Effect Algebras as a Simplicially Enriched Category

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Abstract

Effect algebras are certain partial monoids that arise naturally in the study of quantum logic. Our work is based on two key observations. First, every effect algebra can be viewed as a Frobenius algebra internal to the category of sets and relations. Second, the category of such Frobenius algebras embeds fully faithfully into a certain category of simplicial sets. Using these observations, we organize effect algebras into a simplicially enriched category, denoted **EA**. We investigate properties of a mapping space $\mathrm{Map}(E, F)$, for $E, F \in \mathbf{EA}$. Additionally, we consider the non-commutative generalization to pseudo effect algebras, **PEA**, which admits a well-behaved concept of conjugation. We show that several properties of conjugation have a natural interpretation from the combinatorial perspective of simplicial sets.