Decompositions of Posets with least elements

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Abstract

We show that direct product decompositions (into two factors) of a poset $(P, \leq, \bot)$ with a least element $\bot$ bijectively correspond to complement neutral elements A of the lattice of finitely stable subposets of P (where a subposet is finitely stable if it is an order ideal closed under all existing finite suprema) such that A and its complement B satisfy the following two conditions: (i) for each $p \in P$, sets $\{a \in A\colon\ a \leq p\}$ and $\{b \in B\colon\ b \leq p\}$ have greatest elements; (ii) a supremum of a and b exists in P for all $a \in A$ and $b \in B$. The same result holds also for the lattice of stable subposets of P, where a subposet is stable if it is an order ideal closed under all existing suprema. These results generalize the well-known analogous result from lattice theory. We also prove that if we take the lattice of stable subposets of P and omit (ii), then each element of P has a unique representation as a supremum of two elements, one from A and the other from B. This kind of decompositions is introduced in Scott-domain theory and our result is a generalization of an analogous theorem for Scott-domains. Moreover, we describe what kinds of decompositions of a poset P with a least element correspond to complement neutral elements of lattices of finitely stable and stable subposets of P if we assume no additional conditions for these elements.