An introduction of a new operation on a free barycentric algebra

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Abstract

Barycentric algebras, originating from Möbius's use of barycentric coordinates, were introduced in the 1940s as an axiomatization of real convex sets, presented algebraically via binary operations. They unify the concepts of convexity (every convex set is a barycentric algebra) and order (the variety of semilattices is a subvariety of the variety of barycentric algebras). The free barycentric algebra over a finite set X can be interpreted as the set of all probability distributions of random variables on X. Moreover, the free barycentric algebra over a finite set $X \cup \{\perp\}$ corresponds to the set of all subprobability distributions of random variables on X. Our aim is to study the structures we have built based on free barycentric algebras by introducing a new operation. Initially, we focused on algebraic properties, sets of isomorphisms, and order relations within this new structure. However, the idea emerged to explore, more specifically, the space of probability functions over a group. The results we obtained reveal surprising applications and interpretations, particularly concerning the entropy (in the sense of information theory) of random distributions over groups.