overbrace{\underbrace{mult(asoc(x3, x4, i(i(x3))), x3)}})$

$=$ by Lemma 2103 LR with $\{x3 \leftarrow i(i(x3)), x2 \leftarrow x4, x1 \leftarrow x3\}$

$op_r(x1, x2, \overbrace{mult(x3, asoc(x3, x4, \underbrace{i(i(x3))}))})$

$=$ by Lemma 3 LR with $\{x1 \leftarrow x3\}$

$op_r(x1, x2, \underbrace{mult(x3, asoc(x3, x4, \overbrace{x3}))})$

$=$ by Lemma 1295 LR with $\{x3 \leftarrow x4, x2 \leftarrow x3, x1 \leftarrow x3\}$

$op_r(x1, x2, \overbrace{rd(x3, asoc(x4, x3, x3))})$

Lemma 2394: $op_r(x1, x2, rd(x3, i(x4))) = op_r(x1, x2, mult(x4, x3))$

$op_r(x1, x2, \underbrace{rd(x3, i(x4))})$

$=$ by Lemma 359 RL with $\{x2 \leftarrow x3, x1 \leftarrow x4\}$

$\underbrace{op_r(x1, x2, \overbrace{op_r(mult(x4, x3), x3, x3)})}$

$=$ by Lemma 2391 LR with $\{x4 \leftarrow x3, x3 \leftarrow mult(x4, x3), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_r(x1, x2, mult(x4, x3))}$

Lemma 2395: $op_r(x1, x2, mult(x3, i(x4))) = op_r(x1, x2, rd(x3, x4))$

$$
\begin{aligned}
& \overbrace{op_r(x1, x2, mult(x3, i(x4)))} \\
= \quad & \text{by Lemma 2391 RL with } \{x4 \leftarrow x4, x3 \leftarrow mult(x3, i(x4)), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, x2, \underbrace{op_r(mult(x3, i(x4)), x4, x4)})} \\
= \quad & \text{by Lemma 364 LR with } \{x2 \leftarrow x4, x1 \leftarrow x3\} \\
& op_r(x1, x2, \overbrace{rd(x3, x4)})
\end{aligned}
$$

Lemma 2396: $op_r(x1, x2, mult(i(x3), x4)) = op_r(x1, x2, op_r(mult(i(x3), x4), x4, op_t(x4, x3)))$

$$
\begin{aligned}
& \overbrace{op_r(x1, x2, mult(i(x3), x4))} \\
= \quad & \text{by Lemma 2391 RL with } \{x4 \leftarrow op_t(x4, x3), x3 \leftarrow mult(i(x3), x4), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, x2, op_r(mult(i(x3), x4), \underbrace{op_t(x4, x3)}, op_t(x4, x3)))} \\
= \quad & \text{by Lemma 1721 RL with } \{x3 \leftarrow x4, x2 \leftarrow x3, x1 \leftarrow x4\} \\
& op_r(x1, x2, op_r(mult(i(x3), x4), \overbrace{op_t(x4, \underbrace{op_r(x3, x4, x4)})}, op_t(x4, x3))) \\
= \quad & \text{by Lemma 353 RL with } \{x2 \leftarrow x3, x1 \leftarrow x4\} \\
& op_r(x1, x2, op_r(mult(i(x3), x4), \underbrace{op_t(x4, \overbrace{rd(x4, rd(x4, x3))})}, op_t(x4, x3))) \\
= \quad & \text{by Lemma 1287 LR with } \{x2 \leftarrow x3, x3 \leftarrow x4, x1 \leftarrow x4\} \\
& op_r(x1, x2, op_r(mult(i(x3), x4), \overbrace{op_t(x4, \underbrace{rd(rd(i(x4), x4), x3)})}, op_t(x4, x3))) \\
= \quad & \text{by Lemma 10 RL with } \{x2 \leftarrow rd(i(x4), x4), x1 \leftarrow x3\} \\
& op_r(x1, x2, op_r(mult(i(x3), x4), \underbrace{op_t(x4, \overbrace{i(rd(x3, rd(i(x4), x4)))})}, op_t(x4, x3))) \\
= \quad & \text{by Lemma 1290 RL with } \{x3 \leftarrow x3, x2 \leftarrow x4, x1 \leftarrow x4\} \\
& op_r(x1, x2, op_r(mult(i(x3), x4), \overbrace{op_t(\underbrace{x4}, rd(x4, mult(i(x3), x4)))}, op_t(x4, x3))) \\
= \quad & \text{by Axiom 4 LR with } \{x2 \leftarrow mult(i(x3), x4), x1 \leftarrow x4\} \\
& op_r(x1, x2, op_r(mult(i(x3), x4), \underbrace{op_t(\overbrace{mult(rd(x4, mult(i(x3), x4)), mult(i(x3), x4))}, rd(x4, mult(i(x3), x4)))}, op_t(x4, x3))) \\
= \quad & \text{by Lemma 264 RL with } \{x2 \leftarrow rd(x4, mult(i(x3), x4)), x1 \leftarrow mult(i(x3), x4)\} \\
& op_r(x1, x2, op_r(mult(i(x3), x4), \overbrace{mult(mult(i(x3), x4), \underbrace{op_l(rd(x4, mult(i(x3), x4)), rd(x4, mult(i(x3), x4)), mult(i(x3), x4))}}) \\
= \quad & \text{by Lemma 292 LR with } \{x3 \leftarrow mult(i(x3), x4), x2 \leftarrow x4, x1 \leftarrow rd(x4, mult(i(x3), x4))\} \\
& op_r(x1, x2, \underbrace{op_r(mult(i(x3), x4), mult(mult(i(x3), x4), \overbrace{op_l(rd(x4, mult(i(x3), x4)), x4, mult(i(x3), x4))}), op_t(x4, x3))}) \\
= \quad & \text{by Lemma 1953 RL with } \{x3 \leftarrow op_t(x4, x3), x2 \leftarrow x4, x1 \leftarrow mult(i(x3), x4)\} \\
& op_r(x1, x2, \overbrace{\underbrace{op_r(mult(i(x3), x4), op_t(x4, i(mult(i(x3), x4))))}, op_t(x4, x3))}) \\
= \quad & \text{by Lemma 1950 LR with } \{x3 \leftarrow op_t(x4, x3), x2 \leftarrow x4, x1 \leftarrow mult(i(x3), x4)\} \\
& op_r(x1, x2, \overbrace{op_r(mult(i(x3), x4), x4, op_t(x4, x3))})
\end{aligned}
$$

Lemma 2397: $op_r(x1, x2, mult(i(x3), x4)) = op_r(x1, x2, rd(x4, x3))$

$$
\begin{aligned}
& \underbrace{op_r(x1, x2, mult(i(x3), x4))} \\
= \quad & \text{by Lemma 2396 LR with } \{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, x2, \underbrace{op_r(mult(i(x3), x4), x4, op_t(x4, x3))})} \\
= \quad & \text{by Axiom 5 LR with } \{x2 \leftarrow x4, x1 \leftarrow op_r(mult(i(x3), x4), x4, op_t(x4, x3))\} \\
& op_r(x1, x2, \overbrace{rd(mult(\underbrace{op_r(mult(i(x3), x4), x4, op_t(x4, x3))}, x4), x4)}) \\
= \quad & \text{by Axiom 16 LR with } \{x3 \leftarrow op_t(x4, x3), x2 \leftarrow x4, x1 \leftarrow mult(i(x3), x4)\} \\
& op_r(x1, x2, rd(\underbrace{mult(\overbrace{rd(mult(mult(mult(i(x3), x4), x4), op_t(x4, x3)), mult(x4, op_t(x4, x3)))}, x4)}, x4)) \\
= \quad & \text{by Lemma 2059 LR with } \{x2 \leftarrow x3, x1 \leftarrow x4, x3 \leftarrow mult(mult(mult(i(x3), x4), x4), op_t(x4, x3))\} \\
& op_r(x1, x2, rd(\overbrace{rd(op_r(mult(mult(mult(i(x3), x4), x4), op_t(x4, x3)), x4, x4), op_t(x4, x3))}, x4)) \\
= \quad & \text{by Lemma 1672 LR with } \{x3 \leftarrow x3, x2 \leftarrow x4, x1 \leftarrow mult(mult(mult(i(x3), x4), x4), op_t(x4, x3))\} \\
& op_r(x1, x2, rd(\underbrace{op_r(\overbrace{rd(mult(mult(mult(i(x3), x4), x4), op_t(x4, x3)), op_t(x4, x3))}, x4, x4)}, x4)) \\
= \quad & \text{by Axiom 5 RL with } \{x2 \leftarrow op_t(x4, x3), x1 \leftarrow mult(mult(i(x3), x4), x4)\} \\
& op_r(x1, x2, rd(\underbrace{op_r(\overbrace{mult(mult(i(x3), x4), x4)}, x4, x4)}, x4)) \\
= \quad & \text{by Lemma 359 LR with } \{x2 \leftarrow x4, x1 \leftarrow mult(i(x3), x4)\} \\
& op_r(x1, x2, \underbrace{rd(\overbrace{rd(x4, i(mult(i(x3), x4)))}, x4)}) \\
= \quad & \text{by Lemma 354 LR with } \{x2 \leftarrow i(mult(i(x3), x4)), x1 \leftarrow x4\} \\
& op_r(x1, x2, \overbrace{op_r(\underbrace{i(i(mult(i(x3), x4)))}, x4, x4)}) \\
= \quad & \text{by Lemma 3 LR with } \{x1 \leftarrow mult(i(x3), x4)\} \\
& op_r(x1, x2, \underbrace{op_r(\overbrace{mult(i(x3), x4)}, x4, x4)}) \\
= \quad & \text{by Lemma 359 LR with } \{x2 \leftarrow x4, x1 \leftarrow i(x3)\} \\
& op_r(x1, x2, \overbrace{rd(x4, \underbrace{i(i(x3))})}) \\
= \quad & \text{by Lemma 3 LR with } \{x1 \leftarrow x3\} \\
& op_r(x1, x2, rd(x4, \overbrace{x3})))
\end{aligned}
$$

Lemma 2398: $op_r(x1, x2, x3) = op_r(x1, x2, mult(x3, asoc(x4, x3, i(i(x4)))))$

$$
\begin{aligned}
& \underbrace{op_r(x1, x2, x3)} \\
= \quad & \text{by Lemma 2389 RL with } \{x5 \leftarrow i(x3), x4 \leftarrow i(x4), x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, x2, \underbrace{rd(x3, asoc(i(x4), i(x3), i(x4)))})} \\
= \quad & \text{by Lemma 1871 LR with } \{x3 \leftarrow i(x4), x2 \leftarrow x4, x1 \leftarrow x3\} \\
& op_r(x1, x2, \overbrace{\underbrace{mult(asoc(x4, x3, i(i(x4))), x3)}}) \\
= \quad & \text{by Lemma 2104 RL with } \{x3 \leftarrow i(i(x4)), x2 \leftarrow x4, x1 \leftarrow x3\} \\
& op_r(x1, x2, \overbrace{mult(x3, asoc(x4, x3, i(i(x4))))})
\end{aligned}
$$

Lemma 2399: $op_r(x1, x2, x3) = op_r(x1, x2, rd(x3, asoc(x3, x4, x4)))$

$$
\begin{aligned}
& \underbrace{op_r(x1, x2, x3)} \\
= \quad & \text{by Lemma 2398 LR with } \{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, x2, mult(x3, asoc(x4, x3, \underbrace{i(i(x4))})))} \\
= \quad & \text{by Lemma 3 LR with } \{x1 \leftarrow x4\} \\
& op_r(x1, x2, \underbrace{mult(x3, asoc(x4, x3, \overbrace{x4}))}) \\
= \quad & \text{by Lemma 1295 LR with } \{x3 \leftarrow x3, x2 \leftarrow x4, x1 \leftarrow x3\} \\
& op_r(x1, x2, \overbrace{rd(x3, asoc(x3, x4, x4))})
\end{aligned}
$$

Lemma 2400: $op_r(x1, x2, i(mult(x3, x4))) = op_r(x1, x2, rd(i(x3), x4))$

$$
\begin{aligned}
& \underbrace{op_r(x1, x2, i(mult(x3, x4)))} \\
= \quad & \text{by Lemma 2391 RL with } \{x4 \leftarrow x4, x3 \leftarrow i(mult(x3, x4)), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, x2, \underbrace{op_r(i(mult(x3, x4)), x4, x4)})} \\
= \quad & \text{by Lemma 371 LR with } \{x2 \leftarrow x4, x1 \leftarrow x3\} \\
& op_r(x1, x2, \overbrace{rd(i(x3), x4)})
\end{aligned}
$$

Lemma 2401: $op_r(x1, x2, i(x3)) = op_r(x1, x2, mult(i(x3), asoc(i(i(x3)), x4, x3)))$

$$
\begin{aligned}
& \underbrace{op_r(x1, x2, i(x3))} \\
= \quad & \text{by Lemma 2389 RL with } \{x5 \leftarrow i(x4), x4 \leftarrow i(x3), x3 \leftarrow i(x3), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, x2, rd(i(x3), \underbrace{asoc(i(x3), i(x4), i(x3))}))} \\
= \quad & \text{by Lemma 1479 RL with } \{x3 \leftarrow x3, x2 \leftarrow x4, x1 \leftarrow i(x3)\} \\
& op_r(x1, x2, \underbrace{rd(i(x3), \overbrace{i(asoc(i(i(x3)), x4, x3))})}) \\
= \quad & \text{by Lemma 359 RL with } \{x2 \leftarrow i(x3), x1 \leftarrow asoc(i(i(x3)), x4, x3)\} \\
& op_r(x1, x2, \overbrace{op_r(\underbrace{mult(asoc(i(i(x3)), x4, x3), i(x3))}, i(x3), i(x3))}) \\
= \quad & \text{by Lemma 1869 RL with } \{x3 \leftarrow x3, x2 \leftarrow x4, x1 \leftarrow i(x3)\} \\
& op_r(x1, x2, op_r(\overbrace{rd(i(x3), \underbrace{asoc(i(x3), i(x4), i(x3))})}, i(x3), i(x3))) \\
= \quad & \text{by Lemma 1479 RL with } \{x3 \leftarrow x3, x2 \leftarrow x4, x1 \leftarrow i(x3)\} \\
& op_r(x1, x2, op_r(\underbrace{rd(i(x3), \overbrace{i(asoc(i(i(x3)), x4, x3))})}, i(x3), i(x3))) \\
= \quad & \text{by Lemma 23 RL with } \{x2 \leftarrow i(x3), x1 \leftarrow i(asoc(i(i(x3)), x4, x3))\} \\
& op_r(x1, x2, \underbrace{op_r(\overbrace{rd(i(i(asoc(i(i(x3)), x4, x3))), i(i(x3)))}, i(x3), i(x3))}) \\
= \quad & \text{by Lemma 361 RL with } \{x2 \leftarrow i(x3), x1 \leftarrow rd(i(i(asoc(i(i(x3)), x4, x3))), i(i(x3)))\} \\
& op_r(x1, x2, \overbrace{\underbrace{op_r(rd(i(i(asoc(i(i(x3)), x4, x3))), i(i(x3)))}, i(i(x3)), i(i(x3)))}) \\
= \quad & \text{by Lemma 840 RL with } \{x1 \leftarrow i(asoc(i(i(x3)), x4, x3)), x2 \leftarrow i(x3)\} \\
& op_r(x1, x2, \overbrace{\underbrace{rd(mult(i(x3), i(x3)), mult(i(asoc(i(i(x3)), x4, x3)), i(x3)))}}) \\
= \quad & \text{by Lemma 2105 RL with } \{x3 \leftarrow x3, x2 \leftarrow x4, x1 \leftarrow i(x3)\} \\
& op_r(x1, x2, \overbrace{mult(i(x3), asoc(i(i(x3)), x4, x3))})
\end{aligned}
$$

Lemma 2402: $op_r(x1, x2, i(x3)) = op_r(x1, x2, rd(asoc(i(i(x3)), x4, x3), x3))$

$$
\begin{aligned}
& \overbrace{op_r(x1, x2, i(x3))} \\
= \quad & \text{by Lemma 2401 LR with } \{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, x2, \underbrace{mult(i(x3), asoc(i(i(x3)), x4, x3))})} \\
= \quad & \text{by Lemma 380 RL with } \{x2 \leftarrow x3, x1 \leftarrow asoc(i(i(x3)), x4, x3)\} \\
& op_r(x1, x2, \overbrace{rd(asoc(i(i(x3)), x4, x3), \underbrace{op_t(x3, rd(asoc(i(i(x3)), x4, x3), x3))})}) \\
= \quad & \text{by Lemma 2108 LR with } \{x3 \leftarrow x4, x2 \leftarrow i(i(x3)), x1 \leftarrow x3\} \\
& op_r(x1, x2, rd(asoc(i(i(x3)), x4, x3), \overbrace{x3}))
\end{aligned}
$$

Lemma 2403: $op_r(x1, x2, i(x3)) = op_r(x1, x2, op_l(i(x3), x4, x3))$

$$
\begin{aligned}
& \overbrace{op_r(x1, x2, i(x3))} \\
= \quad & \text{by Lemma 2402 LR with } \{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, x2, rd(asoc(\underbrace{i(i(x3))}, x4, x3), x3))} \\
= \quad & \text{by Lemma 3 LR with } \{x1 \leftarrow x3\} \\
& op_r(x1, x2, \underbrace{rd(asoc(\overbrace{x3}, x4, x3), x3)}) \\
= \quad & \text{by Lemma 627 LR with } \{x2 \leftarrow x4, x1 \leftarrow x3\} \\
& op_r(x1, x2, \overbrace{op_l(i(x3), x4, x3)})
\end{aligned}
$$

Lemma 2404: $op_r(x1, x2, i(x3)) = op_r(x1, x2, i(rd(mult(x4, x3), x4)))$

$$
\begin{aligned}
& \overbrace{op_r(x1, x2, i(x3))} \\
= \quad & \text{by Lemma 2391 RL with } \{x4 \leftarrow mult(x3, x4), x3 \leftarrow i(x3), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, x2, \underbrace{op_r(i(x3), mult(x3, x4), mult(x3, x4))})} \\
= \quad & \text{by Lemma 654 LR with } \{x3 \leftarrow mult(x3, x4), x2 \leftarrow x4, x1 \leftarrow x3\} \\
& op_r(x1, x2, \overbrace{i(\underbrace{op_r(x3, x4, mult(x3, x4))})}) \\
= \quad & \text{by Lemma 822 LR with } \{x2 \leftarrow x4, x1 \leftarrow x3\} \\
& op_r(x1, x2, i(\overbrace{rd(mult(x4, x3), x4)}))
\end{aligned}
$$

Lemma 2405: $op_r(x1, x2, rd(op_t(x3, x4), x5)) = op_r(x1, x2, rd(x3, x5))$

$$
\begin{aligned}
& \overbrace{op_r(x1, x2, rd(op_t(x3, x4), x5))} \\
= \quad & \text{by Lemma 2389 RL with } \{x5 \leftarrow op_t(x3, x4), x4 \leftarrow mult(x3, x4), x3 \leftarrow rd(op_t(x3, x4), x5), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, x2, \underbrace{rd(rd(op_t(x3, x4), x5), asoc(mult(x3, x4), op_t(x3, x4), mult(x3, x4)))})} \\
= \quad & \text{by Lemma 1248 LR with } \{x3 \leftarrow mult(x3, x4), x2 \leftarrow x5, x1 \leftarrow op_t(x3, x4)\} \\
& op_r(x1, x2, \overbrace{rd(\underbrace{op_r(op_t(x3, x4), mult(x3, x4), mult(x3, x4))}, x5)}) \\
= \quad & \text{by Lemma 889 LR with } \{x2 \leftarrow x4, x1 \leftarrow x3\} \\
& op_r(x1, x2, rd(\overbrace{x3}, x5))
\end{aligned}
$$

Lemma 2406: $op_r(x1, x2, i(rd(x3, x4))) = op_r(x1, x2, i(rd(op_t(x3, x5), x4)))$

$$
\begin{aligned}
& \underbrace{op_r(x1, x2, i(rd(x3, x4)))} \\
= \quad & \text{by Lemma 818 RL with } \{x3 \leftarrow rd(x3, x4), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_r(x1, i(mult(x1, x2)), rd(x3, x4))}} \\
= \quad & \text{by Lemma 2405 RL with } \{x5 \leftarrow x4, x4 \leftarrow x5, x3 \leftarrow x3, x2 \leftarrow i(mult(x1, x2)), x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_r(x1, i(mult(x1, x2)), rd(op_t(x3, x5), x4))}} \\
= \quad & \text{by Lemma 818 LR with } \{x3 \leftarrow rd(op_t(x3, x5), x4), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, x2, \underbrace{i(rd(op_t(x3, x5), x4))})}
\end{aligned}
$$

Lemma 2407: $op_r(x1, x2, rd(x4, x3)) = op_r(x1, x2, rd(x4, op_t(x3, x5)))$

$$
\begin{aligned}
& op_r(x1, x2, \underbrace{rd(x4, x3)}) \\
= \quad & \text{by Lemma 10 RL with } \{x2 \leftarrow x4, x1 \leftarrow x3\} \\
& \underbrace{op_r(x1, x2, \overbrace{i(rd(x3, x4))})} \\
= \quad & \text{by Lemma 2406 LR with } \{x5 \leftarrow x5, x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, x2, \underbrace{i(rd(op_t(x3, x5), x4))})} \\
= \quad & \text{by Lemma 10 LR with } \{x2 \leftarrow x4, x1 \leftarrow op_t(x3, x5)\} \\
& op_r(x1, x2, \overbrace{rd(x4, op_t(x3, x5))})
\end{aligned}
$$

Lemma 2408: $op_r(x1, x2, i(rd(x3, x4))) = op_r(x1, x2, rd(rd(x4, op_r(x3, x5, x5)), asoc(x5, x3, x5)))$

$$
\begin{aligned}
& \underbrace{op_r(x1, x2, i(rd(x3, x4)))} \\
= \quad & \text{by Lemma 2404 LR with } \{x4 \leftarrow rd(x4, op_r(x3, x5, x5)), x3 \leftarrow rd(x3, x4), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, x2, \underbrace{i(rd(mult(rd(x4, op_r(x3, x5, x5)), rd(x3, x4)), rd(x4, op_r(x3, x5, x5))))})} \\
= \quad & \text{by Lemma 10 LR with } \{x2 \leftarrow rd(x4, op_r(x3, x5, x5)), x1 \leftarrow mult(rd(x4, op_r(x3, x5, x5)), rd(x3, x4))\} \\
& op_r(x1, x2, \overbrace{rd(rd(x4, op_r(x3, x5, x5)), \underbrace{mult(rd(x4, op_r(x3, x5, x5)), rd(x3, x4))})}) \\
= \quad & \text{by Lemma 1626 LR with } \{x3 \leftarrow x5, x2 \leftarrow x3, x1 \leftarrow x4\} \\
& op_r(x1, x2, rd(rd(x4, op_r(x3, x5, x5)), \overbrace{asoc(x5, x3, x5)}))
\end{aligned}
$$

Lemma 2409: $op_r(x1, x2, rd(x4, x3)) = op_r(x1, x2, rd(x4, op_r(x3, x5, x5)))$

$$
\begin{aligned}
& op_r(x1, x2, \underbrace{rd(x4, x3)}) \\
= \quad & \text{by Lemma 10 RL with } \{x2 \leftarrow x4, x1 \leftarrow x3\} \\
& \underbrace{op_r(x1, x2, \overbrace{i(rd(x3, x4))})} \\
= \quad & \text{by Lemma 2408 LR with } \{x5 \leftarrow x5, x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, x2, rd(rd(x4, op_r(x3, x5, x5)), asoc(x5, x3, x5)))} \\
= \quad & \text{by Lemma 2389 LR with } \{x5 \leftarrow x3, x4 \leftarrow x5, x3 \leftarrow rd(x4, op_r(x3, x5, x5)), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, x2, rd(x4, op_r(x3, x5, x5)))}
\end{aligned}
$$

Lemma 2410: $op_r(x1, x2, rd(rd(x3, x4), x5)) = op_r(x1, x2, rd(mult(i(x4), x3), x5))$

$\underbrace{op_r(x1, x2, rd(rd(x3, x4), x5))}$

$=$ by Lemma 2389 RL with $\{x5 \leftarrow x4, x4 \leftarrow x3, x3 \leftarrow rd(rd(x3, x4), x5), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_r(x1, x2, rd(rd(rd(x3, x4), x5), \underbrace{asoc(x3, x4, x3)}))}$

$=$ by Lemma 704 RL with $\{x2 \leftarrow x4, x1 \leftarrow x3\}$

$op_r(x1, x2, rd(rd(rd(x3, x4), x5), \overbrace{rd(rd(x3, x4), \underbrace{mult(i(x4), x3)})}))$

$=$ by Lemma 14 RL with $\{x2 \leftarrow x4, x1 \leftarrow x3\}$

$op_r(x1, x2, \underbrace{rd(rd(rd(x3, x4), x5), rd(rd(x3, x4), \overbrace{op_t(rd(x3, x4), x4)}))})$

$=$ by Lemma 1390 RL with $\{x1 \leftarrow x5, x3 \leftarrow x4, x2 \leftarrow rd(x3, x4)\}$

$op_r(x1, x2, \overbrace{rd(\underbrace{op_t(rd(x3, x4), x4)}, x5)})$

$=$ by Lemma 14 LR with $\{x2 \leftarrow x4, x1 \leftarrow x3\}$

$op_r(x1, x2, rd(\overbrace{mult(i(x4), x3)}, x5))$

Lemma 2411: $rd(mult(op_t(x1, i(x2)), x1), op_t(x2, op_t(x1, i(x2)))) = mult(op_t(x1, i(x2)), mult(op_t(i(x2), x1), op_t(x1, i(x2))))$

$\underbrace{rd(mult(op_t(x1, i(x2)), x1), op_t(x2, op_t(x1, i(x2))))}$

$=$ by Lemma 1152 RL with $\{x3 \leftarrow op_t(x2, op_t(x1, i(x2))), x2 \leftarrow x1, x1 \leftarrow op_t(x1, i(x2))\}$

$\overbrace{mult(op_t(x1, i(x2)), mult(rd(x1, \underbrace{mult(op_t(x1, i(x2)), op_t(x2, op_t(x1, i(x2))))}), op_t(x1, i(x2))))}$

$=$ by Lemma 13 LR with $\{x2 \leftarrow x2, x1 \leftarrow op_t(x1, i(x2))\}$

$mult(op_t(x1, i(x2)), mult(rd(x1, \overbrace{mult(\underbrace{x2}, op_t(x1, i(x2)))}), op_t(x1, i(x2))))$

$=$ by Lemma 3 RL with $\{x1 \leftarrow x2\}$

$mult(op_t(x1, i(x2)), mult(\underbrace{rd(x1, mult(\overbrace{i(\underbrace{i(x2)})}, op_t(x1, i(x2))))}, op_t(x1, i(x2))))$

$=$ by Lemma 670 RL with $\{x2 \leftarrow x1, x1 \leftarrow i(x2)\}$

$mult(op_t(x1, i(x2)), mult(\overbrace{op_t(i(x2), x1)}, op_t(x1, i(x2))))$

Lemma 2412: $rd(rd(x1, op_t(i(x1), x2)), op_t(x2, op_t(x1, i(x2)))) = mult(op_t(x1, i(x2)), mult(op_t(i(x2), x1), op_t(x1, i(x2))))$

$rd(\underbrace{rd(x1, op_t(i(x1), x2))}, op_t(x2, op_t(x1, i(x2))))$

$=$ by Lemma 103 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$rd(\overbrace{\underbrace{mult(x1, op_t(x1, i(x2)))}}, op_t(x2, op_t(x1, i(x2))))$

$=$ by Lemma 27 RL with $\{x2 \leftarrow i(x2), x1 \leftarrow x1\}$

$\underbrace{rd(\overbrace{mult(op_t(x1, i(x2)), x1)}, op_t(x2, op_t(x1, i(x2))))}$

$=$ by Lemma 2411 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{mult(op_t(x1, i(x2)), mult(op_t(i(x2), x1), op_t(x1, i(x2))))}$

Lemma 2413: $mult(op_r(rd(mult(x1,x2),x1),x3,x4),x1) = mult(x1,op_r(x2,x3,x4))$

$\overbrace{mult(op_r(rd(mult(x1,x2),x1),x3,x4),x1)}$

$=$ by Lemma 1967 RL with $\{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow mult(x1,x2), x1 \leftarrow x1\}$

$\overbrace{mult(x1,op_r(\underbrace{mult(i(x1),mult(x1,x2))}_{},x3,x4))}$

$=$ by Axiom 2 RL with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$mult(x1,op_r(\overbrace{x2}^{},x3,x4))$

Lemma 2414: $mult(op_r(x2,x3,x1),x1) = op_l(mult(x1,op_r(x2,x3,x1)),x1,rd(mult(x1,x2),x1))$

$\overbrace{mult(op_r(x2,x3,x1),x1)}$

$=$ by Lemma 155 RL with $\{x4 \leftarrow x1, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{mult(x1,\underbrace{op_r(op_t(x2,x1),x3,x1)}_{})}$

$=$ by Lemma 2413 RL with $\{x4 \leftarrow x1, x3 \leftarrow x3, x2 \leftarrow op_t(x2,x1), x1 \leftarrow x1\}$

$\overbrace{mult(op_r(\underbrace{rd(mult(x1,op_t(x2,x1)),x1)}_{},x3,x1),x1)}$

$=$ by Lemma 1208 RL with $\{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x1\}$

$mult(\underbrace{op_r(\overbrace{op_t(rd(mult(x1,x2),x1),x1)}^{},x3,x1)}_{},x1)$

$=$ by Axiom 14 RL with $\{x4 \leftarrow x1, x3 \leftarrow x1, x2 \leftarrow x3, x1 \leftarrow rd(mult(x1,x2),x1)\}$

$\overbrace{mult(\underbrace{op_t(op_r(rd(mult(x1,x2),x1),x3,x1),x1)}_{},x1)}$

$=$ by Lemma 341 RL with $\{x2 \leftarrow op_r(rd(mult(x1,x2),x1),x3,x1), x1 \leftarrow x1\}$

$\overbrace{op_t(\underbrace{rd(x1,i(op_r(rd(mult(x1,x2),x1),x3,x1)))}_{},x1)}$

$=$ by Lemma 1969 RL with $\{x4 \leftarrow x1, x3 \leftarrow x3, x2 \leftarrow rd(mult(x1,x2),x1), x1 \leftarrow x1\}$

$op_t(\overbrace{mult(x1,\underbrace{op_l(op_r(rd(mult(x1,x2),x1),x3,x1),x1,rd(mult(x1,x2),x1))}_{})}^{},x1)$

$=$ by Axiom 18 RL with $\{x5 \leftarrow x1, x4 \leftarrow x3, x3 \leftarrow rd(mult(x1,x2),x1), x2 \leftarrow x1, x1 \leftarrow rd(mult(x1,x2),x1)\}$

$\underbrace{op_t(mult(x1,\overbrace{op_r(op_l(rd(mult(x1,x2),x1),x1,rd(mult(x1,x2),x1)),x3,x1)}^{}),x1)}$

$=$ by Lemma 2237 LR with $\{x3 \leftarrow x3, x2 \leftarrow rd(mult(x1,x2),x1), x1 \leftarrow x1\}$

$\overbrace{mult(\underbrace{op_r(op_l(rd(mult(x1,x2),x1),x1,rd(mult(x1,x2),x1)),x3,x1)}_{},op_l(x1,x1,rd(mult(x1,x2),x1)))}$

$=$ by Axiom 18 LR with $\{x5 \leftarrow x1, x4 \leftarrow x3, x3 \leftarrow rd(mult(x1,x2),x1), x2 \leftarrow x1, x1 \leftarrow rd(mult(x1,x2),x1)\}$

$\underbrace{mult(\overbrace{op_l(op_r(rd(mult(x1,x2),x1),x3,x1),x1,rd(mult(x1,x2),x1))}^{},op_l(x1,x1,rd(mult(x1,x2),x1)))}$

$=$ by Lemma 1788 LR with $\{x2 \leftarrow rd(mult(x1,x2),x1), x3 \leftarrow x1, x1 \leftarrow op_r(rd(mult(x1,x2),x1),x3,x1)\}$

$\overbrace{op_l(\underbrace{mult(op_r(rd(mult(x1,x2),x1),x3,x1),x1)}_{},x1,rd(mult(x1,x2),x1))}$

$=$ by Lemma 2413 LR with $\{x4 \leftarrow x1, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$op_l(\overbrace{mult(x1,op_r(x2,x3,x1))}^{},x1,rd(mult(x1,x2),x1))$

Lemma 2415: $op_l(mult(x1, op_r(x2, x3, x1)), x1, x2) = mult(op_r(x2, x3, x1), x1)$

$$
\begin{aligned}
& \overbrace{op_l(mult(x1, op_r(x2, x3, x1)), x1, x2)} \\
= \quad & \text{by Lemma 1351 RL with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow mult(x1, op_r(x2, x3, x1))\} \\
& \overbrace{op_l(mult(x1, op_r(x2, x3, x1)), x1, rd(x2, mult(x1, x1)))} \\
= \quad & \text{by Lemma 2043 LR with } \{x4 \leftarrow x1, x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow mult(x1, op_r(x2, x3, x1))\} \\
& \overbrace{op_l(mult(x1, op_r(x2, x3, x1)), x1, rd(mult(x1, x2), x1))} \\
= \quad & \text{by Lemma 2414 RL with } \{x1 \leftarrow x1, x3 \leftarrow x3, x2 \leftarrow x2\} \\
& \overbrace{mult(op_r(x2, x3, x1), x1)}
\end{aligned}
$$

Lemma 2416: $mult(op_r(x2, x1, x3), x1) = op_l(mult(x1, op_r(x2, x1, x3)), x1, rd(mult(x1, x2), x1))$

$$
\begin{aligned}
& \overbrace{mult(op_r(x2, x1, x3), x1)} \\
= \quad & \text{by Lemma 155 RL with } \{x4 \leftarrow x3, x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{mult(x1, op_r(op_t(x2, x1), x1, x3))} \\
= \quad & \text{by Lemma 2413 RL with } \{x4 \leftarrow x3, x3 \leftarrow x1, x2 \leftarrow op_t(x2, x1), x1 \leftarrow x1\} \\
& \overbrace{mult(op_r(\underbrace{rd(mult(x1, op_t(x2, x1)), x1)}, x1, x3), x1)} \\
= \quad & \text{by Lemma 1208 RL with } \{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& mult(\underbrace{op_r(\overbrace{op_t(rd(mult(x1, x2), x1), x1)}, x1, x3)}, x1) \\
= \quad & \text{by Axiom 14 RL with } \{x4 \leftarrow x1, x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow rd(mult(x1, x2), x1)\} \\
& \underbrace{mult(\overbrace{op_t(op_r(rd(mult(x1, x2), x1), x1, x3), x1)}, x1)} \\
= \quad & \text{by Lemma 341 RL with } \{x2 \leftarrow op_r(rd(mult(x1, x2), x1), x1, x3), x1 \leftarrow x1\} \\
& \overbrace{op_t(\underbrace{rd(x1, i(op_r(rd(mult(x1, x2), x1), x1, x3)))}, x1)} \\
= \quad & \text{by Lemma 1969 RL with } \{x4 \leftarrow x3, x3 \leftarrow x1, x2 \leftarrow rd(mult(x1, x2), x1), x1 \leftarrow x1\} \\
& op_t(\overbrace{mult(x1, \underbrace{op_l(op_r(rd(mult(x1, x2), x1), x1, x3), x1, rd(mult(x1, x2), x1))})}, x1) \\
= \quad & \text{by Axiom 18 RL with } \{x5 \leftarrow x3, x4 \leftarrow x1, x3 \leftarrow rd(mult(x1, x2), x1), x2 \leftarrow x1, x1 \leftarrow rd(mult(x1, x2), x1)\} \\
& \underbrace{op_t(mult(x1, \overbrace{op_r(op_l(rd(mult(x1, x2), x1), x1, rd(mult(x1, x2), x1)), x1, x3)}), x1)} \\
= \quad & \text{by Lemma 2238 LR with } \{x3 \leftarrow x3, x2 \leftarrow rd(mult(x1, x2), x1), x1 \leftarrow x1\} \\
& \overbrace{mult(\underbrace{op_r(op_l(rd(mult(x1, x2), x1), x1, rd(mult(x1, x2), x1)), x1, x3)}, op_l(x1, x1, rd(mult(x1, x2), x1)))} \\
= \quad & \text{by Axiom 18 LR with } \{x5 \leftarrow x3, x4 \leftarrow x1, x3 \leftarrow rd(mult(x1, x2), x1), x2 \leftarrow x1, x1 \leftarrow rd(mult(x1, x2), x1)\} \\
& \underbrace{mult(\overbrace{op_l(op_r(rd(mult(x1, x2), x1), x1, x3), x1, rd(mult(x1, x2), x1))}, op_l(x1, x1, rd(mult(x1, x2), x1)))} \\
= \quad & \text{by Lemma 1788 LR with } \{x2 \leftarrow rd(mult(x1, x2), x1), x3 \leftarrow x1, x1 \leftarrow op_r(rd(mult(x1, x2), x1), x1, x3)\} \\
& \overbrace{op_l(\underbrace{mult(op_r(rd(mult(x1, x2), x1), x1, x3), x1)}, x1, rd(mult(x1, x2), x1))} \\
= \quad & \text{by Lemma 2413 LR with } \{x4 \leftarrow x3, x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_l(\overbrace{mult(x1, op_r(x2, x1, x3))}, x1, rd(mult(x1, x2), x1))
\end{aligned}
$$

Lemma 2417: $mult(op_l(op_r(i(x2),x1,x1),x3,x4),x1) = mult(x1,op_l(op_l(i(x2),x2,x1),x3,x4))$

$$
\begin{aligned}
& mult(op_l(\underbrace{op_r(i(x2),x1,x1)},x3,x4),x1) \\
= & \quad \text{by Lemma 354 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{mult(op_l(\overbrace{rd(rd(x1,x2),x1)},x3,x4),x1)} \\
= & \quad \text{by Lemma 1973 RL with } \{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow rd(x1,x2), x1 \leftarrow x1\} \\
& \overbrace{mult(x1,op_l(\underbrace{mult(i(x1),rd(x1,x2))},x3,x4))} \\
= & \quad \text{by Lemma 310 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& mult(x1,op_l(\overbrace{op_l(i(x2),x2,x1)},x3,x4))
\end{aligned}
$$

Lemma 2418: $mult(rd(op_r(x1,x2,x3),x1),mult(x3,x2)) = mult(rd(x2,x1),mult(x1,op_t(x3,x2)))$

$$
\begin{aligned}
& \underbrace{mult(rd(op_r(x1,x2,x3),x1),mult(x3,x2))} \\
= & \quad \text{by Lemma 2004 RL with } \{x4 \leftarrow mult(x3,x2), x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{mult(i(x1),\underbrace{mult(op_r(x1,x2,x3),mult(x3,x2))})} \\
= & \quad \text{by Lemma 1889 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{mult(i(x1),\overbrace{mult(mult(x1,x2),op_t(x3,x2))})} \\
= & \quad \text{by Lemma 1045 LR with } \{x3 \leftarrow op_t(x3,x2), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{mult(rd(x2,x1),mult(x1,op_t(x3,x2)))}
\end{aligned}
$$

Lemma 2419: $rd(op_r(x1,x2,x3),x1) = rd(op_l(x3,x1,x2),x3)$

$$
\begin{aligned}
& \underbrace{rd(op_r(x1,x2,x3),x1)} \\
= & \quad \text{by Axiom 5 LR with } \{x2 \leftarrow mult(x3,x2), x1 \leftarrow rd(op_r(x1,x2,x3),x1)\} \\
& \overbrace{rd(\underbrace{mult(rd(op_r(x1,x2,x3),x1),mult(x3,x2))},mult(x3,x2))} \\
= & \quad \text{by Lemma 2418 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& rd(\overbrace{\underbrace{mult(rd(x2,x1),mult(x1,op_t(x3,x2)))}},mult(x3,x2)) \\
= & \quad \text{by Lemma 1498 LR with } \{x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \underbrace{rd(\overbrace{mult(op_l(x3,x1,x2),x2)},mult(x3,x2))} \\
= & \quad \text{by Lemma 2379 LR with } \{x4 \leftarrow x2, x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x3\} \\
& \overbrace{rd(op_l(x3,x1,x2),x3)}
\end{aligned}
$$

Lemma 2420: $x1 = op_l(x1,x2,rd(op_r(x3,x4,x2),x3))$

$$
\begin{aligned}
& \underbrace{x1} \\
= & \quad \text{by Lemma 1422 RL with } \{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1,x2,op_l(x2,x3,x4))}} \\
= & \quad \text{by Lemma 286 RL with } \{x3 \leftarrow op_l(x2,x3,x4), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1,x2,\underbrace{rd(op_l(x2,x3,x4),x2)})} \\
= & \quad \text{by Lemma 2419 RL with } \{x3 \leftarrow x2, x2 \leftarrow x4, x1 \leftarrow x3\} \\
& op_l(x1,x2,\overbrace{rd(op_r(x3,x4,x2),x3)})
\end{aligned}
$$

Lemma 2421: $op_r(x1, x2, rd(op_r(x3, x4, x2), x3)) = x1$

$op_r(x1, \underbrace{x2}, rd(op_r(x3, x4, x2), x3))$

$=$ by Lemma 25 RL with $\{x2 \leftarrow x2, x1 \leftarrow x2\}$

$op_r(x1, \overbrace{op_l(x2, x2, x2)}, rd(op_r(x3, x4, x2), x3))$

$=$ by Lemma 601 RL with $\{x2 \leftarrow x2, x1 \leftarrow x2\}$

$op_r(x1, \overbrace{op_r(x2, x2, x2)}, rd(op_r(x3, x4, x2), x3))$

$=$ by Lemma 2133 RL with $\{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x2\}$

$op_r(x1, \overbrace{op_r(x2, op_l(x2, x3, x4), op_l(x2, x3, x4))}, \underbrace{rd(op_r(x3, x4, x2), x3)})$

$=$ by Lemma 2419 LR with $\{x3 \leftarrow x2, x2 \leftarrow x4, x1 \leftarrow x3\}$

$op_r(x1, \underbrace{op_r(x2, op_l(x2, x3, x4), op_l(x2, x3, x4))}, \overbrace{rd(op_l(x2, x3, x4), x2)})$

$=$ by Lemma 353 RL with $\{x2 \leftarrow x2, x1 \leftarrow op_l(x2, x3, x4)\}$

$op_r(x1, \overbrace{rd(\underbrace{op_l(x2, x3, x4)}, rd(op_l(x2, x3, x4), x2))}, rd(op_l(x2, x3, x4), x2))$

$=$ by Axiom 4 LR with $\{x2 \leftarrow x2, x1 \leftarrow op_l(x2, x3, x4)\}$

$\underbrace{op_r(x1, rd(\overbrace{mult(rd(op_l(x2, x3, x4), x2), x2)}, rd(op_l(x2, x3, x4), x2)), rd(op_l(x2, x3, x4), x2))}$

$=$ by Lemma 1896 LR with $\{x3 \leftarrow x2, x2 \leftarrow rd(op_l(x2, x3, x4), x2), x1 \leftarrow x1\}$

$\overbrace{op_r(x1, x2, \underbrace{rd(op_l(x2, x3, x4), x2)})}$

$=$ by Lemma 1074 RL with $\{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x2\}$

$\underbrace{op_r(x1, x2, \overbrace{rd(i(x2), op_l(i(x2), x3, x4))})}$

$=$ by Lemma 2369 RL with $\{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{x1}$

Lemma 2422: $op_r(x1, rd(op_r(x2, x3, x1), x2), x4) = x1$

$op_r(x1, \underbrace{rd(op_r(x2, x3, x1), x2)}, x4)$

$=$ by Lemma 2419 LR with $\{x3 \leftarrow x1, x2 \leftarrow x3, x1 \leftarrow x2\}$

$\underbrace{op_r(x1, \overbrace{rd(op_l(x1, x2, x3), x1)}, x4)}$

$=$ by Lemma 1940 LR with $\{x3 \leftarrow x4, x2 \leftarrow op_l(x1, x2, x3), x1 \leftarrow x1\}$

$\overbrace{op_r(x1, \underbrace{i(op_l(x1, x2, x3))}, i(x4))}$

$=$ by Lemma 1049 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{op_r(x1, \overbrace{op_l(i(x1), x2, x3)}, i(x4))}$

$=$ by Lemma 138 RL with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow op_r(x1, op_l(i(x1), x2, x3), i(x4))\}$

$\overbrace{op_l(op_l(op_r(x1, \underbrace{op_l(i(x1), x2, x3)}, i(x4)), x2, x3), x3, x2)}$

$=$ by Lemma 1049 RL with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$op_l(\underbrace{op_l(op_r(x1, \overbrace{i(op_l(x1, x2, x3))}, i(x4)), x2, x3)}, x3, x2)$

$=$ by Axiom 18 RL with $\{x5 \leftarrow i(x4), x4 \leftarrow i(op_l(x1, x2, x3)), x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$op_l(\overbrace{op_r(op_l(x1, x2, x3), i(op_l(x1, x2, x3)), i(x4))}, x3, x2)$

$=$ by Lemma 52 LR with $\{x2 \leftarrow i(x4), x1 \leftarrow op_l(x1, x2, x3)\}$

$\underbrace{op_l(\overbrace{op_l(x1, x2, x3)}, x3, x2)}$

$=$ by Lemma 138 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{x1}$

Lemma 2423: $op_l(x1, rd(op_l(x1, x2, x3), x1), x4) = op_l(x1, rd(op_r(x2, x3, x1), x2), op_t(x4, op_l(x1, x2, x3)))$

$\underbrace{op_l(x1, rd(op_l(x1, x2, x3), x1), x4)}$

$=$ by Lemma 2327 LR with $\{x2 \leftarrow x4, x3 \leftarrow rd(op_l(x1, x2, x3), x1), x1 \leftarrow x1\}$

$\overbrace{op_l(x1, \underbrace{op_t(rd(op_l(x1, x2, x3), x1), x1)}, x4)}$

$=$ by Lemma 14 LR with $\{x2 \leftarrow x1, x1 \leftarrow op_l(x1, x2, x3)\}$

$\underbrace{op_l(x1, \overbrace{mult(i(x1), op_l(x1, x2, x3))}, x4)}$

$=$ by Lemma 2336 LR with $\{x3 \leftarrow x4, x2 \leftarrow mult(i(x1), op_l(x1, x2, x3)), x1 \leftarrow x1\}$

$\overbrace{op_l(x1, mult(i(x1), op_l(x1, x2, x3)), \underbrace{op_t(x4, op_r(\underbrace{mult(x1, mult(i(x1), op_l(x1, x2, x3)))}, mult(i(x1), op_l(x1, x2, x3)), x1))})}$

$=$ by Lemma 1721 RL with $\{x3 \leftarrow x1, x2 \leftarrow op_r(mult(x1, mult(i(x1), op_l(x1, x2, x3))), mult(i(x1), op_l(x1, x2, x3)), x1), x1 \leftarrow x$

$op_l(x1, mult(i(x1), op_l(x1, x2, x3)), \overbrace{op_t(x4, \underbrace{op_r(op_r(\underbrace{mult(x1, mult(i(x1), op_l(x1, x2, x3)))}, mult(i(x1), op_l(x1, x2, x3)), x1),}$

$=$ by Lemma 353 RL with $\{x2 \leftarrow op_r(mult(x1, mult(i(x1), op_l(x1, x2, x3))), mult(i(x1), op_l(x1, x2, x3)), x1), x1 \leftarrow x1\}$

$op_l(x1, mult(i(x1), op_l(x1, x2, x3)), op_t(x4, \overbrace{rd(x1, \underbrace{rd(x1, op_r(\underbrace{mult(x1, mult(i(x1), op_l(x1, x2, x3)))}, mult(i(x1), op_l(x1, x2,}$

$=$ by Lemma 1501 LR with $\{x2 \leftarrow mult(i(x1), op_l(x1, x2, x3)), x1 \leftarrow x1\}$

$op_l(x1, mult(i(x1), op_l(x1, x2, x3)), op_t(x4, \underbrace{rd(x1, \overbrace{op_l(i(\underbrace{mult(i(x1), op_l(x1, x2, x3))}), mult(i(x1), op_l(x1, x2, x3)), x1)})}))$

$=$ by Lemma 231 RL with $\{x2 \leftarrow mult(i(x1), op_l(x1, x2, x3)), x1 \leftarrow x1\}$

$op_l(x1, mult(i(x1), op_l(x1, x2, x3)), op_t(x4, \overbrace{\underbrace{mult(x1, mult(i(x1)}, op_l(x1, x2, x3))})))$

$=$ by Lemma 12 LR with $\{x2 \leftarrow op_l(x1, x2, x3), x1 \leftarrow x1\}$

$op_l(x1, \underbrace{mult(i(x1), op_l(x1, x2, x3))}, op_t(x4, \overbrace{op_l(x1, x2, x3)}))$

$=$ by Lemma 14 RL with $\{x2 \leftarrow x1, x1 \leftarrow op_l(x1, x2, x3)\}$

$\underbrace{op_l(x1, \overbrace{op_t(rd(op_l(x1, x2, x3), x1), x1)}, op_t(x4, op_l(x1, x2, x3)))}$

$=$ by Lemma 2327 RL with $\{x2 \leftarrow op_t(x4, op_l(x1, x2, x3)), x3 \leftarrow rd(op_l(x1, x2, x3), x1), x1 \leftarrow x1\}$

$\overbrace{op_l(x1, \underbrace{rd(op_l(x1, x2, x3), x1)}, op_t(x4, op_l(x1, x2, x3)))}$

$=$ by Lemma 2419 RL with $\{x3 \leftarrow x1, x2 \leftarrow x3, x1 \leftarrow x2\}$

$op_l(x1, \overbrace{rd(op_r(x2, x3, x1), x2)}, op_t(x4, op_l(x1, x2, x3)))$

Lemma 2424: $x1 = op_r(x1, x2, mult(x3, i(op_r(x3, x4, x2))))$

$\underbrace{x1}$

$=$ by Lemma 2421 RL with $\{x4 \leftarrow mult(x3, x4), x3 \leftarrow i(x3), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_r(x1, x2, rd(\underbrace{op_r(i(x3), mult(x3, x4), x2)}, i(x3)))}$

$=$ by Lemma 654 LR with $\{x3 \leftarrow x2, x2 \leftarrow x4, x1 \leftarrow x3\}$

$\underbrace{op_r(x1, x2, rd(\overbrace{i(op_r(x3, x4, x2))}, i(x3)))}$

$=$ by Lemma 2394 LR with $\{x4 \leftarrow x3, x3 \leftarrow i(op_r(x3, x4, x2)), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_r(x1, x2, mult(x3, i(op_r(x3, x4, x2))))}$

Lemma 2425: $rd(x1, rd(x2, op_l(x2, x1, x3))) = op_r(x1, x3, x2)$

$$
\begin{aligned}
& \underbrace{rd(x1, rd(x2, op_l(x2, x1, x3)))} \\
= & \quad \text{by Lemma 2314 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{rd(mult(x1, op_l(x2, x1, x3)), x2)}} \\
= & \quad \text{by Lemma 2155 RL with } \{x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \overbrace{mult(\underbrace{rd(op_l(x2, x1, x3), x2)}, x1)} \\
= & \quad \text{by Lemma 2419 RL with } \{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \underbrace{mult(\overbrace{rd(op_r(x1, x3, x2), x1)}, x1)} \\
= & \quad \text{by Axiom 4 RL with } \{x2 \leftarrow x1, x1 \leftarrow op_r(x1, x3, x2)\} \\
& \overbrace{op_r(x1, x3, x2)}
\end{aligned}
$$

Lemma 2426: $op_r(x1, i(x2), x3) = rd(x1, rd(i(x3), op_l(i(x3), x1, x2)))$

$$
\begin{aligned}
& op_r(x1, i(x2), \underbrace{x3}) \\
= & \quad \text{by Lemma 3 RL with } \{x1 \leftarrow x3\} \\
& \underbrace{op_r(x1, i(x2), \overbrace{i(i(x3))})} \\
= & \quad \text{by Lemma 2168 LR with } \{x4 \leftarrow x3, x3 \leftarrow i(x3), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_r(x1, rd(x2, op_t(x1, x3)), i(x3))}} \\
= & \quad \text{by Lemma 2425 RL with } \{x3 \leftarrow rd(x2, op_t(x1, x3)), x2 \leftarrow i(x3), x1 \leftarrow x1\} \\
& \overbrace{rd(x1, rd(i(x3), \underbrace{op_l(i(x3), x1, rd(x2, op_t(x1, x3)))}))} \\
= & \quad \text{by Lemma 2343 LR with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x3\} \\
& rd(x1, rd(i(x3), \overbrace{op_l(i(x3), x1, x2)}))
\end{aligned}
$$

Lemma 2427: $op_r(x1, i(x2), x3) = op_r(x1, x2, i(x3))$

$$
\begin{aligned}
& \underbrace{op_r(x1, i(x2), x3)} \\
= & \quad \text{by Lemma 2426 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{rd(x1, rd(i(x3), op_l(i(x3), x1, x2)))}} \\
= & \quad \text{by Lemma 2425 LR with } \{x3 \leftarrow x2, x2 \leftarrow i(x3), x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, x2, i(x3))}
\end{aligned}
$$

Lemma 2428: $op_r(i(x1), x2, x3) = i(op_r(x1, x2, x3))$

$$
\begin{aligned}
& op_r(i(x1), x2, \underbrace{x3}) \\
= & \quad \text{by Lemma 3 RL with } \{x1 \leftarrow x3\} \\
& \underbrace{op_r(i(x1), x2, \overbrace{i(i(x3))})} \\
= & \quad \text{by Lemma 2427 RL with } \{x3 \leftarrow i(x3), x2 \leftarrow x2, x1 \leftarrow i(x1)\} \\
& \overbrace{\underbrace{op_r(i(x1), i(x2), i(x3))}} \\
= & \quad \text{by Lemma 815 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{i(op_r(x1, x2, x3))}
\end{aligned}
$$

Lemma 2429: $op_r(x1, rd(x2, x1), x3) = op_r(x1, x2, x3)$

$$
\begin{aligned}
& \underbrace{op_r(x1, rd(x2, x1), x3)} \\
= \quad & \text{by Lemma 1940 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_r(x1, i(x2), i(x3))}} \\
= \quad & \text{by Lemma 2427 LR with } \{x3 \leftarrow i(x3), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, x2, \underbrace{i(i(x3))})} \\
= \quad & \text{by Lemma 3 LR with } \{x1 \leftarrow x3\} \\
& op_r(x1, x2, \overbrace{x3}^{})
\end{aligned}
$$

Lemma 2430: $op_r(x1, mult(x2, x1), x3) = op_r(x1, x2, x3)$

$$
\begin{aligned}
& \underbrace{op_r(x1, mult(x2, x1), x3)} \\
= \quad & \text{by Lemma 2425 RL with } \{x3 \leftarrow mult(x2, x1), x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{rd(x1, rd(x3, \underbrace{op_l(x3, x1, mult(x2, x1))}))} \\
= \quad & \text{by Lemma 291 LR with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x3\} \\
& \underbrace{rd(x1, rd(x3, \overbrace{op_l(x3, x1, x2)}))} \\
= \quad & \text{by Lemma 2425 LR with } \{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, x2, x3)}
\end{aligned}
$$

Lemma 2431: $op_r(x1, mult(x1, x2), x3) = op_r(x1, x2, x3)$

$$
\begin{aligned}
& \underbrace{op_r(x1, mult(x1, x2), x3)} \\
= \quad & \text{by Lemma 1954 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, op_t(mult(x2, x1), \underbrace{x2}), x3)} \\
= \quad & \text{by Axiom 5 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& op_r(x1, op_t(\underbrace{mult(x2, x1)}, \overbrace{rd(mult(x2, x1), x1)}), x3) \\
= \quad & \text{by Axiom 4 LR with } \{x2 \leftarrow x1, x1 \leftarrow mult(x2, x1)\} \\
& op_r(x1, \underbrace{op_t(\overbrace{mult(rd(mult(x2, x1), x1), x1)}, rd(mult(x2, x1), x1))}, x3) \\
= \quad & \text{by Lemma 264 RL with } \{x2 \leftarrow rd(mult(x2, x1), x1), x1 \leftarrow x1\} \\
& op_r(x1, \overbrace{mult(x1, \underbrace{op_l(rd(mult(x2, x1), x1), rd(mult(x2, x1), x1), x1)})}, x3) \\
= \quad & \text{by Lemma 292 LR with } \{x3 \leftarrow x1, x2 \leftarrow mult(x2, x1), x1 \leftarrow rd(mult(x2, x1), x1)\} \\
& \underbrace{op_r(x1, mult(x1, \overbrace{op_l(rd(mult(x2, x1), x1), mult(x2, x1), x1)}), x3)} \\
= \quad & \text{by Lemma 1953 RL with } \{x3 \leftarrow x3, x2 \leftarrow mult(x2, x1), x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_r(x1, op_t(mult(x2, x1), i(x1)), x3)}} \\
= \quad & \text{by Lemma 1950 LR with } \{x3 \leftarrow x3, x2 \leftarrow mult(x2, x1), x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_r(x1, mult(x2, x1), x3)}} \\
= \quad & \text{by Lemma 2430 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, x2, x3)}
\end{aligned}
$$

Lemma 2432: $op_r(x1, rd(x1, x2), x3) = op_r(x1, x2, i(x3))$

$$
\begin{aligned}
& op_r(x1, rd(x1, \underbrace{x2}), x3) \\
= & \quad \text{by Lemma 3 RL with } \{x1 \leftarrow x2\} \\
& \underbrace{op_r(x1, rd(x1, \overbrace{i(i(x2))}), x3)} \\
= & \quad \text{by Lemma 1949 LR with } \{x3 \leftarrow x3, x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& \underbrace{\overbrace{op_r(x1, mult(i(x2), x1), x3)}} \\
= & \quad \text{by Lemma 2430 LR with } \{x3 \leftarrow x3, x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& \underbrace{\overbrace{op_r(x1, i(x2), x3)}} \\
= & \quad \text{by Lemma 2427 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, x2, i(x3))}
\end{aligned}
$$

Lemma 2433: $op_r(x1, mult(i(x1), x2), x3) = op_r(x1, x2, x3)$

$$
\begin{aligned}
& \underbrace{op_r(x1, mult(i(x1), x2), x3)} \\
= & \quad \text{by Lemma 819 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{\overbrace{op_r(x1, i(x2), i(x3))}} \\
= & \quad \text{by Lemma 2427 LR with } \{x3 \leftarrow i(x3), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, x2, \underbrace{i(i(x3))})} \\
= & \quad \text{by Lemma 3 LR with } \{x1 \leftarrow x3\} \\
& op_r(x1, x2, \overbrace{x3})
\end{aligned}
$$

Lemma 2434: $mult(x1, op_r(i(x1), x2, x3)) = rd(x1, op_r(x1, x2, x3))$

$$
\begin{aligned}
& \underbrace{mult(x1, op_r(i(x1), x2, x3))} \\
= & \quad \text{by Lemma 159 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{mult(\underbrace{op_r(i(x1), x2, x3)}, x1)} \\
= & \quad \text{by Lemma 2428 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{mult(\overbrace{i(op_r(x1, x2, x3))}, x1)} \\
= & \quad \text{by Lemma 113 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{rd(x1, op_r(x1, x2, x3))}
\end{aligned}
$$

Lemma 2435: $mult(op_r(i(x1), x2, x3), x1) = rd(x1, op_r(x1, x2, x3))$

$$
\begin{aligned}
& \underbrace{mult(op_r(i(x1), x2, x3), x1)} \\
= & \quad \text{by Lemma 159 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{mult(x1, op_r(i(x1), x2, x3))}} \\
= & \quad \text{by Lemma 2434 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{rd(x1, op_r(x1, x2, x3))}
\end{aligned}
$$

Lemma 2436: $op_r(x1, rd(x2, x3), i(x4)) = op_r(x1, rd(x3, x2), x4)$

$$
\begin{aligned}
& \underbrace{op_r(x1, rd(x2, x3), i(x4))} \\
= \quad & \text{by Lemma 2427 RL with } \{x3 \leftarrow x4, x2 \leftarrow rd(x2, x3), x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, \underbrace{i(rd(x2, x3))}, x4)} \\
= \quad & \text{by Lemma 10 LR with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& op_r(x1, \overbrace{rd(x3, x2)}, x4)
\end{aligned}
$$

Lemma 2437: $rd(x1, rd(x2, op_r(x2, x3, x1))) = op_l(x1, x2, x3)$

$$
\begin{aligned}
& \underbrace{rd(x1, rd(x2, op_r(x2, x3, x1)))} \\
= \quad & \text{by Lemma 2305 RL with } \{x3 \leftarrow x1, x2 \leftarrow x3, x1 \leftarrow x2\} \\
& \overbrace{mult(\underbrace{rd(op_r(x2, x3, x1), x2)}, x1)} \\
= \quad & \text{by Lemma 2419 LR with } \{x3 \leftarrow x1, x2 \leftarrow x3, x1 \leftarrow x2\} \\
& \underbrace{mult(\overbrace{rd(op_l(x1, x2, x3), x1)}, x1)} \\
= \quad & \text{by Axiom 4 RL with } \{x2 \leftarrow x1, x1 \leftarrow op_l(x1, x2, x3)\} \\
& \overbrace{op_l(x1, x2, x3)}
\end{aligned}
$$

Lemma 2438: $op_l(x1, op_r(x2, x3, x4), mult(x2, x2)) = rd(x1, rd(op_r(x2, x3, x4), op_r(x2, x3, x4)))$

$\underbrace{op_l(x1, op_r(x2, x3, x4), mult(x2, x2))}$

$=$ by Lemma 2437 RL with $\{x3 \leftarrow mult(x2, x2), x2 \leftarrow op_r(x2, x3, x4), x1 \leftarrow x1\}$

$\overbrace{rd(x1, rd(op_r(x2, x3, x4), \underbrace{op_r(op_r(x2, x3, x4), mult(x2, x2), x1)}))}$

$=$ by Axiom 17 RL with $\{x5 \leftarrow x4, x4 \leftarrow x3, x3 \leftarrow x1, x2 \leftarrow mult(x2, x2), x1 \leftarrow x2\}$

$rd(x1, rd(op_r(x2, x3, x4), \overbrace{op_r(op_r(x2, mult(x2, \underbrace{x2}), x1), x3, x4)}))$

$=$ by Lemma 3 RL with $\{x1 \leftarrow x2\}$

$rd(x1, rd(op_r(x2, x3, x4), op_r(\underbrace{op_r(x2, mult(x2, \overbrace{i(i(x2))}), x1)}, x3, x4)))$

$=$ by Axiom 16 LR with $\{x3 \leftarrow x1, x2 \leftarrow mult(x2, i(i(x2))), x1 \leftarrow x2\}$

$rd(x1, rd(op_r(x2, x3, x4), op_r(\overbrace{rd(mult(\underbrace{mult(x2, mult(x2, i(i(x2))))}, x1), mult(mult(x2, i(i(x2))), x1))}, x3, x4)))$

$=$ by Lemma 24 RL with $\{x2 \leftarrow i(i(x2)), x1 \leftarrow x2\}$

$rd(x1, rd(op_r(x2, x3, x4), op_r(rd(mult(\overbrace{mult(mult(x2, \underbrace{x2}), i(i(x2)))}, x1), mult(mult(x2, i(i(x2))), x1)), x3, x4)))$

$=$ by Lemma 3 RL with $\{x1 \leftarrow x2\}$

$rd(x1, rd(op_r(x2, x3, x4), op_r(rd(mult(mult(mult(x2, \overbrace{i(i(x2))}), \underbrace{i(i(x2))}), x1), mult(mult(x2, i(i(x2))), x1)), x3, x4)))$

$=$ by Axiom 2 LR with $\{x2 \leftarrow x2, x1 \leftarrow i(i(x2))\}$

$rd(x1, rd(op_r(x2, x3, x4), op_r(rd(\underbrace{mult(mult(mult(x2, i(i(x2))), \overbrace{mult(i(x2), mult(x2, i(i(x2))))}), x1)}, mult(mult(x2, i(i(x2$

$=$ by Axiom 6 RL with $\{x3 \leftarrow x1, x2 \leftarrow i(x2), x1 \leftarrow mult(x2, i(i(x2)))\}$

$rd(x1, rd(op_r(x2, x3, x4), op_r(\underbrace{rd(\overbrace{mult(mult(x2, i(i(x2))), mult(i(x2), mult(mult(x2, i(i(x2))), x1)))}), mult(mult(x2, i(i(x2$

$=$ by Lemma 665 LR with $\{x3 \leftarrow mult(mult(x2, i(i(x2))), x1), x2 \leftarrow i(x2), x1 \leftarrow x2\}$

$rd(x1, rd(op_r(x2, x3, x4), op_r(\overbrace{op_r(x2, \underbrace{i(i(x2))}, mult(i(x2), mult(mult(x2, i(i(x2))), x1)))}, x3, x4)))$

$=$ by Lemma 3 LR with $\{x1 \leftarrow x2\}$

$rd(x1, rd(op_r(x2, x3, x4), op_r(\underbrace{op_r(x2, \overbrace{x2}, mult(i(x2), mult(mult(x2, i(i(x2))), x1)))}, x3, x4)))$

$=$ by Lemma 26 LR with $\{x2 \leftarrow mult(i(x2), mult(mult(x2, i(i(x2))), x1)), x1 \leftarrow x2\}$

$rd(x1, rd(op_r(x2, x3, x4), op_r(\overbrace{x2}, x3, x4)))$

Lemma 2439: $op_l(x1, op_r(x2, x3, x4), mult(x2, x2)) = x1$

$\underbrace{op_l(x1, op_r(x2, x3, x4), mult(x2, x2))}$

$=$ by Lemma 2438 LR with $\{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{rd(x1, \underbrace{rd(op_r(x2, x3, x4), op_r(x2, x3, x4))})}$

$=$ by Lemma 2 LR with $\{x1 \leftarrow op_r(x2, x3, x4)\}$

$\underbrace{rd(x1, \overbrace{unit()})}$

$=$ by Lemma 1 LR with $\{x1 \leftarrow x1\}$

$\overbrace{x1}$

Lemma 2440: $mult(x1, rd(op_r(x2, x3, x1), x2)) = op_l(x1, x2, x3)$

$$
\begin{aligned}
& \overbrace{mult(x1, rd(op_r(x2, x3, x1), x2))} \\
= & \quad \text{by Lemma 2302 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{rd(x1, rd(x2, op_r(x2, x3, x1)))} \\
= & \quad \text{by Lemma 2437 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x2, x3)}
\end{aligned}
$$

Lemma 2441: $mult(rd(op_r(x1, x2, x3), x1), x3) = op_l(x3, x1, x2)$

$$
\begin{aligned}
& \overbrace{mult(rd(op_r(x1, x2, x3), x1), x3)} \\
= & \quad \text{by Lemma 2305 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{rd(x3, rd(x1, op_r(x1, x2, x3)))} \\
= & \quad \text{by Lemma 2437 LR with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x3\} \\
& \overbrace{op_l(x3, x1, x2)}
\end{aligned}
$$

Lemma 2442: $mult(rd(x1, x2), op_r(x2, x3, x1)) = op_l(x1, x2, x3)$

$$
\begin{aligned}
& mult(\overbrace{rd(x1, x2)}, op_r(x2, x3, x1)) \\
= & \quad \text{by Lemma 23 RL with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& mult(\overbrace{rd(i(x2), i(x1))}, \overbrace{op_r(x2, x3, x1)}) \\
= & \quad \text{by Axiom 2 LR with } \{x2 \leftarrow x1, x1 \leftarrow op_r(x2, x3, x1)\} \\
& \overbrace{mult(rd(i(x2), i(x1)), \overbrace{mult(i(x1), mult(x1, op_r(x2, x3, x1)))})} \\
= & \quad \text{by Lemma 1091 LR with } \{x3 \leftarrow mult(x1, op_r(x2, x3, x1)), x2 \leftarrow i(x1), x1 \leftarrow i(x2)\} \\
& \overbrace{mult(i(x2), \overbrace{op_l(mult(x1, op_r(x2, x3, x1)), i(x1), i(x2))})} \\
= & \quad \text{by Lemma 1050 LR with } \{x3 \leftarrow i(x2), x2 \leftarrow x1, x1 \leftarrow mult(x1, op_r(x2, x3, x1))\} \\
& mult(i(x2), \overbrace{op_l(mult(x1, op_r(x2, x3, x1)), x1, \overbrace{i(i(x2))})}) \\
= & \quad \text{by Lemma 3 LR with } \{x1 \leftarrow x2\} \\
& mult(i(x2), \overbrace{op_l(mult(x1, op_r(x2, x3, x1)), x1, \overbrace{x2})}) \\
= & \quad \text{by Lemma 2415 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{mult(i(x2), \overbrace{mult(op_r(x2, x3, x1), x1)})} \\
= & \quad \text{by Lemma 2004 LR with } \{x4 \leftarrow x1, x3 \leftarrow x1, x2 \leftarrow x3, x1 \leftarrow x2\} \\
& \overbrace{mult(rd(op_r(x2, x3, x1), x2), x1)} \\
= & \quad \text{by Lemma 2305 LR with } \{x3 \leftarrow x1, x2 \leftarrow x3, x1 \leftarrow x2\} \\
& \overbrace{rd(x1, rd(x2, op_r(x2, x3, x1)))} \\
= & \quad \text{by Lemma 2437 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x2, x3)}
\end{aligned}
$$

Lemma 2443: $op_r(x1, x3, x2) = rd(x1, rd(op_l(x2, x3, x1), x2))$

$op_r(x1, x3, \underbrace{x2}_{})$

$=$ by Lemma 3 RL with $\{x1 \leftarrow x2\}$

$\underbrace{op_r(x1, x3, \overbrace{i(i(x2))}^{})}_{}$

$=$ by Lemma 2169 RL with $\{x4 \leftarrow i(x2), x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_r(x1, rd(op_t(i(x1), x2), x3), i(x2))}_{}}^{}$

$=$ by Lemma 2425 RL with $\{x3 \leftarrow rd(op_t(i(x1), x2), x3), x2 \leftarrow i(x2), x1 \leftarrow x1\}$

$\overbrace{rd(x1, rd(i(x2), op_l(i(x2), x1, \underbrace{rd(op_t(i(x1), x2), x3)}_{})))}^{}$

$=$ by Lemma 66 RL with $\{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x3\}$

$rd(x1, rd(i(x2), \underbrace{op_l(i(x2), x1, \overbrace{rd(i(x3), op_t(x1, i(x2)))}^{})}_{}))$

$=$ by Lemma 2327 LR with $\{x2 \leftarrow rd(i(x3), op_t(x1, i(x2))), x3 \leftarrow x1, x1 \leftarrow i(x2)\}$

$rd(x1, rd(i(x2), \overbrace{\underbrace{op_l(i(x2), op_t(x1, i(x2)), rd(i(x3), op_t(x1, i(x2))))}_{}}^{}))$

$=$ by Lemma 286 LR with $\{x3 \leftarrow i(x3), x2 \leftarrow op_t(x1, i(x2)), x1 \leftarrow i(x2)\}$

$rd(x1, rd(i(x2), \overbrace{\underbrace{op_l(i(x2), op_t(x1, i(x2)), i(x3))}_{}}^{}))$

$=$ by Lemma 2327 RL with $\{x2 \leftarrow i(x3), x3 \leftarrow x1, x1 \leftarrow i(x2)\}$

$rd(x1, rd(i(x2), \overbrace{\underbrace{op_l(i(x2), x1, i(x3))}_{}}^{}))$

$=$ by Lemma 2030 LR with $\{x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow i(x2)\}$

$rd(x1, \underbrace{rd(i(x2), \overbrace{op_l(i(x2), x3, x1)}^{})}_{})$

$=$ by Lemma 1074 LR with $\{x4 \leftarrow x1, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x2\}$

$rd(x1, \overbrace{rd(op_l(x2, x3, x1), x2)}^{})$

Lemma 2444: $mult(rd(x1, op_l(x1, x2, x3)), x3) = op_r(x3, x2, x1)$

$\underbrace{mult(rd(x1, op_l(x1, x2, x3)), x3)}_{}$

$=$ by Lemma 2154 RL with $\{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x3\}$

$\overbrace{\underbrace{rd(mult(x3, x1), op_l(x1, x2, x3))}_{}}^{}$

$=$ by Lemma 2316 LR with $\{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x3\}$

$\overbrace{\underbrace{rd(x3, rd(op_l(x1, x2, x3), x1))}_{}}^{}$

$=$ by Lemma 2443 RL with $\{x2 \leftarrow x1, x3 \leftarrow x2, x1 \leftarrow x3\}$

$\overbrace{op_r(x3, x2, x1)}^{}$

Lemma 2445: $op_l(x1, x2, rd(x3, op_t(x2, x4))) = rd(x1, rd(x2, op_r(x2, x3, x1)))$

$$
\begin{aligned}
& \underbrace{op_l(x1, x2, rd(x3, op_t(x2, x4)))} \\
= \quad & \text{by Lemma 2437 RL with } \{x3 \leftarrow rd(x3, op_t(x2, x4)), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{rd(x1, rd(x2, \underbrace{op_r(x2, rd(x3, op_t(x2, x4)), x1)}))} \\
= \quad & \text{by Lemma 2168 RL with } \{x4 \leftarrow x4, x3 \leftarrow x1, x2 \leftarrow x3, x1 \leftarrow x2\} \\
& rd(x1, rd(x2, \overbrace{\underbrace{op_r(x2, i(x3), i(x1))}})) \\
= \quad & \text{by Lemma 2427 LR with } \{x3 \leftarrow i(x1), x2 \leftarrow x3, x1 \leftarrow x2\} \\
& rd(x1, rd(x2, \overbrace{op_r(x2, x3, \underbrace{i(i(x1))})})) \\
= \quad & \text{by Lemma 3 LR with } \{x1 \leftarrow x1\} \\
& rd(x1, rd(x2, op_r(x2, x3, \overbrace{x1}^{}))))
\end{aligned}
$$

Lemma 2446: $op_l(x1, x2, rd(x3, op_t(x2, x4))) = op_l(x1, x2, x3)$

$$
\begin{aligned}
& \underbrace{op_l(x1, x2, rd(x3, op_t(x2, x4)))} \\
= \quad & \text{by Lemma 2445 LR with } \{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{rd(x1, rd(x2, op_r(x2, x3, x1)))}} \\
= \quad & \text{by Lemma 2437 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x2, x3)}
\end{aligned}
$$

Lemma 2447: $op_l(x1, x3, x2) = op_l(x1, mult(x3, op_t(x2, x4)), x2)$

$$
\begin{aligned}
& \underbrace{op_l(x1, x3, x2)} \\
= \quad & \text{by Lemma 2030 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, x2, i(x3))}} \\
= \quad & \text{by Lemma 2446 RL with } \{x4 \leftarrow x4, x3 \leftarrow i(x3), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, x2, rd(i(x3), op_t(x2, x4)))}} \\
= \quad & \text{by Lemma 2183 LR with } \{x3 \leftarrow x4, x4 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, mult(x3, op_t(x2, x4)), x2)}
\end{aligned}
$$

Lemma 2448: $rd(x1, rd(op_r(x2, x3, x1), x2)) = op_l(x1, x3, x2)$

$\overbrace{rd(x1, rd(op_r(x2, x3, x1), x2))}$

$=$ by Lemma 2295 RL with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{rd(\overbrace{mult(x1, x2)}, op_r(x2, x3, x1))}$

$=$ by Lemma 2046 RL with $\{x4 \leftarrow x1, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{mult(x1, rd(x2, op_r(\underbrace{x2}, x3, x1)))}$

$=$ by Lemma 3 RL with $\{x1 \leftarrow x2\}$

$mult(x1, rd(x2, \underbrace{op_r(\overbrace{i(i(x2))}, x3, x1)}))$

$=$ by Lemma 2428 LR with $\{x3 \leftarrow x1, x2 \leftarrow x3, x1 \leftarrow i(x2)\}$

$mult(x1, \underbrace{rd(x2, \overbrace{i(op_r(i(x2), x3, x1))})})$

$=$ by Lemma 38 RL with $\{x1 \leftarrow x2, x2 \leftarrow op_r(i(x2), x3, x1)\}$

$\underbrace{mult(x1, \overbrace{rd(op_r(i(x2), x3, x1), i(x2))})}$

$=$ by Lemma 2440 LR with $\{x3 \leftarrow x3, x2 \leftarrow i(x2), x1 \leftarrow x1\}$

$\overbrace{op_l(x1, i(x2), x3)}$

$=$ by Lemma 2031 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_l(x1, x3, x2)}$

Lemma 2449: $rd(op_l(x1, x2, x3), x1) = rd(x1, op_l(x1, x3, x2))$

$rd(\underbrace{op_l(x1, x2, x3)}, x1)$

$=$ by Lemma 2440 RL with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{rd(\overbrace{mult(x1, rd(op_r(x2, x3, x1), x2))}, x1)}$

$=$ by Lemma 453 RL with $\{x2 \leftarrow x1, x1 \leftarrow rd(op_r(x2, x3, x1), x2)\}$

$\underbrace{\overbrace{op_l(op_r(rd(op_r(x2, x3, x1), x2), x1, x1), rd(op_r(x2, x3, x1), x2), x1)}}$

$=$ by Axiom 18 RL with $\{x5 \leftarrow x1, x4 \leftarrow x1, x3 \leftarrow x1, x2 \leftarrow rd(op_r(x2, x3, x1), x2), x1 \leftarrow rd(op_r(x2, x3, x1), x2)\}$

$\underbrace{\overbrace{op_r(op_l(rd(op_r(x2, x3, x1), x2), rd(op_r(x2, x3, x1), x2), x1), x1, x1)}}$

$=$ by Lemma 866 RL with $\{x2 \leftarrow rd(op_r(x2, x3, x1), x2), x1 \leftarrow x1\}$

$\overbrace{rd(x1, op_r(\underbrace{i(rd(rd(op_r(x2, x3, x1), x2), x1))}, \overbrace{rd(op_r(x2, x3, x1), x2), rd(op_r(x2, x3, x1), x2)}))}$

$=$ by Lemma 10 LR with $\{x2 \leftarrow x1, x1 \leftarrow rd(op_r(x2, x3, x1), x2)\}$

$rd(x1, op_r(\underbrace{\overbrace{rd(x1, rd(op_r(x2, x3, x1), x2))}}, rd(op_r(x2, x3, x1), x2), rd(op_r(x2, x3, x1), x2)))$

$=$ by Lemma 2448 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$rd(x1, \underbrace{op_r(\overbrace{op_l(x1, x3, x2)}, rd(op_r(x2, x3, x1), x2), rd(op_r(x2, x3, x1), x2)))}$

$=$ by Axiom 18 LR with $\{x5 \leftarrow rd(op_r(x2, x3, x1), x2), x4 \leftarrow rd(op_r(x2, x3, x1), x2), x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x1\}$

$rd(x1, \overbrace{op_l(\underbrace{op_r(x1, rd(op_r(x2, x3, x1), x2), rd(op_r(x2, x3, x1), x2))}, x3, x2)})$

$=$ by Lemma 2422 LR with $\{x4 \leftarrow rd(op_r(x2, x3, x1), x2), x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$rd(x1, op_l(\overbrace{x1}, x3, x2))$

Lemma 2450: $mult(rd(x1, op_r(x1, x2, x3)), x3) = op_l(x3, x2, x1)$

$$
\begin{aligned}
& \overbrace{mult(rd(x1, op_r(x1, x2, x3)), x3)} \\
= \quad & \text{by Lemma 2296 RL with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x3\} \\
& \overbrace{rd(\underbrace{mult(x3, x1)}, op_r(x1, x2, x3))} \\
= \quad & \text{by Lemma 2295 LR with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x3\} \\
& \overbrace{rd(x3, rd(op_r(x1, x2, x3), x1))} \\
= \quad & \text{by Lemma 2448 LR with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x3\} \\
& \overbrace{op_l(x3, x2, x1)}
\end{aligned}
$$

Lemma 2451: $op_r(x1, mult(x2, x3), i(x2)) = op_r(x1, rd(i(x2), x3), x2)$

$$
\begin{aligned}
& \overbrace{op_r(x1, mult(x2, x3), i(x2))} \\
= \quad & \text{by Lemma 2427 RL with } \{x3 \leftarrow x2, x2 \leftarrow mult(x2, x3), x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, \underbrace{i(mult(x2, x3))}, x2)} \\
= \quad & \text{by Lemma 33 RL with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& \underbrace{op_r(x1, \overbrace{op_t(rd(i(x2), x3), x2)}, x2)} \\
= \quad & \text{by Lemma 1894 LR with } \{x3 \leftarrow x2, x2 \leftarrow rd(i(x2), x3), x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, rd(i(x2), x3), x2)}
\end{aligned}
$$

Lemma 2452: $op_l(x1, op_r(x1, x2, x3), x4) = op_r(x1, x4, op_r(x1, x2, x3))$

$$
\begin{aligned}
& \underbrace{op_l(x1, op_r(x1, x2, x3), x4)} \\
= \quad & \text{by Lemma 2437 RL with } \{x3 \leftarrow x4, x2 \leftarrow op_r(x1, x2, x3), x1 \leftarrow x1\} \\
& \overbrace{rd(x1, rd(op_r(x1, x2, x3), \underbrace{op_r(op_r(x1, x2, x3), x4, x1)}))} \\
= \quad & \text{by Lemma 739 LR with } \{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{rd(x1, rd(op_r(x1, x2, x3), \overbrace{op_l(op_r(x1, x2, x3), x1, x4)}))} \\
= \quad & \text{by Lemma 2425 LR with } \{x3 \leftarrow x4, x2 \leftarrow op_r(x1, x2, x3), x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, x4, op_r(x1, x2, x3))}
\end{aligned}
$$

Lemma 2453: $rd(x1, mult(x2, op_l(x1, x2, x3))) = i(op_r(x2, x3, x1))$

$$
\begin{aligned}
& \underbrace{rd(x1, mult(x2, op_l(x1, x2, x3)))} \\
= \quad & \text{by Lemma 2356 RL with } \{x4 \leftarrow x2, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{\overbrace{rd(rd(x1, op_l(x1, x2, x3)), x2)}} \\
= \quad & \text{by Lemma 10 RL with } \{x2 \leftarrow rd(x1, op_l(x1, x2, x3)), x1 \leftarrow x2\} \\
& \overbrace{i(\underbrace{rd(x2, rd(x1, op_l(x1, x2, x3)))})} \\
= \quad & \text{by Lemma 2425 LR with } \{x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow x2\} \\
& i(\overbrace{op_r(x2, x3, x1)})
\end{aligned}
$$

Lemma 2454: $mult(i(x1), rd(op_l(x2, i(x1), x3), x2)) = op_t(op_r(i(x1), x3, x2), x1)$

$$
\begin{aligned}
& \overbrace{mult(i(x1), rd(op_l(x2, i(x1), x3), x2))} \\
= & \quad \text{by Lemma 45 RL with } \{x3 \leftarrow op_l(x2, i(x1), x3), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{i(mult(x1, rd(x2, op_l(x2, i(x1), x3))))} \\
= & \quad \text{by Lemma 33 RL with } \{x2 \leftarrow rd(x2, op_l(x2, i(x1), x3)), x1 \leftarrow x1\} \\
& \overbrace{op_t(\underbrace{rd(i(x1), rd(x2, op_l(x2, i(x1), x3)))}, x1)} \\
= & \quad \text{by Lemma 2425 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow i(x1)\} \\
& op_t(\overbrace{op_r(i(x1), x3, x2)}, x1)
\end{aligned}
$$

Lemma 2455: $rd(op_l(x2, x3, x1), mult(x1, x2)) = op_t(op_r(i(x1), x3, x2), x1)$

$$
\begin{aligned}
& rd(\underbrace{op_l(x2, x3, x1)}, mult(x1, x2)) \\
= & \quad \text{by Lemma 2031 RL with } \{x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \underbrace{rd(\overbrace{op_l(x2, i(x1), x3)}, mult(x1, x2))} \\
= & \quad \text{by Lemma 2358 RL with } \{x4 \leftarrow x1, x3 \leftarrow x3, x2 \leftarrow i(x1), x1 \leftarrow x2\} \\
& \underbrace{\overbrace{rd(rd(op_l(x2, i(x1), x3), x2), x1)}} \\
= & \quad \text{by Lemma 40 RL with } \{x3 \leftarrow op_l(x2, i(x1), x3), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{\overbrace{rd(i(x1), rd(x2, op_l(x2, i(x1), x3)))}} \\
= & \quad \text{by Lemma 2363 LR with } \{x3 \leftarrow x3, x2 \leftarrow i(x1), x1 \leftarrow x2, x4 \leftarrow i(x1)\} \\
& \underbrace{\overbrace{mult(i(x1), rd(op_l(x2, i(x1), x3), x2))}} \\
= & \quad \text{by Lemma 2454 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_t(op_r(i(x1), x3, x2), x1)}
\end{aligned}
$$

Lemma 2456: $op_r(x1, x2, i(x3)) = rd(x1, rd(x3, op_l(x3, x2, x1)))$

$$
\begin{aligned}
& \underbrace{op_r(x1, x2, i(x3))} \\
= & \quad \text{by Lemma 2425 RL with } \{x3 \leftarrow x2, x2 \leftarrow i(x3), x1 \leftarrow x1\} \\
& \overbrace{rd(x1, \underbrace{rd(i(x3), op_l(i(x3), x1, x2))})} \\
= & \quad \text{by Lemma 1074 LR with } \{x4 \leftarrow x2, x3 \leftarrow x1, x2 \leftarrow x3, x1 \leftarrow x3\} \\
& rd(x1, \underbrace{\overbrace{rd(op_l(x3, x1, x2), x3)}}) \\
= & \quad \text{by Lemma 2449 LR with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x3\} \\
& rd(x1, \overbrace{rd(x3, op_l(x3, x2, x1))})
\end{aligned}
$$

Lemma 2457: $rd(x1, op_r(x1, x3, x2)) = rd(op_r(x1, x3, i(x2)), op_t(x1, rd(x2, op_l(x2, x3, x1))))$

$rd(x1, op_r(\underbrace{x1}, x3, x2))$

$=$ by Lemma 2311 RL with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$rd(x1, \overbrace{op_r(op_t(x1, rd(x2, op_l(x2, x3, x1))), x3, x2)})$

$=$ by Axiom 14 RL with $\{x4 \leftarrow rd(x2, op_l(x2, x3, x1)), x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x1\}$

$rd(x1, \overbrace{op_t(\underbrace{op_r(x1, x3, x2)}, rd(x2, op_l(x2, x3, x1)))})$

$=$ by Lemma 2444 RL with $\{x3 \leftarrow x1, x2 \leftarrow x3, x1 \leftarrow x2\}$

$\underbrace{rd(x1, op_t(\overbrace{mult(rd(x2, op_l(x2, x3, x1)), x1)}, rd(x2, op_l(x2, x3, x1))))}$

$=$ by Lemma 1113 LR with $\{x1 \leftarrow rd(x2, op_l(x2, x3, x1)), x2 \leftarrow x1\}$

$\overbrace{mult(i(op_t(x1, rd(x2, op_l(x2, x3, x1)))), \underbrace{rd(i(rd(x2, op_l(x2, x3, x1))), i(x1))})}$

$=$ by Lemma 23 LR with $\{x2 \leftarrow x1, x1 \leftarrow rd(x2, op_l(x2, x3, x1))\}$

$mult(i(op_t(x1, rd(x2, op_l(x2, x3, x1)))), \overbrace{rd(x1, rd(x2, op_l(x2, x3, x1)))})$

$=$ by Lemma 2456 RL with $\{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x1\}$

$mult(i(op_t(x1, rd(x2, op_l(x2, x3, x1)))), \overbrace{op_r(\underbrace{x1}, x3, i(x2))})$

$=$ by Lemma 157 RL with $\{x3 \leftarrow i(op_t(x1, rd(x2, op_l(x2, x3, x1)))), x2 \leftarrow rd(x2, op_l(x2, x3, x1)), x1 \leftarrow x1\}$

$mult(i(op_t(x1, rd(x2, op_l(x2, x3, x1)))), \overbrace{op_r(\overbrace{op_r(x1, op_t(x1, rd(x2, op_l(x2, x3, x1))), i(op_t(x1, rd(x2, op_l(x2, x3, x1)))))}, x3, x}$

$=$ by Axiom 17 LR with $\{x5 \leftarrow i(x2), x4 \leftarrow x3, x3 \leftarrow i(op_t(x1, rd(x2, op_l(x2, x3, x1)))), x2 \leftarrow op_t(x1, rd(x2, op_l(x2, x3, x1))), x1$

$\underbrace{mult(i(op_t(x1, rd(x2, op_l(x2, x3, x1)))), \overbrace{op_r(op_r(x1, x3, i(x2)), op_t(x1, rd(x2, op_l(x2, x3, x1))), i(op_t(x1, rd(x2, op_l(x2, x3, x}$

$=$ by Lemma 139 LR with $\{x2 \leftarrow op_r(x1, x3, i(x2)), x1 \leftarrow op_t(x1, rd(x2, op_l(x2, x3, x1)))\}$

$\overbrace{rd(\underbrace{op_t(op_r(x1, x3, i(x2)), op_t(x1, rd(x2, op_l(x2, x3, x1))))}, op_t(x1, rd(x2, op_l(x2, x3, x1))))}$

$=$ by Axiom 14 LR with $\{x4 \leftarrow op_t(x1, rd(x2, op_l(x2, x3, x1))), x3 \leftarrow i(x2), x2 \leftarrow x3, x1 \leftarrow x1\}$

$rd(\overbrace{op_r(\underbrace{op_t(x1, op_t(x1, rd(x2, op_l(x2, x3, x1))))}, x3, i(x2))}, op_t(x1, rd(x2, op_l(x2, x3, x1))))$

$=$ by Axiom 10 RL with $\{x2 \leftarrow x1, x1 \leftarrow op_t(x1, rd(x2, op_l(x2, x3, x1)))\}$

$rd(op_r(\overbrace{mult(i(op_t(x1, rd(x2, op_l(x2, x3, x1)))), \underbrace{mult(x1, op_t(x1, rd(x2, op_l(x2, x3, x1))))})}, x3, i(x2)), op_t(x1, rd(x2, op_l(x2,$

$=$ by Lemma 27 RL with $\{x2 \leftarrow rd(x2, op_l(x2, x3, x1)), x1 \leftarrow x1\}$

$rd(op_r(\underbrace{mult(i(op_t(x1, rd(x2, op_l(x2, x3, x1))))}, \overbrace{mult(op_t(x1, rd(x2, op_l(x2, x3, x1))), x1)}), x3, i(x2)), op_t(x1, rd(x2, op_l(x2,$

$=$ by Axiom 2 RL with $\{x2 \leftarrow op_t(x1, rd(x2, op_l(x2, x3, x1))), x1 \leftarrow x1\}$

$rd(op_r(\overbrace{x1}, x3, i(x2)), op_t(x1, rd(x2, op_l(x2, x3, x1))))$

Lemma 2458: $rd(op_r(x1, x2, i(x3)), x1) = rd(x1, op_r(x1, x2, x3))$

$rd(op_r(x1, x2, i(x3)), \underbrace{x1})$

$=$ by Lemma 2311 RL with $\{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x1\}$

$\underbrace{rd(op_r(x1, x2, i(x3)), \overbrace{op_t(x1, rd(x3, op_l(x3, x2, x1)))})}$

$=$ by Lemma 2457 RL with $\{x2 \leftarrow x3, x3 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{rd(x1, op_r(x1, x2, x3))}$

Lemma 2459: $i(mult(op_t(op_r(i(x3),x2,x1),rd(x1,op_l(x1,x2,i(x3)))),x3)) = mult(op_t(x3,i(rd(x1,op_l(x1,x2,i(x3))))),op_r(i(x3$

$i(mult(op_t(\overbrace{op_r(i(x3),x2,x1)},rd(x1,op_l(x1,x2,i(x3)))),x3))$

$=$ by Lemma 2428 LR with $\{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x3\}$

$i(mult(op_t(\overbrace{i(op_r(x3,x2,x1))},rd(x1,op_l(x1,x2,i(x3)))),x3))$

$=$ by Lemma 2453 RL with $\{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x1\}$

$i(mult(op_t(\overbrace{rd(x1,mult(x3,\overbrace{op_l(x1,x3,x2)}))},rd(x1,op_l(x1,x2,i(x3)))),x3))$

$=$ by Lemma 2030 RL with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$i(mult(op_t(\overbrace{rd(x1,mult(x3,\overbrace{op_l(x1,x2,i(x3))}))},rd(x1,op_l(x1,x2,i(x3)))),x3))$

$=$ by Lemma 2356 RL with $\{x4 \leftarrow x3, x3 \leftarrow i(x3), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{i(mult(op_t(\overbrace{rd(rd(x1,op_l(x1,x2,i(x3))),x3)},rd(x1,op_l(x1,x2,i(x3)))),x3))}$

$=$ by Lemma 1998 RL with $\{x2 \leftarrow rd(x1,op_l(x1,x2,i(x3))),x1 \leftarrow x3\}$

$\overbrace{op_l(\underbrace{i(rd(x3,mult(x3,i(rd(x1,op_l(x1,x2,i(x3))))))},x3,rd(x1,op_l(x1,x2,i(x3))))}$

$=$ by Lemma 10 LR with $\{x2 \leftarrow mult(x3,i(rd(x1,op_l(x1,x2,i(x3))))),x1 \leftarrow x3\}$

$op_l(\overbrace{rd(mult(x3,i(rd(x1,op_l(x1,x2,i(x3))))),x3)},x3,rd(x1,op_l(x1,x2,i(x3))))$

$=$ by Lemma 932 RL with $\{x2 \leftarrow x3, x1 \leftarrow rd(x1,op_l(x1,x2,i(x3)))\}$

$op_l(\overbrace{op_r(op_l(i(rd(x1,op_l(x1,x2,i(x3)))),x3,rd(x1,op_l(x1,x2,i(x3)))),x3,x3)},x3,rd(x1,op_l(x1,x2,i(x3))))$

$=$ by Axiom 18 LR with $\{x5 \leftarrow x3, x4 \leftarrow x3, x3 \leftarrow rd(x1,op_l(x1,x2,i(x3))),x2 \leftarrow x3, x1 \leftarrow i(rd(x1,op_l(x1,x2,i(x3))))\}$

$op_l(\overbrace{op_l(op_r(i(rd(x1,op_l(x1,x2,i(x3)))),x3,x3),x3,rd(x1,op_l(x1,x2,i(x3))))},x3,rd(x1,op_l(x1,x2,i(x3))))$

$=$ by Lemma 1328 RL with $\{x3 \leftarrow rd(x1,op_l(x1,x2,i(x3))),x2 \leftarrow x3, x1 \leftarrow op_r(i(rd(x1,op_l(x1,x2,i(x3)))),x3,x3)\}$

$\underbrace{op_l(\overbrace{op_l(op_r(i(rd(x1,op_l(x1,x2,i(x3)))),x3,x3),x3,rd(rd(x1,op_l(x1,x2,i(x3))),i(x3)))},x3,rd(x1,op_l(x1,x2,i(x3))))}$

$=$ by Lemma 2182 RL with $\{x2 \leftarrow rd(x1,op_l(x1,x2,i(x3))),x1 \leftarrow x3\}$

$\overbrace{mult(op_t(x3,i(rd(x1,op_l(x1,x2,i(x3))))),\underbrace{rd(i(x3),rd(x1,op_l(x1,x2,i(x3))))})}$

$=$ by Lemma 2456 RL with $\{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow i(x3)\}$

$mult(op_t(x3,i(rd(x1,op_l(x1,x2,i(x3))))),\overbrace{op_r(i(x3),x2,i(x1))})$

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Lemma 2460: $rd(x1, op_r(x1, x2, i(x3))) = rd(op_r(x1, x2, x3), x1)$

$\underbrace{rd(x1, op_r(x1, x2, i(x3)))}$

$=$ by Lemma 2434 RL with $\{x3 \leftarrow i(x3), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{mult(\underbrace{x1}, op_r(i(x1), x2, i(x3)))}$

$=$ by Lemma 2311 RL with $\{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x1\}$

$mult(\overbrace{op_t(x1, rd(x3, \underbrace{op_l(x3, x2, x1)}))}, op_r(i(x1), x2, i(x3)))$

$=$ by Lemma 2031 RL with $\{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x3\}$

$mult(op_t(x1, \underbrace{rd(x3, \overbrace{op_l(x3, i(x1), x2)})}), op_r(i(x1), x2, i(x3)))$

$=$ by Lemma 2449 RL with $\{x3 \leftarrow i(x1), x2 \leftarrow x2, x1 \leftarrow x3\}$

$mult(op_t(x1, \overbrace{rd(op_l(x3, x2, i(x1)), x3)}), op_r(i(x1), x2, i(x3)))$

$=$ by Lemma 10 RL with $\{x2 \leftarrow op_l(x3, x2, i(x1)), x1 \leftarrow x3\}$

$\underbrace{mult(op_t(x1, \overbrace{i(rd(x3, op_l(x3, x2, i(x1))))}), op_r(i(x1), x2, i(x3)))}$

$=$ by Lemma 2459 RL with $\{x1 \leftarrow x3, x2 \leftarrow x2, x3 \leftarrow x1\}$

$\overbrace{i(mult(\underbrace{op_t(op_r(i(x1), x2, x3), rd(x3, op_l(x3, x2, i(x1))))}, x1))}$

$=$ by Axiom 14 LR with $\{x4 \leftarrow rd(x3, op_l(x3, x2, i(x1))), x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow i(x1)\}$

$i(mult(\overbrace{op_r(\underbrace{op_t(i(x1), rd(x3, op_l(x3, x2, i(x1))))}, x2, x3)}, x1))$

$=$ by Lemma 2311 LR with $\{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow i(x1)\}$

$i(\underbrace{mult(op_r(\overbrace{i(x1)}, x2, x3), x1)})$

$=$ by Lemma 2435 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{i(\overbrace{rd(x1, op_r(x1, x2, x3))})}$

$=$ by Lemma 10 LR with $\{x2 \leftarrow op_r(x1, x2, x3), x1 \leftarrow x1\}$

$\overbrace{rd(op_r(x1, x2, x3), x1)}$

Lemma 2461: $mult(x1, rd(x2, op_l(x2, x3, x1))) = op_r(x1, x3, x2)$

$\underbrace{mult(x1, rd(x2, op_l(x2, x3, x1)))}$

$=$ by Lemma 1785 LR with $\{x3 \leftarrow x4, x1 \leftarrow rd(x2, op_l(x2, x3, x1)), x2 \leftarrow x1\}$

$\overbrace{op_l(mult(rd(x2, op_l(x2, x3, x1)), x1), x1, op_t(\underbrace{rd(x1, rd(x2, op_l(x2, x3, x1)))}, x4))}$

$=$ by Lemma 2456 RL with $\{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x1\}$

$op_l(\underbrace{mult(rd(x2, op_l(x2, x3, x1)), x1)}, x1, op_t(\overbrace{op_r(x1, x3, i(x2))}, x4))$

$=$ by Lemma 2444 LR with $\{x3 \leftarrow x1, x2 \leftarrow x3, x1 \leftarrow x2\}$

$\underbrace{op_l(\overbrace{op_r(x1, x3, x2)}, x1, op_t(op_r(x1, x3, i(x2)), x4))}$

$=$ by Axiom 18 RL with $\{x5 \leftarrow x2, x4 \leftarrow x3, x3 \leftarrow op_t(op_r(x1, x3, i(x2)), x4), x2 \leftarrow x1, x1 \leftarrow x1\}$

$\overbrace{op_r(\underbrace{op_l(x1, x1, op_t(op_r(x1, x3, i(x2)), x4))}, x3, x2)}$

$=$ by Lemma 1725 LR with $\{x3 \leftarrow x4, x2 \leftarrow op_r(x1, x3, i(x2)), x1 \leftarrow x1\}$

$\underbrace{op_r(\overbrace{op_l(x1, x1, op_r(x1, x3, i(x2)))}, x3, x2)}$

$=$ by Axiom 18 LR with $\{x5 \leftarrow x2, x4 \leftarrow x3, x3 \leftarrow op_r(x1, x3, i(x2)), x2 \leftarrow x1, x1 \leftarrow x1\}$

$\underbrace{\overbrace{op_l(op_r(x1, x3, x2), x1, op_r(x1, x3, i(x2)))}}$

$=$ by Lemma 1444 LR with $\{x4 \leftarrow i(x2), x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow op_r(x1, x3, x2)\}$

$\overbrace{op_r(x1, x3, x2)}$

Lemma 2462: $op_r(rd(x3, rd(x2, x1)), x3, i(i(x1))) = rd(mult(i(i(x3)), x1), x2)$

$op_r(\underbrace{rd(x3, rd(x2, x1))}, x3, i(i(x1)))$

$=$ by Lemma 39 RL with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{op_r(\overbrace{rd(rd(x1, x2), i(x3))}, x3, i(i(x1)))}$

$=$ by Lemma 2427 RL with $\{x3 \leftarrow i(x1), x2 \leftarrow x3, x1 \leftarrow rd(rd(x1, x2), i(x3))\}$

$\underbrace{\overbrace{op_r(rd(rd(x1, x2), i(x3)), i(x3), i(x1))}}$

$=$ by Lemma 1487 LR with $\{x3 \leftarrow i(x3), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{rd(mult(i(i(x3)), x1), x2)}$

Lemma 2463: $op_r(rd(x3, rd(x2, x1)), x3, x1) = rd(mult(x3, x1), x2)$

$op_r(rd(x3, rd(x2, x1)), x3, \underbrace{x1})$

$=$ by Lemma 3 RL with $\{x1 \leftarrow x1\}$

$\underbrace{op_r(rd(x3, rd(x2, x1)), x3, \overbrace{i(i(x1))})}$

$=$ by Lemma 2462 LR with $\{x1 \leftarrow x1, x2 \leftarrow x2, x3 \leftarrow x3\}$

$\overbrace{rd(mult(\underbrace{i(i(x3))}, x1), x2)}$

$=$ by Lemma 3 LR with $\{x1 \leftarrow x3\}$

$rd(mult(\overbrace{x3}, x1), x2)$

Lemma 2464: $rd(x1, mult(x2, op_r(x1, x3, x2))) = op_l(i(x2), x1, x3)$

$$
\begin{aligned}
& \overbrace{rd(x1, mult(x2, op_r(x1, x3, x2)))} \\
= & \quad \text{by Lemma 2190 LR with } \{x4 \leftarrow x2, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{rd(\underbrace{rd(x1, op_r(x1, x3, x2))}, x2)} \\
= & \quad \text{by Lemma 10 RL with } \{x2 \leftarrow rd(x1, op_r(x1, x3, x2)), x1 \leftarrow x2\} \\
& \overbrace{i(\underbrace{rd(x2, rd(x1, op_r(x1, x3, x2)))})} \\
= & \quad \text{by Lemma 2437 LR with } \{x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \underbrace{i(\overbrace{op_l(x2, x1, x3)})} \\
= & \quad \text{by Lemma 1049 LR with } \{x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \overbrace{op_l(i(x2), x1, x3)}
\end{aligned}
$$

Lemma 2465: $rd(op_r(x1, x2, x3), mult(x3, x1)) = i(op_l(x3, x2, x1))$

$$
\begin{aligned}
& \underbrace{rd(op_r(x1, x2, x3), mult(x3, x1))} \\
= & \quad \text{by Lemma 2299 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{rd(\underbrace{rd(op_r(x1, x2, x3), x1)}, x3)} \\
= & \quad \text{by Lemma 10 RL with } \{x2 \leftarrow rd(op_r(x1, x2, x3), x1), x1 \leftarrow x3\} \\
& \overbrace{i(\underbrace{rd(x3, rd(op_r(x1, x2, x3), x1))})} \\
= & \quad \text{by Lemma 2448 LR with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x3\} \\
& i(\overbrace{op_l(x3, x2, x1)})
\end{aligned}
$$

Lemma 2466: $rd(op_r(x1, x2, x3), mult(x3, x1)) = op_l(i(x3), x2, x1)$

$$
\begin{aligned}
& \underbrace{rd(op_r(x1, x2, x3), mult(x3, x1))} \\
= & \quad \text{by Lemma 2465 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{i(op_l(x3, x2, x1))}} \\
= & \quad \text{by Lemma 1049 LR with } \{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x3\} \\
& \overbrace{op_l(i(x3), x2, x1)}
\end{aligned}
$$

Lemma 2467: $rd(x1, rd(op_r(x1, x2, x3), x1)) = op_r(x1, x2, i(x3))$

$$
\begin{aligned}
& \underbrace{rd(x1, rd(op_r(x1, x2, x3), x1))} \\
= & \quad \text{by Lemma 1409 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{mult(\underbrace{rd(x1, op_r(x1, x2, x3))}, x1)} \\
= & \quad \text{by Lemma 2458 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{mult(\overbrace{rd(op_r(x1, x2, i(x3)), x1)}, x1)} \\
= & \quad \text{by Axiom 4 RL with } \{x2 \leftarrow x1, x1 \leftarrow op_r(x1, x2, i(x3))\} \\
& \overbrace{op_r(x1, x2, i(x3))}
\end{aligned}
$$

Lemma 2468: $rd(x2, x1) = rd(op_r(x2, x3, x1), op_l(x1, x2, x3))$

$$
\begin{aligned}
& \underbrace{rd(x2, x1)} \\
= \quad & \text{by Lemma 10 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{i(rd(x1, x2))}} \\
= \quad & \text{by Lemma 9 RL with } \{x2 \leftarrow rd(x1, x2), x1 \leftarrow op_r(x2, x3, x1)\} \\
& \overbrace{rd(op_r(x2, x3, x1), \underbrace{mult(rd(x1, x2), op_r(x2, x3, x1))})} \\
= \quad & \text{by Lemma 2442 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& rd(op_r(x2, x3, x1), \overbrace{op_l(x1, x2, x3)})
\end{aligned}
$$

Lemma 2469: $mult(x1, x2) = mult(op_r(x1, x3, x2), op_l(x2, x3, x1))$

$$
\begin{aligned}
& \underbrace{mult(x1, x2)} \\
= \quad & \text{by Lemma 1471 RL with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow op_r(x1, x3, x2)\} \\
& \overbrace{op_l(mult(op_r(x1, x3, x2), \underbrace{mult(rd(x1, op_r(x1, x3, x2)), x2)}), op_r(x1, x3, x2), x1)} \\
= \quad & \text{by Lemma 2450 LR with } \{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \underbrace{op_l(mult(op_r(x1, x3, x2), \overbrace{op_l(x2, x3, x1)}), op_r(x1, x3, x2), x1)} \\
= \quad & \text{by Lemma 1443 LR with } \{x4 \leftarrow x2, x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow mult(op_r(x1, x3, x2), op_l(x2, x3, x1))\} \\
& \overbrace{mult(op_r(x1, x3, x2), op_l(x2, x3, x1))}
\end{aligned}
$$

Lemma 2470: $op_t(x1, rd(x2, mult(x1, op_r(x2, x3, x4)))) = x1$

$\overbrace{op_t(x1, rd(x2, mult(x1, op_r(x2, x3, x4))))}$

$=$ by Lemma 2078 LR with $\{x3 \leftarrow op_r(x2, x3, x4), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_t(x1, rd(\underbrace{rd(op_r(x2, x3, x4), x2)}, x1))}$

$=$ by Lemma 712 RL with $\{x2 \leftarrow rd(op_r(x2, x3, x4), x2), x1 \leftarrow x1\}$

$\overbrace{op_t(x1, rd(\underbrace{i(rd(op_r(x2, x3, x4), x2))}, x1))}$

$=$ by Lemma 1013 LR with $\{x1 \leftarrow x4, x2 \leftarrow rd(op_r(x2, x3, x4), x2)\}$

$\underbrace{op_t(x1, rd(\overbrace{op_t(rd(x4, mult(x4, rd(op_r(x2, x3, x4), x2))), op_t(i(x4), rd(op_r(x2, x3, x4), x2)))}, x1))}$

$=$ by Lemma 1634 LR with $\{x3 \leftarrow op_t(i(x4), rd(op_r(x2, x3, x4), x2)), x2 \leftarrow rd(x4, mult(x4, rd(op_r(x2, x3, x4), x2))), x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_t(x1, rd(rd(x4, mult(x4, rd(op_r(x2, x3, x4), x2))), x1))}}$

$=$ by Lemma 2078 RL with $\{x3 \leftarrow x4, x2 \leftarrow mult(x4, rd(op_r(x2, x3, x4), x2)), x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_t(x1, rd(mult(x4, rd(op_r(x2, x3, x4), x2)), mult(x1, x4)))}}$

$=$ by Lemma 1767 LR with $\{x3 \leftarrow x4, x2 \leftarrow mult(x4, rd(op_r(x2, x3, x4), x2)), x1 \leftarrow x1\}$

$\overbrace{op_t(x1, rd(x4, mult(x1, \underbrace{mult(x4, rd(op_r(x2, x3, x4), x2))})))}$

$=$ by Lemma 2440 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x4\}$

$\underbrace{op_t(x1, rd(x4, mult(x1, \overbrace{op_l(x4, x2, x3)})))}$

$=$ by Lemma 2078 LR with $\{x3 \leftarrow op_l(x4, x2, x3), x2 \leftarrow x4, x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_t(x1, rd(rd(op_l(x4, x2, x3), x4), x1))}}$

$=$ by Lemma 1509 LR with $\{x2 \leftarrow rd(op_l(x4, x2, x3), x4), x1 \leftarrow x1\}$

$\overbrace{mult(i(rd(op_l(x4, x2, x3), x4)), op_r(\underbrace{mult(x1, rd(op_l(x4, x2, x3), x4))}, rd(op_l(x4, x2, x3), x4), x1))}$

$=$ by Lemma 2363 RL with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x4, x4 \leftarrow x1\}$

$mult(\underbrace{i(rd(op_l(x4, x2, x3), x4))}, op_r(\overbrace{rd(x1, rd(x4, op_l(x4, x2, x3)))}, rd(op_l(x4, x2, x3), x4), x1))$

$=$ by Lemma 10 LR with $\{x2 \leftarrow x4, x1 \leftarrow op_l(x4, x2, x3)\}$

$mult(\overbrace{rd(x4, op_l(x4, x2, x3))}, \underbrace{op_r(rd(x1, rd(x4, op_l(x4, x2, x3))), rd(op_l(x4, x2, x3), x4), x1)})$

$=$ by Lemma 1920 LR with $\{x3 \leftarrow op_l(x4, x2, x3), x2 \leftarrow x4, x1 \leftarrow x1\}$

$mult(\underbrace{rd(x4, op_l(x4, x2, x3))}, \overbrace{mult(rd(op_l(x4, x2, x3), x4), x1)})$

$=$ by Lemma 10 RL with $\{x2 \leftarrow x4, x1 \leftarrow op_l(x4, x2, x3)\}$

$\underbrace{mult(\overbrace{i(rd(op_l(x4, x2, x3), x4))}, mult(rd(op_l(x4, x2, x3), x4), x1))}$

$=$ by Axiom 2 RL with $\{x2 \leftarrow rd(op_l(x4, x2, x3), x4), x1 \leftarrow x1\}$

$\overbrace{x1}$

Lemma 2471: $op_r(op_t(x1,x2),rd(x3,x1),x4) = op_r(op_t(x1,x2),x3,x4)$

$\underbrace{op_r(op_t(x1,x2),rd(x3,x1),x4)}$

$=$ by Axiom 14 RL with $\{x4 \leftarrow x2, x3 \leftarrow x4, x2 \leftarrow rd(x3,x1), x1 \leftarrow x1\}$

$\overbrace{op_t(\underbrace{op_r(x1,rd(x3,x1),x4)},x2)}$

$=$ by Lemma 1940 LR with $\{x3 \leftarrow x4, x2 \leftarrow x3, x1 \leftarrow x1\}$

$\underbrace{op_t(\overbrace{op_r(x1,i(x3),i(x4))},x2)}$

$=$ by Axiom 14 LR with $\{x4 \leftarrow x2, x3 \leftarrow i(x4), x2 \leftarrow i(x3), x1 \leftarrow x1\}$

$\underbrace{\overbrace{op_r(op_t(x1,x2),i(x3),i(x4))}}$

$=$ by Lemma 2427 LR with $\{x3 \leftarrow i(x4), x2 \leftarrow x3, x1 \leftarrow op_t(x1,x2)\}$

$\overbrace{op_r(op_t(x1,x2),x3,\underbrace{i(i(x4))})}$

$=$ by Lemma 3 LR with $\{x1 \leftarrow x4\}$

$op_r(op_t(x1,x2),x3,\overbrace{x4})$

Lemma 2472: $op_l(x1, x2, rd(i(x3), x3)) = op_l(x1, x3, mult(x2, mult(x3, x2)))$

$$
\begin{aligned}
& \underbrace{op_l(x1, x2, rd(i(x3), x3))} \\
= \quad & \text{by Lemma 2437 RL with } \{x3 \leftarrow rd(i(x3), x3), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{rd(x1, rd(x2, \underbrace{op_r(x2, rd(i(x3), x3), x1)}))} \\
= \quad & \text{by Lemma 2172 LR with } \{x4 \leftarrow x1, x2 \leftarrow x3, x3 \leftarrow i(x3), x1 \leftarrow x2\} \\
& \underbrace{rd(x1, rd(x2, \overbrace{op_r(x2, rd(op_t(i(x3), x2), x3), x1)}))} \\
= \quad & \text{by Lemma 2437 LR with } \{x3 \leftarrow rd(op_t(i(x3), x2), x3), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, x2, rd(op_t(i(x3), x2), x3))}} \\
= \quad & \text{by Lemma 2052 LR with } \{x4 \leftarrow x3, x3 \leftarrow op_t(i(x3), x2), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, rd(x3, op_t(i(x3), x2)), x2)}} \\
= \quad & \text{by Lemma 2070 LR with } \{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, op_t(mult(x3, mult(x2, x3)), i(x2)), \underbrace{op_t(x2, mult(x3, mult(x2, x3)))})} \\
= \quad & \text{by Lemma 1722 RL with } \{x3 \leftarrow i(x2), x2 \leftarrow mult(x3, mult(x2, x3)), x1 \leftarrow x2\} \\
& \underbrace{op_l(x1, op_t(mult(x3, mult(x2, x3)), i(x2)), \overbrace{op_t(x2, op_t(mult(x3, mult(x2, x3)), i(x2)))})} \\
= \quad & \text{by Lemma 1315 LR with } \{x3 \leftarrow x2, x2 \leftarrow op_t(mult(x3, mult(x2, x3)), i(x2)), x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x2, \underbrace{i(op_t(mult(x3, mult(x2, x3)), i(x2)))})} \\
= \quad & \text{by Lemma 19 LR with } \{x2 \leftarrow x2, x1 \leftarrow mult(x3, mult(x2, x3))\} \\
& \underbrace{op_l(x1, x2, \overbrace{op_t(i(mult(x3, mult(x2, x3))), x2)})} \\
= \quad & \text{by Lemma 1315 LR with } \{x3 \leftarrow i(mult(x3, mult(x2, x3))), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, i(mult(x3, mult(x2, x3))), i(x2))}} \\
= \quad & \text{by Lemma 1050 LR with } \{x3 \leftarrow i(x2), x2 \leftarrow mult(x3, mult(x2, x3)), x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, mult(x3, mult(x2, x3)), \underbrace{i(i(x2))})} \\
= \quad & \text{by Lemma 3 LR with } \{x1 \leftarrow x2\} \\
& \underbrace{op_l(x1, mult(x3, mult(x2, x3)), \overbrace{x2})} \\
= \quad & \text{by Lemma 1407 LR with } \{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x3, mult(x2, mult(x3, x2)))}
\end{aligned}
$$

Lemma 2473: $op_l(x1, x2, rd(x3, op_t(x4, x2))) = op_l(x1, x2, rd(x3, x4))$

$$
\begin{aligned}
& \underbrace{op_l(x1, x2, rd(x3, op_t(x4, x2)))} \\
= \quad & \text{by Lemma 2437 RL with } \{x3 \leftarrow rd(x3, op_t(x4, x2)), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{rd(x1, rd(x2, \underbrace{op_r(x2, rd(x3, op_t(x4, x2)), x1)}))} \\
= \quad & \text{by Lemma 2176 LR with } \{x4 \leftarrow x1, x3 \leftarrow x4, x2 \leftarrow x3, x1 \leftarrow x2\} \\
& \underbrace{rd(x1, rd(x2, \overbrace{op_r(x2, rd(x3, x4), x1)}))} \\
= \quad & \text{by Lemma 2437 LR with } \{x3 \leftarrow rd(x3, x4), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, x2, rd(x3, x4))}}
\end{aligned}
$$

Lemma 2474: $rd(op_l(mult(x1,x2),x1,x3),x2) = op_r(x1,x3,mult(x1,x2))$

$rd(op_l(mult(x1,x2),x1,x3),\underbrace{x2}_{})$

$=$ by Lemma 1444 RL with $\{x4 \leftarrow mult(x1,x2), x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow x2\}$

$rd(\underbrace{op_l(mult(x1,x2),x1,x3)}_{},\overbrace{op_l(x2,x1,op_r(x1,x3,mult(x1,x2)))}^{})$

$=$ by Lemma 2441 RL with $\{x3 \leftarrow mult(x1,x2), x2 \leftarrow x3, x1 \leftarrow x1\}$

$rd(\overbrace{mult(rd(op_r(x1,x3,mult(x1,x2)),x1),mult(x1,x2))}^{},\underbrace{op_l(x2,x1,op_r(x1,x3,mult(x1,x2)))}_{})$

$=$ by Lemma 286 RL with $\{x3 \leftarrow op_r(x1,x3,mult(x1,x2)), x2 \leftarrow x1, x1 \leftarrow x2\}$

$\underbrace{rd(mult(rd(op_r(x1,x3,mult(x1,x2)),x1),mult(x1,x2)),\overbrace{op_l(x2,x1,rd(op_r(x1,x3,mult(x1,x2)),x1))}^{})}_{}$

$=$ by Lemma 888 LR with $\{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow rd(op_r(x1,x3,mult(x1,x2)),x1)\}$

$\overbrace{\underbrace{mult(rd(op_r(x1,x3,mult(x1,x2)),x1),x1)}_{}}^{}$

$=$ by Axiom 4 RL with $\{x2 \leftarrow x1, x1 \leftarrow op_r(x1,x3,mult(x1,x2))\}$

$\overbrace{op_r(x1,x3,mult(x1,x2))}^{}$

Lemma 2475: $op_l(x1, x2, mult(x3, mult(x2, x3))) = op_l(x1, x2, mult(x3, mult(x3, i(x2))))$

$$
\begin{aligned}
& op_l(x1, x2, \underbrace{mult(x3, mult(x2, x3))}) \\
= & \quad \text{by Lemma 140 RL with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& \underbrace{op_l(x1, x2, \overbrace{rd(x2, rd(op_t(i(x3), x2), x3))})} \\
= & \quad \text{by Lemma 1321 LR with } \{x3 \leftarrow rd(op_t(i(x3), x2), x3), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x2, \underbrace{i(rd(op_t(i(x3), x2), x3))})} \\
= & \quad \text{by Lemma 10 LR with } \{x2 \leftarrow x3, x1 \leftarrow op_t(i(x3), x2)\} \\
& \underbrace{op_l(x1, x2, \overbrace{rd(x3, op_t(i(x3), x2))})} \\
= & \quad \text{by Lemma 2446 RL with } \{x4 \leftarrow x3, x3 \leftarrow rd(x3, op_t(i(x3), x2)), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x2, rd(rd(x3, op_t(i(x3), x2)), \underbrace{op_t(x2, x3)}))} \\
= & \quad \text{by Lemma 1722 RL with } \{x3 \leftarrow i(x2), x2 \leftarrow x3, x1 \leftarrow x2\} \\
& op_l(x1, x2, \underbrace{rd(rd(x3, op_t(i(x3), x2)), \overbrace{op_t(x2, op_t(x3, i(x2)))})}) \\
= & \quad \text{by Lemma 2412 LR with } \{x2 \leftarrow x2, x1 \leftarrow x3\} \\
& op_l(x1, x2, \underbrace{\overbrace{mult(op_t(x3, i(x2)), mult(op_t(i(x2), x3), op_t(x3, i(x2))))}}) \\
= & \quad \text{by Lemma 1263 LR with } \{x3 \leftarrow op_t(x3, i(x2)), x2 \leftarrow i(x2), x1 \leftarrow x3\} \\
& op_l(x1, x2, \underbrace{\overbrace{mult(x3, mult(i(x2), op_t(x3, i(x2))))}}) \\
= & \quad \text{by Lemma 1776 LR with } \{x3 \leftarrow i(x2), x2 \leftarrow x2, x1 \leftarrow x3\} \\
& op_l(x1, x2, \overbrace{rd(x3, mult(\underbrace{i(op_t(x3, i(x2)))}, x2))}) \\
= & \quad \text{by Lemma 19 LR with } \{x2 \leftarrow x2, x1 \leftarrow x3\} \\
& op_l(x1, x2, rd(x3, \underbrace{mult(\overbrace{op_t(i(x3), x2)}, x2)})) \\
= & \quad \text{by Lemma 340 LR with } \{x2 \leftarrow x2, x1 \leftarrow x3\} \\
& op_l(x1, x2, \underbrace{rd(x3, \overbrace{op_t(rd(x2, x3), x2)})}) \\
= & \quad \text{by Lemma 464 LR with } \{x2 \leftarrow x2, x1 \leftarrow x3\} \\
& op_l(x1, x2, \overbrace{mult(x3, mult(x3, i(x2)))})
\end{aligned}
$$

Lemma 2476: $op_l(x1, x2, mult(x3, mult(x2, x3))) = op_l(x1, x2, mult(x3, x3))$

$$
\begin{aligned}
& \underbrace{op_l(x1, x2, mult(x3, mult(x2, x3)))} \\
= & \quad \text{by Lemma 2475 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x2, \underbrace{mult(x3, mult(x3, i(x2)))})} \\
= & \quad \text{by Lemma 24 RL with } \{x2 \leftarrow i(x2), x1 \leftarrow x3\} \\
& \underbrace{op_l(x1, x2, \overbrace{mult(mult(x3, x3), i(x2))})} \\
= & \quad \text{by Lemma 1325 LR with } \{x3 \leftarrow mult(x3, x3), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x2, mult(x3, x3))}
\end{aligned}
$$

Lemma 2477: $op_l(x1, mult(x2, x2), x3) = op_l(x1, x2, mult(x3, x3))$

$$
\begin{aligned}
& \overbrace{op_l(x1, mult(x2, x2), x3)} \\
= \quad & \text{by Lemma 2039 RL with } \{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{\overbrace{op_l(x1, x3, rd(i(x2), x2))}} \\
= \quad & \text{by Lemma 2472 LR with } \{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{\overbrace{op_l(x1, x2, mult(x3, mult(x2, x3)))}} \\
= \quad & \text{by Lemma 2476 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x2, mult(x3, x3))}
\end{aligned}
$$

Lemma 2478: $op_l(x1, x3, mult(x2, x2)) = op_l(x1, rd(rd(x3, x2), x2), x3)$

$$
\begin{aligned}
& \overbrace{op_l(x1, x3, mult(x2, x2))} \\
= \quad & \text{by Lemma 2038 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\overbrace{op_l(x1, rd(i(x2), x2), x3)}} \\
= \quad & \text{by Lemma 2447 LR with } \{x4 \leftarrow rd(x3, x2), x2 \leftarrow x3, x3 \leftarrow rd(i(x2), x2), x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, \underbrace{mult(rd(i(x2), x2), op_t(x3, rd(x3, x2)))}, x3)} \\
= \quad & \text{by Lemma 1923 LR with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& op_l(x1, \overbrace{rd(rd(x3, x2), x2)}, x3)
\end{aligned}
$$

Lemma 2479: $op_l(x1, rd(x2, x3), mult(x2, x3)) = op_l(x1, x2, mult(x3, x3))$

$$
\begin{aligned}
& \overbrace{op_l(x1, rd(x2, x3), mult(x2, x3))} \\
= \quad & \text{by Lemma 2048 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\overbrace{op_l(x1, x2, rd(x3, rd(x2, x3)))}} \\
= \quad & \text{by Lemma 2052 LR with } \{x4 \leftarrow rd(x2, x3), x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\overbrace{op_l(x1, rd(rd(x2, x3), x3), x2)}} \\
= \quad & \text{by Lemma 2478 RL with } \{x2 \leftarrow x3, x3 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x2, mult(x3, x3))}
\end{aligned}
$$

Lemma 2480: $op_l(x1, x2, rd(rd(x2, x3), x3)) = op_l(x1, mult(x3, x3), x2)$

$$
\begin{aligned}
& op_l(x1, x2, \underbrace{rd(rd(x2, x3), x3)}) \\
= & \quad \text{by Lemma 1923 RL with } \{x2 \leftarrow x2, x1 \leftarrow x3\} \\
& \underbrace{op_l(x1, x2, \overbrace{mult(rd(i(x3), x3), op_t(x2, rd(x2, x3)))})} \\
= & \quad \text{by Lemma 2437 RL with } \{x3 \leftarrow mult(rd(i(x3), x3), op_t(x2, rd(x2, x3))), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{rd(x1, rd(x2, \underbrace{op_r(x2, mult(rd(i(x3), x3), op_t(x2, rd(x2, x3))), x1)}))} \\
= & \quad \text{by Lemma 2174 RL with } \{x2 \leftarrow rd(x2, x3), x4 \leftarrow x1, x3 \leftarrow rd(i(x3), x3), x1 \leftarrow x2\} \\
& rd(x1, rd(x2, \underbrace{\overbrace{op_r(x2, rd(x2, i(rd(i(x3), x3))), x1)}})) \\
= & \quad \text{by Lemma 1949 LR with } \{x3 \leftarrow x1, x2 \leftarrow rd(i(x3), x3), x1 \leftarrow x2\} \\
& rd(x1, rd(x2, \underbrace{\overbrace{op_r(x2, mult(rd(i(x3), x3), x2), x1)}})) \\
= & \quad \text{by Lemma 2430 LR with } \{x3 \leftarrow x1, x2 \leftarrow rd(i(x3), x3), x1 \leftarrow x2\} \\
& \underbrace{rd(x1, rd(x2, \overbrace{op_r(x2, rd(i(x3), x3), x1)}))} \\
= & \quad \text{by Lemma 2437 LR with } \{x3 \leftarrow rd(i(x3), x3), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, x2, rd(i(x3), x3))}} \\
= & \quad \text{by Lemma 2039 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, mult(x3, x3), x2)}
\end{aligned}
$$

Lemma 2481: $op_l(x1, rd(x2, x3), mult(x3, x2)) = op_l(x1, mult(x2, x2), x3)$

$$
\begin{aligned}
& \underbrace{op_l(x1, rd(x2, x3), mult(x3, x2))} \\
= & \quad \text{by Lemma 2042 RL with } \{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, mult(mult(x3, x2), x2), x3)}} \\
= & \quad \text{by Lemma 1439 LR with } \{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, x3, rd(rd(x3, x2), x2))}} \\
= & \quad \text{by Lemma 2480 LR with } \{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, mult(x2, x2), x3)}
\end{aligned}
$$

Lemma 2482: $i(op_r(x1, x2, i(x3))) = mult(op_l(i(x3), x2, x1), mult(x3, i(x1)))$

$$
\begin{aligned}
& \underbrace{i(op_r(x1, x2, i(x3)))} \\
= & \quad \text{by Lemma 1003 RL with } \{x3 \leftarrow op_r(x1, x2, i(x3)), x2 \leftarrow x1, x1 \leftarrow x3\} \\
& \overbrace{mult(\underbrace{rd(mult(i(x3), x1), op_r(x1, x2, i(x3)))}, mult(x3, i(x1)))} \\
= & \quad \text{by Lemma 2295 LR with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow i(x3)\} \\
& mult(\underbrace{\overbrace{rd(i(x3), rd(op_r(x1, x2, i(x3)), x1))}}, mult(x3, i(x1))) \\
= & \quad \text{by Lemma 2448 LR with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow i(x3)\} \\
& mult(\overbrace{op_l(i(x3), x2, x1)}, mult(x3, i(x1)))
\end{aligned}
$$

Lemma 2483: $op_l(x1, op_t(x2, x3), rd(x4, x2)) = op_l(x1, op_t(x2, x3), x4)$

$$
\begin{aligned}
& \underbrace{op_l(x1, op_t(x2, x3), rd(x4, x2))} \\
= & \quad \text{by Lemma 2437 RL with } \{x3 \leftarrow rd(x4, x2), x2 \leftarrow op_t(x2, x3), x1 \leftarrow x1\} \\
& \overbrace{rd(x1, rd(op_t(x2, x3), \underbrace{op_r(op_t(x2, x3), rd(x4, x2), x1)}))} \\
= & \quad \text{by Lemma 2471 LR with } \{x4 \leftarrow x1, x3 \leftarrow x4, x2 \leftarrow x3, x1 \leftarrow x2\} \\
& \underbrace{rd(x1, rd(op_t(x2, x3), \overbrace{op_r(op_t(x2, x3), x4, x1)}))} \\
= & \quad \text{by Lemma 2437 LR with } \{x3 \leftarrow x4, x2 \leftarrow op_t(x2, x3), x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, op_t(x2, x3), x4)}
\end{aligned}
$$

Lemma 2484: $op_l(x1, op_t(x2, x3), i(x4)) = op_l(x1, mult(x4, x2), op_t(x2, x3))$

$$
\begin{aligned}
& \underbrace{op_l(x1, op_t(x2, x3), i(x4))} \\
= & \quad \text{by Lemma 2483 RL with } \{x4 \leftarrow i(x4), x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, op_t(x2, x3), rd(i(x4), x2))}} \\
= & \quad \text{by Lemma 2052 LR with } \{x4 \leftarrow x2, x3 \leftarrow i(x4), x2 \leftarrow op_t(x2, x3), x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, rd(x2, i(x4)), op_t(x2, x3))}} \\
= & \quad \text{by Lemma 2027 RL with } \{x4 \leftarrow x3, x3 \leftarrow x2, x2 \leftarrow x4, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, mult(x4, x2), op_t(x2, x3))}
\end{aligned}
$$

Lemma 2485: $op_l(x1, mult(x2, x3), op_t(x3, x4)) = op_l(x1, x2, op_t(x3, x4))$

$$
\begin{aligned}
& \underbrace{op_l(x1, mult(x2, x3), op_t(x3, x4))} \\
= & \quad \text{by Lemma 2484 RL with } \{x4 \leftarrow x2, x3 \leftarrow x4, x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, op_t(x3, x4), i(x2))}} \\
= & \quad \text{by Lemma 2030 LR with } \{x3 \leftarrow x2, x2 \leftarrow op_t(x3, x4), x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x2, op_t(x3, x4))}
\end{aligned}
$$

Lemma 2486: $op_l(x1, i(x2), op_t(x3, x4)) = op_l(x1, rd(x3, x2), op_t(x3, x4))$

$$
\begin{aligned}
& \underbrace{op_l(x1, i(x2), op_t(x3, x4))} \\
= & \quad \text{by Lemma 2485 RL with } \{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, mult(i(x2), x3), op_t(x3, x4))}} \\
= & \quad \text{by Lemma 2026 LR with } \{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, mult(rd(x3, x2), op_t(x3, x4)), op_t(x3, x4))}} \\
= & \quad \text{by Lemma 285 LR with } \{x3 \leftarrow op_t(x3, x4), x2 \leftarrow rd(x3, x2), x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, rd(x3, x2), op_t(x3, x4))}
\end{aligned}
$$

Lemma 2487: $op_l(x1, rd(x2, x3), op_t(x3, x4)) = op_l(x1, x2, op_t(x3, x4))$

$$
\begin{aligned}
& \underbrace{op_l(x1, rd(x2, x3), op_t(x3, x4))} \\
= \quad & \text{by Lemma 2448 RL with } \{x3 \leftarrow rd(x2, x3), x2 \leftarrow op_t(x3, x4), x1 \leftarrow x1\} \\
& \overbrace{rd(x1, rd(\underbrace{op_r(op_t(x3, x4), rd(x2, x3), x1)}, op_t(x3, x4)))} \\
= \quad & \text{by Lemma 2471 LR with } \{x4 \leftarrow x1, x3 \leftarrow x2, x2 \leftarrow x4, x1 \leftarrow x3\} \\
& \underbrace{rd(x1, rd(\overbrace{op_r(op_t(x3, x4), x2, x1)}, op_t(x3, x4)))} \\
= \quad & \text{by Lemma 2448 LR with } \{x3 \leftarrow x2, x2 \leftarrow op_t(x3, x4), x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x2, op_t(x3, x4))}
\end{aligned}
$$

Lemma 2488: $op_l(x1, i(x2), op_t(x3, x4)) = op_l(x1, op_t(x3, x4), mult(x2, x3))$

$$
\begin{aligned}
& \underbrace{op_l(x1, i(x2), op_t(x3, x4))} \\
= \quad & \text{by Lemma 2487 RL with } \{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& \underbrace{\overbrace{op_l(x1, rd(i(x2), x3), op_t(x3, x4))}} \\
= \quad & \text{by Lemma 2052 RL with } \{x4 \leftarrow i(x2), x3 \leftarrow x3, x2 \leftarrow op_t(x3, x4), x1 \leftarrow x1\} \\
& \underbrace{\overbrace{op_l(x1, op_t(x3, x4), rd(x3, i(x2)))}} \\
= \quad & \text{by Lemma 2028 RL with } \{x4 \leftarrow x2, x3 \leftarrow x4, x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, op_t(x3, x4), mult(x2, x3))}
\end{aligned}
$$

Lemma 2489: $op_r(mult(x1, x1), x2, x3) = mult(x1, mult(x1, rd(x3, op_l(x3, mult(x2, x2), x1))))$

$$
\begin{aligned}
& \underbrace{op_r(mult(x1, x1), x2, x3)} \\
= \quad & \text{by Lemma 2461 RL with } \{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow mult(x1, x1)\} \\
& \overbrace{mult(mult(x1, x1), rd(x3, \underbrace{op_l(x3, x2, mult(x1, x1))}))} \\
= \quad & \text{by Lemma 2477 RL with } \{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x3\} \\
& \underbrace{mult(mult(x1, x1), rd(x3, \overbrace{op_l(x3, mult(x2, x2), x1)}))} \\
= \quad & \text{by Lemma 24 LR with } \{x2 \leftarrow rd(x3, op_l(x3, mult(x2, x2), x1)), x1 \leftarrow x1\} \\
& \overbrace{mult(x1, mult(x1, rd(x3, op_l(x3, mult(x2, x2), x1))))}
\end{aligned}
$$

Lemma 2490: $mult(x1, op_r(x1, mult(x2, x2), x3)) = op_r(mult(x1, x1), x2, x3)$

$$
\begin{aligned}
& mult(x1, \underbrace{op_r(x1, mult(x2, x2), x3)}) \\
= \quad & \text{by Lemma 2461 RL with } \{x3 \leftarrow mult(x2, x2), x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \underbrace{mult(x1, \overbrace{mult(x1, rd(x3, op_l(x3, mult(x2, x2), x1)))})} \\
= \quad & \text{by Lemma 2489 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_r(mult(x1, x1), x2, x3)}
\end{aligned}
$$

Lemma 2491: $op_l(x1, mult(x2, x2), mult(x3, x2)) = op_l(x1, mult(mult(x3, x2), x2), x3)$

$\underbrace{op_l(x1, mult(x2, x2), mult(x3, x2))}$

$=$ by Lemma 2481 RL with $\{x3 \leftarrow mult(x3, x2), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_l(x1, \underbrace{rd(x2, mult(x3, x2))}, mult(mult(x3, x2), x2))}$

$=$ by Lemma 9 LR with $\{x2 \leftarrow x3, x1 \leftarrow x2\}$

$\underbrace{op_l(x1, \overbrace{i(x3)}, mult(mult(x3, x2), x2))}$

$=$ by Lemma 2031 LR with $\{x3 \leftarrow mult(mult(x3, x2), x2), x2 \leftarrow x3, x1 \leftarrow x1\}$

$\overbrace{op_l(x1, mult(mult(x3, x2), x2), x3)}$

Lemma 2492: $op_l(x1, mult(x2, x2), mult(x3, x2)) = op_l(x1, mult(x2, x2), x3)$

$\underbrace{op_l(x1, mult(x2, x2), mult(x3, x2))}$

$=$ by Lemma 2491 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_l(x1, mult(mult(x3, x2), x2), x3)}}$

$=$ by Lemma 1439 LR with $\{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_l(x1, x3, rd(rd(x3, x2), x2))}}$

$=$ by Lemma 2480 LR with $\{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x1\}$

$\overbrace{op_l(x1, mult(x2, x2), x3)}$

Lemma 2493: $op_l(x1, mult(x2, x2), op_t(x3, x2)) = rd(x1, rd(mult(x2, x2), op_r(mult(x2, x2), x3, x1)))$

$$
\begin{aligned}
& \underbrace{op_l(x1, mult(x2, x2), op_t(x3, x2))} \\
= \quad & \text{by Lemma 2437 RL with } \{x3 \leftarrow op_t(x3, x2), x2 \leftarrow mult(x2, x2), x1 \leftarrow x1\} \\
& \overbrace{rd(x1, rd(mult(x2, x2), \underbrace{op_r(mult(x2, x2), op_t(x3, x2), x1)}))} \\
= \quad & \text{by Lemma 2490 RL with } \{x3 \leftarrow x1, x2 \leftarrow op_t(x3, x2), x1 \leftarrow x2\} \\
& rd(x1, rd(mult(x2, x2), \overbrace{mult(x2, op_r(x2, \underbrace{mult(op_t(x3, x2), op_t(x3, x2))}, x1))})) \\
= \quad & \text{by Lemma 231 LR with } \{x2 \leftarrow op_t(x3, x2), x1 \leftarrow op_t(x3, x2)\} \\
& rd(x1, rd(mult(x2, x2), mult(x2, \underbrace{op_r(x2, \overbrace{rd(op_t(x3, x2), op_l(i(op_t(x3, x2)), op_t(x3, x2), op_t(x3, x2)))}, x1)}))) \\
= \quad & \text{by Lemma 2172 RL with } \{x4 \leftarrow x1, x2 \leftarrow op_l(i(op_t(x3, x2)), op_t(x3, x2), op_t(x3, x2)), x3 \leftarrow x3, x1 \leftarrow x2\} \\
& rd(x1, rd(mult(x2, x2), mult(x2, \overbrace{op_r(x2, rd(x3, \underbrace{op_l(i(op_t(x3, x2)), op_t(x3, x2), op_t(x3, x2))}), x1)}))) \\
= \quad & \text{by Lemma 1740 LR with } \{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow op_t(x3, x2)\} \\
& rd(x1, rd(mult(x2, x2), mult(x2, op_r(x2, \underbrace{rd(x3, \overbrace{op_l(i(op_t(x3, x2)), op_t(x3, x2), x3)}), x1)}))) \\
= \quad & \text{by Lemma 231 RL with } \{x2 \leftarrow op_t(x3, x2), x1 \leftarrow x3\} \\
& rd(x1, rd(mult(x2, x2), mult(x2, op_r(x2, \overbrace{mult(x3, op_t(x3, x2))}, x1)))) \\
= \quad & \text{by Lemma 386 RL with } \{x2 \leftarrow x3, x1 \leftarrow op_t(x3, x2)\} \\
& rd(x1, rd(mult(x2, x2), mult(x2, \underbrace{op_r(x2, \overbrace{rd(op_t(x3, x2), op_t(i(x3), rd(x3, op_t(x3, x2))))}, x1)}))) \\
= \quad & \text{by Lemma 2172 RL with } \{x4 \leftarrow x1, x2 \leftarrow op_t(i(x3), rd(x3, op_t(x3, x2))), x3 \leftarrow x3, x1 \leftarrow x2\} \\
& rd(x1, rd(mult(x2, x2), mult(x2, \overbrace{op_r(x2, rd(x3, \underbrace{op_t(i(x3), rd(x3, op_t(x3, x2)))}), x1)}))) \\
= \quad & \text{by Lemma 1639 LR with } \{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x3\} \\
& rd(x1, rd(mult(x2, x2), mult(x2, op_r(x2, \underbrace{rd(x3, \overbrace{op_t(i(x3), rd(x3, x3))}), x1)}))) \\
= \quad & \text{by Lemma 386 LR with } \{x2 \leftarrow x3, x1 \leftarrow x3\} \\
& rd(x1, rd(mult(x2, x2), \underbrace{mult(x2, op_r(x2, \overbrace{mult(x3, x3)}, x1))})) \\
= \quad & \text{by Lemma 2490 LR with } \{x3 \leftarrow x1, x2 \leftarrow x3, x1 \leftarrow x2\} \\
& rd(x1, rd(mult(x2, x2), \overbrace{op_r(mult(x2, x2), x3, x1)})))
\end{aligned}
$$

Lemma 2494: $op_l(x1, mult(x2, x2), op_t(x3, x2)) = op_l(x1, mult(x2, x2), x3)$

$$
\begin{aligned}
& \underbrace{op_l(x1, mult(x2, x2), op_t(x3, x2))} \\
= \quad & \text{by Lemma 2493 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{\overbrace{rd(x1, rd(mult(x2, x2), op_r(mult(x2, x2), x3, x1)))}} \\
= \quad & \text{by Lemma 2437 LR with } \{x3 \leftarrow x3, x2 \leftarrow mult(x2, x2), x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, \overbrace{mult(x2, x2)}, x3)}
\end{aligned}
$$

Lemma 2495: $rd(x1, mult(i(x2), op_l(x1, x3, x2))) = op_r(x2, x3, x1)$

$$
\begin{aligned}
& rd(x1, mult(i(x2), \underbrace{op_l(x1, x3, x2)})) \\
= & \quad \text{by Lemma 2031 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{rd(x1, mult(i(x2), \overbrace{op_l(x1, i(x2), x3)}))} \\
= & \quad \text{by Lemma 2453 LR with } \{x3 \leftarrow x3, x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& \overbrace{\underbrace{i(op_r(i(x2), x3, x1))}} \\
= & \quad \text{by Lemma 2428 RL with } \{x3 \leftarrow x1, x2 \leftarrow x3, x1 \leftarrow i(x2)\} \\
& \overbrace{op_r(\underbrace{i(i(x2))}, x3, x1)} \\
= & \quad \text{by Lemma 3 LR with } \{x1 \leftarrow x2\} \\
& op_r(\overbrace{x2}, x3, x1)
\end{aligned}
$$

Lemma 2496: $op_l(op_r(x1, x2, mult(x1, x3)), x2, x1) = op_r(x1, x2, x3)$

$$
\begin{aligned}
& op_l(\underbrace{op_r(x1, x2, mult(x1, x3))}, x2, x1) \\
= & \quad \text{by Lemma 2389 RL with } \{x5 \leftarrow x2, x4 \leftarrow x1, x3 \leftarrow mult(x1, x3), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_l(\overbrace{op_r(x1, x2, \underbrace{rd(mult(x1, x3), asoc(x1, x2, x1))})}, x2, x1) \\
= & \quad \text{by Lemma 1708 RL with } \{x3 \leftarrow mult(x1, x3), x2 \leftarrow x1, x1 \leftarrow x2\} \\
& op_l(op_r(x1, x2, \overbrace{mult(asoc(x2, x1, x1), mult(x1, x3))}), x2, x1) \\
= & \quad \text{by Lemma 959 LR with } \{x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow x2\} \\
& op_l(\underbrace{op_r(x1, x2, \overbrace{mult(op_l(x1, x2, x1), x3)})}, x2, x1) \\
= & \quad \text{by Lemma 2431 RL with } \{x3 \leftarrow mult(op_l(x1, x2, x1), x3), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{op_l(\overbrace{op_r(x1, mult(x1, x2), mult(op_l(x1, x2, x1), x3))}, x2, x1)} \\
= & \quad \text{by Axiom 18 RL with } \{x5 \leftarrow mult(op_l(x1, x2, x1), x3), x4 \leftarrow mult(x1, x2), x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_r(op_l(x1, x2, x1), mult(x1, x2), mult(op_l(x1, x2, x1), x3))}} \\
= & \quad \text{by Lemma 2474 RL with } \{x3 \leftarrow mult(x1, x2), x2 \leftarrow x3, x1 \leftarrow op_l(x1, x2, x1)\} \\
& \overbrace{rd(\underbrace{op_l(mult(op_l(x1, x2, x1), x3), op_l(x1, x2, x1), mult(x1, x2))}, x3)} \\
= & \quad \text{by Lemma 1077 RL with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow mult(op_l(x1, x2, x1), x3)\} \\
& rd(\overbrace{op_l(\underbrace{mult(op_l(x1, x2, x1), x3)}, x1, x2)}, x3) \\
= & \quad \text{by Lemma 1880 RL with } \{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x1\} \\
& rd(\underbrace{op_l(\overbrace{op_l(mult(x1, op_l(x3, x1, x2)), x2, x1)}, x1, x2)}, x3) \\
= & \quad \text{by Lemma 138 LR with } \{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow mult(x1, op_l(x3, x1, x2))\} \\
& \underbrace{rd(\overbrace{mult(x1, op_l(x3, x1, x2))}, x3)} \\
= & \quad \text{by Lemma 2361 RL with } \{x4 \leftarrow x2, x3 \leftarrow x1, x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{rd(x1, rd(x3, op_l(x3, x1, x2)))}} \\
= & \quad \text{by Lemma 2425 LR with } \{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, x2, x3)}
\end{aligned}
$$

Lemma 2497: $op_l(op_r(x1, x2, x3), x2, x1) = op_r(x1, x2, rd(x3, x1))$

$$
\begin{aligned}
& op_l(op_r(x1, x2, \underbrace{x3}), x2, x1) \\
= \quad & \text{by Lemma 12 RL with } \{x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \underbrace{op_l(op_r(x1, x2, \overbrace{mult(x1, mult(i(x1), x3))}), x2, x1)} \\
= \quad & \text{by Lemma 2496 LR with } \{x3 \leftarrow mult(i(x1), x3), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{\overbrace{op_r(x1, x2, mult(i(x1), x3))}} \\
= \quad & \text{by Lemma 2397 LR with } \{x4 \leftarrow x3, x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, x2, rd(x3, x1))}
\end{aligned}
$$

Lemma 2498: $op_r(x1, x2, i(rd(x3, x1))) = op_l(op_r(x1, op_r(rd(i(x2), x1), x2, x2), x3), x1, x2)$

$$
\begin{aligned}
& \underbrace{op_r(x1, x2, i(rd(x3, x1)))} \\
= \quad & \text{by Lemma 1966 RL with } \{x3 \leftarrow rd(x3, x1), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{\overbrace{op_r(x1, op_r(rd(i(x2), x1), x2, x2), rd(x3, x1))}} \\
= \quad & \text{by Lemma 2497 RL with } \{x3 \leftarrow x3, x2 \leftarrow op_r(rd(i(x2), x1), x2, x2), x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(op_r(x1, op_r(rd(i(x2), x1), x2, x2), x3), op_r(rd(i(x2), x1), x2, x2), x1)}} \\
= \quad & \text{by Lemma 1799 LR with } \{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow op_r(x1, op_r(rd(i(x2), x1), x2, x2), x3)\} \\
& \overbrace{op_l(op_r(x1, op_r(rd(i(x2), x1), x2, x2), x3), x1, x2)}
\end{aligned}
$$

Lemma 2499: $op_r(x1, x2, rd(x1, x3)) = op_l(op_r(x1, x2, i(x3)), x1, x2)$

$$
\begin{aligned}
& op_r(x1, x2, \underbrace{rd(x1, x3)}) \\
= \quad & \text{by Lemma 10 RL with } \{x2 \leftarrow x1, x1 \leftarrow x3\} \\
& \underbrace{op_r(x1, x2, \overbrace{i(rd(x3, x1))})} \\
= \quad & \text{by Lemma 2498 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(\underbrace{op_r(x1, op_r(rd(i(x2), x1), x2, x2), x3)}, x1, x2)} \\
= \quad & \text{by Lemma 1966 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_l(\overbrace{op_r(x1, x2, i(x3))}, x1, x2)
\end{aligned}
$$

Lemma 2500: $op_r(x1, x2, mult(x3, x1)) = op_l(op_r(x1, x2, x3), x1, x2)$

$$
\begin{aligned}
& \underbrace{op_r(x1, x2, mult(x3, x1))} \\
= \quad & \text{by Lemma 2394 RL with } \{x4 \leftarrow x3, x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{\overbrace{op_r(x1, x2, rd(x1, i(x3)))}} \\
= \quad & \text{by Lemma 2499 LR with } \{x3 \leftarrow i(x3), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(op_r(x1, x2, \underbrace{i(i(x3))}), x1, x2)} \\
= \quad & \text{by Lemma 3 LR with } \{x1 \leftarrow x3\} \\
& op_l(op_r(x1, x2, \overbrace{x3}), x1, x2)
\end{aligned}
$$

Lemma 2501: $op_l(x1, x2, rd(x3, op_l(x2, x2, x4))) = rd(x1, rd(x2, op_r(x2, x3, x1)))$

$\underbrace{op_l(x1, x2, rd(x3, op_l(x2, x2, x4)))}$

$=$ by Lemma 2437 RL with $\{x3 \leftarrow rd(x3, op_l(x2, x2, x4)), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{rd(x1, rd(x2, op_r(x2, \underbrace{rd(x3, op_l(x2, x2, x4))}, x1)))}$

$=$ by Lemma 1582 LR with $\{x2 \leftarrow x4, x1 \leftarrow x2, x3 \leftarrow x3\}$

$rd(x1, rd(x2, \underbrace{op_r(x2, \overbrace{rd(asoc(x4, x2, x2), rd(x2, x3))}, x1)}))$

$=$ by Lemma 2179 RL with $\{x4 \leftarrow x4, x3 \leftarrow x1, x2 \leftarrow rd(x2, x3), x1 \leftarrow x2\}$

$rd(x1, rd(x2, \overbrace{op_r(x2, \underbrace{i(rd(x2, x3))}, x1)}))$

$=$ by Lemma 10 LR with $\{x2 \leftarrow x3, x1 \leftarrow x2\}$

$rd(x1, rd(x2, \underbrace{op_r(x2, \overbrace{rd(x3, x2)}, x1)}))$

$=$ by Lemma 1940 LR with $\{x3 \leftarrow x1, x2 \leftarrow x3, x1 \leftarrow x2\}$

$rd(x1, rd(x2, \overbrace{\underbrace{op_r(x2, i(x3), i(x1))}}))$

$=$ by Lemma 2427 LR with $\{x3 \leftarrow i(x1), x2 \leftarrow x3, x1 \leftarrow x2\}$

$rd(x1, rd(x2, \overbrace{op_r(x2, x3, \underbrace{i(i(x1))})}))$

$=$ by Lemma 3 LR with $\{x1 \leftarrow x1\}$

$rd(x1, rd(x2, op_r(x2, x3, \overbrace{x1})))$

Lemma 2502: $rd(mult(x1, x2), op_l(x2, x2, x3)) = rd(x1, rd(i(x2), op_r(i(x2), x3, x2)))$

$rd(mult(x1, x2), \underbrace{op_l(x2, x2, x3)})$

$=$ by Lemma 601 RL with $\{x2 \leftarrow x3, x1 \leftarrow x2\}$

$\underbrace{rd(mult(x1, x2), \overbrace{op_r(x2, x3, x2)})}$

$=$ by Lemma 2046 RL with $\{x4 \leftarrow x2, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{mult(x1, \underbrace{rd(x2, op_r(x2, x3, x2))})}$

$=$ by Lemma 23 RL with $\{x2 \leftarrow x2, x1 \leftarrow op_r(x2, x3, x2)\}$

$mult(x1, \overbrace{rd(\underbrace{i(op_r(x2, x3, x2))}, i(x2))})$

$=$ by Lemma 2428 RL with $\{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x2\}$

$\underbrace{mult(x1, rd(\overbrace{op_r(i(x2), x3, x2)}, i(x2)))}$

$=$ by Lemma 1914 RL with $\{x3 \leftarrow i(x2), x2 \leftarrow op_r(i(x2), x3, x2), x1 \leftarrow x1\}$

$\overbrace{rd(mult(x1, op_r(i(x2), x3, x2)), \underbrace{op_l(i(x2), rd(op_r(i(x2), x3, x2), i(x2)), x1)})}$

$=$ by Lemma 1049 RL with $\{x3 \leftarrow x1, x2 \leftarrow rd(op_r(i(x2), x3, x2), i(x2)), x1 \leftarrow x2\}$

$rd(mult(x1, op_r(i(x2), x3, x2)), \overbrace{i(\underbrace{op_l(x2, rd(op_r(i(x2), x3, x2), i(x2)), x1)})})$

$=$ by Lemma 2339 RL with $\{x5 \leftarrow x3, x4 \leftarrow i(x2), x3 \leftarrow x1, x2 \leftarrow rd(op_r(i(x2), x3, x2), i(x2)), x1 \leftarrow x2\}$

$rd(mult(x1, op_r(i(x2), x3, x2)), i(\overbrace{\underbrace{op_l(x2, rd(op_r(i(x2), x3, x2), i(x2)), op_t(x1, op_l(x2, i(x2), x3)))}}))$

$=$ by Lemma 2423 RL with $\{x4 \leftarrow x1, x3 \leftarrow x3, x2 \leftarrow i(x2), x1 \leftarrow x2\}$

$rd(mult(x1, op_r(i(x2), x3, x2)), i(\overbrace{\underbrace{op_l(x2, rd(op_l(x2, i(x2), x3), x2), x1)}}))$

$=$ by Lemma 2372 LR with $\{x4 \leftarrow x1, x3 \leftarrow x3, x2 \leftarrow i(x2), x1 \leftarrow x2\}$

$rd(mult(x1, op_r(i(x2), x3, x2)), i(\overbrace{mult(rd(x2, mult(x1, op_l(x2, i(x2), x3))), mult(x1, op_l(x2, i(x2), x3)))}))$

$=$ by Axiom 4 RL with $\{x2 \leftarrow mult(x1, op_l(x2, i(x2), x3)), x1 \leftarrow x2\}$

$\underbrace{rd(mult(x1, op_r(i(x2), x3, x2)), i(\overbrace{x2}))}$

$=$ by Lemma 2189 LR with $\{x4 \leftarrow x2, x3 \leftarrow x3, x2 \leftarrow i(x2), x1 \leftarrow x1\}$

$\overbrace{rd(x1, rd(i(x2), op_r(i(x2), x3, x2)))}$

Lemma 2503: $rd(mult(x1, x2), op_l(x2, x2, x3)) = rd(x1, asoc(x2, x3, x2))$

$$
\begin{aligned}
& \overbrace{rd(mult(x1, x2), op_l(x2, x2, x3))} \\
= & \quad \text{by Lemma 2502 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{rd(x1, rd(i(x2), \underbrace{op_r(i(x2), x3, x2)}))} \\
= & \quad \text{by Lemma 2428 LR with } \{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x2\} \\
& rd(x1, \underbrace{rd(i(x2), \overbrace{i(op_r(x2, x3, x2))})}) \\
= & \quad \text{by Lemma 23 LR with } \{x2 \leftarrow op_r(x2, x3, x2), x1 \leftarrow x2\} \\
& rd(x1, \overbrace{rd(\underbrace{op_r(x2, x3, x2)}, x2)}) \\
= & \quad \text{by Lemma 601 LR with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& rd(x1, \underbrace{rd(\overbrace{op_l(x2, x2, x3)}, x2)}) \\
= & \quad \text{by Lemma 2449 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x2\} \\
& rd(x1, \overbrace{rd(x2, op_l(x2, x3, x2))}) \\
= & \quad \text{by Lemma 617 LR with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& rd(x1, \overbrace{asoc(x2, x3, x2)})
\end{aligned}
$$

Lemma 2504: $i(mult(op_r(x1, x2, i(i(x3))), x4)) = mult(op_r(i(x1), x2, x3), i(x4))$

$$
\begin{aligned}
& i(mult(\underbrace{op_r(x1, x2, i(i(x3)))}, x4)) \\
= & \quad \text{by Lemma 2427 RL with } \{x3 \leftarrow i(x3), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& i(mult(\overbrace{op_r(x1, i(x2), i(x3))}, x4)) \\
= & \quad \text{by Lemma 813 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{i(mult(\overbrace{i(op_r(i(x1), x2, x3))}, x4))} \\
= & \quad \text{by Lemma 16 LR with } \{x2 \leftarrow x4, x1 \leftarrow op_r(i(x1), x2, x3)\} \\
& \overbrace{mult(op_r(i(x1), x2, x3), i(x4))}
\end{aligned}
$$

Lemma 2505: $op_l(x1, x2, op_r(x3, x4, i(i(x5)))) = op_l(x1, op_r(i(x3), x4, x5), x2)$

$$
\begin{aligned}
& op_l(x1, x2, \underbrace{op_r(x3, x4, i(i(x5)))}) \\
= & \quad \text{by Lemma 2427 RL with } \{x3 \leftarrow i(x5), x2 \leftarrow x4, x1 \leftarrow x3\} \\
& op_l(x1, x2, \overbrace{op_r(x3, i(x4), i(x5))}) \\
= & \quad \text{by Lemma 813 RL with } \{x3 \leftarrow x5, x2 \leftarrow x4, x1 \leftarrow x3\} \\
& \underbrace{op_l(x1, x2, \overbrace{i(op_r(i(x3), x4, x5))})} \\
= & \quad \text{by Lemma 2030 LR with } \{x3 \leftarrow op_r(i(x3), x4, x5), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, op_r(i(x3), x4, x5), x2)}
\end{aligned}
$$

Lemma 2506: $op_l(x1, op_r(i(x2), x3, x4), x5) = op_l(x1, x5, op_r(x2, x3, x4))$

$$
\begin{aligned}
& \underbrace{op_l(x1, op_r(i(x2), x3, x4), x5)} \\
= \quad & \text{by Lemma 2505 RL with } \{x5 \leftarrow x4, x4 \leftarrow x3, x3 \leftarrow x2, x2 \leftarrow x5, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x5, op_r(x2, x3, \underbrace{i(i(x4))}))} \\
= \quad & \text{by Lemma 3 LR with } \{x1 \leftarrow x4\} \\
& op_l(x1, x5, op_r(x2, x3, \overbrace{x4}))
\end{aligned}
$$

Lemma 2507: $op_l(x1, op_r(x2, x3, i(i(x4))), x5) = op_l(x1, x5, op_r(i(x2), x3, x4))$

$$
\begin{aligned}
& op_l(x1, \underbrace{op_r(x2, x3, i(i(x4)))}, x5) \\
= \quad & \text{by Lemma 2427 RL with } \{x3 \leftarrow i(x4), x2 \leftarrow x3, x1 \leftarrow x2\} \\
& op_l(x1, \overbrace{op_r(x2, i(x3), i(x4))}, x5) \\
= \quad & \text{by Lemma 813 RL with } \{x3 \leftarrow x4, x2 \leftarrow x3, x1 \leftarrow x2\} \\
& \underbrace{op_l(x1, \overbrace{i(op_r(i(x2), x3, x4))}, x5)} \\
= \quad & \text{by Lemma 2031 LR with } \{x3 \leftarrow x5, x2 \leftarrow op_r(i(x2), x3, x4), x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x5, op_r(i(x2), x3, x4))}
\end{aligned}
$$

Lemma 2508: $op_l(x1, x2, op_r(i(x3), x4, x5)) = op_l(x1, op_r(x3, x4, x5), x2)$

$$
\begin{aligned}
& \underbrace{op_l(x1, x2, op_r(i(x3), x4, x5))} \\
= \quad & \text{by Lemma 2507 RL with } \{x5 \leftarrow x2, x4 \leftarrow x5, x3 \leftarrow x4, x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, op_r(x3, x4, \underbrace{i(i(x5))}), x2)} \\
= \quad & \text{by Lemma 3 LR with } \{x1 \leftarrow x5\} \\
& op_l(x1, op_r(x3, x4, \overbrace{x5}), x2)
\end{aligned}
$$

Lemma 2509: $op_r(x1, mult(i(x2), x3), i(x4)) = op_r(x1, mult(x2, i(x3)), x4)$

$$
\begin{aligned}
& \underbrace{op_r(x1, mult(i(x2), x3), i(x4))} \\
= \quad & \text{by Lemma 2427 RL with } \{x3 \leftarrow x4, x2 \leftarrow mult(i(x2), x3), x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, \underbrace{i(mult(i(x2), x3))}, x4)} \\
= \quad & \text{by Lemma 16 LR with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& op_r(x1, \overbrace{mult(x2, i(x3))}, x4)
\end{aligned}
$$

Lemma 2510: $op_r(i(x1), x2, mult(x3, x2)) = op_r(rd(rd(x2, x1), x2), x2, x3)$

$$
\begin{aligned}
& op_r(i(x1), x2, \underbrace{mult(x3, x2)}) \\
= & \quad \text{by Lemma 3 RL with } \{x1 \leftarrow mult(x3, x2)\} \\
& \underbrace{op_r(i(x1), x2, \overbrace{i(i(mult(x3, x2)))})} \\
= & \quad \text{by Lemma 2428 LR with } \{x3 \leftarrow i(i(mult(x3, x2))), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{\overbrace{i(op_r(x1, x2, i(i(mult(x3, x2)))))}} \\
= & \quad \text{by Lemma 2482 LR with } \{x3 \leftarrow i(mult(x3, x2)), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{\overbrace{mult(op_l(i(i(mult(x3, x2))), x2, x1), mult(i(mult(x3, x2)), i(x1)))}} \\
= & \quad \text{by Lemma 2003 LR with } \{x4 \leftarrow i(x1), x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow i(mult(x3, x2))\} \\
& \underbrace{\overbrace{mult(rd(i(mult(x3, x2)), op_l(i(mult(x3, x2)), x2, x1)), i(x1))}} \\
= & \quad \text{by Lemma 2236 LR with } \{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow i(mult(x3, x2))\} \\
& \overbrace{rd(i(mult(x3, x2)), \underbrace{mult(x1, op_l(i(mult(x3, x2)), x2, x1))})} \\
= & \quad \text{by Lemma 1091 RL with } \{x3 \leftarrow i(mult(x3, x2)), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& rd(i(mult(x3, x2)), \overbrace{mult(rd(x1, x2), \underbrace{mult(x2, i(mult(x3, x2)))})}) \\
= & \quad \text{by Lemma 31 LR with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& \underbrace{rd(i(mult(x3, x2)), mult(rd(x1, x2), \overbrace{i(op_t(x3, x2))}))} \\
= & \quad \text{by Lemma 62 LR with } \{x3 \leftarrow op_t(x3, x2), x2 \leftarrow rd(x1, x2), x1 \leftarrow mult(x3, x2)\} \\
& \overbrace{\underbrace{rd(mult(i(rd(x1, x2)), op_t(x3, x2)), mult(x3, x2))}} \\
= & \quad \text{by Lemma 1965 RL with } \{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow mult(i(rd(x1, x2)), op_t(x3, x2))\} \\
& \overbrace{op_r(rd(\underbrace{rd(mult(i(rd(x1, x2)), op_t(x3, x2)), op_t(x3, x2))}, x2), x2, x3)} \\
= & \quad \text{by Axiom 5 RL with } \{x2 \leftarrow op_t(x3, x2), x1 \leftarrow i(rd(x1, x2))\} \\
& op_r(rd(\overbrace{i(rd(x1, x2))}, x2), x2, x3) \\
= & \quad \text{by Lemma 10 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_r(rd(\overbrace{rd(x2, x1)}, x2), x2, x3)
\end{aligned}
$$

Lemma 2511: $op_r(i(i(x1)), x2, mult(x3, x2)) = op_r(op_r(x1, x2, x2), x2, x3)$

$$
\begin{aligned}
& \underbrace{op_r(i(i(x1)), x2, mult(x3, x2))} \\
= & \quad \text{by Lemma 2510 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow i(x1)\} \\
& \overbrace{op_r(\underbrace{rd(rd(x2, i(x1)), x2)}, x2, x3)} \\
= & \quad \text{by Lemma 354 LR with } \{x2 \leftarrow i(x1), x1 \leftarrow x2\} \\
& op_r(\overbrace{op_r(\underbrace{i(i(x1))}, x2, x2)}, x2, x3) \\
= & \quad \text{by Lemma 3 LR with } \{x1 \leftarrow x1\} \\
& op_r(op_r(\overbrace{x1}, x2, x2), x2, x3)
\end{aligned}
$$

Lemma 2512: $op_r(op_r(x1, x2, x2), x2, x3) = op_r(x1, x2, mult(x3, x2))$

$$
\begin{aligned}
& \underbrace{op_r(op_r(x1, x2, x2), x2, x3)} \\
= \quad & \text{by Lemma 2511 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_r(\underbrace{i(i(x1))}, x2, mult(x3, x2))} \\
= \quad & \text{by Lemma 3 LR with } \{x1 \leftarrow x1\} \\
& op_r(\overbrace{x1}, x2, mult(x3, x2))
\end{aligned}
$$

Lemma 2513: $op_r(rd(i(x1), i(x2)), i(x1), rd(x1, x3)) = op_r(mult(x2, i(x1)), i(x1), i(x3))$

$$
\begin{aligned}
& op_r(\underbrace{rd(i(x1), i(x2))}, i(x1), rd(x1, x3)) \\
= \quad & \text{by Lemma 359 RL with } \{x2 \leftarrow i(x1), x1 \leftarrow x2\} \\
& \underbrace{op_r(\overbrace{op_r(mult(x2, i(x1)), i(x1), i(x1))}, i(x1), rd(x1, x3))} \\
= \quad & \text{by Lemma 2512 LR with } \{x3 \leftarrow rd(x1, x3), x2 \leftarrow i(x1), x1 \leftarrow mult(x2, i(x1))\} \\
& \overbrace{op_r(mult(x2, i(x1)), i(x1), \underbrace{mult(rd(x1, x3), i(x1))})} \\
= \quad & \text{by Lemma 22 LR with } \{x2 \leftarrow x3, x1 \leftarrow x1\} \\
& op_r(mult(x2, i(x1)), i(x1), \overbrace{i(x3)})
\end{aligned}
$$

Lemma 2514: $op_r(rd(x2, x1), x1, i(rd(x1, x3))) = op_r(mult(x2, i(x1)), i(x1), i(x3))$

$$
\begin{aligned}
& op_r(\underbrace{rd(x2, x1)}, x1, i(rd(x1, x3))) \\
= \quad & \text{by Lemma 23 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{op_r(\overbrace{rd(i(x1), i(x2))}, x1, i(rd(x1, x3)))} \\
= \quad & \text{by Lemma 2427 RL with } \{x3 \leftarrow rd(x1, x3), x2 \leftarrow x1, x1 \leftarrow rd(i(x1), i(x2))\} \\
& \overbrace{\underbrace{op_r(rd(i(x1), i(x2)), i(x1), rd(x1, x3))}} \\
= \quad & \text{by Lemma 2513 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_r(mult(x2, i(x1)), i(x1), i(x3))}
\end{aligned}
$$

Lemma 2515: $op_r(rd(x2, x1), x1, rd(x3, x1)) = op_r(mult(x2, i(x1)), x1, i(i(x3)))$

$$
\begin{aligned}
& op_r(rd(x2, x1), x1, \underbrace{rd(x3, x1)}) \\
= \quad & \text{by Lemma 10 RL with } \{x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \underbrace{op_r(rd(x2, x1), x1, \overbrace{i(rd(x1, x3))})} \\
= \quad & \text{by Lemma 2514 LR with } \{x3 \leftarrow x3, x1 \leftarrow x1, x2 \leftarrow x2\} \\
& \overbrace{\underbrace{op_r(mult(x2, i(x1)), i(x1), i(x3))}} \\
= \quad & \text{by Lemma 2427 LR with } \{x3 \leftarrow i(x3), x2 \leftarrow x1, x1 \leftarrow mult(x2, i(x1))\} \\
& \overbrace{op_r(mult(x2, i(x1)), x1, i(i(x3)))}
\end{aligned}
$$

Lemma 2516: $op_r(rd(x1,x2),x2,rd(x3,x2)) = op_r(mult(x1,i(x2)),x2,x3)$

$$
\begin{aligned}
& \overbrace{op_r(rd(x1,x2),x2,rd(x3,x2))} \\
= & \quad \text{by Lemma 2515 LR with } \{x3 \leftarrow x3, x1 \leftarrow x2, x2 \leftarrow x1\} \\
& \overbrace{op_r(mult(x1,i(x2)),x2,\underbrace{i(i(x3))})} \\
= & \quad \text{by Lemma 3 LR with } \{x1 \leftarrow x3\} \\
& op_r(mult(x1,i(x2)),x2,\overbrace{x3}^{})
\end{aligned}
$$

Lemma 2517: $op_r(op_r(x1,x2,i(x2)),x2,x3) = op_r(x1,x2,rd(x3,x2))$

$$
\begin{aligned}
& op_r(\underbrace{op_r(x1,x2,i(x2))},x2,x3) \\
= & \quad \text{by Lemma 92 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{op_r(\overbrace{mult(mult(x1,x2),i(x2))},x2,x3)} \\
= & \quad \text{by Lemma 2516 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow mult(x1,x2)\} \\
& \overbrace{op_r(\underbrace{rd(mult(x1,x2),x2)},x2,rd(x3,x2))} \\
= & \quad \text{by Axiom 5 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_r(\overbrace{x1}^{},x2,rd(x3,x2))
\end{aligned}
$$

Lemma 2518: $op_r(mult(x1,rd(x2,op_t(x1,x2))),op_t(x1,x2),x3) = op_r(x2,op_t(x1,x2),rd(x3,op_t(x1,x2)))$

$$
\begin{aligned}
& op_r(\underbrace{mult(x1,rd(x2,op_t(x1,x2)))},op_t(x1,x2),x3) \\
= & \quad \text{by Lemma 1253 RL with } \{x3 \leftarrow x2, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{op_r(\overbrace{mult(mult(x1,x2),i(op_t(x1,x2)))},op_t(x1,x2),x3)} \\
= & \quad \text{by Lemma 2516 RL with } \{x3 \leftarrow x3, x2 \leftarrow op_t(x1,x2), x1 \leftarrow mult(x1,x2)\} \\
& \overbrace{op_r(\underbrace{rd(mult(x1,x2),op_t(x1,x2))},op_t(x1,x2),rd(x3,op_t(x1,x2)))} \\
= & \quad \text{by Lemma 29 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_r(\overbrace{x2}^{},op_t(x1,x2),rd(x3,op_t(x1,x2)))
\end{aligned}
$$

Lemma 2519: $op_r(op_t(x2,i(x1)),x1,x3) = op_r(op_l(x2,x2,x1),x1,rd(x3,x1))$

$$
\begin{aligned}
& op_r(\underbrace{op_t(x2,i(x1))},x1,x3) \\
= & \quad \text{by Lemma 1499 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{op_r(\overbrace{mult(op_r(mult(x1,x2),x2,x1),i(x1))},x1,x3)} \\
= & \quad \text{by Lemma 2516 RL with } \{x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow op_r(mult(x1,x2),x2,x1)\} \\
& \overbrace{op_r(\underbrace{rd(op_r(mult(x1,x2),x2,x1),x1)},x1,rd(x3,x1))} \\
= & \quad \text{by Lemma 1500 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_r(\overbrace{op_l(x2,x2,x1)},x1,rd(x3,x1))
\end{aligned}
$$

Lemma 2520: $op_t(x1, i(x2)) = op_r(x1, x2, rd(x1, op_l(x2, x3, x4)))$

$\overbrace{op_t(x1, i(x2))}$

$=$ by Lemma 2424 LR with $\{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow op_t(x1, i(x2))\}$

$\overbrace{op_r(op_t(x1, i(x2)), x2, mult(x3, i(op_r(x3, x4, x2))))}$

$=$ by Lemma 2395 LR with $\{x4 \leftarrow op_r(x3, x4, x2), x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow op_t(x1, i(x2))\}$

$\overbrace{op_r(op_t(x1, i(x2)), x2, rd(x3, op_r(x3, x4, x2)))}$

$=$ by Lemma 2519 LR with $\{x3 \leftarrow rd(x3, op_r(x3, x4, x2)), x1 \leftarrow x2, x2 \leftarrow x1\}$

$\overbrace{op_r(op_l(x1, x1, x2), x2, rd(rd(x3, op_r(x3, x4, x2)), x2))}$

$=$ by Axiom 18 LR with $\{x5 \leftarrow rd(rd(x3, op_r(x3, x4, x2)), x2), x4 \leftarrow x2, x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x1\}$

$\overbrace{op_l(op_r(x1, x2, rd(rd(x3, op_r(x3, x4, x2)), x2)), x1, x2)}$

$=$ by Lemma 2500 RL with $\{x3 \leftarrow rd(rd(x3, op_r(x3, x4, x2)), x2), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_r(x1, x2, mult(rd(rd(x3, op_r(x3, x4, x2)), x2), \underbrace{x1}))}$

$=$ by Lemma 3 RL with $\{x1 \leftarrow x1\}$

$op_r(x1, x2, \underbrace{mult(rd(rd(x3, op_r(x3, x4, x2)), x2), \overbrace{i(i(x1))})})$

$=$ by Lemma 44 RL with $\{x3 \leftarrow i(x1), x2 \leftarrow rd(x3, op_r(x3, x4, x2)), x1 \leftarrow x2\}$

$op_r(x1, x2, \overbrace{i(\underbrace{mult(rd(x2, rd(x3, op_r(x3, x4, x2))), i(x1))})})$

$=$ by Lemma 1426 LR with $\{x3 \leftarrow x1, x1 \leftarrow rd(x3, op_r(x3, x4, x2)), x2 \leftarrow x2\}$

$op_r(x1, x2, \overbrace{i(\underbrace{op_t(rd(rd(x2, rd(x3, op_r(x3, x4, x2))), x1), rd(rd(x3, op_r(x3, x4, x2)), x2))})})$

$=$ by Lemma 15 RL with $\{x2 \leftarrow rd(rd(x3, op_r(x3, x4, x2)), x2), x1 \leftarrow rd(rd(x2, rd(x3, op_r(x3, x4, x2))), x1)\}$

$\overbrace{op_r(x1, x2, \underbrace{op_t(i(rd(rd(x2, rd(x3, op_r(x3, x4, x2))), x1)), i(rd(rd(x3, op_r(x3, x4, x2)), x2)))})}$

$=$ by Lemma 2392 LR with $\{x4 \leftarrow i(rd(rd(x3, op_r(x3, x4, x2)), x2)), x3 \leftarrow i(rd(rd(x2, rd(x3, op_r(x3, x4, x2))), x1)), x2 \leftarrow x2, x1$

$\overbrace{op_r(x1, x2, \underbrace{i(rd(rd(x2, rd(x3, op_r(x3, x4, x2))), x1))})}$

$=$ by Lemma 10 LR with $\{x2 \leftarrow x1, x1 \leftarrow rd(x2, rd(x3, op_r(x3, x4, x2)))\}$

$op_r(x1, x2, \overbrace{rd(x1, \underbrace{rd(x2, rd(x3, op_r(x3, x4, x2)))})})$

$=$ by Lemma 2437 LR with $\{x3 \leftarrow x4, x2 \leftarrow x3, x1 \leftarrow x2\}$

$op_r(x1, x2, rd(x1, \overbrace{op_l(x2, x3, x4)}))$

Lemma 2521: $op_r(x1, x2, rd(x3, rd(x2, x1))) = op_r(op_r(x1, x2, i(rd(x2, x1))), rd(x2, x1), x3)$

$\overbrace{op_r(x1, x2, rd(x3, rd(x2, x1)))}$

$=$ by Lemma 2429 RL with $\{x3 \leftarrow rd(x3, rd(x2, x1)), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_r(x1, rd(x2, x1), rd(x3, rd(x2, x1)))}$

$=$ by Lemma 2517 RL with $\{x3 \leftarrow x3, x2 \leftarrow rd(x2, x1), x1 \leftarrow x1\}$

$\overbrace{op_r(\underbrace{op_r(x1, rd(x2, x1), i(rd(x2, x1)))}, rd(x2, x1), x3)}$

$=$ by Lemma 2429 LR with $\{x3 \leftarrow i(rd(x2, x1)), x2 \leftarrow x2, x1 \leftarrow x1\}$

$op_r(\overbrace{op_r(x1, x2, i(rd(x2, x1)))}, rd(x2, x1), x3)$

Lemma 2522: $op_r(x1, x2, rd(x3, rd(x2, x1))) = op_r(op_t(x1, i(x2)), rd(x2, x1), x3)$

$$
\begin{aligned}
& \underbrace{op_r(x1, x2, rd(x3, rd(x2, x1)))} \\
= & \quad \text{by Lemma 2521 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_r(op_r(x1, x2, \underbrace{i(rd(x2, x1))}), rd(x2, x1), x3)} \\
= & \quad \text{by Lemma 10 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& op_r(\underbrace{op_r(x1, x2, \overbrace{rd(x1, x2)})}, rd(x2, x1), x3) \\
= & \quad \text{by Lemma 1188 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_r(\overbrace{op_t(x1, i(x2))}, rd(x2, x1), x3)
\end{aligned}
$$

Lemma 2523: $op_r(mult(i(x2), x1), x2, x3) = op_r(rd(x1, x2), x2, rd(x3, x1))$

$$
\begin{aligned}
& op_r(\underbrace{mult(i(x2), x1)}, x2, x3) \\
= & \quad \text{by Lemma 14 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{op_r(\overbrace{op_t(rd(x1, x2), x2)}, x2, x3)} \\
= & \quad \text{by Axiom 14 RL with } \{x4 \leftarrow x2, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow rd(x1, x2)\} \\
& \overbrace{op_t(\underbrace{op_r(rd(x1, x2), x2, x3)}, x2)} \\
= & \quad \text{by Lemma 2430 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow rd(x1, x2)\} \\
& \underbrace{op_t(\overbrace{op_r(rd(x1, x2), mult(x2, rd(x1, x2)), x3)}, x2)} \\
= & \quad \text{by Axiom 14 LR with } \{x4 \leftarrow x2, x3 \leftarrow x3, x2 \leftarrow mult(x2, rd(x1, x2)), x1 \leftarrow rd(x1, x2)\} \\
& \overbrace{op_r(\underbrace{op_t(rd(x1, x2), x2)}, mult(x2, rd(x1, x2)), x3)} \\
= & \quad \text{by Lemma 1416 LR with } \{x2 \leftarrow x2, x1 \leftarrow rd(x1, x2)\} \\
& op_r(\overbrace{op_r(rd(x1, x2), x2, \underbrace{rd(i(x2), rd(x1, x2))})}, mult(x2, rd(x1, x2)), x3) \\
= & \quad \text{by Lemma 2400 RL with } \{x4 \leftarrow rd(x1, x2), x3 \leftarrow x2, x2 \leftarrow x2, x1 \leftarrow rd(x1, x2)\} \\
& op_r(\overbrace{op_r(rd(x1, x2), x2, \underbrace{i(mult(x2, rd(x1, x2)))})}, mult(x2, rd(x1, x2)), x3) \\
= & \quad \text{by Lemma 2430 RL with } \{x3 \leftarrow i(mult(x2, rd(x1, x2))), x2 \leftarrow x2, x1 \leftarrow rd(x1, x2)\} \\
& \underbrace{op_r(\overbrace{op_r(rd(x1, x2), mult(x2, rd(x1, x2)), \underbrace{i(mult(x2, rd(x1, x2)))})}, mult(x2, rd(x1, x2)), x3)} \\
= & \quad \text{by Lemma 2517 LR with } \{x3 \leftarrow x3, x2 \leftarrow mult(x2, rd(x1, x2)), x1 \leftarrow rd(x1, x2)\} \\
& \overbrace{\underbrace{op_r(rd(x1, x2), mult(x2, rd(x1, x2)), rd(x3, mult(x2, rd(x1, x2))))}} \\
= & \quad \text{by Lemma 2430 LR with } \{x3 \leftarrow rd(x3, mult(x2, rd(x1, x2))), x2 \leftarrow x2, x1 \leftarrow rd(x1, x2)\} \\
& \overbrace{op_r(rd(x1, x2), x2, rd(x3, \underbrace{mult(x2, rd(x1, x2))}))} \\
= & \quad \text{by Lemma 427 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_r(rd(x1, x2), x2, \underbrace{rd(x3, \overbrace{op_l(x1, x1, x2)})}) \\
= & \quad \text{by Lemma 1575 LR with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x3\} \\
& \underbrace{op_r(rd(x1, x2), x2, \overbrace{rd(rd(x3, x1), asoc(x1, x2, x1))})} \\
= & \quad \text{by Lemma 2389 LR with } \{x5 \leftarrow x2, x4 \leftarrow x1, x3 \leftarrow rd(x3, x1), x2 \leftarrow x2, x1 \leftarrow rd(x1, x2)\} \\
& \overbrace{op_r(rd(x1, x2), x2, rd(x3, x1))}
\end{aligned}
$$

Lemma 2524: $op_r(op_t(rd(x1,x2),i(x1)),x1,x3) = op_r(rd(x1,x2),x1,rd(x3,op_r(x2,x1,x1)))$

$\underbrace{op_r(op_t(rd(x1,x2),i(x1)),x1,x3)}$

$=$ by Lemma 2471 RL with $\{x4 \leftarrow x3, x3 \leftarrow x1, x2 \leftarrow i(x1), x1 \leftarrow rd(x1,x2)\}$

$\overbrace{op_r(op_t(rd(x1,x2),i(x1)),rd(x1,rd(x1,x2)),x3)}$

$=$ by Lemma 2522 RL with $\{x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow rd(x1,x2)\}$

$\overbrace{op_r(rd(x1,x2),x1,rd(x3,\underbrace{rd(x1,rd(x1,x2))}))}$

$=$ by Lemma 353 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$op_r(rd(x1,x2),x1,rd(x3,\overbrace{op_r(x2,x1,x1)}))$

Lemma 2525: $op_r(mult(x1,i(x2)),x1,x3) = op_r(rd(x1,x2),x1,rd(x3,x2))$

$op_r(\underbrace{mult(x1,i(x2))},x1,x3)$

$=$ by Lemma 34 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{op_r(\overbrace{op_t(rd(x1,x2),i(x1))},x1,x3)}$

$=$ by Lemma 2524 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{\overbrace{op_r(rd(x1,x2),x1,rd(x3,op_r(x2,x1,x1)))}}$

$=$ by Lemma 2409 RL with $\{x5 \leftarrow x1, x3 \leftarrow x2, x4 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow rd(x1,x2)\}$

$\overbrace{op_r(rd(x1,x2),x1,rd(x3,x2))}$

Lemma 2526: $op_r(mult(x1,rd(x3,x2)),x1,x2) = op_r(rd(x1,rd(x2,x3)),x1,op_r(x3,x2,x2))$

$op_r(mult(x1,\underbrace{rd(x3,x2)}),x1,x2)$

$=$ by Lemma 10 RL with $\{x2 \leftarrow x3, x1 \leftarrow x2\}$

$\underbrace{op_r(mult(x1,\overbrace{i(rd(x2,x3))}),x1,x2)}$

$=$ by Lemma 2525 LR with $\{x3 \leftarrow x2, x2 \leftarrow rd(x2,x3), x1 \leftarrow x1\}$

$\overbrace{op_r(rd(x1,rd(x2,x3)),x1,\underbrace{rd(x2,rd(x2,x3))})}$

$=$ by Lemma 353 LR with $\{x2 \leftarrow x3, x1 \leftarrow x2\}$

$op_r(rd(x1,rd(x2,x3)),x1,\overbrace{op_r(x3,x2,x2)})$

Lemma 2527: $op_r(mult(x1,rd(x2,x3)),x1,x3) = rd(mult(x1,x2),x3)$

$\underbrace{op_r(mult(x1,rd(x2,x3)),x1,x3)}$

$=$ by Lemma 2526 LR with $\{x2 \leftarrow x3, x3 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{\overbrace{op_r(rd(x1,rd(x3,x2)),x1,op_r(x2,x3,x3))}}$

$=$ by Lemma 2391 LR with $\{x4 \leftarrow x3, x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow rd(x1,rd(x3,x2))\}$

$\underbrace{\overbrace{op_r(rd(x1,rd(x3,x2)),x1,x2)}}$

$=$ by Lemma 2463 LR with $\{x1 \leftarrow x2, x2 \leftarrow x3, x3 \leftarrow x1\}$

$\overbrace{rd(mult(x1,x2),x3)}$

Lemma 2528: $op_r(x1, x2, i(x3)) = mult(op_r(i(x1), x2, x3), mult(x1, op_l(x1, op_r(x1, x2, i(x3)), x1)))$

$\overbrace{op_r(x1, x2, i(x3))}$

$=$ by Axiom 4 LR with $\{x2 \leftarrow mult(x1, op_l(x1, op_r(x1, x2, i(x3)), x1)), x1 \leftarrow op_r(x1, x2, i(x3))\}$

$\overbrace{mult(rd(op_r(x1, x2, i(x3)), mult(x1, \underbrace{op_l(x1, op_r(x1, x2, i(x3)), x1)})), mult(x1, op_l(x1, op_r(x1, x2, i(x3)), x1)))}$

$=$ by Lemma 960 LR with $\{x2 \leftarrow op_r(x1, x2, i(x3)), x1 \leftarrow x1\}$

$mult(rd(op_r(x1, x2, i(x3)), \underbrace{mult(x1, \overbrace{mult(op_l(i(op_r(x1, x2, i(x3))), op_r(x1, x2, i(x3)), x1), mult(x1, op_r(x1, x2, i(x3))))})}), n$

$=$ by Axiom 6 LR with $\{x3 \leftarrow op_r(x1, x2, i(x3)), x2 \leftarrow op_l(i(op_r(x1, x2, i(x3))), op_r(x1, x2, i(x3)), x1), x1 \leftarrow x1\}$

$mult(\underbrace{rd(op_r(x1, x2, i(x3)), \overbrace{mult(mult(x1, mult(op_l(i(op_r(x1, x2, i(x3))), op_r(x1, x2, i(x3)), x1), x1)), op_r(x1, x2, i(x3)))})}, n$

$=$ by Lemma 9 LR with $\{x2 \leftarrow mult(x1, mult(op_l(i(op_r(x1, x2, i(x3))), op_r(x1, x2, i(x3)), x1), x1)), x1 \leftarrow op_r(x1, x2, i(x3))\}$

$mult(\overbrace{i(mult(x1, mult(op_l(i(op_r(x1, x2, i(x3))), op_r(x1, x2, i(x3)), x1), x1)))}, mult(x1, op_l(x1, op_r(x1, x2, i(x3)), x1)))$

$=$ by Lemma 59 LR with $\{x2 \leftarrow op_l(i(op_r(x1, x2, i(x3))), op_r(x1, x2, i(x3)), x1), x1 \leftarrow x1\}$

$mult(\overbrace{rd(i(x1), \underbrace{mult(x1, op_l(i(op_r(x1, x2, i(x3))), op_r(x1, x2, i(x3)), x1))})}, mult(x1, op_l(x1, op_r(x1, x2, i(x3)), x1)))$

$=$ by Lemma 232 LR with $\{x2 \leftarrow op_r(x1, x2, i(x3)), x1 \leftarrow x1\}$

$mult(\underbrace{rd(i(x1), \overbrace{rd(x1, op_r(x1, x2, i(x3)))})}, mult(x1, op_l(x1, op_r(x1, x2, i(x3)), x1)))$

$=$ by Lemma 40 LR with $\{x3 \leftarrow op_r(x1, x2, i(x3)), x2 \leftarrow x1, x1 \leftarrow x1\}$

$mult(\overbrace{rd(\underbrace{rd(op_r(x1, x2, i(x3)), x1)}, x1)}, mult(x1, op_l(x1, op_r(x1, x2, i(x3)), x1)))$

$=$ by Lemma 2458 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$mult(\underbrace{rd(\overbrace{rd(x1, op_r(x1, x2, x3))}, x1)}, mult(x1, op_l(x1, op_r(x1, x2, i(x3)), x1)))$

$=$ by Lemma 2190 RL with $\{x4 \leftarrow x3, x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x1\}$

$mult(\underbrace{rd(x1, mult(x1, op_r(x1, x2, x3)))}, mult(x1, op_l(x1, op_r(x1, x2, i(x3)), x1)))$

$=$ by Lemma 146 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$mult(\underbrace{i(op_r(x1, x2, x3))}, mult(x1, op_l(x1, op_r(x1, x2, i(x3)), x1)))$

$=$ by Lemma 2428 RL with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$mult(\overbrace{op_r(i(x1), x2, x3)}, mult(x1, op_l(x1, op_r(x1, x2, i(x3)), x1)))$

Lemma 2529: $op_r(mult(i(x1), rd(x2, x3)), x1, x2) = op_r(rd(rd(x2, x3), x1), x1, x3)$

$\underbrace{op_r(mult(i(x1), rd(x2, x3)), x1, x2)}$

$=$ by Lemma 2523 LR with $\{x3 \leftarrow x2, x1 \leftarrow rd(x2, x3), x2 \leftarrow x1\}$

$\overbrace{op_r(\underbrace{rd(rd(x2, x3), x1)}, x1, \underbrace{rd(x2, rd(x2, x3))})}$

$=$ by Lemma 353 LR with $\{x2 \leftarrow x3, x1 \leftarrow x2\}$

$\underbrace{op_r(\underbrace{rd(rd(x2, x3), x1)}, x1, \overbrace{op_r(x3, x2, x2)})}$

$=$ by Lemma 2391 LR with $\{x4 \leftarrow x2, x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow rd(rd(x2, x3), x1)\}$

$\overbrace{op_r(\underbrace{rd(rd(x2, x3), x1)}, x1, x3)}$

Lemma 2530: $op_r(mult(i(x1), rd(x2, x3)), x1, x2) = rd(x2, mult(x1, x3))$

$$
\begin{aligned}
& \overbrace{op_r(mult(i(x1), rd(x2, x3)), x1, x2)} \\
= & \quad \text{by Lemma 2529 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_r(\underbrace{rd(rd(x2, x3), x1)}, x1, x3)} \\
= & \quad \text{by Lemma 839 LR with } \{x3 \leftarrow x1, x2 \leftarrow x3, x1 \leftarrow x2\} \\
& \overbrace{rd(x2, mult(x1, x3))}
\end{aligned}
$$

Lemma 2531: $mult(rd(x1, op_l(x2, x3, x4)), x2) = rd(x1, rd(x2, op_l(x2, x4, x3)))$

$$
\begin{aligned}
& \overbrace{mult(rd(x1, op_l(x2, x3, x4)), x2)} \\
= & \quad \text{by Lemma 2354 LR with } \{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{rd(x1, \underbrace{rd(op_l(x2, x3, x4), x2)})} \\
= & \quad \text{by Lemma 2449 LR with } \{x3 \leftarrow x4, x2 \leftarrow x3, x1 \leftarrow x2\} \\
& rd(x1, \overbrace{rd(x2, op_l(x2, x4, x3))})
\end{aligned}
$$

Lemma 2532: $rd(x1, mult(x4, op_l(x1, x3, x2))) = rd(op_l(x1, x2, x3), mult(x4, x1))$

$$
\begin{aligned}
& \overbrace{rd(x1, mult(x4, op_l(x1, x3, x2)))} \\
= & \quad \text{by Lemma 2356 RL with } \{x4 \leftarrow x4, x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{rd(\underbrace{rd(x1, op_l(x1, x3, x2))}, x4)} \\
= & \quad \text{by Lemma 2449 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{rd(\overbrace{rd(op_l(x1, x2, x3), x1)}, x4)} \\
= & \quad \text{by Lemma 2358 LR with } \{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{rd(op_l(x1, x2, x3), mult(x4, x1))}
\end{aligned}
$$

Lemma 2533: $op_r(x1, rd(mult(x1, x2), x3), x4) = op_r(x1, rd(x2, mult(x1, x3)), x4)$

$$
\begin{aligned}
& \underbrace{op_r(x1, rd(mult(x1, x2), x3), x4)} \\
= & \quad \text{by Lemma 2425 RL with } \{x3 \leftarrow rd(mult(x1, x2), x3), x2 \leftarrow x4, x1 \leftarrow x1\} \\
& \overbrace{rd(x1, rd(x4, \underbrace{op_l(x4, x1, rd(mult(x1, x2), x3))}))} \\
= & \quad \text{by Lemma 2043 RL with } \{x4 \leftarrow x3, x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x4\} \\
& \underbrace{rd(x1, rd(x4, \overbrace{op_l(x4, x1, rd(x2, mult(x1, x3)))}))} \\
= & \quad \text{by Lemma 2425 LR with } \{x3 \leftarrow rd(x2, mult(x1, x3)), x2 \leftarrow x4, x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, rd(x2, mult(x1, x3)), x4)}
\end{aligned}
$$

Lemma 2534: $op_r(x1, rd(x2, rd(x1, x3)), x2) = rd(x1, rd(x2, op_l(x2, x1, mult(mult(x2, x1), x3))))$

$$
\begin{aligned}
& \overbrace{op_r(x1, rd(x2, rd(x1, x3)), x2)} \\
= & \quad \text{by Lemma 2240 LR with } \{x4 \leftarrow x2, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, \underbrace{rd(x2, mult(x1, i(x3)))}, x2)} \\
= & \quad \text{by Lemma 2425 RL with } \{x3 \leftarrow rd(x2, mult(x1, i(x3))), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{rd(x1, rd(x2, \underbrace{op_l(x2, x1, rd(x2, mult(x1, i(x3))))}))} \\
= & \quad \text{by Lemma 2350 LR with } \{x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow x2\} \\
& rd(x1, rd(x2, \overbrace{op_l(x2, x1, mult(mult(x2, x1), x3))}))
\end{aligned}
$$

Lemma 2535: $op_r(x1, mult(mult(x2, x1), x3), x2) = op_r(x1, rd(x2, rd(x1, x3)), x2)$

$$
\begin{aligned}
& \overbrace{op_r(x1, mult(mult(x2, x1), x3), x2)} \\
= & \quad \text{by Lemma 2425 RL with } \{x3 \leftarrow mult(mult(x2, x1), x3), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{rd(x1, rd(x2, \underbrace{op_l(x2, x1, mult(mult(x2, x1), x3))}))} \\
= & \quad \text{by Lemma 2534 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, rd(x2, rd(x1, x3)), x2)}
\end{aligned}
$$

Lemma 2536: $op_r(x1, rd(rd(x2, x1), x3), x4) = op_r(x1, rd(x2, rd(x3, x1)), x4)$

$$
\begin{aligned}
& \overbrace{op_r(x1, rd(rd(x2, x1), x3), x4)} \\
= & \quad \text{by Lemma 2172 LR with } \{x4 \leftarrow x4, x2 \leftarrow x3, x3 \leftarrow rd(x2, x1), x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, rd(\underbrace{op_t(rd(x2, x1), x1)}, x3), x4)} \\
= & \quad \text{by Lemma 14 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \overbrace{op_r(x1, rd(\overbrace{mult(i(x1), x2)}, x3), x4)} \\
= & \quad \text{by Lemma 2430 RL with } \{x3 \leftarrow x4, x2 \leftarrow rd(mult(i(x1), x2), x3), x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_r(x1, mult(rd(mult(i(x1), x2), x3), x1), x4)}} \\
= & \quad \text{by Lemma 2431 RL with } \{x3 \leftarrow x4, x2 \leftarrow mult(rd(mult(i(x1), x2), x3), x1), x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, \underbrace{mult(x1, mult(rd(mult(i(x1), x2), x3), x1))}, x4)} \\
= & \quad \text{by Lemma 2352 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{op_r(x1, \overbrace{rd(x2, mult(i(x1), x3))}, x4)} \\
= & \quad \text{by Lemma 2241 LR with } \{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, rd(x2, rd(x3, x1)), x4)}
\end{aligned}
$$

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Lemma 2537: $mult(op_t(x1,x2),op_l(x1,x3,x4)) = mult(x1,op_l(op_t(x1,x2),x3,x4))$

$\underbrace{mult(op_t(x1,x2),op_l(x1,x3,x4))}$

$=$ by Axiom 5 LR with $\{x2 \leftarrow op_t(i(op_t(x1,x2)),op_l(x1,x3,x4)), x1 \leftarrow mult(op_t(x1,x2),op_l(x1,x3,x4))\}$

$\overbrace{rd(\underbrace{mult(mult(op_t(x1,x2),op_l(x1,x3,x4)),op_t(i(op_t(x1,x2)),op_l(x1,x3,x4)))}, op_t(i(op_t(x1,x2)),op_l(x1,x3,x4)))}$

$=$ by Lemma 695 LR with $\{x2 \leftarrow op_l(x1,x3,x4), x1 \leftarrow op_t(x1,x2)\}$

$rd(\overbrace{op_r(op_l(x1,x3,x4),op_t(x1,x2),op_t(x1,x2))}, op_t(i(op_t(x1,x2)),op_l(\underbrace{x1},x3,x4)))$

$=$ by Lemma 3 RL with $\{x1 \leftarrow x1\}$

$rd(op_r(op_l(x1,x3,x4),op_t(x1,x2),op_t(x1,x2)),op_t(\underbrace{i(op_t(x1,x2))},op_l(\overbrace{i(\overbrace{i(x1)})},x3,x4)))$

$=$ by Lemma 15 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$rd(op_r(op_l(x1,x3,x4),op_t(x1,x2),op_t(x1,x2)),\underbrace{op_t(\overbrace{op_t(i(x1),i(x2))},op_l(i(i(x1)),x3,x4))})$

$=$ by Axiom 13 LR with $\{x3 \leftarrow op_l(i(i(x1)),x3,x4), x2 \leftarrow i(x2), x1 \leftarrow i(x1)\}$

$rd(op_r(op_l(x1,x3,x4),op_t(x1,x2),op_t(x1,x2)),\overbrace{op_t(\underbrace{op_t(i(x1),op_l(i(i(x1)),x3,x4))},i(x2))})$

$=$ by Lemma 189 LR with $\{x3 \leftarrow x4, x2 \leftarrow x3, x1 \leftarrow i(x1)\}$

$rd(op_r(op_l(x1,x3,x4),op_t(x1,x2),op_t(x1,x2)),\underbrace{op_t(\overbrace{i(x1)},i(x2))})$

$=$ by Lemma 15 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$rd(\underbrace{op_r(op_l(x1,x3,x4),op_t(x1,x2),op_t(x1,x2))},\overbrace{i(\overbrace{op_t(x1,x2)})})$

$=$ by Axiom 18 LR with $\{x5 \leftarrow op_t(x1,x2), x4 \leftarrow op_t(x1,x2), x3 \leftarrow x4, x2 \leftarrow x3, x1 \leftarrow x1\}$

$rd(\overbrace{op_l(\underbrace{op_r(x1,op_t(x1,x2),op_t(x1,x2))},x3,x4)},i(op_t(x1,x2)))$

$=$ by Lemma 157 LR with $\{x3 \leftarrow op_t(x1,x2), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{rd(op_l(\overbrace{x1},x3,x4),i(op_t(x1,x2)))}$

$=$ by Lemma 1996 LR with $\{x4 \leftarrow x2, x3 \leftarrow x4, x2 \leftarrow x3, x1 \leftarrow x1\}$

$\overbrace{mult(\underbrace{op_l(x1,x3,x4)},op_t(x1,x2))}$

$=$ by Lemma 2437 RL with $\{x3 \leftarrow x4, x2 \leftarrow x3, x1 \leftarrow x1\}$

$mult(\overbrace{rd(x1,rd(x3,\underbrace{op_r(x3,x4,x1)}))},op_t(x1,x2))$

$=$ by Lemma 2392 RL with $\{x4 \leftarrow x2, x3 \leftarrow x1, x2 \leftarrow x4, x1 \leftarrow x3\}$

$\underbrace{mult(rd(x1,rd(x3,\overbrace{op_r(x3,x4,op_t(x1,x2))})),op_t(x1,x2))}$

$=$ by Lemma 1261 RL with $\{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow rd(x1,rd(x3,op_r(x3,x4,op_t(x1,x2))))\}$

$\overbrace{mult(\underbrace{rd(rd(x1,rd(x3,op_r(x3,x4,op_t(x1,x2)))),rd(x1,op_t(x1,x2)))},x1)}$

$=$ by Lemma 1390 RL with $\{x1 \leftarrow rd(x3,op_r(x3,x4,op_t(x1,x2))), x3 \leftarrow x2, x2 \leftarrow x1\}$

$mult(\overbrace{\underbrace{rd(op_t(x1,x2),rd(x3,op_r(x3,x4,op_t(x1,x2))))}},x1)$

$=$ by Lemma 2437 LR with $\{x3 \leftarrow x4, x2 \leftarrow x3, x1 \leftarrow op_t(x1,x2)\}$

$\underbrace{mult(\overbrace{op_l(op_t(x1,x2),x3,x4)},x1)}$

$=$ by Lemma 167 RL with $\{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow op_t(x1,x2), x1 \leftarrow x1\}$

$\overbrace{mult(x1,op_l(\underbrace{op_t(op_t(x1,x2),x1)},x3,x4))}$

$=$ by Lemma 11 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$mult(x1,op_l(\overbrace{op_t(x1,x2)},x3,x4))$

Lemma 2538: $op_t(rd(mult(x2, op_r(x1, x3, x4)), x1), i(x2)) = rd(x2, mult(rd(x1, mult(x2, op_r(x1, x3, x4))), x2))$

$\underbrace{op_t(rd(mult(x2, op_r(x1, x3, x4)), x1), i(x2))}$

$=$ by Lemma 46 RL with $\{x3 \leftarrow x2, x2 \leftarrow mult(x2, op_r(x1, x3, x4)), x1 \leftarrow x1\}$

$\overbrace{\underbrace{i(op_t(rd(x1, mult(x2, op_r(x1, x3, x4))), x2))}}$

$=$ by Lemma 373 RL with $\{x2 \leftarrow rd(x1, mult(x2, op_r(x1, x3, x4))), x1 \leftarrow x2\}$

$\overbrace{rd(\underbrace{op_t(x2, rd(x1, mult(x2, op_r(x1, x3, x4))))}, mult(rd(x1, mult(x2, op_r(x1, x3, x4))), x2))}$

$=$ by Lemma 2470 LR with $\{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow x2\}$

$rd(\overbrace{x2}, mult(rd(x1, mult(x2, op_r(x1, x3, x4))), x2))$

Lemma 2539: $mult(x2, i(rd(x1, op_r(x1, x3, x4)))) = rd(x2, mult(rd(x1, mult(x2, op_r(x1, x3, x4))), x2))$

$\underbrace{mult(x2, i(rd(x1, op_r(x1, x3, x4))))}$

$=$ by Lemma 34 RL with $\{x2 \leftarrow rd(x1, op_r(x1, x3, x4)), x1 \leftarrow x2\}$

$\overbrace{op_t(\underbrace{rd(x2, rd(x1, op_r(x1, x3, x4)))}, i(x2))}$

$=$ by Lemma 2189 RL with $\{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow x2\}$

$\underbrace{op_t(\overbrace{rd(mult(x2, op_r(x1, x3, x4)), x1)}, i(x2))}$

$=$ by Lemma 2538 LR with $\{x4 \leftarrow x4, x3 \leftarrow x3, x1 \leftarrow x1, x2 \leftarrow x2\}$

$\overbrace{rd(x2, mult(rd(x1, mult(x2, op_r(x1, x3, x4))), x2))}$

Lemma 2540: $mult(x2, rd(op_r(x1, x3, x4), x1)) = i(rd(x1, mult(x2, op_r(x1, x3, x4))))$

$mult(x2, \underbrace{rd(op_r(x1, x3, x4), x1)})$

$=$ by Lemma 10 RL with $\{x2 \leftarrow op_r(x1, x3, x4), x1 \leftarrow x1\}$

$\underbrace{mult(x2, \overbrace{i(rd(x1, op_r(x1, x3, x4)))})}$

$=$ by Lemma 2539 LR with $\{x4 \leftarrow x4, x3 \leftarrow x3, x1 \leftarrow x1, x2 \leftarrow x2\}$

$\overbrace{\underbrace{rd(x2, mult(rd(x1, mult(x2, op_r(x1, x3, x4))), x2))}}$

$=$ by Lemma 9 LR with $\{x2 \leftarrow rd(x1, mult(x2, op_r(x1, x3, x4))), x1 \leftarrow x2\}$

$\overbrace{i(rd(x1, mult(x2, op_r(x1, x3, x4))))}$

Lemma 2541: $mult(x2, rd(op_r(x1, x3, x4), x1)) = rd(x2, rd(x1, op_r(x1, x3, x4)))$

$\underbrace{mult(x2, rd(op_r(x1, x3, x4), x1))}$

$=$ by Lemma 2540 LR with $\{x4 \leftarrow x4, x3 \leftarrow x3, x1 \leftarrow x1, x2 \leftarrow x2\}$

$\overbrace{\underbrace{i(rd(x1, mult(x2, op_r(x1, x3, x4))))}}$

$=$ by Lemma 10 LR with $\{x2 \leftarrow mult(x2, op_r(x1, x3, x4)), x1 \leftarrow x1\}$

$\overbrace{\underbrace{rd(mult(x2, op_r(x1, x3, x4)), x1)}}$

$=$ by Lemma 2189 LR with $\{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow x2\}$

$\overbrace{rd(x2, rd(x1, op_r(x1, x3, x4)))}$

Lemma 2542: $rd(rd(op_r(x1,x2,x3),x1),x4) = rd(op_r(x1,x2,x3),mult(x4,x1))$

$$
\begin{aligned}
& \overbrace{rd(rd(op_r(x1,x2,x3),x1),x4)} \\
= & \quad \text{by Lemma 40 RL with } \{x3 \leftarrow op_r(x1,x2,x3), x2 \leftarrow x1, x1 \leftarrow x4\} \\
& \overbrace{rd(i(x4),rd(x1,op_r(x1,x2,x3)))} \\
= & \quad \text{by Lemma 2541 RL with } \{x4 \leftarrow x3, x3 \leftarrow x2, x1 \leftarrow x1, x2 \leftarrow i(x4)\} \\
& \overbrace{mult(i(x4),rd(op_r(x1,x2,x3),x1))} \\
= & \quad \text{by Lemma 2187 LR with } \{x4 \leftarrow x3, x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x4\} \\
& \overbrace{rd(op_r(x1,x2,x3),mult(x4,x1))}
\end{aligned}
$$

Lemma 2543: $rd(x2,rd(op_r(x1,x3,x4),x1)) = op_t(rd(x2,rd(op_r(x1,x3,x4),x1)),i(x2))$

$$
\begin{aligned}
& \overbrace{rd(x2,rd(op_r(x1,x3,x4),x1))} \\
= & \quad \text{by Lemma 2245 LR with } \{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \overbrace{rd(x1,mult(i(x2),op_r(x1,x3,x4)))} \\
= & \quad \text{by Lemma 1511 RL with } \{x2 \leftarrow x2, x1 \leftarrow rd(x1,mult(i(x2),op_r(x1,x3,x4)))\} \\
& \overbrace{mult(op_r(mult(rd(x1,mult(i(x2),op_r(x1,x3,x4))),x2),x2,rd(x1,mult(i(x2),op_r(x1,x3,x4)))),\underbrace{op_t(i(x2),rd(x1,mult(i(x}} \\
= & \quad \text{by Lemma 2470 LR with } \{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow i(x2)\} \\
& \underbrace{mult(op_r(mult(rd(x1,mult(i(x2),op_r(x1,x3,x4))),x2),x2,rd(x1,mult(i(x2),op_r(x1,x3,x4)))),\overbrace{i(x2)}} \\
= & \quad \text{by Lemma 1505 LR with } \{x2 \leftarrow x2, x1 \leftarrow rd(x1,mult(i(x2),op_r(x1,x3,x4)))\} \\
& \overbrace{op_t(\underbrace{rd(x1,mult(i(x2),op_r(x1,x3,x4)))},i(x2))} \\
= & \quad \text{by Lemma 2245 RL with } \{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow x2\} \\
& op_t(\overbrace{rd(x2,rd(op_r(x1,x3,x4),x1))},i(x2))
\end{aligned}
$$

Lemma 2544: $rd(x2,rd(op_r(x1,x3,x4),x1)) = mult(x2,rd(x1,op_r(x1,x3,x4)))$

$$
\begin{aligned}
& \overbrace{rd(x2,rd(op_r(x1,x3,x4),x1))} \\
= & \quad \text{by Lemma 2543 LR with } \{x4 \leftarrow x4, x3 \leftarrow x3, x1 \leftarrow x1, x2 \leftarrow x2\} \\
& \overbrace{op_t(rd(x2,rd(op_r(x1,x3,x4),x1)),i(x2))} \\
= & \quad \text{by Lemma 34 LR with } \{x2 \leftarrow rd(op_r(x1,x3,x4),x1), x1 \leftarrow x2\} \\
& \overbrace{mult(x2,\underbrace{i(rd(op_r(x1,x3,x4),x1))})} \\
= & \quad \text{by Lemma 10 LR with } \{x2 \leftarrow x1, x1 \leftarrow op_r(x1,x3,x4)\} \\
& mult(x2,\overbrace{rd(x1,op_r(x1,x3,x4))})
\end{aligned}
$$

Lemma 2545: $rd(mult(x1,x2),op_r(x2,x3,x4)) = rd(x1,rd(op_r(x2,x3,x4),x2))$

$$
\begin{aligned}
& \overbrace{rd(mult(x1,x2),op_r(x2,x3,x4))} \\
= & \quad \text{by Lemma 2046 RL with } \{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{mult(x1,rd(x2,op_r(x2,x3,x4)))} \\
= & \quad \text{by Lemma 2544 RL with } \{x4 \leftarrow x4, x3 \leftarrow x3, x1 \leftarrow x2, x2 \leftarrow x1\} \\
& \overbrace{rd(x1,rd(op_r(x2,x3,x4),x2))}
\end{aligned}
$$

Lemma 2546: $rd(op_r(x1,x2,x3),mult(i(x3),x1)) = mult(x3,rd(op_r(x1,x2,x3),x1))$

$rd(op_r(x1,x2,x3),mult(i(\overbrace{x3}),x1))$

$=$ by Lemma 2422 RL with $\{x4 \leftarrow i(x4), x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x3\}$

$rd(op_r(x1,x2,x3),mult(\overbrace{i(op_r(x3,rd(op_r(x1,x2,x3),x1),i(x4)))},x1))$

$=$ by Lemma 2428 RL with $\{x3 \leftarrow i(x4), x2 \leftarrow rd(op_r(x1,x2,x3),x1), x1 \leftarrow x3\}$

$\underbrace{rd(op_r(x1,x2,x3),mult(\overbrace{op_r(i(x3),rd(op_r(x1,x2,x3),x1),i(x4))},x1))}$

$=$ by Lemma 2542 RL with $\{x4 \leftarrow op_r(i(x3),rd(op_r(x1,x2,x3),x1),i(x4)), x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{rd(rd(op_r(x1,x2,x3),x1),\underbrace{op_r(i(x3),rd(op_r(x1,x2,x3),x1),i(x4))})}$

$=$ by Lemma 2403 LR with $\{x4 \leftarrow rd(op_r(x1,x2,x3),x1), x3 \leftarrow x4, x2 \leftarrow rd(op_r(x1,x2,x3),x1), x1 \leftarrow i(x3)\}$

$rd(rd(op_r(x1,x2,x3),x1),\overbrace{op_r(i(x3),rd(op_r(x1,x2,x3),x1),op_l(i(x4),rd(op_r(x1,x2,x3),x1),x4))})$

$=$ by Lemma 2428 LR with $\{x3 \leftarrow op_l(i(x4),rd(op_r(x1,x2,x3),x1),x4), x2 \leftarrow rd(op_r(x1,x2,x3),x1), x1 \leftarrow x3\}$

$\underbrace{rd(rd(op_r(x1,x2,x3),x1),\overbrace{i(op_r(x3,rd(op_r(x1,x2,x3),x1),op_l(i(x4),rd(op_r(x1,x2,x3),x1),x4)))})}$

$=$ by Lemma 357 RL with $\{x2 \leftarrow rd(op_r(x1,x2,x3),x1), x1 \leftarrow op_r(x3,rd(op_r(x1,x2,x3),x1),op_l(i(x4),rd(op_r(x1,x2,x3),x1),$

$\overbrace{mult(\underbrace{op_r(op_r(x3,rd(op_r(x1,x2,x3),x1),op_l(i(x4),rd(op_r(x1,x2,x3),x1),x4)),rd(op_r(x1,x2,x3),x1),rd(op_r(x1,x2,x3),$

$=$ by Axiom 17 RL with $\{x5 \leftarrow op_l(i(x4),rd(op_r(x1,x2,x3),x1),x4), x4 \leftarrow rd(op_r(x1,x2,x3),x1), x3 \leftarrow rd(op_r(x1,x2,x3),x1)$

$mult(\overbrace{op_r(op_r(x3,rd(op_r(x1,x2,x3),x1),rd(op_r(x1,x2,x3),x1)),rd(op_r(x1,x2,x3),x1),op_l(i(x4),rd(op_r(x1,x2,x3),x1),$

$=$ by Lemma 2512 LR with $\{x3 \leftarrow op_l(i(x4),rd(op_r(x1,x2,x3),x1),x4), x2 \leftarrow rd(op_r(x1,x2,x3),x1), x1 \leftarrow x3\}$

$mult(\overbrace{op_r(x3,rd(op_r(x1,x2,x3),x1),\underbrace{mult(op_l(i(x4),rd(op_r(x1,x2,x3),x1),x4),rd(op_r(x1,x2,x3),x1))}),rd(op_r(x1,x2,x3$

$=$ by Lemma 256 LR with $\{x2 \leftarrow rd(op_r(x1,x2,x3),x1), x1 \leftarrow x4\}$

$mult(\underbrace{op_r(x3,rd(op_r(x1,x2,x3),x1),\overbrace{rd(rd(op_r(x1,x2,x3),x1),op_t(x4,rd(op_r(x1,x2,x3),x1))))},rd(op_r(x1,x2,x3),x1))$

$=$ by Lemma 2407 RL with $\{x5 \leftarrow rd(op_r(x1,x2,x3),x1), x3 \leftarrow x4, x4 \leftarrow rd(op_r(x1,x2,x3),x1), x2 \leftarrow rd(op_r(x1,x2,x3),x1), x$

$mult(\overbrace{\underbrace{op_r(x3,rd(op_r(x1,x2,x3),x1),rd(rd(op_r(x1,x2,x3),x1),x4))},rd(op_r(x1,x2,x3),x1))}$

$=$ by Lemma 2422 LR with $\{x4 \leftarrow rd(rd(op_r(x1,x2,x3),x1),x4), x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x3\}$

$mult(\overbrace{x3},rd(op_r(x1,x2,x3),x1))$

Lemma 2547: $rd(op_t(op_r(x1,x2,x3),i(i(x3))),x1) = rd(op_t(i(x3),i(op_r(x1,x2,x3))),op_l(i(x3),x1,x2))$

$\overbrace{rd(op_t(op_r(x1,x2,x3),i(i(x3))),x1)}$

$=$ by Lemma 2378 LR with $\{x1 \leftarrow x1, x3 \leftarrow i(x3), x2 \leftarrow op_r(x1,x2,x3)\}$

$\overbrace{rd(op_t(i(x3),i(op_r(x1,x2,x3))),\underbrace{mult(rd(x1,op_r(x1,x2,x3)),i(x3))}}$

$=$ by Lemma 2307 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$rd(op_t(i(x3),i(op_r(x1,x2,x3))),\overbrace{rd(\underbrace{i(rd(op_r(x1,x2,x3),x1))},x3)})$

$=$ by Lemma 10 LR with $\{x2 \leftarrow x1, x1 \leftarrow op_r(x1,x2,x3)\}$

$rd(op_t(i(x3),i(op_r(x1,x2,x3))),\underbrace{rd(\overbrace{rd(x1,op_r(x1,x2,x3))},x3)})$

$=$ by Lemma 2190 RL with $\{x4 \leftarrow x3, x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x1\}$

$rd(op_t(i(x3),i(op_r(x1,x2,x3))),\overbrace{rd(x1,mult(x3,op_r(x1,x2,x3)))})$

$=$ by Lemma 2464 LR with $\{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x1\}$

$rd(op_t(i(x3),i(op_r(x1,x2,x3))),\overbrace{op_l(i(x3),x1,x2)})$

Lemma 2548: $rd(op_r(op_t(x1,x3),x2,x3),x1) = rd(x3,op_l(op_t(x3,x1),x2,x1))$

$rd(op_r(op_t(x1,\underbrace{x3}),x2,x3),x1)$

$=$ by Lemma 3 RL with $\{x1 \leftarrow x3\}$

$rd(\underbrace{op_r(op_t(x1,\overbrace{i(i(x3))}),x2,x3)},x1)$

$=$ by Axiom 14 RL with $\{x4 \leftarrow i(i(x3)), x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{rd(\overbrace{op_t(op_r(x1,x2,x3),i(i(x3)))},x1)}$

$=$ by Lemma 2547 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{rd(\underbrace{op_t(i(x3),i(op_r(x1,x2,x3)))},op_l(i(x3),x1,x2))}$

$=$ by Lemma 15 LR with $\{x2 \leftarrow op_r(x1,x2,x3), x1 \leftarrow x3\}$

$\underbrace{rd(\overbrace{i(op_t(x3,op_r(x1,x2,x3)))},op_l(i(x3),x1,x2))}$

$=$ by Lemma 1074 LR with $\{x4 \leftarrow x2, x3 \leftarrow x1, x2 \leftarrow x3, x1 \leftarrow op_t(x3,op_r(x1,x2,x3))\}$

$\overbrace{rd(op_l(x3,x1,x2),op_t(x3,\underbrace{op_r(x1,x2,x3)}))}$

$=$ by Axiom 16 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$rd(op_l(x3,x1,x2),\underbrace{op_t(x3,\overbrace{rd(mult(mult(x1,x2),x3),mult(x2,x3))})})$

$=$ by Lemma 2085 LR with $\{x3 \leftarrow mult(x2,x3), x2 \leftarrow mult(x1,x2), x1 \leftarrow x3\}$

$rd(op_l(x3,x1,x2),\overbrace{op_t(x3,rd(mult(x1,x2),\underbrace{rd(mult(x2,x3),x3)}))})$

$=$ by Axiom 5 RL with $\{x2 \leftarrow x3, x1 \leftarrow x2\}$

$rd(op_l(x3,x1,x2),op_t(x3,\underbrace{rd(mult(x1,x2),\overbrace{x2})}))$

$=$ by Axiom 5 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$rd(\underbrace{op_l(x3,x1,x2)},op_t(x3,\overbrace{x1}))$

$=$ by Lemma 2437 RL with $\{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x3\}$

$rd(\overbrace{rd(x3,rd(x1,\underbrace{op_r(x1,x2,x3)}))},op_t(x3,x1))$

$=$ by Lemma 2392 RL with $\{x4 \leftarrow x1, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$rd(rd(x3,\underbrace{rd(x1,\overbrace{op_r(x1,x2,op_t(x3,x1))})}),op_t(x3,x1))$

$=$ by Lemma 2458 RL with $\{x3 \leftarrow op_t(x3,x1), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{rd(rd(x3,\overbrace{rd(op_r(x1,x2,i(op_t(x3,x1))),x1)}),op_t(x3,x1))}$

$=$ by Lemma 1651 RL with $\{x3 \leftarrow rd(op_r(x1,x2,i(op_t(x3,x1))),x1), x2 \leftarrow x1, x1 \leftarrow x3\}$

$\overbrace{rd(x3,\underbrace{rd(op_t(x3,x1),i(\overbrace{rd(op_r(x1,x2,i(op_t(x3,x1))),x1)}))})}$

$=$ by Lemma 38 LR with $\{x1 \leftarrow rd(op_r(x1,x2,i(op_t(x3,x1))),x1), x2 \leftarrow op_t(x3,x1)\}$

$rd(x3,\underbrace{\overbrace{rd(rd(op_r(x1,x2,i(op_t(x3,x1))),x1),i(op_t(x3,x1)))}})$

$=$ by Lemma 2299 LR with $\{x3 \leftarrow i(op_t(x3,x1)), x2 \leftarrow x2, x1 \leftarrow x1\}$

$rd(x3,\underbrace{\overbrace{rd(op_r(x1,x2,i(op_t(x3,x1))),mult(i(op_t(x3,x1)),x1))}})$

$=$ by Lemma 2466 LR with $\{x3 \leftarrow i(op_t(x3,x1)), x2 \leftarrow x2, x1 \leftarrow x1\}$

$rd(x3,\overbrace{op_l(\underbrace{i(i(op_t(x3,x1)))},x2,x1)})$

$=$ by Lemma 3 LR with $\{x1 \leftarrow op_t(x3,x1)\}$

$rd(x3,op_l(\overbrace{op_t(x3,x1)},x2,x1))$

Lemma 2549: $rd(rd(x1,x2),op_l(x1,x3,x4)) = rd(op_l(x1,x4,x3),rd(x1,i(x2)))$

$$
\begin{aligned}
& \underbrace{rd(rd(x1,x2),op_l(x1,x3,x4))} \\
= \quad & \text{by Lemma 2377 RL with } \{x3 \leftarrow x4, x2 \leftarrow x3, x4 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{rd(x1,\underbrace{rd(x2,op_l(i(x1),x3,x4))})} \\
= \quad & \text{by Lemma 2075 RL with } \{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \underbrace{rd(x1,\overbrace{mult(op_r(x2,x1,x1),op_l(x1,x3,x4))})} \\
= \quad & \text{by Lemma 2532 LR with } \{x2 \leftarrow x4, x3 \leftarrow x3, x4 \leftarrow op_r(x2,x1,x1), x1 \leftarrow x1\} \\
& \overbrace{rd(op_l(x1,x4,x3),\underbrace{mult(op_r(x2,x1,x1),x1)})} \\
= \quad & \text{by Lemma 357 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& rd(op_l(x1,x4,x3),\overbrace{rd(x1,i(x2))})
\end{aligned}
$$

Lemma 2550: $rd(x1,rd(op_l(x1,x3,x2),x4)) = rd(op_l(x1,x2,x3),rd(x1,x4))$

$$
\begin{aligned}
& \underbrace{rd(x1,rd(op_l(x1,x3,x2),x4))} \\
= \quad & \text{by Lemma 10 RL with } \{x2 \leftarrow x1, x1 \leftarrow rd(op_l(x1,x3,x2),x4)\} \\
& \overbrace{i(\underbrace{rd(rd(op_l(x1,x3,x2),x4),x1)})} \\
= \quad & \text{by Lemma 2376 RL with } \{x4 \leftarrow x4, x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \underbrace{i(\overbrace{rd(op_l(x1,x3,x2),rd(x1,i(x4)))})} \\
= \quad & \text{by Lemma 10 LR with } \{x2 \leftarrow rd(x1,i(x4)), x1 \leftarrow op_l(x1,x3,x2)\} \\
& \overbrace{\underbrace{rd(rd(x1,i(x4)),op_l(x1,x3,x2))}} \\
= \quad & \text{by Lemma 2549 LR with } \{x4 \leftarrow x2, x3 \leftarrow x3, x2 \leftarrow i(x4), x1 \leftarrow x1\} \\
& \overbrace{rd(op_l(x1,x2,x3),rd(x1,\underbrace{i(i(x4))}))} \\
= \quad & \text{by Lemma 3 LR with } \{x1 \leftarrow x4\} \\
& rd(op_l(x1,x2,x3),rd(x1,\overbrace{x4}))
\end{aligned}
$$

Lemma 2551: $op_l(x1,x2,mult(mult(x1,x2),x3)) = op_l(x1,x2,rd(x1,rd(x2,x3)))$

$$
\begin{aligned}
& \underbrace{op_l(x1,x2,mult(mult(x1,x2),x3))} \\
= \quad & \text{by Lemma 2437 RL with } \{x3 \leftarrow mult(mult(x1,x2),x3), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{rd(x1,rd(x2,\underbrace{op_r(x2,mult(mult(x1,x2),x3),x1)}))} \\
= \quad & \text{by Lemma 2535 LR with } \{x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \underbrace{rd(x1,rd(x2,\overbrace{op_r(x2,rd(x1,rd(x2,x3)),x1)}))} \\
= \quad & \text{by Lemma 2437 LR with } \{x3 \leftarrow rd(x1,rd(x2,x3)), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1,x2,rd(x1,rd(x2,x3)))}
\end{aligned}
$$

Lemma 2552: $op_r(x1, x2, rd(i(x2), op_r(x1, x3, x2))) = rd(x1, rd(op_t(x2, op_r(x1, x3, x2)), x2))$

$\overbrace{op_r(x1, x2, rd(i(x2), op_r(x1, x3, x2)))}$

$=$ by Lemma 2400 RL with $\{x4 \leftarrow op_r(x1, x3, x2), x3 \leftarrow x2, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_r(x1, x2, \underbrace{i(mult(x2, op_r(x1, x3, x2)))}}$

$=$ by Lemma 2456 LR with $\{x3 \leftarrow mult(x2, op_r(x1, x3, x2)), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{rd(x1, rd(mult(x2, op_r(x1, x3, x2)), \underbrace{op_l(mult(x2, op_r(x1, x3, x2)), x2, x1)}))}$

$=$ by Lemma 2415 LR with $\{x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow x2\}$

$rd(x1, \underbrace{rd(mult(x2, op_r(x1, x3, x2)), \overbrace{mult(op_r(x1, x3, x2), x2)})})$

$=$ by Lemma 862 LR with $\{x1 \leftarrow op_r(x1, x3, x2), x2 \leftarrow x2\}$

$rd(x1, \overbrace{rd(rd(op_t(x2, op_r(x1, x3, x2)), x2), unit())})$

$=$ by Lemma 1 LR with $\{x1 \leftarrow rd(op_t(x2, op_r(x1, x3, x2)), x2)\}$

$rd(x1, \overbrace{rd(op_t(x2, op_r(x1, x3, x2)), x2)})$

Lemma 2553: $mult(x1, x2) = mult(op_r(x1, x3, i(x2)), op_l(x2, x1, x3))$

$\underbrace{mult(x1, x2)}$

$=$ by Lemma 2469 LR with $\{x3 \leftarrow op_r(rd(i(x3), x1), x3, x3), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{mult(op_r(x1, op_r(rd(i(x3), x1), x3, x3), x2), \underbrace{op_l(x2, op_r(rd(i(x3), x1), x3, x3), x1)})}$

$=$ by Lemma 1799 LR with $\{x3 \leftarrow x1, x2 \leftarrow x3, x1 \leftarrow x2\}$

$mult(\underbrace{op_r(x1, op_r(rd(i(x3), x1), x3, x3), x2)}, \overbrace{op_l(x2, x1, x3)})$

$=$ by Lemma 1966 LR with $\{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x1\}$

$mult(\overbrace{op_r(x1, x3, i(x2))}, op_l(x2, x1, x3))$

Lemma 2554: $op_t(x1,x2) = op_r(x1,x2,i(rd(op_l(x2,x3,x4),i(x1))))$

$$
\begin{aligned}
& op_t(x1,\underbrace{x2}) \\
= \quad & \text{by Lemma 3 RL with } \{x1 \leftarrow x2\} \\
& \underbrace{op_t(x1,\overbrace{i(i(x2))})} \\
= \quad & \text{by Lemma 2520 LR with } \{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& \overbrace{op_r(x1,i(x2),\underbrace{rd(x1,op_l(i(x2),x3,x4))})} \\
= \quad & \text{by Lemma 2075 RL with } \{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_r(x1,i(x2),\overbrace{\underbrace{mult(op_r(x1,x2,x2),op_l(x2,x3,x4))}}) \\
= \quad & \text{by Lemma 1719 RL with } \{x3 \leftarrow x2, x2 \leftarrow op_l(x2,x3,x4), x1 \leftarrow x1\} \\
& op_r(x1,i(x2),\overbrace{\underbrace{rd(mult(x1,op_l(x2,x3,x4)),asoc(x2,x1,x2))}}) \\
= \quad & \text{by Lemma 1708 RL with } \{x3 \leftarrow mult(x1,op_l(x2,x3,x4)), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_r(x1,i(x2),\overbrace{\underbrace{mult(asoc(x1,x2,x2),mult(x1,op_l(x2,x3,x4)))}}) \\
= \quad & \text{by Lemma 2138 RL with } \{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_r(x1,i(x2),\overbrace{\underbrace{mult(mult(x1,op_l(x2,x3,x4)),asoc(mult(x1,op_l(x2,x3,x4)),x2,x2))}}) \\
= \quad & \text{by Lemma 1300 RL with } \{x2 \leftarrow mult(x1,op_l(x2,x3,x4)), x3 \leftarrow x2, x1 \leftarrow mult(x1,op_l(x2,x3,x4))\} \\
& op_r(x1,i(x2),\overbrace{\underbrace{rd(mult(x1,op_l(x2,x3,x4)),asoc(x2,mult(x1,op_l(x2,x3,x4)),x2))}}) \\
= \quad & \text{by Lemma 641 LR with } \{x2 \leftarrow x2, x1 \leftarrow mult(x1,op_l(x2,x3,x4))\} \\
& op_r(x1,i(x2),\overbrace{\underbrace{op_r(mult(x1,op_l(x2,x3,x4)),x2,x2)}}) \\
= \quad & \text{by Lemma 2133 RL with } \{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow mult(x1,op_l(x2,x3,x4))\} \\
& op_r(x1,i(x2),\overbrace{\underbrace{op_r(mult(x1,op_l(x2,x3,x4)),op_l(x2,x3,x4),op_l(x2,x3,x4))}}) \\
= \quad & \text{by Lemma 359 LR with } \{x2 \leftarrow op_l(x2,x3,x4), x1 \leftarrow x1\} \\
& \underbrace{op_r(x1,i(x2),\overbrace{rd(op_l(x2,x3,x4),i(x1))})} \\
= \quad & \text{by Lemma 2427 LR with } \{x3 \leftarrow rd(op_l(x2,x3,x4),i(x1)), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_r(x1,x2,i(rd(op_l(x2,x3,x4),i(x1))))}
\end{aligned}
$$

Lemma 2555: $x1 = op_r(x1,x2,rd(op_r(x3,x4,mult(x2,x4)),x3))$

$$
\begin{aligned}
& \underbrace{x1} \\
= \quad & \text{by Lemma 2421 RL with } \{x4 \leftarrow x4, x3 \leftarrow op_r(x3,x4,x4), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_r(x1,x2,rd(\underbrace{op_r(op_r(x3,x4,x4),x4,x2)},op_r(x3,x4,x4)))} \\
= \quad & \text{by Lemma 2512 LR with } \{x3 \leftarrow x2, x2 \leftarrow x4, x1 \leftarrow x3\} \\
& \underbrace{op_r(x1,x2,rd(\overbrace{op_r(x3,x4,mult(x2,x4))},op_r(x3,x4,x4)))} \\
= \quad & \text{by Lemma 2409 RL with } \{x5 \leftarrow x4, x3 \leftarrow x3, x4 \leftarrow op_r(x3,x4,mult(x2,x4)), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_r(x1,x2,rd(op_r(x3,x4,mult(x2,x4)),x3))}
\end{aligned}
$$

Lemma 2556: $rd(op_r(x2,x1,x3),mult(x1,x2)) = i(mult(x3,mult(mult(i(x3),x1),rd(x2,op_r(x2,x1,x3)))))$

$\underbrace{rd(op_r(x2,x1,x3),mult(x1,x2))}$

$=$ by Lemma 2542 RL with $\{x4 \leftarrow x1, x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow x2\}$

$\overbrace{rd(rd(op_r(x2,x1,x3),x2),x1)}$

$=$ by Lemma 40 RL with $\{x3 \leftarrow op_r(x2,x1,x3), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{rd(i(x1),rd(x2,op_r(x2,x1,x3)))}$

$=$ by Lemma 2541 RL with $\{x4 \leftarrow x3, x3 \leftarrow x1, x1 \leftarrow x2, x2 \leftarrow i(x1)\}$

$\overbrace{mult(i(x1),rd(op_r(x2,x1,x3),x2))}$

$=$ by Lemma 45 RL with $\{x3 \leftarrow op_r(x2,x1,x3), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{i(mult(x1,rd(x2,op_r(x2,x1,x3))))}$

$=$ by Lemma 22 RL with $\{x2 \leftarrow mult(x1,rd(x2,op_r(x2,x1,x3))), x1 \leftarrow mult(mult(i(x3),x1),rd(x2,op_r(x2,x1,x3)))\}$

$\overbrace{mult(\underbrace{rd(mult(mult(i(x3),x1),rd(x2,op_r(x2,x1,x3))),mult(x1,rd(x2,op_r(x2,x1,x3))))}, i(mult(mult(i(x3),x1),rd(x2,op$

$=$ by Axiom 16 RL with $\{x3 \leftarrow rd(x2,op_r(x2,x1,x3)), x2 \leftarrow x1, x1 \leftarrow i(x3)\}$

$mult(\overbrace{op_r(i(x3),x1,\underbrace{rd(x2,op_r(x2,x1,x3))})}, i(mult(mult(i(x3),x1),rd(x2,op_r(x2,x1,x3)))))$

$=$ by Lemma 10 RL with $\{x2 \leftarrow x2, x1 \leftarrow op_r(x2,x1,x3)\}$

$mult(\underbrace{op_r(i(x3),x1,\overbrace{i(rd(op_r(x2,x1,x3),x2))})}, i(mult(mult(i(x3),x1),rd(x2,op_r(x2,x1,x3)))))$

$=$ by Lemma 1948 RL with $\{x3 \leftarrow rd(op_r(x2,x1,x3),x2), x2 \leftarrow x1, x1 \leftarrow x3\}$

$mult(\overbrace{op_r(i(x3),rd(x3,x1),rd(op_r(x2,x1,\underbrace{x3}),x2))}, i(mult(mult(i(x3),x1),rd(x2,op_r(x2,x1,x3)))))$

$=$ by Axiom 4 LR with $\{x2 \leftarrow x1, x1 \leftarrow x3\}$

$mult(\underbrace{op_r(i(x3),rd(x3,x1),rd(op_r(x2,x1,\overbrace{mult(rd(x3,x1),x1)}),x2))}, i(mult(mult(i(x3),x1),rd(x2,op_r(x2,x1,x3)))))$

$=$ by Lemma 2555 RL with $\{x4 \leftarrow x1, x3 \leftarrow x2, x2 \leftarrow rd(x3,x1), x1 \leftarrow i(x3)\}$

$\underbrace{mult(\overbrace{i(x3)}, i(mult(mult(i(x3),x1),rd(x2,op_r(x2,x1,x3)))))}$

$=$ by Axiom 11 LR with $\{x2 \leftarrow mult(mult(i(x3),x1),rd(x2,op_r(x2,x1,x3))), x1 \leftarrow x3\}$

$\overbrace{i(mult(x3,mult(mult(i(x3),x1),rd(x2,op_r(x2,x1,x3)))))}$

Lemma 2557: $rd(op_r(x2,x1,x3),mult(x1,x2)) = i(mult(x3,rd(mult(i(x3),x1),rd(op_r(x2,x1,x3),x2))))$

$\underbrace{rd(op_r(x2,x1,x3),mult(x1,x2))}$

$=$ by Lemma 2556 LR with $\{x3 \leftarrow x3, x1 \leftarrow x1, x2 \leftarrow x2\}$

$\overbrace{i(mult(x3,\underbrace{mult(mult(i(x3),x1),rd(x2,op_r(x2,x1,x3)))}))}$

$=$ by Lemma 2046 LR with $\{x4 \leftarrow x3, x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow mult(i(x3),x1)\}$

$i(mult(x3,\overbrace{rd(mult(mult(i(x3),x1),x2),op_r(x2,x1,x3))}))$

$=$ by Lemma 2545 LR with $\{x4 \leftarrow x3, x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow mult(i(x3),x1)\}$

$i(mult(x3,\overbrace{rd(mult(i(x3),x1),rd(op_r(x2,x1,x3),x2))}))$

Lemma 2558: $rd(op_r(x2,x1,x3),mult(x1,x2)) = mult(rd(rd(rd(op_r(x2,x1,x3),x2),x3),x1),x3)$

$\overbrace{rd(op_r(x2,x1,x3),mult(x1,x2))}$

$=$ by Lemma 2557 LR with $\{x3 \leftarrow x3, x1 \leftarrow x1, x2 \leftarrow x2\}$

$\overbrace{i(mult(x3,rd(mult(i(x3),x1),rd(op_r(x2,x1,x3),x2))))}$

$=$ by Lemma 45 LR with $\{x3 \leftarrow rd(op_r(x2,x1,x3),x2), x2 \leftarrow mult(i(x3),x1), x1 \leftarrow x3\}$

$\overbrace{mult(i(x3),rd(rd(op_r(x2,x1,x3),x2),mult(i(x3),x1)))}$

$=$ by Lemma 467 RL with $\{x2 \leftarrow x3, x1 \leftarrow rd(rd(op_r(x2,x1,x3),x2),mult(i(x3),x1))\}$

$\overbrace{op_r(\overbrace{rd(op_t(rd(rd(op_r(x2,x1,x3),x2),mult(i(x3),x1)),x3),x3)},x3,x3)}$

$=$ by Lemma 324 RL with $\{x2 \leftarrow x3, x1 \leftarrow rd(rd(op_r(x2,x1,x3),x2),mult(i(x3),x1))\}$

$\overbrace{op_r(\overbrace{op_t(mult(rd(rd(op_r(x2,x1,x3),x2),mult(i(x3),x1)),i(x3)),x3)},x3,x3)}$

$=$ by Lemma 361 RL with $\{x2 \leftarrow x3, x1 \leftarrow op_t(mult(rd(rd(op_r(x2,x1,x3),x2),mult(i(x3),x1)),i(x3)),x3)\}$

$\overbrace{op_r(op_t(mult(rd(rd(op_r(x2,x1,x3),x2),mult(i(x3),x1)),i(x3)),x3),i(x3),i(x3))}$

$=$ by Axiom 14 RL with $\{x4 \leftarrow x3, x3 \leftarrow i(x3), x2 \leftarrow i(x3), x1 \leftarrow mult(rd(rd(op_r(x2,x1,x3),x2),mult(i(x3),x1)),i(x3))\}$

$\overbrace{op_t(op_r(mult(rd(rd(op_r(x2,x1,x3),x2),mult(i(x3),x1)),i(x3)),i(x3),i(x3)),x3)}$

$=$ by Lemma 908 RL with $\{x2 \leftarrow mult(rd(rd(op_r(x2,x1,x3),x2),mult(i(x3),x1)),i(x3)), x1 \leftarrow x3\}$

$\overbrace{mult(i(x3),rd(mult(rd(rd(op_r(x2,x1,x3),x2),mult(i(x3),x1)),i(x3)),i(x3)))}$

$=$ by Lemma 69 LR with $\{x2 \leftarrow mult(rd(rd(op_r(x2,x1,x3),x2),mult(i(x3),x1)),i(x3)), x1 \leftarrow x3\}$

$\overbrace{mult(\overbrace{mult(i(x3),mult(rd(rd(op_r(x2,x1,x3),x2),mult(i(x3),x1)),i(x3)))},x3)}$

$=$ by Lemma 1152 LR with $\{x3 \leftarrow x1, x2 \leftarrow rd(op_r(x2,x1,x3),x2), x1 \leftarrow i(x3)\}$

$mult(\overbrace{rd(\overbrace{mult(i(x3),rd(op_r(x2,x1,x3),x2))},x1)},x3)$

$=$ by Lemma 2541 LR with $\{x4 \leftarrow x3, x3 \leftarrow x1, x1 \leftarrow x2, x2 \leftarrow i(x3)\}$

$mult(rd(\overbrace{rd(i(x3),rd(x2,op_r(x2,x1,x3)))},x1),x3)$

$=$ by Lemma 40 LR with $\{x3 \leftarrow op_r(x2,x1,x3), x2 \leftarrow x2, x1 \leftarrow x3\}$

$mult(rd(\overbrace{rd(rd(op_r(x2,x1,x3),x2),x3)},x1),x3)$

Lemma 2559: $rd(op_r(x2,x1,x3),mult(x1,x2)) = mult(rd(op_l(i(x3),x1,x2),x1),x3)$

$\overbrace{rd(op_r(x2,x1,x3),mult(x1,x2))}$

$=$ by Lemma 2558 LR with $\{x3 \leftarrow x3, x1 \leftarrow x1, x2 \leftarrow x2\}$

$\overbrace{mult(rd(\overbrace{rd(rd(op_r(x2,x1,x3),x2),x3)},x1),x3)}$

$=$ by Lemma 2542 LR with $\{x4 \leftarrow x3, x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow x2\}$

$mult(rd(\overbrace{rd(op_r(x2,x1,x3),mult(x3,x2))},x1),x3)$

$=$ by Lemma 2466 LR with $\{x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow x2\}$

$mult(rd(\overbrace{op_l(i(x3),x1,x2)},x1),x3)$

Lemma 2560: $rd(op_r(x2,x1,x3),mult(x1,x2)) = mult(rd(i(x1),op_l(x3,x1,x2)),x3)$

$$
\begin{aligned}
& \underbrace{rd(op_r(x2,x1,x3),mult(x1,x2))} \\
= & \quad \text{by Lemma 2559 LR with } \{x3 \leftarrow x3, x1 \leftarrow x1, x2 \leftarrow x2\} \\
& \overbrace{mult(\underbrace{rd(op_l(i(x3),x1,x2),x1)},x3)} \\
= & \quad \text{by Lemma 10 RL with } \{x2 \leftarrow op_l(i(x3),x1,x2), x1 \leftarrow x1\} \\
& mult(\overbrace{\underbrace{i(rd(x1,op_l(i(x3),x1,x2)))}},x3) \\
= & \quad \text{by Lemma 2137 RL with } \{x3 \leftarrow x2, x1 \leftarrow x3, x2 \leftarrow x1\} \\
& mult(\overbrace{rd(i(x1),op_l(x3,x1,x2))},x3)
\end{aligned}
$$

Lemma 2561: $rd(op_r(x2,x1,x3),mult(x1,x2)) = rd(rd(op_l(x3,x2,x1),x3),x1)$

$$
\begin{aligned}
& \underbrace{rd(op_r(x2,x1,x3),mult(x1,x2))} \\
= & \quad \text{by Lemma 2560 LR with } \{x3 \leftarrow x3, x1 \leftarrow x1, x2 \leftarrow x2\} \\
& \overbrace{\underbrace{mult(rd(i(x1),op_l(x3,x1,x2)),x3)}} \\
= & \quad \text{by Lemma 2531 LR with } \{x4 \leftarrow x2, x3 \leftarrow x1, x2 \leftarrow x3, x1 \leftarrow i(x1)\} \\
& \overbrace{\underbrace{rd(i(x1),rd(x3,op_l(x3,x2,x1)))}} \\
= & \quad \text{by Lemma 40 LR with } \{x3 \leftarrow op_l(x3,x2,x1), x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{rd(rd(op_l(x3,x2,x1),x3),x1)}
\end{aligned}
$$

Lemma 2562: $rd(op_r(x2,x1,x3),mult(x1,x2)) = rd(x3,mult(x1,op_l(x3,x1,x2)))$

$$
\begin{aligned}
& \underbrace{rd(op_r(x2,x1,x3),mult(x1,x2))} \\
= & \quad \text{by Lemma 2561 LR with } \{x3 \leftarrow x3, x1 \leftarrow x1, x2 \leftarrow x2\} \\
& \overbrace{rd(\underbrace{rd(op_l(x3,x2,x1),x3)},x1)} \\
= & \quad \text{by Lemma 2449 LR with } \{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x3\} \\
& \underbrace{rd(\overbrace{rd(x3,op_l(x3,x1,x2))},x1)} \\
= & \quad \text{by Lemma 2356 LR with } \{x4 \leftarrow x1, x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x3\} \\
& \overbrace{rd(x3,mult(x1,op_l(x3,x1,x2)))}
\end{aligned}
$$

Lemma 2563: $rd(x1, rd(op_r(x2, x1, x3), x2)) = op_r(x1, x2, x3)$

$\underbrace{rd(x1, rd(op_r(x2, x1, x3), x2))}$

$=$ by Lemma 2545 RL with $\{x4 \leftarrow x3, x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{\underbrace{rd(mult(x1, x2), op_r(x2, x1, x3))}}$

$=$ by Lemma 10 RL with $\{x2 \leftarrow mult(x1, x2), x1 \leftarrow op_r(x2, x1, x3)\}$

$\overbrace{i(\underbrace{rd(op_r(x2, x1, x3), mult(x1, x2))})}$

$=$ by Lemma 2562 LR with $\{x3 \leftarrow x3, x1 \leftarrow x1, x2 \leftarrow x2\}$

$i(\overbrace{\underbrace{rd(x3, mult(x1, op_l(x3, x1, x2)))}})$

$=$ by Lemma 2453 LR with $\{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x3\}$

$i(\overbrace{\underbrace{i(op_r(x1, x2, x3))}})$

$=$ by Lemma 2428 RL with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{i(\overbrace{op_r(i(x1), x2, x3)})}$

$=$ by Lemma 2428 RL with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow i(x1)\}$

$\overbrace{op_r(\underbrace{i(i(x1))}, x2, x3)}$

$=$ by Lemma 3 LR with $\{x1 \leftarrow x1\}$

$op_r(\overbrace{x1}, x2, x3)$

Lemma 2564: $op_r(x1, op_r(x2, x3, x4), x2) = rd(x1, rd(op_r(x2, x3, x4), op_l(op_r(x2, x3, x4), x1, x2)))$

$\underbrace{op_r(x1, op_r(x2, x3, x4), x2)}$

$=$ by Lemma 2563 RL with $\{x3 \leftarrow x2, x2 \leftarrow op_r(x2, x3, x4), x1 \leftarrow x1\}$

$\overbrace{rd(x1, rd(\underbrace{op_r(op_r(x2, x3, x4), x1, x2)}, op_r(x2, x3, x4)))}$

$=$ by Lemma 739 LR with $\{x4 \leftarrow x1, x3 \leftarrow x4, x2 \leftarrow x3, x1 \leftarrow x2\}$

$rd(x1, \underbrace{rd(\overbrace{op_l(op_r(x2, x3, x4), x2, x1)}, op_r(x2, x3, x4)))}$

$=$ by Lemma 2449 LR with $\{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow op_r(x2, x3, x4)\}$

$rd(x1, \overbrace{rd(op_r(x2, x3, x4), op_l(op_r(x2, x3, x4), x1, x2))})$

Lemma 2565: $op_r(x1, op_r(x2, x3, x4), x2) = op_r(x1, x2, op_r(x2, x3, x4))$

$\underbrace{op_r(x1, op_r(x2, x3, x4), x2)}$

$=$ by Lemma 2564 LR with $\{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{\underbrace{rd(x1, rd(op_r(x2, x3, x4), op_l(op_r(x2, x3, x4), x1, x2)))}}$

$=$ by Lemma 2425 LR with $\{x3 \leftarrow x2, x2 \leftarrow op_r(x2, x3, x4), x1 \leftarrow x1\}$

$\overbrace{op_r(x1, x2, op_r(x2, x3, x4))}$

Lemma 2566: $op_r(x1, x2, i(x3)) = rd(x1, rd(x2, op_r(x2, x1, x3)))$

$op_r(\underbrace{x1}, x2, i(x3))$

$=$ by Lemma 3 RL with $\{x1 \leftarrow x1\}$

$\underbrace{op_r(\overbrace{i(i(x1))}, x2, i(x3))}$

$=$ by Lemma 2427 RL with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow i(i(x1))\}$

$\overbrace{\underbrace{op_r(i(i(x1)), i(x2), x3)}}$

$=$ by Lemma 2428 LR with $\{x3 \leftarrow x3, x2 \leftarrow i(x2), x1 \leftarrow i(x1)\}$

$\overbrace{\underbrace{i(op_r(i(x1), i(x2), x3))}}$

$=$ by Lemma 2453 RL with $\{x3 \leftarrow i(x2), x2 \leftarrow i(x1), x1 \leftarrow x3\}$

$\overbrace{\underbrace{rd(x3, mult(i(x1), op_l(x3, i(x1), i(x2))))}}$

$=$ by Lemma 2562 RL with $\{x3 \leftarrow x3, x1 \leftarrow i(x1), x2 \leftarrow i(x2)\}$

$\overbrace{rd(op_r(i(x2), i(x1), x3), \underbrace{mult(i(x1), i(x2))})}$

$=$ by Axiom 11 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$rd(\underbrace{op_r(i(x2), i(x1), x3)}, \overbrace{i(mult(x1, x2))})$

$=$ by Lemma 2428 LR with $\{x3 \leftarrow x3, x2 \leftarrow i(x1), x1 \leftarrow x2\}$

$\underbrace{rd(\overbrace{i(op_r(x2, i(x1), x3))}, i(mult(x1, x2)))}$

$=$ by Lemma 23 LR with $\{x2 \leftarrow mult(x1, x2), x1 \leftarrow op_r(x2, i(x1), x3)\}$

$\overbrace{\underbrace{rd(mult(x1, x2), op_r(x2, i(x1), x3))}}$

$=$ by Lemma 2545 LR with $\{x4 \leftarrow x3, x3 \leftarrow i(x1), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{rd(x1, rd(\underbrace{op_r(x2, i(x1), x3)}, x2))}$

$=$ by Lemma 2427 LR with $\{x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow x2\}$

$rd(x1, \underbrace{rd(\overbrace{op_r(x2, x1, i(x3))}, x2)})$

$=$ by Lemma 2458 LR with $\{x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow x2\}$

$rd(x1, \overbrace{rd(x2, op_r(x2, x1, x3))})$

Lemma 2567: $mult(rd(x1, x2), op_r(x2, x1, x3)) = op_r(x1, x2, i(x3))$

$\underbrace{mult(rd(x1, x2), op_r(x2, x1, x3))}$

$=$ by Lemma 2269 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{\underbrace{rd(x1, rd(x2, op_r(x2, x1, x3)))}}$

$=$ by Lemma 2566 RL with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_r(x1, x2, i(x3))}$

Lemma 2568: $op_r(mult(x1,x2),x1,i(i(x3))) = mult(x1,op_r(x2,x1,x3))$

$\overbrace{op_r(mult(x1,x2),x1,i(i(x3)))}$

$=$ by Lemma 2427 RL with $\{x3 \leftarrow i(x3), x2 \leftarrow x1, x1 \leftarrow mult(x1,x2)\}$

$\overbrace{op_r(mult(x1,x2),i(x1),i(x3))}$

$=$ by Lemma 12 RL with $\{x2 \leftarrow op_r(mult(x1,x2),i(x1),i(x3)), x1 \leftarrow mult(x1,mult(x2,x1))\}$

$\overbrace{mult(mult(x1,mult(x2,x1)),mult(i(mult(x1,mult(x2,x1))),op_r(mult(x1,x2),i(x1),i(x3))))}$

$=$ by Axiom 6 RL with $\{x3 \leftarrow mult(i(mult(x1,mult(x2,x1))),op_r(mult(x1,x2),i(x1),i(x3))), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{mult(x1,mult(x2,mult(x1,mult(\underbrace{i(mult(x1,mult(x2,x1)))}, op_r(mult(x1,x2),i(x1),i(x3))))))}$

$=$ by Lemma 59 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$mult(x1,mult(x2,mult(x1,\underbrace{mult(\overbrace{rd(i(x1),mult(x1,x2))}, op_r(mult(x1,x2),i(x1),i(x3))))}))$

$=$ by Lemma 2567 LR with $\{x3 \leftarrow i(x3), x2 \leftarrow mult(x1,x2), x1 \leftarrow i(x1)\}$

$mult(x1,mult(x2,\underbrace{mult(x1,\overbrace{op_r(i(x1),mult(x1,x2),i(i(x3)))})}))$

$=$ by Lemma 2434 LR with $\{x3 \leftarrow i(i(x3)), x2 \leftarrow mult(x1,x2), x1 \leftarrow x1\}$

$mult(x1,mult(x2,\overbrace{rd(x1,op_r(x1,mult(x1,x2),i(i(x3))))}))$

$=$ by Lemma 2460 LR with $\{x3 \leftarrow i(x3), x2 \leftarrow mult(x1,x2), x1 \leftarrow x1\}$

$mult(x1,\underbrace{mult(x2,\overbrace{rd(op_r(x1,mult(x1,x2),i(x3)),x1)})})$

$=$ by Lemma 2541 LR with $\{x4 \leftarrow i(x3), x3 \leftarrow mult(x1,x2), x1 \leftarrow x1, x2 \leftarrow x2\}$

$mult(x1,\overbrace{rd(x2,rd(x1,\underbrace{op_r(x1,mult(x1,x2),i(x3))}))})$

$=$ by Lemma 2431 LR with $\{x3 \leftarrow i(x3), x2 \leftarrow x2, x1 \leftarrow x1\}$

$mult(x1,\underbrace{rd(x2,rd(x1,\overbrace{op_r(x1,x2,i(x3))}))})$

$=$ by Lemma 2566 RL with $\{x3 \leftarrow i(x3), x2 \leftarrow x1, x1 \leftarrow x2\}$

$mult(x1,\overbrace{op_r(x2,x1,\underbrace{i(i(x3))})})$

$=$ by Lemma 3 LR with $\{x1 \leftarrow x3\}$

$mult(x1,op_r(x2,x1,\overbrace{x3}))$

Lemma 2569: $mult(x1,op_r(x2,x1,x3)) = op_r(mult(x1,x2),x1,x3)$

$\overbrace{mult(x1,op_r(x2,x1,x3))}$

$=$ by Lemma 2568 RL with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_r(mult(x1,x2),x1,\underbrace{i(i(x3))})}$

$=$ by Lemma 3 LR with $\{x1 \leftarrow x3\}$

$op_r(mult(x1,x2),x1,\overbrace{x3})$

Lemma 2570: $mult(op_r(x1, x2, x3), x2) = op_r(mult(x1, x2), x2, x3)$

$\underbrace{mult(op_r(x1, x2, x3), x2)}$

$=$ by Lemma 2416 LR with $\{x3 \leftarrow x3, x1 \leftarrow x2, x2 \leftarrow x1\}$

$\overbrace{op_l(mult(x2, op_r(x1, x2, x3)), x2, rd(mult(x2, x1), x2))}$

$=$ by Lemma 2043 RL with $\{x4 \leftarrow x2, x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow mult(x2, op_r(x1, x2, x3))\}$

$\overbrace{op_l(mult(x2, op_r(x1, x2, x3)), x2, rd(x1, mult(x2, x2)))}$

$=$ by Lemma 1351 LR with $\{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow mult(x2, op_r(x1, x2, x3))\}$

$\overbrace{op_l(\underbrace{mult(x2, op_r(x1, x2, x3))}, x2, x1)}$

$=$ by Lemma 2569 LR with $\{x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow x2\}$

$\underbrace{op_l(\overbrace{op_r(mult(x2, x1), x2, x3)}, x2, x1)}$

$=$ by Axiom 18 RL with $\{x5 \leftarrow x3, x4 \leftarrow x2, x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow mult(x2, x1)\}$

$\overbrace{op_r(\underbrace{op_l(mult(x2, x1), x2, x1)}, x2, x3)}$

$=$ by Lemma 435 LR with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$op_r(\overbrace{mult(x1, x2)}, x2, x3)$

Lemma 2571: $rd(op_r(x1, x2, x3), x2) = op_r(rd(x1, x2), x2, x3)$

$rd(op_r(\underbrace{x1}, x2, x3), x2)$

$=$ by Axiom 4 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$rd(op_r(\overbrace{mult(rd(x1, x2), x2)}, x2, x3), x2)$

$=$ by Lemma 435 RL with $\{x2 \leftarrow rd(x1, x2), x1 \leftarrow x2\}$

$rd(\underbrace{op_r(\overbrace{op_l(mult(x2, rd(x1, x2)), x2, rd(x1, x2))}, x2, x3)}, x2)$

$=$ by Axiom 18 LR with $\{x5 \leftarrow x3, x4 \leftarrow x2, x3 \leftarrow rd(x1, x2), x2 \leftarrow x2, x1 \leftarrow mult(x2, rd(x1, x2))\}$

$\overbrace{rd(\underbrace{op_l(op_r(mult(x2, rd(x1, x2)), x2, x3), x2, rd(x1, x2))}, x2)}$

$=$ by Lemma 2032 RL with $\{x3 \leftarrow rd(x1, x2), x2 \leftarrow x2, x1 \leftarrow op_r(mult(x2, rd(x1, x2)), x2, x3)\}$

$\underbrace{rd(\overbrace{op_l(op_r(mult(x2, rd(x1, x2)), x2, x3), x2, mult(x2, rd(x1, x2)))}, x2)}$

$=$ by Lemma 1971 LR with $\{x4 \leftarrow x2, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow mult(x2, rd(x1, x2))\}$

$\overbrace{mult(i(x2), \underbrace{op_r(mult(x2, rd(x1, x2)), x2, x3)})}$

$=$ by Lemma 2569 RL with $\{x3 \leftarrow x3, x2 \leftarrow rd(x1, x2), x1 \leftarrow x2\}$

$\underbrace{mult(i(x2), \overbrace{mult(x2, op_r(rd(x1, x2), x2, x3))})}$

$=$ by Axiom 2 RL with $\{x2 \leftarrow x2, x1 \leftarrow op_r(rd(x1, x2), x2, x3)\}$

$\overbrace{op_r(rd(x1, x2), x2, x3)}$

Lemma 2572: $op_r(rd(x1,x2),x2,rd(x3,x1)) = mult(i(x2),op_r(x1,x2,x3))$

$$
\begin{aligned}
& \underbrace{op_r(rd(x1,x2),x2,rd(x3,x1))} \\
= & \quad \text{by Lemma 2571 RL with } \{x3 \leftarrow rd(x3,x1), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{rd(\underbrace{op_r(x1,x2,rd(x3,x1))},x2)} \\
= & \quad \text{by Lemma 2497 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{rd(\overbrace{op_l(op_r(x1,x2,x3),x2,x1)},x2)} \\
= & \quad \text{by Lemma 1971 LR with } \{x4 \leftarrow x2, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{mult(i(x2),op_r(x1,x2,x3))}
\end{aligned}
$$

Lemma 2573: $mult(op_l(x1,x1,op_r(x1,x2,x3)),mult(op_r(i(x1),x2,x3),x2)) = op_l(op_r(x2,i(x1),i(x3)),op_r(i(x1),x2,x3),x1)$

$$
\begin{aligned}
& mult(\underbrace{op_l(x1,x1,op_r(x1,x2,x3))},mult(op_r(i(x1),x2,x3),x2)) \\
= & \quad \text{by Lemma 2506 RL with } \{x5 \leftarrow x1, x4 \leftarrow x3, x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x1\} \\
& mult(\overbrace{op_l(\underbrace{x1},op_r(i(x1),x2,x3),x1)},mult(op_r(i(x1),x2,x3),x2)) \\
= & \quad \text{by Lemma 3 RL with } \{x1 \leftarrow x1\} \\
& \underbrace{mult(op_l(\overbrace{i(i(x1))},op_r(i(x1),x2,x3),x1),mult(op_r(i(x1),x2,x3),x2))} \\
= & \quad \text{by Lemma 2185 RL with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow op_r(i(x1),x2,x3)\} \\
& \overbrace{op_l(\underbrace{mult(rd(op_r(i(x1),x2,x3),i(x1)),x2)},op_r(i(x1),x2,x3),x1)} \\
= & \quad \text{by Lemma 2149 RL with } \{x3 \leftarrow x3, x2 \leftarrow i(x1), x1 \leftarrow x2\} \\
& op_l(\underbrace{\overbrace{mult(rd(x2,i(x1)),op_r(i(x1),x2,x3))}},op_r(i(x1),x2,x3),x1) \\
= & \quad \text{by Lemma 2269 LR with } \{x3 \leftarrow x3, x2 \leftarrow i(x1), x1 \leftarrow x2\} \\
& op_l(\underbrace{\overbrace{rd(x2,rd(i(x1),op_r(i(x1),x2,x3)))}},op_r(i(x1),x2,x3),x1) \\
= & \quad \text{by Lemma 2566 RL with } \{x3 \leftarrow x3, x2 \leftarrow i(x1), x1 \leftarrow x2\} \\
& op_l(\overbrace{op_r(x2,i(x1),i(x3))},op_r(i(x1),x2,x3),x1)
\end{aligned}
$$

Lemma 2574: $mult(x1,op_r(mult(i(x1),x2),x2,x3)) = op_l(op_r(x2,i(x1),i(x3)),op_r(i(x1),x2,x3),x1)$

$$
\begin{aligned}
& mult(x1,\underbrace{op_r(mult(i(x1),x2),x2,x3)}) \\
= & \quad \text{by Lemma 2570 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow i(x1)\} \\
& mult(\underbrace{x1},\overbrace{mult(op_r(i(x1),x2,x3),x2)}) \\
= & \quad \text{by Lemma 1444 RL with } \{x4 \leftarrow x3, x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x1\} \\
& \underbrace{mult(\overbrace{op_l(x1,x1,op_r(x1,x2,x3))},mult(op_r(i(x1),x2,x3),x2))} \\
= & \quad \text{by Lemma 2573 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(op_r(x2,i(x1),i(x3)),op_r(i(x1),x2,x3),x1)}
\end{aligned}
$$

Lemma 2575: $mult(op_r(rd(x2,x1),x2,x3),x1) = op_l(op_r(x2,i(x1),i(x3)),x1,op_r(x1,x2,x3))$

$\overbrace{mult(op_r(rd(x2,x1),x2,x3),x1)}$

$=$ by Lemma 1967 RL with $\{x4 \leftarrow x3, x3 \leftarrow x2, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{mult(x1,\overbrace{op_r(mult(i(x1),x2),x2,x3)})}$

$=$ by Lemma 2574 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{\overbrace{op_l(op_r(x2,i(x1),i(x3)),op_r(i(x1),x2,x3),x1)}}$

$=$ by Lemma 2506 LR with $\{x5 \leftarrow x1, x4 \leftarrow x3, x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow op_r(x2,i(x1),i(x3))\}$

$\overbrace{op_l(op_r(x2,i(x1),i(x3)),x1,op_r(x1,x2,x3))}$

Lemma 2576: $mult(op_r(rd(x2,x1),x2,x3),x1) = op_r(x2,x1,i(i(x3)))$

$\overbrace{mult(op_r(rd(x2,x1),x2,x3),x1)}$

$=$ by Lemma 2575 LR with $\{x3 \leftarrow x3, x1 \leftarrow x1, x2 \leftarrow x2\}$

$\overbrace{\overbrace{op_l(op_r(x2,i(x1),i(x3)),x1,op_r(x1,x2,x3))}}$

$=$ by Lemma 1444 LR with $\{x4 \leftarrow x3, x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow op_r(x2,i(x1),i(x3))\}$

$\overbrace{\overbrace{op_r(x2,i(x1),i(x3))}}$

$=$ by Lemma 2427 LR with $\{x3 \leftarrow i(x3), x2 \leftarrow x1, x1 \leftarrow x2\}$

$\overbrace{op_r(x2,x1,i(i(x3)))}$

Lemma 2577: $op_r(op_l(x1,x1,x2),rd(x1,x2),i(x3)) = mult(rd(op_l(x1,x1,x2),rd(x1,x2)),op_r(rd(x1,x2),x1,x3))$

$\overbrace{op_r(op_l(x1,x1,x2),rd(x1,x2),i(x3))}$

$=$ by Lemma 2567 RL with $\{x3 \leftarrow x3, x2 \leftarrow rd(x1,x2), x1 \leftarrow op_l(x1,x1,x2)\}$

$\overbrace{mult(rd(\underbrace{op_l(x1,x1,x2)}),rd(x1,x2)),op_r(rd(x1,x2),\underbrace{op_l(x1,x1,x2)},x3))}$

$=$ by Lemma 606 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$mult(rd(\underbrace{op_l(x1,x1,x2)}),rd(x1,x2)),op_r(rd(x1,x2),\overbrace{\underbrace{op_l(x1,x2,i(x1))}},x3))$

$=$ by Lemma 1317 RL with $\{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x1\}$

$mult(rd(\underbrace{op_l(x1,x1,x2)}),rd(x1,x2)),\underbrace{op_r(rd(x1,x2),\overbrace{op_l(x1,rd(x1,x2),x1)},x3)})$

$=$ by Lemma 1944 RL with $\{x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow rd(x1,x2)\}$

$mult(rd(\underbrace{op_l(x1,x1,x2)}),rd(x1,x2)),\overbrace{op_r(rd(x1,x2),x1,x3)})$

Lemma 2578: $op_r(i(rd(x2,x3)),x1,mult(x3,x1)) = rd(rd(mult(x1,x3),x2),x1)$

$$
\begin{aligned}
& \overbrace{op_r(i(rd(x2,x3)),x1,mult(x3,x1))} \\
= \quad & \text{by Lemma 2512 RL with } \{x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow i(rd(x2,x3))\} \\
& \overbrace{op_r(\underbrace{op_r(i(rd(x2,x3)),x1,x1)},x1,x3)} \\
= \quad & \text{by Lemma 354 RL with } \{x2 \leftarrow rd(x2,x3), x1 \leftarrow x1\} \\
& \underbrace{op_r(\overbrace{rd(rd(x1,rd(x2,x3)),x1)},x1,x3)} \\
= \quad & \text{by Lemma 2571 RL with } \{x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow rd(x1,rd(x2,x3))\} \\
& \overbrace{rd(\underbrace{op_r(rd(x1,rd(x2,x3)),x1,x3)},x1)} \\
= \quad & \text{by Lemma 2463 LR with } \{x1 \leftarrow x3, x2 \leftarrow x2, x3 \leftarrow x1\} \\
& rd(\overbrace{rd(mult(x1,x3),x2)},x1)
\end{aligned}
$$

Lemma 2579: $rd(op_r(x1,x2,x3),mult(i(op_t(x2,rd(x2,rd(x1,op_r(x1,x2,x3))))),x1)) = rd(x2,rd(op_r(x2,x1,x3),op_t(x2,rd(op_r$

$$
\begin{aligned}
& rd(op_r(x1,x2,x3),mult(i(\underbrace{op_t(x2,rd(x2,rd(x1,op_r(x1,x2,x3))))}),x1)) \\
= \quad & \text{by Lemma 1753 RL with } \{x4 \leftarrow x1, x3 \leftarrow op_r(x1,x2,x3), x2 \leftarrow x2, x1 \leftarrow x2\} \\
& \underbrace{rd(op_r(x1,x2,x3),mult(i(\overbrace{op_t(x2,mult(x2,rd(op_r(x1,x2,x3),x1)))}),x1))} \\
= \quad & \text{by Lemma 2542 RL with } \{x4 \leftarrow i(op_t(x2,mult(x2,rd(op_r(x1,x2,x3),x1)))), x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{rd(rd(op_r(x1,x2,x3),x1),i(op_t(x2,\underbrace{mult(x2,rd(op_r(x1,x2,x3),x1))})))}} \\
= \quad & \text{by Lemma 38 RL with } \{x1 \leftarrow rd(op_r(x1,x2,x3),x1), x2 \leftarrow op_t(x2,mult(x2,rd(op_r(x1,x2,x3),x1)))\} \\
& \overbrace{\underbrace{rd(op_t(x2,mult(x2,rd(op_r(x1,x2,x3),x1))),i(rd(op_r(x1,x2,x3),x1)))}} \\
= \quad & \text{by Lemma 2023 RL with } \{x2 \leftarrow rd(op_r(x1,x2,x3),x1), x1 \leftarrow x2\} \\
& \overbrace{rd(x2,rd(\underbrace{op_l(x2,x2,rd(op_r(x1,x2,x3),x1))},mult(x2,rd(op_r(x1,x2,x3),x1))))} \\
= \quad & \text{by Lemma 606 RL with } \{x2 \leftarrow rd(op_r(x1,x2,x3),x1), x1 \leftarrow x2\} \\
& rd(x2,rd(\overbrace{op_l(x2,rd(op_r(x1,x2,x3),x1),i(x2))},mult(x2,rd(op_r(x1,x2,x3),x1)))) \\
= \quad & \text{by Lemma 1387 RL with } \{x3 \leftarrow rd(op_r(x1,x2,x3),x1), x2 \leftarrow x2, x1 \leftarrow x2\} \\
& rd(x2,rd(\overbrace{op_l(x2,x2,\underbrace{rd(op_t(x2,rd(op_r(x1,x2,x3),x1)),i(rd(op_r(x1,x2,x3),x1))))}},mult(x2,rd(op_r(x1,x2,x3),x1)))) \\
= \quad & \text{by Lemma 38 LR with } \{x1 \leftarrow rd(op_r(x1,x2,x3),x1), x2 \leftarrow op_t(x2,rd(op_r(x1,x2,x3),x1))\} \\
& rd(x2,\underbrace{rd(op_l(x2,x2,\overbrace{rd(rd(op_r(x1,x2,x3),x1),i(op_t(x2,rd(op_r(x1,x2,x3),x1)))))},mult(x2,rd(op_r(x1,x2,x3),x1)))}) \\
= \quad & \text{by Lemma 1470 RL with } \{x2 \leftarrow rd(op_r(x1,x2,x3),x1), x1 \leftarrow x2\} \\
& rd(x2,\overbrace{rd(x2,\underbrace{rd(rd(op_r(x1,x2,x3),x1),i(op_t(x2,rd(op_r(x1,x2,x3),x1))))})}) \\
= \quad & \text{by Lemma 38 RL with } \{x1 \leftarrow rd(op_r(x1,x2,x3),x1), x2 \leftarrow op_t(x2,rd(op_r(x1,x2,x3),x1))\} \\
& rd(x2,\underbrace{rd(x2,\overbrace{rd(op_t(x2,rd(op_r(x1,x2,x3),x1)),i(rd(op_r(x1,x2,x3),x1)))})}) \\
= \quad & \text{by Lemma 1651 LR with } \{x3 \leftarrow rd(op_r(x1,x2,x3),x1), x2 \leftarrow rd(op_r(x1,x2,x3),x1), x1 \leftarrow x2\} \\
& rd(x2,\overbrace{rd(\underbrace{rd(x2,rd(op_r(x1,x2,x3),x1))},op_t(x2,rd(op_r(x1,x2,x3),x1)))}) \\
= \quad & \text{by Lemma 2563 LR with } \{x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow x2\} \\
& rd(x2,rd(\overbrace{op_r(x2,x1,x3)},op_t(x2,rd(op_r(x1,x2,x3),x1))))
\end{aligned}
$$

Lemma 2580: $rd(op_r(x1,x2,x3),mult(i(x2),x1)) = rd(x2,rd(op_r(x2,x1,x3),x2))$

$rd(op_r(x1,x2,x3),mult(i(\underbrace{x2}),x1))$

$=$ by Lemma 2261 LR with $\{x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow x2\}$

$rd(op_r(x1,x2,x3),mult(i(\overbrace{op_l(x2,x2,rd(x1,op_r(x1,x2,x3)))}),x1))$

$=$ by Lemma 2268 RL with $\{x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow x2\}$

$\underbrace{rd(op_r(x1,x2,x3),mult(i(\overbrace{op_t(x2,rd(x2,rd(x1,op_r(x1,x2,x3))))}),x1))}$

$=$ by Lemma 2579 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{rd(x2,rd(op_r(x2,x1,x3),\underbrace{op_t(x2,rd(op_r(x1,x2,x3),x1))}))}$

$=$ by Lemma 2263 LR with $\{x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow x2\}$

$rd(x2,rd(op_r(x2,x1,x3),\overbrace{x2}))$

Lemma 2581: $op_r(op_r(x1,x2,x2),x2,i(x3)) = rd(op_r(x2,x1,x3),mult(i(op_r(x1,x2,x2)),x2))$

$\underbrace{op_r(op_r(x1,x2,x2),x2,i(x3))}$

$=$ by Lemma 2467 RL with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow op_r(x1,x2,x2)\}$

$\overbrace{\underbrace{rd(op_r(x1,x2,x2),rd(op_r(op_r(x1,x2,x2),x2,x3),op_r(x1,x2,x2)))}}$

$=$ by Lemma 2580 RL with $\{x3 \leftarrow x3, x2 \leftarrow op_r(x1,x2,x2), x1 \leftarrow x2\}$

$\overbrace{rd(\underbrace{op_r(x2,op_r(x1,x2,x2),x3)},mult(i(op_r(x1,x2,x2)),x2))}$

$=$ by Lemma 1943 LR with $\{x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow x2\}$

$rd(\overbrace{op_r(x2,x1,x3)},mult(i(op_r(x1,x2,x2)),x2))$

Lemma 2582: $op_r(x1,x2,mult(x2,op_l(x1,x3,x4))) = rd(mult(x2,x1),x2)$

$\underbrace{op_r(x1,x2,mult(x2,op_l(x1,x3,x4)))}$

$=$ by Lemma 2394 RL with $\{x4 \leftarrow x2, x3 \leftarrow op_l(x1,x3,x4), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_r(x1,x2,\underbrace{rd(op_l(x1,x3,x4),i(x2))})}$

$=$ by Lemma 10 RL with $\{x2 \leftarrow op_l(x1,x3,x4), x1 \leftarrow i(x2)\}$

$\underbrace{op_r(x1,x2,\overbrace{i(rd(i(x2),op_l(x1,x3,x4)))})}$

$=$ by Lemma 2566 LR with $\{x3 \leftarrow rd(i(x2),op_l(x1,x3,x4)), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{rd(x1,rd(x2,op_r(x2,x1,\underbrace{rd(i(x2),op_l(x1,x3,x4))})))}$

$=$ by Lemma 10 RL with $\{x2 \leftarrow i(x2), x1 \leftarrow op_l(x1,x3,x4)\}$

$rd(x1,rd(x2,\underbrace{op_r(x2,x1,\overbrace{i(rd(op_l(x1,x3,x4),i(x2)))})}))$

$=$ by Lemma 2554 RL with $\{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow x2\}$

$\underbrace{rd(x1,rd(x2,\overbrace{op_t(x2,x1)}))}$

$=$ by Lemma 398 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{rd(mult(x2,x1),x2)}$

Lemma 2583: $i(x3) = op_r(op_r(i(x3), x2, x1), x2, i(x1))$

$$
\begin{aligned}
& \overbrace{i(x3)} \\
= \quad & \text{by Lemma 9 RL with } \{x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{rd(x1, \underbrace{mult(x3, x1)})} \\
= \quad & \text{by Lemma 12 RL with } \{x2 \leftarrow mult(x3, x1), x1 \leftarrow x2\} \\
& \underbrace{rd(x1, \overbrace{mult(x2, mult(i(x2), mult(x3, x1)))})} \\
= \quad & \text{by Lemma 2530 RL with } \{x3 \leftarrow mult(i(x2), mult(x3, x1)), x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \overbrace{op_r(\underbrace{mult(i(x2), rd(x1, mult(i(x2), mult(x3, x1))))}, x2, x1)} \\
= \quad & \text{by Lemma 1155 RL with } \{x3 \leftarrow mult(x3, x1), x2 \leftarrow x1, x1 \leftarrow i(x2)\} \\
& op_r(\overbrace{\underbrace{mult(rd(mult(i(x2), x1), mult(x3, x1)), i(i(x2)))}}, x2, x1) \\
= \quad & \text{by Lemma 44 RL with } \{x3 \leftarrow i(x2), x2 \leftarrow mult(i(x2), x1), x1 \leftarrow mult(x3, x1)\} \\
& op_r(\overbrace{i(mult(\underbrace{rd(mult(x3, x1), mult(i(x2), x1))}, i(x2)))}, x2, x1) \\
= \quad & \text{by Lemma 676 RL with } \{x3 \leftarrow x1, x2 \leftarrow i(x2), x1 \leftarrow x3\} \\
& op_r(i(\underbrace{mult(\overbrace{op_r(rd(x3, i(x2)), i(x2), x1)}, i(x2)))}, x2, x1) \\
= \quad & \text{by Lemma 2570 LR with } \{x3 \leftarrow x1, x2 \leftarrow i(x2), x1 \leftarrow rd(x3, i(x2))\} \\
& op_r(i(\overbrace{op_r(\underbrace{mult(rd(x3, i(x2)), i(x2))}, i(x2), x1)}), x2, x1) \\
= \quad & \text{by Axiom 4 RL with } \{x2 \leftarrow i(x2), x1 \leftarrow x3\} \\
& op_r(\underbrace{i(op_r(\overbrace{x3}, i(x2), x1))}, x2, x1) \\
= \quad & \text{by Lemma 2428 RL with } \{x3 \leftarrow x1, x2 \leftarrow i(x2), x1 \leftarrow x3\} \\
& op_r(\overbrace{\underbrace{op_r(i(x3), i(x2), x1)}}, x2, x1) \\
= \quad & \text{by Lemma 2427 LR with } \{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow i(x3)\} \\
& \underbrace{op_r(\overbrace{op_r(i(x3), x2, i(x1))}, x2, x1)} \\
= \quad & \text{by Axiom 17 LR with } \{x5 \leftarrow x1, x4 \leftarrow x2, x3 \leftarrow i(x1), x2 \leftarrow x2, x1 \leftarrow i(x3)\} \\
& \overbrace{op_r(op_r(i(x3), x2, x1), x2, i(x1))}
\end{aligned}
$$

Lemma 2584: $x1 = op_r(op_r(x1, x2, x3), x2, i(x3))$

$$
\begin{aligned}
& \overbrace{x1} \\
= \quad & \text{by Lemma 3 RL with } \{x1 \leftarrow x1\} \\
& \overbrace{\underbrace{i(i(x1))}} \\
= \quad & \text{by Lemma 2583 LR with } \{x1 \leftarrow x3, x2 \leftarrow x2, x3 \leftarrow i(x1)\} \\
& \overbrace{op_r(op_r(\underbrace{i(i(x1))}, x2, x3), x2, i(x3))} \\
= \quad & \text{by Lemma 3 LR with } \{x1 \leftarrow x1\} \\
& op_r(op_r(\overbrace{x1}, x2, x3), x2, i(x3))
\end{aligned}
$$

Lemma 2585: $op_l(x1, op_r(x2, x3, x1), x3) = op_l(x1, x2, x3)$

$$
\begin{aligned}
& op_l(\underbrace{x1}, op_r(x2, x3, x1), x3) \\
= & \quad \text{by Lemma 3 RL with } \{x1 \leftarrow x1\} \\
& \underbrace{op_l(\overbrace{i(i(x1))}, op_r(x2, x3, x1), x3)} \\
= & \quad \text{by Lemma 2464 RL with } \{x3 \leftarrow x3, x2 \leftarrow i(x1), x1 \leftarrow op_r(x2, x3, x1)\} \\
& \overbrace{rd(op_r(x2, x3, x1), mult(i(x1), \underbrace{op_r(op_r(x2, x3, x1), x3, i(x1))}))} \\
= & \quad \text{by Lemma 2584 RL with } \{x3 \leftarrow x1, x2 \leftarrow x3, x1 \leftarrow x2\} \\
& \underbrace{rd(op_r(x2, x3, x1), mult(i(x1), \overbrace{x2}))} \\
= & \quad \text{by Lemma 2546 LR with } \{x3 \leftarrow x1, x2 \leftarrow x3, x1 \leftarrow x2\} \\
& \overbrace{\underbrace{mult(x1, rd(op_r(x2, x3, x1), x2))}} \\
= & \quad \text{by Lemma 2541 LR with } \{x4 \leftarrow x1, x3 \leftarrow x3, x1 \leftarrow x2, x2 \leftarrow x1\} \\
& \overbrace{\underbrace{rd(x1, rd(x2, op_r(x2, x3, x1)))}} \\
= & \quad \text{by Lemma 2437 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x2, x3)}
\end{aligned}
$$

Lemma 2586: $op_l(x1, x2, mult(x2, rd(x3, x1))) = op_l(x1, rd(x1, mult(x2, x3)), x2)$

$$
\begin{aligned}
& \underbrace{op_l(x1, x2, mult(x2, rd(x3, x1)))} \\
= & \quad \text{by Lemma 2031 RL with } \{x3 \leftarrow x2, x2 \leftarrow mult(x2, rd(x3, x1)), x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, \underbrace{i(mult(x2, rd(x3, x1)))}, x2)} \\
= & \quad \text{by Lemma 45 LR with } \{x3 \leftarrow x1, x2 \leftarrow x3, x1 \leftarrow x2\} \\
& \underbrace{op_l(x1, \overbrace{mult(i(x2), rd(x1, x3))}, x2)} \\
= & \quad \text{by Lemma 2585 RL with } \{x3 \leftarrow x2, x2 \leftarrow mult(i(x2), rd(x1, x3)), x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, \underbrace{op_r(mult(i(x2), rd(x1, x3)), x2, x1)}, x2)} \\
= & \quad \text{by Lemma 2530 LR with } \{x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow x2\} \\
& op_l(x1, \overbrace{rd(x1, mult(x2, x3))}, x2)
\end{aligned}
$$

Lemma 2587: $op_l(x1, x2, rd(x3, x1)) = op_l(x1, x2, rd(mult(x2, x3), x1))$

$$
\begin{aligned}
& \underbrace{op_l(x1, x2, rd(x3, x1))} \\
= & \quad \text{by Lemma 2032 RL with } \{x3 \leftarrow rd(x3, x1), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, x2, mult(x2, rd(x3, x1)))}} \\
= & \quad \text{by Lemma 2586 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, rd(x1, mult(x2, x3)), x2)}} \\
= & \quad \text{by Lemma 2052 RL with } \{x4 \leftarrow x1, x3 \leftarrow mult(x2, x3), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x2, rd(mult(x2, x3), x1))}
\end{aligned}
$$

Lemma 2588: $op_l(x1, x2, rd(mult(x3, x1), mult(x2, x1))) = op_l(x1, x2, op_r(x3, x1, x2))$

$\overbrace{op_l(x1, x2, rd(mult(x3, x1), mult(x2, x1)))}$

$=$ by Lemma 2043 LR with $\{x4 \leftarrow x1, x3 \leftarrow mult(x3, x1), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_l(x1, x2, rd(mult(x2, mult(x3, x1)), x1))}$

$=$ by Lemma 2446 RL with $\{x4 \leftarrow mult(x3, x1), x3 \leftarrow rd(mult(x2, mult(x3, x1)), x1), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_l(x1, x2, \underbrace{rd(rd(mult(x2, mult(x3, x1)), x1), op_t(x2, mult(x3, x1)))}}$

$=$ by Lemma 2007 LR with $\{x3 \leftarrow x1, x2 \leftarrow mult(x3, x1), x1 \leftarrow x2\}$

$\underbrace{op_l(x1, x2, \overbrace{rd(rd(mult(mult(x3, x1), x2), x1), x2)})}$

$=$ by Lemma 286 LR with $\{x3 \leftarrow rd(mult(mult(x3, x1), x2), x1), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_l(x1, x2, rd(mult(mult(x3, x1), x2), x1))}$

$=$ by Lemma 2473 RL with $\{x4 \leftarrow x1, x3 \leftarrow mult(mult(x3, x1), x2), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_l(x1, x2, rd(mult(mult(x3, x1), x2), op_t(x1, x2)))}$

$=$ by Lemma 286 RL with $\{x3 \leftarrow rd(mult(mult(x3, x1), x2), op_t(x1, x2)), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_l(x1, x2, rd(rd(mult(mult(x3, x1), x2), op_t(x1, x2)), x2))}$

$=$ by Lemma 2448 RL with $\{x3 \leftarrow x2, x2 \leftarrow rd(rd(mult(mult(x3, x1), x2), op_t(x1, x2)), x2), x1 \leftarrow x1\}$

$\overbrace{rd(x1, rd(op_r(rd(rd(mult(mult(x3, x1), x2), op_t(x1, x2)), x2), x2, x1), rd(rd(mult(mult(x3, x1), x2), op_t(x1, x2)), x2)))}$

$=$ by Lemma 2245 LR with $\{x4 \leftarrow x1, x3 \leftarrow x2, x2 \leftarrow rd(rd(mult(mult(x3, x1), x2), op_t(x1, x2)), x2), x1 \leftarrow x1\}$

$\overbrace{rd(\underbrace{rd(rd(mult(mult(x3, x1), x2), op_t(x1, x2)), x2)}, mult(i(x1), op_r(rd(rd(mult(mult(x3, x1), x2), op_t(x1, x2)), x2), x2, x1))}$

$=$ by Lemma 2584 LR with $\{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow rd(rd(mult(mult(x3, x1), x2), op_t(x1, x2)), x2)\}$

$\underbrace{rd(\overbrace{op_r(op_r(rd(rd(mult(mult(x3, x1), x2), op_t(x1, x2)), x2), x2, x1), x2, i(x1))}, mult(i(x1), op_r(rd(rd(mult(mult(x3, x1), x2$

$=$ by Lemma 2466 LR with $\{x3 \leftarrow i(x1), x2 \leftarrow x2, x1 \leftarrow op_r(rd(rd(mult(mult(x3, x1), x2), op_t(x1, x2)), x2), x2, x1)\}$

$\overbrace{op_l(\underbrace{i(i(x1))}, x2, op_r(rd(rd(mult(mult(x3, x1), x2), op_t(x1, x2)), x2), x2, x1))}$

$=$ by Lemma 3 LR with $\{x1 \leftarrow x1\}$

$op_l(\overbrace{x1}, x2, \underbrace{op_r(rd(rd(mult(mult(x3, x1), x2), op_t(x1, x2)), x2), x2, x1)})$

$=$ by Lemma 1965 LR with $\{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow mult(mult(x3, x1), x2)\}$

$op_l(x1, x2, \overbrace{rd(\underbrace{mult(mult(x3, x1), x2), mult(x1, x2)})})$

$=$ by Axiom 16 RL with $\{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x3\}$

$op_l(x1, x2, \overbrace{op_r(x3, x1, x2)})$

Lemma 2589: $op_r(x1, mult(rd(x1, x2), x3), x2) = op_r(x1, rd(x3, x2), x2)$

$\underbrace{op_r(x1, mult(rd(x1, x2), x3), x2)}$

$=$ by Lemma 2425 RL with $\{x3 \leftarrow mult(rd(x1, x2), x3), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{rd(x1, rd(x2, \underbrace{op_l(x2, x1, mult(rd(x1, x2), x3))}))}$

$=$ by Lemma 2031 RL with $\{x3 \leftarrow x1, x2 \leftarrow mult(rd(x1, x2), x3), x1 \leftarrow x2\}$

$rd(x1, rd(x2, \overbrace{op_l(x2, \underbrace{i(mult(rd(x1, x2), x3))}, x1)}))$

$=$ by Lemma 33 RL with $\{x2 \leftarrow x3, x1 \leftarrow rd(x1, x2)\}$

$rd(x1, rd(x2, op_l(x2, \overbrace{\underbrace{op_t(rd(\underbrace{i(rd(x1, x2))}, x3), rd(x1, x2))}, x1})))$

$=$ by Lemma 1722 RL with $\{x3 \leftarrow x1, x2 \leftarrow rd(x1, x2), x1 \leftarrow rd(i(rd(x1, x2)), x3)\}$

$rd(x1, rd(x2, \underbrace{op_l(x2, \overbrace{op_t(rd(i(rd(x1, x2)), x3), op_t(rd(x1, x2), x1))}, x1)}))$

$=$ by Lemma 2031 RL with $\{x3 \leftarrow op_t(rd(i(rd(x1, x2)), x3), op_t(rd(x1, x2), x1)), x2 \leftarrow x1, x1 \leftarrow x2\}$

$rd(x1, rd(x2, \overbrace{\underbrace{op_l(x2, i(x1), op_t(rd(i(rd(x1, x2)), x3), op_t(rd(x1, x2), x1)))}}))$

$=$ by Lemma 2337 RL with $\{x2 \leftarrow x1, x3 \leftarrow rd(i(rd(x1, x2)), x3), x1 \leftarrow x2\}$

$rd(x1, rd(x2, \overbrace{op_l(x2, rd(\underbrace{i(rd(x1, x2))}, x3), x1)}))$

$=$ by Lemma 10 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$rd(x1, rd(x2, \underbrace{op_l(x2, rd(\overbrace{rd(x2, x1)}, x3), x1)}))$

$=$ by Lemma 2052 RL with $\{x4 \leftarrow rd(x2, x1), x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow x2\}$

$\underbrace{rd(x1, rd(x2, \overbrace{op_l(x2, x1, rd(x3, rd(x2, x1)))}))}$

$=$ by Lemma 2425 LR with $\{x3 \leftarrow rd(x3, rd(x2, x1)), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_r(x1, rd(x3, rd(x2, x1)), x2)}}$

$=$ by Lemma 2241 RL with $\{x4 \leftarrow x2, x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_r(x1, rd(x3, mult(i(x1), x2)), x2)}}$

$=$ by Lemma 2436 RL with $\{x4 \leftarrow x2, x3 \leftarrow x3, x2 \leftarrow mult(i(x1), x2), x1 \leftarrow x1\}$

$\overbrace{op_r(x1, \underbrace{rd(mult(i(x1), x2), x3)}, i(x2))}$

$=$ by Lemma 1487 RL with $\{x3 \leftarrow x1, x2 \leftarrow x3, x1 \leftarrow x2\}$

$op_r(x1, \overbrace{op_r(rd(rd(rd(x2, x3), x1), x1, i(x2))}, \underbrace{i(x2)})$

$=$ by Lemma 3 RL with $\{x1 \leftarrow i(x2)\}$

$\underbrace{op_r(x1, op_r(rd(rd(x2, x3), x1), x1, i(x2)), \overbrace{i(i(i(x2)))})}$

$=$ by Lemma 2566 LR with $\{x3 \leftarrow i(i(x2)), x2 \leftarrow op_r(rd(rd(x2, x3), x1), x1, i(x2)), x1 \leftarrow x1\}$

$\overbrace{rd(x1, rd(op_r(rd(rd(x2, x3), x1), x1, i(x2)), \underbrace{op_r(op_r(rd(rd(x2, x3), x1), x1, i(x2)), x1, i(i(x2)))}))}$

$=$ by Lemma 2584 RL with $\{x3 \leftarrow i(x2), x2 \leftarrow x1, x1 \leftarrow rd(rd(x2, x3), x1)\}$

$\underbrace{rd(x1, rd(op_r(rd(rd(x2, x3), x1), x1, i(x2)), \overbrace{rd(rd(x2, x3), x1)}))}$

$=$ by Lemma 2563 LR with $\{x3 \leftarrow i(x2), x2 \leftarrow rd(rd(x2, x3), x1), x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_r(x1, rd(rd(x2, x3), x1), i(x2))}}$

$=$ by Lemma 2436 LR with $\{x4 \leftarrow x2, x3 \leftarrow x1, x2 \leftarrow rd(x2, x3), x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_r(x1, rd(x1, rd(x2, x3)), x2)}}$

$=$ by Lemma 2432 LR with $\{x3 \leftarrow x2, x2 \leftarrow rd(x2, x3), x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_r(x1, rd(x2, x3), i(x2))}}$

$=$ by Lemma 2436 LR with $\{x4 \leftarrow x2, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_r(x1, rd(x3, x2), x2)}$

Lemma 2590: $op_r(x1, mult(x3, x2), x2) = op_r(x1, rd(x2, mult(x1, i(x3))), x2)$

$\underbrace{op_r(x1, mult(x3, x2), x2)}$

$=$ by Lemma 2225 LR with $\{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x1\}$

$\overbrace{op_r(x1, rd(x3, i(x2)), x2)}$

$=$ by Lemma 2436 RL with $\{x4 \leftarrow x2, x3 \leftarrow x3, x2 \leftarrow i(x2), x1 \leftarrow x1\}$

$\overbrace{op_r(x1, rd(i(x2), x3), i(x2))}$

$=$ by Lemma 2432 RL with $\{x3 \leftarrow x2, x2 \leftarrow rd(i(x2), x3), x1 \leftarrow x1\}$

$\overbrace{op_r(x1, \underbrace{rd(x1, rd(i(x2), x3))}, x2)}$

$=$ by Lemma 2436 RL with $\{x4 \leftarrow x2, x3 \leftarrow x1, x2 \leftarrow rd(i(x2), x3), x1 \leftarrow x1\}$

$\overbrace{op_r(x1, rd(\underbrace{rd(i(x2), x3)}, x1), i(x2))}$

$=$ by Lemma 2566 LR with $\{x3 \leftarrow x2, x2 \leftarrow rd(rd(i(x2), x3), x1), x1 \leftarrow x1\}$

$\overbrace{rd(x1, rd(\underbrace{rd(rd(i(x2), x3), x1)}, \underbrace{op_r(rd(rd(i(x2), x3), x1), x1, x2)}))}$

$=$ by Lemma 2584 LR with $\{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow rd(rd(i(x2), x3), x1)\}$

$\underbrace{rd(x1, rd(\overbrace{op_r(op_r(rd(rd(i(x2), x3), x1), x1, x2), x1, i(x2))}, \underbrace{op_r(rd(rd(i(x2), x3), x1), x1, x2)}))}$

$=$ by Lemma 2563 LR with $\{x3 \leftarrow i(x2), x2 \leftarrow op_r(rd(rd(i(x2), x3), x1), x1, x2), x1 \leftarrow x1\}$

$\overbrace{op_r(x1, \underbrace{op_r(rd(rd(i(x2), x3), x1), x1, x2)}, i(x2))}$

$=$ by Lemma 676 LR with $\{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow rd(i(x2), x3)\}$

$op_r(x1, \overbrace{rd(\underbrace{mult(rd(i(x2), x3), x2)}, mult(x1, x2))}, i(x2))$

$=$ by Lemma 21 LR with $\{x2 \leftarrow x3, x1 \leftarrow x2\}$

$\underbrace{op_r(x1, rd(\overbrace{i(x3)}, mult(x1, x2)), i(x2))}$

$=$ by Lemma 2436 LR with $\{x4 \leftarrow x2, x3 \leftarrow mult(x1, x2), x2 \leftarrow i(x3), x1 \leftarrow x1\}$

$\overbrace{op_r(x1, \underbrace{rd(mult(x1, x2), i(x3))}, x2)}$

$=$ by Lemma 2533 LR with $\{x4 \leftarrow x2, x3 \leftarrow i(x3), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_r(x1, rd(x2, mult(x1, i(x3))), x2)}$

Lemma 2591: $op_r(x1, mult(mult(x1, x2), x3), x2) = op_r(x1, mult(x3, x2), x2)$

$\underbrace{op_r(x1, mult(mult(x1, x2), x3), x2)}$

$=$ by Lemma 2250 LR with $\{x4 \leftarrow x2, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_r(x1, mult(mult(x2, x1), x3), x2)}$

$=$ by Lemma 2535 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_r(x1, \underbrace{rd(x2, rd(x1, x3))}, x2)}$

$=$ by Lemma 2240 LR with $\{x4 \leftarrow x2, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_r(x1, rd(x2, mult(x1, i(x3))), x2)}$

$=$ by Lemma 2590 RL with $\{x2 \leftarrow x2, x3 \leftarrow x3, x1 \leftarrow x1\}$

$\overbrace{op_r(x1, mult(x3, x2), x2)}$

Lemma 2592: $op_r(x1, op_r(i(x1), x2, x3), mult(x2, x3)) = op_r(x1, i(asoc(x1, x2, x3)), mult(x2, x3))$

$op_r(x1, \underbrace{op_r(i(x1), x2, x3)}, mult(x2, x3))$

$=$ by Lemma 2428 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$op_r(x1, \overbrace{i(\underbrace{op_r(x1, x2, x3)})}, mult(x2, x3))$

$=$ by Lemma 891 RL with $\{x3 \leftarrow x1, x2 \leftarrow x3, x1 \leftarrow x2\}$

$op_r(x1, \overbrace{rd(mult(x2, x3), \underbrace{mult(mult(x1, x2), x3)})}, \underbrace{mult(x2, x3)})$

$=$ by Lemma 441 RL with $\{x2 \leftarrow mult(x2, x3), x1 \leftarrow mult(mult(x1, x2), x3)\}$

$op_r(x1, rd(mult(x2, x3), mult(mult(x1, x2), x3)), \overbrace{rd(\underbrace{op_l(mult(mult(x1, x2), x3), mult(mult(x1, x2), x3), mult(x2, x3))}, rd($

$=$ by Lemma 1477 LR with $\{x1 \leftarrow mult(x2, x3), x2 \leftarrow mult(mult(x1, x2), x3)\}$

$op_r(x1, rd(mult(x2, x3), mult(mult(x1, x2), x3)), \overbrace{op_r(\underbrace{op_t(mult(x2, x3), i(mult(mult(x1, x2), x3)))}, rd(mult(x2, x3), mult($

$=$ by Lemma 358 RL with $\{x2 \leftarrow rd(mult(x2, x3), mult(mult(x1, x2), x3)), x1 \leftarrow op_t(mult(x2, x3), i(mult(mult(x1, x2), x3)))\}$

$\underbrace{op_r(x1, rd(mult(x2, x3), mult(mult(x1, x2), x3)), \overbrace{rd(rd(\underbrace{op_t(mult(x2, x3), i(mult(mult(x1, x2), x3)))}, i(rd(mult(x2, x3), m}}$

$=$ by Lemma 2218 RL with $\{x3 \leftarrow rd(op_t(mult(x2, x3), i(mult(mult(x1, x2), x3))), i(rd(mult(x2, x3), mult(mult(x1, x2), x3))))\}$

$\overbrace{op_r(x1, rd(mult(x2, x3), mult(mult(x1, x2), x3)), \underbrace{mult(rd(\underbrace{op_t(mult(x2, x3), i(mult(mult(x1, x2), x3)))}, i(rd(mult(x2, x3),}}$

$=$ by Axiom 4 RL with $\{x2 \leftarrow i(rd(mult(x2, x3), mult(mult(x1, x2), x3))), x1 \leftarrow op_t(mult(x2, x3), i(mult(mult(x1, x2), x3)))\}$

$\underbrace{op_r(x1, rd(mult(x2, x3), mult(mult(x1, x2), x3)), \overbrace{op_t(mult(x2, x3), i(\underbrace{mult(mult(x1, x2), x3)}))})}$

$=$ by Lemma 2017 LR with $\{x3 \leftarrow mult(mult(x1, x2), x3), x2 \leftarrow mult(x2, x3), x1 \leftarrow x1\}$

$\overbrace{op_r(x1, \underbrace{op_t(rd(mult(x2, x3), mult(mult(x1, x2), x3)), mult(mult(x1, x2), x3))}, mult(x2, x3))}$

$=$ by Lemma 14 LR with $\{x2 \leftarrow mult(mult(x1, x2), x3), x1 \leftarrow mult(x2, x3)\}$

$\underbrace{op_r(x1, \overbrace{mult(i(mult(mult(x1, x2), x3)), mult(x2, x3))}, mult(x2, x3))}$

$=$ by Lemma 2591 RL with $\{x3 \leftarrow i(mult(mult(x1, x2), x3)), x2 \leftarrow mult(x2, x3), x1 \leftarrow x1\}$

$\overbrace{op_r(x1, \underbrace{mult(mult(x1, mult(x2, x3)), i(mult(mult(x1, x2), x3)))}, mult(x2, x3))}$

$=$ by Lemma 964 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$op_r(x1, \overbrace{i(asoc(x1, x2, x3))}, mult(x2, x3))$

Lemma 2593: $op_r(x1, asoc(x1, x2, x3), rd(i(x2), x3)) = x1$

$$
\begin{aligned}
& \underbrace{op_r(x1, asoc(x1, x2, x3), rd(i(x2), x3))} \\
= \quad & \text{by Lemma 2400 RL with } \{x4 \leftarrow x3, x3 \leftarrow x2, x2 \leftarrow asoc(x1, x2, x3), x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_r(x1, asoc(x1, x2, x3), i(mult(x2, x3)))}} \\
= \quad & \text{by Lemma 2427 RL with } \{x3 \leftarrow mult(x2, x3), x2 \leftarrow asoc(x1, x2, x3), x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_r(x1, i(asoc(x1, x2, x3)), mult(x2, x3))}} \\
= \quad & \text{by Lemma 2592 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, \underbrace{op_r(i(x1), x2, x3)}, mult(x2, x3))} \\
= \quad & \text{by Lemma 2428 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{op_r(x1, \overbrace{i(op_r(x1, x2, x3))}, mult(x2, x3))} \\
= \quad & \text{by Lemma 2427 LR with } \{x3 \leftarrow mult(x2, x3), x2 \leftarrow op_r(x1, x2, x3), x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_r(x1, op_r(x1, x2, x3), i(mult(x2, x3)))}} \\
= \quad & \text{by Axiom 16 LR with } \{x3 \leftarrow i(mult(x2, x3)), x2 \leftarrow op_r(x1, x2, x3), x1 \leftarrow x1\} \\
& \overbrace{rd(\underbrace{mult(mult(x1, op_r(x1, x2, x3)), i(mult(x2, x3)))}, mult(op_r(x1, x2, x3), i(mult(x2, x3))))} \\
= \quad & \text{by Lemma 1442 LR with } \{x4 \leftarrow i(mult(x2, x3)), x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{rd(\overbrace{mult(x1, mult(op_r(x1, x2, x3), i(mult(x2, x3))))}, mult(op_r(x1, x2, x3), i(mult(x2, x3))))} \\
= \quad & \text{by Axiom 5 RL with } \{x2 \leftarrow mult(op_r(x1, x2, x3), i(mult(x2, x3))), x1 \leftarrow x1\} \\
& \overbrace{x1}
\end{aligned}
$$

Lemma 2594: $op_r(x1, asoc(x1, x2, x3), rd(x3, i(x2))) = rd(x1, rd(x1, x1))$

$$
\begin{aligned}
& op_r(x1, asoc(x1, x2, x3), \underbrace{rd(x3, i(x2))}) \\
= \quad & \text{by Lemma 10 RL with } \{x2 \leftarrow x3, x1 \leftarrow i(x2)\} \\
& \underbrace{op_r(x1, asoc(x1, x2, x3), \overbrace{i(rd(i(x2), x3))})} \\
= \quad & \text{by Lemma 2467 RL with } \{x3 \leftarrow rd(i(x2), x3), x2 \leftarrow asoc(x1, x2, x3), x1 \leftarrow x1\} \\
& \overbrace{rd(x1, rd(\underbrace{op_r(x1, asoc(x1, x2, x3), rd(i(x2), x3))}, x1))} \\
= \quad & \text{by Lemma 2593 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& rd(x1, rd(\overbrace{x1}, x1))
\end{aligned}
$$

Lemma 2595: $op_r(x1, asoc(x1, x2, x3), mult(x2, x3)) = op_r(x1, x1, x1)$

$$
\begin{aligned}
& \underbrace{op_r(x1, asoc(x1, x2, x3), mult(x2, x3))} \\
= \quad & \text{by Lemma 2394 RL with } \{x4 \leftarrow x2, x3 \leftarrow x3, x2 \leftarrow asoc(x1, x2, x3), x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_r(x1, asoc(x1, x2, x3), rd(x3, i(x2)))}} \\
= \quad & \text{by Lemma 2594 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{rd(x1, rd(x1, x1))}} \\
= \quad & \text{by Lemma 353 LR with } \{x2 \leftarrow x1, x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, x1, x1)}
\end{aligned}
$$

Lemma 2596: $op_r(x1, asoc(x1, x2, x3), mult(x2, x3)) = x1$

$$
\begin{aligned}
& \underbrace{op_r(x1, asoc(x1, x2, x3), mult(x2, x3))} \\
= \quad & \text{by Lemma 2595 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_r(x1, x1, x1)}} \\
= \quad & \text{by Lemma 601 LR with } \{x2 \leftarrow x1, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, x1, x1)}} \\
= \quad & \text{by Lemma 25 LR with } \{x2 \leftarrow x1, x1 \leftarrow x1\} \\
& \overbrace{x1}
\end{aligned}
$$

Lemma 2597: $op_l(mult(x1, x2), x3, asoc(x3, x1, x2)) = rd(mult(x1, x2), unit())$

$$
\begin{aligned}
& \underbrace{op_l(mult(x1, x2), x3, asoc(x3, x1, x2))} \\
= \quad & \text{by Lemma 2437 RL with } \{x3 \leftarrow asoc(x3, x1, x2), x2 \leftarrow x3, x1 \leftarrow mult(x1, x2)\} \\
& \overbrace{rd(mult(x1, x2), rd(x3, \underbrace{op_r(x3, asoc(x3, x1, x2), mult(x1, x2))}))} \\
= \quad & \text{by Lemma 2596 LR with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x3\} \\
& rd(mult(x1, x2), \underbrace{rd(x3, \overbrace{x3})}) \\
= \quad & \text{by Lemma 2 LR with } \{x1 \leftarrow x3\} \\
& rd(mult(x1, x2), \overbrace{unit()})
\end{aligned}
$$

Lemma 2598: $op_l(x1, x2, mult(x3, mult(x2, x1))) = op_l(x1, x2, mult(x1, x3))$

$op_l(x1, \underbrace{x2}, mult(x3, mult(x2, x1)))$

$=$ by Axiom 5 LR with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$\underbrace{op_l(x1, \overbrace{rd(mult(x2, x1), x1)}, mult(x3, mult(x2, x1)))}$

$=$ by Lemma 2031 RL with $\{x3 \leftarrow rd(mult(x2, x1), x1), x2 \leftarrow mult(x3, mult(x2, x1)), x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_l(x1, i(mult(x3, mult(x2, x1))), rd(mult(x2, x1), x1))}}$

$=$ by Lemma 2351 LR with $\{x3 \leftarrow mult(x2, x1), x2 \leftarrow x3, x1 \leftarrow x1\}$

$\overbrace{op_l(x1, rd(i(x3), mult(x2, x1)), \underbrace{rd(mult(x2, x1), x1)})}$

$=$ by Axiom 5 RL with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$\underbrace{op_l(x1, rd(i(x3), mult(x2, x1)), \overbrace{x2})}$

$=$ by Lemma 2052 RL with $\{x4 \leftarrow i(x3), x3 \leftarrow mult(x2, x1), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_l(x1, x2, rd(mult(x2, x1), i(x3)))}}$

$=$ by Lemma 2043 RL with $\{x4 \leftarrow i(x3), x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_l(x1, x2, rd(x1, mult(x2, i(x3))))}}$

$=$ by Lemma 2350 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_l(x1, x2, mult(mult(x1, x2), x3))}}$

$=$ by Lemma 2551 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_l(x1, x2, rd(x1, rd(x2, x3)))}}$

$=$ by Lemma 2331 LR with $\{x3 \leftarrow rd(x2, x3), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_l(x1, x2, mult(x1, \underbrace{i(rd(x2, x3))}))}$

$=$ by Lemma 10 LR with $\{x2 \leftarrow x3, x1 \leftarrow x2\}$

$op_l(x1, x2, \underbrace{mult(x1, \overbrace{rd(x3, x2)})})$

$=$ by Axiom 5 LR with $\{x2 \leftarrow x1, x1 \leftarrow mult(x1, rd(x3, x2))\}$

$\underbrace{op_l(x1, x2, \overbrace{rd(mult(mult(x1, rd(x3, x2)), x1), x1)})}$

$=$ by Lemma 2587 LR with $\{x3 \leftarrow mult(mult(x1, rd(x3, x2)), x1), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_l(x1, x2, rd(mult(x2, mult(mult(x1, rd(x3, x2)), x1)), x1))}}$

$=$ by Lemma 2043 RL with $\{x4 \leftarrow x1, x3 \leftarrow mult(mult(x1, rd(x3, x2)), x1), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_l(x1, x2, rd(mult(mult(x1, rd(x3, x2)), x1), mult(x2, x1)))}}$

$=$ by Lemma 2588 LR with $\{x3 \leftarrow mult(x1, rd(x3, x2)), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_l(x1, x2, \underbrace{op_r(mult(x1, rd(x3, x2)), x1, x2)})}$

$=$ by Lemma 2527 LR with $\{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x1\}$

$\underbrace{op_l(x1, x2, \overbrace{rd(mult(x1, x3), x2)})}$

$=$ by Lemma 286 LR with $\{x3 \leftarrow mult(x1, x3), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_l(x1, x2, mult(x1, x3))}$

Lemma 2599: $op_l(x1, mult(x2, x3), asoc(x2, x3, x1)) = x1$

$$
\begin{aligned}
& op_l(x1, mult(x2, x3), \underbrace{asoc(x2, x3, x1)}) \\
= & \quad \text{by Lemma 108 RL with } \{x3 \leftarrow x1, x2 \leftarrow x3, x1 \leftarrow x2\} \\
& \underbrace{op_l(x1, mult(x2, x3), \overbrace{mult(i(mult(x2, mult(x3, x1))), mult(mult(x2, x3), x1))})} \\
= & \quad \text{by Lemma 2598 LR with } \{x3 \leftarrow i(mult(x2, mult(x3, x1))), x2 \leftarrow mult(x2, x3), x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, mult(x2, x3), \underbrace{mult(x1, i(mult(x2, mult(x3, x1))))})} \\
= & \quad \text{by Lemma 2331 RL with } \{x3 \leftarrow mult(x2, mult(x3, x1)), x2 \leftarrow mult(x2, x3), x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, mult(x2, x3), rd(x1, \underbrace{mult(x2, mult(x3, x1))}))} \\
= & \quad \text{by Lemma 132 RL with } \{x3 \leftarrow x1, x2 \leftarrow x3, x1 \leftarrow x2\} \\
& op_l(x1, mult(x2, x3), rd(x1, \overbrace{mult(mult(x2, x3), \underbrace{op_l(x1, x3, x2)})})) \\
= & \quad \text{by Lemma 291 RL with } \{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \underbrace{op_l(x1, mult(x2, x3), rd(x1, mult(mult(x2, x3), \overbrace{op_l(x1, x3, mult(x2, x3))})))} \\
= & \quad \text{by Lemma 2043 LR with } \{x4 \leftarrow op_l(x1, x3, mult(x2, x3)), x3 \leftarrow x1, x2 \leftarrow mult(x2, x3), x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, mult(x2, x3), rd(mult(mult(x2, x3), x1), op_l(x1, x3, mult(x2, x3))))}} \\
= & \quad \text{by Lemma 2344 RL with } \{x2 \leftarrow x3, x3 \leftarrow mult(x2, x3), x1 \leftarrow x1\} \\
& \overbrace{x1}
\end{aligned}
$$

Lemma 2600: $op_r(x1, x2, op_r(x2, x3, x4)) = op_r(x1, x2, x2)$

$$
\begin{aligned}
& \underbrace{op_r(x1, x2, op_r(x2, x3, x4))} \\
= & \quad \text{by Lemma 2565 RL with } \{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_r(x1, op_r(x2, x3, x4), x2)}} \\
= & \quad \text{by Lemma 2495 RL with } \{x3 \leftarrow op_r(x2, x3, x4), x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \overbrace{\underbrace{rd(x2, mult(i(x1), op_l(x2, op_r(x2, x3, x4), x1)))}} \\
= & \quad \text{by Lemma 2532 LR with } \{x2 \leftarrow x1, x3 \leftarrow op_r(x2, x3, x4), x4 \leftarrow i(x1), x1 \leftarrow x2\} \\
& \overbrace{rd(op_l(x2, x1, op_r(x2, x3, x4)), mult(i(x1), \underbrace{x2}))} \\
= & \quad \text{by Lemma 2584 LR with } \{x3 \leftarrow x4, x2 \leftarrow x3, x1 \leftarrow x2\} \\
& rd(op_l(x2, x1, op_r(x2, x3, x4)), \underbrace{mult(i(x1), \overbrace{op_r(op_r(x2, x3, x4), x3, i(x4))})}) \\
= & \quad \text{by Lemma 1971 RL with } \{x4 \leftarrow x1, x3 \leftarrow i(x4), x2 \leftarrow x3, x1 \leftarrow op_r(x2, x3, x4)\} \\
& rd(op_l(x2, x1, op_r(x2, x3, x4)), \overbrace{rd(op_l(\underbrace{op_r(op_r(x2, x3, x4), x3, i(x4))}, x1, op_r(x2, x3, x4)), x1)}) \\
= & \quad \text{by Lemma 2584 RL with } \{x3 \leftarrow x4, x2 \leftarrow x3, x1 \leftarrow x2\} \\
& \underbrace{rd(op_l(x2, x1, op_r(x2, x3, x4)), rd(op_l(\overbrace{x2}, x1, op_r(x2, x3, x4)), x1))} \\
= & \quad \text{by Lemma 353 LR with } \{x2 \leftarrow x1, x1 \leftarrow op_l(x2, x1, op_r(x2, x3, x4))\} \\
& \overbrace{\underbrace{op_r(x1, op_l(x2, x1, op_r(x2, x3, x4)), op_l(x2, x1, op_r(x2, x3, x4)))}} \\
= & \quad \text{by Lemma 2133 LR with } \{x4 \leftarrow op_r(x2, x3, x4), x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, x2, x2)}
\end{aligned}
$$

Lemma 2601: $op_l(x1, x2, op_r(x1, x3, x4)) = op_l(x1, x2, x1)$

$$
\begin{aligned}
& \underbrace{op_l(x1, x2, op_r(x1, x3, x4))} \\
= \quad & \text{by Lemma 442 RL with } \{x2 \leftarrow op_l(x1, x2, op_r(x1, x3, x4)), x1 \leftarrow x2\} \\
& \overbrace{op_l(mult(x2, rd(op_l(\underbrace{x1}, x2, op_r(x1, x3, x4)), x2)), x2, op_l(x1, x2, op_r(x1, x3, x4)))} \\
= \quad & \text{by Lemma 2584 LR with } \{x3 \leftarrow x4, x2 \leftarrow x3, x1 \leftarrow x1\} \\
& op_l(mult(x2, \underbrace{rd(op_l(\overbrace{op_r(op_r(x1, x3, x4), x3, i(x4))}, x2, op_r(x1, x3, x4)), x2)}), x2, op_l(x1, x2, op_r(x1, x3, x4))) \\
= \quad & \text{by Lemma 1971 LR with } \{x4 \leftarrow x2, x3 \leftarrow i(x4), x2 \leftarrow x3, x1 \leftarrow op_r(x1, x3, x4)\} \\
& op_l(mult(x2, \overbrace{mult(i(x2), \underbrace{op_r(op_r(x1, x3, x4), x3, i(x4))})}), x2, op_l(x1, x2, op_r(x1, x3, x4))) \\
= \quad & \text{by Lemma 2584 RL with } \{x3 \leftarrow x4, x2 \leftarrow x3, x1 \leftarrow x1\} \\
& op_l(\underbrace{mult(x2, mult(i(x2), \overbrace{x1}))}, x2, op_l(x1, x2, op_r(x1, x3, x4))) \\
= \quad & \text{by Lemma 12 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \underbrace{op_l(\overbrace{x1}, x2, op_l(x1, x2, op_r(x1, x3, x4)))} \\
= \quad & \text{by Lemma 1819 LR with } \{x4 \leftarrow op_r(x1, x3, x4), x3 \leftarrow x2, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x2, x1)}
\end{aligned}
$$

Lemma 2602: $op_l(x1, op_r(x1, x3, x4), x2) = rd(x1, rd(op_r(x2, x1, x1), x2))$

$$
\begin{aligned}
& \underbrace{op_l(x1, op_r(x1, x3, x4), x2)} \\
= \quad & \text{by Lemma 2452 LR with } \{x4 \leftarrow x2, x3 \leftarrow x4, x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \underbrace{\overbrace{op_r(x1, x2, op_r(x1, x3, x4))}} \\
= \quad & \text{by Lemma 2563 RL with } \{x3 \leftarrow op_r(x1, x3, x4), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{rd(x1, rd(\underbrace{op_r(x2, x1, op_r(x1, x3, x4))}, x2))} \\
= \quad & \text{by Lemma 2600 LR with } \{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow x2\} \\
& rd(x1, rd(\overbrace{op_r(x2, x1, x1)}, x2))
\end{aligned}
$$

Lemma 2603: $op_r(x1, x2, rd(x2, x2)) = op_r(x1, x2, rd(op_r(x2, x3, x4), x2))$

$$
\begin{aligned}
& \underbrace{op_r(x1, x2, rd(x2, x2))} \\
= \quad & \text{by Lemma 2517 RL with } \{x3 \leftarrow x2, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{\overbrace{op_r(op_r(x1, x2, i(x2)), x2, x2)}} \\
= \quad & \text{by Lemma 2600 RL with } \{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow op_r(x1, x2, i(x2))\} \\
& \underbrace{\overbrace{op_r(op_r(x1, x2, i(x2)), x2, op_r(x2, x3, x4))}} \\
= \quad & \text{by Lemma 2517 LR with } \{x3 \leftarrow op_r(x2, x3, x4), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, x2, rd(op_r(x2, x3, x4), x2))}
\end{aligned}
$$

Lemma 2604: $x1 = op_r(x1, x2, rd(op_r(x2, x3, x4), x2))$

$$
\begin{aligned}
& \overbrace{x1} \\
= \quad & \text{by Lemma 6 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, x2, \underbrace{unit()}_{})} \\
= \quad & \text{by Lemma 2 RL with } \{x1 \leftarrow x2\} \\
& \underbrace{op_r(x1, x2, \overbrace{rd(x2, x2)}^{})}_{} \\
= \quad & \text{by Lemma 2603 LR with } \{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, x2, rd(op_r(x2, x3, x4), x2))}^{}
\end{aligned}
$$

Lemma 2605: $rd(rd(mult(i(x1), mult(x1, x2)), x3), i(x1)) = op_r(rd(mult(x1, x2), x3), i(x1), mult(x1, rd(x2, x1)))$

$$
\begin{aligned}
& \underbrace{rd(rd(mult(i(x1), mult(x1, x2)), x3), i(x1))}_{} \\
= \quad & \text{by Lemma 2578 RL with } \{x1 \leftarrow i(x1), x3 \leftarrow mult(x1, x2), x2 \leftarrow x3\} \\
& \overbrace{op_r(\underbrace{i(rd(x3, mult(x1, x2)))}_{}, i(x1), mult(mult(x1, x2), i(x1)))}^{} \\
= \quad & \text{by Lemma 10 LR with } \{x2 \leftarrow mult(x1, x2), x1 \leftarrow x3\} \\
& op_r(\overbrace{rd(mult(x1, x2), x3)}^{}, i(x1), \underbrace{mult(mult(x1, x2), i(x1))}_{}) \\
= \quad & \text{by Lemma 55 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_r(rd(mult(x1, x2), x3), i(x1), \overbrace{mult(x1, rd(x2, x1))}^{})
\end{aligned}
$$

Lemma 2606: $rd(rd(x3, x2), x1) = op_r(rd(x3, mult(x1, x2)), x1, i(mult(x1, rd(x2, x1))))$

$rd(rd(x3, \underbrace{x2}), x1)$

$=$ by Axiom 2 LR with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$\underbrace{rd(rd(x3, \overbrace{mult(i(x1), mult(x1, x2))}), x1)}$

$=$ by Lemma 40 RL with $\{x3 \leftarrow x3, x2 \leftarrow mult(i(x1), mult(x1, x2)), x1 \leftarrow x1\}$

$\overbrace{rd(i(x1), \underbrace{rd(mult(i(x1), mult(x1, x2)), x3)})}$

$=$ by Lemma 2463 RL with $\{x1 \leftarrow mult(x1, x2), x2 \leftarrow x3, x3 \leftarrow i(x1)\}$

$\underbrace{rd(i(x1), \overbrace{op_r(rd(i(x1), rd(x3, mult(x1, x2))), i(x1), mult(x1, x2))})}$

$=$ by Lemma 10 RL with $\{x2 \leftarrow i(x1), x1 \leftarrow op_r(rd(i(x1), rd(x3, mult(x1, x2))), i(x1), mult(x1, x2))\}$

$\overbrace{i(\underbrace{rd(op_r(rd(i(x1), rd(x3, mult(x1, x2))), i(x1), mult(x1, x2)), i(x1))})}$

$=$ by Lemma 2571 LR with $\{x3 \leftarrow mult(x1, x2), x2 \leftarrow i(x1), x1 \leftarrow rd(i(x1), rd(x3, mult(x1, x2)))\}$

$\underbrace{i(\overbrace{op_r(rd(rd(i(x1), rd(x3, mult(x1, x2))), i(x1)), i(x1), mult(x1, x2))})}$

$=$ by Lemma 2428 RL with $\{x3 \leftarrow mult(x1, x2), x2 \leftarrow i(x1), x1 \leftarrow rd(rd(i(x1), rd(x3, mult(x1, x2))), i(x1))\}$

$\overbrace{op_r(\underbrace{i(rd(rd(i(x1), rd(x3, mult(x1, x2))), i(x1)))}, i(x1), mult(x1, x2))}$

$=$ by Lemma 10 LR with $\{x2 \leftarrow i(x1), x1 \leftarrow rd(i(x1), rd(x3, mult(x1, x2)))\}$

$op_r(\overbrace{rd(i(x1), rd(i(x1), rd(x3, mult(x1, x2))))}, i(x1), mult(x1, x2))$

$=$ by Lemma 353 LR with $\{x2 \leftarrow rd(x3, mult(x1, x2)), x1 \leftarrow i(x1)\}$

$\underbrace{op_r(\overbrace{op_r(rd(x3, mult(x1, x2)), i(x1), i(x1))}, i(x1), mult(x1, x2))}$

$=$ by Lemma 2512 LR with $\{x3 \leftarrow mult(x1, x2), x2 \leftarrow i(x1), x1 \leftarrow rd(x3, mult(x1, x2))\}$

$\overbrace{op_r(rd(x3, mult(x1, x2)), i(x1), \underbrace{mult(mult(x1, x2), i(x1))})}$

$=$ by Lemma 55 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{op_r(rd(x3, mult(x1, x2)), i(x1), \overbrace{mult(x1, rd(x2, x1))})}$

$=$ by Lemma 2427 LR with $\{x3 \leftarrow mult(x1, rd(x2, x1)), x2 \leftarrow x1, x1 \leftarrow rd(x3, mult(x1, x2))\}$

$\overbrace{op_r(rd(x3, mult(x1, x2)), x1, i(mult(x1, rd(x2, x1))))}$

Lemma 2607: $rd(rd(x3, x2), x1) = op_r(rd(x3, mult(x1, x2)), x1, rd(rd(x1, x2), x1))$

$\underbrace{rd(rd(x3, x2), x1)}$

$=$ by Lemma 2606 LR with $\{x1 \leftarrow x1, x2 \leftarrow x2, x3 \leftarrow x3\}$

$\underbrace{\overbrace{op_r(rd(x3, mult(x1, x2)), x1, i(mult(x1, rd(x2, x1))))}}$

$=$ by Lemma 2400 LR with $\{x4 \leftarrow rd(x2, x1), x3 \leftarrow x1, x2 \leftarrow x1, x1 \leftarrow rd(x3, mult(x1, x2))\}$

$\overbrace{op_r(rd(x3, mult(x1, x2)), x1, \underbrace{rd(i(x1), rd(x2, x1))})}$

$=$ by Lemma 40 LR with $\{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x1\}$

$op_r(rd(x3, mult(x1, x2)), x1, \overbrace{rd(rd(x1, x2), x1)})$

Lemma 2608: $rd(rd(x3, x2), x1) = op_r(rd(x3, mult(x1, x2)), x1, i(x2))$

$\underbrace{rd(rd(x3, x2), x1)}$

$=$ by Lemma 2607 LR with $\{x1 \leftarrow x1, x2 \leftarrow x2, x3 \leftarrow x3\}$

$\overbrace{op_r(rd(x3, mult(x1, x2)), x1, \underbrace{rd(rd(x1, x2), x1)})}$

$=$ by Lemma 354 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{op_r(rd(x3, mult(x1, x2)), x1, \overbrace{op_r(i(x2), x1, x1)})}$

$=$ by Lemma 2391 LR with $\{x4 \leftarrow x1, x3 \leftarrow i(x2), x2 \leftarrow x1, x1 \leftarrow rd(x3, mult(x1, x2))\}$

$\overbrace{op_r(rd(x3, mult(x1, x2)), x1, i(x2))}$

Lemma 2609: $mult(rd(x1, op_r(x2, x3, x4)), x2) = rd(x1, rd(op_r(x2, x3, x4), x2))$

$mult(rd(x1, \underbrace{op_r(x2, x3, x4)}), x2)$

$=$ by Axiom 4 LR with $\{x2 \leftarrow x2, x1 \leftarrow op_r(x2, x3, x4)\}$

$mult(\underbrace{rd(x1, \overbrace{mult(rd(op_r(x2, x3, x4), x2), x2)})}, x2)$

$=$ by Lemma 1965 RL with $\{x3 \leftarrow x2, x2 \leftarrow rd(op_r(x2, x3, x4), x2), x1 \leftarrow x1\}$

$\underbrace{mult(\overbrace{op_r(rd(rd(x1, op_t(rd(op_r(x2, x3, x4), x2), x2)), x2), x2, rd(op_r(x2, x3, x4), x2))}, x2)}$

$=$ by Lemma 2570 LR with $\{x3 \leftarrow rd(op_r(x2, x3, x4), x2), x2 \leftarrow x2, x1 \leftarrow rd(rd(x1, op_t(rd(op_r(x2, x3, x4), x2), x2)), x2)\}$

$\overbrace{op_r(\underbrace{mult(rd(rd(x1, op_t(rd(op_r(x2, x3, x4), x2), x2)), x2), x2)}, x2, rd(op_r(x2, x3, x4), x2))}$

$=$ by Axiom 4 RL with $\{x2 \leftarrow x2, x1 \leftarrow rd(x1, op_t(rd(op_r(x2, x3, x4), x2), x2))\}$

$\underbrace{op_r(\overbrace{rd(x1, op_t(rd(op_r(x2, x3, x4), x2), x2))}, x2, rd(op_r(x2, x3, x4), x2))}$

$=$ by Lemma 2604 RL with $\{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow rd(x1, op_t(rd(op_r(x2, x3, x4), x2), x2))\}$

$\overbrace{rd(x1, \underbrace{op_t(rd(op_r(x2, x3, x4), x2), x2)})}$

$=$ by Lemma 14 LR with $\{x2 \leftarrow x2, x1 \leftarrow op_r(x2, x3, x4)\}$

$rd(x1, \overbrace{\underbrace{mult(i(x2), op_r(x2, x3, x4))})}$

$=$ by Lemma 117 LR with $\{x3 \leftarrow x4, x2 \leftarrow x3, x1 \leftarrow x2\}$

$rd(x1, \overbrace{rd(op_r(x2, x3, x4), x2)})$

Lemma 2610: $mult(rd(x1, x2), op_r(x2, x3, x4)) = rd(x1, rd(x2, op_r(x2, x3, x4)))$

$mult(rd(x1, \underbrace{x2}), op_r(x2, x3, x4))$

$=$ by Lemma 2584 LR with $\{x3 \leftarrow x4, x2 \leftarrow mult(i(x2), x3), x1 \leftarrow x2\}$

$mult(rd(x1, \overbrace{op_r(\underbrace{op_r(x2, mult(i(x2), x3), x4)}, mult(i(x2), x3), i(x4))}), op_r(x2, x3, x4))$

$=$ by Lemma 2433 LR with $\{x3 \leftarrow x4, x2 \leftarrow x3, x1 \leftarrow x2\}$

$mult(rd(x1, \underbrace{op_r(\overbrace{op_r(x2, x3, x4)}, mult(i(x2), x3), i(x4))}), op_r(x2, x3, x4))$

$=$ by Lemma 2509 LR with $\{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow op_r(x2, x3, x4)\}$

$\underbrace{mult(rd(x1, \overbrace{op_r(op_r(x2, x3, x4), mult(x2, i(x3)), x4)}), op_r(x2, x3, x4))}$

$=$ by Lemma 2609 LR with $\{x4 \leftarrow x4, x3 \leftarrow mult(x2, i(x3)), x2 \leftarrow op_r(x2, x3, x4), x1 \leftarrow x1\}$

$\overbrace{rd(x1, rd(\underbrace{op_r(op_r(x2, x3, x4), mult(x2, i(x3)), x4)}, op_r(x2, x3, x4)))}$

$=$ by Lemma 2509 RL with $\{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow op_r(x2, x3, x4)\}$

$rd(x1, rd(\overbrace{op_r(\underbrace{op_r(x2, x3, x4)}, mult(i(x2), x3), i(x4))}, op_r(x2, x3, x4)))$

$=$ by Lemma 2433 RL with $\{x3 \leftarrow x4, x2 \leftarrow x3, x1 \leftarrow x2\}$

$rd(x1, rd(\underbrace{op_r(\overbrace{op_r(x2, mult(i(x2), x3), x4)}, mult(i(x2), x3), i(x4))}, op_r(x2, x3, x4)))$

$=$ by Lemma 2584 RL with $\{x3 \leftarrow x4, x2 \leftarrow mult(i(x2), x3), x1 \leftarrow x2\}$

$rd(x1, rd(\overbrace{x2}, op_r(x2, x3, x4)))$

Lemma 2611: $rd(x1, x2) = op_r(op_r(rd(x1, x2), x1, i(x3)), x2, i(i(x3)))$

$\overbrace{rd(x1, x2)}$

$=$ by Lemma 2584 LR with $\{x3 \leftarrow x3, x2 \leftarrow mult(op_r(i(x1), x2, x2), rd(x1, x2)), x1 \leftarrow rd(x1, x2)\}$

$\overbrace{op_r(\underbrace{op_r(rd(x1, x2), mult(op_r(i(x1), x2, x2), rd(x1, x2)), x3)}, mult(op_r(i(x1), x2, x2), rd(x1, x2)), i(x3))}$

$=$ by Lemma 2430 LR with $\{x3 \leftarrow x3, x2 \leftarrow op_r(i(x1), x2, x2), x1 \leftarrow rd(x1, x2)\}$

$op_r(\overbrace{op_r(rd(x1, x2), op_r(i(x1), x2, x2), x3)}, mult(\underbrace{op_r(i(x1), x2, x2)}, rd(x1, x2)), i(x3))$

$=$ by Lemma 356 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$op_r(op_r(rd(x1, x2), op_r(i(x1), x2, x2), x3), mult(\overbrace{i(op_r(x1, x2, x2))}, \underbrace{rd(x1, x2)}), i(x3))$

$=$ by Lemma 352 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$op_r(op_r(rd(x1, x2), op_r(i(x1), x2, x2), x3), \underbrace{mult(i(op_r(x1, x2, x2)), \overbrace{mult(op_r(x1, x2, x2), i(x2))})}, i(x3))$

$=$ by Axiom 2 RL with $\{x2 \leftarrow op_r(x1, x2, x2), x1 \leftarrow i(x2)\}$

$\underbrace{op_r(op_r(rd(x1, x2), op_r(i(x1), x2, x2), x3), \overbrace{i(x2)}, i(x3))}$

$=$ by Lemma 2427 LR with $\{x3 \leftarrow i(x3), x2 \leftarrow x2, x1 \leftarrow op_r(rd(x1, x2), op_r(i(x1), x2, x2), x3)\}$

$\overbrace{op_r(op_r(rd(x1, x2), \underbrace{op_r(i(x1), x2, x2)}, x3), x2, i(i(x3)))}$

$=$ by Lemma 2428 LR with $\{x3 \leftarrow x2, x2 \leftarrow x2, x1 \leftarrow x1\}$

$op_r(\underbrace{op_r(rd(x1, x2), \overbrace{i(op_r(x1, x2, x2))}, x3)}, x2, i(i(x3)))$

$=$ by Lemma 2427 LR with $\{x3 \leftarrow x3, x2 \leftarrow op_r(x1, x2, x2), x1 \leftarrow rd(x1, x2)\}$

$op_r(\overbrace{op_r(rd(x1, x2), op_r(x1, x2, x2), i(x3))}, x2, i(i(x3)))$

$=$ by Lemma 1956 LR with $\{x3 \leftarrow i(x3), x2 \leftarrow x2, x1 \leftarrow x1\}$

$op_r(\overbrace{op_r(rd(x1, x2), rd(\underbrace{mult(x2, op_t(x1, x2))}, x2), i(x3))}, x2, i(i(x3)))$

$=$ by Lemma 13 LR with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$op_r(op_r(rd(x1, x2), \underbrace{rd(\overbrace{mult(x1, x2)}, x2)}, i(x3)), x2, i(i(x3)))$

$=$ by Axiom 5 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$op_r(op_r(rd(x1, x2), \overbrace{x1}, i(x3)), x2, i(i(x3)))$

Lemma 2612: $rd(mult(x2, x1), x1) = op_r(op_t(x2, x1), x1, mult(op_l(x2, x3, x4), x1))$

$rd(\underbrace{mult(x2, x1)}, x1)$

$=$ by Lemma 13 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{rd(\overbrace{mult(x1, op_t(x2, x1))}, x1)}$

$=$ by Lemma 2582 RL with $\{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow op_t(x2, x1)\}$

$\overbrace{op_r(op_t(x2, x1), x1, \underbrace{mult(x1, op_l(op_t(x2, x1), x3, x4))})}$

$=$ by Lemma 167 LR with $\{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$op_r(op_t(x2, x1), x1, \overbrace{mult(op_l(x2, x3, x4), x1)})$

Lemma 2613: $op_r(x1, rd(x2, op_l(x2, x4, x3)), x4) = op_r(x1, mult(x4, op_r(op_t(i(x4), rd(x2, op_l(x2, x3, x4))), x3, i(x2))), i(x4))$

$op_r(x1, \underbrace{rd(x2, op_l(x2, x4, x3))}, x4)$

$=$ by Lemma 2449 RL with $\{x3 \leftarrow x4, x2 \leftarrow x3, x1 \leftarrow x2\}$

$\underbrace{op_r(x1, \overbrace{rd(op_l(x2, x3, x4), x2)}, x4)}$

$=$ by Lemma 2436 RL with $\{x4 \leftarrow x4, x3 \leftarrow op_l(x2, x3, x4), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_r(x1, rd(x2, op_l(x2, x3, x4)), i(x4))}}$

$=$ by Lemma 2230 RL with $\{x2 \leftarrow i(x4), x3 \leftarrow rd(x2, op_l(x2, x3, x4)), x1 \leftarrow x1\}$

$\overbrace{op_r(x1, rd(op_t(\underbrace{mult(rd(x2, op_l(x2, x3, x4)), i(x4))}, rd(x2, op_l(x2, x3, x4))), i(x4)), i(x4))}$

$=$ by Lemma 2003 RL with $\{x4 \leftarrow i(x4), x3 \leftarrow x4, x2 \leftarrow x3, x1 \leftarrow x2\}$

$op_r(x1, rd(op_t(\overbrace{mult(op_l(i(x2), x3, x4), mult(x2, i(x4)))}, rd(x2, op_l(x2, x3, x4))), i(x4)), i(x4))$

$=$ by Lemma 2482 RL with $\{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x4\}$

$op_r(x1, rd(op_t(\underbrace{i(op_r(x4, x3, i(x2)))}, rd(x2, op_l(x2, x3, x4))), i(x4)), i(x4))$

$=$ by Lemma 2428 RL with $\{x3 \leftarrow i(x2), x2 \leftarrow x3, x1 \leftarrow x4\}$

$\underbrace{op_r(x1, rd(op_t(\overbrace{op_r(i(x4), x3, i(x2))}, rd(x2, op_l(x2, x3, x4))), i(x4)), i(x4))}$

$=$ by Lemma 2436 LR with $\{x4 \leftarrow x4, x3 \leftarrow i(x4), x2 \leftarrow op_t(op_r(i(x4), x3, i(x2)), rd(x2, op_l(x2, x3, x4))), x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_r(x1, rd(i(x4), op_t(op_r(i(x4), x3, i(x2)), rd(x2, op_l(x2, x3, x4)))), x4)}}$

$=$ by Lemma 2451 RL with $\{x3 \leftarrow op_t(op_r(i(x4), x3, i(x2)), rd(x2, op_l(x2, x3, x4))), x2 \leftarrow x4, x1 \leftarrow x1\}$

$\overbrace{op_r(x1, mult(x4, \underbrace{op_t(op_r(i(x4), x3, i(x2)), rd(x2, op_l(x2, x3, x4)))}), i(x4))}$

$=$ by Axiom 14 LR with $\{x4 \leftarrow rd(x2, op_l(x2, x3, x4)), x3 \leftarrow i(x2), x2 \leftarrow x3, x1 \leftarrow i(x4)\}$

$op_r(x1, mult(x4, \overbrace{op_r(op_t(i(x4), rd(x2, op_l(x2, x3, x4))), x3, i(x2))}), i(x4))$

Lemma 2614: $op_r(x1, rd(x2, op_l(x2, x4, x3)), x4) = op_r(x1, rd(x4, op_r(x4, x3, i(x2))), i(x4))$

$\underbrace{op_r(x1, rd(x2, op_l(x2, x4, x3)), x4)}$

$=$ by Lemma 2613 LR with $\{x3 \leftarrow x3, x4 \leftarrow x4, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_r(x1, mult(x4, op_r(op_t(i(x4), rd(x2, \underbrace{op_l(x2, x3, x4)})), x3, i(x2))), i(x4))}$

$=$ by Lemma 2031 RL with $\{x3 \leftarrow x3, x2 \leftarrow x4, x1 \leftarrow x2\}$

$op_r(x1, mult(x4, op_r(\underbrace{op_t(i(x4), rd(x2, \overbrace{op_l(x2, i(x4), x3)}))}, x3, i(x2))), i(x4))$

$=$ by Lemma 2309 RL with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow i(x4)\}$

$op_r(x1, \underbrace{mult(x4, op_r(\overbrace{i(x4)}, x3, i(x2)))}, i(x4))$

$=$ by Lemma 2434 LR with $\{x3 \leftarrow i(x2), x2 \leftarrow x3, x1 \leftarrow x4\}$

$op_r(x1, \overbrace{rd(x4, op_r(x4, x3, i(x2)))}, i(x4))$

Lemma 2615: $op_r(x1, rd(x2, op_l(x2, x4, x3)), x4) = op_r(x1, rd(x4, op_r(x4, x3, x2)), x4)$

$$
\begin{aligned}
& \underbrace{op_r(x1, rd(x2, op_l(x2, x4, x3)), x4)} \\
= & \quad \text{by Lemma 2614 LR with } \{x3 \leftarrow x3, x4 \leftarrow x4, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, \underbrace{rd(x4, op_r(x4, x3, i(x2)))}, i(x4))} \\
= & \quad \text{by Lemma 2460 LR with } \{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x4\} \\
& \underbrace{op_r(x1, \overbrace{rd(op_r(x4, x3, x2), x4)}, i(x4))} \\
= & \quad \text{by Lemma 2436 LR with } \{x4 \leftarrow x4, x3 \leftarrow x4, x2 \leftarrow op_r(x4, x3, x2), x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, rd(x4, op_r(x4, x3, x2)), x4)}
\end{aligned}
$$

Lemma 2616: $x1 = op_r(x1, rd(x2, op_l(x2, x5, op_t(x3, x4))), x3)$

$$
\begin{aligned}
& \underbrace{x1} \\
= & \quad \text{by Axiom 5 LR with } \{x2 \leftarrow rd(op_r(op_t(op_t(x3, x2), x4), x5, i(i(x2))), op_t(op_t(x3, x2), x4)), x1 \leftarrow x1\} \\
& \overbrace{rd(mult(x1, rd(op_r(op_t(op_t(x3, x2), x4), x5, i(i(x2))), op_t(op_t(x3, x2), x4))), rd(op_r(op_t(op_t(x3, x2), x4), x5, i(i(x2))), op_t(o} \\
= & \quad \text{by Lemma 2545 RL with } \{x4 \leftarrow i(i(x2)), x3 \leftarrow x5, x2 \leftarrow op_t(op_t(x3, x2), x4), x1 \leftarrow mult(x1, rd(op_r(op_t(op_t(x3, x2), x4), x5, \\
& \overbrace{rd(mult(mult(x1, rd(op_r(op_t(op_t(x3, x2), x4), x5, i(i(x2))), op_t(op_t(x3, x2), x4))), op_t(op_t(x3, x2), x4)), op_r(op_t(op_t(x3, x2)} \\
= & \quad \text{by Lemma 693 LR with } \{x3 \leftarrow op_t(op_t(x3, x2), x4), x2 \leftarrow op_r(op_t(op_t(x3, x2), x4), x5, i(i(x2))), x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, \underbrace{rd(op_r(op_t(op_t(x3, x2), x4), x5, i(i(x2))), op_t(op_t(x3, x2), x4))}, op_t(op_t(x3, x2), x4))} \\
= & \quad \text{by Lemma 2458 LR with } \{x3 \leftarrow i(x2), x2 \leftarrow x5, x1 \leftarrow op_t(op_t(x3, x2), x4)\} \\
& \underbrace{op_r(x1, \overbrace{rd(op_t(op_t(x3, x2), x4), op_r(op_t(op_t(x3, x2), x4), x5, i(x2)))}, op_t(op_t(x3, x2), x4))} \\
= & \quad \text{by Lemma 2615 RL with } \{x3 \leftarrow x5, x4 \leftarrow op_t(op_t(x3, x2), x4), x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, rd(i(x2), op_l(i(x2), \underbrace{op_t(op_t(x3, x2), x4)}, x5)), op_t(op_t(x3, x2), x4))} \\
= & \quad \text{by Axiom 13 RL with } \{x3 \leftarrow x2, x2 \leftarrow x4, x1 \leftarrow x3\} \\
& op_r(x1, rd(i(x2), \underbrace{op_l(i(x2), \overbrace{op_t(op_t(x3, x4), x2)}, x5)}), op_t(op_t(x3, x2), x4)) \\
= & \quad \text{by Lemma 2330 LR with } \{x3 \leftarrow x5, x2 \leftarrow op_t(x3, x4), x1 \leftarrow x2\} \\
& \underbrace{op_r(x1, rd(i(x2), \overbrace{op_l(i(x2), op_t(x3, x4), x5)}), op_t(op_t(x3, x2), x4))} \\
= & \quad \text{by Lemma 2392 LR with } \{x4 \leftarrow x4, x3 \leftarrow op_t(x3, x2), x2 \leftarrow rd(i(x2), op_l(i(x2), op_t(x3, x4), x5)), x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_r(x1, rd(i(x2), op_l(i(x2), op_t(x3, x4), x5)), op_t(x3, x2))}} \\
= & \quad \text{by Lemma 2392 LR with } \{x4 \leftarrow x2, x3 \leftarrow x3, x2 \leftarrow rd(i(x2), op_l(i(x2), op_t(x3, x4), x5)), x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, \underbrace{rd(i(x2), op_l(i(x2), op_t(x3, x4), x5))}, x3)} \\
= & \quad \text{by Lemma 1074 LR with } \{x4 \leftarrow x5, x3 \leftarrow op_t(x3, x4), x2 \leftarrow x2, x1 \leftarrow x2\} \\
& op_r(x1, \underbrace{\overbrace{rd(op_l(x2, op_t(x3, x4), x5), x2)}}, x3) \\
= & \quad \text{by Lemma 2449 LR with } \{x3 \leftarrow x5, x2 \leftarrow op_t(x3, x4), x1 \leftarrow x2\} \\
& op_r(x1, \overbrace{rd(x2, op_l(x2, x5, op_t(x3, x4)))}, x3)
\end{aligned}
$$

Lemma 2617: $op_r(rd(op_r(x1,x2,x3),x1),x3,x4) = mult(unit(),rd(op_r(x1,x2,x3),x1))$

$$
\begin{aligned}
& \underbrace{op_r(rd(op_r(x1,x2,x3),x1),x3,x4)} \\
= \quad & \text{by Lemma 2444 RL with } \{x3 \leftarrow rd(op_r(x1,x2,x3),x1), x2 \leftarrow x3, x1 \leftarrow x4\} \\
& \overbrace{mult(rd(x4,\underbrace{op_l(x4,x3,rd(op_r(x1,x2,x3),x1))}),rd(op_r(x1,x2,x3),x1))} \\
= \quad & \text{by Lemma 2420 RL with } \{x4 \leftarrow x2, x3 \leftarrow x1, x2 \leftarrow x3, x1 \leftarrow x4\} \\
& mult(\underbrace{rd(x4,\overbrace{x4})},rd(op_r(x1,x2,x3),x1)) \\
= \quad & \text{by Lemma 2 LR with } \{x1 \leftarrow x4\} \\
& mult(\overbrace{unit()},rd(op_r(x1,x2,x3),x1))
\end{aligned}
$$

Lemma 2618: $rd(x1,op_l(x1,x2,x3)) = rd(op_r(x3,x2,x1),x3)$

$$
\begin{aligned}
& \underbrace{rd(x1,op_l(x1,x2,x3))} \\
= \quad & \text{by Lemma 2449 RL with } \{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{rd(op_l(x1,x3,x2),x1)}} \\
= \quad & \text{by Lemma 2379 RL with } \{x4 \leftarrow x4, x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{rd(mult(\underbrace{op_l(x1,x3,x2)},x4),mult(x1,x4))} \\
= \quad & \text{by Lemma 2441 RL with } \{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x3\} \\
& \underbrace{rd(mult(\overbrace{mult(rd(op_r(x3,x2,x1),x3),x1)},x4),mult(x1,x4))} \\
= \quad & \text{by Axiom 16 RL with } \{x3 \leftarrow x4, x2 \leftarrow x1, x1 \leftarrow rd(op_r(x3,x2,x1),x3)\} \\
& \overbrace{\underbrace{op_r(rd(op_r(x3,x2,x1),x3),x1,x4)}} \\
= \quad & \text{by Lemma 2617 LR with } \{x4 \leftarrow x4, x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x3\} \\
& \overbrace{\underbrace{mult(unit(),rd(op_r(x3,x2,x1),x3))}} \\
= \quad & \text{by Axiom 3 RL with } \{x1 \leftarrow rd(op_r(x3,x2,x1),x3)\} \\
& \overbrace{rd(op_r(x3,x2,x1),x3)}
\end{aligned}
$$

Lemma 2619: $rd(x1,op_l(x1,rd(x2,x3),x2)) = rd(x2,op_r(x2,x3,x1))$

$$
\begin{aligned}
& \underbrace{rd(x1,op_l(x1,rd(x2,x3),x2))} \\
= \quad & \text{by Lemma 2618 LR with } \{x3 \leftarrow x2, x2 \leftarrow rd(x2,x3), x1 \leftarrow x1\} \\
& \overbrace{rd(\underbrace{op_r(x2,rd(x2,x3),x1)},x2)} \\
= \quad & \text{by Lemma 2432 LR with } \{x3 \leftarrow x1, x2 \leftarrow x3, x1 \leftarrow x2\} \\
& \underbrace{rd(\overbrace{op_r(x2,x3,i(x1))},x2)} \\
= \quad & \text{by Lemma 2458 LR with } \{x3 \leftarrow x1, x2 \leftarrow x3, x1 \leftarrow x2\} \\
& \overbrace{rd(x2,op_r(x2,x3,x1))}
\end{aligned}
$$

Lemma 2620: $rd(x1, op_l(x1, x2, x3)) = rd(x2, op_r(x2, x3, x1))$

$rd(x1, \underbrace{op_l(x1, x2, x3)})$

$=$ by Lemma 286 RL with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$rd(x1, \overbrace{op_l(x1, x2, \underbrace{rd(x3, x2)})})$

$=$ by Lemma 2052 LR with $\{x4 \leftarrow x2, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{rd(x1, \overbrace{op_l(x1, \underbrace{rd(x2, x3)}, x2)})}$

$=$ by Lemma 2619 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{rd(x2, \underbrace{op_r(x2, x3, x1)})}$

Lemma 2621: $op_r(x1, mult(x2, x3), x2) = op_r(x1, mult(op_l(x2, x4, x2), x3), x2)$

$\underbrace{op_r(x1, mult(x2, x3), x2)}$

$=$ by Axiom 16 LR with $\{x3 \leftarrow x2, x2 \leftarrow mult(x2, x3), x1 \leftarrow x1\}$

$\overbrace{rd(\underbrace{mult(mult(x1, \underbrace{mult(x2, x3)}), x2)}, mult(mult(x2, x3), x2))}$

$=$ by Axiom 4 LR with $\{x2 \leftarrow op_l(x2, x2, x4), x1 \leftarrow mult(mult(x1, mult(x2, x3)), x2)\}$

$rd(\overbrace{mult(\underbrace{rd(mult(mult(x1, mult(x2, x3)), x2), op_l(x2, x2, x4))}, op_l(x2, x2, x4))}, mult(mult(x2, x3), x2))$

$=$ by Lemma 1575 LR with $\{x3 \leftarrow x4, x2 \leftarrow x2, x1 \leftarrow mult(mult(x1, mult(x2, x3)), x2)\}$

$rd(mult(\overbrace{rd(\underbrace{rd(mult(mult(x1, mult(x2, x3)), x2), x2)}, asoc(x2, x4, x2))}, op_l(x2, x2, x4)), mult(mult(x2, x3), x2))$

$=$ by Axiom 5 RL with $\{x2 \leftarrow x2, x1 \leftarrow mult(x1, mult(x2, x3))\}$

$rd(mult(\underbrace{rd(\overbrace{mult(x1, mult(x2, x3))}, asoc(x2, x4, x2))}, op_l(x2, x2, x4)), mult(mult(x2, x3), x2))$

$=$ by Lemma 2383 RL with $\{x4 \leftarrow x4, x3 \leftarrow x2, x2 \leftarrow mult(x2, x3), x1 \leftarrow x1\}$

$rd(mult(\overbrace{mult(x1, \underbrace{rd(mult(x2, x3), asoc(x2, x4, x2))})}, op_l(x2, x2, x4)), mult(mult(x2, x3), x2))$

$=$ by Lemma 2503 RL with $\{x3 \leftarrow x4, x2 \leftarrow x2, x1 \leftarrow mult(x2, x3)\}$

$\underbrace{rd(mult(mult(x1, \overbrace{rd(mult(mult(x2, x3), x2), op_l(x2, x2, x4))}), op_l(x2, x2, x4)), mult(mult(x2, x3), x2))}$

$=$ by Lemma 693 LR with $\{x3 \leftarrow op_l(x2, x2, x4), x2 \leftarrow mult(mult(x2, x3), x2), x1 \leftarrow x1\}$

$\overbrace{op_r(x1, rd(mult(mult(x2, x3), x2), op_l(x2, x2, x4)), \underbrace{op_l(x2, x2, x4)})}$

$=$ by Lemma 773 LR with $\{x2 \leftarrow x4, x1 \leftarrow x2\}$

$\underbrace{op_r(x1, rd(mult(mult(x2, x3), x2), op_l(x2, x2, x4)), \overbrace{rd(x2, asoc(x4, x2, x2))})}$

$=$ by Lemma 2393 RL with $\{x4 \leftarrow x4, x3 \leftarrow x2, x2 \leftarrow rd(mult(mult(x2, x3), x2), op_l(x2, x2, x4)), x1 \leftarrow x1\}$

$\overbrace{op_r(x1, \underbrace{rd(mult(mult(x2, x3), x2), op_l(x2, x2, x4))}, x2)}$

$=$ by Lemma 2503 LR with $\{x3 \leftarrow x4, x2 \leftarrow x2, x1 \leftarrow mult(x2, x3)\}$

$op_r(x1, \overbrace{\underbrace{rd(mult(x2, x3), asoc(x2, x4, x2))}, x2})$

$=$ by Lemma 1708 RL with $\{x3 \leftarrow mult(x2, x3), x2 \leftarrow x2, x1 \leftarrow x4\}$

$op_r(x1, \overbrace{\underbrace{mult(asoc(x4, x2, x2), mult(x2, x3))}, x2})$

$=$ by Lemma 959 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x4\}$

$op_r(x1, \overbrace{mult(op_l(x2, x4, x2), x3)}, x2)$

Lemma 2622: $op_l(rd(x1,x2),x1,op_l(x3,x1,rd(x1,x2))) = rd(op_r(x1,x3,rd(x1,x2)),x2)$

$op_l(\underbrace{rd(x1,x2)}, x1, op_l(x3,x1,rd(x1,x2)))$

$=$ by Lemma 364 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{op_l(\overbrace{op_r(mult(x1,i(x2)),x2,x2)}, x1, op_l(x3,x1,rd(x1,x2)))}$

$=$ by Axiom 18 RL with $\{x5 \leftarrow x2, x4 \leftarrow x2, x3 \leftarrow op_l(x3,x1,rd(x1,x2)), x2 \leftarrow x1, x1 \leftarrow mult(x1,i(x2))\}$

$\overbrace{op_r(\underbrace{op_l(mult(x1,i(x2)),x1,op_l(x3,x1,rd(x1,x2)))}, x2, x2)}$

$=$ by Lemma 358 RL with $\{x2 \leftarrow x2, x1 \leftarrow op_l(mult(x1,i(x2)),x1,op_l(x3,x1,rd(x1,x2)))\}$

$\overbrace{rd(\underbrace{rd(op_l(mult(x1,i(x2)),x1,\underbrace{op_l(x3,x1,rd(x1,x2))}, i(x2))}, x2)}$

$=$ by Lemma 2474 LR with $\{x3 \leftarrow op_l(x3,x1,rd(x1,x2)), x2 \leftarrow i(x2), x1 \leftarrow x1\}$

$rd(\overbrace{\underbrace{op_r(x1,op_l(x3,x1,rd(x1,x2)),mult(x1,i(x2)))}, x2)}$

$=$ by Lemma 2395 LR with $\{x4 \leftarrow x2, x3 \leftarrow x1, x2 \leftarrow op_l(x3,x1,rd(x1,x2)), x1 \leftarrow x1\}$

$rd(\overbrace{\underbrace{op_r(x1,op_l(x3,x1,rd(x1,x2)),rd(x1,x2))}, x2)}$

$=$ by Lemma 1896 RL with $\{x3 \leftarrow op_l(x3,x1,rd(x1,x2)), x2 \leftarrow rd(x1,x2), x1 \leftarrow x1\}$

$rd(\overbrace{\underbrace{op_r(x1,rd(mult(rd(x1,x2),op_l(x3,x1,rd(x1,x2))),rd(x1,x2)),rd(x1,x2))}, x2)}$

$=$ by Lemma 2589 RL with $\{x3 \leftarrow mult(rd(x1,x2),op_l(x3,x1,rd(x1,x2))), x2 \leftarrow rd(x1,x2), x1 \leftarrow x1\}$

$rd(\overbrace{\underbrace{op_r(x1,mult(rd(x1,rd(x1,x2)),mult(rd(x1,x2),\underbrace{op_l(x3,x1,rd(x1,x2))})),rd(x1,x2))}, x2)}$

$=$ by Lemma 292 RL with $\{x3 \leftarrow rd(x1,x2), x2 \leftarrow x1, x1 \leftarrow x3\}$

$rd(op_r(x1,\underbrace{mult(rd(x1,rd(x1,x2)),mult(rd(x1,x2),\overbrace{op_l(x3,rd(x1,rd(x1,x2)),rd(x1,x2))})}),rd(x1,x2)),x2)$

$=$ by Lemma 137 LR with $\{x3 \leftarrow x3, x2 \leftarrow rd(x1,x2), x1 \leftarrow rd(x1,rd(x1,x2))\}$

$rd(op_r(x1,\overbrace{\underbrace{mult(\underbrace{mult(rd(x1,rd(x1,x2)),rd(x1,x2))}, x3)}),rd(x1,x2)),x2)$

$=$ by Axiom 4 RL with $\{x2 \leftarrow rd(x1,x2), x1 \leftarrow x1\}$

$rd(\underbrace{op_r(x1,mult(\overbrace{x1}, x3),rd(x1,x2))}, x2)$

$=$ by Lemma 2431 LR with $\{x3 \leftarrow rd(x1,x2), x2 \leftarrow x3, x1 \leftarrow x1\}$

$rd(\overbrace{op_r(x1,x3,rd(x1,x2))}, x2)$

Lemma 2623: $rd(op_r(x1,x2,x3),x1) = rd(op_r(op_r(x1,x2,x3),x4,x1),op_l(x1,x1,x4))$

$\underbrace{rd(op_r(x1,x2,x3),x1)}$

$=$ by Lemma 2468 LR with $\{x3 \leftarrow x4, x1 \leftarrow x1, x2 \leftarrow op_r(x1,x2,x3)\}$

$\overbrace{rd(op_r(op_r(x1,x2,x3),x4,x1),\underbrace{op_l(x1,op_r(x1,x2,x3),x4)})}$

$=$ by Lemma 2602 LR with $\{x2 \leftarrow x4, x4 \leftarrow x3, x3 \leftarrow x2, x1 \leftarrow x1\}$

$rd(op_r(op_r(x1,x2,x3),x4,x1),\overbrace{\underbrace{rd(x1,rd(op_r(x4,x1,x1),x4))})$

$=$ by Lemma 2448 LR with $\{x3 \leftarrow x1, x2 \leftarrow x4, x1 \leftarrow x1\}$

$rd(op_r(op_r(x1,x2,x3),x4,x1),\overbrace{op_l(x1,x1,x4)})$

Lemma 2624: $op_r(x1, op_l(op_l(x2, x3, x4), x5, x6), x2) = rd(x1, rd(x2, op_l(x2, x1, x2)))$

$\overbrace{op_r(x1, op_l(op_l(x2, x3, x4), x5, x6), x2)}$

$=$ by Lemma 2425 RL with $\{x3 \leftarrow op_l(op_l(x2, x3, x4), x5, x6), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{rd(x1, rd(x2, \underbrace{op_l(x2, x1, op_l(op_l(x2, x3, x4), x5, x6))}))}$

$=$ by Lemma 1873 LR with $\{x6 \leftarrow x6, x5 \leftarrow x5, x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow x2\}$

$rd(x1, rd(x2, \overbrace{\underbrace{op_l(x2, x1, op_l(x2, x3, x4))}}))$

$=$ by Lemma 1819 LR with $\{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow x2\}$

$rd(x1, rd(x2, \overbrace{op_l(x2, x1, x2)}))$

Lemma 2625: $op_l(x1, x2, x3) = rd(mult(op_r(x2, x3, x1), x1), op_t(x2, x1))$

$\overbrace{op_l(x1, x2, x3)}$

$=$ by Lemma 2437 RL with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{rd(x1, rd(x2, op_r(x2, x3, x1)))}$

$=$ by Lemma 2189 RL with $\{x4 \leftarrow x1, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{rd(mult(x1, op_r(x2, x3, x1)), x2)}$

$=$ by Axiom 5 LR with $\{x2 \leftarrow op_r(x2, op_r(rd(i(x1), x2), x1, x1), mult(x1, op_r(x2, x3, x1))), x1 \leftarrow rd(mult(x1, op_r(x2, x3, x1)), x$

$\overbrace{rd(\underbrace{mult(rd(mult(x1, op_r(x2, x3, x1)), x2), op_r(x2, op_r(rd(i(x1), x2), x1, x1), mult(x1, op_r(x2, x3, x1))))}, op_r(x2, op_r(rd(i(x$

$=$ by Lemma 2442 LR with $\{x3 \leftarrow op_r(rd(i(x1), x2), x1, x1), x2 \leftarrow x2, x1 \leftarrow mult(x1, op_r(x2, x3, x1))\}$

$rd(\overbrace{\underbrace{op_l(mult(x1, op_r(x2, x3, x1)), x2, op_r(rd(i(x1), x2), x1, x1))}, op_r(x2, op_r(rd(i(x1), x2), x1, x1), mult(x1, op_r(x2, x3, x1))$

$=$ by Lemma 1795 LR with $\{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow mult(x1, op_r(x2, x3, x1))\}$

$rd(\overbrace{op_l(mult(x1, op_r(x2, x3, x1)), x1, x2)}, \underbrace{op_r(x2, op_r(rd(i(x1), x2), x1, x1), mult(x1, op_r(x2, x3, x1)))})$

$=$ by Lemma 1966 LR with $\{x3 \leftarrow mult(x1, op_r(x2, x3, x1)), x2 \leftarrow x1, x1 \leftarrow x2\}$

$rd(\underbrace{op_l(mult(x1, op_r(x2, x3, x1)), x1, x2)}, \overbrace{op_r(x2, x1, i(mult(x1, op_r(x2, x3, x1))))})$

$=$ by Lemma 2415 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$rd(\overbrace{mult(op_r(x2, x3, x1), x1)}, \underbrace{op_r(x2, x1, i(mult(x1, op_r(x2, x3, x1))))})$

$=$ by Lemma 2400 LR with $\{x4 \leftarrow op_r(x2, x3, x1), x3 \leftarrow x1, x2 \leftarrow x1, x1 \leftarrow x2\}$

$rd(mult(op_r(x2, x3, x1), x1), \overbrace{\underbrace{op_r(x2, x1, rd(i(x1), op_r(x2, x3, x1)))})$

$=$ by Lemma 2552 LR with $\{x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow x2\}$

$rd(mult(op_r(x2, x3, x1), x1), \overbrace{rd(x2, rd(op_t(x1, \underbrace{op_r(x2, x3, x1)}), x1)))})$

$=$ by Axiom 16 LR with $\{x3 \leftarrow x1, x2 \leftarrow x3, x1 \leftarrow x2\}$

$rd(mult(op_r(x2, x3, x1), x1), rd(x2, rd(\underbrace{op_t(x1, \overbrace{rd(mult(mult(x2, x3), x1), mult(x3, x1)))}), x1)))$

$=$ by Lemma 2085 LR with $\{x3 \leftarrow mult(x3, x1), x2 \leftarrow mult(x2, x3), x1 \leftarrow x1\}$

$rd(mult(op_r(x2, x3, x1), x1), rd(x2, rd(\overbrace{op_t(x1, rd(mult(x2, x3), \underbrace{rd(mult(x3, x1), x1)}))}, x1)))$

$=$ by Axiom 5 RL with $\{x2 \leftarrow x1, x1 \leftarrow x3\}$

$rd(mult(op_r(x2, x3, x1), x1), rd(x2, rd(op_t(x1, \underbrace{rd(mult(x2, x3), \overbrace{x3})}), x1)))$

$=$ by Axiom 5 RL with $\{x2 \leftarrow x3, x1 \leftarrow x2\}$

$rd(mult(op_r(x2, x3, x1), x1), \underbrace{rd(x2, rd(op_t(x1, \overbrace{x2}), x1)))})$

$=$ by Lemma 390 LR with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$rd(mult(op_r(x2, x3, x1), x1), \overbrace{op_t(x2, x1)})$

Lemma 2626: $op_r(asoc(x1,x2,x3),x1,rd(i(x2),x3)) = rd(asoc(x1,x2,x3),unit())$

$$
\begin{aligned}
& \overbrace{op_r(asoc(x1,x2,x3),x1,rd(i(x2),x3))} \\
= & \quad \text{by Lemma 2563 RL with } \{x3 \leftarrow rd(i(x2),x3), x2 \leftarrow x1, x1 \leftarrow asoc(x1,x2,x3)\} \\
& \overbrace{rd(asoc(x1,x2,x3),rd(\underbrace{op_r(x1,asoc(x1,x2,x3),rd(i(x2),x3))}, x1))} \\
= & \quad \text{by Lemma 2593 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& rd(asoc(x1,x2,x3),\underbrace{rd(\overbrace{x1}, x1)}) \\
= & \quad \text{by Lemma 2 LR with } \{x1 \leftarrow x1\} \\
& rd(asoc(x1,x2,x3),\overbrace{unit()})
\end{aligned}
$$

Lemma 2627: $op_r(x1,op_r(x2,x3,x4),op_r(x2,x3,x4)) = op_r(x1,x2,x2)$

$$
\begin{aligned}
& \overbrace{op_r(x1,op_r(x2,x3,x4),op_r(x2,x3,x4))} \\
= & \quad \text{by Lemma 2600 RL with } \{x4 \leftarrow i(x4), x3 \leftarrow x3, x2 \leftarrow op_r(x2,x3,x4), x1 \leftarrow x1\} \\
& \overbrace{op_r(x1,\underbrace{op_r(x2,x3,x4)},op_r(op_r(x2,x3,x4),x3,i(x4)))} \\
= & \quad \text{by Lemma 2565 RL with } \{x4 \leftarrow i(x4), x3 \leftarrow x3, x2 \leftarrow op_r(x2,x3,x4), x1 \leftarrow x1\} \\
& \overbrace{op_r(x1,\underbrace{op_r(op_r(x2,x3,x4),x3,i(x4))},op_r(x2,x3,x4))} \\
= & \quad \text{by Lemma 2584 RL with } \{x3 \leftarrow x4, x2 \leftarrow x3, x1 \leftarrow x2\} \\
& \underbrace{op_r(x1,\overbrace{x2},op_r(x2,x3,x4))} \\
= & \quad \text{by Lemma 2600 LR with } \{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_r(x1,x2,x2)}
\end{aligned}
$$

Lemma 2628: $op_r(x1,op_l(x2,x3,x4),op_l(x2,x3,x4)) = op_r(x1,op_l(x2,x5,x6),op_l(x2,x3,x4))$

$$
\begin{aligned}
& \underbrace{op_r(x1,op_l(x2,x3,x4),op_l(x2,x3,x4))} \\
= & \quad \text{by Lemma 2425 RL with } \{x3 \leftarrow op_l(x2,x3,x4), x2 \leftarrow op_l(x2,x3,x4), x1 \leftarrow x1\} \\
& \underbrace{\overbrace{rd(x1,rd(op_l(x2,x3,x4),op_l(op_l(x2,x3,x4),x1,op_l(x2,x3,x4))))}} \\
= & \quad \text{by Lemma 2624 RL with } \{x6 \leftarrow x6, x5 \leftarrow x5, x4 \leftarrow x3, x3 \leftarrow x4, x2 \leftarrow op_l(x2,x3,x4), x1 \leftarrow x1\} \\
& \overbrace{op_r(x1,op_l(\underbrace{op_l(op_l(x2,x3,x4),x4,x3)},x5,x6),op_l(x2,x3,x4))} \\
= & \quad \text{by Lemma 138 LR with } \{x3 \leftarrow x4, x2 \leftarrow x3, x1 \leftarrow x2\} \\
& op_r(x1,op_l(\overbrace{x2},x5,x6),op_l(x2,x3,x4))
\end{aligned}
$$

Lemma 2629: $asoc(op_r(x2,x3,x4),x1,op_r(x2,x3,x4)) = asoc(x2,x1,x2)$

$\overbrace{asoc(op_r(x2,x3,x4),x1,op_r(x2,x3,x4))}$

$=$ by Lemma 640 LR with $\{x2 \leftarrow x1, x1 \leftarrow op_r(x2,x3,x4)\}$

$\overbrace{rd(x1,op_r(x1,op_r(x2,x3,x4),op_r(x2,x3,x4)))}$

$=$ by Lemma 113 RL with $\{x3 \leftarrow op_r(x2,x3,x4), x2 \leftarrow op_r(x2,x3,x4), x1 \leftarrow x1\}$

$\overbrace{mult(i(op_r(x1,op_r(x2,x3,x4),op_r(x2,x3,x4))),\underbrace{x1}\,)}$

$=$ by Axiom 1 LR with $\{x1 \leftarrow x1\}$

$mult(i(op_r(x1,op_r(x2,x3,x4),op_r(x2,x3,x4))),\overbrace{mult(x1,\underbrace{unit()}\,)})$

$=$ by Lemma 274 RL with $\{x3 \leftarrow op_r(x2,x3,x4), x2 \leftarrow op_r(x2,x3,x4), x1 \leftarrow x1\}$

$mult(i(op_r(x1,op_r(x2,x3,x4),op_r(x2,x3,x4))),\underbrace{mult(x1,\overbrace{asoc(x1,i(op_r(x1,op_r(x2,x3,x4),op_r(x2,x3,x4))),x1)}\,)})$

$=$ by Lemma 158 LR with $\{x2 \leftarrow i(op_r(x1,op_r(x2,x3,x4),op_r(x2,x3,x4))), x1 \leftarrow x1\}$

$\underbrace{mult(i(op_r(x1,op_r(x2,x3,x4),op_r(x2,x3,x4))),\overbrace{op_l(x1,x1,i(op_r(x1,op_r(x2,x3,x4),op_r(x2,x3,x4))))}\,)}$

$=$ by Lemma 264 LR with $\{x2 \leftarrow x1, x1 \leftarrow i(op_r(x1,op_r(x2,x3,x4),op_r(x2,x3,x4)))\}$

$\overbrace{op_t(\underbrace{mult(x1,i(op_r(x1,op_r(x2,x3,x4),op_r(x2,x3,x4))))}\,),x1)}$

$=$ by Lemma 115 LR with $\{x3 \leftarrow op_r(x2,x3,x4), x2 \leftarrow op_r(x2,x3,x4), x1 \leftarrow x1\}$

$op_t(\overbrace{rd(x1,\underbrace{op_r(x1,op_r(x2,x3,x4),op_r(x2,x3,x4))}\,)},x1)$

$=$ by Lemma 2627 LR with $\{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{op_t(rd(x1,\overbrace{op_r(x1,x2,x2)}\,),x1)}$

$=$ by Lemma 1765 LR with $\{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x1\}$

$\overbrace{\underbrace{mult(op_r(i(x1),x2,x2),x1)}}$

$=$ by Lemma 2435 LR with $\{x3 \leftarrow x2, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{\underbrace{rd(x1,op_r(x1,x2,x2))}}$

$=$ by Lemma 640 RL with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$\overbrace{asoc(x2,x1,x2)}$

Lemma 2630: $mult(x1, i(mult(x2, op_t(x1, op_l(x2, x3, x4))))) = rd(op_t(op_l(x2, x3, x4), x1), mult(x2, op_l(op_t(x2, x1), x3, x4)))$

$mult(x1, \underbrace{i(mult(x2, op_t(x1, op_l(x2, x3, x4))))}$

$=$ by Lemma 163 RL with $\{x3 \leftarrow op_l(x2, x3, x4), x2 \leftarrow x1, x1 \leftarrow i(mult(x2, op_t(x1, op_l(x2, x3, x4))))\}$

$\underbrace{mult(x1, \overbrace{op_l(i(mult(x2, op_t(x1, op_l(x2, x3, x4))))), op_t(x1, op_l(x2, x3, x4)), x1)})$

$=$ by Lemma 1091 RL with $\{x3 \leftarrow i(mult(x2, op_t(x1, op_l(x2, x3, x4)))), x2 \leftarrow op_t(x1, op_l(x2, x3, x4)), x1 \leftarrow x1\}$

$\overbrace{mult(\underbrace{rd(x1, op_t(x1, op_l(x2, x3, x4)))}, mult(op_t(x1, op_l(x2, x3, x4)), i(mult(x2, op_t(x1, op_l(x2, x3, x4))))))}$

$=$ by Lemma 395 RL with $\{x1 \leftarrow x1, x2 \leftarrow op_l(x2, x3, x4)\}$

$mult(\overbrace{rd(op_t(op_l(x2, x3, x4), x1), op_l(x2, x3, x4))}, mult(op_t(x1, op_l(x2, x3, x4)), i(mult(x2, op_t(x1, op_l(x2, x3, x4))))))$

$=$ by Lemma 858 RL with $\{x2 \leftarrow op_l(x2, x3, x4), x1 \leftarrow x1\}$

$\underbrace{mult(\overbrace{asoc(x1, mult(op_l(x2, x3, x4), x1), mult(op_l(x2, x3, x4), x1))}, mult(op_t(x1, op_l(x2, x3, x4)), i(mult(x2, op_t(x1, op_l(x2,$

$=$ by Lemma 1708 LR with $\{x3 \leftarrow mult(op_t(x1, op_l(x2, x3, x4)), i(mult(x2, op_t(x1, op_l(x2, x3, x4))))), x2 \leftarrow mult(op_l(x2, x3, x4$

$\overbrace{rd(mult(op_t(x1, op_l(x2, x3, x4)), i(mult(x2, op_t(x1, op_l(x2, x3, x4))))), \underbrace{asoc(mult(op_l(x2, x3, x4), x1), x1, mult(op_l(x2, x3,}$

$=$ by Lemma 838 LR with $\{x2 \leftarrow x1, x1 \leftarrow op_l(x2, x3, x4)\}$

$rd(\underbrace{mult(op_t(x1, op_l(x2, x3, x4)), i(mult(x2, op_t(x1, op_l(x2, x3, x4)))))}, \overbrace{rd(op_l(x2, x3, x4), op_t(op_l(x2, x3, x4), x1))})$

$=$ by Lemma 31 LR with $\{x2 \leftarrow x2, x1 \leftarrow op_t(x1, op_l(x2, x3, x4))\}$

$\underbrace{rd(\overbrace{i(op_t(x2, op_t(x1, op_l(x2, x3, x4))))}, rd(op_l(x2, x3, x4), op_t(op_l(x2, x3, x4), x1)))}$

$=$ by Lemma 40 LR with $\{x3 \leftarrow op_t(op_l(x2, x3, x4), x1), x2 \leftarrow op_l(x2, x3, x4), x1 \leftarrow op_t(x2, op_t(x1, op_l(x2, x3, x4)))\}$

$\overbrace{\underbrace{rd(rd(op_t(op_l(x2, x3, x4), x1), op_l(x2, x3, x4)), op_t(x2, op_t(x1, op_l(x2, x3, x4))))}}$

$=$ by Lemma 1291 LR with $\{x3 \leftarrow op_t(x2, op_t(x1, op_l(x2, x3, x4))), x2 \leftarrow x1, x1 \leftarrow op_l(x2, x3, x4)\}$

$\overbrace{rd(op_t(op_l(x2, x3, x4), x1), \underbrace{mult(op_t(x2, op_t(x1, op_l(x2, x3, x4))))}, op_l(x2, x3, x4)))}$

$=$ by Lemma 1722 LR with $\{x3 \leftarrow op_l(x2, x3, x4), x2 \leftarrow x1, x1 \leftarrow x2\}$

$rd(op_t(op_l(x2, x3, x4), x1), \underbrace{mult(\overbrace{op_t(x2, x1)}, op_l(x2, x3, x4)))}$

$=$ by Lemma 2537 LR with $\{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow x2\}$

$rd(op_t(op_l(x2, x3, x4), x1), \overbrace{mult(x2, op_l(op_t(x2, x1), x3, x4))})$

Lemma 2631: $op_r(x1, x2, rd(i(x2), x1)) = mult(i(x2), i(i(mult(x2, op_t(x1, op_l(x2, x3, x4))))))$

$\overbrace{op_r(x1, x2, rd(i(x2), x1))}$

$=$ by Lemma 2407 LR with $\{x5 \leftarrow op_l(x2, x3, x4), x3 \leftarrow x1, x4 \leftarrow i(x2), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_r(x1, x2, rd(i(x2), op_t(x1, op_l(x2, x3, x4))))}$

$=$ by Lemma 2400 RL with $\{x4 \leftarrow op_t(x1, op_l(x2, x3, x4)), x3 \leftarrow x2, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_r(x1, x2, \overbrace{i(mult(x2, op_t(x1, op_l(x2, x3, x4))))})}$

$=$ by Lemma 3 RL with $\{x1 \leftarrow i(mult(x2, op_t(x1, op_l(x2, x3, x4))))\}$

$\overbrace{op_r(x1, x2, \overbrace{i(i(i(mult(x2, op_t(x1, op_l(x2, x3, x4)))))))})$

$=$ by Lemma 2430 RL with $\{x3 \leftarrow i(i(i(mult(x2, op_t(x1, op_l(x2, x3, x4)))))), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_r(x1, mult(x2, x1), i(i(i(mult(x2, op_t(x1, op_l(x2, x3, x4)))))))}$

$=$ by Lemma 1949 RL with $\{x3 \leftarrow i(i(i(mult(x2, op_t(x1, op_l(x2, x3, x4)))))), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_r(x1, rd(x1, i(x2)), i(i(i(mult(x2, op_t(x1, op_l(x2, x3, x4)))))))}$

$=$ by Lemma 2174 LR with $\{x2 \leftarrow op_l(x2, x3, x4), x4 \leftarrow i(i(i(mult(x2, op_t(x1, op_l(x2, x3, x4)))))), x3 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_r(x1, mult(x2, op_t(x1, op_l(x2, x3, x4))), i(i(i(mult(x2, op_t(x1, op_l(x2, x3, x4)))))))}$

$=$ by Lemma 2427 RL with $\{x3 \leftarrow i(i(mult(x2, op_t(x1, op_l(x2, x3, x4))))), x2 \leftarrow mult(x2, op_t(x1, op_l(x2, x3, x4))), x1 \leftarrow x1\}$

$\overbrace{op_r(x1, \overbrace{i(mult(x2, op_t(x1, op_l(x2, x3, x4))))}, i(i(mult(x2, op_t(x1, op_l(x2, x3, x4))))))}$

$=$ by Lemma 92 RL with $\{x2 \leftarrow i(mult(x2, op_t(x1, op_l(x2, x3, x4)))), x1 \leftarrow x1\}$

$\overbrace{mult(\overbrace{mult(x1, i(mult(x2, op_t(x1, op_l(x2, x3, x4)))))}, i(i(mult(x2, op_t(x1, op_l(x2, x3, x4))))))}$

$=$ by Lemma 2630 LR with $\{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$mult(\overbrace{rd(\overbrace{op_t(op_l(x2, x3, x4), x1)}, mult(x2, op_l(op_t(x2, x1), x3, x4)))}, i(i(mult(x2, op_t(x1, op_l(x2, x3, x4))))))$

$=$ by Axiom 15 LR with $\{x4 \leftarrow x1, x3 \leftarrow x4, x2 \leftarrow x3, x1 \leftarrow x2\}$

$mult(\overbrace{rd(\overbrace{op_l(op_t(x2, x1), x3, x4)}, mult(x2, op_l(op_t(x2, x1), x3, x4)))}, i(i(mult(x2, op_t(x1, op_l(x2, x3, x4))))))$

$=$ by Lemma 9 LR with $\{x2 \leftarrow x2, x1 \leftarrow op_l(op_t(x2, x1), x3, x4)\}$

$mult(\overbrace{i(x2)}, i(i(mult(x2, op_t(x1, op_l(x2, x3, x4))))))$

Lemma 2632: $op_t(x1, op_l(x2, x3, x4)) = op_t(x1, x2)$

$$
\begin{aligned}
& \underbrace{op_t(x1, op_l(x2, x3, x4))} \\
= \quad & \text{by Axiom 2 LR with } \{x2 \leftarrow x2, x1 \leftarrow op_t(x1, op_l(x2, x3, x4))\} \\
& \overbrace{\underbrace{mult(i(x2), mult(x2, op_t(x1, op_l(x2, x3, x4))))}} \\
= \quad & \text{by Lemma 17 RL with } \{x2 \leftarrow mult(x2, op_t(x1, op_l(x2, x3, x4))), x1 \leftarrow x2\} \\
& \overbrace{\underbrace{i(mult(x2, i(mult(x2, op_t(x1, op_l(x2, x3, x4))))))}} \\
= \quad & \text{by Axiom 11 RL with } \{x2 \leftarrow i(mult(x2, op_t(x1, op_l(x2, x3, x4)))), x1 \leftarrow x2\} \\
& \overbrace{\underbrace{mult(i(x2), i(i(mult(x2, op_t(x1, op_l(x2, x3, x4))))))}} \\
= \quad & \text{by Lemma 2631 RL with } \{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_r(x1, x2, rd(i(x2), x1))}} \\
= \quad & \text{by Lemma 1416 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, x2)}
\end{aligned}
$$

Lemma 2633: $asoc(x1, op_l(x2, x3, x4), x1) = rd(x1, op_r(op_t(x1, x2), x2, x2))$

$$
\begin{aligned}
& \underbrace{asoc(x1, op_l(x2, x3, x4), x1)} \\
= \quad & \text{by Lemma 747 RL with } \{x2 \leftarrow op_l(x2, x3, x4), x1 \leftarrow x1\} \\
& \overbrace{rd(x1, op_r(\underbrace{op_t(x1, op_l(x2, x3, x4))}, op_l(x2, x3, x4), op_l(x2, x3, x4)))} \\
= \quad & \text{by Lemma 2632 LR with } \{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& rd(x1, \underbrace{op_r(\overbrace{op_t(x1, x2)}, op_l(x2, x3, x4), op_l(x2, x3, x4)))} \\
= \quad & \text{by Lemma 2133 LR with } \{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow op_t(x1, x2)\} \\
& rd(x1, \overbrace{op_r(op_t(x1, x2), x2, x2)})
\end{aligned}
$$

Lemma 2634: $op_t(x1, mult(mult(x2, x3), x4)) = op_t(x1, mult(x3, mult(x2, x4)))$

$$
\begin{aligned}
& op_t(x1, \underbrace{mult(mult(x2, x3), x4)}) \\
= \quad & \text{by Lemma 1047 RL with } \{x3 \leftarrow x4, x2 \leftarrow x2, x1 \leftarrow x3\} \\
& \underbrace{op_t(x1, \overbrace{op_l(mult(x3, mult(x2, x4)), x3, x2)})} \\
= \quad & \text{by Lemma 2632 LR with } \{x4 \leftarrow x2, x3 \leftarrow x3, x2 \leftarrow mult(x3, mult(x2, x4)), x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, mult(x3, mult(x2, x4)))}
\end{aligned}
$$

Lemma 2635: $rd(i(x1), asoc(x1, op_l(x2, x3, x4), x1)) = i(mult(x1, asoc(x1, x2, x1)))$

$$
\begin{aligned}
& \underbrace{rd(i(x1), asoc(x1, op_l(x2, x3, x4), x1))} \\
= \quad & \text{by Lemma 1845 RL with } \{x3 \leftarrow x1, x2 \leftarrow op_l(x2, x3, x4), x1 \leftarrow x1\} \\
& \overbrace{i(\underbrace{mult(asoc(x1, op_l(x2, x3, x4), x1), x1)})} \\
= \quad & \text{by Lemma 2103 LR with } \{x3 \leftarrow x1, x2 \leftarrow op_l(x2, x3, x4), x1 \leftarrow x1\} \\
& i(\overbrace{mult(x1, \underbrace{asoc(x1, op_l(x2, x3, x4), x1)})}) \\
= \quad & \text{by Lemma 2633 LR with } \{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& i(mult(x1, \overbrace{rd(x1, op_r(op_t(x1, x2), x2, x2))})) \\
= \quad & \text{by Lemma 747 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& i(mult(x1, \overbrace{asoc(x1, x2, x1)}))
\end{aligned}
$$

Lemma 2636: $op_l(i(x1), x1, op_l(x2, x3, x4)) = i(mult(x1, asoc(x1, x2, x1)))$

$$
\begin{aligned}
& \underbrace{op_l(i(x1), x1, op_l(x2, x3, x4))} \\
= \quad & \text{by Lemma 776 LR with } \{x2 \leftarrow op_l(x2, x3, x4), x1 \leftarrow x1\} \\
& \overbrace{\underbrace{rd(asoc(op_l(x2, x3, x4), x1, x1), x1)}} \\
= \quad & \text{by Lemma 785 RL with } \{x3 \leftarrow op_l(x2, x3, x4), x2 \leftarrow x1, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{rd(i(x1), asoc(x1, op_l(x2, x3, x4), x1))}} \\
= \quad & \text{by Lemma 2635 LR with } \{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{i(mult(x1, asoc(x1, x2, x1)))}
\end{aligned}
$$

Lemma 2637: $op_l(i(x1), x1, op_l(x2, x3, x4)) = rd(asoc(x2, x1, x1), x1)$

$$
\begin{aligned}
& \underbrace{op_l(i(x1), x1, op_l(x2, x3, x4))} \\
= \quad & \text{by Lemma 2636 LR with } \{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{\overbrace{i(mult(x1, asoc(x1, x2, x1)))}} \\
= \quad & \text{by Lemma 2109 LR with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x1\} \\
& \underbrace{\overbrace{rd(i(x1), asoc(x1, x2, x1))}} \\
= \quad & \text{by Lemma 785 LR with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x1\} \\
& \overbrace{rd(asoc(x2, x1, x1), x1)}
\end{aligned}
$$

Lemma 2638: $op_t(rd(x1, op_l(x2, x3, x4)), x2) = mult(op_l(i(x2), x3, x4), x1)$

$$
\begin{aligned}
& \underbrace{op_t(rd(x1, op_l(x2, x3, x4)), x2)} \\
= \quad & \text{by Lemma 2632 RL with } \{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow rd(x1, op_l(x2, x3, x4))\} \\
& \overbrace{\underbrace{op_t(rd(x1, op_l(x2, x3, x4)), op_l(x2, x3, x4))}} \\
= \quad & \text{by Lemma 14 LR with } \{x2 \leftarrow op_l(x2, x3, x4), x1 \leftarrow x1\} \\
& \overbrace{mult(\underbrace{i(op_l(x2, x3, x4))}, x1)} \\
= \quad & \text{by Lemma 1049 LR with } \{x3 \leftarrow x4, x2 \leftarrow x3, x1 \leftarrow x2\} \\
& mult(\overbrace{op_l(i(x2), x3, x4)}, x1)
\end{aligned}
$$

Lemma 2639: $op_t(x1, mult(x3, mult(x2, mult(x4, i(mult(mult(x4, x2), x3))))))) = op_t(x1, i(asoc(x4, x2, x3)))$

$op_t(x1, \overbrace{mult(x3, mult(x2, mult(x4, i(mult(mult(x4, x2), x3)))))}^{})$

$=$ by Lemma 2634 RL with $\{x4 \leftarrow mult(x4, i(mult(mult(x4, x2), x3))), x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_t(x1, \underbrace{mult(mult(x2, x3), mult(x4, i(mult(mult(x4, x2), x3))))}_{})}^{}$

$=$ by Lemma 2634 RL with $\{x4 \leftarrow i(mult(mult(x4, x2), x3)), x3 \leftarrow mult(x2, x3), x2 \leftarrow x4, x1 \leftarrow x1\}$

$\overbrace{op_t(x1, \underbrace{mult(mult(x4, mult(x2, x3)), i(mult(mult(x4, x2), x3)))}_{})}^{}$

$=$ by Lemma 964 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x4\}$

$op_t(x1, \overbrace{i(\underbrace{asoc(x4, x2, x3)}_{})}^{})$

Lemma 2640: $op_t(x1, mult(x3, op_l(i(x3), x4, x2))) = op_t(x1, i(asoc(x4, x2, x3)))$

$op_t(x1, mult(x3, \underbrace{op_l(i(x3), x4, x2)}_{}))$

$=$ by Lemma 1912 LR with $\{x3 \leftarrow x2, x2 \leftarrow x4, x1 \leftarrow x3\}$

$op_t(x1, mult(x3, \overbrace{\underbrace{mult(mult(x2, x4), i(mult(x2, mult(x4, x3))))}_{}}^{}))$

$=$ by Lemma 1400 LR with $\{x3 \leftarrow mult(x4, x3), x2 \leftarrow x4, x1 \leftarrow x2\}$

$op_t(x1, mult(x3, \overbrace{mult(x2, \underbrace{mult(rd(x4, x2), i(mult(x4, x3)))}_{})}^{}))$

$=$ by Lemma 1399 LR with $\{x3 \leftarrow x3, x1 \leftarrow x2, x2 \leftarrow x4\}$

$\overbrace{op_t(x1, mult(x3, mult(x2, \overbrace{mult(x4, i(mult(mult(x4, x2), x3)))}^{})))}^{}$

$=$ by Lemma 2639 LR with $\{x4 \leftarrow x4, x2 \leftarrow x2, x3 \leftarrow x3, x1 \leftarrow x1\}$

$\overbrace{op_t(x1, i(asoc(x4, x2, x3)))}^{}$

Lemma 2641: $op_t(x1, rd(x2, op_l(x2, x3, x4))) = op_t(x1, i(asoc(x3, x4, x2)))$

$op_t(x1, \underbrace{rd(x2, op_l(x2, x3, x4))}_{})$

$=$ by Lemma 1056 RL with $\{x3 \leftarrow x4, x2 \leftarrow x3, x1 \leftarrow x2\}$

$\underbrace{op_t(x1, \overbrace{mult(x2, op_l(i(x2), x3, x4))}^{})}_{}$

$=$ by Lemma 2640 LR with $\{x2 \leftarrow x4, x4 \leftarrow x3, x3 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_t(x1, i(asoc(x3, x4, x2)))}^{}$

Lemma 2642: $op_t(x1, rd(op_l(x2, x3, x4), x2)) = i(op_t(i(x1), i(asoc(x3, x4, x2))))$

$op_t(x1, \underbrace{rd(op_l(x2, x3, x4), x2)}_{})$

$=$ by Lemma 10 RL with $\{x2 \leftarrow op_l(x2, x3, x4), x1 \leftarrow x2\}$

$\underbrace{op_t(x1, \overbrace{i(\overbrace{rd(x2, op_l(x2, x3, x4))}^{})}^{})}_{}$

$=$ by Lemma 18 RL with $\{x2 \leftarrow rd(x2, op_l(x2, x3, x4)), x1 \leftarrow x1\}$

$\overbrace{i(\underbrace{op_t(i(x1), rd(x2, op_l(x2, x3, x4)))}_{})}^{}$

$=$ by Lemma 2641 LR with $\{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow i(x1)\}$

$i(\overbrace{op_t(i(x1), i(asoc(x3, x4, x2)))}^{})$

Lemma 2643: $op_t(x1, i(asoc(x4, x3, x2))) = i(op_t(i(x1), i(asoc(x3, x4, x2))))$

$$
\begin{aligned}
& \underbrace{op_t(x1, i(asoc(x4, x3, x2)))} \\
= & \quad \text{by Lemma 2641 RL with } \{x4 \leftarrow x3, x3 \leftarrow x4, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, \underbrace{rd(x2, op_l(x2, x4, x3))})} \\
= & \quad \text{by Lemma 2449 RL with } \{x3 \leftarrow x4, x2 \leftarrow x3, x1 \leftarrow x2\} \\
& \underbrace{op_t(x1, \overbrace{rd(op_l(x2, x3, x4), x2)})} \\
= & \quad \text{by Lemma 2642 LR with } \{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{i(op_t(i(x1), i(asoc(x3, x4, x2))))}
\end{aligned}
$$

Lemma 2644: $op_t(x1, i(asoc(x4, x3, x2))) = op_t(x1, asoc(x3, x4, x2))$

$$
\begin{aligned}
& \underbrace{op_t(x1, i(asoc(x4, x3, x2)))} \\
= & \quad \text{by Lemma 2643 LR with } \{x2 \leftarrow x2, x3 \leftarrow x3, x4 \leftarrow x4, x1 \leftarrow x1\} \\
& \underbrace{\overbrace{i(op_t(i(x1), i(asoc(x3, x4, x2))))}} \\
= & \quad \text{by Lemma 18 LR with } \{x2 \leftarrow i(asoc(x3, x4, x2)), x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, \underbrace{i(i(asoc(x3, x4, x2)))})} \\
= & \quad \text{by Lemma 3 LR with } \{x1 \leftarrow asoc(x3, x4, x2)\} \\
& op_t(x1, \overbrace{asoc(x3, x4, x2)})
\end{aligned}
$$

Lemma 2645: $op_t(x1, rd(x2, op_l(x2, x3, x4))) = op_t(x1, asoc(x4, x3, x2))$

$$
\begin{aligned}
& \underbrace{op_t(x1, rd(x2, op_l(x2, x3, x4)))} \\
= & \quad \text{by Lemma 2641 LR with } \{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{\overbrace{op_t(x1, i(asoc(x3, x4, x2)))}} \\
= & \quad \text{by Lemma 2644 LR with } \{x2 \leftarrow x2, x3 \leftarrow x4, x4 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, asoc(x4, x3, x2))}
\end{aligned}
$$

Lemma 2646: $op_t(x1, rd(x2, mult(i(x3), mult(x3, x4)))) = op_t(x1, mult(x3, mult(rd(x2, x3), i(x4))))$

$$
\begin{aligned}
& \underbrace{op_t(x1, rd(x2, mult(i(x3), mult(x3, x4))))} \\
= & \quad \text{by Lemma 1774 RL with } \{x4 \leftarrow mult(x3, x4), x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_t(x1, mult(x2, mult(x3, i(mult(x3, x4)))))}} \\
= & \quad \text{by Lemma 2634 RL with } \{x4 \leftarrow i(mult(x3, x4)), x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, \underbrace{mult(mult(x3, x2), i(mult(x3, x4)))})} \\
= & \quad \text{by Lemma 1400 LR with } \{x3 \leftarrow x4, x2 \leftarrow x2, x1 \leftarrow x3\} \\
& op_t(x1, \overbrace{mult(x3, mult(rd(x2, x3), i(x4)))})
\end{aligned}
$$

Lemma 2647: $rd(x1, rd(x2, op_l(x2, x4, x3))) = mult(x2, mult(op_l(i(x2), x3, x4), x1))$

$$
\begin{aligned}
& \underbrace{rd(x1, rd(x2, op_l(x2, x4, x3)))} \\
= & \quad \text{by Lemma 2531 RL with } \{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{mult(rd(x1, op_l(x2, x3, x4)), x2)}} \\
= & \quad \text{by Lemma 13 RL with } \{x2 \leftarrow rd(x1, op_l(x2, x3, x4)), x1 \leftarrow x2\} \\
& \overbrace{mult(x2, \underbrace{op_t(rd(x1, op_l(x2, x3, x4)), x2)})} \\
= & \quad \text{by Lemma 2638 LR with } \{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& mult(x2, \overbrace{mult(op_l(i(x2), x3, x4), x1)})
\end{aligned}
$$

Lemma 2648: $mult(rd(x1, op_l(x1, x2, x3)), x4) = rd(x4, rd(x1, op_l(x1, x3, x2)))$

$$
\begin{aligned}
& \underbrace{mult(rd(x1, op_l(x1, x2, x3)), x4)} \\
= & \quad \text{by Lemma 2000 LR with } \{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{mult(x1, mult(op_l(i(x1), x2, x3), x4)}} \\
= & \quad \text{by Lemma 2647 RL with } \{x3 \leftarrow x2, x4 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow x4\} \\
& \overbrace{rd(x4, rd(x1, op_l(x1, x3, x2)))}
\end{aligned}
$$

Lemma 2649: $op_t(x1, rd(op_l(x2, x3, x4), x2)) = op_t(rd(rd(x1, rd(x2, op_l(x2, x3, x4))), rd(x2, op_l(x2, x4, x3))), rd(rd(x2, op_l(x2, x$

$$
\begin{aligned}
& op_t(x1, \underbrace{rd(op_l(x2, x3, x4), x2)}) \\
= & \quad \text{by Lemma 10 RL with } \{x2 \leftarrow op_l(x2, x3, x4), x1 \leftarrow x2\} \\
& \underbrace{op_t(x1, \overbrace{i(rd(x2, op_l(x2, x3, x4)))})} \\
= & \quad \text{by Lemma 1006 RL with } \{x2 \leftarrow x1, x1 \leftarrow rd(x2, op_l(x2, x3, x4))\} \\
& \overbrace{op_t(\underbrace{mult(rd(x2, op_l(x2, x3, x4)), rd(x1, rd(x2, op_l(x2, x3, x4))))}, rd(rd(x2, op_l(x2, x3, x4)), x1))} \\
= & \quad \text{by Lemma 2648 LR with } \{x4 \leftarrow rd(x1, rd(x2, op_l(x2, x3, x4))), x3 \leftarrow x4, x2 \leftarrow x3, x1 \leftarrow x2\} \\
& op_t(\overbrace{rd(rd(x1, rd(x2, op_l(x2, x3, x4))), rd(x2, op_l(x2, x4, x3)))}, rd(rd(x2, op_l(x2, x3, x4)), x1))
\end{aligned}
$$

Lemma 2650: $op_t(x1, asoc(x3, x4, x2)) = op_t(rd(rd(x1, rd(x2, op_l(x2, x3, x4))), rd(x2, op_l(x2, x4, x3))), rd(rd(x2, op_l(x2, x3, x4))$

$$
\begin{aligned}
& \underbrace{op_t(x1, asoc(x3, x4, x2))} \\
= & \quad \text{by Lemma 2645 RL with } \{x4 \leftarrow x3, x3 \leftarrow x4, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_t(x1, \underbrace{rd(x2, op_l(x2, x4, x3))})} \\
= & \quad \text{by Lemma 2449 RL with } \{x3 \leftarrow x4, x2 \leftarrow x3, x1 \leftarrow x2\} \\
& \underbrace{op_t(x1, \overbrace{rd(op_l(x2, x3, x4), x2)})} \\
= & \quad \text{by Lemma 2649 LR with } \{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_t(rd(rd(x1, rd(x2, op_l(x2, x3, x4))), rd(x2, op_l(x2, x4, x3))), rd(rd(x2, op_l(x2, x3, x4)), x1))}
\end{aligned}
$$

Lemma 2651: $op_t(x1, asoc(x3, x4, x2)) = mult(rd(x1, rd(x2, op_l(x2, x3, x4))), rd(x2, op_l(x2, x3, x4)))$

$$
\begin{aligned}
& \underbrace{op_t(x1, asoc(x3, x4, x2))} \\
= & \quad \text{by Lemma 2650 LR with } \{x2 \leftarrow x2, x4 \leftarrow x4, x3 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_t(rd(rd(x1, rd(x2, op_l(x2, x3, x4))), rd(x2, op_l(x2, x4, x3))), rd(rd(x2, op_l(x2, x3, x4)), x1))}}^{} \\
= & \quad \text{by Lemma 1426 RL with } \{x3 \leftarrow rd(x2, op_l(x2, x4, x3)), x1 \leftarrow rd(x2, op_l(x2, x3, x4)), x2 \leftarrow x1\} \\
& \overbrace{mult(rd(x1, rd(x2, op_l(x2, x3, x4))), \underbrace{i(rd(x2, op_l(x2, x4, x3)))})}^{} \\
= & \quad \text{by Lemma 10 LR with } \{x2 \leftarrow op_l(x2, x4, x3), x1 \leftarrow x2\} \\
& mult(rd(x1, rd(x2, op_l(x2, x3, x4))), \overbrace{\underbrace{rd(op_l(x2, x4, x3), x2)}}^{}) \\
= & \quad \text{by Lemma 2449 LR with } \{x3 \leftarrow x3, x2 \leftarrow x4, x1 \leftarrow x2\} \\
& mult(rd(x1, rd(x2, op_l(x2, x3, x4))), \overbrace{rd(x2, op_l(x2, x3, x4))}^{})
\end{aligned}
$$

Lemma 2652: $op_t(x1, asoc(x2, x3, x4)) = x1$

$$
\begin{aligned}
& \underbrace{op_t(x1, asoc(x2, x3, x4))} \\
= & \quad \text{by Lemma 2651 LR with } \{x2 \leftarrow x4, x4 \leftarrow x3, x3 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{mult(rd(x1, rd(x4, op_l(x4, x2, x3))), rd(x4, op_l(x4, x2, x3)))}}^{} \\
= & \quad \text{by Axiom 4 RL with } \{x2 \leftarrow rd(x4, op_l(x4, x2, x3)), x1 \leftarrow x1\} \\
& \overbrace{x1}^{}
\end{aligned}
$$

Lemma 2653: $op_t(asoc(x1, x2, x3), x4) = rd(asoc(x1, x2, x3), unit())$

$$
\begin{aligned}
& \underbrace{op_t(asoc(x1, x2, x3), x4)} \\
= & \quad \text{by Lemma 390 RL with } \{x2 \leftarrow x4, x1 \leftarrow asoc(x1, x2, x3)\} \\
& \overbrace{rd(asoc(x1, x2, x3), rd(\underbrace{op_t(x4, asoc(x1, x2, x3))}, x4))}^{} \\
= & \quad \text{by Lemma 2652 LR with } \{x4 \leftarrow x3, x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x4\} \\
& rd(asoc(x1, x2, x3), \underbrace{rd(\overbrace{x4}^{}, x4)}) \\
= & \quad \text{by Lemma 2 LR with } \{x1 \leftarrow x4\} \\
& rd(asoc(x1, x2, x3), \overbrace{unit()}^{})
\end{aligned}
$$

Lemma 2654: $op_t(asoc(x1, x2, x3), x4) = asoc(x1, x2, x3)$

$$
\begin{aligned}
& \underbrace{op_t(asoc(x1, x2, x3), x4)} \\
= & \quad \text{by Lemma 2653 LR with } \{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{rd(asoc(x1, x2, x3), unit())}}^{} \\
= & \quad \text{by Lemma 1 LR with } \{x1 \leftarrow asoc(x1, x2, x3)\} \\
& \overbrace{asoc(x1, x2, x3)}^{}
\end{aligned}
$$

Lemma 2655: $op_t(x1, rd(op_r(x2, x3, x4), x2)) = x1$

$$
\begin{aligned}
& \overbrace{op_t(x1, \underbrace{rd(op_r(x2, x3, x4), x2)}_{})}^{} \\
= \quad & \text{by Lemma 2618 RL with } \{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x4\} \\
& \underbrace{op_t(x1, \overbrace{rd(x4, op_l(x4, x3, x2))}^{})}_{} \\
= \quad & \text{by Lemma 2645 LR with } \{x4 \leftarrow x2, x3 \leftarrow x3, x2 \leftarrow x4, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_t(x1, asoc(x2, x3, x4))}_{}}^{} \\
= \quad & \text{by Lemma 2652 LR with } \{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{x1}^{}
\end{aligned}
$$

Lemma 2656: $asoc(x1, x2, x3) = rd(op_l(x4, asoc(x1, x2, x3), x4), mult(i(asoc(x1, x2, x3)), x4))$

$$
\begin{aligned}
& \underbrace{asoc(x1, x2, x3)}_{} \\
= \quad & \text{by Lemma 3 RL with } \{x1 \leftarrow asoc(x1, x2, x3)\} \\
& \overbrace{\underbrace{i(i(asoc(x1, x2, x3)))}_{}}^{} \\
= \quad & \text{by Lemma 20 RL with } \{x2 \leftarrow i(asoc(x1, x2, x3)), x1 \leftarrow x4\} \\
& \overbrace{\underbrace{mult(x4, i(mult(x4, i(asoc(x1, x2, x3)))))}_{}}^{} \\
= \quad & \text{by Lemma 16 RL with } \{x2 \leftarrow mult(x4, i(asoc(x1, x2, x3))), x1 \leftarrow x4\} \\
& \overbrace{\underbrace{i(mult(i(x4), mult(x4, i(asoc(x1, x2, x3)))))}_{}}^{} \\
= \quad & \text{by Axiom 5 LR with } \{x2 \leftarrow mult(i(x4), mult(mult(x4, i(asoc(x1, x2, x3))), x4)), x1 \leftarrow i(mult(i(x4), mult(x4, i(asoc(x1, x2, x \\
& \overbrace{rd(\underbrace{mult(i(mult(i(x4), mult(x4, i(asoc(x1, x2, x3))))), mult(i(x4), mult(mult(x4, i(asoc(x1, x2, x3))), x4)))}_{}, mult(i(x4), mu}^{} \\
= \quad & \text{by Axiom 12 LR with } \{x3 \leftarrow x4, x2 \leftarrow mult(x4, i(asoc(x1, x2, x3))), x1 \leftarrow i(x4)\} \\
& rd(\overbrace{\underbrace{op_l(x4, mult(x4, i(asoc(x1, x2, x3))), i(x4))}_{}}^{}, mult(i(x4), mult(mult(x4, i(asoc(x1, x2, x3))), x4))) \\
= \quad & \text{by Lemma 606 LR with } \{x2 \leftarrow mult(x4, i(asoc(x1, x2, x3))), x1 \leftarrow x4\} \\
& rd(\overbrace{op_l(x4, x4, mult(x4, i(asoc(x1, x2, x3))))}^{}, \underbrace{mult(i(x4), mult(mult(x4, i(asoc(x1, x2, x3))), x4))}_{}) \\
= \quad & \text{by Axiom 10 LR with } \{x2 \leftarrow mult(x4, i(asoc(x1, x2, x3))), x1 \leftarrow x4\} \\
& rd(op_l(x4, x4, mult(x4, i(asoc(x1, x2, x3)))), \overbrace{\underbrace{op_t(mult(x4, i(asoc(x1, x2, x3))), x4)}_{}}^{}) \\
= \quad & \text{by Lemma 673 RL with } \{x2 \leftarrow x4, x1 \leftarrow asoc(x1, x2, x3)\} \\
& rd(\underbrace{op_l(x4, x4, mult(x4, i(asoc(x1, x2, x3))))}_{}, \overbrace{op_t(mult(i(asoc(x1, x2, x3)), x4), asoc(x1, x2, x3))}^{}) \\
= \quad & \text{by Lemma 624 LR with } \{x2 \leftarrow i(asoc(x1, x2, x3)), x1 \leftarrow x4\} \\
& rd(\overbrace{op_l(x4, x4, i(asoc(x1, x2, x3)))}^{}, op_t(mult(i(asoc(x1, x2, x3)), x4), asoc(x1, x2, x3))) \\
= \quad & \text{by Lemma 608 LR with } \{x2 \leftarrow asoc(x1, x2, x3), x1 \leftarrow x4\} \\
& rd(\overbrace{op_l(x4, asoc(x1, x2, x3), x4)}^{}, \underbrace{op_t(mult(i(asoc(x1, x2, x3)), x4), asoc(x1, x2, x3))}_{}) \\
= \quad & \text{by Lemma 2652 LR with } \{x4 \leftarrow x3, x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow mult(i(asoc(x1, x2, x3)), x4)\} \\
& rd(op_l(x4, asoc(x1, x2, x3), x4), \overbrace{mult(i(asoc(x1, x2, x3)), x4)}^{})
\end{aligned}
$$

Lemma 2657: $op_r(asoc(x1,x2,x3),x4,x4) = asoc(x1,x2,x3)$

$$
\begin{aligned}
& \underbrace{op_r(asoc(x1,x2,x3),x4,x4)} \\
= \quad & \text{by Lemma 2495 RL with } \{x3 \leftarrow x4, x2 \leftarrow asoc(x1,x2,x3), x1 \leftarrow x4\} \\
& \overbrace{\underbrace{rd(x4, mult(i(asoc(x1,x2,x3)), op_l(x4,x4,asoc(x1,x2,x3))))}} \\
= \quad & \text{by Lemma 2532 LR with } \{x2 \leftarrow asoc(x1,x2,x3), x3 \leftarrow x4, x4 \leftarrow i(asoc(x1,x2,x3)), x1 \leftarrow x4\} \\
& \overbrace{\underbrace{rd(op_l(x4,asoc(x1,x2,x3),x4), mult(i(asoc(x1,x2,x3)),x4))}} \\
= \quad & \text{by Lemma 2656 RL with } \{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{asoc(x1,x2,x3)}
\end{aligned}
$$

Lemma 2658: $mult(i(asoc(x1,x2,x3)),x4) = rd(x4,asoc(x1,x2,x3))$

$$
\begin{aligned}
& \underbrace{mult(i(asoc(x1,x2,x3)),x4)} \\
= \quad & \text{by Lemma 14 RL with } \{x2 \leftarrow asoc(x1,x2,x3), x1 \leftarrow x4\} \\
& \overbrace{\underbrace{op_t(rd(x4,asoc(x1,x2,x3)),asoc(x1,x2,x3))}} \\
= \quad & \text{by Lemma 2652 LR with } \{x4 \leftarrow x3, x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow rd(x4,asoc(x1,x2,x3))\} \\
& \overbrace{rd(x4,asoc(x1,x2,x3))}
\end{aligned}
$$

Lemma 2659: $rd(asoc(x1,x2,x3),x4) = mult(i(x4),asoc(x1,x2,x3))$

$$
\begin{aligned}
& \underbrace{rd(asoc(x1,x2,x3),x4)} \\
= \quad & \text{by Lemma 2652 RL with } \{x4 \leftarrow x3, x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow rd(asoc(x1,x2,x3),x4)\} \\
& \overbrace{\underbrace{op_t(rd(asoc(x1,x2,x3),x4),asoc(x1,x2,x3))}} \\
= \quad & \text{by Lemma 340 RL with } \{x2 \leftarrow asoc(x1,x2,x3), x1 \leftarrow x4\} \\
& \overbrace{mult(\underbrace{op_t(i(x4),asoc(x1,x2,x3))},asoc(x1,x2,x3))} \\
= \quad & \text{by Lemma 2652 LR with } \{x4 \leftarrow x3, x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow i(x4)\} \\
& mult(\overbrace{i(x4)},asoc(x1,x2,x3))
\end{aligned}
$$

Lemma 2660: $rd(asoc(x1,x2,x3),i(x4)) = mult(x4,asoc(x1,x2,x3))$

$$
\begin{aligned}
& \underbrace{rd(asoc(x1,x2,x3),i(x4))} \\
= \quad & \text{by Lemma 2652 RL with } \{x4 \leftarrow x3, x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow rd(asoc(x1,x2,x3),i(x4))\} \\
& \overbrace{\underbrace{op_t(rd(asoc(x1,x2,x3),i(x4)),asoc(x1,x2,x3))}} \\
= \quad & \text{by Lemma 341 LR with } \{x2 \leftarrow x4, x1 \leftarrow asoc(x1,x2,x3)\} \\
& \overbrace{mult(\underbrace{op_t(x4,asoc(x1,x2,x3))},asoc(x1,x2,x3))} \\
= \quad & \text{by Lemma 2652 LR with } \{x4 \leftarrow x3, x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x4\} \\
& mult(\overbrace{x4},asoc(x1,x2,x3))
\end{aligned}
$$

Lemma 2661: $mult(asoc(x1, x2, x3), x4) = rd(asoc(x1, x2, x3), i(op_t(x4, asoc(x1, x2, x3))))$

$\underbrace{mult(asoc(x1, x2, x3), x4)}$

$=$ by Lemma 2652 RL with $\{x4 \leftarrow x3, x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow mult(asoc(x1, x2, x3), x4)\}$

$\overbrace{\underbrace{op_t(mult(asoc(x1, x2, x3), x4), asoc(x1, x2, x3))}}$

$=$ by Lemma 750 RL with $\{x2 \leftarrow x4, x1 \leftarrow asoc(x1, x2, x3)\}$

$\overbrace{rd(asoc(x1, x2, x3), \underbrace{op_t(i(op_t(x4, asoc(x1, x2, x3))), asoc(x1, x2, x3))})}$

$=$ by Lemma 2652 LR with $\{x4 \leftarrow x3, x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow i(op_t(x4, asoc(x1, x2, x3)))\}$

$rd(asoc(x1, x2, x3), \overbrace{i(op_t(x4, asoc(x1, x2, x3))))}$

Lemma 2662: $mult(asoc(x1, x2, x3), x4) = mult(x4, asoc(x1, x2, x3))$

$\underbrace{mult(asoc(x1, x2, x3), x4)}$

$=$ by Lemma 2661 LR with $\{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{rd(asoc(x1, x2, x3), i(\underbrace{op_t(x4, asoc(x1, x2, x3))}))}$

$=$ by Lemma 2652 LR with $\{x4 \leftarrow x3, x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x4\}$

$\underbrace{rd(asoc(x1, x2, x3), i(\overbrace{x4}))}$

$=$ by Lemma 2660 LR with $\{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{mult(x4, asoc(x1, x2, x3))}$

Lemma 2663: $rd(asoc(x1,x2,x3), op_t(x4, asoc(x1,x2,x3))) = mult(asoc(x1,x2,x3), rd(asoc(x1,x2,x3), mult(x4, asoc(x1,x2,x3$

$rd(asoc(x1,x2,x3), \underbrace{op_t(x4, asoc(x1,x2,x3))})$

$=$ by Lemma 2652 RL with $\{x4 \leftarrow x3, x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow op_t(x4, asoc(x1,x2,x3))\}$

$rd(asoc(x1,x2,x3), \overbrace{op_t(\underbrace{op_t(x4, asoc(x1,x2,x3))}, asoc(x1,x2,x3))})$

$=$ by Lemma 3 RL with $\{x1 \leftarrow op_t(x4, asoc(x1,x2,x3))\}$

$\underbrace{rd(asoc(x1,x2,x3), op_t(\overbrace{i(i(op_t(x4, asoc(x1,x2,x3))))}, asoc(x1,x2,x3)))}$

$=$ by Lemma 402 RL with $\{x2 \leftarrow i(i(op_t(x4, asoc(x1,x2,x3)))), x1 \leftarrow asoc(x1,x2,x3)\}$

$\overbrace{mult(asoc(x1,x2,x3), \underbrace{op_r(i(op_t(x4, asoc(x1,x2,x3))), asoc(x1,x2,x3), i(asoc(x1,x2,x3)))})}$

$=$ by Lemma 321 RL with $\{x2 \leftarrow asoc(x1,x2,x3), x1 \leftarrow i(op_t(x4, asoc(x1,x2,x3)))\}$

$mult(asoc(x1,x2,x3), \overbrace{op_r(i(op_t(x4, asoc(x1,x2,x3))), i(asoc(x1,x2,x3)), \underbrace{asoc(x1,x2,x3)})})$

$=$ by Lemma 3 RL with $\{x1 \leftarrow asoc(x1,x2,x3)\}$

$mult(asoc(x1,x2,x3), op_r(\underbrace{i(op_t(x4, asoc(x1,x2,x3)))}, i(asoc(x1,x2,x3)), \overbrace{i(i(asoc(x1,x2,x3)))}))$

$=$ by Lemma 15 RL with $\{x2 \leftarrow asoc(x1,x2,x3), x1 \leftarrow x4\}$

$mult(asoc(x1,x2,x3), \underbrace{op_r(\overbrace{op_t(i(x4), i(asoc(x1,x2,x3)))}, i(asoc(x1,x2,x3)), i(i(asoc(x1,x2,x3))))})$

$=$ by Lemma 319 RL with $\{x2 \leftarrow i(asoc(x1,x2,x3)), x1 \leftarrow i(x4)\}$

$mult(asoc(x1,x2,x3), \overbrace{rd(op_t(\underbrace{mult(i(x4), i(asoc(x1,x2,x3)))}, i(asoc(x1,x2,x3))), i(asoc(x1,x2,x3)))})$

$=$ by Axiom 11 LR with $\{x2 \leftarrow asoc(x1,x2,x3), x1 \leftarrow x4\}$

$mult(asoc(x1,x2,x3), \underbrace{rd(op_t(\overbrace{i(mult(x4, asoc(x1,x2,x3)))}, i(asoc(x1,x2,x3))), i(asoc(x1,x2,x3))))}$

$=$ by Lemma 67 LR with $\{x3 \leftarrow asoc(x1,x2,x3), x2 \leftarrow i(asoc(x1,x2,x3)), x1 \leftarrow mult(x4, asoc(x1,x2,x3))\}$

$mult(asoc(x1,x2,x3), \overbrace{rd(asoc(x1,x2,x3), op_t(mult(x4, asoc(x1,x2,x3)), \underbrace{i(i(asoc(x1,x2,x3)))}))})$

$=$ by Lemma 3 LR with $\{x1 \leftarrow asoc(x1,x2,x3)\}$

$mult(asoc(x1,x2,x3), rd(asoc(x1,x2,x3), \underbrace{op_t(mult(x4, asoc(x1,x2,x3)), \overbrace{asoc(x1,x2,x3)})}))$

$=$ by Lemma 2652 LR with $\{x4 \leftarrow x3, x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow mult(x4, asoc(x1,x2,x3))\}$

$mult(asoc(x1,x2,x3), rd(asoc(x1,x2,x3), \overbrace{mult(x4, asoc(x1,x2,x3))}))$

Lemma 2664: $rd(asoc(x1,x2,x3), x4) = mult(asoc(x1,x2,x3), i(x4))$

$rd(asoc(x1,x2,x3), \underbrace{x4})$

$=$ by Lemma 2652 RL with $\{x4 \leftarrow x3, x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x4\}$

$\underbrace{rd(asoc(x1,x2,x3), \overbrace{op_t(x4, asoc(x1,x2,x3))})}$

$=$ by Lemma 2663 LR with $\{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{mult(asoc(x1,x2,x3), \underbrace{rd(asoc(x1,x2,x3), mult(x4, asoc(x1,x2,x3)))})}$

$=$ by Lemma 9 LR with $\{x2 \leftarrow x4, x1 \leftarrow asoc(x1,x2,x3)\}$

$mult(asoc(x1,x2,x3), \overbrace{i(x4)})$

Lemma 2665: $rd(mult(asoc(x1,x2,x3),x4),x4) = mult(x4, rd(asoc(x1,x2,x3),x4))$

$$
\begin{aligned}
& \underbrace{rd(mult(asoc(x1,x2,x3),x4),x4)} \\
= & \quad \text{by Lemma 2652 RL with } \{x4 \leftarrow x3, x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow mult(asoc(x1,x2,x3),x4)\} \\
& \overbrace{rd(\underbrace{op_t(mult(asoc(x1,x2,x3),x4),asoc(x1,x2,x3))},x4)} \\
= & \quad \text{by Lemma 895 RL with } \{x2 \leftarrow asoc(x1,x2,x3), x1 \leftarrow x4\} \\
& \overbrace{mult(\underbrace{op_t(x4,asoc(x1,x2,x3))}, rd(asoc(x1,x2,x3),x4))} \\
= & \quad \text{by Lemma 2652 LR with } \{x4 \leftarrow x3, x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x4\} \\
& mult(\overbrace{x4}, rd(asoc(x1,x2,x3),x4))
\end{aligned}
$$

Lemma 2666: $mult(asoc(x1,x2,x3), mult(x4,x4)) = mult(mult(x4,asoc(x1,x2,x3)),x4)$

$$
\begin{aligned}
& \underbrace{mult(asoc(x1,x2,x3), mult(x4,x4))} \\
= & \quad \text{by Lemma 2652 RL with } \{x4 \leftarrow x3, x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow mult(asoc(x1,x2,x3), mult(x4,x4))\} \\
& \overbrace{\underbrace{op_t(mult(asoc(x1,x2,x3), mult(x4,x4)), asoc(x1,x2,x3))}} \\
= & \quad \text{by Lemma 320 RL with } \{x2 \leftarrow asoc(x1,x2,x3), x1 \leftarrow mult(x4,x4)\} \\
& \overbrace{op_r(\underbrace{mult(mult(x4,x4),asoc(x1,x2,x3))}, asoc(x1,x2,x3), i(asoc(x1,x2,x3)))} \\
= & \quad \text{by Lemma 24 LR with } \{x2 \leftarrow asoc(x1,x2,x3), x1 \leftarrow x4\} \\
& op_r(\overbrace{mult(x4,\underbrace{mult(x4,asoc(x1,x2,x3))})}, asoc(x1,x2,x3), i(asoc(x1,x2,x3))) \\
= & \quad \text{by Lemma 13 RL with } \{x2 \leftarrow x4, x1 \leftarrow asoc(x1,x2,x3)\} \\
& \underbrace{op_r(mult(x4,\overbrace{mult(asoc(x1,x2,x3), op_t(x4,asoc(x1,x2,x3)))}), asoc(x1,x2,x3), i(asoc(x1,x2,x3)))} \\
= & \quad \text{by Lemma 1911 RL with } \{x2 \leftarrow asoc(x1,x2,x3), x1 \leftarrow x4\} \\
& \overbrace{mult(\underbrace{op_t(mult(x4,asoc(x1,x2,x3)),asoc(x1,x2,x3))}, op_t(x4,asoc(x1,x2,x3)))} \\
= & \quad \text{by Lemma 2652 LR with } \{x4 \leftarrow x3, x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow mult(x4,asoc(x1,x2,x3))\} \\
& mult(\overbrace{mult(x4,asoc(x1,x2,x3))}, \underbrace{op_t(x4,asoc(x1,x2,x3))}) \\
= & \quad \text{by Lemma 2652 LR with } \{x4 \leftarrow x3, x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x4\} \\
& mult(mult(x4,asoc(x1,x2,x3)), \overbrace{x4})
\end{aligned}
$$

Lemma 2667: $rd(x1, rd(op_r(x2,x3,x4),x2)) = mult(rd(x2,op_r(x2,x3,x4)),x1)$

$$
\begin{aligned}
& \underbrace{rd(x1, rd(op_r(x2,x3,x4),x2))} \\
= & \quad \text{by Lemma 2655 RL with } \{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow rd(x1, rd(op_r(x2,x3,x4),x2))\} \\
& \overbrace{\underbrace{op_t(rd(x1, rd(op_r(x2,x3,x4),x2)), rd(op_r(x2,x3,x4),x2))}} \\
= & \quad \text{by Lemma 14 LR with } \{x2 \leftarrow rd(op_r(x2,x3,x4),x2), x1 \leftarrow x1\} \\
& \overbrace{mult(\underbrace{i(rd(op_r(x2,x3,x4),x2))}, x1)} \\
= & \quad \text{by Lemma 10 LR with } \{x2 \leftarrow x2, x1 \leftarrow op_r(x2,x3,x4)\} \\
& mult(\overbrace{rd(x2,op_r(x2,x3,x4))}, x1)
\end{aligned}
$$

Lemma 2668: $op_t(x1, x2) = op_t(x1, rd(mult(op_r(x3, x4, x2), x2), x3))$

$\overbrace{op_t(x1, x2)}$

$=$ by Lemma 1731 RL with $\{x3 \leftarrow x2, x2 \leftarrow rd(x3, x4), x1 \leftarrow x1\}$

$\overbrace{op_t(x1, rd(mult(rd(x3, x4), x2), \underbrace{rd(x3, x4)}))}$

$=$ by Axiom 2 LR with $\{x2 \leftarrow mult(op_r(x3, x4, x2), x2), x1 \leftarrow rd(x3, x4)\}$

$\underbrace{op_t(x1, rd(mult(rd(x3, x4), x2), \overbrace{mult(i(mult(op_r(x3, x4, x2), x2)), mult(mult(op_r(x3, x4, x2), x2), rd(x3, x4)))}))}$

$=$ by Lemma 2646 LR with $\{x4 \leftarrow rd(x3, x4), x3 \leftarrow mult(op_r(x3, x4, x2), x2), x2 \leftarrow mult(rd(x3, x4), x2), x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_t(x1, mult(mult(op_r(x3, x4, x2), x2), mult(rd(mult(rd(x3, x4), x2), mult(op_r(x3, x4, x2), x2)), i(rd(x3, x4)))))}}$

$=$ by Lemma 1774 LR with $\{x4 \leftarrow rd(x3, x4), x3 \leftarrow rd(mult(rd(x3, x4), x2), mult(op_r(x3, x4, x2), x2)), x2 \leftarrow mult(op_r(x3, x4,$

$\overbrace{op_t(x1, rd(mult(op_r(x3, x4, x2), x2), mult(\underbrace{i(rd(mult(rd(x3, x4), x2), mult(op_r(x3, x4, x2), x2)))}, rd(x3, x4))))}$

$=$ by Lemma 10 LR with $\{x2 \leftarrow mult(op_r(x3, x4, x2), x2), x1 \leftarrow mult(rd(x3, x4), x2)\}$

$op_t(x1, rd(mult(op_r(x3, x4, x2), x2), mult(\underbrace{rd(mult(op_r(x3, x4, x2), x2), mult(rd(x3, x4), x2))}, rd(x3, x4))))$

$=$ by Lemma 676 RL with $\{x3 \leftarrow x2, x2 \leftarrow rd(x3, x4), x1 \leftarrow op_r(x3, x4, x2)\}$

$op_t(x1, rd(mult(op_r(x3, x4, x2), x2), \underbrace{mult(\overbrace{op_r(rd(op_r(x3, x4, x2), rd(x3, x4)), rd(x3, x4), x2)}, rd(x3, x4))))$

$=$ by Lemma 2570 LR with $\{x3 \leftarrow x2, x2 \leftarrow rd(x3, x4), x1 \leftarrow rd(op_r(x3, x4, x2), rd(x3, x4))\}$

$op_t(x1, rd(mult(op_r(x3, x4, x2), x2), \overbrace{op_r(\underbrace{mult(rd(op_r(x3, x4, x2), rd(x3, x4)), rd(x3, x4))}, rd(x3, x4), x2)}))$

$=$ by Axiom 4 RL with $\{x2 \leftarrow rd(x3, x4), x1 \leftarrow op_r(x3, x4, x2)\}$

$op_t(x1, rd(mult(op_r(x3, x4, x2), x2), \underbrace{op_r(\overbrace{op_r(x3, x4, x2)}, rd(x3, x4), x2)}))$

$=$ by Lemma 2436 RL with $\{x4 \leftarrow x2, x3 \leftarrow x3, x2 \leftarrow x4, x1 \leftarrow op_r(x3, x4, x2)\}$

$op_t(x1, rd(mult(op_r(x3, x4, x2), x2), \overbrace{op_r(\underbrace{op_r(x3, x4, x2)}, rd(x4, x3), i(x2))}))$

$=$ by Lemma 2429 RL with $\{x3 \leftarrow x2, x2 \leftarrow x4, x1 \leftarrow x3\}$

$op_t(x1, rd(mult(op_r(x3, x4, x2), x2), \underbrace{op_r(\overbrace{op_r(x3, rd(x4, x3), x2)}, rd(x4, x3), i(x2))}))$

$=$ by Lemma 2584 RL with $\{x3 \leftarrow x2, x2 \leftarrow rd(x4, x3), x1 \leftarrow x3\}$

$op_t(x1, rd(mult(op_r(x3, x4, x2), x2), \overbrace{x3}))$

Lemma 2669: $mult(i(x1), mult(op_r(x1, x2, x3), x3)) = op_l(x3, x1, x2)$

$\underbrace{mult(i(x1), mult(op_r(x1, x2, x3), x3))}$

$=$ by Lemma 380 RL with $\{x2 \leftarrow x1, x1 \leftarrow mult(op_r(x1, x2, x3), x3)\}$

$\overbrace{rd(mult(op_r(x1, x2, x3), x3), \underbrace{op_t(x1, rd(mult(op_r(x1, x2, x3), x3), x1))})}$

$=$ by Lemma 2668 RL with $\{x4 \leftarrow x2, x3 \leftarrow x1, x2 \leftarrow x3, x1 \leftarrow x1\}$

$\underbrace{rd(mult(op_r(x1, x2, x3), x3), \overbrace{op_t(x1, x3)})}$

$=$ by Lemma 2625 RL with $\{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x3\}$

$\overbrace{op_l(x3, x1, x2)}$

Lemma 2670: $mult(x1, x2) = op_l(mult(x2, x1), op_l(x2, x3, x4), x1)$

$mult(x1, \underbrace{x2}_{})$

= by Lemma 138 RL with $\{x3 \leftarrow x4, x2 \leftarrow x3, x1 \leftarrow x2\}$

$\underbrace{mult(x1, \overbrace{op_l(op_l(x2, x3, x4), x4, x3)}^{})}_{}$

= by Lemma 13 RL with $\{x2 \leftarrow x1, x1 \leftarrow op_l(op_l(x2, x3, x4), x4, x3)\}$

$\overbrace{mult(op_l(op_l(x2, x3, x4), x4, x3), \underbrace{op_t(x1, op_l(op_l(x2, x3, x4), x4, x3))}_{})}^{}$

= by Lemma 2632 LR with $\{x4 \leftarrow x3, x3 \leftarrow x4, x2 \leftarrow op_l(x2, x3, x4), x1 \leftarrow x1\}$

$\underbrace{mult(op_l(op_l(x2, x3, x4), x4, x3), \overbrace{op_t(x1, op_l(x2, x3, x4))}^{})}_{}$

= by Lemma 1664 RL with $\{x3 \leftarrow op_l(x2, x3, x4), x2 \leftarrow x1, x1 \leftarrow op_l(op_l(x2, x3, x4), x4, x3)\}$

$\overbrace{rd(mult(op_l(op_l(x2, x3, x4), x4, x3), x1), rd(op_t(op_l(x2, x3, x4), x1), op_l(x2, x3, x4)))}^{}$

= by Lemma 1717 RL with $\{x3 \leftarrow mult(op_l(op_l(x2, x3, x4), x4, x3), x1), x2 \leftarrow x1, x1 \leftarrow op_l(x2, x3, x4)\}$

$\overbrace{mult(rd(op_l(x2, x3, x4), op_t(op_l(x2, x3, x4), x1)), mult(op_l(op_l(x2, x3, x4), x4, x3), x1))}^{}$

= by Lemma 1095 RL with $\{x3 \leftarrow mult(op_l(op_l(x2, x3, x4), x4, x3), x1), x2 \leftarrow x1, x1 \leftarrow op_l(x2, x3, x4)\}$

$\overbrace{mult(i(op_t(op_l(x2, x3, x4), x1)), mult(\underbrace{op_l(x2, x3, x4)}_{}, mult(op_l(op_l(x2, x3, x4), x4, x3), x1)))}^{}$

= by Axiom 5 LR with $\{x2 \leftarrow x1, x1 \leftarrow op_l(x2, x3, x4)\}$

$mult(i(op_t(op_l(x2, x3, x4), x1)), mult(\overbrace{rd(mult(op_l(x2, x3, x4), x1), x1)}^{}, mult(op_l(op_l(x2, x3, x4), x4, x3), x1)))$

= by Lemma 2612 LR with $\{x4 \leftarrow x3, x3 \leftarrow x4, x1 \leftarrow x1, x2 \leftarrow op_l(x2, x3, x4)\}$

$\underbrace{mult(i(op_t(op_l(x2, x3, x4), x1)), mult(\overbrace{op_r(op_t(op_l(x2, x3, x4), x1), x1, mult(op_l(op_l(x2, x3, x4), x4, x3), x1))}^{}, mult(op_l(op_l(x$

= by Lemma 2669 LR with $\{x3 \leftarrow mult(op_l(op_l(x2, x3, x4), x4, x3), x1), x2 \leftarrow x1, x1 \leftarrow op_t(op_l(x2, x3, x4), x1)\}$

$\overbrace{op_l(mult(op_l(op_l(x2, x3, x4), x4, x3), x1), op_t(op_l(x2, x3, x4), x1), x1)}^{}$

= by Lemma 2034 LR with $\{x3 \leftarrow x1, x2 \leftarrow op_l(x2, x3, x4), x1 \leftarrow mult(op_l(op_l(x2, x3, x4), x4, x3), x1)\}$

$\overbrace{op_l(mult(\underbrace{op_l(op_l(x2, x3, x4), x4, x3)}_{}, x1), op_l(x2, x3, x4), x1)}^{}$

= by Lemma 138 LR with $\{x3 \leftarrow x4, x2 \leftarrow x3, x1 \leftarrow x2\}$

$op_l(mult(\overbrace{x2}^{}, x1), op_l(x2, x3, x4), x1)$

Lemma 2671: $op_l(x1, rd(x2, x3), op_t(i(x2), x4)) = op_l(x1, op_t(i(x2), x4), x3)$

$\underbrace{op_l(x1, rd(x2, x3), op_t(i(x2), x4))}_{}$

= by Lemma 2485 RL with $\{x4 \leftarrow x4, x3 \leftarrow i(x2), x2 \leftarrow rd(x2, x3), x1 \leftarrow x1\}$

$\overbrace{op_l(x1, \underbrace{mult(rd(x2, x3), i(x2))}_{}, op_t(i(x2), x4))}^{}$

= by Lemma 22 LR with $\{x2 \leftarrow x3, x1 \leftarrow x2\}$

$\underbrace{op_l(x1, \overbrace{i(x3)}^{}, op_t(i(x2), x4))}_{}$

= by Lemma 2031 LR with $\{x3 \leftarrow op_t(i(x2), x4), x2 \leftarrow x3, x1 \leftarrow x1\}$

$\overbrace{op_l(x1, op_t(i(x2), x4), x3)}^{}$

Lemma 2672: $op_l(x1, x2, op_l(x3, x4, x2)) = op_l(x1, x2, rd(op_l(mult(x3, x2), x4, x2), x2))$

$\overbrace{op_l(x1, x2, op_l(x3, x4, x2))}$

$=$ by Lemma 2437 RL with $\{x3 \leftarrow op_l(x3, x4, x2), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{rd(x1, rd(x2, op_r(x2, \underbrace{op_l(x3, x4, x2)}, x1)))}$

$=$ by Lemma 3 RL with $\{x1 \leftarrow op_l(x3, x4, x2)\}$

$rd(x1, rd(x2, \underbrace{op_r(x2, \overbrace{i(i(op_l(x3, x4, x2)))}, x1)}))$

$=$ by Lemma 2179 LR with $\{x4 \leftarrow x4, x3 \leftarrow x1, x2 \leftarrow i(op_l(x3, x4, x2)), x1 \leftarrow x2\}$

$rd(x1, rd(x2, \overbrace{op_r(x2, \underbrace{rd(asoc(x4, x2, x2), i(op_l(x3, x4, x2)))}, x1)}))$

$=$ by Lemma 38 LR with $\{x1 \leftarrow op_l(x3, x4, x2), x2 \leftarrow asoc(x4, x2, x2)\}$

$rd(x1, rd(x2, op_r(x2, \overbrace{rd(op_l(x3, x4, x2), \underbrace{i(asoc(x4, x2, x2))})}, x1)))$

$=$ by Lemma 759 LR with $\{x2 \leftarrow x2, x1 \leftarrow x4\}$

$\underbrace{rd(x1, rd(x2, op_r(x2, rd(op_l(x3, x4, x2), \overbrace{asoc(x2, x4, x2)}), x1)))}$

$=$ by Lemma 2437 LR with $\{x3 \leftarrow rd(op_l(x3, x4, x2)), asoc(x2, x4, x2)), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_l(x1, x2, \underbrace{rd(op_l(x3, x4, x2), asoc(x2, x4, x2))})}$

$=$ by Lemma 1300 LR with $\{x2 \leftarrow x4, x3 \leftarrow x2, x1 \leftarrow op_l(x3, x4, x2)\}$

$op_l(x1, x2, \overbrace{mult(op_l(x3, x4, x2), \underbrace{asoc(x4, x2, x2)})})$

$=$ by Axiom 4 LR with $\{x2 \leftarrow op_l(x3, x4, x2), x1 \leftarrow asoc(x4, x2, x2)\}$

$op_l(x1, x2, mult(op_l(x3, x4, x2), \overbrace{mult(rd(asoc(x4, x2, x2), \underbrace{op_l(x3, x4, x2)}), op_l(x3, x4, x2))}))$

$=$ by Axiom 5 LR with $\{x2 \leftarrow x2, x1 \leftarrow op_l(x3, x4, x2)\}$

$op_l(x1, x2, mult(op_l(x3, x4, x2), mult(\underbrace{rd(asoc(x4, x2, x2), \overbrace{rd(mult(op_l(x3, x4, x2), x2), x2)})}, op_l(x3, x4, x2))))$

$=$ by Lemma 1569 LR with $\{x3 \leftarrow mult(op_l(x3, x4, x2), x2), x2 \leftarrow x2, x1 \leftarrow x4\}$

$op_l(x1, x2, \underbrace{mult(op_l(x3, x4, x2), mult(\overbrace{rd(op_l(x2, x4, x2), mult(op_l(x3, x4, x2), x2))}, op_l(x3, x4, x2)))})$

$=$ by Lemma 1152 LR with $\{x3 \leftarrow x2, x2 \leftarrow op_l(x2, x4, x2), x1 \leftarrow op_l(x3, x4, x2)\}$

$op_l(x1, x2, \overbrace{rd(\underbrace{mult(op_l(x3, x4, x2), op_l(x2, x4, x2))}, x2)})$

$=$ by Lemma 1789 LR with $\{x2 \leftarrow x2, x3 \leftarrow x4, x1 \leftarrow x3\}$

$op_l(x1, x2, rd(\overbrace{op_l(mult(x3, x2), x4, x2)}, x2))$

Lemma 2673: $rd(x1, op_r(x1, rd(x3, x1), x4)) = rd(x4, op_l(x4, op_l(x1, x2, x1), x3))$

$\underbrace{rd(x1, op_r(x1, rd(x3, x1), x4))}$

$=$ by Axiom 5 LR with $\{x2 \leftarrow op_r(op_r(x1, rd(x3, x1), x4), op_r(rd(i(x2), op_r(x1, rd(x3, x1), x4)), x2, x2), x1), x1 \leftarrow rd(x1, op_r(x$

$\overbrace{rd(\underbrace{mult(rd(x1, op_r(x1, rd(x3, x1), x4)), op_r(op_r(x1, rd(x3, x1), x4), op_r(rd(i(x2), op_r(x1, rd(x3, x1), x4)), x2, x2), x1))}, op_r($

$=$ by Lemma 2442 LR with $\{x3 \leftarrow op_r(rd(i(x2), op_r(x1, rd(x3, x1), x4)), x2, x2), x2 \leftarrow op_r(x1, rd(x3, x1), x4), x1 \leftarrow x1\}$

$rd(\overbrace{\underbrace{op_l(x1, op_r(x1, rd(x3, x1), x4), op_r(rd(i(x2), op_r(x1, rd(x3, x1), x4)), x2, x2))}, op_r(op_r(x1, rd(x3, x1), x4), op_r(rd(i(x2), o}$

$=$ by Lemma 1795 LR with $\{x3 \leftarrow x2, x2 \leftarrow op_r(x1, rd(x3, x1), x4), x1 \leftarrow x1\}$

$rd(\overbrace{op_l(x1, x2, op_r(x1, rd(x3, x1), x4))}, \underbrace{op_r(op_r(x1, rd(x3, x1), x4), op_r(rd(i(x2), op_r(x1, rd(x3, x1), x4)), x2, x2), x1))}$

$=$ by Lemma 1966 LR with $\{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow op_r(x1, rd(x3, x1), x4)\}$

$rd(\underbrace{op_l(x1, x2, op_r(x1, rd(x3, x1), x4))}, \overbrace{op_r(op_r(x1, rd(x3, x1), x4), x2, i(x1)))}$

$=$ by Lemma 2601 LR with $\{x4 \leftarrow x4, x3 \leftarrow rd(x3, x1), x2 \leftarrow x2, x1 \leftarrow x1\}$

$rd(\overbrace{op_l(x1, x2, x1)}, \underbrace{op_r(op_r(x1, rd(x3, x1), x4), x2, i(x1)))}$

$=$ by Axiom 17 RL with $\{x5 \leftarrow x4, x4 \leftarrow rd(x3, x1), x3 \leftarrow i(x1), x2 \leftarrow x2, x1 \leftarrow x1\}$

$rd(op_l(x1, x2, x1), \overbrace{op_r(\underbrace{op_r(x1, x2, i(x1))}, rd(x3, x1), x4)})$

$=$ by Lemma 658 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{rd(op_l(x1, x2, x1), op_r(\overbrace{op_l(x1, x2, x1)}, rd(x3, x1), x4))}$

$=$ by Lemma 2620 RL with $\{x3 \leftarrow rd(x3, x1), x2 \leftarrow op_l(x1, x2, x1), x1 \leftarrow x4\}$

$\overbrace{rd(x4, op_l(x4, op_l(x1, x2, x1), rd(x3, \underbrace{x1})))}$

$=$ by Lemma 138 RL with $\{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x1\}$

$rd(x4, op_l(x4, op_l(x1, x2, x1), rd(x3, \overbrace{op_l(op_l(x1, x2, x1), x1, x2)})))$

$=$ by Lemma 1077 LR with $\{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow op_l(x1, x2, x1)\}$

$rd(x4, \underbrace{op_l(x4, op_l(x1, x2, x1), rd(x3, \overbrace{op_l(op_l(x1, x2, x1), op_l(x1, x2, x1), mult(x1, x2))}))})$

$=$ by Lemma 2501 LR with $\{x4 \leftarrow mult(x1, x2), x3 \leftarrow x3, x2 \leftarrow op_l(x1, x2, x1), x1 \leftarrow x4\}$

$rd(x4, \overbrace{rd(x4, rd(op_l(x1, x2, x1), op_r(op_l(x1, x2, x1), x3, x4)))})$

$=$ by Lemma 2437 LR with $\{x3 \leftarrow x3, x2 \leftarrow op_l(x1, x2, x1), x1 \leftarrow x4\}$

$rd(x4, \overbrace{op_l(x4, op_l(x1, x2, x1), x3)})$

Lemma 2674: $i(op_l(i(x1), op_l(x2, x3, x2), x4)) = mult(rd(x2, op_r(x2, x4, i(x1))), x1)$

$\underbrace{i(op_l(i(x1), op_l(x2, x3, x2), x4))}$

$=$ by Lemma 21 RL with $\{x2 \leftarrow op_l(i(x1), op_l(x2, x3, x2), x4), x1 \leftarrow x1\}$

$\overbrace{mult(\underbrace{rd(i(x1), op_l(i(x1), op_l(x2, x3, x2), x4))}, x1)}$

$=$ by Lemma 2673 RL with $\{x2 \leftarrow x3, x4 \leftarrow i(x1), x3 \leftarrow x4, x1 \leftarrow x2\}$

$mult(\overbrace{rd(x2, \underbrace{op_r(x2, rd(x4, x2), i(x1)))}), x1)}$

$=$ by Lemma 2429 LR with $\{x3 \leftarrow i(x1), x2 \leftarrow x4, x1 \leftarrow x2\}$

$mult(rd(x2, \overbrace{op_r(x2, x4, i(x1))}), x1)$

Lemma 2675: $op_l(x1, op_l(x2, x3, x2), x4) = mult(rd(x2, op_r(x2, x4, i(x1))), x1)$

$$
\begin{aligned}
& op_l(\underbrace{x1}, op_l(x2, x3, x2), x4) \\
= & \quad \text{by Lemma 3 RL with } \{x1 \leftarrow x1\} \\
& \underbrace{op_l(\overbrace{i(i(x1))}, op_l(x2, x3, x2), x4)} \\
= & \quad \text{by Lemma 1049 RL with } \{x3 \leftarrow x4, x2 \leftarrow op_l(x2, x3, x2), x1 \leftarrow i(x1)\} \\
& \overbrace{\underbrace{i(op_l(i(x1), op_l(x2, x3, x2), x4))}} \\
= & \quad \text{by Lemma 2674 LR with } \{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{mult(rd(x2, op_r(x2, x4, i(x1))), x1)}
\end{aligned}
$$

Lemma 2676: $op_l(x1, op_l(x2, x3, x2), x4) = rd(x1, rd(x2, op_r(x2, x4, x1)))$

$$
\begin{aligned}
& \underbrace{op_l(x1, op_l(x2, x3, x2), x4)} \\
= & \quad \text{by Lemma 2675 LR with } \{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{\overbrace{mult(rd(x2, op_r(x2, x4, i(x1))), x1)}} \\
= & \quad \text{by Lemma 2667 RL with } \{x4 \leftarrow i(x1), x3 \leftarrow x4, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{rd(x1, \underbrace{rd(op_r(x2, x4, i(x1)), x2)})} \\
= & \quad \text{by Lemma 2458 LR with } \{x3 \leftarrow x1, x2 \leftarrow x4, x1 \leftarrow x2\} \\
& rd(x1, \overbrace{rd(x2, op_r(x2, x4, x1))})
\end{aligned}
$$

Lemma 2677: $op_l(x1, op_l(x2, x3, x2), x4) = op_l(x1, x2, x4)$

$$
\begin{aligned}
& \underbrace{op_l(x1, op_l(x2, x3, x2), x4)} \\
= & \quad \text{by Lemma 2676 LR with } \{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{\overbrace{rd(x1, rd(x2, op_r(x2, x4, x1)))}} \\
= & \quad \text{by Lemma 2437 LR with } \{x3 \leftarrow x4, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x2, x4)}
\end{aligned}
$$

Lemma 2678: $op_r(x1, op_l(x2, x3, x2), x4) = op_r(x1, x2, x4)$

$$
\begin{aligned}
& \underbrace{op_r(x1, op_l(x2, x3, x2), x4)} \\
= & \quad \text{by Lemma 2495 RL with } \{x3 \leftarrow op_l(x2, x3, x2), x2 \leftarrow x1, x1 \leftarrow x4\} \\
& \overbrace{rd(x4, mult(i(x1), \underbrace{op_l(x4, op_l(x2, x3, x2), x1)}))} \\
= & \quad \text{by Lemma 2677 LR with } \{x4 \leftarrow x1, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x4\} \\
& \underbrace{rd(x4, mult(i(x1), \overbrace{op_l(x4, x2, x1)}))} \\
= & \quad \text{by Lemma 2495 LR with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x4\} \\
& \overbrace{op_r(x1, x2, x4)}
\end{aligned}
$$

Lemma 2679: $op_r(x1, op_l(x2, x2, x3), x4) = op_r(x1, x2, x4)$

$\underbrace{op_r(x1, op_l(x2, x2, x3), x4)}$

$=$ by Lemma 2495 RL with $\{x3 \leftarrow op_l(x2, x2, x3), x2 \leftarrow x1, x1 \leftarrow x4\}$

$\overbrace{rd(x4, mult(i(x1), op_l(x4, \underbrace{op_l(x2, x2, x3)}, x1)))}$

$=$ by Lemma 1076 RL with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x2\}$

$rd(x4, mult(i(x1), \underbrace{op_l(x4, \overbrace{op_l(x2, op_l(rd(x2, x3), x3, x2), x2)}, x1)}))$

$=$ by Lemma 2677 LR with $\{x4 \leftarrow x1, x3 \leftarrow op_l(rd(x2, x3), x3, x2), x2 \leftarrow x2, x1 \leftarrow x4\}$

$\underbrace{rd(x4, mult(i(x1), \overbrace{op_l(x4, x2, x1)}))}$

$=$ by Lemma 2495 LR with $\{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x4\}$

$\overbrace{op_r(x1, x2, x4)}$

Lemma 2680: $op_l(x1, x2, op_t(i(x3), x4)) = op_l(x1, op_t(x3, x4), x2)$

$\underbrace{op_l(x1, x2, op_t(i(x3), x4))}$

$=$ by Lemma 2234 LR with $\{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_l(x1, op_t(x3, i(x4)), x2)}}$

$=$ by Lemma 2677 RL with $\{x4 \leftarrow x2, x3 \leftarrow mult(x4, x4), x2 \leftarrow op_t(x3, i(x4)), x1 \leftarrow x1\}$

$\overbrace{op_l(x1, \underbrace{op_l(op_t(x3, i(x4)), mult(x4, x4), op_t(x3, i(x4)))}, x2)}$

$=$ by Lemma 2049 RL with $\{x4 \leftarrow i(x4), x3 \leftarrow x3, x2 \leftarrow mult(x4, x4), x1 \leftarrow op_t(x3, i(x4))\}$

$op_l(x1, \overbrace{op_l(op_t(x3, i(x4)), mult(x4, x4), \underbrace{op_l(op_t(x3, i(x4)), mult(x4, x4), x3)})}, x2)$

$=$ by Axiom 15 RL with $\{x4 \leftarrow i(x4), x3 \leftarrow x3, x2 \leftarrow mult(x4, x4), x1 \leftarrow x3\}$

$op_l(x1, op_l(op_t(x3, i(x4)), mult(x4, x4), \overbrace{\underbrace{op_t(op_l(x3, mult(x4, x4), x3), i(x4))}}), x2)$

$=$ by Lemma 1129 RL with $\{x2 \leftarrow x3, x1 \leftarrow x4\}$

$op_l(x1, op_l(op_t(x3, i(x4)), mult(x4, x4), \overbrace{op_t(\underbrace{mult(x4, mult(i(x4), x3))}, x4)}), x2)$

$=$ by Lemma 12 LR with $\{x2 \leftarrow x3, x1 \leftarrow x4\}$

$op_l(x1, \underbrace{op_l(op_t(x3, i(x4)), mult(x4, x4), op_t(\overbrace{x3}, x4))}, x2)$

$=$ by Lemma 2494 LR with $\{x3 \leftarrow x3, x2 \leftarrow x4, x1 \leftarrow op_t(x3, i(x4))\}$

$op_l(x1, \overbrace{\underbrace{op_l(op_t(x3, i(x4)), mult(x4, x4), x3)}}, x2)$

$=$ by Axiom 15 RL with $\{x4 \leftarrow i(x4), x3 \leftarrow x3, x2 \leftarrow mult(x4, x4), x1 \leftarrow x3\}$

$op_l(x1, \underbrace{\overbrace{op_t(op_l(x3, mult(x4, x4), x3), i(x4))}}, x2)$

$=$ by Lemma 1129 RL with $\{x2 \leftarrow x3, x1 \leftarrow x4\}$

$op_l(x1, \overbrace{\underbrace{op_t(mult(x4, mult(i(x4), x3)), x4)}}, x2)$

$=$ by Lemma 12 LR with $\{x2 \leftarrow x3, x1 \leftarrow x4\}$

$op_l(x1, op_t(\overbrace{x3}, x4), x2)$

Lemma 2681: $op_l(x1, x3, op_t(x2, x4)) = op_l(x1, op_t(i(x2), x4), x3)$

$\overbrace{op_l(x1, x3, op_t(x2, x4))}$

$=$ by Lemma 2030 RL with $\{x3 \leftarrow x3, x2 \leftarrow op_t(x2, x4), x1 \leftarrow x1\}$

$\overbrace{op_l(x1, op_t(x2, x4), i(x3))}$

$=$ by Lemma 2031 RL with $\{x3 \leftarrow op_t(x2, x4), x2 \leftarrow i(x3), x1 \leftarrow x1\}$

$\overbrace{op_l(x1, i(i(x3)), op_t(x2, x4))}$

$=$ by Lemma 2488 LR with $\{x4 \leftarrow x4, x3 \leftarrow x2, x2 \leftarrow i(x3), x1 \leftarrow x1\}$

$\overbrace{op_l(x1, op_t(x2, x4), \underbrace{mult(i(x3), x2)})}$

$=$ by Lemma 17 RL with $\{x2 \leftarrow x2, x1 \leftarrow x3\}$

$\underbrace{op_l(x1, op_t(x2, x4), \overbrace{i(mult(x3, i(x2)))})}$

$=$ by Lemma 1321 RL with $\{x3 \leftarrow mult(x3, i(x2)), x2 \leftarrow op_t(x2, x4), x1 \leftarrow x1\}$

$\overbrace{op_l(x1, op_t(x2, x4), \underbrace{rd(op_t(x2, x4), mult(x3, i(x2)))})}$

$=$ by Lemma 1677 LR with $\{x3 \leftarrow x3, x2 \leftarrow x4, x1 \leftarrow x2\}$

$\underbrace{op_l(x1, op_t(x2, x4), \overbrace{mult(rd(x2, x3), op_t(x2, x4))})}$

$=$ by Lemma 291 LR with $\{x3 \leftarrow rd(x2, x3), x2 \leftarrow op_t(x2, x4), x1 \leftarrow x1\}$

$\overbrace{op_l(x1, op_t(x2, x4), rd(x2, x3))}$

$=$ by Lemma 2680 RL with $\{x4 \leftarrow x4, x3 \leftarrow x2, x2 \leftarrow rd(x2, x3), x1 \leftarrow x1\}$

$\overbrace{op_l(x1, rd(x2, x3), op_t(i(x2), x4))}$

$=$ by Lemma 2671 LR with $\{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_l(x1, op_t(i(x2), x4), x3)}$

Lemma 2682: $op_l(x1, mult(x2, rd(x3, x2)), x4) = op_l(x1, op_l(mult(x2, rd(x3, x2)), x2, x3), x4)$

$\overbrace{op_l(x1, mult(x2, rd(x3, x2)), x4)}$

$=$ by Lemma 2677 RL with $\{x4 \leftarrow x4, x3 \leftarrow mult(x3, x2), x2 \leftarrow mult(x2, rd(x3, x2)), x1 \leftarrow x1\}$

$\overbrace{op_l(x1, \underbrace{op_l(mult(x2, rd(x3, x2)), mult(x3, x2), mult(x2, rd(x3, x2)))}, x4)}$

$=$ by Lemma 1332 RL with $\{x3 \leftarrow mult(x3, x2), x2 \leftarrow mult(x2, rd(x3, x2)), x1 \leftarrow mult(x2, rd(x3, x2))\}$

$op_l(x1, \overbrace{op_l(mult(x2, rd(x3, x2)), op_t(mult(x2, rd(\underbrace{x3}, x2)), mult(x3, x2)), i(mult(x3, x2)))}, x4)$

$=$ by Lemma 3 RL with $\{x1 \leftarrow x3\}$

$op_l(x1, op_l(mult(x2, rd(x3, x2)), op_t(\underbrace{mult(x2, rd(\overbrace{i(i(x3))}, x2))}, mult(x3, x2)), i(mult(x3, x2))), x4)$

$=$ by Lemma 413 RL with $\{x2 \leftarrow x2, x1 \leftarrow i(x3)\}$

$op_l(x1, op_l(mult(x2, rd(x3, x2)), op_t(\overbrace{op_r(i(op_t(i(x3), x2)), x2, x2)}, mult(x3, x2)), i(mult(x3, x2))), x4)$

$=$ by Lemma 354 RL with $\{x2 \leftarrow op_t(i(x3), x2), x1 \leftarrow x2\}$

$op_l(x1, op_l(mult(x2, rd(x3, x2)), op_t(\overbrace{rd(rd(x2, op_t(i(x3), x2)), x2)}, mult(x3, x2)), i(mult(x3, x2))), x4)$

$=$ by Lemma 23 RL with $\{x2 \leftarrow rd(x2, op_t(i(x3), x2)), x1 \leftarrow x2\}$

$op_l(x1, op_l(mult(x2, rd(x3, x2)), op_t(\overbrace{rd(i(x2), i(\underbrace{rd(x2, op_t(i(x3), x2))}))}, mult(x3, x2)), i(mult(x3, x2))), x4)$

$=$ by Lemma 326 RL with $\{x2 \leftarrow x2, x1 \leftarrow x3\}$

$op_l(x1, op_l(mult(x2, rd(x3, x2)), \underbrace{op_t(rd(i(x2), i(\overbrace{op_t(mult(x3, x2), i(x2))})), mult(x3, x2))}, i(mult(x3, x2))), x4)$

$=$ by Lemma 909 LR with $\{x2 \leftarrow mult(x3, x2), x1 \leftarrow i(x2)\}$

$op_l(x1, op_l(mult(x2, rd(x3, x2)), \overbrace{mult(i(mult(x3, x2)), mult(mult(x3, x2), mult(i(x2), mult(x3, x2))))}, i(mult(x3, x2))), x4$

$=$ by Axiom 2 RL with $\{x2 \leftarrow mult(x3, x2), x1 \leftarrow mult(i(x2), mult(x3, x2))\}$

$op_l(x1, op_l(mult(x2, rd(x3, x2)), \overbrace{mult(i(x2), mult(x3, x2))}, i(mult(x3, x2))), x4)$

$=$ by Axiom 10 LR with $\{x2 \leftarrow x3, x1 \leftarrow x2\}$

$op_l(x1, \underbrace{op_l(mult(x2, rd(x3, x2)), \overbrace{op_t(x3, x2)}, i(mult(x3, x2)))}, x4)$

$=$ by Lemma 1312 LR with $\{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow mult(x2, rd(x3, x2))\}$

$op_l(x1, \overbrace{op_l(mult(x2, rd(x3, x2)), x2, x3)}, x4)$

Lemma 2683: $op_r(x1, mult(x2, rd(x3, x2)), x4) = op_r(x1, x3, x4)$

$op_r(x1, \underbrace{mult(x2, rd(x3, x2))}, x4)$

$=$ by Lemma 427 RL with $\{x2 \leftarrow x2, x1 \leftarrow x3\}$

$\underbrace{op_r(x1, \overbrace{op_l(x3, x3, x2)}, x4)}$

$=$ by Lemma 2679 LR with $\{x4 \leftarrow x4, x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x1\}$

$\overbrace{op_r(x1, x3, x4)}$

Lemma 2684: $op_r(x1, mult(x2, x3), x4) = op_r(x1, mult(x3, x2), x4)$

$\underbrace{op_r(x1, mult(x2, x3), x4)}$

$=$ by Lemma 2678 RL with $\{x4 \leftarrow x4, x3 \leftarrow rd(x2, x3), x2 \leftarrow mult(x2, x3), x1 \leftarrow x1\}$

$\overbrace{op_r(x1, \underbrace{op_l(mult(x2, x3), rd(x2, x3), mult(x2, x3))}, x4)}$

$=$ by Lemma 2479 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow mult(x2, x3)\}$

$op_r(x1, \overbrace{\underbrace{op_l(mult(x2, x3), x2, mult(x3, x3))}}, x4)$

$=$ by Lemma 1614 LR with $\{x1 \leftarrow x3, x2 \leftarrow x2\}$

$op_r(x1, \overbrace{\underbrace{mult(op_t(x3, x2), x2)}}, x4)$

$=$ by Lemma 636 RL with $\{x2 \leftarrow x2, x1 \leftarrow x3\}$

$op_r(x1, \overbrace{\underbrace{op_l(mult(x3, x2), x2, x3)}}, x4)$

$=$ by Lemma 291 RL with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow mult(x3, x2)\}$

$\underbrace{op_r(x1, \overbrace{\underbrace{op_l(mult(x3, x2), x2, mult(x3, x2))}}, x4)}$

$=$ by Lemma 2678 LR with $\{x4 \leftarrow x4, x3 \leftarrow x2, x2 \leftarrow mult(x3, x2), x1 \leftarrow x1\}$

$\overbrace{op_r(x1, mult(x3, x2), x4)}$

Lemma 2685: $op_r(x1, op_r(x2, x3, x3), x4) = op_r(x1, rd(mult(x3, x2), x3), x4)$

$\underbrace{op_r(x1, op_r(x2, x3, x3), x4)}$

$=$ by Lemma 2678 RL with $\{x4 \leftarrow x4, x3 \leftarrow rd(x2, x3), x2 \leftarrow op_r(x2, x3, x3), x1 \leftarrow x1\}$

$\overbrace{op_r(x1, \underbrace{op_l(op_r(x2, x3, x3), rd(x2, x3), op_r(x2, x3, x3))}, x4)}$

$=$ by Lemma 1362 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow op_r(x2, x3, x3)\}$

$op_r(x1, \overbrace{op_l(op_r(x2, x3, x3), x2, x3)}, x4)$

$=$ by Lemma 2500 RL with $\{x3 \leftarrow x3, x2 \leftarrow x3, x1 \leftarrow x2\}$

$op_r(x1, \overbrace{op_r(x2, \underbrace{x3}, mult(x3, x2))}, x4)$

$=$ by Axiom 5 LR with $\{x2 \leftarrow x2, x1 \leftarrow x3\}$

$op_r(x1, op_r(x2, \overbrace{rd(mult(x3, x2), x2)}, mult(x3, x2)), x4)$

$=$ by Lemma 10 RL with $\{x2 \leftarrow mult(x3, x2), x1 \leftarrow x2\}$

$op_r(x1, op_r(\underbrace{x2}, \overbrace{i(rd(x2, mult(x3, x2)))}, mult(x3, x2)), x4)$

$=$ by Lemma 368 RL with $\{x2 \leftarrow mult(x3, x2), x1 \leftarrow x2\}$

$op_r(x1, op_r(\overbrace{op_r(op_t(x2, rd(mult(x3, x2), x2)), mult(x3, x2), mult(x3, x2))}, i(rd(x2, mult(x3, x2))), mult(x3, x2)), x4)$

$=$ by Axiom 14 RL with $\{x4 \leftarrow rd(mult(x3, x2), x2), x3 \leftarrow mult(x3, x2), x2 \leftarrow mult(x3, x2), x1 \leftarrow x2\}$

$op_r(x1, op_r(\overbrace{op_t(\underbrace{op_r(x2, mult(x3, x2), mult(x3, x2))}, rd(mult(x3, x2), x2))}, i(rd(x2, mult(x3, x2))), mult(x3, x2)), x4)$

$=$ by Lemma 361 RL with $\{x2 \leftarrow mult(x3, x2), x1 \leftarrow x2\}$

$op_r(x1, op_r(op_t(\overbrace{op_r(x2, i(mult(x3, x2)), i(mult(x3, x2)))}, rd(mult(x3, x2), x2)), i(rd(x2, mult(x3, x2))), mult(x3, x2)), x4)$

$=$ by Lemma 813 RL with $\{x3 \leftarrow mult(x3, x2), x2 \leftarrow mult(x3, x2), x1 \leftarrow x2\}$

$op_r(x1, \underbrace{op_r(op_t(\overbrace{i(op_r(i(x2), mult(x3, x2), mult(x3, x2)))}, rd(mult(x3, x2), x2)), i(rd(x2, mult(x3, x2))), mult(x3, x2))}, x4)$

$=$ by Lemma 975 RL with $\{x2 \leftarrow mult(x3, x2), x1 \leftarrow x2\}$

$op_r(x1, \overbrace{op_r(x2, mult(x3, x2), mult(x3, x2))}, x4)$

$=$ by Lemma 800 LR with $\{x2 \leftarrow x3, x1 \leftarrow x2\}$

$op_r(x1, \overbrace{rd(mult(x3, x2), x3)}, x4)$

Lemma 2686: $op_r(x1, op_t(rd(x2, x3), x2), x4) = op_r(x1, mult(i(x3), x2), x4)$

$op_r(x1, \underbrace{op_t(rd(x2, x3), x2)}, x4)$

$=$ by Lemma 249 RL with $\{x2 \leftarrow x2, x1 \leftarrow x3\}$

$op_r(x1, \overbrace{op_l(mult(i(x3), x2), x3, x2)}, x4)$

$=$ by Lemma 1791 RL with $\{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow mult(i(x3), x2)\}$

$\underbrace{op_r(x1, \overbrace{op_l(mult(i(x3), x2), op_r(x3, x2, x2), mult(i(x3), x2))}, x4)}$

$=$ by Lemma 2678 LR with $\{x4 \leftarrow x4, x3 \leftarrow op_r(x3, x2, x2), x2 \leftarrow mult(i(x3), x2), x1 \leftarrow x1\}$

$\overbrace{op_r(x1, mult(i(x3), x2), x4)}$

Lemma 2687: $op_r(x1, rd(x2, x3), x4) = op_r(x1, op_l(rd(x2, x3), op_t(x2, x5), x3), x4)$

$$
\begin{aligned}
& \underbrace{op_r(x1, rd(x2, x3), x4)} \\
= \quad & \text{by Lemma 2679 RL with } \{x4 \leftarrow x4, x3 \leftarrow op_t(x2, x5), x2 \leftarrow rd(x2, x3), x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, \underbrace{op_l(rd(x2, x3), rd(x2, x3), op_t(x2, x5))}, x4)} \\
= \quad & \text{by Lemma 2486 RL with } \{x4 \leftarrow x5, x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow rd(x2, x3)\} \\
& op_r(x1, \overbrace{\underbrace{op_l(rd(x2, x3), i(x3), op_t(x2, x5))}}, x4) \\
= \quad & \text{by Lemma 2031 LR with } \{x3 \leftarrow op_t(x2, x5), x2 \leftarrow x3, x1 \leftarrow rd(x2, x3)\} \\
& op_r(x1, \overbrace{op_l(rd(x2, x3), op_t(x2, x5), x3)}, x4)
\end{aligned}
$$

Lemma 2688: $op_r(x1, rd(x2, x3), x4) = op_r(x1, rd(op_t(x2, i(x3)), x3), x4)$

$$
\begin{aligned}
& \underbrace{op_r(x1, rd(x2, x3), x4)} \\
= \quad & \text{by Lemma 2687 LR with } \{x5 \leftarrow x5, x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, \underbrace{op_l(rd(x2, x3), op_t(x2, x5), x3)}, x4)} \\
= \quad & \text{by Lemma 292 RL with } \{x3 \leftarrow x3, x2 \leftarrow op_t(x2, x5), x1 \leftarrow rd(x2, x3)\} \\
& op_r(x1, \overbrace{\underbrace{op_l(rd(x2, x3), rd(op_t(x2, x5), x3), x3)}}, x4) \\
= \quad & \text{by Lemma 1060 RL with } \{x4 \leftarrow x3, x3 \leftarrow op_t(x2, x5), x2 \leftarrow x3, x1 \leftarrow rd(x2, x3)\} \\
& op_r(x1, \overbrace{\underbrace{op_l(rd(x2, x3), rd(x3, op_t(x2, x5)), i(x3))}}, x4) \\
= \quad & \text{by Lemma 243 RL with } \{x3 \leftarrow rd(x3, op_t(x2, x5)), x2 \leftarrow x3, x1 \leftarrow rd(x2, x3)\} \\
& op_r(x1, \overbrace{op_l(rd(x2, x3), x3, \underbrace{mult(i(x3), rd(x3, op_t(x2, x5)))})}, x4) \\
= \quad & \text{by Lemma 45 RL with } \{x3 \leftarrow x3, x2 \leftarrow op_t(x2, x5), x1 \leftarrow x3\} \\
& op_r(x1, op_l(rd(x2, x3), x3, \overbrace{i(\underbrace{mult(x3, rd(op_t(x2, x5), x3))})}), x4) \\
= \quad & \text{by Lemma 1209 LR with } \{x3 \leftarrow x5, x2 \leftarrow x2, x1 \leftarrow x3\} \\
& op_r(x1, \underbrace{op_l(rd(x2, x3), x3, i(\overbrace{op_t(mult(x3, rd(x2, x3)), x5)}))}, x4) \\
= \quad & \text{by Lemma 987 RL with } \{x3 \leftarrow rd(x2, x3), x2 \leftarrow x3, x1 \leftarrow op_t(mult(x3, rd(x2, x3)), x5)\} \\
& op_r(x1, \overbrace{\underbrace{mult(op_t(mult(x3, rd(x2, x3)), x5), mult(rd(i(x3), op_t(mult(x3, rd(x2, x3)), x5)), mult(x3, rd(x2, x3))))}}, x4) \\
= \quad & \text{by Lemma 1760 LR with } \{x3 \leftarrow rd(i(x3), op_t(mult(x3, rd(x2, x3)), x5)), x2 \leftarrow x5, x1 \leftarrow mult(x3, rd(x2, x3))\} \\
& op_r(x1, \overbrace{mult(mult(x3, rd(x2, x3)), \underbrace{mult(rd(i(x3), op_t(mult(x3, rd(x2, x3)), x5)), op_t(mult(x3, rd(x2, x3)), x5))})}, x4) \\
= \quad & \text{by Axiom 4 RL with } \{x2 \leftarrow op_t(mult(x3, rd(x2, x3)), x5), x1 \leftarrow i(x3)\} \\
& op_r(x1, \underbrace{mult(mult(x3, rd(x2, x3)), \overbrace{i(x3)})}, x4) \\
= \quad & \text{by Lemma 55 LR with } \{x2 \leftarrow rd(x2, x3), x1 \leftarrow x3\} \\
& op_r(x1, \overbrace{\underbrace{mult(x3, rd(rd(x2, x3), x3))}}, x4) \\
= \quad & \text{by Lemma 417 LR with } \{x2 \leftarrow x2, x1 \leftarrow x3\} \\
& op_r(x1, \overbrace{rd(op_t(x2, i(x3)), x3)}, x4)
\end{aligned}
$$

Lemma 2689: $op_r(op_t(asoc(x1,x2,x3),mult(mult(x4,x4),x5)),mult(x4,x4),x5) = mult(i(mult(mult(x4,x4),x5)),mult(mult($

$\underbrace{op_r(op_t(asoc(x1,x2,x3),mult(mult(x4,x4),x5)),mult(x4,x4),x5)}$

$=$ by Axiom 14 RL with $\{x4 \leftarrow mult(mult(x4,x4),x5), x3 \leftarrow x5, x2 \leftarrow mult(x4,x4), x1 \leftarrow asoc(x1,x2,x3)\}$

$\overbrace{op_t(\underbrace{op_r(asoc(x1,x2,x3),mult(x4,x4),x5)},mult(mult(x4,x4),x5))}$

$=$ by Axiom 16 LR with $\{x3 \leftarrow x5, x2 \leftarrow mult(x4,x4), x1 \leftarrow asoc(x1,x2,x3)\}$

$\underbrace{op_t(\overbrace{rd(mult(mult(asoc(x1,x2,x3),\underbrace{mult(x4,x4)}),x5),mult(mult(x4,x4),x5))},mult(mult(x4,x4),x5))}$

$=$ by Lemma 14 LR with $\{x2 \leftarrow mult(mult(x4,x4),x5), x1 \leftarrow mult(mult(asoc(x1,x2,x3),mult(x4,x4)),x5)\}$

$\overbrace{mult(i(mult(mult(x4,x4),x5)),mult(\underbrace{mult(asoc(x1,x2,x3),mult(x4,x4))},x5))}$

$=$ by Lemma 2666 LR with $\{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$mult(i(mult(mult(x4,x4),x5)),mult(\overbrace{mult(mult(x4,asoc(x1,x2,x3)),\underbrace{x4}_{})},x5))$

$=$ by Lemma 2652 RL with $\{x4 \leftarrow x3, x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x4\}$

$mult(i(mult(mult(x4,x4),x5)),mult(mult(mult(x4,asoc(x1,x2,x3)),\overbrace{\underbrace{op_t(x4,asoc(x1,x2,x3))}}),x5))$

$=$ by Lemma 2652 RL with $\{x4 \leftarrow x3, x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow op_t(x4,asoc(x1,x2,x3))\}$

$mult(i(mult(mult(x4,x4),x5)),mult(\underbrace{mult(mult(x4,asoc(x1,x2,x3)),\overbrace{op_t(op_t(x4,asoc(x1,x2,x3)),asoc(x1,x2,x3))}},x5)$

$=$ by Lemma 137 RL with $\{x3 \leftarrow op_t(op_t(x4,asoc(x1,x2,x3)),asoc(x1,x2,x3)), x2 \leftarrow asoc(x1,x2,x3), x1 \leftarrow x4\}$

$mult(i(mult(mult(x4,x4),x5)),mult(\overbrace{mult(x4,\underbrace{mult(asoc(x1,x2,x3),op_l(op_t(op_t(x4,asoc(x1,x2,x3)),asoc(x1,x2,x3)),x$

$=$ by Lemma 167 LR with $\{x4 \leftarrow asoc(x1,x2,x3), x3 \leftarrow x4, x2 \leftarrow op_t(x4,asoc(x1,x2,x3)), x1 \leftarrow asoc(x1,x2,x3)\}$

$mult(i(mult(mult(x4,x4),x5)),mult(mult(x4,\overbrace{mult(\underbrace{op_l(op_t(x4,asoc(x1,x2,x3)),x4,asoc(x1,x2,x3))},asoc(x1,x2,x3))),$

$=$ by Lemma 421 RL with $\{x2 \leftarrow asoc(x1,x2,x3), x1 \leftarrow x4\}$

$mult(i(mult(mult(x4,x4),x5)),mult(mult(x4,\underbrace{mult(\overbrace{op_r(x4,asoc(x1,x2,x3)),i(asoc(x1,x2,x3)))},asoc(x1,x2,x3))},x5))$

$=$ by Lemma 318 LR with $\{x2 \leftarrow asoc(x1,x2,x3), x1 \leftarrow x4\}$

$mult(i(mult(mult(x4,x4),x5)),mult(mult(x4,\overbrace{\underbrace{op_t(mult(asoc(x1,x2,x3),x4),asoc(x1,x2,x3))}}),x5))$

$=$ by Lemma 2652 LR with $\{x4 \leftarrow x3, x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow mult(asoc(x1,x2,x3),x4)\}$

$mult(i(mult(mult(x4,x4),x5)),mult(mult(x4,\overbrace{mult(asoc(x1,x2,x3),x4)}),x5))$

Lemma 2690: $op_r(asoc(x1,x2,x3),mult(x4,x4),x5) = mult(i(mult(x4,mult(x4,x5))),mult(mult(x4,mult(asoc(x1,x2,x3),x4$

$op_r(\underbrace{asoc(x1,x2,x3)},mult(x4,x4),x5)$

$=$ by Lemma 2654 RL with $\{x4 \leftarrow mult(mult(x4,x4),x5), x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{op_r(\overbrace{op_t(asoc(x1,x2,x3),mult(mult(x4,x4),x5))},mult(x4,x4),x5)}$

$=$ by Lemma 2689 LR with $\{x5 \leftarrow x5, x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{mult(i(\underbrace{mult(mult(x4,x4),x5)}),mult(mult(x4,mult(asoc(x1,x2,x3),x4)),x5))}$

$=$ by Lemma 24 LR with $\{x2 \leftarrow x5, x1 \leftarrow x4\}$

$mult(i(\overbrace{mult(x4,mult(x4,x5))}),mult(mult(x4,mult(asoc(x1,x2,x3),x4)),x5))$

Lemma 2691: $op_l(x1, i(x2), op_l(rd(x2, x3), x4, i(x2))) = op_l(x1, i(x2), op_l(i(x3), x4, i(x2)))$

$$
\begin{aligned}
& \underbrace{op_l(x1, i(x2), op_l(rd(x2, x3), x4, i(x2)))} \\
= \quad & \text{by Lemma 2672 LR with } \{x4 \leftarrow x4, x3 \leftarrow rd(x2, x3), x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, i(x2), \underbrace{rd(op_l(mult(rd(x2, x3), i(x2)), x4, i(x2)), i(x2))})} \\
= \quad & \text{by Lemma 286 LR with } \{x3 \leftarrow op_l(mult(rd(x2, x3), i(x2)), x4, i(x2)), x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, i(x2), op_l(\underbrace{mult(rd(x2, x3), i(x2))}, x4, i(x2)))} \\
= \quad & \text{by Lemma 22 LR with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& op_l(x1, i(x2), op_l(\overbrace{i(x3)}, x4, i(x2)))
\end{aligned}
$$

Lemma 2692: $op_l(x1, op_l(rd(x2, x3), x2, x4), x2) = op_l(x1, i(x2), op_l(i(x3), x4, i(x2)))$

$$
\begin{aligned}
& op_l(x1, \underbrace{op_l(rd(x2, x3), x2, x4)}, x2) \\
= \quad & \text{by Lemma 2030 RL with } \{x3 \leftarrow x2, x2 \leftarrow x4, x1 \leftarrow rd(x2, x3)\} \\
& \underbrace{op_l(x1, \overbrace{op_l(rd(x2, x3), x4, i(x2))}, x2)} \\
= \quad & \text{by Lemma 2031 RL with } \{x3 \leftarrow op_l(rd(x2, x3), x4, i(x2)), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, i(x2), op_l(rd(x2, x3), x4, i(x2)))}} \\
= \quad & \text{by Lemma 2691 LR with } \{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, i(x2), op_l(i(x3), x4, i(x2)))}
\end{aligned}
$$

Lemma 2693: $op_l(x1, op_l(rd(x2, x3), x2, x4), x2) = op_l(x1, x2, op_l(x3, x4, i(x2)))$

$$
\begin{aligned}
& \underbrace{op_l(x1, op_l(rd(x2, x3), x2, x4), x2)} \\
= \quad & \text{by Lemma 2692 LR with } \{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, i(x2), op_l(i(x3), x4, i(x2)))}} \\
= \quad & \text{by Lemma 2031 LR with } \{x3 \leftarrow op_l(i(x3), x4, i(x2)), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, op_l(i(x3), x4, i(x2)), x2)}} \\
= \quad & \text{by Lemma 1419 LR with } \{x5 \leftarrow x2, x4 \leftarrow i(x2), x3 \leftarrow x4, x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, op_l(x3, x4, i(x2)), i(x2))}} \\
= \quad & \text{by Lemma 2030 LR with } \{x3 \leftarrow x2, x2 \leftarrow op_l(x3, x4, i(x2)), x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x2, op_l(x3, x4, i(x2)))}
\end{aligned}
$$

Lemma 2694: $op_r(x1, rd(x2, i(x3)), x4) = op_r(x1, op_r(mult(x3, x2), x3, x3), x4)$

$$
\begin{aligned}
& \underbrace{op_r(x1, rd(x2, i(x3)), x4)} \\
= \quad & \text{by Lemma 2683 RL with } \{x4 \leftarrow x4, x3 \leftarrow rd(x2, i(x3)), x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, mult(x3, \underbrace{rd(rd(x2, i(x3)), x3)}), x4)} \\
= \quad & \text{by Lemma 358 LR with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& op_r(x1, \underbrace{mult(x3, \overbrace{op_r(x2, x3, x3)})}, x4) \\
= \quad & \text{by Lemma 2569 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x3\} \\
& op_r(x1, \overbrace{op_r(mult(x3, x2), x3, x3)}, x4)
\end{aligned}
$$

Lemma 2695: $op_r(x1, op_r(x2, x3, x4), x5) = op_r(x1, op_r(x2, x3, rd(x4, x2)), x5)$

$\underbrace{op_r(x1, op_r(x2, x3, x4), x5)}$

$=$ by Lemma 2683 RL with $\{x4 \leftarrow x5, x3 \leftarrow op_r(x2, x3, x4), x2 \leftarrow rd(x2, x3), x1 \leftarrow x1\}$

$\overbrace{op_r(x1, mult(rd(x2, x3), rd(op_r(x2, x3, x4), rd(x2, \underbrace{x3}))), x5)}$

$=$ by Lemma 3 RL with $\{x1 \leftarrow x3\}$

$op_r(x1, mult(rd(x2, x3), rd(op_r(x2, x3, x4), \underbrace{rd(x2, \overbrace{i(i(x3))})}))), x5)$

$=$ by Lemma 359 RL with $\{x2 \leftarrow x2, x1 \leftarrow i(x3)\}$

$op_r(x1, mult(rd(x2, x3), rd(op_r(x2, x3, x4), \underbrace{\overbrace{op_r(mult(i(x3), x2), x2, x2)}})), x5)$

$=$ by Lemma 2570 RL with $\{x3 \leftarrow x2, x2 \leftarrow x2, x1 \leftarrow i(x3)\}$

$op_r(x1, mult(rd(x2, x3), rd(op_r(x2, x3, x4), \overbrace{mult(\underbrace{op_r(i(x3), x2, x2)}, x2)})), x5)$

$=$ by Lemma 2428 LR with $\{x3 \leftarrow x2, x2 \leftarrow x2, x1 \leftarrow x3\}$

$op_r(x1, mult(rd(x2, x3), \underbrace{rd(op_r(x2, x3, x4), mult(\overbrace{i(op_r(x3, x2, x2))}, x2))}), x5)$

$=$ by Lemma 2581 RL with $\{x3 \leftarrow x4, x2 \leftarrow x2, x1 \leftarrow x3\}$

$op_r(x1, mult(rd(x2, x3), \underbrace{\overbrace{op_r(op_r(x3, x2, x2), x2, i(x4))}}), x5)$

$=$ by Lemma 2512 LR with $\{x3 \leftarrow i(x4), x2 \leftarrow x2, x1 \leftarrow x3\}$

$op_r(x1, mult(rd(x2, x3), \overbrace{op_r(x3, x2, mult(i(x4), x2))}), x5)$

$=$ by Lemma 2397 LR with $\{x4 \leftarrow x2, x3 \leftarrow x4, x2 \leftarrow x2, x1 \leftarrow x3\}$

$op_r(x1, \underbrace{mult(rd(x2, x3), \overbrace{op_r(x3, x2, rd(x2, x4))})}, x5)$

$=$ by Lemma 2610 LR with $\{x4 \leftarrow rd(x2, x4), x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x2\}$

$op_r(x1, \overbrace{\underbrace{rd(x2, rd(x3, op_r(x3, x2, rd(x2, x4))))}}, x5)$

$=$ by Lemma 2566 RL with $\{x3 \leftarrow rd(x2, x4), x2 \leftarrow x3, x1 \leftarrow x2\}$

$op_r(x1, \overbrace{op_r(x2, x3, \underbrace{i(rd(x2, x4))})}, x5)$

$=$ by Lemma 10 LR with $\{x2 \leftarrow x4, x1 \leftarrow x2\}$

$op_r(x1, op_r(x2, x3, \overbrace{rd(x4, x2)}), x5)$

Lemma 2696: $op_r(x1, x2, op_r(x3, x4, x5)) = op_r(x1, x2, op_r(x3, x4, mult(x4, x5)))$

$$
\begin{aligned}
& op_r(x1, x2, \underbrace{op_r(x3, x4, x5)}) \\
= & \quad \text{by Lemma 1894 RL with } \{x3 \leftarrow x5, x2 \leftarrow x4, x1 \leftarrow x3\} \\
& \underbrace{op_r(x1, x2, \overbrace{op_r(x3, op_t(x4, x5), x5)})} \\
= & \quad \text{by Lemma 2391 RL with } \{x4 \leftarrow op_t(x4, x5), x3 \leftarrow op_r(x3, op_t(x4, x5), x5), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, x2, \underbrace{op_r(op_r(x3, op_t(x4, x5), x5), op_t(x4, x5), op_t(x4, x5))})} \\
= & \quad \text{by Axiom 17 RL with } \{x5 \leftarrow x5, x4 \leftarrow op_t(x4, x5), x3 \leftarrow op_t(x4, x5), x2 \leftarrow op_t(x4, x5), x1 \leftarrow x3\} \\
& op_r(x1, x2, \overbrace{\underbrace{op_r(op_r(x3, op_t(x4, x5), op_t(x4, x5)), op_t(x4, x5), x5)}}) \\
= & \quad \text{by Lemma 2512 LR with } \{x3 \leftarrow x5, x2 \leftarrow op_t(x4, x5), x1 \leftarrow x3\} \\
& op_r(x1, x2, \overbrace{op_r(x3, op_t(x4, x5), \underbrace{mult(x5, op_t(x4, x5))})}) \\
= & \quad \text{by Lemma 13 LR with } \{x2 \leftarrow x4, x1 \leftarrow x5\} \\
& op_r(x1, x2, \underbrace{op_r(x3, op_t(x4, x5), \overbrace{mult(x4, x5)})}) \\
= & \quad \text{by Lemma 1893 RL with } \{x4 \leftarrow mult(x4, x5), x3 \leftarrow x5, x2 \leftarrow x4, x1 \leftarrow x3\} \\
& op_r(x1, x2, \overbrace{op_r(x3, op_t(x4, x5), \underbrace{op_t(mult(x4, x5), x4)})}) \\
= & \quad \text{by Lemma 1922 LR with } \{x2 \leftarrow x5, x1 \leftarrow x4\} \\
& op_r(x1, x2, \underbrace{op_r(x3, op_t(x4, x5), \overbrace{mult(mult(mult(x4, x5), x4), i(op_t(x4, x5)))})}) \\
= & \quad \text{by Lemma 2218 LR with } \{x3 \leftarrow mult(mult(x4, x5), x4), x2 \leftarrow op_t(x4, x5), x1 \leftarrow x3\} \\
& op_r(x1, x2, \overbrace{op_r(x3, op_t(x4, x5), \underbrace{rd(mult(mult(x4, x5), x4), op_t(x4, x5))})}) \\
= & \quad \text{by Lemma 2198 RL with } \{x3 \leftarrow x4, x1 \leftarrow x4, x2 \leftarrow x5\} \\
& op_r(x1, x2, op_r(x3, op_t(x4, x5), \overbrace{rd(mult(mult(x5, x4), x4), x4)})) \\
= & \quad \text{by Axiom 5 RL with } \{x2 \leftarrow x4, x1 \leftarrow mult(x5, x4)\} \\
& op_r(x1, x2, \underbrace{op_r(x3, op_t(x4, x5), \overbrace{mult(x5, x4)})}) \\
= & \quad \text{by Lemma 2009 LR with } \{x3 \leftarrow x5, x2 \leftarrow x4, x1 \leftarrow x3\} \\
& op_r(x1, x2, \overbrace{op_r(x3, x4, mult(x4, x5))})
\end{aligned}
$$

Lemma 2697: $op_l(x1, op_r(op_t(i(x2), x3), x3, x4), x3) = op_l(x1, op_r(op_t(rd(x3, x2), x3), x3, x4), x3)$

$$
\begin{aligned}
& \underbrace{op_l(x1, op_r(op_t(i(x2), x3), x3, x4), x3)} \\
= & \quad \text{by Lemma 285 RL with } \{x3 \leftarrow x3, x2 \leftarrow op_r(op_t(i(x2), x3), x3, x4), x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, \underbrace{mult(op_r(op_t(i(x2), x3), x3, x4), x3)}, x3)} \\
= & \quad \text{by Lemma 2570 LR with } \{x3 \leftarrow x4, x2 \leftarrow x3, x1 \leftarrow op_t(i(x2), x3)\} \\
& op_l(x1, \overbrace{op_r(\underbrace{mult(op_t(i(x2), x3), x3)}, x3, x4)}, x3) \\
= & \quad \text{by Lemma 340 LR with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& op_l(x1, op_r(\overbrace{op_t(rd(x3, x2), x3)}, x3, x4), x3)
\end{aligned}
$$

Lemma 2698: $op_r(x1, mult(x2, asoc(x3, x4, x4)), x3) = op_r(x1, x2, x3)$

$op_r(\overbrace{x1}, mult(x2, asoc(x3, x4, x4)), x3)$

$=$ by Lemma 3 RL with $\{x1 \leftarrow x1\}$

$\underbrace{op_r(\overbrace{i(i(x1))}, mult(x2, asoc(x3, x4, x4)), x3)}$

$=$ by Lemma 2392 RL with $\{x4 \leftarrow rd(x4, x3), x3 \leftarrow x3, x2 \leftarrow mult(x2, asoc(x3, x4, x4)), x1 \leftarrow i(i(x1))\}$

$\overbrace{\underbrace{op_r(i(i(x1)), mult(x2, asoc(x3, x4, x4)), op_t(x3, rd(x4, x3)))}}$

$=$ by Lemma 2428 LR with $\{x3 \leftarrow op_t(x3, rd(x4, x3)), x2 \leftarrow mult(x2, asoc(x3, x4, x4)), x1 \leftarrow i(x1)\}$

$\overbrace{\underbrace{i(op_r(i(x1), mult(x2, asoc(x3, x4, x4)), op_t(x3, rd(x4, x3))))}}$

$=$ by Lemma 891 RL with $\{x3 \leftarrow i(x1), x2 \leftarrow op_t(x3, rd(x4, x3)), x1 \leftarrow mult(x2, asoc(x3, x4, x4))\}$

$\overbrace{rd(mult(mult(x2, asoc(x3, x4, x4)), op_t(x3, rd(x4, x3))), \underbrace{mult(mult(i(x1), mult(x2, asoc(x3, x4, x4))), op_t(x3, rd(x4, x3)))}}$

$=$ by Lemma 1262 LR with $\{x3 \leftarrow x4, x2 \leftarrow x3, x1 \leftarrow mult(i(x1), mult(x2, asoc(x3, x4, x4)))\}$

$rd(mult(mult(x2, asoc(x3, x4, x4)), op_t(x3, rd(x4, x3))), \overbrace{mult(\underbrace{rd(mult(i(x1), mult(x2, asoc(x3, x4, x4))), asoc(x3, x4, x4))},$

$=$ by Lemma 2384 LR with $\{x3 \leftarrow x4, x4 \leftarrow x3, x2 \leftarrow mult(x2, asoc(x3, x4, x4)), x1 \leftarrow i(x1)\}$

$rd(\underbrace{mult(mult(x2, asoc(x3, x4, x4)), op_t(x3, rd(x4, x3)))}, mult(\overbrace{mult(i(x1), rd(mult(x2, asoc(x3, x4, x4)), asoc(x3, x4, x4)))},$

$=$ by Lemma 1262 LR with $\{x3 \leftarrow x4, x2 \leftarrow x3, x1 \leftarrow mult(x2, asoc(x3, x4, x4))\}$

$\underbrace{rd(\overbrace{mult(rd(mult(x2, asoc(x3, x4, x4)), asoc(x3, x4, x4)), x3)}, mult(mult(i(x1), rd(mult(x2, asoc(x3, x4, x4)), asoc(x3, x4, x$

$=$ by Lemma 891 LR with $\{x3 \leftarrow i(x1), x2 \leftarrow x3, x1 \leftarrow rd(mult(x2, asoc(x3, x4, x4)), asoc(x3, x4, x4))\}$

$\overbrace{\underbrace{i(op_r(i(x1), rd(mult(x2, asoc(x3, x4, x4)), asoc(x3, x4, x4)), x3))}}$

$=$ by Lemma 2428 RL with $\{x3 \leftarrow x3, x2 \leftarrow rd(mult(x2, asoc(x3, x4, x4)), asoc(x3, x4, x4)), x1 \leftarrow i(x1)\}$

$\overbrace{op_r(\underbrace{i(i(x1))}, rd(mult(x2, asoc(x3, x4, x4)), asoc(x3, x4, x4)), x3)}$

$=$ by Lemma 3 LR with $\{x1 \leftarrow x1\}$

$op_r(\overbrace{x1}, \underbrace{rd(mult(x2, asoc(x3, x4, x4)), asoc(x3, x4, x4))}, x3)$

$=$ by Axiom 5 RL with $\{x2 \leftarrow asoc(x3, x4, x4), x1 \leftarrow x2\}$

$op_r(x1, \overbrace{x2}, x3)$

Lemma 2699: $op_r(x1, mult(op_t(x2, x3), x4), x2) = op_r(x1, mult(op_l(x2, x3, x2), x4), x2)$

$\underbrace{op_r(x1, mult(op_t(x2, x3), x4), x2)}$

$=$ by Lemma 2698 RL with $\{x4 \leftarrow x3, x3 \leftarrow x2, x2 \leftarrow mult(op_t(x2, x3), x4), x1 \leftarrow x1\}$

$\overbrace{op_r(x1, \underbrace{mult(mult(op_t(x2, x3), x4), asoc(x2, x3, x3))}, x2)}$

$=$ by Lemma 1300 RL with $\{x2 \leftarrow x2, x3 \leftarrow x3, x1 \leftarrow mult(op_t(x2, x3), x4)\}$

$op_r(x1, \overbrace{\underbrace{rd(mult(op_t(x2, x3), x4), asoc(x3, x2, x3))}, x2})$

$=$ by Lemma 1708 RL with $\{x3 \leftarrow mult(op_t(x2, x3), x4), x2 \leftarrow x3, x1 \leftarrow x2\}$

$op_r(x1, \overbrace{\underbrace{mult(asoc(x2, x3, x3)}, mult(op_t(x2, x3), x4))}, x2)$

$=$ by Lemma 795 RL with $\{x2 \leftarrow x3, x1 \leftarrow x2\}$

$op_r(x1, \underbrace{mult(\overbrace{rd(op_l(x2, x3, x2), op_t(x2, x3))}, mult(op_t(x2, x3), x4))}, x2)$

$=$ by Lemma 1091 LR with $\{x3 \leftarrow x4, x2 \leftarrow op_t(x2, x3), x1 \leftarrow op_l(x2, x3, x2)\}$

$op_r(x1, \overbrace{mult(op_l(x2, x3, x2), op_l(x4, op_t(x2, x3), \underbrace{op_l(x2, x3, x2)}))}, x2)$

$=$ by Lemma 608 RL with $\{x2 \leftarrow x3, x1 \leftarrow x2\}$

$op_r(x1, mult(op_l(x2, x3, x2), op_l(x4, \underbrace{op_t(x2, x3)}, \overbrace{op_l(x2, x2, i(x3))})), x2)$

$=$ by Lemma 597 RL with $\{x2 \leftarrow x3, x1 \leftarrow x2\}$

$op_r(x1, mult(op_l(x2, x3, x2), op_l(x4, \overbrace{mult(rd(x2, op_t(\underbrace{x3}, x2)), x3)}, op_l(x2, x2, i(x3)))), x2)$

$=$ by Lemma 3 RL with $\{x1 \leftarrow x3\}$

$op_r(x1, mult(op_l(x2, x3, x2), op_l(x4, mult(rd(x2, op_t(\overbrace{i(i(x3))}, x2)), x3)), \underbrace{op_l(x2, x2, i(x3))})), x2)$

$=$ by Lemma 684 LR with $\{x2 \leftarrow i(x3), x1 \leftarrow x2\}$

$op_r(x1, mult(op_l(x2, x3, x2), op_l(x4, mult(\underbrace{rd(x2, op_t(i(i(x3)), x2))}, x3), \overbrace{rd(rd(x2, op_t(i(i(x3)), x2)), i(x3))})), x2)$

$=$ by Lemma 3 RL with $\{x1 \leftarrow rd(x2, op_t(i(i(x3)), x2))\}$

$op_r(x1, mult(op_l(x2, x3, x2), op_l(x4, mult(\overbrace{i(i(rd(x2, op_t(i(i(x3)), x2))))}, x3), \underbrace{rd(rd(x2, op_t(i(i(x3)), x2)), i(x3))})), x2)$

$=$ by Lemma 38 RL with $\{x1 \leftarrow rd(x2, op_t(i(i(x3)), x2)), x2 \leftarrow x3\}$

$op_r(x1, mult(op_l(x2, x3, x2), \underbrace{op_l(x4, mult(i(i(rd(x2, op_t(i(i(x3)), x2)))), x3), \overbrace{rd(x3, i(rd(x2, op_t(i(i(x3)), x2))))})}), x2)$

$=$ by Lemma 164 LR with $\{x3 \leftarrow x3, x2 \leftarrow i(rd(x2, op_t(i(i(x3)), x2))), x1 \leftarrow x4\}$

$op_r(x1, mult(op_l(x2, x3, x2), \overbrace{x4}), x2)$

Lemma 2700: $op_r(x1, mult(x2, i(x4)), x2) = op_r(x1, rd(op_t(x2, x3), x4), x2)$

$$
\begin{aligned}
& \underbrace{op_r(x1, mult(x2, i(x4)), x2)} \\
= & \quad \text{by Lemma 2621 LR with } \{x4 \leftarrow x3, x3 \leftarrow i(x4), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_r(x1, mult(op_l(x2, x3, x2), i(x4)), x2)}} \\
= & \quad \text{by Lemma 2699 RL with } \{x4 \leftarrow i(x4), x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, \underbrace{mult(op_t(x2, x3), i(x4))}, x2)} \\
= & \quad \text{by Lemma 1756 RL with } \{x3 \leftarrow x4, x2 \leftarrow x3, x1 \leftarrow x2\} \\
& \underbrace{op_r(x1, \overbrace{op_t(rd(op_t(x2, x3), x4), i(x2))}, x2)} \\
= & \quad \text{by Lemma 2224 LR with } \{x3 \leftarrow x2, x2 \leftarrow rd(op_t(x2, x3), x4), x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, rd(op_t(x2, x3), x4), x2)}
\end{aligned}
$$

Lemma 2701: $op_r(x1, rd(x2, x4), x2) = op_r(x1, rd(op_t(x2, x3), x4), x2)$

$$
\begin{aligned}
& \underbrace{op_r(x1, rd(x2, x4), x2)} \\
= & \quad \text{by Lemma 2227 LR with } \{x3 \leftarrow x4, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, \underbrace{i(mult(i(x2), x4))}, x2)} \\
= & \quad \text{by Lemma 16 LR with } \{x2 \leftarrow x4, x1 \leftarrow x2\} \\
& \underbrace{op_r(x1, \overbrace{mult(x2, i(x4))}, x2)} \\
= & \quad \text{by Lemma 2700 LR with } \{x3 \leftarrow x3, x4 \leftarrow x4, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, rd(op_t(x2, x3), x4), x2)}
\end{aligned}
$$

Lemma 2702: $op_r(x1, rd(x4, op_t(x2, x3)), x2) = rd(x1, rd(op_r(x1, rd(x2, x4), x2), x1))$

$$
\begin{aligned}
& \underbrace{op_r(x1, rd(x4, op_t(x2, x3)), x2)} \\
= & \quad \text{by Lemma 2436 RL with } \{x4 \leftarrow x2, x3 \leftarrow x4, x2 \leftarrow op_t(x2, x3), x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_r(x1, rd(op_t(x2, x3), x4), i(x2))}} \\
= & \quad \text{by Lemma 2467 RL with } \{x3 \leftarrow x2, x2 \leftarrow rd(op_t(x2, x3), x4), x1 \leftarrow x1\} \\
& \overbrace{rd(x1, rd(\underbrace{op_r(x1, rd(op_t(x2, x3), x4), x2)}, x1))} \\
= & \quad \text{by Lemma 2701 RL with } \{x3 \leftarrow x3, x4 \leftarrow x4, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& rd(x1, rd(\overbrace{op_r(x1, rd(x2, x4), x2)}, x1))
\end{aligned}
$$

Lemma 2703: $op_r(x1, rd(x4, op_t(x2, x3)), x2) = op_r(x1, rd(x4, x2), x2)$

$$
\begin{aligned}
& \underbrace{op_r(x1, rd(x4, op_t(x2, x3)), x2)} \\
= & \quad \text{by Lemma 2702 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x4 \leftarrow x4, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{rd(x1, rd(op_r(x1, rd(x2, x4), x2), x1))}} \\
= & \quad \text{by Lemma 2467 LR with } \{x3 \leftarrow x2, x2 \leftarrow rd(x2, x4), x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_r(x1, rd(x2, x4), i(x2))}} \\
= & \quad \text{by Lemma 2436 LR with } \{x4 \leftarrow x2, x3 \leftarrow x4, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, rd(x4, x2), x2)}
\end{aligned}
$$

Lemma 2704: $op_r(x1, rd(op_r(op_t(x2, x3), x4, x5), x2), x2) = x1$

$$
\begin{aligned}
& \underbrace{op_r(x1, rd(op_r(op_t(x2, x3), x4, x5), x2), x2)} \\
= & \quad \text{by Lemma 2703 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x4 \leftarrow op_r(op_t(x2, x3), x4, x5), x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, \underbrace{rd(op_r(op_t(x2, x3), x4, x5), op_t(x2, x3))}, x2)} \\
= & \quad \text{by Lemma 2618 RL with } \{x3 \leftarrow op_t(x2, x3), x2 \leftarrow x4, x1 \leftarrow x5\} \\
& \underbrace{op_r(x1, \overbrace{rd(x5, op_l(x5, x4, op_t(x2, x3)))}, x2)} \\
= & \quad \text{by Lemma 2616 RL with } \{x4 \leftarrow x3, x3 \leftarrow x2, x5 \leftarrow x4, x2 \leftarrow x5, x1 \leftarrow x1\} \\
& \overbrace{x1}
\end{aligned}
$$

Lemma 2705: $mult(rd(mult(x1, x2), asoc(x1, x2, x3)), x3) = mult(x1, mult(x2, x3))$

$$
\begin{aligned}
& mult(\underbrace{rd(mult(x1, x2), asoc(x1, x2, x3))}, x3) \\
= & \quad \text{by Lemma 2658 RL with } \{x4 \leftarrow mult(x1, x2), x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{mult(\overbrace{mult(i(asoc(x1, x2, x3)), mult(x1, x2))}, x3)} \\
= & \quad \text{by Lemma 91 RL with } \{x3 \leftarrow x3, x2 \leftarrow mult(x1, x2), x1 \leftarrow i(asoc(x1, x2, x3))\} \\
& \overbrace{mult(op_r(\underbrace{i(asoc(x1, x2, x3))}, mult(x1, x2), x3), mult(mult(x1, x2), x3))} \\
= & \quad \text{by Lemma 7 RL with } \{x1 \leftarrow asoc(x1, x2, x3)\} \\
& mult(\underbrace{op_r(\overbrace{op_t(i(asoc(x1, x2, x3)), asoc(x1, x2, x3))}, mult(x1, x2), x3)}, mult(mult(x1, x2), x3)) \\
= & \quad \text{by Axiom 14 RL with } \{x4 \leftarrow asoc(x1, x2, x3), x3 \leftarrow x3, x2 \leftarrow mult(x1, x2), x1 \leftarrow i(asoc(x1, x2, x3))\} \\
& mult(\underbrace{\overbrace{op_t(op_r(i(asoc(x1, x2, x3)), mult(x1, x2), x3), asoc(x1, x2, x3))}}, mult(mult(x1, x2), x3)) \\
= & \quad \text{by Lemma 2455 RL with } \{x1 \leftarrow asoc(x1, x2, x3), x3 \leftarrow mult(x1, x2), x2 \leftarrow x3\} \\
& mult(\overbrace{rd(op_l(x3, mult(x1, x2), asoc(x1, x2, x3)), mult(asoc(x1, x2, x3), x3))}, mult(mult(x1, x2), x3)) \\
= & \quad \text{by Lemma 2239 RL with } \{x3 \leftarrow asoc(x1, x2, x3), x2 \leftarrow mult(x1, x2), x1 \leftarrow x3\} \\
& mult(\overbrace{mult(\underbrace{op_l(x3, mult(x1, x2), asoc(x1, x2, x3))}, i(mult(x3, asoc(x1, x2, x3))))}, mult(mult(x1, x2), x3)) \\
= & \quad \text{by Lemma 2599 LR with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x3\} \\
& mult(\underbrace{mult(\overbrace{x3}, i(mult(x3, asoc(x1, x2, x3))))}, mult(mult(x1, x2), x3)) \\
= & \quad \text{by Lemma 20 LR with } \{x2 \leftarrow asoc(x1, x2, x3), x1 \leftarrow x3\} \\
& \underbrace{mult(\overbrace{i(asoc(x1, x2, x3))}, mult(mult(x1, x2), x3))} \\
= & \quad \text{by Lemma 2658 LR with } \{x4 \leftarrow mult(mult(x1, x2), x3), x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{rd(\underbrace{mult(mult(x1, x2), x3)}, asoc(x1, x2, x3))} \\
= & \quad \text{by Axiom 8 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{rd(\overbrace{mult(mult(x1, mult(x2, x3)), asoc(x1, x2, x3))}, asoc(x1, x2, x3))} \\
= & \quad \text{by Axiom 5 RL with } \{x2 \leftarrow asoc(x1, x2, x3), x1 \leftarrow mult(x1, mult(x2, x3))\} \\
& \overbrace{mult(x1, mult(x2, x3))}
\end{aligned}
$$

Lemma 2706: $mult(asoc(x1,x2,x3),mult(x1,mult(x2,x3))) = mult(mult(x1,x2),x3)$

$mult(\underbrace{asoc(x1,x2,x3)}, mult(x1,mult(x2,x3)))$

= by Lemma 2657 RL with $\{x4 \leftarrow mult(x1,x2), x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$mult(\overbrace{op_r(asoc(x1,x2,x3),mult(x1,x2),mult(x1,x2))}, \underbrace{mult(x1,mult(x2,x3))})$

= by Lemma 2705 RL with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{mult(op_r(asoc(x1,x2,x3),mult(x1,x2),mult(x1,x2)), \overbrace{mult(rd(mult(x1,x2),asoc(x1,x2,x3)),x3)})}$

= by Lemma 1719 RL with $\{x3 \leftarrow mult(x1,x2), x2 \leftarrow mult(rd(mult(x1,x2),asoc(x1,x2,x3)),x3), x1 \leftarrow asoc(x1,x2,x3)\}$

$\overbrace{rd(\underbrace{mult(asoc(x1,x2,x3),mult(rd(mult(x1,x2),asoc(x1,x2,x3)),x3))}, asoc(mult(x1,x2),asoc(x1,x2,x3),mult(x1,x2)))}$

= by Lemma 1874 LR with $\{x3 \leftarrow x3, x2 \leftarrow mult(x1,x2), x1 \leftarrow asoc(x1,x2,x3)\}$

$\underbrace{rd(\overbrace{mult(op_l(mult(x1,x2),mult(x1,x2),asoc(x1,x2,x3)),op_l(x3,mult(x1,x2),asoc(x1,x2,x3)))}, asoc(mult(x1,x2),asoc($

= by Lemma 1708 RL with $\{x3 \leftarrow mult(op_l(mult(x1,x2),mult(x1,x2),asoc(x1,x2,x3)),op_l(x3,mult(x1,x2),asoc(x1,x2,x3)$

$\overbrace{mult(asoc(asoc(x1,x2,x3),mult(x1,x2),mult(x1,x2)),\underbrace{mult(op_l(mult(x1,x2),mult(x1,x2),asoc(x1,x2,x3)),op_l(x3,mu}$

= by Lemma 1994 RL with $\{x3 \leftarrow asoc(x1,x2,x3), x2 \leftarrow op_l(x3,mult(x1,x2),asoc(x1,x2,x3)), x1 \leftarrow mult(x1,x2)\}$

$\overbrace{mult(mult(x1,x2),\underbrace{op_l(x3,mult(x1,x2),asoc(x1,x2,x3))})}$

= by Lemma 2599 LR with $\{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x3\}$

$mult(mult(x1,x2),\overbrace{x3})$

Lemma 2707: $mult(asoc(x1,x2,x3),x1) = rd(mult(mult(x1,x2),x3),mult(x2,x3))$

$\underbrace{mult(asoc(x1,x2,x3),x1)}$

= by Lemma 888 RL with $\{x3 \leftarrow mult(x2,x3), x2 \leftarrow x1, x1 \leftarrow asoc(x1,x2,x3)\}$

$\overbrace{rd(\underbrace{mult(asoc(x1,x2,x3),mult(x1,mult(x2,x3)))}, op_l(mult(x2,x3),x1,asoc(x1,x2,x3)))}$

= by Lemma 2706 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$rd(\overbrace{mult(mult(x1,x2),x3)}, \underbrace{op_l(mult(x2,x3),x1,asoc(x1,x2,x3))})$

= by Lemma 2597 LR with $\{x3 \leftarrow x1, x2 \leftarrow x3, x1 \leftarrow x2\}$

$rd(mult(mult(x1,x2),x3),\overbrace{\underbrace{rd(mult(x2,x3),unit())}})$

= by Lemma 1 LR with $\{x1 \leftarrow mult(x2,x3)\}$

$rd(mult(mult(x1,x2),x3),\overbrace{mult(x2,x3)})$

Lemma 2708: $mult(x1,asoc(x1,x2,x3)) = op_r(x1,x2,x3)$

$\underbrace{mult(x1,asoc(x1,x2,x3))}$

= by Lemma 2662 RL with $\{x4 \leftarrow x1, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{\underbrace{mult(asoc(x1,x2,x3),x1)}}$

= by Lemma 2707 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{\underbrace{rd(mult(mult(x1,x2),x3),mult(x2,x3))}}$

= by Axiom 16 RL with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_r(x1,x2,x3)}$

Lemma 2709: $op_r(x1, x2, asoc(x2, x3, x4)) = op_l(x1, op_r(x2, x3, x4), mult(x2, x2))$

$\overbrace{op_r(x1, x2, asoc(x2, x3, x4))}$

$=$ by Lemma 2425 RL with $\{x3 \leftarrow x2, x2 \leftarrow asoc(x2, x3, x4), x1 \leftarrow x1\}$

$\overbrace{rd(x1, rd(asoc(x2, x3, x4), op_l(\underbrace{asoc(x2, x3, x4)}, x1, x2)))}$

$=$ by Lemma 2654 RL with $\{x4 \leftarrow mult(x2, x1), x3 \leftarrow x4, x2 \leftarrow x3, x1 \leftarrow x2\}$

$rd(x1, rd(asoc(x2, x3, x4), \underbrace{op_l(\overbrace{op_t(asoc(x2, x3, x4), mult(x2, x1))}, x1, x2)}))$

$=$ by Lemma 2033 RL with $\{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow op_t(asoc(x2, x3, x4), mult(x2, x1))\}$

$rd(x1, rd(asoc(x2, x3, x4), \overbrace{op_l(op_t(asoc(x2, x3, x4), mult(x2, x1)), mult(x2, x1), x2)}))$

$=$ by Lemma 2321 LR with $\{x3 \leftarrow x2, x2 \leftarrow mult(x2, x1), x1 \leftarrow asoc(x2, x3, x4)\}$

$rd(x1, rd(asoc(x2, x3, x4), \overbrace{\underbrace{mult(mult(i(x2), i(mult(x2, x1)))}, mult(x2, mult(asoc(x2, x3, x4), mult(x2, x1))))}))$

$=$ by Axiom 11 LR with $\{x2 \leftarrow mult(x2, x1), x1 \leftarrow x2\}$

$rd(x1, rd(asoc(x2, x3, x4), mult(\overbrace{i(mult(x2, mult(x2, x1)))}, \underbrace{mult(x2, mult(asoc(x2, x3, x4), mult(x2, x1)))})))$

$=$ by Axiom 6 LR with $\{x3 \leftarrow x1, x2 \leftarrow asoc(x2, x3, x4), x1 \leftarrow x2\}$

$rd(x1, rd(asoc(x2, x3, x4), \underbrace{mult(i(mult(x2, mult(x2, x1))), \overbrace{mult(mult(x2, mult(asoc(x2, x3, x4), x2)), x1)})}))$

$=$ by Lemma 2690 RL with $\{x5 \leftarrow x1, x4 \leftarrow x2, x3 \leftarrow x4, x2 \leftarrow x3, x1 \leftarrow x2\}$

$\underbrace{rd(x1, rd(asoc(x2, x3, x4), \overbrace{op_r(asoc(x2, x3, x4), mult(x2, x2), x1)}))}$

$=$ by Lemma 2437 LR with $\{x3 \leftarrow mult(x2, x2), x2 \leftarrow asoc(x2, x3, x4), x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_l(x1, asoc(x2, x3, x4), mult(x2, x2))}}$

$=$ by Lemma 2030 RL with $\{x3 \leftarrow asoc(x2, x3, x4), x2 \leftarrow mult(x2, x2), x1 \leftarrow x1\}$

$\overbrace{op_l(x1, mult(x2, x2), \underbrace{i(asoc(x2, x3, x4))})}$

$=$ by Axiom 4 LR with $\{x2 \leftarrow x2, x1 \leftarrow i(asoc(x2, x3, x4))\}$

$\underbrace{op_l(x1, mult(x2, x2), \overbrace{mult(rd(i(asoc(x2, x3, x4)), x2), x2)})}$

$=$ by Lemma 2492 LR with $\{x3 \leftarrow rd(i(asoc(x2, x3, x4)), x2), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_l(x1, mult(x2, x2), rd(i(asoc(x2, x3, x4)), x2))}}$

$=$ by Lemma 2494 RL with $\{x3 \leftarrow rd(i(asoc(x2, x3, x4)), x2), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_l(x1, mult(x2, x2), \underbrace{op_t(rd(i(asoc(x2, x3, x4)), x2), x2)})}$

$=$ by Lemma 14 LR with $\{x2 \leftarrow x2, x1 \leftarrow i(asoc(x2, x3, x4))\}$

$op_l(x1, mult(x2, x2), \overbrace{\underbrace{mult(i(x2), i(asoc(x2, x3, x4)))}})$

$=$ by Lemma 13 RL with $\{x2 \leftarrow i(x2), x1 \leftarrow i(asoc(x2, x3, x4))\}$

$\underbrace{op_l(x1, mult(x2, x2), \overbrace{mult(i(asoc(x2, x3, x4)), op_t(i(x2), i(asoc(x2, x3, x4))))})}$

$=$ by Lemma 2243 RL with $\{x3 \leftarrow i(asoc(x2, x3, x4)), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_l(x1, mult(x2, x2), mult(x2, i(asoc(x2, x3, x4))))}}$

$=$ by Lemma 2246 LR with $\{x3 \leftarrow asoc(x2, x3, x4), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_l(x1, \underbrace{mult(x2, asoc(x2, x3, x4))}, mult(x2, x2))}$

$=$ by Lemma 2708 LR with $\{x3 \leftarrow x4, x2 \leftarrow x3, x1 \leftarrow x2\}$

$op_l(x1, \overbrace{op_r(x2, x3, x4)}, mult(x2, x2))$

Lemma 2710: $op_r(x1, x2, asoc(x2, x3, x4)) = x1$

$$
\begin{aligned}
& \underbrace{op_r(x1, x2, asoc(x2, x3, x4))} \\
= & \quad \text{by Lemma 2709 LR with } \{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_l(x1, op_r(x2, x3, x4), mult(x2, x2))}} \\
= & \quad \text{by Lemma 2439 LR with } \{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{x1}
\end{aligned}
$$

Lemma 2711: $rd(op_r(x1, x2, x3), x1) = asoc(x1, x2, x3)$

$$
\begin{aligned}
& rd(\underbrace{op_r(x1, x2, x3)}, x1) \\
= & \quad \text{by Axiom 16 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& rd(\overbrace{rd(\underbrace{mult(mult(x1, x2), x3)}, mult(x2, x3))}, x1) \\
= & \quad \text{by Lemma 2706 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{rd(rd(\overbrace{mult(asoc(x1, x2, x3), mult(x1, mult(x2, x3)))}, mult(x2, x3)), x1)} \\
= & \quad \text{by Lemma 2608 LR with } \{x1 \leftarrow x1, x2 \leftarrow mult(x2, x3), x3 \leftarrow mult(asoc(x1, x2, x3), mult(x1, mult(x2, x3)))\} \\
& \overbrace{op_r(\underbrace{rd(mult(asoc(x1, x2, x3), mult(x1, mult(x2, x3))), mult(x1, mult(x2, x3)))}, x1, i(mult(x2, x3)))} \\
= & \quad \text{by Axiom 5 RL with } \{x2 \leftarrow mult(x1, mult(x2, x3)), x1 \leftarrow asoc(x1, x2, x3)\} \\
& \underbrace{op_r(\overbrace{asoc(x1, x2, x3)}, x1, i(mult(x2, x3)))} \\
= & \quad \text{by Lemma 2400 with } \{x4 \leftarrow x3, x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow asoc(x1, x2, x3)\} \\
& \overbrace{\underbrace{op_r(asoc(x1, x2, x3), x1, rd(i(x2), x3))}} \\
= & \quad \text{by Lemma 2626 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{rd(asoc(x1, x2, x3), unit())}} \\
= & \quad \text{by Lemma 1 LR with } \{x1 \leftarrow asoc(x1, x2, x3)\} \\
& \overbrace{asoc(x1, x2, x3)}
\end{aligned}
$$

Lemma 2712: $asoc(x1, i(x2), x3) = asoc(x1, x2, i(x3))$

$$
\begin{aligned}
& \underbrace{asoc(x1, i(x2), x3)} \\
= & \quad \text{by Lemma 2711 RL with } \{x3 \leftarrow x3, x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& \overbrace{rd(\underbrace{op_r(x1, i(x2), x3)}, x1)} \\
= & \quad \text{by Lemma 2427 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{rd(\overbrace{op_r(x1, x2, i(x3))}, x1)} \\
= & \quad \text{by Lemma 2711 LR with } \{x3 \leftarrow i(x3), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{asoc(x1, x2, i(x3))}
\end{aligned}
$$

Lemma 2713: $i(asoc(x1, x2, x3)) = asoc(i(x1), x2, x3)$

$$
\begin{aligned}
& \overbrace{i(asoc(x1, x2, x3))} \\
= \quad & \text{by Lemma 1480 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{asoc(i(x1), i(x2), i(x3))} \\
= \quad & \text{by Lemma 2712 LR with } \{x3 \leftarrow i(x3), x2 \leftarrow x2, x1 \leftarrow i(x1)\} \\
& \overbrace{asoc(i(x1), x2, \underbrace{i(i(x3))})} \\
= \quad & \text{by Lemma 3 LR with } \{x1 \leftarrow x3\} \\
& asoc(i(x1), x2, \overbrace{x3})
\end{aligned}
$$

Lemma 2714: $op_r(x1, asoc(x2, x3, x4), x4) = x1$

$$
\begin{aligned}
& op_r(x1, \underbrace{asoc(x2, x3, x4)}, x4) \\
= \quad & \text{by Lemma 2711 RL with } \{x3 \leftarrow x4, x2 \leftarrow x3, x1 \leftarrow x2\} \\
& op_r(x1, \overbrace{\underbrace{rd(op_r(x2, x3, x4), x2)}}, x4) \\
= \quad & \text{by Lemma 2306 RL with } \{x3 \leftarrow x4, x2 \leftarrow x3, x1 \leftarrow x2\} \\
& op_r(x1, \overbrace{\underbrace{op_r(rd(op_r(x2, x3, x4), x2), x4, x4)}}, x4) \\
= \quad & \text{by Lemma 2133 RL with } \{x4 \leftarrow x3, x3 \leftarrow x2, x2 \leftarrow x4, x1 \leftarrow rd(op_r(x2, x3, x4), x2)\} \\
& op_r(x1, \overbrace{\underbrace{op_r(rd(op_r(x2, x3, x4), x2)}, op_l(x4, x2, x3), op_l(x4, x2, x3))}, x4) \\
= \quad & \text{by Lemma 2419 LR with } \{x3 \leftarrow x4, x2 \leftarrow x3, x1 \leftarrow x2\} \\
& op_r(x1, \underbrace{op_r(\overbrace{rd(op_l(x4, x2, x3), x4)}, op_l(x4, x2, x3), op_l(x4, x2, x3))}, x4) \\
= \quad & \text{by Lemma 1495 LR with } \{x1 \leftarrow x4, x2 \leftarrow op_l(x4, x2, x3)\} \\
& op_r(x1, \overbrace{\underbrace{op_r(i(rd(x4, op_l(x4, x2, x3)))}, op_l(x4, x4, op_l(x4, x2, x3)), op_l(x4, x2, x3))}, x4) \\
= \quad & \text{by Lemma 10 LR with } \{x2 \leftarrow op_l(x4, x2, x3), x1 \leftarrow x4\} \\
& op_r(x1, op_r(\overbrace{rd(op_l(x4, x2, x3), x4)}, \underbrace{op_l(x4, x4, op_l(x4, x2, x3))}, op_l(x4, x2, x3)), x4) \\
= \quad & \text{by Lemma 286 RL with } \{x3 \leftarrow op_l(x4, x2, x3), x2 \leftarrow x4, x1 \leftarrow x4\} \\
& op_r(x1, op_r(rd(op_l(x4, x2, x3), x4), \overbrace{op_l(x4, x4, rd(op_l(x4, x2, x3), x4))}, \underbrace{op_l(x4, x2, x3)}), x4) \\
= \quad & \text{by Axiom 4 LR with } \{x2 \leftarrow x4, x1 \leftarrow op_l(x4, x2, x3)\} \\
& op_r(x1, \underbrace{op_r(rd(op_l(x4, x2, x3), x4), op_l(x4, x4, rd(op_l(x4, x2, x3), x4))}, \overbrace{mult(rd(op_l(x4, x2, x3), x4), x4)}), x4) \\
= \quad & \text{by Lemma 1205 RL with } \{x1 \leftarrow rd(op_l(x4, x2, x3), x4), x2 \leftarrow x4\} \\
& \underbrace{op_r(x1, \overbrace{rd(mult(x4, rd(op_l(x4, x2, x3), x4)), x4)}, x4)} \\
= \quad & \text{by Lemma 1896 LR with } \{x3 \leftarrow rd(op_l(x4, x2, x3), x4), x2 \leftarrow x4, x1 \leftarrow x1\} \\
& \underbrace{\overbrace{op_r(x1, rd(op_l(x4, x2, x3), x4), x4)}} \\
= \quad & \text{by Lemma 2360 LR with } \{x4 \leftarrow x3, x3 \leftarrow x2, x2 \leftarrow x4, x1 \leftarrow x1\} \\
& \overbrace{x1}
\end{aligned}
$$

Lemma 2715: $rd(x1, asoc(x2, x3, x1)) = op_l(x1, x3, x2)$

$$
\begin{aligned}
& \underbrace{rd(x1, \underbrace{asoc(x2, x3, x1)}) } \\
= & \quad \text{by Lemma 2711 RL with } \{x3 \leftarrow x1, x2 \leftarrow x3, x1 \leftarrow x2\} \\
& \underbrace{rd(x1, \overbrace{rd(op_r(x2, x3, x1), x2)}) } \\
= & \quad \text{by Lemma 2448 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x3, x2)}
\end{aligned}
$$

Lemma 2716: $rd(x1, asoc(x2, x1, x3)) = op_r(x1, x2, x3)$

$$
\begin{aligned}
& \underbrace{rd(x1, \underbrace{asoc(x2, x1, x3)}) } \\
= & \quad \text{by Lemma 2711 RL with } \{x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \underbrace{rd(x1, \overbrace{rd(op_r(x2, x1, x3), x2)}) } \\
= & \quad \text{by Lemma 2563 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, x2, x3)}
\end{aligned}
$$

Lemma 2717: $asoc(i(x1), x2, x3) = asoc(x2, x1, x3)$

$$
\begin{aligned}
& \underbrace{asoc(i(x1), x2, x3)} \\
= & \quad \text{by Lemma 2654 RL with } \{x4 \leftarrow x2, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow i(x1)\} \\
& \overbrace{op_t(\underbrace{asoc(i(x1), x2, x3)}, x2)} \\
= & \quad \text{by Lemma 2713 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{op_t(\overbrace{i(asoc(x1, x2, x3))}, x2)} \\
= & \quad \text{by Lemma 345 RL with } \{x2 \leftarrow asoc(x1, x2, x3), x1 \leftarrow x2\} \\
& \overbrace{rd(op_t(\underbrace{rd(x2, asoc(x1, x2, x3))}, x2), x2)} \\
= & \quad \text{by Lemma 2716 LR with } \{x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow x2\} \\
& rd(\underbrace{op_t(\overbrace{op_r(x2, x1, x3)}, x2)}, x2) \\
= & \quad \text{by Axiom 14 LR with } \{x4 \leftarrow x2, x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow x2\} \\
& rd(\overbrace{op_r(\underbrace{op_t(x2, x2)}, x1, x3)}, x2) \\
= & \quad \text{by Lemma 4 LR with } \{x1 \leftarrow x2\} \\
& \underbrace{rd(op_r(\overbrace{x2}, x1, x3), x2)} \\
= & \quad \text{by Lemma 2711 LR with } \{x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow x2\} \\
& \overbrace{asoc(x2, x1, x3)}
\end{aligned}
$$

Lemma 2718: $i(asoc(x1, x2, x3)) = asoc(x2, x1, x3)$

$$
\begin{aligned}
& \underbrace{i(asoc(x1, x2, x3))} \\
= & \quad \text{by Lemma 2713 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{\overbrace{asoc(i(x1), x2, x3)}} \\
= & \quad \text{by Lemma 2717 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{asoc(x2, x1, x3)}
\end{aligned}
$$

Lemma 2719: $asoc(x1, x2, i(x3)) = asoc(x2, x1, x3)$

$$
\begin{aligned}
& \overbrace{asoc(x1, x2, i(x3))} \\
= & \quad \text{by Lemma 2712 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\overbrace{asoc(x1, i(x2), x3)}} \\
= & \quad \text{by Lemma 2717 RL with } \{x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow i(x2)\} \\
& \overbrace{asoc(\overbrace{i(i(x2))}, x1, x3)} \\
= & \quad \text{by Lemma 3 LR with } \{x1 \leftarrow x2\} \\
& asoc(\overbrace{x2}, x1, x3)
\end{aligned}
$$

Lemma 2720: $asoc(x1, i(x2), x3) = asoc(x2, x1, x3)$

$$
\begin{aligned}
& \overbrace{asoc(x1, i(x2), x3)} \\
= & \quad \text{by Lemma 2712 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\overbrace{asoc(x1, x2, i(x3))}} \\
= & \quad \text{by Lemma 2719 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{asoc(x2, x1, x3)}
\end{aligned}
$$

Lemma 2721: $rd(x1, op_l(x1, x2, x3)) = asoc(x3, x2, x1)$

$$
\begin{aligned}
& \overbrace{rd(x1, op_l(x1, x2, x3))} \\
= & \quad \text{by Lemma 2618 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\overbrace{rd(op_r(x3, x2, x1), x3)}} \\
= & \quad \text{by Lemma 2711 LR with } \{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x3\} \\
& \overbrace{asoc(x3, x2, x1)}
\end{aligned}
$$

Lemma 2722: $rd(op_l(x1, x2, x3), x1) = asoc(x2, x3, x1)$

$$
\begin{aligned}
& \overbrace{rd(op_l(x1, x2, x3), x1)} \\
= & \quad \text{by Lemma 2449 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\overbrace{rd(x1, op_l(x1, x3, x2))}} \\
= & \quad \text{by Lemma 2721 LR with } \{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{asoc(x2, x3, x1)}
\end{aligned}
$$

Lemma 2723: $rd(x1, op_r(x1, x2, x3)) = asoc(x2, x1, x3)$

$$
\begin{aligned}
& \overbrace{rd(x1, op_r(x1, x2, x3))} \\
= & \quad \text{by Lemma 2620 RL with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x3\} \\
& \overbrace{\overbrace{rd(x3, op_l(x3, x1, x2))}} \\
= & \quad \text{by Lemma 2721 LR with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x3\} \\
& \overbrace{asoc(x2, x1, x3)}
\end{aligned}
$$

Lemma 2724: $asoc(x1, x2, op_t(x3, x4)) = asoc(x1, x2, x3)$

$$
\begin{aligned}
& \underbrace{asoc(x1, x2, op_t(x3, x4))} \\
= & \quad \text{by Lemma 2711 RL with } \{x3 \leftarrow op_t(x3, x4), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{rd(\underbrace{op_r(x1, x2, op_t(x3, x4))}, x1)} \\
= & \quad \text{by Lemma 2392 LR with } \{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{rd(\overbrace{op_r(x1, x2, x3)}, x1)} \\
= & \quad \text{by Lemma 2711 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{asoc(x1, x2, x3)}
\end{aligned}
$$

Lemma 2725: $asoc(x1, rd(x2, x1), x3) = asoc(x1, x2, x3)$

$$
\begin{aligned}
& \underbrace{asoc(x1, rd(x2, x1), x3)} \\
= & \quad \text{by Lemma 2711 RL with } \{x3 \leftarrow x3, x2 \leftarrow rd(x2, x1), x1 \leftarrow x1\} \\
& \overbrace{rd(\underbrace{op_r(x1, rd(x2, x1), x3)}, x1)} \\
= & \quad \text{by Lemma 2429 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{rd(\overbrace{op_r(x1, x2, x3)}, x1)} \\
= & \quad \text{by Lemma 2711 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{asoc(x1, x2, x3)}
\end{aligned}
$$

Lemma 2726: $asoc(x1, rd(x1, x2), x3) = asoc(x2, x1, x3)$

$$
\begin{aligned}
& \underbrace{asoc(x1, rd(x1, x2), x3)} \\
= & \quad \text{by Lemma 2722 RL with } \{x3 \leftarrow rd(x1, x2), x2 \leftarrow x1, x1 \leftarrow x3\} \\
& \overbrace{rd(\underbrace{op_l(x3, x1, rd(x1, x2))}, x3)} \\
= & \quad \text{by Lemma 2035 LR with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x3\} \\
& \underbrace{rd(\overbrace{op_l(x3, x2, x1)}, x3)} \\
= & \quad \text{by Lemma 2722 LR with } \{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x3\} \\
& \overbrace{asoc(x2, x1, x3)}
\end{aligned}
$$

Lemma 2727: $op_r(x1, asoc(x2, x3, x4), i(x4)) = mult(i(x1), mult(x1, x1))$

$$
\begin{aligned}
& \underbrace{op_r(x1, asoc(x2, x3, x4), i(x4))} \\
= & \quad \text{by Lemma 2528 LR with } \{x3 \leftarrow x4, x2 \leftarrow asoc(x2, x3, x4), x1 \leftarrow x1\} \\
& \overbrace{mult(op_r(i(x1), asoc(x2, x3, x4), x4), mult(x1, \underbrace{op_l(x1, op_r(x1, asoc(x2, x3, x4), i(x4)), x1)}))} \\
= & \quad \text{by Lemma 1443 LR with } \{x4 \leftarrow i(x4), x3 \leftarrow asoc(x2, x3, x4), x2 \leftarrow x1, x1 \leftarrow x1\} \\
& mult(\underbrace{op_r(i(x1), asoc(x2, x3, x4), x4)}, mult(x1, \overbrace{x1})) \\
= & \quad \text{by Lemma 2714 LR with } \{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow i(x1)\} \\
& mult(\overbrace{i(x1)}, mult(x1, x1))
\end{aligned}
$$

Lemma 2728: $mult(x1, asoc(x2, x3, x4)) = rd(x1, asoc(x3, x2, x4))$

$$
\begin{aligned}
& mult(x1, \underbrace{asoc(x2, x3, x4)}) \\
= & \quad \text{by Lemma 2721 RL with } \{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x4\} \\
& mult(x1, \overbrace{rd(x4, op_l(x4, x3, x2))}) \\
= & \quad \text{by Lemma 2449 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x4\} \\
& \underbrace{mult(x1, \overbrace{rd(op_l(x4, x2, x3), x4)})} \\
= & \quad \text{by Lemma 2363 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x4, x4 \leftarrow x1\} \\
& \overbrace{rd(x1, \underbrace{rd(x4, op_l(x4, x2, x3))})} \\
= & \quad \text{by Lemma 2721 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x4\} \\
& rd(x1, \overbrace{asoc(x3, x2, x4)})
\end{aligned}
$$

Lemma 2729: $mult(asoc(x3, x2, x1), x4) = rd(x4, asoc(x2, x3, x1))$

$$
\begin{aligned}
& mult(\underbrace{asoc(x3, x2, x1)}, x4) \\
= & \quad \text{by Lemma 2721 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{mult(\overbrace{rd(x1, op_l(x1, x2, x3))}, x4)} \\
= & \quad \text{by Lemma 2648 LR with } \{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{rd(x4, \underbrace{rd(x1, op_l(x1, x3, x2))})} \\
= & \quad \text{by Lemma 2721 LR with } \{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x1\} \\
& rd(x4, \overbrace{asoc(x2, x3, x1)})
\end{aligned}
$$

Lemma 2730: $mult(op_l(i(x1), x2, x3), x1) = asoc(x3, x2, x1)$

$$
\begin{aligned}
& \underbrace{mult(op_l(i(x1), x2, x3), x1)} \\
= & \quad \text{by Lemma 1058 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{rd(x1, op_l(x1, x2, x3))}} \\
= & \quad \text{by Lemma 2721 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{asoc(x3, x2, x1)}
\end{aligned}
$$

Lemma 2731: $asoc(x1, x2, op_l(x3, x4, x3)) = asoc(x1, x2, x3)$

$$
\begin{aligned}
& \underbrace{asoc(x1, x2, op_l(x3, x4, x3))} \\
= & \quad \text{by Lemma 2711 RL with } \{x3 \leftarrow op_l(x3, x4, x3), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{rd(\underbrace{op_r(x1, x2, op_l(x3, x4, x3))}, x1)} \\
= & \quad \text{by Lemma 2390 LR with } \{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{rd(\overbrace{op_r(x1, x2, x3)}, x1)} \\
= & \quad \text{by Lemma 2711 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{asoc(x1, x2, x3)}
\end{aligned}
$$

Lemma 2732: $asoc(x1, x2, op_r(x3, x4, x4)) = asoc(x1, x2, x3)$

$$
\begin{aligned}
& \underbrace{asoc(x1, x2, op_r(x3, x4, x4))} \\
= & \quad \text{by Lemma 2711 RL with } \{x3 \leftarrow op_r(x3, x4, x4), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{rd(\underbrace{op_r(x1, x2, op_r(x3, x4, x4))}, x1)} \\
= & \quad \text{by Lemma 2391 LR with } \{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{rd(\overbrace{op_r(x1, x2, x3)}, x1)} \\
= & \quad \text{by Lemma 2711 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{asoc(x1, x2, x3)}
\end{aligned}
$$

Lemma 2733: $asoc(rd(x1, x2), x3, x4) = asoc(x3, rd(x2, x1), x4)$

$$
\begin{aligned}
& asoc(rd(x1, x2), x3, \underbrace{x4}) \\
= & \quad \text{by Lemma 3 RL with } \{x1 \leftarrow x4\} \\
& \underbrace{asoc(rd(x1, x2), x3, \overbrace{i(i(x4))})} \\
= & \quad \text{by Lemma 2712 RL with } \{x3 \leftarrow i(x4), x2 \leftarrow x3, x1 \leftarrow rd(x1, x2)\} \\
& \overbrace{\underbrace{asoc(rd(x1, x2), i(x3), i(x4))}} \\
= & \quad \text{by Lemma 1478 LR with } \{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{i(asoc(rd(x2, x1), x3, x4))}} \\
= & \quad \text{by Lemma 2718 LR with } \{x3 \leftarrow x4, x2 \leftarrow x3, x1 \leftarrow rd(x2, x1)\} \\
& \overbrace{asoc(x3, rd(x2, x1), x4)}
\end{aligned}
$$

Lemma 2734: $rd(i(x1), asoc(x2, x3, x4)) = rd(asoc(x3, x2, x4), x1)$

$$
\begin{aligned}
& \underbrace{rd(i(x1), asoc(x2, x3, x4))} \\
= & \quad \text{by Lemma 2728 RL with } \{x4 \leftarrow x4, x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow i(x1)\} \\
& \overbrace{\underbrace{mult(i(x1), asoc(x3, x2, x4))}} \\
= & \quad \text{by Lemma 2659 RL with } \{x4 \leftarrow x1, x3 \leftarrow x4, x2 \leftarrow x2, x1 \leftarrow x3\} \\
& \overbrace{rd(asoc(x3, x2, x4), x1)}
\end{aligned}
$$

Lemma 2735: $asoc(x1, op_t(x2, i(x3)), x4) = rd(op_r(x1, op_t(x2, x3), x4), x1)$

$$
\begin{aligned}
& \overbrace{asoc(x1, op_t(x2, i(x3)), x4)} \\
= \quad & \text{by Lemma 2711 RL with } \{x3 \leftarrow x4, x2 \leftarrow op_t(x2, i(x3)), x1 \leftarrow x1\} \\
& \overbrace{rd(op_r(x1, \underbrace{op_t(x2, i(x3))}, x4), x1)} \\
= \quad & \text{by Lemma 1130 RL with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& rd(op_r(x1, \overbrace{op_l(op_t(x2, x3), x2, mult(x3, x3))}, x4), x1) \\
= \quad & \text{by Lemma 2030 RL with } \{x3 \leftarrow x2, x2 \leftarrow mult(x3, x3), x1 \leftarrow op_t(x2, x3)\} \\
& rd(op_r(x1, \overbrace{op_l(op_t(x2, x3), mult(x3, x3), i(x2))}, x4), x1) \\
= \quad & \text{by Lemma 2494 RL with } \{x3 \leftarrow i(x2), x2 \leftarrow x3, x1 \leftarrow op_t(x2, x3)\} \\
& rd(op_r(x1, \overbrace{op_l(op_t(x2, x3), mult(x3, x3), \underbrace{op_t(i(x2), x3)})}, x4), x1) \\
= \quad & \text{by Lemma 19 RL with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& rd(op_r(x1, \underbrace{op_l(op_t(x2, x3), mult(x3, x3), \overbrace{i(op_t(x2, i(x3)))})}, x4), x1) \\
= \quad & \text{by Lemma 1315 RL with } \{x3 \leftarrow mult(x3, x3), x2 \leftarrow op_t(x2, i(x3)), x1 \leftarrow op_t(x2, x3)\} \\
& rd(op_r(x1, \overbrace{op_l(op_t(x2, x3), op_t(x2, i(x3)), \underbrace{op_t(mult(x3, x3), op_t(x2, i(x3)))})}, x4), x1) \\
= \quad & \text{by Lemma 1722 LR with } \{x3 \leftarrow i(x3), x2 \leftarrow x2, x1 \leftarrow mult(x3, x3)\} \\
& rd(op_r(x1, op_l(op_t(x2, x3), op_t(x2, i(x3)), \overbrace{op_t(mult(x3, x3), x2)}), x4), x1) \\
= \quad & \text{by Lemma 1722 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow mult(x3, x3)\} \\
& rd(op_r(x1, op_l(op_t(x2, x3), op_t(\underbrace{x2}, i(x3)), \overbrace{op_t(mult(x3, x3), op_t(x2, x3))}), x4), x1) \\
= \quad & \text{by Lemma 12 RL with } \{x2 \leftarrow x2, x1 \leftarrow x3\} \\
& rd(op_r(x1, \underbrace{op_l(op_t(x2, x3), op_t(\overbrace{mult(x3, mult(i(x3), x2))}, i(x3)), op_t(mult(x3, x3), op_t(x2, x3)))}, x4), x1) \\
= \quad & \text{by Lemma 2072 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow op_t(x2, x3)\} \\
& \underbrace{rd(op_r(x1, \overbrace{op_l(op_t(x2, x3), op_t(x2, x3), mult(x3, x3))}, x4)}, x1) \\
= \quad & \text{by Lemma 2679 LR with } \{x4 \leftarrow x4, x3 \leftarrow mult(x3, x3), x2 \leftarrow op_t(x2, x3), x1 \leftarrow x1\} \\
& \overbrace{rd(op_r(x1, op_t(x2, x3), x4), x1)}
\end{aligned}
$$

Lemma 2736: $asoc(rd(i(x1), x2), x3, x1) = asoc(x3, mult(x1, x2), x1)$

$$
\begin{aligned}
& \overbrace{asoc(rd(i(x1), x2), x3, x1)} \\
= \quad & \text{by Lemma 2721 RL with } \{x3 \leftarrow rd(i(x1), x2), x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{rd(x1, \underbrace{op_l(x1, x3, rd(i(x1), x2))})} \\
= \quad & \text{by Lemma 2333 LR with } \{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \underbrace{rd(x1, \overbrace{op_l(x1, mult(x1, x2), x3)})} \\
= \quad & \text{by Lemma 2721 LR with } \{x3 \leftarrow x3, x2 \leftarrow mult(x1, x2), x1 \leftarrow x1\} \\
& \overbrace{asoc(x3, mult(x1, x2), x1)}
\end{aligned}
$$

Lemma 2737: $asoc(op_t(i(x1), x2), x3, x4) = asoc(x3, op_t(x1, x2), x4)$

$$
\begin{aligned}
& \underbrace{asoc(op_t(i(x1), x2), x3, x4)} \\
= \quad & \text{by Lemma 2721 RL with } \{x3 \leftarrow op_t(i(x1), x2), x2 \leftarrow x3, x1 \leftarrow x4\} \\
& \overbrace{rd(x4, \underbrace{op_l(x4, x3, op_t(i(x1), x2))})} \\
= \quad & \text{by Lemma 2680 LR with } \{x4 \leftarrow x2, x3 \leftarrow x1, x2 \leftarrow x3, x1 \leftarrow x4\} \\
& \underbrace{rd(x4, \overbrace{op_l(x4, op_t(x1, x2), x3)})} \\
= \quad & \text{by Lemma 2721 LR with } \{x3 \leftarrow x3, x2 \leftarrow op_t(x1, x2), x1 \leftarrow x4\} \\
& \overbrace{asoc(x3, op_t(x1, x2), x4)}
\end{aligned}
$$

Lemma 2738: $mult(rd(x1, op_l(x2, x3, x4)), x2) = rd(x1, asoc(x3, x4, x2))$

$$
\begin{aligned}
& \underbrace{mult(rd(x1, op_l(x2, x3, x4)), x2)} \\
= \quad & \text{by Lemma 2531 LR with } \{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{rd(x1, \underbrace{rd(x2, op_l(x2, x4, x3))})} \\
= \quad & \text{by Lemma 2721 LR with } \{x3 \leftarrow x3, x2 \leftarrow x4, x1 \leftarrow x2\} \\
& rd(x1, \overbrace{asoc(x3, x4, x2)})
\end{aligned}
$$

Lemma 2739: $rd(op_l(x1, x2, x3), mult(x4, x1)) = rd(asoc(x2, x3, x1), x4)$

$$
\begin{aligned}
& \underbrace{rd(op_l(x1, x2, x3), mult(x4, x1))} \\
= \quad & \text{by Lemma 2532 RL with } \{x2 \leftarrow x2, x3 \leftarrow x3, x4 \leftarrow x4, x1 \leftarrow x1\} \\
& \underbrace{\overbrace{rd(x1, mult(x4, op_l(x1, x3, x2)))}} \\
= \quad & \text{by Lemma 2356 RL with } \{x4 \leftarrow x4, x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{rd(\underbrace{rd(x1, op_l(x1, x3, x2))}, x4)} \\
= \quad & \text{by Lemma 2721 LR with } \{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x1\} \\
& rd(\overbrace{asoc(x2, x3, x1)}, x4)
\end{aligned}
$$

Lemma 2740: $mult(rd(x1, x2), op_r(x2, x3, x4)) = rd(x1, asoc(x3, x2, x4))$

$$
\begin{aligned}
& \underbrace{mult(rd(x1, x2), op_r(x2, x3, x4))} \\
= \quad & \text{by Lemma 2610 LR with } \{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{rd(x1, \underbrace{rd(x2, op_r(x2, x3, x4))})} \\
= \quad & \text{by Lemma 2723 LR with } \{x3 \leftarrow x4, x2 \leftarrow x3, x1 \leftarrow x2\} \\
& rd(x1, \overbrace{asoc(x3, x2, x4)})
\end{aligned}
$$

Lemma 2741: $mult(x1, rd(x4, asoc(x2, x1, x3))) = mult(op_r(x1, x2, x3), x4)$

$mult(x1, \underbrace{rd(x4, asoc(x2, x1, x3))}$

$=$ by Lemma 2729 RL with $\{x4 \leftarrow x4, x1 \leftarrow x3, x2 \leftarrow x2, x3 \leftarrow x1\}$

$\underbrace{mult(x1, \overbrace{mult(asoc(x1, x2, x3), x4)})}$

$=$ by Lemma 132 RL with $\{x3 \leftarrow x4, x2 \leftarrow asoc(x1, x2, x3), x1 \leftarrow x1\}$

$\overbrace{mult(mult(x1, asoc(x1, x2, x3)), \underbrace{op_l(x4, asoc(x1, x2, x3), x1)})}$

$=$ by Lemma 1815 LR with $\{x2 \leftarrow x1, x3 \leftarrow asoc(x1, x2, x3), x1 \leftarrow x4\}$

$mult(mult(x1, asoc(x1, x2, x3)), \overbrace{op_l(x4, \underbrace{op_t(mult(mult(x1, x1), asoc(x1, x2, x3)), rd(asoc(x1, x2, x3), rd(op_t(asoc(x1, x2, x$

$=$ by Lemma 1722 RL with $\{x3 \leftarrow mult(x1, x1), x2 \leftarrow rd(asoc(x1, x2, x3), rd(op_t(asoc(x1, x2, x3), mult(x1, x1)), asoc(x1, x2, x$

$mult(mult(x1, asoc(x1, x2, x3)), op_l(x4, \overbrace{op_t(mult(mult(x1, x1), asoc(x1, x2, x3)), \underbrace{op_t(rd(asoc(x1, x2, x3), rd(op_t(asoc(x1, x$

$=$ by Lemma 401 LR with $\{x2 \leftarrow mult(x1, x1), x1 \leftarrow asoc(x1, x2, x3)\}$

$mult(mult(x1, asoc(x1, x2, x3)), op_l(x4, op_t(\underbrace{mult(mult(x1, x1), asoc(x1, x2, x3))}, \overbrace{asoc(x1, x2, x3)}), x1))$

$=$ by Lemma 24 LR with $\{x2 \leftarrow asoc(x1, x2, x3), x1 \leftarrow x1\}$

$mult(mult(x1, asoc(x1, x2, x3)), op_l(x4, op_t(\overbrace{mult(x1, \underbrace{mult(x1, asoc(x1, x2, x3))})}, asoc(x1, x2, x3)), x1))$

$=$ by Lemma 2708 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$mult(mult(x1, asoc(x1, x2, x3)), op_l(x4, \underbrace{op_t(mult(x1, \overbrace{op_r(x1, x2, x3)}), asoc(x1, x2, x3))}, x1))$

$=$ by Lemma 2652 LR with $\{x4 \leftarrow x3, x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow mult(x1, op_r(x1, x2, x3))\}$

$mult(mult(x1, asoc(x1, x2, x3)), \underbrace{op_l(x4, \overbrace{mult(x1, op_r(x1, x2, x3))}, x1)})$

$=$ by Lemma 2033 LR with $\{x3 \leftarrow op_r(x1, x2, x3), x2 \leftarrow x1, x1 \leftarrow x4\}$

$mult(mult(x1, asoc(x1, x2, x3)), \overbrace{\underbrace{op_l(x4, op_r(x1, x2, x3), x1)}})$

$=$ by Lemma 1443 LR with $\{x4 \leftarrow x3, x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x4\}$

$mult(\underbrace{mult(x1, asoc(x1, x2, x3))}, \overbrace{x4})$

$=$ by Lemma 2708 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$mult(\overbrace{op_r(x1, x2, x3)}, x4)$

Lemma 2742: $rd(rd(x1, x2), asoc(x2, x3, x4)) = rd(x1, op_r(x2, x3, x4))$

$rd(rd(x1, \underbrace{x2}), asoc(x2, x3, x4))$

$=$ by Lemma 2652 RL with $\{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x2\}$

$\underbrace{rd(rd(x1, \overbrace{op_t(x2, asoc(x2, x3, x4))}), asoc(x2, x3, x4))}$

$=$ by Lemma 2704 RL with $\{x5 \leftarrow x4, x4 \leftarrow x3, x3 \leftarrow asoc(x2, x3, x4), x2 \leftarrow x2, x1 \leftarrow rd(rd(x1, op_t(x2, asoc(x2, x3, x4))), asoc($

$\overbrace{op_r(rd(rd(x1, op_t(x2, asoc(x2, x3, x4))), asoc(x2, x3, x4)), rd(\underbrace{op_r(op_t(x2, asoc(x2, x3, x4)), x3, x4)}, x2), x2)}$

$=$ by Axiom 14 RL with $\{x4 \leftarrow asoc(x2, x3, x4), x3 \leftarrow x4, x2 \leftarrow x3, x1 \leftarrow x2\}$

$op_r(rd(rd(x1, op_t(x2, asoc(x2, x3, x4))), asoc(x2, x3, x4)), rd(\overbrace{op_t(\underbrace{op_r(x2, x3, x4)}, asoc(x2, x3, x4))}, x2), x2)$

$=$ by Lemma 2708 RL with $\{x3 \leftarrow x4, x2 \leftarrow x3, x1 \leftarrow x2\}$

$\underbrace{op_r(rd(rd(x1, op_t(x2, asoc(x2, x3, x4))), asoc(x2, x3, x4)), rd(op_t(\overbrace{mult(x2, asoc(x2, x3, x4))}, asoc(x2, x3, x4)), x2), x2)}$

$=$ by Lemma 2231 RL with $\{x2 \leftarrow x2, x3 \leftarrow asoc(x2, x3, x4), x1 \leftarrow rd(rd(x1, op_t(x2, asoc(x2, x3, x4))), asoc(x2, x3, x4))\}$

$\overbrace{\underbrace{op_r(rd(rd(x1, op_t(x2, asoc(x2, x3, x4))), asoc(x2, x3, x4)), asoc(x2, x3, x4), x2)}}$

$=$ by Lemma 1965 LR with $\{x3 \leftarrow asoc(x2, x3, x4), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{rd(x1, \underbrace{mult(x2, asoc(x2, x3, x4))})}$

$=$ by Lemma 2708 LR with $\{x3 \leftarrow x4, x2 \leftarrow x3, x1 \leftarrow x2\}$

$rd(x1, \overbrace{op_r(x2, x3, x4)})$

Lemma 2743: $mult(mult(x1, x2), asoc(x2, x3, x4)) = mult(x1, op_r(x2, x3, x4))$

$\underbrace{mult(mult(x1, x2), asoc(x2, x3, x4))}$

$=$ by Lemma 91 RL with $\{x3 \leftarrow asoc(x2, x3, x4), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{mult(\underbrace{op_r(x1, x2, asoc(x2, x3, x4))}, mult(x2, asoc(x2, x3, x4)))}$

$=$ by Lemma 2710 LR with $\{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$mult(\overbrace{x1}, \underbrace{mult(x2, asoc(x2, x3, x4))})$

$=$ by Lemma 2708 LR with $\{x3 \leftarrow x4, x2 \leftarrow x3, x1 \leftarrow x2\}$

$mult(x1, \overbrace{op_r(x2, x3, x4)})$

Lemma 2744: $rd(mult(asoc(x1,x2,x3),i(i(x3))),x4) = rd(asoc(x1,x2,x3),rd(x4,i(i(x3))))$

$\underbrace{rd(mult(asoc(x1,x2,x3),i(i(x3))),x4)}$

$=$ by Lemma 4 RL with $\{x1 \leftarrow rd(mult(asoc(x1,x2,x3),i(i(x3))),x4)\}$

$\overbrace{op_t(rd(mult(asoc(x1,x2,x3),i(i(x3))),x4),rd(mult(asoc(x1,x2,x3),i(i(x3))),x4))}$

$=$ by Axiom 10 RL with $\{x2 \leftarrow rd(mult(asoc(x1,x2,x3),i(i(x3))),x4), x1 \leftarrow rd(mult(asoc(x1,x2,x3),i(i(x3))),x4)\}$

$\overbrace{mult(i(rd(mult(asoc(x1,x2,x3),i(i(x3))),x4)),mult(rd(mult(asoc(x1,x2,x3),i(i(x3))),x4),rd(mult(asoc(x1,x2,x3),i(i$

$=$ by Lemma 2727 RL with $\{x4 \leftarrow x3, x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow rd(mult(asoc(x1,x2,x3),i(i(x3))),x4)\}$

$\overbrace{op_r(rd(mult(asoc(x1,x2,x3),i(i(x3))),x4),asoc(x1,x2,x3),\underbrace{i(x3)})}$

$=$ by Lemma 3 RL with $\{x1 \leftarrow i(x3)\}$

$\underbrace{op_r(rd(mult(asoc(x1,x2,x3),i(i(x3))),x4),asoc(x1,x2,x3),\overbrace{i(i(i(x3)))})}$

$=$ by Lemma 2391 RL with $\{x4 \leftarrow asoc(x1,x2,x3), x3 \leftarrow i(i(i(x3))), x2 \leftarrow asoc(x1,x2,x3), x1 \leftarrow rd(mult(asoc(x1,x2,x3),i(i($

$\overbrace{op_r(rd(mult(asoc(x1,x2,x3),i(i(x3))),x4),asoc(x1,x2,x3),\underbrace{op_r(i(i(i(x3))),asoc(x1,x2,x3),asoc(x1,x2,x3))})}$

$=$ by Lemma 354 RL with $\{x2 \leftarrow i(i(x3)), x1 \leftarrow asoc(x1,x2,x3)\}$

$op_r(rd(mult(asoc(x1,x2,x3),i(i(x3))),x4),asoc(x1,x2,x3),\overbrace{rd(rd(asoc(x1,x2,x3),i(i(x3))),asoc(x1,x2,x3))})$

$=$ by Lemma 40 RL with $\{x3 \leftarrow asoc(x1,x2,x3), x2 \leftarrow i(i(x3)), x1 \leftarrow asoc(x1,x2,x3)\}$

$\underbrace{op_r(rd(mult(asoc(x1,x2,x3),i(i(x3))),x4),asoc(x1,x2,x3),\overbrace{rd(i(asoc(x1,x2,x3)),rd(i(i(x3)),asoc(x1,x2,x3)))})}$

$=$ by Lemma 2400 RL with $\{x4 \leftarrow rd(i(i(x3)),asoc(x1,x2,x3)), x3 \leftarrow asoc(x1,x2,x3), x2 \leftarrow asoc(x1,x2,x3), x1 \leftarrow rd(mult(as$

$\overbrace{op_r(rd(mult(asoc(x1,x2,x3),i(i(x3))),x4),asoc(x1,x2,x3),i(mult(asoc(x1,x2,x3),rd(i(i(x3)),asoc(x1,x2,x3)))))}$

$=$ by Lemma 2427 RL with $\{x3 \leftarrow mult(asoc(x1,x2,x3),rd(i(i(x3)),asoc(x1,x2,x3))), x2 \leftarrow asoc(x1,x2,x3), x1 \leftarrow rd(mult(a$

$\overbrace{op_r(rd(mult(asoc(x1,x2,x3),i(i(x3))),x4),i(asoc(x1,x2,x3)),mult(asoc(x1,x2,x3),rd(i(i(x3)),asoc(x1,x2,x3))))}$

$=$ by Lemma 2605 RL with $\{x3 \leftarrow x4, x2 \leftarrow i(i(x3)), x1 \leftarrow asoc(x1,x2,x3)\}$

$\overbrace{rd(rd(mult(i(asoc(x1,x2,x3)),mult(asoc(x1,x2,x3),i(i(x3)))),x4),i(asoc(x1,x2,x3)))}$

$=$ by Lemma 39 LR with $\{x3 \leftarrow asoc(x1,x2,x3), x2 \leftarrow x4, x1 \leftarrow mult(i(asoc(x1,x2,x3)),mult(asoc(x1,x2,x3),i(i(x3))))\}$

$\overbrace{rd(asoc(x1,x2,x3),rd(x4,\underbrace{mult(i(asoc(x1,x2,x3)),mult(asoc(x1,x2,x3),i(i(x3))))}))}$

$=$ by Axiom 2 RL with $\{x2 \leftarrow asoc(x1,x2,x3), x1 \leftarrow i(i(x3))\}$

$rd(asoc(x1,x2,x3),rd(x4,\overbrace{i(i(x3))}))$

Lemma 2745: $rd(op_l(x3,x1,x2),x4) = rd(asoc(x1,x2,x3),rd(x4,i(i(x3))))$

$rd(\underbrace{op_l(x3,x1,x2)},x4)$

$=$ by Lemma 2030 RL with $\{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x3\}$

$rd(\overbrace{op_l(x3,x2,i(x1))},x4)$

$=$ by Lemma 2715 RL with $\{x3 \leftarrow x2, x2 \leftarrow i(x1), x1 \leftarrow x3\}$

$rd(\overbrace{rd(x3,\underbrace{asoc(i(x1),x2,x3)})},x4)$

$=$ by Lemma 2654 RL with $\{x4 \leftarrow x3, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow i(x1)\}$

$rd(rd(x3,\overbrace{op_t(\underbrace{asoc(i(x1),x2,x3)},x3)}),x4)$

$=$ by Lemma 2713 RL with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$rd(rd(x3,\underbrace{op_t(\overbrace{i(asoc(x1,x2,x3))},x3)}),x4)$

$=$ by Axiom 10 RL with $\{x2 \leftarrow i(asoc(x1,x2,x3)), x1 \leftarrow x3\}$

$rd(rd(x3,\overbrace{mult(i(x3),\underbrace{mult(i(asoc(x1,x2,x3)),x3)})}),x4)$

$=$ by Lemma 77 RL with $\{x2 \leftarrow asoc(x1,x2,x3), x1 \leftarrow x3\}$

$rd(\underbrace{rd(x3,mult(i(x3),\overbrace{rd(x3,op_t(asoc(x1,x2,x3),mult(i(asoc(x1,x2,x3)),x3)))})},x4)$

$=$ by Lemma 57 LR with $\{x2 \leftarrow op_t(asoc(x1,x2,x3),mult(i(asoc(x1,x2,x3)),x3)), x1 \leftarrow x3\}$

$rd(\overbrace{mult(x3,\underbrace{op_t(asoc(x1,x2,x3),mult(i(asoc(x1,x2,x3)),x3))})},x4)$

$=$ by Lemma 375 LR with $\{x2 \leftarrow x3, x1 \leftarrow asoc(x1,x2,x3)\}$

$rd(mult(x3,\overbrace{op_t(asoc(x1,x2,x3),\underbrace{rd(x3,asoc(x1,x2,x3))})}),x4)$

$=$ by Lemma 2715 LR with $\{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x3\}$

$rd(mult(x3,\underbrace{op_t(asoc(x1,x2,x3),\overbrace{op_l(x3,x2,x1)})}),x4)$

$=$ by Lemma 2654 LR with $\{x4 \leftarrow op_l(x3,x2,x1), x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$rd(\underbrace{mult(x3,\overbrace{asoc(x1,x2,x3)})},x4)$

$=$ by Lemma 2660 RL with $\{x4 \leftarrow x3, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$rd(\overbrace{\underbrace{rd(asoc(x1,x2,x3),i(x3))}},x4)$

$=$ by Lemma 2664 LR with $\{x4 \leftarrow i(x3), x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{rd(\overbrace{mult(asoc(x1,x2,x3),i(i(x3)))},x4)}$

$=$ by Lemma 2744 LR with $\{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{rd(asoc(x1,x2,x3),rd(x4,i(i(x3))))}$

Lemma 2746: $asoc(op_r(x1,x2,x3),op_t(x4,x1),op_t(x4,x1)) = mult(op_r(op_t(x4,x1),op_r(x1,x2,x3),op_r(x1,x2,x3)),rd(op_r(x1,x$

$asoc(op_r(x1,x2,x3),op_t(x4,x1),op_t(x4,x1))$

$=$ by Lemma 794 LR with $\{x2 \leftarrow op_t(x4,x1), x1 \leftarrow op_r(x1,x2,x3)\}$

$rd(mult(i(op_r(x1,x2,x3)),op_t(x4,x1)),rd(op_t(x4,x1),op_r(x1,x2,x3)))$

$=$ by Lemma 587 LR with $\{x1 \leftarrow op_t(x4,x1), x2 \leftarrow op_r(x1,x2,x3)\}$

$rd(op_t(rd(op_t(x4,x1),op_r(x1,x2,x3)),op_l(op_r(x1,x2,x3),op_t(x4,x1),op_r(x1,x2,x3))),rd(op_t(x4,x1),op_r(x1,x2,x3)))$

$=$ by Lemma 861 RL with $\{x2 \leftarrow op_l(op_r(x1,x2,x3),op_t(x4,x1),op_r(x1,x2,x3)), x1 \leftarrow rd(op_t(x4,x1),op_r(x1,x2,x3))\}$

$mult(mult(rd(op_t(x4,x1),op_r(x1,x2,x3)),op_l(op_r(x1,x2,x3),op_t(x4,x1),op_r(x1,x2,x3))),i(mult(op_l(op_r(x1,x2,x3),op$

$=$ by Axiom 5 LR with $\{x2 \leftarrow op_t(op_r(x1,x2,x3),op_t(x4,x1)), x1 \leftarrow rd(op_t(x4,x1),op_r(x1,x2,x3))\}$

$mult(mult(rd(op_t(x4,x1),op_r(x1,x2,x3)),op_l(op_r(x1,x2,x3),op_t(x4,x1),op_r(x1,x2,x3))),i(mult(op_l(op_r(x1,x2,x3),op$

$=$ by Lemma 650 LR with $\{x2 \leftarrow op_r(x1,x2,x3), x1 \leftarrow op_t(x4,x1)\}$

$mult(mult(rd(op_t(x4,x1),op_r(x1,x2,x3)),op_l(op_r(x1,x2,x3),op_t(x4,x1),op_r(x1,x2,x3))),i(mult(op_l(op_r(x1,x2,x3),op$

$=$ by Lemma 619 RL with $\{x2 \leftarrow op_t(x4,x1), x1 \leftarrow op_r(x1,x2,x3)\}$

$mult(mult(rd(op_t(x4,x1),op_r(x1,x2,x3)),op_l(op_r(x1,x2,x3),op_t(x4,x1),op_r(x1,x2,x3))),i(mult(op_l(op_r(x1,x2,x3),m$

$=$ by Lemma 894 RL with $\{x2 \leftarrow op_t(x4,x1), x1 \leftarrow op_r(x1,x2,x3)\}$

$mult(mult(rd(op_t(x4,x1),op_r(x1,x2,x3)),op_l(op_r(x1,x2,x3),op_t(x4,x1),op_r(x1,x2,x3))),i(rd(mult(op_r(x1,x2,x3),op_t$

$=$ by Lemma 698 LR with $\{x2 \leftarrow op_r(x1,x2,x3), x1 \leftarrow op_t(x4,x1)\}$

$mult(op_r(op_t(x4,x1),op_r(x1,x2,x3),op_r(x1,x2,x3)),i(rd(mult(op_r(x1,x2,x3),op_t(x4,x1)),op_r(x1,x2,x3))))$

$=$ by Lemma 10 LR with $\{x2 \leftarrow op_r(x1,x2,x3), x1 \leftarrow mult(op_r(x1,x2,x3),op_t(x4,x1))\}$

$mult(op_r(op_t(x4,x1),op_r(x1,x2,x3),op_r(x1,x2,x3)),rd(op_r(x1,x2,x3),mult(op_r(x1,x2,x3),op_t(x4,x1))))$

$=$ by Lemma 2741 RL with $\{x3 \leftarrow x3, x2 \leftarrow x2, x4 \leftarrow op_t(x4,x1), x1 \leftarrow x1\}$

$mult(op_r(op_t(x4,x1),op_r(x1,x2,x3),op_r(x1,x2,x3)),rd(op_r(x1,x2,x3),mult(x1,rd(op_t(x4,x1),asoc(x2,x1,x3)))))$

$=$ by Lemma 2584 LR with $\{x3 \leftarrow asoc(x2,x1,x3), x2 \leftarrow x1, x1 \leftarrow mult(x1,rd(op_t(x4,x1),asoc(x2,x1,x3)))\}$

$mult(op_r(op_t(x4,x1),op_r(x1,x2,x3),op_r(x1,x2,x3)),rd(op_r(x1,x2,x3),op_r(op_r(mult(x1,rd(op_t(x4,x1),asoc(x2,x1,x3)$

$=$ by Lemma 2527 LR with $\{x3 \leftarrow asoc(x2,x1,x3), x2 \leftarrow op_t(x4,x1), x1 \leftarrow x1\}$

$mult(op_r(op_t(x4,x1),op_r(x1,x2,x3),op_r(x1,x2,x3)),rd(op_r(x1,x2,x3),op_r(rd(mult(x1,op_t(x4,x1)),asoc(x2,x1,x3)),x$

$=$ by Lemma 13 LR with $\{x2 \leftarrow x4, x1 \leftarrow x1\}$

$mult(op_r(op_t(x4,x1),op_r(x1,x2,x3),op_r(x1,x2,x3)),rd(op_r(x1,x2,x3),op_r(rd(mult(x4,x1),asoc(x2,x1,x3)),x1,i(asoc$

$=$ by Lemma 2728 RL with $\{x4 \leftarrow x3, x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow mult(x4,x1)\}$

$mult(op_r(op_t(x4,x1),op_r(x1,x2,x3),op_r(x1,x2,x3)),rd(op_r(x1,x2,x3),op_r(mult(mult(x4,x1),asoc(x1,x2,x3)),x1,i(as$

$=$ by Lemma 2743 LR with $\{x4 \leftarrow x3, x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x4\}$

$mult(op_r(op_t(x4,x1),op_r(x1,x2,x3),op_r(x1,x2,x3)),rd(op_r(x1,x2,x3),op_r(mult(x4,op_r(x1,x2,x3)),x1,i(asoc(x2,x1,x$

$=$ by Lemma 2718 LR with $\{x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow x2\}$

$mult(op_r(op_t(x4,x1),op_r(x1,x2,x3),op_r(x1,x2,x3)),rd(op_r(x1,x2,x3),op_r(mult(x4,op_r(x1,x2,x3)),x1,asoc(x1,x2,x3$

$=$ by Lemma 2710 LR with $\{x4 \leftarrow x3, x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow mult(x4,op_r(x1,x2,x3))\}$

$mult(op_r(op_t(x4,x1),op_r(x1,x2,x3),op_r(x1,x2,x3)),rd(op_r(x1,x2,x3),mult(x4,op_r(x1,x2,x3))))$

Lemma 2747: $asoc(op_r(x1,x2,x3),x4,x4) = mult(op_r(op_t(x4,x1),op_r(x1,x2,x3),op_r(x1,x2,x3)),rd(op_r(x1,x2,x3),mult(x4,$

$$
\begin{aligned}
& \underbrace{asoc(op_r(x1,x2,x3),x4,x4)} \\
= \quad & \text{by Lemma 1910 RL with } \{x3 \leftarrow x1, x2 \leftarrow x4, x1 \leftarrow op_r(x1,x2,x3)\} \\
& \overbrace{\underbrace{asoc(op_r(x1,x2,x3),op_t(x4,x1),x4)}} \\
= \quad & \text{by Lemma 2724 RL with } \{x4 \leftarrow x1, x3 \leftarrow x4, x2 \leftarrow op_t(x4,x1), x1 \leftarrow op_r(x1,x2,x3)\} \\
& \overbrace{\underbrace{asoc(op_r(x1,x2,x3),op_t(x4,x1),op_t(x4,x1))}} \\
= \quad & \text{by Lemma 2746 LR with } \{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{mult(op_r(op_t(x4,x1),op_r(x1,x2,x3),op_r(x1,x2,x3)),rd(op_r(x1,x2,x3),mult(x4,op_r(x1,x2,x3))))}
\end{aligned}
$$

Lemma 2748: $asoc(op_r(x1,x2,x3),x4,x4) = mult(op_r(op_t(x4,x1),x1,x1),i(x4))$

$$
\begin{aligned}
& \underbrace{asoc(op_r(x1,x2,x3),x4,x4)} \\
= \quad & \text{by Lemma 2747 LR with } \{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{mult(\underbrace{op_r(op_t(x4,x1),op_r(x1,x2,x3),op_r(x1,x2,x3))},rd(op_r(x1,x2,x3),mult(x4,op_r(x1,x2,x3))))} \\
= \quad & \text{by Lemma 2627 LR with } \{x4 \leftarrow x3, x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow op_t(x4,x1)\} \\
& mult(\overbrace{op_r(op_t(x4,x1),x1,x1)},\underbrace{rd(op_r(x1,x2,x3),mult(x4,op_r(x1,x2,x3))))} \\
= \quad & \text{by Lemma 9 LR with } \{x2 \leftarrow x4, x1 \leftarrow op_r(x1,x2,x3)\} \\
& mult(op_r(op_t(x4,x1),x1,x1),\overbrace{i(x4)}) \\
\end{aligned}
$$

Lemma 2749: $asoc(op_r(x1,x2,x3),x4,x4) = asoc(x1,x4,x4)$

$$
\begin{aligned}
& \underbrace{asoc(op_r(x1,x2,x3),x4,x4)} \\
= \quad & \text{by Lemma 2748 LR with } \{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{mult(op_r(op_t(x4,x1),x1,x1),i(x4))}} \\
= \quad & \text{by Lemma 342 RL with } \{x2 \leftarrow x4, x1 \leftarrow op_r(op_t(x4,x1),x1,x1)\} \\
& \overbrace{rd(\underbrace{op_r(op_r(op_t(x4,x1),x1,x1),x4,i(x4))},x4)} \\
= \quad & \text{by Axiom 17 RL with } \{x5 \leftarrow x1, x4 \leftarrow x1, x3 \leftarrow i(x4), x2 \leftarrow x4, x1 \leftarrow op_t(x4,x1)\} \\
& rd(\overbrace{op_r(\underbrace{op_r(op_t(x4,x1),x4,i(x4))},x1,x1)},x4) \\
= \quad & \text{by Lemma 28 LR with } \{x3 \leftarrow i(x4), x2 \leftarrow x1, x1 \leftarrow x4\} \\
& \underbrace{rd(op_r(\overbrace{op_t(x4,x1)},x1,x1),x4)} \\
= \quad & \text{by Lemma 2548 LR with } \{x2 \leftarrow x1, x3 \leftarrow x1, x1 \leftarrow x4\} \\
& \overbrace{rd(x1,\underbrace{op_l(op_t(x1,x4),x1,x4)})} \\
= \quad & \text{by Lemma 421 RL with } \{x2 \leftarrow x4, x1 \leftarrow x1\} \\
& \underbrace{rd(x1,\overbrace{op_r(x1,x4,i(x4))})} \\
= \quad & \text{by Lemma 2723 LR with } \{x3 \leftarrow i(x4), x2 \leftarrow x4, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{asoc(x4,x1,i(x4))}} \\
= \quad & \text{by Lemma 2719 LR with } \{x3 \leftarrow x4, x2 \leftarrow x1, x1 \leftarrow x4\} \\
& \overbrace{asoc(x1,x4,x4)}
\end{aligned}
$$

Lemma 2750: $asoc(x1, op_t(rd(x2, x3), x2), x4) = asoc(x1, mult(i(x3), x2), x4)$

$$
\begin{aligned}
& \overbrace{asoc(x1, op_t(rd(x2, x3), x2), x4)} \\
= \quad & \text{by Lemma 2711 RL with } \{x3 \leftarrow x4, x2 \leftarrow op_t(rd(x2, x3), x2), x1 \leftarrow x1\} \\
& \overbrace{rd(\underbrace{op_r(x1, op_t(rd(x2, x3), x2), x4)}, x1)} \\
= \quad & \text{by Lemma 2686 LR with } \{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{rd(\overbrace{op_r(x1, mult(i(x3), x2), x4)}, x1)} \\
= \quad & \text{by Lemma 2711 LR with } \{x3 \leftarrow x4, x2 \leftarrow mult(i(x3), x2), x1 \leftarrow x1\} \\
& \overbrace{asoc(x1, mult(i(x3), x2), x4)}
\end{aligned}
$$

Lemma 2751: $i(mult(op_l(x1, x2, x3), x4)) = mult(asoc(x3, x2, x1), i(mult(x1, x4)))$

$$
\begin{aligned}
& \underbrace{i(mult(op_l(x1, x2, x3), x4))} \\
= \quad & \text{by Lemma 20 RL with } \{x2 \leftarrow mult(op_l(x1, x2, x3), x4), x1 \leftarrow x1\} \\
& \overbrace{mult(x1, i(\underbrace{mult(x1, mult(op_l(x1, x2, x3), x4))}))} \\
= \quad & \text{by Lemma 1420 RL with } \{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{mult(x1, i(\overbrace{mult(mult(x1, op_l(x1, x2, x3)), x4)}))} \\
= \quad & \text{by Lemma 1399 RL with } \{x3 \leftarrow x4, x1 \leftarrow op_l(x1, x2, x3), x2 \leftarrow x1\} \\
& \overbrace{mult(\underbrace{rd(x1, op_l(x1, x2, x3))}, i(mult(x1, x4)))} \\
= \quad & \text{by Lemma 2721 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& mult(\overbrace{asoc(x3, x2, x1)}, i(mult(x1, x4)))
\end{aligned}
$$

Lemma 2752: $op_t(rd(x1, op_r(x2, x3, x4)), x2) = op_r(mult(op_r(i(x2), x3, x4), x1), x2, asoc(x3, i(x2), x4))$

$op_t(rd(x1, \underbrace{op_r(x2, x3, x4)}), x2)$

$=$ by Lemma 2716 RL with $\{x3 \leftarrow x4, x2 \leftarrow x3, x1 \leftarrow x2\}$

$op_t(rd(x1, \overbrace{rd(x2, \underbrace{asoc(x3, x2, x4)})}), x2)$

$=$ by Lemma 2717 RL with $\{x3 \leftarrow x4, x2 \leftarrow x3, x1 \leftarrow x2\}$

$op_t(rd(x1, \underbrace{rd(x2, \overbrace{asoc(i(x2), x3, x4)})}), x2)$

$=$ by Lemma 2728 RL with $\{x4 \leftarrow x4, x3 \leftarrow i(x2), x2 \leftarrow x3, x1 \leftarrow x2\}$

$op_t(rd(\underbrace{x1}, \overbrace{mult(x2, asoc(x3, i(x2), x4))}), x2)$

$=$ by Axiom 4 LR with $\{x2 \leftarrow asoc(x3, i(x2), x4), x1 \leftarrow x1\}$

$\underbrace{op_t(rd(\overbrace{mult(rd(x1, asoc(x3, i(x2), x4)), asoc(x3, i(x2), x4))}, mult(x2, asoc(x3, i(x2), x4))), x2)}$

$=$ by Axiom 10 RL with $\{x2 \leftarrow rd(mult(rd(x1, asoc(x3, i(x2), x4)), asoc(x3, i(x2), x4)), mult(x2, asoc(x3, i(x2), x4))), x1 \leftarrow x2$

$\overbrace{mult(i(x2), mult(\underbrace{rd(mult(rd(x1, asoc(x3, i(x2), x4)), asoc(x3, i(x2), x4)), mult(x2, asoc(x3, i(x2), x4)))}, x2))}$

$=$ by Lemma 676 RL with $\{x3 \leftarrow asoc(x3, i(x2), x4), x2 \leftarrow x2, x1 \leftarrow rd(x1, asoc(x3, i(x2), x4))\}$

$mult(i(x2), \underbrace{mult(\overbrace{op_r(rd(rd(x1, asoc(x3, i(x2), x4)), x2), x2, asoc(x3, i(x2), x4))}, x2))}$

$=$ by Lemma 2570 LR with $\{x3 \leftarrow asoc(x3, i(x2), x4), x2 \leftarrow x2, x1 \leftarrow rd(rd(x1, asoc(x3, i(x2), x4)), x2)\}$

$mult(i(x2), \overbrace{op_r(\underbrace{mult(rd(rd(x1, asoc(x3, i(x2), x4)), x2), x2)}, x2, asoc(x3, i(x2), x4))})$

$=$ by Axiom 4 RL with $\{x2 \leftarrow x2, x1 \leftarrow rd(x1, asoc(x3, i(x2), x4))\}$

$\underbrace{mult(i(x2), op_r(\overbrace{rd(x1, asoc(x3, i(x2), x4))}, x2, asoc(x3, i(x2), x4)))}$

$=$ by Lemma 2572 RL with $\{x3 \leftarrow asoc(x3, i(x2), x4), x2 \leftarrow x2, x1 \leftarrow rd(x1, asoc(x3, i(x2), x4))\}$

$\overbrace{op_r(rd(rd(x1, asoc(x3, i(x2), x4)), x2), x2, rd(asoc(x3, i(x2), x4), rd(x1, asoc(x3, i(x2), x4))))}$

$=$ by Lemma 2523 RL with $\{x3 \leftarrow asoc(x3, i(x2), x4), x1 \leftarrow rd(x1, asoc(x3, i(x2), x4)), x2 \leftarrow x2\}$

$\overbrace{op_r(\underbrace{mult(i(x2), rd(x1, asoc(x3, i(x2), x4)))}, x2, asoc(x3, i(x2), x4))}$

$=$ by Lemma 2741 LR with $\{x3 \leftarrow x4, x2 \leftarrow x3, x4 \leftarrow x1, x1 \leftarrow i(x2)\}$

$op_r(\overbrace{mult(op_r(i(x2), x3, x4), x1)}, x2, asoc(x3, i(x2), x4))$

Lemma 2753: $op_t(rd(x1, op_r(x2, x3, x4)), x2) = mult(op_r(i(x2), x3, x4), x1)$

$\underbrace{op_t(rd(x1, op_r(x2, x3, x4)), x2)}$

$=$ by Lemma 2752 LR with $\{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_r(mult(op_r(i(x2), x3, x4), x1), x2, \underbrace{asoc(x3, i(x2), x4)})}$

$=$ by Lemma 2720 LR with $\{x3 \leftarrow x4, x2 \leftarrow x2, x1 \leftarrow x3\}$

$\underbrace{op_r(mult(op_r(i(x2), x3, x4), x1), x2, \overbrace{asoc(x2, x3, x4)})}$

$=$ by Lemma 2710 LR with $\{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow mult(op_r(i(x2), x3, x4), x1)\}$

$\overbrace{mult(op_r(i(x2), x3, x4), x1)}$

Lemma 2754: $rd(mult(op_r(i(x2),x3,x4),i(x1)),rd(i(x1),op_r(x2,x3,x4))) = asoc(x2,rd(i(x1),op_r(x2,x3,x4)),op_l(i(x1),op_r(x2$

$rd(\underbrace{mult(op_r(i(x2),x3,x4),i(x1))}, rd(i(x1),op_r(x2,x3,x4)))$

$=$ by Lemma 2753 RL with $\{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow i(x1)\}$

$\underbrace{rd(\overbrace{op_t(rd(i(x1),op_r(x2,x3,x4)),x2)}, rd(i(x1),op_r(x2,x3,x4)))}$

$=$ by Axiom 5 LR with $\{x2 \leftarrow x2, x1 \leftarrow rd(op_t(rd(i(x1),op_r(x2,x3,x4)),x2),rd(i(x1),op_r(x2,x3,x4)))\}$

$\overbrace{rd(\underbrace{mult(rd(op_t(rd(i(x1),op_r(x2,x3,x4)),x2),rd(i(x1),op_r(x2,x3,x4)))}, x2), x2)}$

$=$ by Lemma 394 LR with $\{x2 \leftarrow x2, x1 \leftarrow rd(i(x1),op_r(x2,x3,x4))\}$

$rd(\overbrace{rd(mult(rd(i(x1),op_r(x2,x3,x4)),x2),rd(i(x1),op_r(x2,x3,x4)))}, x2)$

$=$ by Lemma 2584 LR with $\{x3 \leftarrow x4, x2 \leftarrow x3, x1 \leftarrow rd(mult(rd(i(x1),op_r(x2,x3,x4)),x2),rd(i(x1),op_r(x2,x3,x4)))\}$

$rd(\overbrace{op_r(\underbrace{op_r(rd(mult(rd(i(x1),op_r(x2,x3,x4)),x2),rd(i(x1),op_r(x2,x3,x4))),x3,x4)}, x3, i(x4))}, x2)$

$=$ by Axiom 5 LR with $\{x2 \leftarrow rd(i(x1),op_r(x2,x3,x4)), x1 \leftarrow op_r(rd(mult(rd(i(x1),op_r(x2,x3,x4)),x2),rd(i(x1),op_r(x2,x3,$

$rd(op_r(\overbrace{rd(\underbrace{mult(op_r(rd(mult(rd(i(x1),op_r(x2,x3,x4)),x2),rd(i(x1),op_r(x2,x3,x4))),x3,x4)}, rd(i(x1),op_r(x2,x3,x4)))}, r$

$=$ by Lemma 2413 LR with $\{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow rd(i(x1),op_r(x2,x3,x4))\}$

$rd(op_r(\underbrace{rd(\overbrace{mult(rd(i(x1),op_r(x2,x3,x4)),op_r(x2,x3,x4))}, rd(i(x1),op_r(x2,x3,x4)))}, x3, i(x4)), x2)$

$=$ by Lemma 822 RL with $\{x2 \leftarrow rd(i(x1),op_r(x2,x3,x4)), x1 \leftarrow op_r(x2,x3,x4)\}$

$rd(\underbrace{op_r(\overbrace{op_r(op_r(x2,x3,x4),rd(i(x1),op_r(x2,x3,x4)),mult(op_r(x2,x3,x4),rd(i(x1),op_r(x2,x3,x4))))}, x3, i(x4))}, x2)$

$=$ by Axiom 17 LR with $\{x5 \leftarrow i(x4), x4 \leftarrow x3, x3 \leftarrow mult(op_r(x2,x3,x4),rd(i(x1),op_r(x2,x3,x4))), x2 \leftarrow rd(i(x1),op_r(x2,x3$

$rd(\overbrace{op_r(\underbrace{op_r(op_r(x2,x3,x4),x3,i(x4))}, rd(i(x1),op_r(x2,x3,x4)),mult(op_r(x2,x3,x4),rd(i(x1),op_r(x2,x3,x4))))}, x2)$

$=$ by Lemma 2584 RL with $\{x3 \leftarrow x4, x2 \leftarrow x3, x1 \leftarrow x2\}$

$\underbrace{rd(op_r(\overbrace{x2}, rd(i(x1),op_r(x2,x3,x4)),mult(op_r(x2,x3,x4),rd(i(x1),op_r(x2,x3,x4)))), x2)}$

$=$ by Lemma 2711 LR with $\{x3 \leftarrow mult(op_r(x2,x3,x4),rd(i(x1),op_r(x2,x3,x4))), x2 \leftarrow rd(i(x1),op_r(x2,x3,x4)), x1 \leftarrow x2\}$

$\overbrace{asoc(x2,rd(i(x1),op_r(x2,x3,x4)),\underbrace{mult(op_r(x2,x3,x4),rd(i(x1),op_r(x2,x3,x4)))})}$

$=$ by Lemma 904 LR with $\{x2 \leftarrow x1, x1 \leftarrow op_r(x2,x3,x4)\}$

$asoc(x2,rd(i(x1),op_r(x2,x3,x4)),\overbrace{op_l(i(x1),op_r(x2,x3,x4),x1)})$

Lemma 2755: $rd(i(mult(op_r(x2,x3,x4),x1)),rd(i(x1),op_r(x2,x3,x4))) = asoc(x2,rd(i(x1),op_r(x2,x3,x4)),op_l(i(x1),op_r(x2,x$

$rd(i(mult(op_r(x2,x3,\underbrace{x4}),x1)),rd(i(x1),op_r(x2,x3,x4)))$

$=$ by Lemma 3 RL with $\{x1 \leftarrow x4\}$

$rd(\underbrace{i(mult(op_r(x2,x3,\overbrace{i(i(x4))}),x1))}, rd(i(x1),op_r(x2,x3,x4)))$

$=$ by Lemma 2504 LR with $\{x4 \leftarrow x1, x3 \leftarrow x4, x2 \leftarrow x3, x1 \leftarrow x2\}$

$\underbrace{rd(\overbrace{mult(op_r(i(x2),x3,x4),i(x1))}, rd(i(x1),op_r(x2,x3,x4)))}$

$=$ by Lemma 2754 LR with $\{x1 \leftarrow x1, x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2\}$

$\overbrace{asoc(x2,rd(i(x1),op_r(x2,x3,x4)),op_l(i(x1),op_r(x2,x3,x4),x1))}$

Lemma 2756: $asoc(x1, x4, x4) = asoc(op_r(x1, x2, x3), mult(x4, x1), x4)$

$\overbrace{asoc(x1, x4, x4)}$

$=$ by Lemma 2749 RL with $\{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{asoc(op_r(x1, x2, x3), x4, x4)}$

$=$ by Lemma 2749 RL with $\{x4 \leftarrow x4, x3 \leftarrow i(x3), x2 \leftarrow x2, x1 \leftarrow op_r(x1, x2, x3)\}$

$\overbrace{asoc(op_r(op_r(x1, x2, x3), x2, i(x3)), x4, x4)}$

$=$ by Lemma 789 LR with $\{x2 \leftarrow x4, x1 \leftarrow op_r(op_r(x1, x2, x3), x2, i(x3))\}$

$\overbrace{rd(\underbrace{rd(x4, i(op_r(op_r(x1, x2, x3), x2, i(x3))))}, mult(op_r(op_r(x1, x2, x3), x2, i(x3)), x4))}$

$=$ by Lemma 38 LR with $\{x1 \leftarrow op_r(op_r(x1, x2, x3), x2, i(x3)), x2 \leftarrow x4\}$

$\underbrace{rd(\overbrace{rd(op_r(op_r(x1, x2, x3), x2, i(x3)), i(x4))}, mult(op_r(op_r(x1, x2, x3), x2, i(x3)), x4))}$

$=$ by Lemma 40 RL with $\{x3 \leftarrow op_r(op_r(x1, x2, x3), x2, i(x3)), x2 \leftarrow i(x4), x1 \leftarrow mult(op_r(op_r(x1, x2, x3), x2, i(x3)), x4)\}$

$\overbrace{\underbrace{rd(i(mult(op_r(op_r(x1, x2, x3), x2, i(x3)), x4)), rd(i(x4), op_r(op_r(x1, x2, x3), x2, i(x3))))}}$

$=$ by Lemma 2755 LR with $\{x1 \leftarrow x4, x4 \leftarrow i(x3), x3 \leftarrow x2, x2 \leftarrow op_r(x1, x2, x3)\}$

$\overbrace{asoc(op_r(x1, x2, x3), rd(i(x4), op_r(op_r(x1, x2, x3), x2, i(x3))), \underbrace{op_l(i(x4), op_r(op_r(x1, x2, x3), x2, i(x3)), x4)})$

$=$ by Lemma 1049 RL with $\{x3 \leftarrow x4, x2 \leftarrow op_r(op_r(x1, x2, x3), x2, i(x3)), x1 \leftarrow x4\}$

$\underbrace{asoc(op_r(x1, x2, x3), rd(i(x4), op_r(op_r(x1, x2, x3), x2, i(x3))), \overbrace{i(op_l(x4, op_r(op_r(x1, x2, x3), x2, i(x3)), x4))})}$

$=$ by Lemma 2719 LR with $\{x3 \leftarrow op_l(x4, op_r(op_r(x1, x2, x3), x2, i(x3)), x4), x2 \leftarrow rd(i(x4), op_r(op_r(x1, x2, x3), x2, i(x3))), x1$

$\overbrace{asoc(rd(i(x4), op_r(op_r(x1, x2, x3), x2, i(x3))), op_r(x1, x2, x3), op_l(x4, op_r(op_r(x1, x2, x3), x2, i(x3)), x4))}$

$=$ by Lemma 2731 LR with $\{x4 \leftarrow op_r(op_r(x1, x2, x3), x2, i(x3)), x3 \leftarrow x4, x2 \leftarrow op_r(x1, x2, x3), x1 \leftarrow rd(i(x4), op_r(op_r(x1, x2,$

$\overbrace{asoc(rd(i(x4), op_r(op_r(x1, x2, x3), x2, i(x3))), op_r(x1, x2, x3), x4)}$

$=$ by Lemma 2736 LR with $\{x3 \leftarrow op_r(x1, x2, x3), x2 \leftarrow op_r(op_r(x1, x2, x3), x2, i(x3)), x1 \leftarrow x4\}$

$\overbrace{asoc(op_r(x1, x2, x3), mult(x4, \underbrace{op_r(op_r(x1, x2, x3), x2, i(x3))}), x4)}$

$=$ by Lemma 2584 RL with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$asoc(op_r(x1, x2, x3), mult(x4, \overbrace{x1}), x4)$

Lemma 2757: $op_l(x1, mult(x1, x2), op_r(x2, x3, x4)) = op_l(x1, x1, x2)$

$\underbrace{op_l(x1, mult(x1, x2), op_r(x2, x3, x4))}$

$=$ by Lemma 2715 RL with $\{x3 \leftarrow mult(x1, x2), x2 \leftarrow op_r(x2, x3, x4), x1 \leftarrow x1\}$

$\overbrace{rd(x1, \underbrace{asoc(op_r(x2, x3, x4), mult(x1, x2), x1)})}$

$=$ by Lemma 2756 RL with $\{x3 \leftarrow x4, x2 \leftarrow x3, x4 \leftarrow x1, x1 \leftarrow x2\}$

$\underbrace{rd(x1, \overbrace{asoc(x2, x1, x1)})}$

$=$ by Lemma 2715 LR with $\{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_l(x1, x1, x2)}$

Lemma 2758: $op_l(mult(x1,x2),x1,mult(x1,x2)) = op_l(mult(x1,x2),mult(x1,rd(x2,x1)),op_r(i(x1),x3,x4))$

$$
\begin{aligned}
& \overbrace{op_l(mult(x1,x2),x1,mult(x1,x2))} \\
= \quad & \text{by Lemma 2030 RL with } \{x3 \leftarrow x1, x2 \leftarrow mult(x1,x2), x1 \leftarrow mult(x1,x2)\} \\
& \overbrace{\overbrace{op_l(mult(x1,x2),mult(x1,x2),i(x1))}} \\
= \quad & \text{by Lemma 2757 RL with } \{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow i(x1), x1 \leftarrow mult(x1,x2)\} \\
& \overbrace{op_l(mult(x1,x2),\overbrace{mult(mult(x1,x2),i(x1))},op_r(i(x1),x3,x4))} \\
= \quad & \text{by Lemma 55 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_l(mult(x1,x2),\overbrace{mult(x1,rd(x2,x1))},op_r(i(x1),x3,x4))
\end{aligned}
$$

Lemma 2759: $mult(x2,x1) = op_l(mult(x1,x2),mult(x1,rd(x2,x1)),op_r(i(x1),x3,x4))$

$$
\begin{aligned}
& \overbrace{mult(x2,x1)} \\
= \quad & \text{by Lemma 435 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\overbrace{op_l(mult(x1,x2),x1,x2)}} \\
= \quad & \text{by Lemma 2032 RL with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow mult(x1,x2)\} \\
& \overbrace{\overbrace{op_l(mult(x1,x2),x1,mult(x1,x2))}} \\
= \quad & \text{by Lemma 2758 LR with } \{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(mult(x1,x2),mult(x1,rd(x2,x1)),op_r(i(x1),x3,x4))}
\end{aligned}
$$

Lemma 2760: $mult(x2,x1) = op_l(mult(x1,x2),x2,op_r(i(x1),x3,x4))$

$$
\begin{aligned}
& \overbrace{mult(x2,x1)} \\
= \quad & \text{by Lemma 2759 LR with } \{x4 \leftarrow x4, x3 \leftarrow x3, x1 \leftarrow x1, x2 \leftarrow x2\} \\
& \overbrace{\overbrace{op_l(mult(x1,x2),mult(x1,rd(x2,x1)),op_r(i(x1),x3,x4))}} \\
= \quad & \text{by Lemma 2682 LR with } \{x4 \leftarrow op_r(i(x1),x3,x4), x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow mult(x1,x2)\} \\
& \overbrace{op_l(mult(x1,x2),\underbrace{op_l(mult(x1,rd(x2,x1)),x1,x2)},op_r(i(x1),x3,x4))} \\
= \quad & \text{by Lemma 442 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& op_l(mult(x1,x2),\overbrace{x2},op_r(i(x1),x3,x4))
\end{aligned}
$$

Lemma 2761: $mult(x1,x2) = op_l(mult(x2,x1),x2,op_r(x1,x3,x4))$

$$
\begin{aligned}
& \overbrace{mult(x1,x2)} \\
= \quad & \text{by Lemma 138 RL with } \{x3 \leftarrow x2, x2 \leftarrow op_r(x1,x3,x4), x1 \leftarrow mult(x1,x2)\} \\
& \overbrace{op_l(\underbrace{op_l(mult(x1,x2),op_r(x1,x3,x4),x2)},x2,op_r(x1,x3,x4))} \\
= \quad & \text{by Lemma 2508 RL with } \{x5 \leftarrow x4, x4 \leftarrow x3, x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow mult(x1,x2)\} \\
& op_l(\overbrace{op_l(mult(x1,x2),x2,op_r(i(x1),x3,x4))},x2,op_r(x1,x3,x4)) \\
= \quad & \text{by Lemma 2760 RL with } \{x4 \leftarrow x4, x3 \leftarrow x3, x1 \leftarrow x1, x2 \leftarrow x2\} \\
& op_l(\overbrace{mult(x2,x1)},x2,op_r(x1,x3,x4))
\end{aligned}
$$

Lemma 2762: $mult(op_l(op_l(i(x1),x2,x3),x1,op_l(i(x1),x2,x3)),mult(x1,x4)) = op_l(mult(asoc(x2,x3,i(x1)),x4),x1,op_l(i(x1),$

$mult(op_l(op_l(i(x1),x2,x3),x1,op_l(i(x1),x2,x3)),mult(\underbrace{x1},x4))$

= by Lemma 3 RL with $\{x1 \leftarrow x1\}$

$\underbrace{mult(op_l(op_l(i(x1),x2,x3),x1,op_l(i(x1),x2,x3)),mult(\overbrace{i(i(x1))},x4))}$

= by Lemma 2184 RL with $\{x3 \leftarrow x4, x2 \leftarrow x1, x1 \leftarrow op_l(i(x1),x2,x3)\}$

$\overbrace{op_l(mult(\underbrace{rd(op_l(i(x1),x2,x3),i(x1))},x4),x1,op_l(i(x1),x2,x3))}$

= by Lemma 2722 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow i(x1)\}$

$op_l(mult(\overbrace{asoc(x2,x3,i(x1))},x4),x1,op_l(i(x1),x2,x3))$

Lemma 2763: $mult(op_l(i(x1),x2,x3),mult(x1,x4)) = op_l(mult(asoc(x2,x3,i(x1)),x4),x1,op_l(i(x1),x2,x3))$

$mult(\underbrace{op_l(i(x1),x2,x3)},mult(x1,x4))$

= by Lemma 1421 RL with $\{x4 \leftarrow x3, x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow op_l(i(x1),x2,x3)\}$

$mult(\overbrace{op_l(op_l(i(x1),x2,x3),op_l(x1,x2,x3),x1)},mult(x1,x4))$

= by Lemma 2041 RL with $\{x5 \leftarrow x3, x4 \leftarrow x2, x3 \leftarrow x1, x2 \leftarrow x1, x1 \leftarrow op_l(i(x1),x2,x3)\}$

$\underbrace{mult(\overbrace{op_l(op_l(i(x1),x2,x3),x1,op_l(i(x1),x2,x3))},mult(x1,x4))}$

= by Lemma 2762 LR with $\{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_l(mult(asoc(x2,x3,i(x1)),x4),x1,op_l(i(x1),x2,x3))}$

Lemma 2764: $mult(op_l(i(x1),x2,x3),mult(x1,x4)) = mult(asoc(x2,x3,i(x1)),x4)$

$\underbrace{mult(op_l(i(x1),x2,x3),mult(x1,x4))}$

= by Lemma 2763 LR with $\{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{\overbrace{op_l(mult(asoc(x2,x3,i(x1)),x4),x1,op_l(i(x1),x2,x3))}}$

= by Lemma 2041 LR with $\{x5 \leftarrow x3, x4 \leftarrow x2, x3 \leftarrow x1, x2 \leftarrow x1, x1 \leftarrow mult(asoc(x2,x3,i(x1)),x4)\}$

$\underbrace{\overbrace{op_l(mult(asoc(x2,x3,i(x1)),x4),op_l(x1,x2,x3),x1)}}$

= by Lemma 1421 LR with $\{x4 \leftarrow x3, x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow mult(asoc(x2,x3,i(x1)),x4)\}$

$\overbrace{mult(asoc(x2,x3,i(x1)),x4)}$

Lemma 2765: $mult(rd(op_r(x1,x2,x3),x4),i(x1)) = i(rd(x4,asoc(x1,x2,x3)))$

$\underbrace{mult(rd(op_r(x1,x2,x3),x4),i(x1))}$

= by Lemma 44 RL with $\{x3 \leftarrow x1, x2 \leftarrow op_r(x1,x2,x3), x1 \leftarrow x4\}$

$\overbrace{i(\underbrace{mult(rd(x4,op_r(x1,x2,x3)),x1)})}$

= by Lemma 2609 LR with $\{x4 \leftarrow x3, x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x4\}$

$i(\overbrace{rd(x4,\underbrace{rd(op_r(x1,x2,x3),x1)})})$

= by Lemma 2711 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$i(rd(x4,\overbrace{asoc(x1,x2,x3)}))$

Lemma 2766: $rd(x2, op_t(x1, i(x2))) = op_l(rd(x2, x1), x2, op_r(op_l(i(x1), x1, x2), x3, x4))$

$$
\begin{aligned}
& rd(x2, \underbrace{op_t(x1, i(x2))}) \\
= & \quad \text{by Lemma 18 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{rd(x2, \overbrace{i(op_t(i(x1), x2))})} \\
= & \quad \text{by Lemma 359 RL with } \{x2 \leftarrow x2, x1 \leftarrow op_t(i(x1), x2)\} \\
& \overbrace{op_r(\underbrace{mult(op_t(i(x1), x2), x2)}, x2, x2)} \\
= & \quad \text{by Lemma 341 RL with } \{x2 \leftarrow i(x1), x1 \leftarrow x2\} \\
& op_r(\overbrace{op_t(rd(x2, \underbrace{i(i(x1))}), x2)}, x2, x2) \\
= & \quad \text{by Axiom 2 LR with } \{x2 \leftarrow x2, x1 \leftarrow i(i(x1))\} \\
& op_r(op_t(rd(x2, \overbrace{mult(i(x2), \underbrace{mult(x2, i(i(x1)))})}), x2), x2, x2) \\
= & \quad \text{by Lemma 218 RL with } \{x2 \leftarrow i(x1), x1 \leftarrow x2\} \\
& op_r(op_t(\underbrace{rd(x2, mult(i(x2), \overbrace{rd(x2, op_l(i(x1), mult(i(i(x1)), x2), i(x1)))}))}, x2), x2, x2) \\
= & \quad \text{by Lemma 57 LR with } \{x2 \leftarrow op_l(i(x1), mult(i(i(x1)), x2), i(x1)), x1 \leftarrow x2\} \\
& op_r(op_t(\overbrace{mult(x2, \underbrace{op_l(i(x1), mult(i(i(x1)), x2), i(x1))})}, x2), x2, x2) \\
= & \quad \text{by Lemma 229 LR with } \{x2 \leftarrow x2, x1 \leftarrow i(x1)\} \\
& \underbrace{op_r(op_t(mult(x2, \overbrace{op_l(i(x1), i(i(x1)), x2)}), x2), x2, x2)} \\
= & \quad \text{by Lemma 385 LR with } \{x2 \leftarrow op_l(i(x1), i(i(x1)), x2), x1 \leftarrow x2\} \\
& \overbrace{mult(op_l(i(x1), \underbrace{i(i(x1))}, x2), x2)} \\
= & \quad \text{by Lemma 3 LR with } \{x1 \leftarrow x1\} \\
& \underbrace{mult(op_l(i(x1), \overbrace{x1}, x2), x2)} \\
= & \quad \text{by Lemma 2761 LR with } \{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow op_l(i(x1), x1, x2)\} \\
& \overbrace{op_l(\underbrace{mult(x2, op_l(i(x1), x1, x2))}, x2, op_r(op_l(i(x1), x1, x2), x3, x4))} \\
= & \quad \text{by Lemma 232 LR with } \{x2 \leftarrow x1, x1 \leftarrow x2\} \\
& op_l(\overbrace{rd(x2, x1)}, x2, op_r(op_l(i(x1), x1, x2), x3, x4))
\end{aligned}
$$

Lemma 2767: $mult(rd(x1,x2),op_r(x2,x3,x4)) = mult(rd(x1,op_t(x2,i(x1))),op_r(op_r(x2,x3,x4),x1,i(rd(x1,x2))))$

$\underbrace{mult(rd(x1,x2),op_r(x2,x3,x4))}$

$=$ by Lemma 1471 RL with $\{x3 \leftarrow op_r(x2,x3,x4), x2 \leftarrow rd(x1,x2), x1 \leftarrow op_l(rd(x1,x2),op_r(x2,x3,x4),x1)\}$

$\overbrace{op_l(mult(op_l(rd(x1,x2),op_r(x2,x3,x4),x1),mult(\underbrace{rd(rd(x1,x2),op_l(rd(x1,x2),op_r(x2,x3,x4),x1))}, op_r(x2,x3,x4))), op_l($

$=$ by Lemma 2449 RL with $\{x3 \leftarrow op_r(x2,x3,x4), x2 \leftarrow x1, x1 \leftarrow rd(x1,x2)\}$

$op_l(mult(op_l(rd(x1,x2),op_r(x2,x3,x4),x1),mult(\overbrace{rd(op_l(rd(x1,x2),x1,op_r(x2,x3,x4)),rd(x1,x2))}, op_r(x2,x3,x4))), op_l($

$=$ by Lemma 10 RL with $\{x2 \leftarrow op_l(rd(x1,x2),x1,op_r(x2,x3,x4)), x1 \leftarrow rd(x1,x2)\}$

$op_l(mult(op_l(rd(x1,x2),op_r(x2,x3,x4),x1),\underbrace{mult(i(\overbrace{rd(rd(x1,x2),op_l(rd(x1,x2),x1,op_r(x2,x3,x4)))}), op_r(x2,x3,x4))}, o$

$=$ by Lemma 14 RL with $\{x2 \leftarrow rd(rd(x1,x2),op_l(rd(x1,x2),x1,op_r(x2,x3,x4))), x1 \leftarrow op_r(x2,x3,x4)\}$

$op_l(mult(op_l(rd(x1,x2),op_r(x2,x3,x4),x1),\overbrace{op_t(\underbrace{rd(op_r(x2,x3,x4),rd(rd(x1,x2),op_l(rd(x1,x2),x1,op_r(x2,x3,x4))))}, rd($

$=$ by Lemma 2456 RL with $\{x3 \leftarrow rd(x1,x2), x2 \leftarrow x1, x1 \leftarrow op_r(x2,x3,x4)\}$

$op_l(mult(op_l(rd(x1,x2),op_r(x2,x3,x4),x1),\underbrace{op_t(\overbrace{op_r(op_r(x2,x3,x4),x1,i(rd(x1,x2)))}), rd(rd(x1,x2),op_l(rd(x1,x2),x1,op$

$=$ by Axiom 14 LR with $\{x4 \leftarrow rd(rd(x1,x2),op_l(rd(x1,x2),x1,op_r(x2,x3,x4))), x3 \leftarrow i(rd(x1,x2)), x2 \leftarrow x1, x1 \leftarrow op_r(x2,x3$

$op_l(mult(op_l(rd(x1,x2),op_r(x2,x3,x4),x1),\overbrace{op_r(\underbrace{op_t(op_r(x2,x3,x4),rd(rd(x1,x2),op_l(rd(x1,x2),x1,op_r(x2,x3,x4))))}, x1$

$=$ by Lemma 2311 LR with $\{x3 \leftarrow x1, x2 \leftarrow rd(x1,x2), x1 \leftarrow op_r(x2,x3,x4)\}$

$\underbrace{op_l(mult(op_l(rd(x1,x2),op_r(x2,x3,x4),x1),op_r(\overbrace{op_r(x2,x3,x4)},x1,i(rd(x1,x2)))), op_l(rd(x1,x2),op_r(x2,x3,x4),x1),rd($

$=$ by Lemma 1421 LR with $\{x4 \leftarrow x1, x3 \leftarrow op_r(x2,x3,x4), x2 \leftarrow rd(x1,x2), x1 \leftarrow mult(op_l(rd(x1,x2),op_r(x2,x3,x4),x1),op_$

$\overbrace{mult(\underbrace{op_l(rd(x1,x2),op_r(x2,x3,x4),x1)}, op_r(op_r(x2,x3,x4),x1,i(rd(x1,x2))))}$

$=$ by Lemma 2508 RL with $\{x5 \leftarrow x4, x4 \leftarrow x3, x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow rd(x1,x2)\}$

$mult(\overbrace{op_l(\underbrace{rd(x1,x2)},x1,op_r(i(x2),x3,x4))}, op_r(op_r(x2,x3,x4),x1,i(rd(x1,x2))))$

$=$ by Lemma 364 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$mult(\underbrace{op_l(\overbrace{op_r(mult(x1,i(x2)),x2,x2)},x1,op_r(i(x2),x3,x4))}, op_r(op_r(x2,x3,x4),x1,i(rd(x1,x2))))$

$=$ by Axiom 18 RL with $\{x5 \leftarrow x2, x4 \leftarrow x2, x3 \leftarrow op_r(i(x2),x3,x4), x2 \leftarrow x1, x1 \leftarrow mult(x1,i(x2))\}$

$\overbrace{op_r(\underbrace{op_l(mult(x1,i(x2)),x1,op_r(i(x2),x3,x4))},x2,x2)}, op_r(op_r(x2,x3,x4),x1,i(rd(x1,x2))))$

$=$ by Lemma 358 RL with $\{x2 \leftarrow x2, x1 \leftarrow op_l(mult(x1,i(x2)),x1,op_r(i(x2),x3,x4))\}$

$mult(\overbrace{rd(\underbrace{rd(op_l(mult(x1,i(x2)),x1,op_r(i(x2),x3,x4)),i(x2))},x2)}, op_r(op_r(x2,x3,x4),x1,i(rd(x1,x2))))$

$=$ by Lemma 2474 LR with $\{x3 \leftarrow op_r(i(x2),x3,x4), x2 \leftarrow i(x2), x1 \leftarrow x1\}$

$mult(rd(\overbrace{op_r(x1,op_r(i(x2),x3,x4),mult(x1,i(x2)))},x2), op_r(op_r(x2,x3,x4),x1,i(rd(x1,x2))))$

$=$ by Lemma 2395 LR with $\{x4 \leftarrow x2, x3 \leftarrow x1, x2 \leftarrow op_r(i(x2),x3,x4), x1 \leftarrow x1\}$

$mult(\underbrace{rd(\overbrace{op_r(x1,op_r(i(x2),x3,x4),rd(x1,x2))},x2)}, op_r(op_r(x2,x3,x4),x1,i(rd(x1,x2))))$

$=$ by Lemma 2622 RL with $\{x3 \leftarrow op_r(i(x2),x3,x4), x2 \leftarrow x2, x1 \leftarrow x1\}$

$mult(\overbrace{op_l(rd(x1,x2),x1,\underbrace{op_l(op_r(i(x2),x3,x4),x1,rd(x1,x2))})}, op_r(op_r(x2,x3,x4),x1,i(rd(x1,x2))))$

$=$ by Lemma 2035 LR with $\{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow op_r(i(x2),x3,x4)\}$

$mult(op_l(rd(x1,x2),x1,\overbrace{op_l(op_r(i(x2),x3,x4),x2,x1)}), op_r(op_r(x2,x3,x4),x1,i(rd(x1,x2))))$

$=$ by Axiom 18 RL with $\{x5 \leftarrow x4, x4 \leftarrow x3, x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow i(x2)\}$

$mult(\underbrace{op_l(rd(x1,x2),x1,\overbrace{op_r(op_l(i(x2),x2,x1),x3,x4)})}, op_r(op_r(x2,x3,x4),x1,i(rd(x1,x2))))$

$=$ by Lemma 2766 RL with $\{x4 \leftarrow x4, x3 \leftarrow x3, x1 \leftarrow x2, x2 \leftarrow x1\}$

$mult(\overbrace{rd(x1,op_t(x2,i(x1)))}, op_r(op_r(x2,x3,x4),x1,i(rd(x1,x2))))$

Lemma 2768: $rd(x1, asoc(x3, x2, x4)) = mult(rd(x1, op_t(x2, i(x1))), op_r(op_r(x2, x3, x4), x1, rd(x2, x1)))$

$$
\begin{aligned}
& \underbrace{rd(x1, asoc(x3, x2, x4))} \\
= & \quad \text{by Lemma 2740 RL with } \{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{\overbrace{mult(rd(x1, x2), op_r(x2, x3, x4))}} \\
= & \quad \text{by Lemma 2767 LR with } \{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{mult(rd(x1, op_t(x2, i(x1))), op_r(op_r(x2, x3, x4), x1, \underbrace{i(rd(x1, x2))}))} \\
= & \quad \text{by Lemma 10 LR with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& mult(rd(x1, op_t(x2, i(x1))), op_r(op_r(x2, x3, x4), x1, \overbrace{rd(x2, x1)}))
\end{aligned}
$$

Lemma 2769: $asoc(op_t(x2, x3), x1, x4) = mult(rd(i(x3), asoc(x1, x2, x4)), x3)$

$\underbrace{asoc(op_t(x2, x3), x1, x4)}$

$=$ by Lemma 2721 RL with $\{x3 \leftarrow op_t(x2, x3), x2 \leftarrow x1, x1 \leftarrow x4\}$

$\overbrace{rd(x4, \underbrace{op_l(x4, x1, op_t(x2, x3))})}$

$=$ by Lemma 2030 RL with $\{x3 \leftarrow x1, x2 \leftarrow op_t(x2, x3), x1 \leftarrow x4\}$

$rd(x4, \overbrace{op_l(x4, op_t(x2, x3), i(x1))})$

$=$ by Lemma 1319 RL with $\{x3 \leftarrow op_t(x2, x3), x2 \leftarrow x1, x1 \leftarrow x4\}$

$rd(x4, \overbrace{op_l(x4, \underbrace{op_t(x1, op_t(x2, x3))}, op_t(x2, x3))})$

$=$ by Lemma 1722 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$rd(x4, \underbrace{op_l(x4, \overbrace{op_t(x1, x2)}, op_t(x2, x3))})$

$=$ by Lemma 2681 LR with $\{x4 \leftarrow x3, x2 \leftarrow x2, x3 \leftarrow op_t(x1, x2), x1 \leftarrow x4\}$

$rd(x4, \underbrace{op_l(x4, op_t(i(x2), x3), \overbrace{op_t(x1, x2)})})$

$=$ by Lemma 1901 LR with $\{x4 \leftarrow x1, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x4\}$

$\underbrace{rd(x4, \overbrace{op_l(x4, x1, op_t(x2, i(x3)))})}$

$=$ by Lemma 2721 LR with $\{x3 \leftarrow op_t(x2, i(x3)), x2 \leftarrow x1, x1 \leftarrow x4\}$

$\overbrace{\underbrace{asoc(op_t(x2, i(x3)), x1, x4)}}$

$=$ by Lemma 2718 RL with $\{x3 \leftarrow x4, x2 \leftarrow op_t(x2, i(x3)), x1 \leftarrow x1\}$

$\overbrace{\underbrace{i(asoc(x1, op_t(x2, i(x3)), x4))}}$

$=$ by Lemma 21 RL with $\{x2 \leftarrow asoc(x1, op_t(x2, i(x3)), x4), x1 \leftarrow x3\}$

$\overbrace{mult(rd(i(x3), \underbrace{asoc(x1, op_t(x2, i(x3)), x4)}), x3)}$

$=$ by Lemma 2711 RL with $\{x3 \leftarrow x4, x2 \leftarrow op_t(x2, i(x3)), x1 \leftarrow x1\}$

$mult(rd(i(x3), \overbrace{rd(op_r(x1, op_t(x2, i(x3)), x4), x1)}), x3)$

$=$ by Lemma 2735 RL with $\{x4 \leftarrow x4, x3 \leftarrow i(x3), x2 \leftarrow x2, x1 \leftarrow x1\}$

$mult(\underbrace{rd(i(x3), \overbrace{asoc(x1, op_t(x2, i(i(x3))), x4)})}, x3)$

$=$ by Lemma 2740 RL with $\{x4 \leftarrow x4, x3 \leftarrow x1, x2 \leftarrow op_t(x2, i(i(x3))), x1 \leftarrow i(x3)\}$

$mult(\overbrace{mult(rd(i(x3), op_t(x2, i(\underbrace{i(x3)}))), op_r(\underbrace{op_t(x2, i(\underbrace{i(x3)}))}, x1, x4))}, x3)$

$=$ by Lemma 1188 RL with $\{x2 \leftarrow i(x3), x1 \leftarrow x2\}$

$mult(mult(rd(i(x3), op_t(x2, i(\underbrace{i(x3)}))), \underbrace{op_r(\overbrace{op_r(x2, i(x3), rd(x2, i(x3)))}, x1, x4)}), x3)$

$=$ by Axiom 17 LR with $\{x5 \leftarrow x4, x4 \leftarrow x1, x3 \leftarrow rd(x2, i(x3)), x2 \leftarrow i(x3), x1 \leftarrow x2\}$

$\underbrace{mult(mult(rd(i(x3), op_t(x2, i(\underbrace{i(x3)}))), \overbrace{op_r(op_r(x2, x1, x4), i(x3), rd(x2, i(x3)))}), x3)}$

$=$ by Lemma 2768 RL with $\{x4 \leftarrow x4, x2 \leftarrow x2, x3 \leftarrow x1, x1 \leftarrow i(x3)\}$

$mult(\overbrace{rd(i(x3), asoc(x1, x2, x4))}, x3)$

Lemma 2770: $asoc(op_t(x2, x3), x1, x4) = asoc(x2, x1, x4)$

$$
\begin{aligned}
& \underbrace{asoc(op_t(x2, x3), x1, x4)} \\
= \quad & \text{by Lemma 2769 LR with } \{x4 \leftarrow x4, x1 \leftarrow x1, x3 \leftarrow x3, x2 \leftarrow x2\} \\
& \overbrace{\underbrace{mult(rd(i(x3), asoc(x1, x2, x4)), x3)}} \\
= \quad & \text{by Lemma 21 LR with } \{x2 \leftarrow asoc(x1, x2, x4), x1 \leftarrow x3\} \\
& \overbrace{\underbrace{i(asoc(x1, x2, x4))}} \\
= \quad & \text{by Lemma 2718 LR with } \{x3 \leftarrow x4, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{asoc(x2, x1, x4)}
\end{aligned}
$$

Lemma 2771: $op_r(x1, op_t(x2, x3), x4) = op_r(x1, x2, x4)$

$$
\begin{aligned}
& \underbrace{op_r(x1, op_t(x2, x3), x4)} \\
= \quad & \text{by Lemma 2716 RL with } \{x3 \leftarrow x4, x2 \leftarrow op_t(x2, x3), x1 \leftarrow x1\} \\
& \overbrace{rd(x1, \underbrace{asoc(op_t(x2, x3), x1, x4)})} \\
= \quad & \text{by Lemma 2770 LR with } \{x4 \leftarrow x4, x1 \leftarrow x1, x3 \leftarrow x3, x2 \leftarrow x2\} \\
& \underbrace{rd(x1, \overbrace{asoc(x2, x1, x4)})} \\
= \quad & \text{by Lemma 2716 LR with } \{x3 \leftarrow x4, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, x2, x4)}
\end{aligned}
$$

Lemma 2772: $op_r(x1, mult(i(x2), x3), x4) = op_r(x1, rd(x3, x2), x4)$

$$
\begin{aligned}
& \underbrace{op_r(x1, mult(i(x2), x3), x4)} \\
= \quad & \text{by Lemma 2686 RL with } \{x4 \leftarrow x4, x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_r(x1, op_t(rd(x3, x2), x3), x4)}} \\
= \quad & \text{by Lemma 2771 LR with } \{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow rd(x3, x2), x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, rd(x3, x2), x4)}
\end{aligned}
$$

Lemma 2773: $op_r(x1, rd(x2, x3), x4) = op_r(x1, mult(x2, i(x3)), x4)$

$$
\begin{aligned}
& \underbrace{op_r(x1, rd(x2, x3), x4)} \\
= \quad & \text{by Lemma 2436 RL with } \{x4 \leftarrow x4, x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_r(x1, rd(x3, x2), i(x4))}} \\
= \quad & \text{by Lemma 2772 RL with } \{x4 \leftarrow i(x4), x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_r(x1, mult(i(x2), x3), i(x4))}} \\
= \quad & \text{by Lemma 2509 LR with } \{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, mult(x2, i(x3)), x4)}
\end{aligned}
$$

Lemma 2774: $asoc(x1, rd(x2, x3), x4) = asoc(x1, mult(i(x3), x2), x4)$

$$
\begin{aligned}
& \underbrace{asoc(x1, rd(x2, x3), x4)} \\
= \quad & \text{by Lemma 2717 RL with } \{x3 \leftarrow x4, x2 \leftarrow x1, x1 \leftarrow rd(x2, x3)\} \\
& \overbrace{\underbrace{asoc(i(rd(x2, x3)), x1, x4)}} \\
= \quad & \text{by Lemma 2770 RL with } \{x4 \leftarrow x4, x1 \leftarrow x1, x3 \leftarrow x2, x2 \leftarrow i(rd(x2, x3))\} \\
& \overbrace{\underbrace{asoc(op_t(i(rd(x2, x3)), x2), x1, x4)}} \\
= \quad & \text{by Lemma 2737 LR with } \{x4 \leftarrow x4, x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow rd(x2, x3)\} \\
& \overbrace{\underbrace{asoc(x1, op_t(rd(x2, x3), x2), x4)}} \\
= \quad & \text{by Lemma 2750 LR with } \{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{asoc(x1, mult(i(x3), x2), x4)}
\end{aligned}
$$

Lemma 2775: $op_r(x1, rd(i(x2), x3), x4) = op_r(x1, mult(x2, x3), i(x4))$

$$
\begin{aligned}
& \underbrace{op_r(x1, rd(i(x2), x3), x4)} \\
= \quad & \text{by Lemma 2771 RL with } \{x4 \leftarrow x4, x3 \leftarrow x2, x2 \leftarrow rd(i(x2), x3), x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, \underbrace{op_t(rd(i(x2), x3), x2)}, x4)} \\
= \quad & \text{by Lemma 33 LR with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& \underbrace{op_r(x1, \overbrace{i(mult(x2, x3))}, x4)} \\
= \quad & \text{by Lemma 2427 LR with } \{x3 \leftarrow x4, x2 \leftarrow mult(x2, x3), x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, mult(x2, x3), i(x4))}
\end{aligned}
$$

Lemma 2776: $op_r(x1, mult(rd(x2, x3), x4), x5) = op_r(x1, rd(x4, rd(x3, x2)), x5)$

$$
\begin{aligned}
& op_r(x1, mult(\underbrace{rd(x2, x3)}, x4), x5) \\
= \quad & \text{by Lemma 10 RL with } \{x2 \leftarrow x2, x1 \leftarrow x3\} \\
& \underbrace{op_r(x1, mult(\overbrace{i(rd(x3, x2))}, x4), x5)} \\
= \quad & \text{by Lemma 2772 LR with } \{x4 \leftarrow x5, x3 \leftarrow x4, x2 \leftarrow rd(x3, x2), x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, rd(x4, rd(x3, x2)), x5)}
\end{aligned}
$$

Lemma 2777: $rd(asoc(x1, x2, x3), asoc(x2, x1, x2)) = asoc(x1, x2, mult(x3, x2))$

$\overbrace{rd(asoc(x1, x2, x3), asoc(x2, x1, x2))}$

$=$ by Lemma 10 RL with $\{x2 \leftarrow asoc(x1, x2, x3), x1 \leftarrow asoc(x2, x1, x2)\}$

$\overbrace{\underbrace{i(rd(asoc(x2, x1, x2), asoc(x1, x2, x3)))}}$

$=$ by Lemma 2765 RL with $\{x4 \leftarrow asoc(x2, x1, x2), x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{mult(rd(op_r(x1, x2, x3), \underbrace{asoc(x2, x1, x2)}), i(x1))}$

$=$ by Lemma 759 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$mult(\underbrace{rd(op_r(x1, x2, x3), \overbrace{i(\underbrace{asoc(x1, x2, x2)})}}), i(x1))$

$=$ by Lemma 1902 RL with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$mult(\overbrace{\underbrace{mult(asoc(op_r(x1, x2, x3), x2, x2), op_r(x1, x2, x3))}}, i(x1))$

$=$ by Lemma 1708 LR with $\{x3 \leftarrow op_r(x1, x2, x3), x2 \leftarrow x2, x1 \leftarrow op_r(x1, x2, x3)\}$

$mult(\overbrace{\underbrace{rd(op_r(x1, x2, x3), asoc(x2, op_r(x1, x2, x3), x2))}}, i(x1))$

$=$ by Lemma 641 LR with $\{x2 \leftarrow x2, x1 \leftarrow op_r(x1, x2, x3)\}$

$mult(\overbrace{\underbrace{op_r(op_r(x1, x2, x3), x2, x2)}}, i(x1))$

$=$ by Axiom 17 RL with $\{x5 \leftarrow x3, x4 \leftarrow x2, x3 \leftarrow x2, x2 \leftarrow x2, x1 \leftarrow x1\}$

$mult(\overbrace{\underbrace{op_r(op_r(x1, x2, x2), x2, x3)}}, i(x1))$

$=$ by Lemma 2512 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{mult(\underbrace{op_r(x1, x2, mult(x3, x2))}, i(x1))}$

$=$ by Lemma 112 LR with $\{x3 \leftarrow mult(x3, x2), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{\underbrace{rd(op_r(x1, x2, mult(x3, x2)), x1)}}$

$=$ by Lemma 2711 LR with $\{x3 \leftarrow mult(x3, x2), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{asoc(x1, x2, mult(x3, x2))}$

Lemma 2778: $asoc(op_l(x1,x2,x3),x4,op_r(x1,x5,x6)) = asoc(x1,x4,x1)$

$\underbrace{asoc(op_l(x1,x2,x3),x4,op_r(x1,x5,x6))}$

$=$ by Lemma 2723 RL with $\{x3 \leftarrow op_r(x1,x5,x6), x2 \leftarrow op_l(x1,x2,x3), x1 \leftarrow x4\}$

$\overbrace{rd(x4,op_r(x4,op_l(\underbrace{x1},x2,x3),op_r(x1,x5,x6)))}$

$=$ by Lemma 2584 LR with $\{x3 \leftarrow x6, x2 \leftarrow x5, x1 \leftarrow x1\}$

$rd(x4,\underbrace{op_r(x4,op_l(\overbrace{op_r(op_r(x1,x5,x6),x5,i(x6))},x2,x3),op_r(x1,x5,x6))})$

$=$ by Lemma 2425 RL with $\{x3 \leftarrow op_l(op_r(op_r(x1,x5,x6),x5,i(x6)),x2,x3), x2 \leftarrow op_r(x1,x5,x6), x1 \leftarrow x4\}$

$rd(x4,\overbrace{rd(x4,rd(op_r(x1,x5,x6),op_l(op_r(x1,x5,x6),x4,\underbrace{op_l(op_r(op_r(x1,x5,x6),x5,i(x6)),x2,x3)})))})$

$=$ by Axiom 18 RL with $\{x5 \leftarrow i(x6), x4 \leftarrow x5, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow op_r(x1,x5,x6)\}$

$rd(x4,rd(x4,rd(op_r(x1,x5,x6),\underbrace{op_l(op_r(x1,x5,x6),x4,\overbrace{op_r(op_l(op_r(x1,x5,x6),x2,x3),x5,i(x6))})})))$

$=$ by Lemma 1615 RL with $\{x5 \leftarrow op_r(op_l(op_r(x1,x5,x6),x2,x3),x5,i(x6)), x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow op_r(x1,x5,x6)\}$

$rd(x4,rd(x4,rd(op_r(x1,x5,x6),\overbrace{op_l(\underbrace{op_l(op_r(x1,x5,x6),x2,x3)},x4,op_r(\underbrace{op_l(op_r(x1,x5,x6),x2,x3)},x5,i(x6)))},x3,x2)))$

$=$ by Lemma 2601 LR with $\{x4 \leftarrow i(x6), x3 \leftarrow x5, x2 \leftarrow x4, x1 \leftarrow op_l(op_r(x1,x5,x6),x2,x3)\}$

$rd(x4,rd(x4,rd(op_r(x1,x5,x6),\underbrace{op_l(\overbrace{op_l(op_l(op_r(x1,x5,x6),x2,x3),x4,op_l(op_r(x1,x5,x6),x2,x3))},x3,x2)})))$

$=$ by Lemma 1615 LR with $\{x5 \leftarrow op_l(op_r(x1,x5,x6),x2,x3), x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow op_r(x1,x5,x6)\}$

$rd(x4,rd(x4,rd(op_r(x1,x5,x6),\overbrace{op_l(op_r(x1,x5,x6),x4,op_l(op_r(x1,x5,x6),x2,x3))})))$

$=$ by Lemma 1819 LR with $\{x4 \leftarrow x3, x3 \leftarrow x2, x2 \leftarrow x4, x1 \leftarrow op_r(x1,x5,x6)\}$

$rd(x4,\underbrace{rd(x4,rd(op_r(x1,x5,x6),\overbrace{op_l(op_r(x1,x5,x6),x4,op_r(x1,x5,x6))}))})$

$=$ by Lemma 2425 LR with $\{x3 \leftarrow op_r(x1,x5,x6), x2 \leftarrow op_r(x1,x5,x6), x1 \leftarrow x4\}$

$rd(x4,\overbrace{op_r(x4,op_r(x1,x5,x6),op_r(x1,x5,x6))})$

$=$ by Lemma 2627 LR with $\{x4 \leftarrow x6, x3 \leftarrow x5, x2 \leftarrow x1, x1 \leftarrow x4\}$

$\underbrace{rd(x4,\overbrace{op_r(x4,x1,x1)})}$

$=$ by Lemma 2723 LR with $\{x3 \leftarrow x1, x2 \leftarrow x1, x1 \leftarrow x4\}$

$\overbrace{asoc(x1,x4,x1)}$

Lemma 2779: $asoc(x2,x1,x3) = mult(rd(x4,op_r(x1,x2,x3)),i(rd(x4,x1)))$

$\underbrace{asoc(x2,x1,x3)}$

$=$ by Lemma 2718 RL with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{\overbrace{i(asoc(x1,x2,x3))}}$

$=$ by Lemma 22 RL with $\{x2 \leftarrow asoc(x1,x2,x3), x1 \leftarrow rd(x4,x1)\}$

$\overbrace{mult(\underbrace{rd(rd(x4,x1),asoc(x1,x2,x3))},i(rd(x4,x1)))}$

$=$ by Lemma 2742 LR with $\{x4 \leftarrow x3, x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x4\}$

$mult(\overbrace{rd(x4,op_r(x1,x2,x3))},i(rd(x4,x1)))$

Lemma 2780: $mult(rd(x1, op_r(x2, x3, x4)), rd(x2, x1)) = asoc(x3, x2, x4)$

$$
\begin{aligned}
& mult(rd(x1, op_r(x2, x3, x4)), \underbrace{rd(x2, x1)}) \\
= & \quad \text{by Lemma 10 RL with } \{x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{mult(rd(x1, op_r(x2, x3, x4)), \overbrace{i(rd(x1, x2))})} \\
= & \quad \text{by Lemma 2779 RL with } \{x4 \leftarrow x1, x3 \leftarrow x4, x1 \leftarrow x2, x2 \leftarrow x3\} \\
& \overbrace{asoc(x3, x2, x4)}
\end{aligned}
$$

Lemma 2781: $asoc(x1, x2, x3) = rd(rd(x4, x1), rd(x4, op_r(x1, x2, x3)))$

$$
\begin{aligned}
& \underbrace{asoc(x1, x2, x3)} \\
= & \quad \text{by Lemma 2657 RL with } \{x4 \leftarrow rd(x4, x1), x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_r(asoc(x1, x2, x3), rd(x4, x1), rd(x4, x1))}} \\
= & \quad \text{by Lemma 353 RL with } \{x2 \leftarrow asoc(x1, x2, x3), x1 \leftarrow rd(x4, x1)\} \\
& \overbrace{rd(rd(x4, x1), \underbrace{rd(rd(x4, x1), asoc(x1, x2, x3))})} \\
= & \quad \text{by Lemma 2742 LR with } \{x4 \leftarrow x3, x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x4\} \\
& rd(rd(x4, x1), \overbrace{rd(x4, op_r(x1, x2, x3))})
\end{aligned}
$$

Lemma 2782: $asoc(x1, x2, op_r(x3, x4, mult(x4, x5))) = asoc(x1, x2, op_r(x3, x4, x5))$

$$
\begin{aligned}
& \underbrace{asoc(x1, x2, op_r(x3, x4, mult(x4, x5)))} \\
= & \quad \text{by Lemma 2711 RL with } \{x3 \leftarrow op_r(x3, x4, mult(x4, x5)), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{rd(\underbrace{op_r(x1, x2, op_r(x3, x4, mult(x4, x5)))}, x1)} \\
= & \quad \text{by Lemma 2696 RL with } \{x5 \leftarrow x5, x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{rd(\overbrace{op_r(x1, x2, op_r(x3, x4, x5))}, x1)} \\
= & \quad \text{by Lemma 2711 LR with } \{x3 \leftarrow op_r(x3, x4, x5), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{asoc(x1, x2, op_r(x3, x4, x5))}
\end{aligned}
$$

Lemma 2783: $asoc(x1, op_l(rd(x1, x2), x1, x3), x4) = rd(x4, op_l(x4, x1, op_l(x2, x1, x3)))$

$$
\begin{aligned}
& \underbrace{asoc(x1, op_l(rd(x1, x2), x1, x3), x4)} \\
= & \quad \text{by Lemma 2721 RL with } \{x3 \leftarrow x1, x2 \leftarrow op_l(rd(x1, x2), x1, x3), x1 \leftarrow x4\} \\
& \overbrace{rd(x4, \underbrace{op_l(x4, op_l(rd(x1, x2), x1, x3), x1)})} \\
= & \quad \text{by Lemma 2693 LR with } \{x4 \leftarrow x3, x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x4\} \\
& rd(x4, \overbrace{op_l(x4, x1, \underbrace{op_l(x2, x3, i(x1))})}) \\
= & \quad \text{by Lemma 2030 LR with } \{x3 \leftarrow x1, x2 \leftarrow x3, x1 \leftarrow x2\} \\
& rd(x4, op_l(x4, x1, \overbrace{op_l(x2, x1, x3)}))
\end{aligned}
$$

Lemma 2784: $asoc(x4, asoc(x1, x2, x3), x5) = asoc(i(x4), rd(asoc(x2, x1, x3), x4), x5)$

$$
\begin{aligned}
& \overbrace{asoc(x4, asoc(x1, x2, x3), x5)} \\
= & \quad \text{by Lemma 2720 RL with } \{x3 \leftarrow x5, x2 \leftarrow x4, x1 \leftarrow asoc(x1, x2, x3)\} \\
& \overbrace{asoc(asoc(x1, x2, x3), i(x4), x5)} \\
= & \quad \text{by Lemma 2726 RL with } \{x3 \leftarrow x5, x2 \leftarrow asoc(x1, x2, x3), x1 \leftarrow i(x4)\} \\
& \overbrace{asoc(i(x4), \underbrace{rd(i(x4), asoc(x1, x2, x3))}, x5)} \\
= & \quad \text{by Lemma 2734 LR with } \{x4 \leftarrow x3, x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x4\} \\
& asoc(i(x4), \overbrace{rd(asoc(x2, x1, x3), x4)}, x5)
\end{aligned}
$$

Lemma 2785: $asoc(x4, asoc(x1, x2, x3), x5) = asoc(x4, rd(x4, asoc(x2, x1, x3)), x5)$

$$
\begin{aligned}
& \overbrace{asoc(x4, asoc(x1, x2, x3), x5)} \\
= & \quad \text{by Lemma 2784 LR with } \{x5 \leftarrow x5, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1, x4 \leftarrow x4\} \\
& \overbrace{asoc(i(x4), rd(asoc(x2, x1, x3), x4), x5)} \\
= & \quad \text{by Lemma 2717 LR with } \{x3 \leftarrow x5, x2 \leftarrow rd(asoc(x2, x1, x3), x4), x1 \leftarrow x4\} \\
& \overbrace{asoc(rd(asoc(x2, x1, x3), x4), x4, x5)} \\
= & \quad \text{by Lemma 2733 LR with } \{x4 \leftarrow x5, x3 \leftarrow x4, x2 \leftarrow x4, x1 \leftarrow asoc(x2, x1, x3)\} \\
& \overbrace{asoc(x4, rd(x4, asoc(x2, x1, x3)), x5)}
\end{aligned}
$$

Lemma 2786: $op_r(x1, mult(op_r(x2, x1, x3), x3), x4) = op_r(x1, mult(rd(x2, x1), x3), x4)$

$op_r(\underbrace{x1}, mult(op_r(x2, x1, x3), x3), x4)$

$=$ by Lemma 3 RL with $\{x1 \leftarrow x1\}$

$\underbrace{op_r(\overbrace{i(i(x1))}, mult(op_r(x2, x1, x3), x3), x4)}$

$=$ by Lemma 2428 LR with $\{x3 \leftarrow x4, x2 \leftarrow mult(op_r(x2, x1, x3), x3), x1 \leftarrow i(x1)\}$

$\overbrace{\underbrace{i(op_r(i(x1), mult(op_r(x2, x1, x3), x3), x4))}}$

$=$ by Lemma 654 RL with $\{x3 \leftarrow x4, x2 \leftarrow mult(op_r(x2, x1, x3), x3), x1 \leftarrow i(x1)\}$

$\overbrace{op_r(i(i(x1)), mult(i(x1), \underbrace{mult(op_r(x2, x1, x3), x3)}), x4)}$

$=$ by Lemma 2553 LR with $\{x3 \leftarrow x1, x2 \leftarrow x3, x1 \leftarrow op_r(x2, x1, x3)\}$

$op_r(i(i(x1)), mult(i(x1), \overbrace{mult(\underbrace{op_r(op_r(x2, x1, x3), x1, i(x3))}, op_l(x3, op_r(x2, x1, x3), x1))}), x4)$

$=$ by Lemma 2584 RL with $\{x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow x2\}$

$op_r(i(i(x1)), mult(i(x1), mult(\overbrace{x2}, \underbrace{op_l(x3, op_r(x2, x1, x3), x1)})), x4)$

$=$ by Lemma 2585 LR with $\{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x3\}$

$op_r(i(i(x1)), mult(i(x1), mult(x2, \overbrace{\underbrace{op_l(x3, x2, x1)}})), x4)$

$=$ by Lemma 2031 RL with $\{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x3\}$

$op_r(i(i(x1)), \underbrace{mult(i(x1), mult(x2, \overbrace{op_l(x3, i(x1), x2)}))}, x4)$

$=$ by Lemma 137 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow i(x1)\}$

$op_r(\underbrace{i(i(x1))}, \overbrace{mult(mult(i(x1), x2), x3)}, x4)$

$=$ by Lemma 3 LR with $\{x1 \leftarrow x1\}$

$op_r(\overbrace{x1}, mult(\underbrace{mult(i(x1), x2)}, x3), x4)$

$=$ by Lemma 14 RL with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$op_r(x1, \underbrace{mult(\overbrace{op_t(rd(x2, x1), x1)}, x3)}, x4)$

$=$ by Lemma 231 LR with $\{x2 \leftarrow x3, x1 \leftarrow op_t(rd(x2, x1), x1)\}$

$\underbrace{op_r(x1, \overbrace{rd(op_t(rd(x2, x1), x1), op_l(i(x3), x3, op_t(rd(x2, x1), x1)))}, x4)}$

$=$ by Lemma 2172 RL with $\{x4 \leftarrow x4, x2 \leftarrow op_l(i(x3), x3, op_t(rd(x2, x1), x1)), x3 \leftarrow rd(x2, x1), x1 \leftarrow x1\}$

$\overbrace{op_r(x1, rd(rd(x2, x1), \underbrace{op_l(i(x3), x3, op_t(rd(x2, x1), x1))}), x4)}$

$=$ by Lemma 1740 LR with $\{x3 \leftarrow x1, x2 \leftarrow rd(x2, x1), x1 \leftarrow x3\}$

$op_r(x1, \underbrace{rd(rd(x2, x1), \overbrace{op_l(i(x3), x3, rd(x2, x1))})}, x4)$

$=$ by Lemma 231 RL with $\{x2 \leftarrow x3, x1 \leftarrow rd(x2, x1)\}$

$op_r(x1, \overbrace{mult(rd(x2, x1), x3)}, x4)$

Lemma 2787: $op_r(x1, mult(x3, op_r(x2, x1, x3)), x4) = op_r(x1, rd(x3, rd(x1, x2)), x4)$

$$
\begin{aligned}
& \underbrace{op_r(x1, mult(x3, op_r(x2, x1, x3)), x4)} \\
= & \quad \text{by Lemma 2684 RL with } \{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow op_r(x2, x1, x3), x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_r(x1, mult(op_r(x2, x1, x3), x3), x4)}} \\
= & \quad \text{by Lemma 2786 LR with } \{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_r(x1, mult(rd(x2, x1), x3), x4)}} \\
= & \quad \text{by Lemma 2776 LR with } \{x5 \leftarrow x4, x4 \leftarrow x3, x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, rd(x3, rd(x1, x2)), x4)}
\end{aligned}
$$

Lemma 2788: $asoc(x1, x2, rd(mult(i(x3), x4), x5)) = asoc(x1, x2, rd(rd(x4, x3), x5))$

$$
\begin{aligned}
& \underbrace{asoc(x1, x2, rd(mult(i(x3), x4), x5))} \\
= & \quad \text{by Lemma 2711 RL with } \{x3 \leftarrow rd(mult(i(x3), x4), x5), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{rd(\underbrace{op_r(x1, x2, rd(mult(i(x3), x4), x5))}, x1)} \\
= & \quad \text{by Lemma 2410 RL with } \{x5 \leftarrow x5, x4 \leftarrow x3, x3 \leftarrow x4, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{rd(\overbrace{op_r(x1, x2, rd(rd(x4, x3), x5))}, x1)} \\
= & \quad \text{by Lemma 2711 LR with } \{x3 \leftarrow rd(rd(x4, x3), x5), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{asoc(x1, x2, rd(rd(x4, x3), x5))}
\end{aligned}
$$

Lemma 2789: $asoc(x1, x2, rd(x3, mult(i(x4), x5))) = rd(op_r(x1, x2, rd(x3, rd(x5, x4))), x1)$

$\overbrace{asoc(x1, x2, rd(x3, mult(i(x4), x5)))}$

$=$ by Lemma 2711 RL with $\{x3 \leftarrow rd(x3, mult(i(x4), x5)), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{rd(op_r(x1, x2, rd(x3, \underbrace{mult(i(x4), x5)})), x1)}$

$=$ by Lemma 788 LR with $\{x1 \leftarrow x5, x2 \leftarrow x4\}$

$rd(op_r(x1, x2, \underbrace{rd(x3, \overbrace{mult(asoc(x4, x5, x5), rd(x5, x4))})}), x1)$

$=$ by Lemma 839 RL with $\{x3 \leftarrow asoc(x4, x5, x5), x2 \leftarrow rd(x5, x4), x1 \leftarrow x3\}$

$rd(op_r(x1, x2, \overbrace{op_r(rd(rd(x3, rd(x5, x4)), asoc(x4, x5, x5)), \underbrace{asoc(x4, x5, x5)}, rd(x5, x4))}), x1)$

$=$ by Lemma 794 LR with $\{x2 \leftarrow x5, x1 \leftarrow x4\}$

$rd(op_r(x1, x2, op_r(rd(rd(x3, rd(x5, x4)), asoc(x4, x5, x5)), \overbrace{rd(\underbrace{mult(i(x4), x5)}, rd(x5, x4))}, rd(x5, x4))), x1)$

$=$ by Lemma 14 RL with $\{x2 \leftarrow x4, x1 \leftarrow x5\}$

$rd(op_r(x1, x2, \underbrace{op_r(rd(rd(x3, rd(x5, x4)), asoc(x4, x5, x5)), rd(\overbrace{op_t(rd(x5, x4), x4)}, rd(x5, x4)), rd(x5, x4))}), x1)$

$=$ by Lemma 693 RL with $\{x3 \leftarrow rd(x5, x4), x2 \leftarrow op_t(rd(x5, x4), x4), x1 \leftarrow rd(rd(x3, rd(x5, x4)), asoc(x4, x5, x5))\}$

$rd(op_r(x1, x2, \overbrace{rd(mult(mult(rd(rd(x3, rd(x5, x4)), asoc(x4, x5, x5)), rd(op_t(rd(x5, x4), x4), rd(x5, x4))), rd(x5, x4)), op_t(rd$

$=$ by Lemma 1292 RL with $\{x3 \leftarrow x4, x2 \leftarrow rd(x5, x4), x1 \leftarrow mult(rd(rd(x3, rd(x5, x4)), asoc(x4, x5, x5)), rd(op_t(rd(x5, x4), x4$

$rd(op_r(x1, x2, \overbrace{rd(mult(rd(rd(x3, rd(x5, x4)), asoc(x4, x5, x5)), rd(op_t(rd(x5, x4), x4), rd(x5, x4))), rd(op_t(rd(x5, x4), x4), r$

$=$ by Axiom 5 RL with $\{x2 \leftarrow rd(op_t(rd(x5, x4), x4), rd(x5, x4)), x1 \leftarrow rd(rd(x3, rd(x5, x4)), asoc(x4, x5, x5))\}$

$rd(op_r(x1, x2, \overbrace{rd(rd(x3, rd(x5, x4)), asoc(x4, x5, x5))}), x1)$

$=$ by Lemma 1295 RL with $\{x3 \leftarrow x4, x2 \leftarrow x5, x1 \leftarrow rd(x3, rd(x5, x4))\}$

$rd(\underbrace{op_r(x1, x2, \overbrace{mult(rd(x3, rd(x5, x4)), asoc(x5, x4, x5))})}, x1)$

$=$ by Lemma 2389 RL with $\{x5 \leftarrow x4, x4 \leftarrow x5, x3 \leftarrow mult(rd(x3, rd(x5, x4)), asoc(x5, x4, x5)), x2 \leftarrow x2, x1 \leftarrow x1\}$

$rd(\overbrace{op_r(x1, x2, \underbrace{rd(mult(rd(x3, rd(x5, x4)), asoc(x5, x4, x5)), asoc(x5, x4, x5))})}, x1)$

$=$ by Axiom 5 RL with $\{x2 \leftarrow asoc(x5, x4, x5), x1 \leftarrow rd(x3, rd(x5, x4))\}$

$rd(op_r(x1, x2, \overbrace{rd(x3, rd(x5, x4))}), x1)$

Lemma 2790: $op_r(x1, mult(rd(i(x1), x3), x2), x4) = op_r(x1, mult(rd(x2, x1), op_r(i(mult(x3, x1)), x1, x2)), x4)$

$op_r(x1, \underbrace{mult(rd(i(x1), x3), x2)}, x4)$

= by Axiom 5 LR with $\{x2 \leftarrow rd(i(x1), x3), x1 \leftarrow mult(rd(i(x1), x3), x2)\}$

$op_r(x1, \overbrace{rd(mult(mult(rd(i(x1), x3), x2), rd(i(x1), x3)), rd(i(x1), x3))}, x4)$

= by Lemma 401 RL with $\{x2 \leftarrow mult(mult(rd(i(x1), x3), x2), rd(i(x1), x3)), x1 \leftarrow rd(mult(mult(rd(i(x1), x3), x2), rd(i(x1), x$

$op_r(x1, \overbrace{op_t(rd(rd(mult(mult(rd(i(x1), x3), x2), rd(i(x1), x3)), rd(i(x1), x3)), \underbrace{rd(op_t(rd(mult(mult(rd(i(x1), x3), x2), rd(i($

= by Lemma 395 LR with $\{x1 \leftarrow mult(mult(rd(i(x1), x3), x2), rd(i(x1), x3)), x2 \leftarrow rd(mult(mult(rd(i(x1), x3), x2), rd(i(x1), x$

$op_r(x1, op_t(\underbrace{rd(rd(mult(mult(rd(i(x1), x3), x2), rd(i(x1), x3)), rd(i(x1), x3)), \overbrace{rd(mult(mult(rd(i(x1), x3), x2), rd(i(x1), x3$

= by Lemma 1390 RL with $\{x1 \leftarrow rd(i(x1), x3), x3 \leftarrow rd(mult(mult(rd(i(x1), x3), x2), rd(i(x1), x3)), rd(i(x1), x3)), x2 \leftarrow mult$

$op_r(x1, op_t(\overbrace{rd(op_t(mult(mult(rd(i(x1), x3), x2), rd(i(x1), x3)), \underbrace{rd(mult(mult(rd(i(x1), x3), x2), rd(i(x1), x3)), rd(i(x1), x3$

= by Axiom 5 RL with $\{x2 \leftarrow rd(i(x1), x3), x1 \leftarrow mult(rd(i(x1), x3), x2)\}$

$op_r(x1, op_t(rd(\underbrace{op_t(mult(mult(rd(i(x1), x3), x2), rd(i(x1), x3)), \overbrace{mult(rd(i(x1), x3), x2)}), rd(i(x1), x3)), mult(mult(rd(i(x1$

= by Lemma 919 RL with $\{x2 \leftarrow mult(rd(i(x1), x3), x2), x1 \leftarrow rd(i(x1), x3)\}$

$\underbrace{op_r(x1, op_t(rd(\overbrace{op_t(mult(rd(i(x1), x3), mult(rd(i(x1), x3), x2)), i(rd(i(x1), x3)))}, rd(i(x1), x3)), mult(mult(rd(i(x1), x3), x$

= by Lemma 2771 LR with $\{x4 \leftarrow x4, x3 \leftarrow mult(mult(rd(i(x1), x3), x2), rd(i(x1), x3)), x2 \leftarrow rd(op_t(mult(rd(i(x1), x3), mult($

$\overbrace{op_r(x1, rd(op_t(mult(rd(i(x1), x3), \overbrace{mult(rd(i(x1), x3), x2)}), i(rd(i(x1), x3))), rd(i(x1), x3)), x4)}$

= by Lemma 2688 RL with $\{x4 \leftarrow x4, x3 \leftarrow rd(i(x1), x3), x2 \leftarrow mult(rd(i(x1), x3), mult(rd(i(x1), x3), x2)), x1 \leftarrow x1\}$

$\overbrace{op_r(x1, rd(mult(rd(i(x1), x3), \overbrace{mult(rd(i(x1), x3), x2)}), rd(i(x1), x3)), x4)}$

= by Lemma 2685 RL with $\{x4 \leftarrow x4, x3 \leftarrow rd(i(x1), x3), x2 \leftarrow mult(rd(i(x1), x3), x2), x1 \leftarrow x1\}$

$\overbrace{op_r(x1, op_r(mult(rd(i(x1), x3), x2), rd(i(x1), x3), rd(i(x1), x3)), x4)}$

= by Lemma 2694 RL with $\{x4 \leftarrow x4, x3 \leftarrow rd(i(x1), x3), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_r(x1, rd(x2, \overbrace{i(rd(i(x1), x3))}), x4)}$

= by Lemma 2180 RL with $\{x4 \leftarrow x4, x3 \leftarrow i(rd(i(x1), x3)), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_r(x1, rd(x2, \underbrace{op_r(i(rd(i(x1), x3)), x1, x1)}), x4)}$

= by Lemma 354 RL with $\{x2 \leftarrow rd(i(x1), x3), x1 \leftarrow x1\}$

$\underbrace{op_r(x1, rd(x2, \overbrace{rd(rd(x1, rd(i(x1), x3)), x1)}), x4)}$

= by Lemma 2536 RL with $\{x4 \leftarrow x4, x3 \leftarrow rd(x1, rd(i(x1), x3)), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_r(x1, rd(\overbrace{rd(x2, x1)}, \overbrace{rd(x1, rd(i(x1), x3))}), x4)}$

= by Lemma 2787 RL with $\{x4 \leftarrow x4, x2 \leftarrow rd(i(x1), x3), x3 \leftarrow rd(x2, x1), x1 \leftarrow x1\}$

$\overbrace{op_r(x1, mult(rd(x2, x1), op_r(\underbrace{rd(i(x1), x3)}, x1, rd(x2, x1))), x4)}$

= by Lemma 37 RL with $\{x2 \leftarrow x1, x1 \leftarrow x3\}$

$op_r(x1, mult(rd(x2, x1), \underbrace{op_r(\overbrace{rd(i(x3), x1)}, x1, rd(x2, x1))}), x4)$

= by Lemma 2516 LR with $\{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow i(x3)\}$

$op_r(x1, mult(rd(x2, x1), \overbrace{op_r(\underbrace{mult(i(x3), i(x1))}, x1, x2)}), x4)$

= by Axiom 11 LR with $\{x2 \leftarrow x1, x1 \leftarrow x3\}$

$op_r(x1, mult(rd(x2, x1), op_r(\overbrace{i(mult(x3, x1))}, x1, x2)), x4)$

Lemma 2791: $op_r(x1, rd(x2, mult(x1, x3)), x4) = op_r(x1, mult(rd(x2, x1), op_r(i(mult(x3, x1)), x1, x2)), x4)$

$op_r(x1, rd(x2, mult(\underbrace{x1}, x3)), x4)$

$=$ by Lemma 3 RL with $\{x1 \leftarrow x1\}$

$op_r(x1, rd(x2, \underbrace{mult(\overbrace{i(i(x1))}, x3)}), x4)$

$=$ by Lemma 14 RL with $\{x2 \leftarrow i(x1), x1 \leftarrow x3\}$

$\underbrace{op_r(x1, rd(x2, \overbrace{op_t(rd(x3, i(x1)), i(x1))}), x4)}$

$=$ by Lemma 2171 RL with $\{x4 \leftarrow x4, x2 \leftarrow rd(x3, i(x1)), x3 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_r(x1, rd(x2, rd(x3, i(x1))), x4)}}$

$=$ by Lemma 2776 RL with $\{x5 \leftarrow x4, x4 \leftarrow x2, x3 \leftarrow x3, x2 \leftarrow i(x1), x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_r(x1, mult(rd(i(x1), x3), x2), x4)}}$

$=$ by Lemma 2790 LR with $\{x4 \leftarrow x4, x2 \leftarrow x2, x3 \leftarrow x3, x1 \leftarrow x1\}$

$\overbrace{op_r(x1, mult(rd(x2, x1), op_r(i(mult(x3, x1)), x1, x2)), x4)}$

Lemma 2792: $asoc(x1, x2, op_r(x5, rd(x4, x3), x4)) = asoc(x1, x2, op_r(x5, x3, i(x4)))$

$asoc(x1, x2, \underbrace{op_r(x5, rd(x4, x3), x4)}_{})$

$=$ by Lemma 2436 RL with $\{x4 \leftarrow x4, x3 \leftarrow x4, x2 \leftarrow x3, x1 \leftarrow x5\}$

$asoc(x1, x2, \overbrace{op_r(x5, rd(x3, x4), i(x4))}^{})$

$=$ by Lemma 2430 RL with $\{x3 \leftarrow i(x4), x2 \leftarrow rd(x3, x4), x1 \leftarrow x5\}$

$asoc(x1, x2, \overbrace{op_r(x5, mult(rd(x3, x4), x5), i(x4))}^{})$

$=$ by Lemma 2567 RL with $\{x3 \leftarrow x4, x2 \leftarrow mult(rd(x3, x4), x5), x1 \leftarrow x5\}$

$asoc(x1, x2, \overbrace{mult(\underbrace{rd(x5, mult(rd(x3, x4), x5))}_{}, op_r(mult(rd(x3, x4), x5), x5, x4))}^{})$

$=$ by Lemma 9 LR with $\{x2 \leftarrow rd(x3, x4), x1 \leftarrow x5\}$

$asoc(x1, x2, \underbrace{mult(\overbrace{i(rd(x3, x4))}^{}, op_r(mult(rd(x3, x4), x5), x5, x4))}_{})$

$=$ by Lemma 1971 RL with $\{x4 \leftarrow rd(x3, x4), x3 \leftarrow x4, x2 \leftarrow x5, x1 \leftarrow mult(rd(x3, x4), x5)\}$

$asoc(x1, x2, \overbrace{rd(\underbrace{op_l(op_r(mult(rd(x3, x4), x5), x5, x4), rd(x3, x4), mult(rd(x3, x4), x5))}_{}, rd(x3, x4))}^{})$

$=$ by Lemma 2032 LR with $\{x3 \leftarrow x5, x2 \leftarrow rd(x3, x4), x1 \leftarrow op_r(mult(rd(x3, x4), x5), x5, x4)\}$

$asoc(x1, x2, rd(\overbrace{op_l(op_r(mult(rd(x3, x4), x5), x5, x4), rd(x3, x4), x5)}^{}, rd(x3, x4)))$

$=$ by Axiom 18 RL with $\{x5 \leftarrow x4, x4 \leftarrow x5, x3 \leftarrow x5, x2 \leftarrow rd(x3, x4), x1 \leftarrow mult(rd(x3, x4), x5)\}$

$asoc(x1, x2, rd(\overbrace{op_r(\underbrace{op_l(mult(rd(x3, x4), x5), rd(x3, x4), x5)}_{}, x5, x4)}^{}, rd(x3, x4)))$

$=$ by Lemma 435 LR with $\{x2 \leftarrow x5, x1 \leftarrow rd(x3, x4)\}$

$asoc(x1, x2, rd(\underbrace{op_r(\overbrace{mult(x5, rd(x3, x4))}^{}, x5, x4)}_{}, rd(x3, x4)))$

$=$ by Lemma 2527 LR with $\{x3 \leftarrow x4, x2 \leftarrow x3, x1 \leftarrow x5\}$

$asoc(x1, x2, \underbrace{rd(\overbrace{rd(mult(x5, x3), x4)}^{}, rd(x3, x4))}_{})$

$=$ by Lemma 1475 LR with $\{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x4, x1 \leftarrow mult(x5, x3)\}$

$\underbrace{asoc(x1, x2, \overbrace{rd(rd(x4, x3), rd(x4, mult(x5, x3)))}^{})}_{}$

$=$ by Lemma 2788 RL with $\{x5 \leftarrow rd(x4, mult(x5, x3)), x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{\underbrace{asoc(x1, x2, rd(\overbrace{mult(i(x3), x4)}^{}, rd(x4, mult(x5, x3))))}_{}}^{}$

$=$ by Lemma 2711 RL with $\{x3 \leftarrow rd(mult(i(x3), x4), rd(x4, mult(x5, x3))), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{\underbrace{rd(op_r(x1, x2, rd(mult(i(x3), x4), rd(x4, mult(x5, x3)))), x1)}_{}}^{}$

$=$ by Lemma 2789 RL with $\{x5 \leftarrow x4, x4 \leftarrow mult(x5, x3), x3 \leftarrow mult(i(x3), x4), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{asoc(x1, x2, rd(mult(\underbrace{i(x3)}_{}, x4), \overbrace{mult(i(mult(x5, x3)), x4)}^{}))}^{}$

$=$ by Lemma 20 RL with $\{x2 \leftarrow x3, x1 \leftarrow x5\}$

$asoc(x1, x2, \underbrace{rd(mult(\overbrace{mult(x5, i(mult(x5, x3)))}^{}, x4), mult(i(mult(x5, x3)), x4))}_{})$

$=$ by Axiom 16 RL with $\{x3 \leftarrow x4, x2 \leftarrow i(mult(x5, x3)), x1 \leftarrow x5\}$

$asoc(x1, x2, \overbrace{op_r(x5, i(mult(x5, x3)), x4)}^{})$

$=$ by Lemma 818 LR with $\{x3 \leftarrow x4, x2 \leftarrow x3, x1 \leftarrow x5\}$

$asoc(x1, x2, \overbrace{op_r(x5, x3, i(x4))}^{})$

Lemma 2793: $asoc(x1, x2, op_r(x3, x4, i(x5))) = asoc(x1, x2, op_r(x3, x5, mult(x5, x4)))$

$\underbrace{asoc(x1, x2, op_r(x3, x4, i(x5)))}$

$=$ by Lemma 2711 RL with $\{x3 \leftarrow op_r(x3, x4, i(x5)), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{rd(\underbrace{op_r(x1, x2, op_r(x3, x4, i(x5)))}, x1)}$

$=$ by Lemma 2399 LR with $\{x4 \leftarrow x4, x3 \leftarrow op_r(x3, x4, i(x5)), x2 \leftarrow x2, x1 \leftarrow x1\}$

$rd(\overbrace{op_r(x1, x2, \underbrace{rd(op_r(x3, x4, i(x5)), asoc(op_r(x3, x4, i(x5)), x4, x4))}), x1)}$

$=$ by Lemma 775 LR with $\{x2 \leftarrow x4, x1 \leftarrow op_r(x3, x4, i(x5))\}$

$rd(op_r(x1, x2, \overbrace{op_r(\underbrace{op_r(x3, x4, i(x5))}, x4, i(x4))}), x1)$

$=$ by Lemma 1939 RL with $\{x3 \leftarrow i(x5), x2 \leftarrow x4, x1 \leftarrow x3\}$

$rd(op_r(x1, x2, \underbrace{op_r(\overbrace{op_r(x3, op_t(x4, x3), i(x5))}, x4, i(x4)))}, x1)$

$=$ by Axiom 17 LR with $\{x5 \leftarrow i(x4), x4 \leftarrow x4, x3 \leftarrow i(x5), x2 \leftarrow op_t(x4, x3), x1 \leftarrow x3\}$

$rd(op_r(x1, x2, \overbrace{op_r(\underbrace{op_r(x3, x4, i(x4))}, op_t(x4, x3), i(x5)))}, x1)$

$=$ by Lemma 604 RL with $\{x2 \leftarrow x3, x1 \leftarrow x4\}$

$rd(op_r(x1, x2, \underbrace{op_r(\overbrace{mult(x4, rd(x3, op_t(x4, x3)))}, op_t(x4, x3), i(x5)))}, x1)$

$=$ by Lemma 2518 LR with $\{x3 \leftarrow i(x5), x2 \leftarrow x3, x1 \leftarrow x4\}$

$rd(op_r(x1, x2, \overbrace{op_r(x3, op_t(x4, x3), \underbrace{rd(i(x5), op_t(x4, x3))})}), x1)$

$=$ by Lemma 1939 LR with $\{x3 \leftarrow rd(i(x5), op_t(x4, x3)), x2 \leftarrow x4, x1 \leftarrow x3\}$

$rd(op_r(x1, x2, \overbrace{op_r(x3, x4, \underbrace{rd(i(x5), op_t(x4, x3))})}), x1)$

$=$ by Lemma 2407 RL with $\{x5 \leftarrow x3, x3 \leftarrow x4, x4 \leftarrow i(x5), x2 \leftarrow x4, x1 \leftarrow x3\}$

$\underbrace{rd(op_r(x1, x2, \overbrace{op_r(x3, x4, rd(i(x5), x4))}), x1)}$

$=$ by Lemma 2711 LR with $\{x3 \leftarrow op_r(x3, x4, rd(i(x5), x4)), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{asoc(x1, x2, \underbrace{op_r(x3, x4, rd(i(x5), x4))})}$

$=$ by Lemma 2400 RL with $\{x4 \leftarrow x4, x3 \leftarrow x5, x2 \leftarrow x4, x1 \leftarrow x3\}$

$\underbrace{asoc(x1, x2, \overbrace{op_r(x3, x4, i(mult(x5, x4)))})}$

$=$ by Lemma 2792 RL with $\{x3 \leftarrow x4, x4 \leftarrow mult(x5, x4), x5 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{asoc(x1, x2, op_r(x3, \underbrace{rd(mult(x5, x4), x4)}, mult(x5, x4)))}$

$=$ by Axiom 5 RL with $\{x2 \leftarrow x4, x1 \leftarrow x5\}$

$asoc(x1, x2, op_r(x3, \overbrace{x5}, mult(x5, x4)))$

Lemma 2794: $op_r(x1, rd(x2, op_r(x3, x1, mult(i(x2), x3))), x4) = op_r(x1, rd(x2, rd(x3, x1)), x4)$

$op_r(x1, rd(x2, \underbrace{op_r(x3, x1, mult(i(x2), x3))}), x4)$

$=$ by Lemma 2500 LR with $\{x3 \leftarrow i(x2), x2 \leftarrow x1, x1 \leftarrow x3\}$

$op_r(x1, rd(x2, \overbrace{op_l(\underbrace{op_r(x3, x1, i(x2))}, x3, x1)}), x4)$

$=$ by Lemma 2429 RL with $\{x3 \leftarrow i(x2), x2 \leftarrow x1, x1 \leftarrow x3\}$

$op_r(x1, rd(x2, op_l(\overbrace{op_r(x3, rd(x1, x3), i(x2))}, x3, x1)), x4)$

$=$ by Lemma 2436 RL with $\{x4 \leftarrow i(x2), x3 \leftarrow x1, x2 \leftarrow x3, x1 \leftarrow x3\}$

$op_r(x1, rd(x2, \underbrace{op_l(\overbrace{op_r(x3, rd(x3, x1), i(i(x2)))}, x3, x1)}), x4)$

$=$ by Axiom 18 RL with $\{x5 \leftarrow i(i(x2)), x4 \leftarrow rd(x3, x1), x3 \leftarrow x1, x2 \leftarrow x3, x1 \leftarrow x3\}$

$op_r(x1, rd(x2, \overbrace{op_r(op_l(x3, x3, x1), rd(x3, x1), i(i(x2)))}), x4)$

$=$ by Lemma 2577 LR with $\{x3 \leftarrow i(x2), x2 \leftarrow x1, x1 \leftarrow x3\}$

$op_r(x1, rd(x2, \overbrace{mult(\underbrace{rd(op_l(x3, x3, x1), rd(x3, x1))}, op_r(rd(x3, x1), x3, i(x2)))}), x4)$

$=$ by Lemma 2550 RL with $\{x4 \leftarrow x1, x2 \leftarrow x3, x3 \leftarrow x1, x1 \leftarrow x3\}$

$op_r(x1, rd(x2, mult(\overbrace{rd(x3, \underbrace{rd(op_l(x3, x1, x3), x1)})}, op_r(rd(x3, x1), x3, i(x2)))), x4)$

$=$ by Lemma 615 LR with $\{x2 \leftarrow x1, x1 \leftarrow x3\}$

$op_r(x1, rd(x2, mult(\underbrace{rd(x3, \overbrace{mult(i(x1), x3)})}, op_r(rd(x3, x1), x3, i(x2)))), x4)$

$=$ by Lemma 9 LR with $\{x2 \leftarrow i(x1), x1 \leftarrow x3\}$

$op_r(x1, rd(x2, mult(\underbrace{i(\overbrace{i(x1)})}, op_r(rd(x3, x1), x3, i(x2)))), x4)$

$=$ by Lemma 3 LR with $\{x1 \leftarrow x1\}$

$\underbrace{op_r(x1, rd(x2, mult(\overbrace{x1}, op_r(rd(x3, x1), x3, i(x2))))), x4)}$

$=$ by Lemma 2791 LR with $\{x4 \leftarrow x4, x3 \leftarrow op_r(rd(x3, x1), x3, i(x2)), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_r(x1, mult(rd(x2, x1), \underbrace{op_r(i(mult(\underbrace{op_r(rd(x3, x1), x3, i(x2))}, x1)), x1, x2)}), x4)}$

$=$ by Lemma 2776 LR with $\{x5 \leftarrow x4, x4 \leftarrow op_r(i(mult(op_r(rd(x3, x1), x3, i(x2)), x1)), x1, x2), x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_r(x1, rd(\underbrace{op_r(i(mult(\underbrace{op_r(rd(x3, x1), x3, i(x2))}, x1)), x1, x2)}, rd(x1, x2)), x4)}$

$=$ by Lemma 2428 LR with $\{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow mult(op_r(rd(x3, x1), x3, i(x2)), x1)\}$

$op_r(x1, \underbrace{rd(\overbrace{i(op_r(mult(\underbrace{op_r(rd(x3, x1), x3, i(x2))}, x1), x1, x2))}, rd(x1, x2))}, x4)$

$=$ by Lemma 40 LR with $\{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow op_r(mult(op_r(rd(x3, x1), x3, i(x2)), x1), x1, x2)\}$

$\underbrace{op_r(x1, \overbrace{rd(rd(x2, x1), op_r(mult(\underbrace{op_r(rd(x3, x1), x3, i(x2))}, x1), x1, x2))}, x4)}$

$=$ by Lemma 2536 LR with $\{x4 \leftarrow x4, x3 \leftarrow op_r(mult(op_r(rd(x3, x1), x3, i(x2)), x1), x1, x2), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_r(x1, rd(x2, \underbrace{rd(op_r(mult(\underbrace{op_r(rd(x3, x1), x3, i(x2))}, x1), x1, x2), x1)}), x4)}$

$=$ by Lemma 2571 LR with $\{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow mult(op_r(rd(x3, x1), x3, i(x2)), x1)\}$

$op_r(x1, rd(x2, \overbrace{op_r(\underbrace{rd(mult(\underbrace{op_r(rd(x3, x1), x3, i(x2))}, x1), x1)}, x1, x2)}), x4)$

$=$ by Axiom 5 RL with $\{x2 \leftarrow x1, x1 \leftarrow op_r(rd(x3, x1), x3, i(x2))\}$

$op_r(x1, rd(x2, op_r(\overbrace{op_r(rd(x3, x1), x3, i(x2))}, x1, \underbrace{x2})), x4)$

$=$ by Lemma 3 RL with $\{x1 \leftarrow x2\}$

$op_r(x1, rd(x2, \underbrace{op_r(op_r(rd(x3, x1), x3, i(x2)), x1, \overbrace{i(i(x2))})}), x4)$

$=$ by Lemma 2611 RL with $\{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x3\}$

$op_r(x1, rd(x2, \overbrace{rd(x3, x1)}), x4)$

Lemma 2795: $op_r(x1, rd(i(x2), rd(i(x3), x1)), x4) = op_r(x1, op_l(rd(x3, x2), i(x3), x1), x4)$

$$
\begin{aligned}
& \overbrace{op_r(x1, rd(i(x2), rd(i(x3), x1)), x4)} \\
= \quad & \text{by Lemma 2794 RL with } \{x4 \leftarrow x4, x3 \leftarrow i(x3), x2 \leftarrow i(x2), x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, rd(i(x2), \underbrace{op_r(i(x3), x1, mult(i(i(x2)), i(x3)))}), x4)} \\
= \quad & \text{by Lemma 2397 LR with } \{x4 \leftarrow i(x3), x3 \leftarrow i(x2), x2 \leftarrow x1, x1 \leftarrow i(x3)\} \\
& op_r(x1, rd(i(x2), \overbrace{\underbrace{op_r(i(x3), x1, rd(i(x3), i(x2)))}}), x4) \\
= \quad & \text{by Lemma 2395 RL with } \{x4 \leftarrow i(x2), x3 \leftarrow i(x3), x2 \leftarrow x1, x1 \leftarrow i(x3)\} \\
& op_r(x1, rd(i(x2), \overbrace{\underbrace{op_r(i(x3), x1, mult(i(x3), i(i(x2))))}}), x4) \\
= \quad & \text{by Lemma 2474 RL with } \{x3 \leftarrow x1, x2 \leftarrow i(i(x2)), x1 \leftarrow i(x3)\} \\
& op_r(x1, \underbrace{rd(i(x2), \overbrace{rd(op_l(mult(i(x3), i(i(x2))), i(x3), x1), i(i(x2))))}}, x4) \\
= \quad & \text{by Lemma 362 LR with } \{x2 \leftarrow op_l(mult(i(x3), i(i(x2))), i(x3), x1), x1 \leftarrow i(x2)\} \\
& op_r(x1, \overbrace{op_r(\underbrace{i(op_l(mult(i(x3), i(i(x2))), i(x3), x1))}, i(x2), i(x2))}, x4) \\
= \quad & \text{by Lemma 1049 LR with } \{x3 \leftarrow x1, x2 \leftarrow i(x3), x1 \leftarrow mult(i(x3), i(i(x2)))\} \\
& op_r(x1, \underbrace{op_r(\overbrace{op_l(i(mult(i(x3), i(i(x2)))), i(x3), x1)}, i(x2), i(x2))}, x4) \\
= \quad & \text{by Axiom 18 LR with } \{x5 \leftarrow i(x2), x4 \leftarrow i(x2), x3 \leftarrow x1, x2 \leftarrow i(x3), x1 \leftarrow i(mult(i(x3), i(i(x2))))\} \\
& op_r(x1, \overbrace{op_l(op_r(\underbrace{i(mult(i(x3), i(i(x2))))}, i(x2), i(x2)), i(x3), x1)}, x4) \\
= \quad & \text{by Lemma 17 LR with } \{x2 \leftarrow i(x2), x1 \leftarrow i(x3)\} \\
& op_r(x1, op_l(\underbrace{op_r(\overbrace{mult(i(i(x3)), i(x2))}, i(x2), i(x2))}, i(x3), x1), x4) \\
= \quad & \text{by Lemma 359 LR with } \{x2 \leftarrow i(x2), x1 \leftarrow i(i(x3))\} \\
& op_r(x1, op_l(\overbrace{rd(i(x2), \underbrace{i(i(i(x3)))})}, i(x3), x1), x4) \\
= \quad & \text{by Lemma 3 LR with } \{x1 \leftarrow i(x3)\} \\
& op_r(x1, op_l(\underbrace{rd(i(x2), \overbrace{i(x3)})}, i(x3), x1), x4) \\
= \quad & \text{by Lemma 23 LR with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& op_r(x1, op_l(\overbrace{rd(x3, x2)}, i(x3), x1), x4)
\end{aligned}
$$

Lemma 2796: $op_r(x1, rd(x2, rd(x1, i(x3))), i(x4)) = op_r(x1, op_l(rd(x3, x2), i(x3), x1), x4)$

$$
\begin{aligned}
& \overbrace{op_r(x1, rd(x2, rd(x1, i(x3))), i(x4))} \\
= \quad & \text{by Lemma 2773 LR with } \{x4 \leftarrow i(x4), x3 \leftarrow rd(x1, i(x3)), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, mult(x2, \underbrace{i(rd(x1, i(x3)))}), i(x4))} \\
= \quad & \text{by Lemma 10 LR with } \{x2 \leftarrow i(x3), x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, mult(x2, \overbrace{rd(i(x3), x1)}), i(x4))} \\
= \quad & \text{by Lemma 2775 RL with } \{x4 \leftarrow x4, x3 \leftarrow rd(i(x3), x1), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_r(x1, rd(i(x2), rd(i(x3), x1)), x4)}} \\
= \quad & \text{by Lemma 2795 LR with } \{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, op_l(rd(x3, x2), i(x3), x1), x4)}
\end{aligned}
$$

Lemma 2797: $op_r(x1, rd(x2, mult(x3, x1)), i(x4)) = op_r(x1, op_l(rd(x3, x2), i(x3), x1), x4)$

$$
\begin{aligned}
& \underbrace{op_r(x1, rd(x2, mult(x3, x1)), i(x4))} \\
= & \quad \text{by Lemma 2180 RL with } \{x4 \leftarrow i(x4), x3 \leftarrow mult(x3, x1), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, rd(x2, \underbrace{op_r(mult(x3, x1), x1, x1)}), i(x4))} \\
= & \quad \text{by Lemma 359 LR with } \{x2 \leftarrow x1, x1 \leftarrow x3\} \\
& \underbrace{op_r(x1, rd(x2, \overbrace{rd(x1, i(x3))}), i(x4))} \\
= & \quad \text{by Lemma 2796 LR with } \{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, op_l(rd(x3, x2), i(x3), x1), x4)}
\end{aligned}
$$

Lemma 2798: $op_r(x1, rd(x3, mult(x1, x2)), x4) = op_r(x1, op_l(rd(x3, x2), i(x3), x1), x4)$

$$
\begin{aligned}
& \underbrace{op_r(x1, rd(x3, mult(x1, x2)), x4)} \\
= & \quad \text{by Lemma 2533 RL with } \{x4 \leftarrow x4, x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_r(x1, rd(mult(x1, x3), x2), x4)}} \\
= & \quad \text{by Lemma 2249 RL with } \{x4 \leftarrow x4, x2 \leftarrow x2, x3 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_r(x1, rd(mult(x3, x1), x2), x4)}} \\
= & \quad \text{by Lemma 2436 RL with } \{x4 \leftarrow x4, x3 \leftarrow mult(x3, x1), x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{\underbrace{op_r(x1, rd(x2, mult(x3, x1)), i(x4))}} \\
= & \quad \text{by Lemma 2797 LR with } \{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, op_l(rd(x3, x2), i(x3), x1), x4)}
\end{aligned}
$$

Lemma 2799: $op_r(x1, rd(mult(x2, x3), mult(x1, op_t(x2, x3))), x4) = op_r(x1, op_l(x3, x1, mult(x2, x3)), x4)$

$$
\begin{aligned}
& \underbrace{op_r(x1, rd(mult(x2, x3), mult(x1, op_t(x2, x3))), x4)} \\
= & \quad \text{by Lemma 2798 LR with } \{x4 \leftarrow x4, x2 \leftarrow op_t(x2, x3), x3 \leftarrow mult(x2, x3), x1 \leftarrow x1\} \\
& \overbrace{op_r(x1, \underbrace{op_l(rd(mult(x2, x3), op_t(x2, x3)), i(mult(x2, x3)), x1)}, x4)} \\
= & \quad \text{by Lemma 2031 LR with } \{x3 \leftarrow x1, x2 \leftarrow mult(x2, x3), x1 \leftarrow rd(mult(x2, x3), op_t(x2, x3))\} \\
& op_r(x1, \overbrace{op_l(\underbrace{rd(mult(x2, x3), op_t(x2, x3))}, x1, mult(x2, x3))}, x4) \\
= & \quad \text{by Lemma 29 LR with } \{x2 \leftarrow x3, x1 \leftarrow x2\} \\
& op_r(x1, op_l(\overbrace{x3}, x1, mult(x2, x3)), x4)
\end{aligned}
$$

Lemma 2800: $op_r(x1, op_r(x2, x1, x3), x4) = op_r(x1, op_l(x2, x1, x3), x4)$

$\underbrace{op_r(x1, op_r(x2, x1, x3), x4)}$

$=$ by Lemma 2695 LR with $\{x5 \leftarrow x4, x4 \leftarrow x3, x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_r(x1, op_r(x2, x1, \underbrace{rd(x3, x2)}), x4)}$

$=$ by Lemma 3 RL with $\{x1 \leftarrow rd(x3, x2)\}$

$op_r(x1, \underbrace{op_r(x2, x1, \overbrace{i(i(rd(x3, x2)))}), x4)}$

$=$ by Lemma 2576 RL with $\{x3 \leftarrow rd(x3, x2), x1 \leftarrow x1, x2 \leftarrow x2\}$

$\underbrace{op_r(x1, \overbrace{mult(op_r(rd(x2, x1), x2, rd(x3, x2)), x1)}, x4)}$

$=$ by Lemma 2430 LR with $\{x3 \leftarrow x4, x2 \leftarrow op_r(rd(x2, x1), x2, rd(x3, x2)), x1 \leftarrow x1\}$

$\overbrace{op_r(x1, op_r(\underbrace{rd(x2, x1)}, x2, rd(x3, x2)), x4)}$

$=$ by Lemma 10 RL with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$op_r(x1, \underbrace{op_r(\overbrace{i(rd(x1, x2))}, x2, rd(x3, x2))}, x4)$

$=$ by Lemma 2428 LR with $\{x3 \leftarrow rd(x3, x2), x2 \leftarrow x2, x1 \leftarrow rd(x1, x2)\}$

$op_r(x1, \overbrace{i(op_r(rd(\underbrace{x1}, x2), x2, rd(x3, x2)))}, x4)$

$=$ by Axiom 5 LR with $\{x2 \leftarrow op_t(rd(x3, x2), x2), x1 \leftarrow x1\}$

$op_r(x1, i(\underbrace{op_r(rd(\overbrace{rd(mult(x1, op_t(rd(x3, x2), x2)), op_t(rd(x3, x2), x2))}), x2), x2, rd(x3, x2))}), x4)$

$=$ by Lemma 1965 LR with $\{x3 \leftarrow x2, x2 \leftarrow rd(x3, x2), x1 \leftarrow mult(x1, op_t(rd(x3, x2), x2))\}$

$op_r(x1, \underbrace{i(\overbrace{rd(mult(x1, op_t(rd(x3, x2), x2)), mult(rd(x3, x2), x2))}), x4)}$

$=$ by Lemma 10 LR with $\{x2 \leftarrow mult(rd(x3, x2), x2), x1 \leftarrow mult(x1, op_t(rd(x3, x2), x2))\}$

$\underbrace{op_r(x1, \overbrace{rd(mult(rd(x3, x2), x2), mult(x1, op_t(rd(x3, x2), x2)))}, x4)}$

$=$ by Lemma 2799 LR with $\{x4 \leftarrow x4, x3 \leftarrow x2, x2 \leftarrow rd(x3, x2), x1 \leftarrow x1\}$

$\overbrace{op_r(x1, op_l(x2, x1, \underbrace{mult(rd(x3, x2), x2)}), x4)}$

$=$ by Axiom 4 RL with $\{x2 \leftarrow x2, x1 \leftarrow x3\}$

$op_r(x1, op_l(x2, x1, \overbrace{x3}), x4)$

Lemma 2801: $asoc(x1, op_r(x2, x1, x3), x4) = asoc(x1, op_l(x2, x1, x3), x4)$

$\underbrace{asoc(x1, op_r(x2, x1, x3), x4)}$

$=$ by Lemma 2711 RL with $\{x3 \leftarrow x4, x2 \leftarrow op_r(x2, x1, x3), x1 \leftarrow x1\}$

$\overbrace{rd(\underbrace{op_r(x1, op_r(x2, x1, x3), x4)}, x1)}$

$=$ by Lemma 2800 LR with $\{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{rd(\overbrace{op_r(x1, op_l(x2, x1, x3), x4)}, x1)}$

$=$ by Lemma 2711 LR with $\{x3 \leftarrow x4, x2 \leftarrow op_l(x2, x1, x3), x1 \leftarrow x1\}$

$\overbrace{asoc(x1, op_l(x2, x1, x3), x4)}$

Lemma 2802: $asoc(x1, op_l(rd(x1, x2), x1, x3), x4) = asoc(op_r(x2, x1, x3), x1, x4)$

$\underbrace{asoc(x1, op_l(rd(x1, x2), x1, x3), x4)}$

$=$ by Lemma 2801 RL with $\{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow rd(x1, x2), x1 \leftarrow x1\}$

$\overbrace{\underbrace{asoc(x1, op_r(rd(x1, x2), x1, x3), x4)}}$

$=$ by Lemma 2721 RL with $\{x3 \leftarrow x1, x2 \leftarrow op_r(rd(x1, x2), x1, x3), x1 \leftarrow x4\}$

$\overbrace{rd(x4, \underbrace{op_l(x4, op_r(rd(x1, x2), x1, x3), x1)})}$

$=$ by Lemma 2030 RL with $\{x3 \leftarrow op_r(rd(x1, x2), x1, x3), x2 \leftarrow x1, x1 \leftarrow x4\}$

$rd(x4, \overbrace{\underbrace{op_l(x4, x1, i(op_r(rd(x1, x2), x1, x3)))}})$

$=$ by Lemma 1319 RL with $\{x3 \leftarrow x1, x2 \leftarrow op_r(rd(x1, x2), x1, x3), x1 \leftarrow x4\}$

$rd(x4, \overbrace{\underbrace{op_l(x4, \underbrace{op_t(op_r(rd(x1, x2), x1, x3), x1)}, x1)}})$

$=$ by Axiom 14 LR with $\{x4 \leftarrow x1, x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow rd(x1, x2)\}$

$rd(x4, \underbrace{op_l(x4, \overbrace{op_r(op_t(rd(x1, x2), x1), x1, x3)}, x1)})$

$=$ by Lemma 2697 RL with $\{x4 \leftarrow x3, x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x4\}$

$rd(x4, \overbrace{\underbrace{op_l(x4, \underbrace{op_r(op_t(i(x2), x1), x1, x3)}, x1)}})$

$=$ by Axiom 14 RL with $\{x4 \leftarrow x1, x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow i(x2)\}$

$rd(x4, \underbrace{op_l(x4, \overbrace{op_t(op_r(i(x2), x1, x3), x1)}, x1)})$

$=$ by Lemma 1319 LR with $\{x3 \leftarrow x1, x2 \leftarrow op_r(i(x2), x1, x3), x1 \leftarrow x4\}$

$rd(x4, \overbrace{\underbrace{op_l(x4, x1, i(op_r(i(x2), x1, x3)))}})$

$=$ by Lemma 2030 LR with $\{x3 \leftarrow op_r(i(x2), x1, x3), x2 \leftarrow x1, x1 \leftarrow x4\}$

$rd(x4, \overbrace{\underbrace{op_l(x4, op_r(i(x2), x1, x3), x1)}})$

$=$ by Lemma 2506 LR with $\{x5 \leftarrow x1, x4 \leftarrow x3, x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x4\}$

$\underbrace{rd(x4, \overbrace{op_l(x4, x1, op_r(x2, x1, x3))})}$

$=$ by Lemma 2721 LR with $\{x3 \leftarrow op_r(x2, x1, x3), x2 \leftarrow x1, x1 \leftarrow x4\}$

$\overbrace{asoc(op_r(x2, x1, x3), x1, x4)}$

Lemma 2803: $asoc(op_l(x2, x1, x3), x1, x4) = asoc(op_r(x2, x1, x3), x1, x4)$

$\underbrace{asoc(op_l(x2, x1, x3), x1, x4)}$

$=$ by Lemma 2721 RL with $\{x3 \leftarrow op_l(x2, x1, x3), x2 \leftarrow x1, x1 \leftarrow x4\}$

$\overbrace{\underbrace{rd(x4, op_l(x4, x1, op_l(x2, x1, x3)))}}$

$=$ by Lemma 2783 RL with $\{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{\underbrace{asoc(x1, op_l(rd(x1, x2), x1, x3), x4)}}$

$=$ by Lemma 2802 LR with $\{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{asoc(op_r(x2, x1, x3), x1, x4)}$

Lemma 2804: $mult(mult(x1, mult(i(x2), mult(mult(x2, i(x1)), x3))), x4) = mult(x1, mult(i(x2), mult(mult(x2, i(x1)), mult(x3,$

$mult(mult(x1, mult(i(x2), \underbrace{mult(mult(x2, i(x1)), x3)}))), x4)$

$=$ by Axiom 4 LR with $\{x2 \leftarrow mult(i(x2), x1), x1 \leftarrow mult(mult(x2, i(x1)), x3)\}$

$mult(mult(x1, \underbrace{mult(i(x2), \overbrace{mult(rd(mult(mult(x2, i(x1)), x3), mult(i(x2), x1)), mult(i(x2), x1))})}), x4)$

$=$ by Axiom 6 LR with $\{x3 \leftarrow x1, x2 \leftarrow rd(mult(mult(x2, i(x1)), x3), mult(i(x2), x1)), x1 \leftarrow i(x2)\}$

$\underbrace{mult(mult(x1, \overbrace{mult(mult(i(x2), mult(rd(\underbrace{mult(mult(x2, i(x1)), x3)}, mult(i(x2), x1)), i(x2))), x1)}), x4)}$

$=$ by Axiom 6 RL with $\{x3 \leftarrow x4, x2 \leftarrow mult(i(x2), mult(rd(mult(mult(x2, i(x1)), x3), mult(i(x2), x1)), i(x2))), x1 \leftarrow x1\}$

$\overbrace{mult(x1, \underbrace{mult(mult(i(x2), mult(rd(\underbrace{mult(mult(x2, i(x1)), x3)}, mult(i(x2), x1)), i(x2))), mult(x1, x4))})}$

$=$ by Axiom 6 RL with $\{x3 \leftarrow mult(x1, x4), x2 \leftarrow rd(mult(mult(x2, i(x1)), x3), mult(i(x2), x1)), x1 \leftarrow i(x2)\}$

$mult(x1, \overbrace{mult(i(x2), mult(\underbrace{rd(mult(mult(x2, i(x1)), x3), mult(i(x2), x1))}, mult(i(x2), mult(x1, x4))))})$

$=$ by Lemma 61 RL with $\{x3 \leftarrow mult(mult(x2, i(x1)), x3), x2 \leftarrow x1, x1 \leftarrow x2\}$

$mult(x1, mult(i(x2), mult(\overbrace{rd(mult(x2, i(x1)), \underbrace{i(mult(mult(x2, i(x1)), x3))})}, mult(i(x2), mult(x1, x4)))))$

$=$ by Lemma 58 LR with $\{x2 \leftarrow x3, x1 \leftarrow mult(x2, i(x1))\}$

$mult(x1, mult(i(x2), mult(\overbrace{mult(mult(x2, i(x1)), mult(x3, mult(x2, i(x1))))}, mult(i(x2), mult(x1, x4)))))$

$=$ by Lemma 982 RL with $\{x3 \leftarrow mult(x3, mult(x2, i(x1))), x2 \leftarrow x1, x1 \leftarrow x2\}$

$mult(x1, mult(i(x2), mult(\overbrace{mult(x2, mult(rd(i(x2), x1), mult(x2, mult(x3, mult(x2, \underbrace{i(x1)}))))))}, mult(i(x2), mult(x1, x4)))))$

$=$ by Lemma 21 RL with $\{x2 \leftarrow x1, x1 \leftarrow x2\}$

$mult(x1, mult(i(x2), mult(\underbrace{mult(x2, mult(rd(i(x2), x1), mult(x2, mult(x3, mult(x2, \overbrace{mult(rd(i(x2), x1), x2)}))))))}, mult(i(x2$

$=$ by Axiom 6 LR with $\{x3 \leftarrow mult(x3, mult(x2, mult(rd(i(x2), x1), x2))), x2 \leftarrow rd(i(x2), x1), x1 \leftarrow x2\}$

$mult(x1, mult(i(x2), \underbrace{mult(\overbrace{mult(mult(x2, mult(rd(i(x2), x1), x2)), mult(x3, mult(x2, mult(rd(i(x2), x1), x2))))}, mult(i(x2$

$=$ by Axiom 6 RL with $\{x3 \leftarrow mult(i(x2), mult(x1, x4)), x2 \leftarrow x3, x1 \leftarrow mult(x2, mult(rd(i(x2), x1), x2))\}$

$mult(x1, mult(i(x2), \overbrace{\underbrace{mult(mult(x2, mult(rd(i(x2), x1), x2)), mult(x3, mult(\underbrace{mult(x2, mult(rd(i(x2), x1), x2)}, mult(i(x2),$

$=$ by Axiom 6 RL with $\{x3 \leftarrow mult(x3, mult(mult(x2, mult(rd(i(x2), x1), x2)), mult(i(x2), mult(x1, x4)))), x2 \leftarrow rd(i(x2), x1)$

$mult(x1, mult(i(x2), \overbrace{mult(x2, mult(rd(i(x2), x1), mult(x2, mult(x3, \underbrace{mult(mult(x2, mult(rd(i(x2), x1), x2)), mult(i(x2), m$

$=$ by Axiom 6 RL with $\{x3 \leftarrow mult(i(x2), mult(x1, x4)), x2 \leftarrow rd(i(x2), x1), x1 \leftarrow x2\}$

$mult(x1, mult(i(x2), \underbrace{mult(x2, mult(rd(i(x2), x1), mult(x2, mult(x3, \overbrace{mult(x2, mult(rd(i(x2), x1), mult(x2, mult(i(x2), mu$

$=$ by Lemma 982 LR with $\{x3 \leftarrow mult(x3, mult(x2, mult(rd(i(x2), x1), mult(x2, mult(i(x2), mult(x1, x4))))))), x2 \leftarrow x1, x1 \leftarrow$

$mult(x1, mult(i(x2), \overbrace{mult(mult(x2, i(x1)), mult(x3, \underbrace{mult(x2, mult(rd(i(x2), x1), mult(x2, mult(i(x2), mult(x1, x4)))))))}))$

$=$ by Lemma 982 LR with $\{x3 \leftarrow mult(i(x2), mult(x1, x4)), x2 \leftarrow x1, x1 \leftarrow x2\}$

$mult(x1, mult(i(x2), mult(mult(x2, i(x1)), mult(x3, \overbrace{mult(mult(x2, i(x1)), mult(i(x2), mult(x1, x4)))})))$

Lemma 2805: $mult(op_l(x3, x1, x2), x4) = mult(x1, mult(i(x2), mult(mult(x2, i(x1)), mult(x3, mult(mult(x2, i(x1)), mult(i(x2),$

$mult(\underbrace{op_l(x3, x1, x2)}, x4)$

$=$ by Lemma 2054 RL with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$mult(\overbrace{mult(x1, \underbrace{mult(rd(i(x1), x2), mult(x2, x3))})}, x4)$

$=$ by Lemma 1045 RL with $\{x3 \leftarrow x3, x2 \leftarrow i(x1), x1 \leftarrow x2\}$

$\underbrace{mult(mult(x1, \overbrace{mult(i(x2), mult(mult(x2, i(x1)), x3))}), x4)}$

$=$ by Lemma 2804 LR with $\{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{mult(x1, mult(i(x2), mult(mult(x2, i(x1)), \underbrace{mult(x3, mult(mult(x2, i(x1)), mult(i(x2), mult(x1, x4))))})))}$

Lemma 2806: $mult(op_l(x3, x1, x2), x4) = op_l(mult(x3, mult(mult(x2, i(x1)), mult(i(x2), mult(x1, x4)))), x1, x2)$

$\underbrace{mult(op_l(x3, x1, x2), x4)}$

$=$ by Lemma 2805 LR with $\{x4 \leftarrow x4, x2 \leftarrow x2, x1 \leftarrow x1, x3 \leftarrow x3\}$

$\overbrace{mult(x1, \underbrace{mult(i(x2), mult(mult(x2, i(x1)), mult(x3, mult(mult(x2, i(x1)), mult(i(x2), mult(x1, x4)))))})}$

$=$ by Lemma 1045 LR with $\{x3 \leftarrow mult(x3, mult(mult(x2, i(x1)), mult(i(x2), mult(x1, x4)))), x2 \leftarrow i(x1), x1 \leftarrow x2\}$

$\underbrace{mult(x1, \overbrace{mult(rd(i(x1), x2), mult(x2, mult(x3, mult(mult(x2, i(x1)), mult(i(x2), mult(x1, x4))))))})$

$=$ by Lemma 2054 LR with $\{x3 \leftarrow mult(x3, mult(mult(x2, i(x1)), mult(i(x2), mult(x1, x4)))), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_l(mult(x3, \underbrace{mult(mult(x2, i(x1)), mult(i(x2), mult(x1, x4)))}), x1, x2)}$

Lemma 2807: $mult(op_l(x3, x1, x2), x4) = op_l(mult(x3, op_l(x4, x2, x1)), x1, x2)$

$\underbrace{mult(op_l(x3, x1, x2), x4)}$

$=$ by Lemma 2806 LR with $\{x4 \leftarrow x4, x2 \leftarrow x2, x1 \leftarrow x1, x3 \leftarrow x3\}$

$\overbrace{op_l(mult(x3, \underbrace{mult(mult(x2, i(x1)), mult(i(x2), mult(x1, x4)))}), x1, x2)}$

$=$ by Lemma 1086 LR with $\{x3 \leftarrow mult(x1, x4), x2 \leftarrow i(x1), x1 \leftarrow x2\}$

$op_l(mult(x3, \overbrace{\underbrace{mult(x2, mult(rd(i(x1), x2), mult(x1, x4)))}}), x1, x2)$

$=$ by Lemma 987 LR with $\{x3 \leftarrow x4, x2 \leftarrow x1, x1 \leftarrow x2\}$

$op_l(mult(x3, \overbrace{op_l(x4, x1, i(x2))}), x1, x2)$

$=$ by Lemma 2030 LR with $\{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x4\}$

$op_l(mult(x3, \overbrace{op_l(x4, x2, x1)}), x1, x2)$

Lemma 2808: $mult(op_l(x1, x2, x3), op_l(x4, x2, x3)) = op_l(mult(x1, x4), x2, x3)$

$\underbrace{mult(op_l(x1, x2, x3), op_l(x4, x2, x3))}$

$=$ by Lemma 2807 LR with $\{x4 \leftarrow op_l(x4, x2, x3), x2 \leftarrow x3, x1 \leftarrow x2, x3 \leftarrow x1\}$

$\overbrace{op_l(mult(x1, \underbrace{op_l(op_l(x4, x2, x3), x3, x2)}), x2, x3)}$

$=$ by Lemma 138 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x4\}$

$op_l(mult(x1, \overbrace{x4}), x2, x3)$

Lemma 2809: $rd(op_l(x1,x2,x3),op_l(x4,x2,x3)) = op_l(rd(x1,x4),x2,x3)$

$$
\begin{aligned}
& rd(op_l(x1,x2,x3),op_l(\underbrace{x4}_{},x2,x3)) \\
= & \quad \text{by Lemma 3 RL with } \{x1 \leftarrow x4\} \\
& rd(op_l(x1,x2,x3),\underbrace{op_l(\overbrace{i(i(x4))}^{},x2,x3)}) \\
= & \quad \text{by Lemma 1049 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow i(x4)\} \\
& \underbrace{rd(op_l(x1,x2,x3),\overbrace{i(op_l(i(x4),x2,x3))}^{})}_{} \\
= & \quad \text{by Lemma 357 RL with } \{x2 \leftarrow op_l(x1,x2,x3), x1 \leftarrow op_l(i(x4),x2,x3)\} \\
& \overbrace{mult(\underbrace{op_r(op_l(i(x4),x2,x3),op_l(x1,x2,x3),op_l(x1,x2,x3))}_{},op_l(x1,x2,x3))}^{} \\
= & \quad \text{by Axiom 18 LR with } \{x5 \leftarrow op_l(x1,x2,x3), x4 \leftarrow op_l(x1,x2,x3), x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow i(x4)\} \\
& \underbrace{mult(\overbrace{op_l(op_r(i(x4),op_l(x1,x2,x3),op_l(x1,x2,x3)),x2,x3)}^{},op_l(x1,x2,x3))}_{} \\
= & \quad \text{by Lemma 2417 LR with } \{x4 \leftarrow x3, x3 \leftarrow x2, x1 \leftarrow op_l(x1,x2,x3), x2 \leftarrow x4\} \\
& \overbrace{\underbrace{mult(op_l(x1,x2,x3),op_l(op_l(i(x4),x4,op_l(x1,x2,x3)),x2,x3))}_{}}^{} \\
= & \quad \text{by Lemma 2808 LR with } \{x4 \leftarrow op_l(i(x4),x4,op_l(x1,x2,x3)), x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{op_l(mult(x1,\underbrace{op_l(i(x4),x4,op_l(x1,x2,x3))}_{}),x2,x3)}^{} \\
= & \quad \text{by Lemma 2637 LR with } \{x4 \leftarrow x3, x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x4\} \\
& op_l(mult(x1,\overbrace{rd(asoc(x1,x4,x4),x4)}^{}),x2,x3) \\
= & \quad \text{by Lemma 776 RL with } \{x2 \leftarrow x1, x1 \leftarrow x4\} \\
& op_l(\underbrace{mult(x1,\overbrace{op_l(i(x4),x4,x1)}^{})}_{},x2,x3) \\
= & \quad \text{by Lemma 232 LR with } \{x2 \leftarrow x4, x1 \leftarrow x1\} \\
& op_l(\overbrace{rd(x1,x4)}^{},x2,x3)
\end{aligned}
$$

Lemma 2810: $op_l(asoc(x1,x2,x3),x4,x5) = asoc(x1,op_l(x2,x4,x5),x3)$

$$
\begin{aligned}
& op_l(\underbrace{asoc(x1,x2,x3)}_{},x4,x5) \\
= & \quad \text{by Lemma 2780 RL with } \{x4 \leftarrow x3, x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x6\} \\
& \underbrace{op_l(\overbrace{mult(rd(x6,op_r(x2,x1,x3)),rd(x2,x6))}^{},x4,x5)}_{} \\
= & \quad \text{by Lemma 2808 RL with } \{x4 \leftarrow rd(x2,x6), x3 \leftarrow x5, x2 \leftarrow x4, x1 \leftarrow rd(x6,op_r(x2,x1,x3))\} \\
& \overbrace{mult(\underbrace{op_l(rd(x6,op_r(x2,x1,x3)),x4,x5)}_{},op_l(rd(x2,x6),x4,x5))}^{} \\
= & \quad \text{by Lemma 2809 RL with } \{x4 \leftarrow op_r(x2,x1,x3), x3 \leftarrow x5, x2 \leftarrow x4, x1 \leftarrow x6\} \\
& mult(\overbrace{rd(op_l(x6,x4,x5),\underbrace{op_l(op_r(x2,x1,x3),x4,x5)}_{})}^{},op_l(rd(x2,x6),x4,x5)) \\
= & \quad \text{by Axiom 18 RL with } \{x5 \leftarrow x3, x4 \leftarrow x1, x3 \leftarrow x5, x2 \leftarrow x4, x1 \leftarrow x2\} \\
& mult(rd(op_l(x6,x4,x5),\overbrace{op_r(op_l(x2,x4,x5),x1,x3)}^{}),\underbrace{op_l(rd(x2,x6),x4,x5)}_{}) \\
= & \quad \text{by Lemma 2809 RL with } \{x4 \leftarrow x6, x3 \leftarrow x5, x2 \leftarrow x4, x1 \leftarrow x2\} \\
& \underbrace{mult(rd(op_l(x6,x4,x5),op_r(op_l(x2,x4,x5),x1,x3)),\overbrace{rd(op_l(x2,x4,x5),op_l(x6,x4,x5))}^{})}_{} \\
= & \quad \text{by Lemma 2780 LR with } \{x4 \leftarrow x3, x3 \leftarrow x1, x2 \leftarrow op_l(x2,x4,x5), x1 \leftarrow op_l(x6,x4,x5)\} \\
& \overbrace{asoc(x1,op_l(x2,x4,x5),x3)}^{}
\end{aligned}
$$

Lemma 2811: $asoc(x1, op_r(x2, x1, x3), x4) = asoc(x1, x2, x4)$

$\underbrace{asoc(x1, op_r(x2, x1, x3), x4)}$

$=$ by Lemma 2801 LR with $\{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{\underbrace{asoc(x1, op_l(x2, x1, x3), x4)}}$

$=$ by Lemma 2810 RL with $\{x5 \leftarrow x3, x4 \leftarrow x1, x3 \leftarrow x4, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_l(\underbrace{asoc(x1, x2, x4)}, x1, x3)}$

$=$ by Lemma 2711 RL with $\{x3 \leftarrow x4, x2 \leftarrow x2, x1 \leftarrow x1\}$

$op_l(\overbrace{rd(op_r(x1, x2, x4), \underbrace{x1}\,)}, x1, x3)$

$=$ by Axiom 1 LR with $\{x1 \leftarrow x1\}$

$\underbrace{op_l(rd(op_r(x1, x2, x4), \overbrace{mult(x1, unit())}), x1, x3)}$

$=$ by Lemma 2068 LR with $\{x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow op_r(x1, x2, x4)\}$

$\overbrace{rd(\underbrace{op_l(op_r(x1, x2, x4), mult(x1, i(x3)), op_l(x3, x1, x3))}, op_l(x1, x1, x3))}$

$=$ by Lemma 2053 LR with $\{x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow op_r(x1, x2, x4)\}$

$rd(\overbrace{\underbrace{op_l(op_r(x1, x2, x4), x1, x3)}}, op_l(x1, x1, x3))$

$=$ by Lemma 739 RL with $\{x4 \leftarrow x3, x3 \leftarrow x4, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{rd(\overbrace{op_r(op_r(x1, x2, x4), x3, x1)}, op_l(x1, x1, x3))}$

$=$ by Lemma 2623 RL with $\{x4 \leftarrow x3, x3 \leftarrow x4, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{\underbrace{rd(op_r(x1, x2, x4), x1)}}$

$=$ by Lemma 2711 RL with $\{x3 \leftarrow x4, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{asoc(x1, x2, x4)}$

Lemma 2812: $asoc(x1, x2, mult(x4, op_r(x2, x1, x3))) = rd(asoc(x1, x2, x4), asoc(op_r(x2, x1, x3), x1, op_r(x2, x1, x3)))$

$\underbrace{asoc(x1, x2, mult(x4, op_r(x2, x1, x3)))}$

$=$ by Lemma 2811 RL with $\{x4 \leftarrow mult(x4, op_r(x2, x1, x3)), x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{\underbrace{asoc(x1, op_r(x2, x1, x3), mult(x4, op_r(x2, x1, x3)))}}$

$=$ by Lemma 2777 RL with $\{x3 \leftarrow x4, x2 \leftarrow op_r(x2, x1, x3), x1 \leftarrow x1\}$

$\overbrace{rd(\underbrace{asoc(x1, op_r(x2, x1, x3), x4)}, asoc(op_r(x2, x1, x3), x1, op_r(x2, x1, x3)))}$

$=$ by Lemma 2811 LR with $\{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$rd(\overbrace{asoc(x1, x2, x4)}, asoc(op_r(x2, x1, x3), x1, op_r(x2, x1, x3)))$

Lemma 2813: $asoc(x1, x2, mult(i(mult(x3, op_r(x2, x1, x4))), x2)) = asoc(x1, x2, op_r(i(x3), op_r(x2, x1, x4), i(op_r(x2, x1, x4))))$

$\underbrace{asoc(x1, x2, mult(i(mult(x3, op_r(x2, x1, x4))), x2))}$

$=$ by Lemma 2777 RL with $\{x3 \leftarrow i(mult(x3, op_r(x2, x1, x4))), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{rd(asoc(x1, x2, i(mult(x3, op_r(x2, x1, x4)))), \underbrace{asoc(x2, x1, x2)})}$

$=$ by Lemma 2629 RL with $\{x1 \leftarrow x1, x4 \leftarrow x4, x3 \leftarrow x1, x2 \leftarrow x2\}$

$\underbrace{rd(asoc(x1, x2, i(mult(x3, op_r(x2, x1, x4)))), \overbrace{asoc(op_r(x2, x1, x4), x1, op_r(x2, x1, x4))})}$

$=$ by Lemma 2812 RL with $\{x3 \leftarrow x4, x4 \leftarrow i(mult(x3, op_r(x2, x1, x4))), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{asoc(x1, x2, \underbrace{mult(i(mult(x3, op_r(x2, x1, x4))), op_r(x2, x1, x4))})}$

$=$ by Lemma 325 LR with $\{x2 \leftarrow op_r(x2, x1, x4), x1 \leftarrow x3\}$

$asoc(x1, x2, \overbrace{op_r(i(x3), op_r(x2, x1, x4), i(op_r(x2, x1, x4)))})$

Lemma 2814: $asoc(x1, x2, rd(x2, mult(x3, op_r(x2, x1, x4)))) = asoc(x1, x2, op_r(i(x3), op_r(x2, x1, x4), i(op_r(x2, x1, x4))))$

$\underbrace{asoc(x1, x2, rd(x2, mult(x3, op_r(x2, x1, x4))))}$

$=$ by Lemma 2711 RL with $\{x3 \leftarrow rd(x2, mult(x3, op_r(x2, x1, x4))), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{rd(\underbrace{op_r(x1, x2, rd(x2, mult(x3, op_r(x2, x1, x4))))}, x1)}$

$=$ by Lemma 2397 RL with $\{x4 \leftarrow x2, x3 \leftarrow mult(x3, op_r(x2, x1, x4)), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{rd(\overbrace{op_r(x1, x2, mult(i(mult(x3, op_r(x2, x1, x4))), x2))}, x1)}$

$=$ by Lemma 2711 LR with $\{x3 \leftarrow mult(i(mult(x3, op_r(x2, x1, x4))), x2), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{\underbrace{asoc(x1, x2, mult(i(mult(x3, op_r(x2, x1, x4))), x2))}}$

$=$ by Lemma 2813 LR with $\{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{asoc(x1, x2, op_r(i(x3), op_r(x2, x1, x4), i(op_r(x2, x1, x4))))}$

Lemma 2815: $asoc(x1, x2, mult(x3, x4)) = asoc(x1, x2, mult(op_l(x3, x1, x2), x4))$

$\underbrace{asoc(x1, x2, mult(x3, x4))}$

$=$ by Lemma 2732 RL with $\{x4 \leftarrow op_r(x1, x2, x3), x3 \leftarrow mult(x3, x4), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{asoc(x1, x2, op_r(mult(x3, x4), op_r(x1, x2, x3), op_r(x1, x2, x3)))}$

$=$ by Lemma 2782 RL with $\{x5 \leftarrow op_r(x1, x2, x3), x4 \leftarrow op_r(x1, x2, x3), x3 \leftarrow mult(x3, x4), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{asoc(x1, x2, op_r(mult(x3, x4), op_r(x1, x2, x3), mult(op_r(x1, x2, x3), op_r(x1, x2, x3))))}$

$=$ by Lemma 2793 RL with $\{x5 \leftarrow op_r(x1, x2, x3), x4 \leftarrow op_r(x1, x2, x3), x3 \leftarrow mult(x3, x4), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{asoc(x1, x2, op_r(mult(x3, x4), op_r(x1, x2, x3), i(op_r(x1, x2, x3))))}$

$=$ by Lemma 2719 RL with $\{x3 \leftarrow op_r(mult(x3, x4), op_r(x1, x2, x3), i(op_r(x1, x2, x3))), x2 \leftarrow x1, x1 \leftarrow x2\}$

$\overbrace{asoc(x2, x1, \underbrace{i(op_r(mult(x3, x4), op_r(x1, x2, x3), i(op_r(x1, x2, x3))))}}$

$=$ by Lemma 2428 RL with $\{x3 \leftarrow i(op_r(x1, x2, x3)), x2 \leftarrow op_r(x1, x2, x3), x1 \leftarrow mult(x3, x4)\}$

$\underbrace{asoc(x2, x1, \overbrace{op_r(i(mult(x3, x4)), op_r(x1, x2, x3), i(op_r(x1, x2, x3)))})}$

$=$ by Lemma 2814 RL with $\{x4 \leftarrow x3, x3 \leftarrow mult(x3, x4), x2 \leftarrow x1, x1 \leftarrow x2\}$

$\overbrace{asoc(x2, x1, \underbrace{rd(x1, mult(mult(x3, x4), op_r(x1, x2, x3)))})}$

$=$ by Lemma 2190 LR with $\{x4 \leftarrow x3, x3 \leftarrow x2, x2 \leftarrow mult(x3, x4), x1 \leftarrow x1\}$

$asoc(x2, x1, \overbrace{rd(\underbrace{rd(x1, op_r(x1, x2, x3))}, mult(x3, x4))})$

$=$ by Lemma 2723 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$asoc(x2, x1, \underbrace{rd(\overbrace{asoc(x2, x1, x3)}, mult(x3, x4))})$

$=$ by Lemma 2664 LR with $\{x4 \leftarrow mult(x3, x4), x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow x2\}$

$asoc(x2, x1, \underbrace{\overbrace{mult(asoc(x2, x1, x3), i(mult(x3, x4)))}})$

$=$ by Lemma 2751 RL with $\{x4 \leftarrow x4, x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x3\}$

$\underbrace{asoc(x2, x1, \overbrace{i(mult(op_l(x3, x1, x2), x4))})}$

$=$ by Lemma 2719 LR with $\{x3 \leftarrow mult(op_l(x3, x1, x2), x4), x2 \leftarrow x1, x1 \leftarrow x2\}$

$\overbrace{asoc(x1, x2, mult(op_l(x3, x1, x2), x4))}$

Lemma 2816: $mult(op_l(i(x1), x4, x5), x1) = op_l(asoc(x5, x4, op_l(x1, x2, x3)), x3, x2)$

$mult(\underbrace{op_l(i(x1), x4, x5)}, x1)$

$=$ by Lemma 1615 RL with $\{x5 \leftarrow x5, x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow i(x1)\}$

$mult(\overbrace{op_l(op_l(\underbrace{op_l(i(x1), x2, x3)}, x4, x5), x3, x2)}, x1)$

$=$ by Lemma 1049 RL with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{mult(op_l(op_l(\overbrace{i(op_l(x1, x2, x3))}, x4, x5), x3, x2), x1)}$

$=$ by Lemma 2807 LR with $\{x4 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x3, x3 \leftarrow op_l(i(op_l(x1, x2, x3)), x4, x5)\}$

$\overbrace{op_l(\underbrace{mult(op_l(i(op_l(x1, x2, x3)), x4, x5), op_l(x1, x2, x3))}, x3, x2)}$

$=$ by Lemma 2730 LR with $\{x3 \leftarrow x5, x2 \leftarrow x4, x1 \leftarrow op_l(x1, x2, x3)\}$

$op_l(\overbrace{asoc(x5, x4, op_l(x1, x2, x3))}, x3, x2)$

Lemma 2817: $asoc(x5, x4, x1) = asoc(x5, op_l(x4, x3, x2), op_l(x1, x2, x3))$

$$
\begin{aligned}
& \overbrace{asoc(x5, x4, x1)} \\
= \quad & \text{by Lemma 2730 RL with } \{x3 \leftarrow x5, x2 \leftarrow x4, x1 \leftarrow x1\} \\
& \overbrace{mult(op_l(i(x1), x4, x5), x1)} \\
= \quad & \text{by Lemma 2816 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x5 \leftarrow x5, x4 \leftarrow x4, x1 \leftarrow x1\} \\
& \overbrace{op_l(asoc(x5, x4, op_l(x1, x2, x3)), x3, x2)} \\
= \quad & \text{by Lemma 2810 LR with } \{x5 \leftarrow x2, x4 \leftarrow x3, x3 \leftarrow op_l(x1, x2, x3), x2 \leftarrow x4, x1 \leftarrow x5\} \\
& \overbrace{asoc(x5, op_l(x4, x3, x2), op_l(x1, x2, x3))}
\end{aligned}
$$

Lemma 2818: $asoc(x1, op_l(x2, x3, x4), x5) = asoc(x1, x2, op_l(x5, x3, x4))$

$$
\begin{aligned}
& \overbrace{asoc(x1, op_l(x2, x3, x4), x5)} \\
= \quad & \text{by Lemma 2817 LR with } \{x2 \leftarrow x3, x3 \leftarrow x4, x1 \leftarrow x5, x4 \leftarrow op_l(x2, x3, x4), x5 \leftarrow x1\} \\
& \overbrace{asoc(x1, \underbrace{op_l(op_l(x2, x3, x4), x4, x3)}, op_l(x5, x3, x4))} \\
= \quad & \text{by Lemma 138 LR with } \{x3 \leftarrow x4, x2 \leftarrow x3, x1 \leftarrow x2\} \\
& asoc(x1, \overbrace{x2}, op_l(x5, x3, x4))
\end{aligned}
$$

Lemma 2819: $op_l(x1, x2, op_l(x5, x3, x4)) = rd(x1, asoc(x5, x2, op_l(x1, x3, x4)))$

$$
\begin{aligned}
& \overbrace{op_l(x1, x2, op_l(x5, x3, x4))} \\
= \quad & \text{by Lemma 2715 RL with } \{x3 \leftarrow x2, x2 \leftarrow op_l(x5, x3, x4), x1 \leftarrow x1\} \\
& \overbrace{rd(x1, \underbrace{asoc(op_l(x5, x3, x4), x2, x1)})} \\
= \quad & \text{by Lemma 2781 LR with } \{x4 \leftarrow op_l(x6, x3, x4), x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow op_l(x5, x3, x4)\} \\
& rd(x1, \overbrace{rd(\underbrace{rd(op_l(x6, x3, x4), op_l(x5, x3, x4))}, rd(op_l(x6, x3, x4), op_r(op_l(x5, x3, x4), x2, x1)))}) \\
= \quad & \text{by Lemma 2809 LR with } \{x4 \leftarrow x5, x3 \leftarrow x4, x2 \leftarrow x3, x1 \leftarrow x6\} \\
& rd(x1, rd(\overbrace{op_l(rd(x6, x5), x3, x4)}, rd(op_l(x6, x3, x4), \underbrace{op_r(op_l(x5, x3, x4), x2, x1)}))) \\
= \quad & \text{by Axiom 18 LR with } \{x5 \leftarrow x1, x4 \leftarrow x2, x3 \leftarrow x4, x2 \leftarrow x3, x1 \leftarrow x5\} \\
& rd(x1, rd(op_l(rd(x6, x5), x3, x4), \underbrace{rd(op_l(x6, x3, x4), \overbrace{op_l(op_r(x5, x2, x1), x3, x4)})})) \\
= \quad & \text{by Lemma 2809 LR with } \{x4 \leftarrow op_r(x5, x2, x1), x3 \leftarrow x4, x2 \leftarrow x3, x1 \leftarrow x6\} \\
& rd(x1, \underbrace{rd(op_l(rd(x6, x5), x3, x4), \overbrace{op_l(rd(x6, op_r(x5, x2, x1)), x3, x4)})}) \\
= \quad & \text{by Lemma 2809 LR with } \{x4 \leftarrow rd(x6, op_r(x5, x2, x1)), x3 \leftarrow x4, x2 \leftarrow x3, x1 \leftarrow rd(x6, x5)\} \\
& rd(x1, \overbrace{op_l(\underbrace{rd(rd(x6, x5), rd(x6, op_r(x5, x2, x1)))}, x3, x4)}) \\
= \quad & \text{by Lemma 2781 RL with } \{x4 \leftarrow x6, x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x5\} \\
& rd(x1, \underbrace{op_l(\overbrace{asoc(x5, x2, x1)}, x3, x4)}) \\
= \quad & \text{by Lemma 2810 LR with } \{x5 \leftarrow x4, x4 \leftarrow x3, x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x5\} \\
& rd(x1, \overbrace{asoc(x5, op_l(x2, x3, x4), x1)}) \\
= \quad & \text{by Lemma 2818 LR with } \{x5 \leftarrow x1, x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x5\} \\
& rd(x1, \overbrace{asoc(x5, x2, op_l(x1, x3, x4))})
\end{aligned}
$$

Lemma 2820: $mult(mult(x1, op_l(i(mult(x1, x2)), x3, x4)), x2) = mult(x1, rd(i(x1), asoc(x3, x4, mult(x1, x2))))$

$mult(mult(x1, \underbrace{op_l(i(mult(x1, x2)), x3, x4)}), x2)$

$=$ by Lemma 1049 RL with $\{x3 \leftarrow x4, x2 \leftarrow x3, x1 \leftarrow mult(x1, x2)\}$

$\underbrace{mult(mult(x1, \overbrace{i(op_l(mult(x1, x2), x3, x4))}), x2)}$

$=$ by Lemma 982 RL with $\{x3 \leftarrow x2, x2 \leftarrow op_l(mult(x1, x2), x3, x4), x1 \leftarrow x1\}$

$\overbrace{mult(x1, \underbrace{mult(rd(i(x1), op_l(mult(x1, x2), x3, x4)), mult(x1, x2))})}$

$=$ by Lemma 2738 LR with $\{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow mult(x1, x2), x1 \leftarrow i(x1)\}$

$mult(x1, \overbrace{rd(i(x1), asoc(x3, x4, mult(x1, x2)))})$

Lemma 2821: $mult(rd(x1, op_l(mult(x2, x1), x3, x4)), x2) = mult(x1, rd(asoc(x4, x3, mult(x1, x2)), x1))$

$mult(\underbrace{rd(x1, op_l(mult(x2, x1), x3, x4))}, x2)$

$=$ by Lemma 2186 RL with $\{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{mult(\overbrace{mult(x1, op_l(i(mult(x1, x2)), x3, x4))}, x2)}$

$=$ by Lemma 2820 LR with $\{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{mult(x1, \underbrace{rd(i(x1), asoc(x3, x4, mult(x1, x2)))})}$

$=$ by Lemma 2734 LR with $\{x4 \leftarrow mult(x1, x2), x3 \leftarrow x4, x2 \leftarrow x3, x1 \leftarrow x1\}$

$mult(x1, \overbrace{rd(asoc(x4, x3, mult(x1, x2)), x1)})$

Lemma 2822: $op_r(op_r(x1, x2, x3), x2, x4) = mult(x1, asoc(x1, x2, mult(x4, x3)))$

$\overbrace{op_r(op_r(x1, x2, x3), x2, x4)}$

$=$ by Lemma 2716 RL with $\{x3 \leftarrow x4, x2 \leftarrow x2, x1 \leftarrow op_r(x1, x2, x3)\}$

$\overbrace{rd(op_r(x1, x2, x3), \underbrace{asoc(x2, op_r(x1, x2, x3), x4)})}$

$=$ by Lemma 2811 LR with $\{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x1, x1 \leftarrow x2\}$

$\underbrace{rd(op_r(x1, x2, x3), \overbrace{asoc(x2, x1, x4)})}$

$=$ by Lemma 2728 RL with $\{x4 \leftarrow x4, x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow op_r(x1, x2, x3)\}$

$\overbrace{mult(\underbrace{op_r(x1, x2, x3)}, asoc(x1, x2, x4))}$

$=$ by Lemma 2741 RL with $\{x3 \leftarrow x3, x2 \leftarrow x2, x4 \leftarrow asoc(x1, x2, x4), x1 \leftarrow x1\}$

$\overbrace{mult(x1, \underbrace{rd(asoc(x1, x2, x4), asoc(x2, x1, x3))})}$

$=$ by Lemma 2738 RL with $\{x4 \leftarrow x1, x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow asoc(x1, x2, x4)\}$

$mult(x1, \overbrace{mult(\underbrace{rd(asoc(x1, x2, x4), op_l(x3, x2, x1))}, x3)})$

$=$ by Lemma 2739 RL with $\{x4 \leftarrow op_l(x3, x2, x1), x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x4\}$

$mult(x1, mult(\overbrace{rd(op_l(x4, x1, x2), \underbrace{mult(op_l(x3, x2, x1), x4)})}, x3))$

$=$ by Lemma 2807 LR with $\{x4 \leftarrow x4, x2 \leftarrow x1, x1 \leftarrow x2, x3 \leftarrow x3\}$

$mult(x1, \underbrace{mult(rd(op_l(x4, x1, x2), \overbrace{op_l(mult(x3, op_l(x4, x1, x2)), x2, x1)}), x3)})$

$=$ by Lemma 2821 LR with $\{x4 \leftarrow x1, x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow op_l(x4, x1, x2)\}$

$mult(x1, \overbrace{mult(op_l(x4, x1, x2), rd(asoc(x1, x2, mult(op_l(x4, x1, x2), x3)), op_l(x4, x1, x2)))})$

$=$ by Lemma 2665 RL with $\{x4 \leftarrow op_l(x4, x1, x2), x3 \leftarrow mult(op_l(x4, x1, x2), x3), x2 \leftarrow x2, x1 \leftarrow x1\}$

$mult(x1, \overbrace{rd(mult(asoc(x1, x2, mult(op_l(x4, x1, x2), x3)), op_l(x4, x1, x2)), op_l(x4, x1, x2))})$

$=$ by Axiom 5 RL with $\{x2 \leftarrow op_l(x4, x1, x2), x1 \leftarrow asoc(x1, x2, mult(op_l(x4, x1, x2), x3))\}$

$mult(x1, \overbrace{asoc(x1, x2, mult(op_l(x4, x1, x2), x3))})$

$=$ by Lemma 2815 RL with $\{x4 \leftarrow x3, x3 \leftarrow x4, x2 \leftarrow x2, x1 \leftarrow x1\}$

$mult(x1, \overbrace{asoc(x1, x2, mult(x4, x3))})$

Lemma 2823: $op_r(op_r(x1, x2, x3), x2, x4) = op_r(x1, x2, mult(x4, x3))$

$\overbrace{op_r(op_r(x1, x2, x3), x2, x4)}$

$=$ by Lemma 2822 LR with $\{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{mult(x1, asoc(x1, x2, mult(x4, x3)))}$

$=$ by Lemma 2728 LR with $\{x4 \leftarrow mult(x4, x3), x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x1\}$

$\overbrace{rd(x1, asoc(x2, x1, mult(x4, x3)))}$

$=$ by Lemma 2716 LR with $\{x3 \leftarrow mult(x4, x3), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_r(x1, x2, mult(x4, x3))}$

Lemma 2824: $op_r(x1, x2, mult(mult(x4, x5), x3)) = op_r(x1, x2, mult(x4, mult(x5, x3)))$

$\overbrace{op_r(x1, x2, mult(mult(x4, x5), x3))}$

$=$ by Lemma 2823 RL with $\{x4 \leftarrow mult(x4, x5), x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_r(op_r(x1, x2, x3), x2, mult(x4, x5))}}$

$=$ by Lemma 2823 RL with $\{x4 \leftarrow x4, x3 \leftarrow x5, x2 \leftarrow x2, x1 \leftarrow op_r(x1, x2, x3)\}$

$\overbrace{op_r(\underbrace{op_r(op_r(x1, x2, x3), x2, x5)}, x2, x4)}$

$=$ by Lemma 2823 LR with $\{x4 \leftarrow x5, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{op_r(\overbrace{op_r(x1, x2, mult(x5, x3))}, x2, x4)}$

$=$ by Lemma 2823 LR with $\{x4 \leftarrow x4, x3 \leftarrow mult(x5, x3), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_r(x1, x2, mult(x4, mult(x5, x3)))}$

Lemma 2825: $op_r(x1, x2, mult(x3, mult(rd(x4, x3), mult(i(x4), x5)))) = op_r(x1, x2, mult(asoc(x4, x3, x4), x5))$

$\underbrace{op_r(x1, x2, mult(x3, mult(rd(x4, x3), mult(i(x4), x5))))}$

$=$ by Lemma 2824 RL with $\{x3 \leftarrow mult(i(x4), x5), x5 \leftarrow rd(x4, x3), x4 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_r(x1, x2, mult(mult(x3, rd(x4, x3)), mult(i(x4), x5)))}}$

$=$ by Lemma 2824 RL with $\{x3 \leftarrow x5, x5 \leftarrow i(x4), x4 \leftarrow mult(x3, rd(x4, x3)), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_r(x1, x2, mult(\underbrace{mult(mult(x3, rd(x4, x3)), i(x4))}, x5))}$

$=$ by Lemma 482 RL with $\{x2 \leftarrow x3, x1 \leftarrow x4\}$

$op_r(x1, x2, mult(\overbrace{asoc(x4, x3, x4)}, x5))$

Lemma 2826: $asoc(x1, x2, op_l(x3, x4, x5)) = asoc(x1, x2, x3)$

$\underbrace{asoc(x1, x2, op_l(x3, x4, x5))}$

$=$ by Lemma 2711 RL with $\{x3 \leftarrow op_l(x3, x4, x5), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{rd(op_r(x1, x2, \underbrace{op_l(x3, x4, x5)}), x1)}$

$=$ by Lemma 1620 RL with $\{x3 \leftarrow x3, x2 \leftarrow x4, x1 \leftarrow x5\}$

$rd(op_r(x1, x2, \overbrace{mult(x5, mult(x4, mult(\underbrace{i(mult(x4, x5))}, x3)))}), x1)$

$=$ by Axiom 11 RL with $\{x2 \leftarrow x5, x1 \leftarrow x4\}$

$rd(op_r(x1, x2, mult(x5, \underbrace{mult(x4, mult(\overbrace{mult(i(x4), i(x5))}, x3))})), x1)$

$=$ by Lemma 1418 RL with $\{x3 \leftarrow x3, x2 \leftarrow x5, x1 \leftarrow x4\}$

$rd(\underbrace{op_r(x1, x2, mult(x5, \overbrace{mult(rd(x4, x5), mult(i(x4), x3))}))}, x1)$

$=$ by Lemma 2825 LR with $\{x5 \leftarrow x3, x4 \leftarrow x4, x3 \leftarrow x5, x2 \leftarrow x2, x1 \leftarrow x1\}$

$rd(\overbrace{op_r(x1, x2, \underbrace{mult(asoc(x4, x5, x4), x3)})}, x1)$

$=$ by Lemma 2729 LR with $\{x4 \leftarrow x3, x1 \leftarrow x4, x2 \leftarrow x5, x3 \leftarrow x4\}$

$rd(op_r(x1, x2, \overbrace{\underbrace{rd(x3, asoc(x5, x4, x4))}}), x1)$

$=$ by Lemma 1295 RL with $\{x3 \leftarrow x5, x2 \leftarrow x4, x1 \leftarrow x3\}$

$rd(\underbrace{op_r(x1, x2, \overbrace{mult(x3, asoc(x4, x5, x4))})}, x1)$

$=$ by Lemma 2389 RL with $\{x5 \leftarrow x5, x4 \leftarrow x4, x3 \leftarrow mult(x3, asoc(x4, x5, x4)), x2 \leftarrow x2, x1 \leftarrow x1\}$

$rd(\overbrace{op_r(x1, x2, \underbrace{rd(mult(x3, asoc(x4, x5, x4)), asoc(x4, x5, x4))})}, x1)$

$=$ by Axiom 5 RL with $\{x2 \leftarrow asoc(x4, x5, x4), x1 \leftarrow x3\}$

$\underbrace{rd(op_r(x1, x2, \overbrace{x3}), x1)}$

$=$ by Lemma 2711 LR with $\{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{asoc(x1, x2, x3)}$

Lemma 2827: $op_l(asoc(x1, x2, x3), x4, x5) = asoc(x1, x2, x3)$

$\underbrace{op_l(asoc(x1, x2, x3), x4, x5)}$

$=$ by Lemma 2810 LR with $\{x5 \leftarrow x5, x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{\underbrace{asoc(x1, op_l(x2, x4, x5), x3)}}$

$=$ by Lemma 2818 LR with $\{x5 \leftarrow x3, x4 \leftarrow x5, x3 \leftarrow x4, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{\underbrace{asoc(x1, x2, op_l(x3, x4, x5))}}$

$=$ by Lemma 2826 LR with $\{x5 \leftarrow x5, x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{asoc(x1, x2, x3)}$

Lemma 2828: $op_l(x1, x3, x2) = op_l(x1, x3, op_l(x2, x4, x5))$

$$
\begin{aligned}
& \underbrace{op_l(x1, x3, x2)} \\
= \quad & \text{by Lemma 2715 RL with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \overbrace{rd(x1, \underbrace{asoc(x2, x3, x1)})} \\
= \quad & \text{by Lemma 2826 RL with } \{x5 \leftarrow x5, x4 \leftarrow x4, x3 \leftarrow x1, x2 \leftarrow x3, x1 \leftarrow x2\} \\
& \underbrace{rd(x1, \overbrace{asoc(x2, x3, op_l(x1, x4, x5))})} \\
= \quad & \text{by Lemma 2819 RL with } \{x4 \leftarrow x5, x3 \leftarrow x4, x5 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x1\} \\
& \overbrace{op_l(x1, x3, \underbrace{op_l(x2, x4, x5)})}
\end{aligned}
$$

Lemma 2829: $rd(x1, rd(x2, asoc(x3, x4, x5))) = rd(rd(x1, asoc(x4, x3, x5)), x2)$

$$
\begin{aligned}
& \underbrace{rd(x1, rd(x2, asoc(x3, x4, x5)))} \\
= \quad & \text{by Lemma 1917 RL with } \{x3 \leftarrow x2, x2 \leftarrow asoc(x3, x4, x5), x1 \leftarrow x1\} \\
& \overbrace{rd(mult(x1, \underbrace{op_l(asoc(x3, x4, x5), rd(x2, asoc(x3, x4, x5)), x1)}), x2)} \\
= \quad & \text{by Lemma 2827 LR with } \{x5 \leftarrow x1, x4 \leftarrow rd(x2, asoc(x3, x4, x5)), x3 \leftarrow x5, x2 \leftarrow x4, x1 \leftarrow x3\} \\
& rd(\underbrace{mult(x1, \overbrace{asoc(x3, x4, x5)})}, x2) \\
= \quad & \text{by Lemma 2728 LR with } \{x4 \leftarrow x5, x3 \leftarrow x4, x2 \leftarrow x3, x1 \leftarrow x1\} \\
& rd(\overbrace{rd(x1, asoc(x4, x3, x5))}, x2)
\end{aligned}
$$

Lemma 2830: $rd(asoc(x1, x2, x3), x4) = rd(op_l(op_t(x3, x4), x1, x2), mult(x3, x4))$

$$
\begin{aligned}
& rd(\underbrace{asoc(x1, x2, x3)}, x4) \\
= \quad & \text{by Lemma 2724 RL with } \{x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{rd(\overbrace{asoc(x1, x2, op_t(x3, x4))}, x4)} \\
= \quad & \text{by Lemma 2739 RL with } \{x4 \leftarrow x4, x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow op_t(x3, x4)\} \\
& \overbrace{rd(op_l(op_t(x3, x4), x1, x2), \underbrace{mult(x4, op_t(x3, x4))})} \\
= \quad & \text{by Lemma 13 LR with } \{x2 \leftarrow x3, x1 \leftarrow x4\} \\
& rd(op_l(op_t(x3, x4), x1, x2), \overbrace{mult(x3, x4)})
\end{aligned}
$$

Lemma 2831: $rd(op_l(x1, x2, x3), op_l(x1, x4, x5)) = rd(asoc(x2, x3, x1), asoc(x4, x5, x1))$

$$
\begin{aligned}
& \underbrace{rd(op_l(x1, x2, x3), op_l(x1, x4, x5))} \\
= \quad & \text{by Lemma 2745 LR with } \{x4 \leftarrow op_l(x1, x4, x5), x2 \leftarrow x3, x1 \leftarrow x2, x3 \leftarrow x1\} \\
& \overbrace{rd(asoc(x2, x3, x1), rd(op_l(x1, x4, x5), \underbrace{i(i(x1))}))} \\
= \quad & \text{by Lemma 3 LR with } \{x1 \leftarrow x1\} \\
& rd(asoc(x2, x3, x1), \underbrace{rd(op_l(x1, x4, x5), \overbrace{x1})}) \\
= \quad & \text{by Lemma 2722 LR with } \{x3 \leftarrow x5, x2 \leftarrow x4, x1 \leftarrow x1\} \\
& rd(asoc(x2, x3, x1), \overbrace{asoc(x4, x5, x1)})
\end{aligned}
$$

Lemma 2832: $rd(asoc(x1, x2, x3), asoc(x4, x5, op_l(x3, x2, x1))) = rd(x3, op_l(op_l(x3, x2, x1), x4, x5))$

$rd(\underbrace{asoc(x1, x2, x3)}, asoc(x4, x5, op_l(x3, x2, x1)))$

$=$ by Lemma 2826 RL with $\{x5 \leftarrow x1, x4 \leftarrow x2, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\underbrace{rd(\overbrace{asoc(x1, x2, op_l(x3, x2, x1))}, asoc(x4, x5, op_l(x3, x2, x1)))}$

$=$ by Lemma 2831 RL with $\{x5 \leftarrow x5, x4 \leftarrow x4, x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow op_l(x3, x2, x1)\}$

$\overbrace{rd(\underbrace{op_l(op_l(x3, x2, x1), x1, x2)}, op_l(op_l(x3, x2, x1), x4, x5))}$

$=$ by Lemma 138 LR with $\{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x3\}$

$rd(\overbrace{x3}, op_l(op_l(x3, x2, x1), x4, x5))$

Lemma 2833: $asoc(x1, rd(x2, x1), x3) = mult(i(mult(x1, mult(mult(i(x1), x2), x3))), mult(x2, x3))$

$\underbrace{asoc(x1, rd(x2, x1), x3)}$

$=$ by Lemma 2774 LR with $\{x4 \leftarrow x3, x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{\underbrace{asoc(x1, mult(i(x1), x2), x3)}}$

$=$ by Lemma 108 RL with $\{x3 \leftarrow x3, x2 \leftarrow mult(i(x1), x2), x1 \leftarrow x1\}$

$\overbrace{mult(i(mult(x1, mult(mult(i(x1), x2), x3))), mult(\underbrace{mult(x1, mult(i(x1), x2))}, x3))}$

$=$ by Lemma 12 LR with $\{x2 \leftarrow x2, x1 \leftarrow x1\}$

$mult(i(mult(x1, mult(mult(i(x1), x2), x3))), mult(\overbrace{x2}, x3))$

Lemma 2834: $op_l(x1, mult(x2, x3), op_t(x2, x3)) = op_l(x1, mult(x2, x3), rd(asoc(x4, x5, x2), x3))$

$\underbrace{op_l(x1, mult(x2, x3), op_t(x2, x3))}$

$=$ by Lemma 2828 LR with $\{x5 \leftarrow x5, x4 \leftarrow x4, x2 \leftarrow op_t(x2, x3), x3 \leftarrow mult(x2, x3), x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_l(x1, mult(x2, x3), op_l(op_t(x2, x3), x4, x5))}}$

$=$ by Lemma 286 RL with $\{x3 \leftarrow op_l(op_t(x2, x3), x4, x5), x2 \leftarrow mult(x2, x3), x1 \leftarrow x1\}$

$\overbrace{op_l(x1, mult(x2, x3), \underbrace{rd(op_l(op_t(x2, x3), x4, x5), mult(x2, x3))})}$

$=$ by Lemma 2830 RL with $\{x4 \leftarrow x3, x3 \leftarrow x2, x2 \leftarrow x5, x1 \leftarrow x4\}$

$op_l(x1, mult(x2, x3), \overbrace{rd(asoc(x4, x5, x2), x3)})$

Lemma 2835: $op_l(x1, mult(x2, x3), x2) = op_l(x1, mult(x2, x3), rd(asoc(x4, x5, x2), x3))$

$\underbrace{op_l(x1, mult(x2, x3), x2)}$

$=$ by Lemma 2715 RL with $\{x3 \leftarrow mult(x2, x3), x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{rd(x1, \underbrace{asoc(x2, mult(x2, x3), x1)})}$

$=$ by Lemma 2770 RL with $\{x4 \leftarrow x1, x1 \leftarrow mult(x2, x3), x3 \leftarrow x3, x2 \leftarrow x2\}$

$\underbrace{rd(x1, \overbrace{asoc(op_t(x2, x3), mult(x2, x3), x1)})}$

$=$ by Lemma 2715 LR with $\{x3 \leftarrow mult(x2, x3), x2 \leftarrow op_t(x2, x3), x1 \leftarrow x1\}$

$\overbrace{\underbrace{op_l(x1, mult(x2, x3), op_t(x2, x3))}}$

$=$ by Lemma 2834 LR with $\{x5 \leftarrow x5, x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{op_l(x1, mult(x2, x3), rd(asoc(x4, x5, x2), x3))}$

Lemma 2836: $mult(rd(x4, asoc(x1, x2, x3)), x5) = mult(rd(x4, mult(op_l(x3, x1, x2), x4)), mult(mult(x3, x4), op_l(x5, x4, x3)))$

$mult(\underbrace{rd(x4, asoc(x1, x2, x3))}, x5)$

$=$ by Lemma 10 RL with $\{x2 \leftarrow x4, x1 \leftarrow asoc(x1, x2, x3)\}$

$mult(\overbrace{i(rd(asoc(x1, x2, x3), x4))}, x5)$

$=$ by Axiom 4 LR with $\{x2 \leftarrow mult(x3, x4), x1 \leftarrow i(rd(asoc(x1, x2, x3), x4))\}$

$\underbrace{mult(\overbrace{mult(rd(i(rd(asoc(x1, x2, x3), x4)), mult(x3, x4)), mult(x3, x4))}, x5)}$

$=$ by Lemma 137 RL with $\{x3 \leftarrow x5, x2 \leftarrow mult(x3, x4), x1 \leftarrow rd(i(rd(asoc(x1, x2, x3), x4)), mult(x3, x4))\}$

$\overbrace{mult(rd(i(rd(asoc(x1, x2, x3), x4)), mult(x3, x4)), mult(mult(x3, x4), \underbrace{op_l(x5, rd(i(rd(asoc(x1, x2, x3), x4)), mult(x3, x4)),}$

$=$ by Lemma 292 LR with $\{x3 \leftarrow mult(x3, x4), x2 \leftarrow i(rd(asoc(x1, x2, x3), x4)), x1 \leftarrow x5\}$

$mult(rd(i(rd(asoc(x1, x2, x3), x4)), mult(x3, x4)), mult(mult(x3, x4), \overbrace{op_l(x5, i(rd(asoc(x1, x2, x3), x4)), mult(x3, x4))}))$

$=$ by Lemma 2031 LR with $\{x3 \leftarrow mult(x3, x4), x2 \leftarrow rd(asoc(x1, x2, x3), x4), x1 \leftarrow x5\}$

$mult(rd(i(rd(asoc(x1, x2, x3), x4)), mult(x3, x4)), mult(mult(x3, x4), \overbrace{op_l(x5, mult(x3, x4), rd(asoc(x1, x2, x3), x4))}))$

$=$ by Lemma 2835 RL with $\{x5 \leftarrow x2, x4 \leftarrow x1, x3 \leftarrow x4, x2 \leftarrow x3, x1 \leftarrow x5\}$

$mult(rd(i(rd(asoc(x1, x2, x3), x4)), mult(x3, x4)), mult(mult(x3, x4), \overbrace{op_l(x5, mult(x3, x4), x3)}))$

$=$ by Lemma 2033 LR with $\{x3 \leftarrow x4, x2 \leftarrow x3, x1 \leftarrow x5\}$

$mult(rd(\underbrace{i(rd(asoc(x1, x2, x3), x4))}, mult(x3, x4)), mult(mult(x3, x4), \overbrace{op_l(x5, x4, x3)}))$

$=$ by Lemma 10 LR with $\{x2 \leftarrow x4, x1 \leftarrow asoc(x1, x2, x3)\}$

$mult(\underbrace{rd(\overbrace{rd(x4, asoc(x1, x2, x3))}, mult(x3, x4))}, mult(mult(x3, x4), op_l(x5, x4, x3)))$

$=$ by Lemma 2829 RL with $\{x5 \leftarrow x3, x4 \leftarrow x1, x3 \leftarrow x2, x2 \leftarrow mult(x3, x4), x1 \leftarrow x4\}$

$mult(\overbrace{rd(x4, rd(\underbrace{mult(x3, x4)}, asoc(x2, x1, x3)))}, mult(mult(x3, x4), op_l(x5, x4, x3)))$

$=$ by Lemma 138 RL with $\{x3 \leftarrow x4, x2 \leftarrow op_l(x3, x1, x2), x1 \leftarrow mult(x3, x4)\}$

$mult(rd(x4, rd(\overbrace{op_l(\underbrace{op_l(mult(x3, x4)}, op_l(x3, x1, x2), x4)}, x4, op_l(x3, x1, x2))}, asoc(x2, x1, x3))), mult(mult(x3, x4), op_l(x5,$

$=$ by Lemma 2670 RL with $\{x4 \leftarrow x2, x3 \leftarrow x1, x2 \leftarrow x3, x1 \leftarrow x4\}$

$mult(rd(x4, rd(op_l(\overbrace{mult(x4, x3)}), x4, \underbrace{op_l(x3, x1, x2)}), asoc(x2, x1, x3))), mult(mult(x3, x4), op_l(x5, x4, x3)))$

$=$ by Lemma 2715 RL with $\{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x3\}$

$mult(rd(x4, \underbrace{rd(op_l(mult(x4, x3), x4, \overbrace{rd(x3, asoc(x2, x1, x3))}), asoc(x2, x1, x3))}), mult(mult(x3, x4), op_l(x5, x4, x3)))$

$=$ by Lemma 1074 RL with $\{x4 \leftarrow rd(x3, asoc(x2, x1, x3)), x3 \leftarrow x4, x2 \leftarrow mult(x4, x3), x1 \leftarrow asoc(x2, x1, x3)\}$

$mult(rd(x4, \overbrace{rd(i(asoc(x2, x1, x3)), op_l(\underbrace{i(mult(x4, x3))}, x4, rd(x3, asoc(x2, x1, x3))))}), mult(mult(x3, x4), op_l(x5, x4, x3)))$

$=$ by Axiom 11 RL with $\{x2 \leftarrow x3, x1 \leftarrow x4\}$

$mult(rd(x4, \underbrace{rd(i(asoc(x2, x1, x3)), op_l(\overbrace{mult(i(x4), i(x3))}, x4, rd(x3, asoc(x2, x1, x3))))}), mult(mult(x3, x4), op_l(x5, x4, x3)$

$=$ by Lemma 1981 RL with $\{x3 \leftarrow x4, x2 \leftarrow asoc(x2, x1, x3), x1 \leftarrow x3\}$

$mult(rd(x4, \overbrace{mult(\underbrace{rd(x3, asoc(x2, x1, x3))}, x4)}), mult(mult(x3, x4), op_l(x5, x4, x3)))$

$=$ by Lemma 2715 LR with $\{x3 \leftarrow x1, x2 \leftarrow x2, x1 \leftarrow x3\}$

$mult(rd(x4, mult(\overbrace{op_l(x3, x1, x2)}, x4)), mult(mult(x3, x4), op_l(x5, x4, x3)))$

Lemma 2837: $mult(rd(x4, asoc(x1, x2, x3)), x5) = mult(op_l(i(x3), x1, x2), mult(mult(x3, x4), op_l(x5, x4, x3)))$

$$
\begin{aligned}
& \underbrace{mult(rd(x4, asoc(x1, x2, x3)), x5)} \\
= \quad & \text{by Lemma 2836 LR with } \{x5 \leftarrow x5, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1, x4 \leftarrow x4\} \\
& \overbrace{mult(\underbrace{rd(x4, mult(op_l(x3, x1, x2), x4))}, mult(mult(x3, x4), op_l(x5, x4, x3)))} \\
= \quad & \text{by Lemma 9 LR with } \{x2 \leftarrow op_l(x3, x1, x2), x1 \leftarrow x4\} \\
& mult(\overbrace{i(op_l(x3, x1, x2))}, mult(mult(x3, x4), op_l(x5, x4, x3))) \\
= \quad & \text{by Lemma 1049 LR with } \{x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x3\} \\
& mult(\overbrace{op_l(i(x3), x1, x2)}, mult(mult(x3, x4), op_l(x5, x4, x3)))
\end{aligned}
$$

Lemma 2838: $mult(rd(x4, asoc(x1, x2, x3)), x5) = rd(mult(x4, x5), asoc(x1, x2, x3))$

$$
\begin{aligned}
& \underbrace{mult(rd(x4, asoc(x1, x2, x3)), x5)} \\
= \quad & \text{by Lemma 2837 LR with } \{x5 \leftarrow x5, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1, x4 \leftarrow x4\} \\
& \overbrace{mult(op_l(i(x3), x1, x2), \underbrace{mult(mult(x3, x4), op_l(x5, x4, x3))})} \\
= \quad & \text{by Lemma 132 LR with } \{x3 \leftarrow x5, x2 \leftarrow x4, x1 \leftarrow x3\} \\
& \underbrace{mult(op_l(i(x3), x1, x2), \overbrace{mult(x3, mult(x4, x5))})} \\
= \quad & \text{by Lemma 2764 LR with } \{x4 \leftarrow mult(x4, x5), x3 \leftarrow x2, x2 \leftarrow x1, x1 \leftarrow x3\} \\
& \overbrace{mult(\underbrace{asoc(x1, x2, i(x3))}, mult(x4, x5))} \\
= \quad & \text{by Lemma 2719 LR with } \{x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\} \\
& \underbrace{mult(\overbrace{asoc(x2, x1, x3)}, mult(x4, x5))} \\
= \quad & \text{by Lemma 2729 LR with } \{x4 \leftarrow mult(x4, x5), x1 \leftarrow x3, x2 \leftarrow x1, x3 \leftarrow x2\} \\
& \overbrace{rd(mult(x4, x5), asoc(x1, x2, x3))}
\end{aligned}
$$

Lemma 2839: $asoc(x1, asoc(x2, x3, x4), x5) = rd(x4, op_l(op_l(x4, x3, x2), x2, x3))$

$\underbrace{asoc(x1, asoc(x2, x3, x4), x5)}$

$=$ by Lemma 2725 RL with $\{x3 \leftarrow x5, x2 \leftarrow asoc(x2, x3, x4), x1 \leftarrow x1\}$

$\overbrace{\underbrace{asoc(x1, rd(asoc(x2, x3, x4), x1), x5)}}$

$=$ by Lemma 2833 LR with $\{x3 \leftarrow x5, x2 \leftarrow asoc(x2, x3, x4), x1 \leftarrow x1\}$

$\overbrace{mult(\underbrace{i(mult(x1, mult(mult(i(x1), asoc(x2, x3, x4)), x5)))}, mult(asoc(x2, x3, x4), x5))}$

$=$ by Axiom 11 RL with $\{x2 \leftarrow mult(mult(i(x1), asoc(x2, x3, x4)), x5), x1 \leftarrow x1\}$

$mult(\overbrace{mult(i(x1), \underbrace{i(mult(mult(i(x1), asoc(x2, x3, x4)), x5)))}}, mult(asoc(x2, x3, x4), x5))$

$=$ by Lemma 1399 RL with $\{x3 \leftarrow x5, x1 \leftarrow asoc(x2, x3, x4), x2 \leftarrow i(x1)\}$

$mult(\overbrace{mult(rd(i(x1), asoc(x2, x3, x4)), \underbrace{i(mult(i(x1), x5))})}, mult(asoc(x2, x3, x4), x5))$

$=$ by Lemma 16 LR with $\{x2 \leftarrow x5, x1 \leftarrow x1\}$

$mult(\underbrace{mult(rd(i(x1), asoc(x2, x3, x4)), \overbrace{mult(x1, i(x5))})}, mult(asoc(x2, x3, x4), x5))$

$=$ by Lemma 2838 LR with $\{x5 \leftarrow mult(x1, i(x5)), x3 \leftarrow x4, x2 \leftarrow x3, x1 \leftarrow x2, x4 \leftarrow i(x1)\}$

$\underbrace{mult(\overbrace{rd(mult(i(x1), mult(x1, i(x5))), asoc(x2, x3, x4))}, mult(asoc(x2, x3, x4), x5))}$

$=$ by Lemma 2838 LR with $\{x5 \leftarrow mult(asoc(x2, x3, x4), x5), x3 \leftarrow x4, x2 \leftarrow x3, x1 \leftarrow x2, x4 \leftarrow mult(i(x1), mult(x1, i(x5)))\}$

$\overbrace{rd(mult(\underbrace{mult(i(x1), mult(x1, i(x5)))}, mult(asoc(x2, x3, x4), x5)), asoc(x2, x3, x4))}$

$=$ by Axiom 2 RL with $\{x2 \leftarrow x1, x1 \leftarrow i(x5)\}$

$rd(mult(\overbrace{i(x5)}, mult(asoc(x2, x3, x4), x5)), asoc(x2, x3, x4))$

$=$ by Axiom 10 LR with $\{x2 \leftarrow asoc(x2, x3, x4), x1 \leftarrow x5\}$

$rd(\overbrace{op_t(asoc(x2, x3, x4), x5)}, asoc(x2, x3, x4))$

$=$ by Lemma 2654 LR with $\{x4 \leftarrow x5, x3 \leftarrow x4, x2 \leftarrow x3, x1 \leftarrow x2\}$

$rd(\overbrace{asoc(x2, x3, x4)}, \underbrace{asoc(x2, x3, x4)})$

$=$ by Lemma 2826 RL with $\{x5 \leftarrow x2, x4 \leftarrow x3, x3 \leftarrow x4, x2 \leftarrow x3, x1 \leftarrow x2\}$

$\underbrace{rd(asoc(x2, x3, x4), \overbrace{asoc(x2, x3, op_l(x4, x3, x2))})}$

$=$ by Lemma 2832 LR with $\{x5 \leftarrow x3, x4 \leftarrow x2, x3 \leftarrow x4, x2 \leftarrow x3, x1 \leftarrow x2\}$

$\overbrace{rd(x4, op_l(op_l(x4, x3, x2), x2, x3))}$

Lemma 2840: $asoc(x1, asoc(x2, x3, x4), x5) = unit()$

$\underbrace{asoc(x1, asoc(x2, x3, x4), x5)}$

$=$ by Lemma 2839 LR with $\{x5 \leftarrow x5, x4 \leftarrow x4, x3 \leftarrow x3, x2 \leftarrow x2, x1 \leftarrow x1\}$

$\overbrace{rd(x4, \underbrace{op_l(op_l(x4, x3, x2), x2, x3)})}$

$=$ by Lemma 138 LR with $\{x3 \leftarrow x2, x2 \leftarrow x3, x1 \leftarrow x4\}$

$\underbrace{rd(x4, \overbrace{x4})}$

$=$ by Lemma 2 LR with $\{x1 \leftarrow x4\}$

$\overbrace{unit()}$

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Theorem 1: $unit() = asoc(asoc(a(), b(), c()), d(), e())$

$$
\begin{aligned}
& \underbrace{unit()} \\
= \quad & \text{by Lemma 2840 RL with } \{x5 \leftarrow e(), x4 \leftarrow c(), x3 \leftarrow a(), x2 \leftarrow b(), x1 \leftarrow d()\} \\
& \underbrace{\overbrace{asoc(d(), asoc(b(), a(), c()), e())}} \\
= \quad & \text{by Lemma 2785 LR with } \{x5 \leftarrow e(), x3 \leftarrow c(), x2 \leftarrow a(), x1 \leftarrow b(), x4 \leftarrow d()\} \\
& \underbrace{\overbrace{asoc(d(), rd(d(), asoc(a(), b(), c())), e())}} \\
= \quad & \text{by Lemma 2726 LR with } \{x3 \leftarrow e(), x2 \leftarrow asoc(a(), b(), c()), x1 \leftarrow d()\} \\
& \overbrace{asoc(asoc(a(), b(), c()), d(), e())}
\end{aligned}
$$