1. $\int_0^1 \sqrt{\frac{1-x}{1+x}}$

$\sqrt{\frac{1-x}{1+x}} = t \qquad dx = \frac{1}{(1+t^2)^2}\left(-2t(1+t^2) - (1-t^2)2t\right) =$

$\frac{1-x}{1+x} = t^2 \qquad = \frac{1}{(1+t^2)^2}\left(-2t - 2t^3 - 2t + 2t^3\right)$

$1-x = t^2 + t^2x \qquad = \frac{-4t}{(1+t^2)^2}$ (+4)

$1-t^2 = x(1+t^2)$

$x = \frac{1-t^2}{1+t^2}$

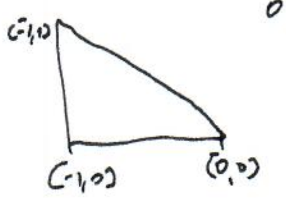
$\to \int_1^0 t\,\frac{-4t}{(1+t^2)^2} = 4\int_0^1 \frac{t^2}{(1+t^2)^2} = 4\int_0^1 \frac{t^2+1-1}{(t^2+1)^2} =$

$= 4\int_0^1 \frac{1}{t^2+1} - 4\int_0^1 \frac{1}{(t^2+1)^2} = 4\left[\operatorname{arctg} t\right]_0^1 - 4\left[\frac{t}{2(1+t^2)} + \frac{1}{2}\operatorname{arctg} t\right]_0^1$ (+3)

$= 4\cdot\frac{\pi}{4} - 4\left(\frac{1}{2\cdot 2} + \frac{1}{2}\,\frac{\pi}{4}\right) = \pi - 1 - \frac{\pi}{2} = \frac{\pi}{2} - 1$ (+3)

2. $\iint_D dx\,dy \overset{(+3)}{=} \int_{\substack{-1<x<0 \\ 0<y<-x}} xy\,dx\,dy = \int_{-1}^{0} x\left[\frac{y^2}{2}\right]_0^{-x} dx = \int_{-1}^{0} x\cdot\frac{1}{2}x^2 =$ (+3)

$= \frac{1}{2}\int_{-1}^{0} x^3 = -\frac{1}{2}\cdot\frac{1}{4} = -\frac{1}{8}$ (+3)




3. $A = \begin{pmatrix} 2 & -3 & 1 \\ 1 & -2 & 1 \\ 1 & -3 & 2 \end{pmatrix} \qquad \lambda I - A = \begin{pmatrix} \lambda-2 & 3 & -1 \\ -1 & \lambda+2 & -1 \\ -1 & 3 & \lambda-2 \end{pmatrix}$ $\uparrow \lambda-2$ $\downarrow -1$

$\Rightarrow \det(\lambda I - A) = \begin{pmatrix} 0 & \lambda^2-1 & 1-\lambda \\ -1 & \lambda+2 & -1 \\ 0 & -\lambda+1 & \lambda-1 \end{pmatrix} = \det\begin{pmatrix} \lambda^2-1 & 1-\lambda \\ 1-\lambda & \lambda-1 \end{pmatrix} =$

$= (\lambda-1)^2\left[\lambda+1-1\right] = \lambda(\lambda-1)^2 \Rightarrow \sigma(A) = \{0, 1\}$ (+4)

$\lambda = 1: \begin{pmatrix} -1 & 3 & -1 \\ -1 & 3 & -1 \\ -1 & 3 & -1 \end{pmatrix} \Rightarrow \operatorname{Ker}(1\cdot I - A) = \operatorname{lin}\left\{\begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}\right\}$ (+3)

$$\lambda = 0: \quad \begin{pmatrix} -2 & 3 & -1 \\ -1 & 2 & -1 \\ -1 & 3 & -2 \end{pmatrix} \sim \begin{pmatrix} 0 & -1 & 1 \\ -1 & 2 & -1 \\ 0 & 1 & -1 \end{pmatrix} \quad (+3)$$

$$\Rightarrow \operatorname{Ker}(0 \cdot I - A) = \operatorname{lin}_{\mathbb{C}} \left\{ \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \right\}$$

5. $$\log(1+x) - \log(1-x) = \frac{\log\frac{1+x}{1-x}}{\frac{1+x}{1-x} - 1} \cdot \left( \frac{1+x}{1-x} - 1 \right) = \frac{\log\frac{1+x}{1-x}}{\frac{1+x}{1-x} - 1} \; \frac{2x}{1-x}$$

$$\frac{\sqrt{1+\sin 2x} - \sqrt[3]{1+\sin 3x}}{x \log\frac{1+x}{1-x}} = \frac{\sqrt{1+\sin 2x} - \sqrt[3]{1+\sin 3x}}{2x^2} \; \frac{1-x}{\frac{\log\frac{1+x}{1-x}}{\frac{1+x}{1-x} - 1}} \quad (+3)$$

$$(1+y)^\alpha = 1 + \binom{\alpha}{1} y + \binom{\alpha}{2} y^2 + \varphi(y), \quad \frac{\varphi(y)}{y^2} \to 0$$

$$(1+\sin 2x)^\alpha = 1 + \binom{\alpha}{1} \sin 2x + \binom{\alpha}{2} (\sin 2x)^2 + o(x^2) \quad \left[ \frac{\varphi(\sin 2x)}{(\sin 2x)^2} \, \frac{(\sin 2x)^2}{x^2} \to 0 \right]$$

$$= 1 + \binom{\alpha}{1} (2x + o(x^2)) + \binom{\alpha}{2} (2x + o(x^2))^2 + o(x^2)$$

$$= 1 + \binom{\alpha}{1} 2x + \binom{\alpha}{2} 4x^2 + o(x^2)$$

$$\sqrt{1+\sin 2x} - \sqrt[3]{1+\sin 3x} = \left(1 + \binom{1/2}{1} 2x + \binom{1/2}{2} 4x^2\right) - \left(1 + \binom{1/3}{1} 3x + \binom{1/3}{2} 9x^2\right) + o(x^2)$$

$$= x^2 \left( \binom{1/2}{2} 4 - \binom{1/3}{2} 9 \right) + o(x^2) = x^2 \cdot \frac{1}{2} + o(x^2) \quad (+5)$$

$$\Rightarrow \lim_{x \to 0} f(x) \overset{(+2)}{=} \lim_{x \to 0} \left( \frac{\frac{1}{2} x^2 + o(x^2)}{2x^2} \cdot \frac{1-x}{\frac{\log\frac{1+x}{1-x}}{\frac{1+x}{1-x} - 1}} \right) = \frac{1}{4}$$

5. $f(x,y) = \log(xy) - 4x - 9y$ , $D_f = \{(x,y) \in \mathbb{R}^2 ; xy > 0\}$

$f_x = \frac{y}{xy} - 4 \Rightarrow \frac{1}{x} - 4 = 0 \Rightarrow x = 1/4$ (+4)

$\frac{1}{y} - 9 = 0 \qquad y = 1/9$

$f_y = \frac{x}{xy} - 9$

$D^2 f(x,y) = \begin{pmatrix} -\frac{1}{x^2} & 0 \\ 0 & -\frac{1}{y^2} \end{pmatrix}$ (+3) $\Rightarrow [\frac{1}{4}, \frac{1}{9}] \Rightarrow \begin{pmatrix} -16 & 0 \\ 0 & -81 \end{pmatrix}$ (+3) ND -- maximum