1. $\int \frac{1}{\sin x + \cos x + 2}$ $\overset{(+4)}{\rightarrow} \int \frac{1}{\frac{2t}{1+t^2} + \frac{1-t^2}{1+t^2} + 2} \cdot \frac{2}{1+t^2} \overset{(+1)}{=} \int \frac{2}{t^2+2t+3} =$

$\operatorname{tg} \frac{x}{2} = t$

$\sin x = \frac{2t}{1+t^2}$ $\quad = \int \frac{2}{(t+1)^2+2} = \int \frac{1}{\left(\frac{t+1}{\sqrt{2}}\right)^2+1} \overset{(+4)}{=} \sqrt{2} \operatorname{arctg} \frac{t+1}{\sqrt{2}}$

$\cos x = \frac{1-t^2}{1+t^2}$

$dx = \frac{2}{1+t^2} dt$ $\quad \overset{(+1)}{\Rightarrow} \int f \overset{c}{=} \sqrt{2} \operatorname{arctg} \frac{\operatorname{tg} \frac{x}{2} + 1}{\sqrt{2}}, \quad x \in (-\pi, \pi)$

2. $\int x \sin y \, dx\, dy \overset{(+2)}{=} \int_0^1 x \left[-\cos y\right]_0^{\pi x} \overset{(+2)}{=} \int_0^1 x(1-\cos \pi x) =$

$0 \le x \le 1$ $\quad$ (+1) $\int$ spojitá, nulkpt… $\quad$ $1 \quad \left(x - \frac{\sin \pi x}{\pi}\right)$

$0 \le y \le \pi x$

$\overset{(+2)}{=} \left[x\left(x - \frac{\sin \pi x}{\pi}\right)\right]_0^1 - \int_0^1 \left(x - \frac{\sin \pi x}{\pi}\right) \overset{(+1)}{=} 1 - \int_0^1 x + \int_0^1 \frac{\sin \pi x}{\pi} =$

$\overset{(+1)}{=} \frac{1}{2} + \frac{1}{\pi} \left[\frac{-\cos \pi x}{\pi}\right]_0^1 \overset{(+1)}{=} \frac{1}{2} + \frac{2}{\pi^2}$

3. $A = \begin{pmatrix} 3 & 1 & -1 \\ 0 & 2 & 0 \\ 1 & 1 & 1 \end{pmatrix}$ $\quad \det(\lambda I - A) = \det \begin{pmatrix} \lambda-3 & -1 & 1 \\ 0 & \lambda-2 & 0 \\ -1 & -1 & \lambda-1 \end{pmatrix} =$

$= (\lambda-2) \det \begin{pmatrix} \lambda-3 & 1 \\ -1 & \lambda-1 \end{pmatrix} = (\lambda-2)(\lambda^2-3\lambda+3+1) = (\lambda-2)^3$ (+4)

$\sigma(A) = \{2\}$ (+1)

$\lambda = 2: \begin{pmatrix} -1 & -1 & 1 \\ 0 & 0 & 0 \\ -1 & -1 & 1 \end{pmatrix} \sim \begin{pmatrix} -1 & -1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ (+2)

$\operatorname{Ker}(2I - A) = \operatorname{lin}_{\mathbb{C}} \left\{ \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} \right\}$ (+3)

4. $(1+y)^{1/3} = 1 + \frac{1}{3}y + \binom{1/3}{2}y^2 + \binom{1/3}{3}y^3 + \varphi(y)$, $\quad \frac{\varphi(y)}{y^3} \xrightarrow[y\to 0]{} 0$ (+2)

$\sin z = z - \frac{z^3}{3!} + o(z^4)$

$\sin x^2 = x^2 + o(x^4)$ (+2)

$(1+\sin x^2)^{1/3} = 1 + \frac{1}{3}(x^2 + o(x^4)) + \binom{1/3}{2}(x^2+o(x^4))^2 + \binom{1/3}{3}(x^2+o(x^4))^3 + \varphi(\sin x^2)$ (+2)

$\left[\frac{\varphi(\sin x^2)}{x^4} = \frac{\varphi(\sin x^2)}{(\sin x^2)^3}\,\frac{(\sin x^2)^3}{x^4} \xrightarrow[x\to 0]{} 0 \right]$

$= 1 + \frac{1}{3}x^2 + \binom{1/3}{2}x^4 + o(x^4) = 1 + \frac{1}{3}x^2 - \frac{1}{9}x^4 + o(x^4)$ (+2)

$\lim_{x\to 0} \frac{1}{x^4}\left((1+\sin x^2)^{1/3} - 1 - \frac{x^2}{3}\right) = \lim_{x\to 0}\left(-\frac{1}{9} + \frac{o(x^4)}{x^4}\right) = -\frac{1}{9}$ (+2)

5. $f(x,y) = xy(2-x^2-y^2)$

$f_x = y(2-3x^2-y^2) \Rightarrow [0,0],\ [0, \pm\sqrt{2}],\ [\pm\sqrt{2}, 0],\ [\pm\sqrt{\tfrac{1}{2}}, \pm\sqrt{\tfrac{1}{2}}]$ (13)

$f_y = x(2-3y^2-x^2)$

$f_{xx} = -6xy$

$f_{yy} = -6xy$ (+2)

$f_{xy} = f_{yx} = 2 - 3x^2 - 3y^2$

$[0,0]$ (+1): $\begin{pmatrix} 0 & 2 \\ 2 & 0 \end{pmatrix} \sim \begin{pmatrix} 4 & 2 \\ 2 & 0 \end{pmatrix} \sim \begin{pmatrix} 4 & 0 \\ 0 & -1 \end{pmatrix}$ ID

$[0, \pm\sqrt{2}]$ (+1): $\begin{pmatrix} 0 & -4 \\ -4 & 0 \end{pmatrix} \sim \begin{pmatrix} -8 & -4 \\ -4 & 0 \end{pmatrix} \sim \begin{pmatrix} -8 & 0 \\ 0 & 2 \end{pmatrix}$ ID

$[\pm\sqrt{2}, 0]$ (+1): $\begin{pmatrix} 0 & -4 \\ -4 & 0 \end{pmatrix} \sim \begin{pmatrix} -8 & 0 \\ 0 & 2 \end{pmatrix}$ ID

$[\pm\sqrt{\tfrac{1}{2}}, \pm\sqrt{\tfrac{1}{2}}]$ (+1): $\begin{pmatrix} -6\cdot\frac{1}{2} & -1 \\ -1 & -6\cdot\frac{1}{2} \end{pmatrix} \sim \begin{pmatrix} -3 & -1 \\ -1 & -3 \end{pmatrix} \sim$

$\begin{pmatrix} -3 & 0 \\ 0 & -8/3 \end{pmatrix}$ ND

$[\pm\sqrt{\tfrac{1}{2}}, \mp\sqrt{\tfrac{1}{2}}]$ (+1): $\begin{pmatrix} 6\cdot\frac{1}{2} & -1 \\ -1 & 6\cdot\frac{1}{2} \end{pmatrix} \sim \begin{pmatrix} 3 & -1 \\ -1 & 3 \end{pmatrix} \sim \begin{pmatrix} 3 & 0 \\ 0 & 8/3 \end{pmatrix}$ PD