Foulis quantales

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Abstract

We have established a novel correspondence between complete orthomodular lattices and a particular class of quantales. In the forward direction, given any complete orthomodular lattice X, we construct an associated Foulis quantale $Lin(X)$ consisting of join-preserving mappings on X. This construction enables us to view X as a left $Lin(X)$-module, thereby providing a new fuzzy-theoretic perspective on complete orthomodular lattices while introducing an exterior implication operation on these structures. In the reverse direction, we show that every Foulis quantale Q gives rise to a complete orthomodular lattice $[Q]$ that naturally carries the structure of a left Q-module. Furthermore, this construction yields a canonical Foulis quantale homomorphism from Q to $Lin([Q])$. This bidirectional relationship creates a bridge between the algebraic theory of orthomodular lattices and the categorical framework of quantales, potentially opening new avenues for investigating quantum logical structures through module-theoretic methods.