Properties of the symmetric difference in lattices with complementation

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Abstract

The symmetric difference in Boolean lattices can be defined in two different but equivalent forms. However, it can be introduced also in every bounded lattice with complementation where these two forms need not coincide. We study lattices with complementation where these two expressions coincide. It is well-known that the symmetric difference is associative in every Boolean lattice. We prove that this is just the property of Boolean lattices, namely the symmetric difference in a lattice with complementation is associative if and only if this lattice is Boolean. Similarly, we prove that a lattice with complementation is Boolean if and only if the symmetric difference satisfies a certain simple identity in two variables.