

# A description of Hilbert spaces as a dagger category

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## Abstract

Complex Hilbert spaces provide the basic mathematical model used in quantum physics. Why this choice is suitable has been a topic of extensive research. In particular, the question has been raised whether the model can be reduced to a simpler one. Indeed, Hilbert spaces can, for instance, be characterised as certain algebraic or relational structures. Moreover, it has recently turned out to be possible to describe Hilbert spaces by purely categorical means.

We elaborate on the latter possibility. We work in the framework of dagger categories; a dagger is an involutory contravariant endofunctor that is the identity on objects. The complex Hilbert spaces and bounded linear maps between them form a category  $\mathcal{Hil}$  and the usual adjoint makes  $\mathcal{Hil}$  into a dagger category. We present a list of categorical axioms characterising  $\mathcal{Hil}$ .

Related results are due to Chris Heunen and Andre Kornell (PNAS 2021), who work, however, in the framework of monoidal categories. A further approach, which characterises the Hilbert spaces over one of the three classical division rings, is due to Stephen Lack and Shay Tobin (Appl. Categor. Struct. 2025).

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