1. $\int \frac{\sqrt{x-1}}{x^2}\,dx \xrightarrow{(+3)} \int \frac{2t^2}{(t^2+1)^2}\,dt = 2\int \frac{t^2+1-1}{(t^2+1)^2} =$

$\sqrt{x-1} = t$

$x = t^2+1 \qquad = 2\int \frac{1}{t^2+1} - 2\int \frac{1}{(t^2+1)^2} =$

$dx = 2t\,dt$

$\overset{(+3)}{=} 2\,\text{arctg}\, t - 2\left(\frac{t}{2(t^2+1)} + \frac{1}{2}\,\text{arctg}\, t\right)$

$\Rightarrow \int_1^\infty \frac{\sqrt{x-1}}{x^2} = \left[2\,\text{arctg}\sqrt{x-1} - 2\left(\frac{\sqrt{x-1}}{2x} + \frac{1}{2}\,\text{arctg}\sqrt{x-1}\right)\right]_1^\infty =$

(+3) $= 2\cdot\frac{\pi}{2} - 2\left(0 + \frac{1}{2}\frac{\pi}{2}\right) - \left(0 - 2\left(0 + \frac{1}{2}\cdot 0\right)\right) =$

$= \pi - \frac{\pi}{2} = \frac{\pi}{2}$

2. $\iint_D dx\,dy = \int_0^1 \log x\ \frac{1}{2}\left[y^2\right]_{-\sqrt{1-x^2}}^{0} dx \overset{(+2)}{=} \frac{1}{2}\int_0^1 (\log x)(x^2-1) =$

D $\int \geq 0$ (+1)

$= -\frac{1}{2}\int_0^1 \log x + \frac{1}{2}\int_0^1 x^2 \log x \overset{(+5)}{=} -\frac{1}{2}\left[x\log x - x\right]_0^1 + \frac{1}{2}\left(\left[\frac{x^3}{3}\log x\right]_0^1 - \int_0^1 \frac{x^2}{3}\right) =$

$\frac{x^3}{3}$ $\quad 1/x$

$\overset{(+3)}{=} \frac{1}{2} + \frac{1}{2}\left(0 - \frac{1}{9}\right) = \frac{1}{2} - \frac{1}{18} = \frac{9-1}{18} = \frac{8}{18} = \frac{4}{9}$

3. $\lambda I - A = \begin{pmatrix} \lambda+2 & 2 & 1 \\ 0 & \lambda+1 & 0 \\ -2 & 1 & \lambda \end{pmatrix} \Rightarrow \det(\lambda I - A) = (\lambda+1)\det\begin{pmatrix} \lambda+2 & 1 \\ -2 & \lambda \end{pmatrix} =$

$\overset{(+3)}{=} (\lambda+1)(\lambda^2+2\lambda+2) \Rightarrow \sigma(A) = \{-1, -1+i, -1-i\}$

$-1:$ (+2) $\begin{pmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -2 & 1 & -1 \end{pmatrix} \sim \begin{pmatrix} 1 & 2 & 1 \\ 0 & 5 & 1 \\ 0 & 0 & 0 \end{pmatrix} \Rightarrow \text{Ker}(-1I - A) = \text{lin}\left\{\begin{pmatrix} -3/5 \\ -1/5 \\ 1 \end{pmatrix}\right\}$

$-1+i:$ (+2.5) $\begin{pmatrix} 1+i & 2 & 1 \\ 0 & i & 0 \\ -2 & 1 & -1+i \end{pmatrix} \underset{\frac{1}{2}(1+i)}{\sim} \begin{pmatrix} 0 & \frac{1}{2}i + \frac{5}{2} & 0 \\ 0 & i & 0 \\ -2 & 1 & -1+i \end{pmatrix} \Rightarrow \text{Ker}((-1+i)I - A) = \text{lin}\left\{\begin{pmatrix} \frac{-1+i}{2} \\ 0 \\ 1 \end{pmatrix}\right\}$

$-1-i:$ $\begin{pmatrix} 1-i & 2 & 1 \\ 0 & -i & 0 \\ -2 & 1 & -1-i \end{pmatrix} \sim \begin{pmatrix} 0 & 2+\frac{1}{2}(1-i) & 0 \\ 0 & -i & 0 \\ -2 & 1 & 1-i \end{pmatrix}$

(+2.5)

$\Rightarrow \ker((-1-i)I-A) = \text{lin}\left\{\begin{pmatrix} \frac{-1-i}{2} \\ 0 \\ 1 \end{pmatrix}\right\}$

4. $\dfrac{1-\cos x^2}{\log(1-x^2-x^4)-\log(1-x^2+x^4)} \overset{(+2)}{=} \dfrac{1-\cos x^2}{x^4}\,\dfrac{x^4}{-2x^4+o(x^4)} =$

$\log(1+y) = y - \frac{y^2}{2} + \frac{y^3}{3} - \frac{y^4}{4} + \varphi(y^4)$, $\frac{\varphi(y)}{y^4} \underset{y\to 0}{\to} 0$

$\log(1-x^2-x^4) = -x^2-x^4-\frac{1}{2}(-x^2-x^4)^2 + o(x^4) + \varphi(-x^2-x^4) = -x^2-x^4-\frac{1}{2}x^4+o(x^4)$

$\left[\dfrac{\varphi(-x^2-x^4)}{x^4} = \dfrac{\varphi(-x^2-x^4)}{(-x^2-x^4)^4}\,\dfrac{(-x^2-x^4)^4}{x^4} \to 0\right]$

$\log(1-x^2+x^4) = -x^2+x^4-\frac{1}{2}(-x^2+x^4)^2 + o(x^4) \overset{(+5)}{=} -x^2+x^4-\frac{1}{2}x^4+o(x^4)$

$\overset{(+2)}{=} \dfrac{1-\cos x^2}{x^4}\,\dfrac{1}{-2+\frac{o(x^4)}{x^4}} \to \frac{1}{2}\cdot\frac{1}{-2} = -\frac{1}{4}$

5. $f = -x^2y + y^2 + x$

$f_x = -2xy + 1 = 0 \Rightarrow 1 = 2x\frac{x^2}{2} = x^3 \Rightarrow x = 1 \Rightarrow [1, 1/2]$ (+4)

$f_y = -x^2 + 2y = 0 \Rightarrow y = \frac{x^2}{2} \Rightarrow y = 1/2$

$D^2 f(x,y) = \begin{pmatrix} -2y & -2x \\ -2x & 2 \end{pmatrix}$ (+2)

$[1, 1/2]:$ $\begin{pmatrix} -1 & -2 \\ -2 & 2 \end{pmatrix} \overset{-2}{\sim} \begin{pmatrix} -1 & -2 \\ 0 & 6 \end{pmatrix} \sim \begin{pmatrix} -1 & 0 \\ 0 & 6 \end{pmatrix}$ ID ... není extrém (+4)