Boolean sublattices in finite partition lattices

Sándor Radeleczki

University of Miskolc

Abstract

Let Part(U) denote the partition lattice of a finite universe U. We show that any highest dimensional Boolean sublattice of Part(U) can be formed using all partitions whose blocks are some subtrees of a tree with vertex set U. We prove that a maximal Boolean sublattice of Part(U) always contains the least $\triangle$ and the largest $\triangledown$ elements of Part(U). We characterize the maximal Boolean sublattices of Part(U) by a certain condition imposed on the circles of a linear hypergaph induced by the atoms of a Boolean sublattice $\mathcal{B} \leq \text{Part}(U)$ with $\triangle, \triangledown \in \mathcal{B}$ on the set U.