

Dimensionalities on monoids in \mathbf{Rel}

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Abstract

Motivated by the fact that a dimension relation on an orthomodular lattice can be considered as a particular internal monad on a (partial) monoid in the 2-category of sets and relations, we examine the following type of equivalences on a monoid (A, \cdot, e) , which we call *dimensionality*:

- If $a.b \sim c$, there exist $a' \sim a$ and $b' \sim b$ such that $a'.b' = c$.
- If $e \sim x$, then $e = x$.

We show that there are many examples of dimensionality arising in mathematics: on groups, modular lattices, orthomodular lattices, etc. In particular, orbits of a group acting on a monoid always give rise to a dimensionality. If the monoid is a group, the quotient monoid is a hypergroup.

We prove that if \sim is a dimensionality on a monoid A , then A/\sim is a *relational monoid*, that means, a monoid in the monoidal category of sets and relation \mathbf{Rel} . This motivates us to extend the notion of dimensionality to all monoids in \mathbf{Rel} .

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