Určete definitnost následujících matic:

$$\textbf{1.}\begin{pmatrix}1&2&1\\2&2&1\\1&1&1\end{pmatrix},\quad \textbf{2.}\begin{pmatrix}-1&-1&-1\\-1&-1&0\\-1&0&1\end{pmatrix},\quad \textbf{3.}\begin{pmatrix}2&1&1\\1&1&2\\1&2&5\end{pmatrix},\quad \textbf{4.}\begin{pmatrix}6&0&3\\0&3&3\\3&3&5\end{pmatrix},\quad \textbf{5.}\begin{pmatrix}7&0&4&2\\0&3&3&2\\4&3&6&3\\2&2&3&2\end{pmatrix}$$

$$\textbf{6.}\begin{pmatrix}2&-3&1&-1\\-3&8&3&0\\1&3&5&-2\\-1&0&-2&1\end{pmatrix},\quad \textbf{7.}\begin{pmatrix}2&1&1&-1\\1&-7&1&-2\\1&1&-10&0\\-1&-2&0&-2\end{pmatrix},\quad \textbf{8.}\begin{pmatrix}3&4&4&3\\4&6&5&6\\4&5&6&2\\3&6&2&11\end{pmatrix},\quad \textbf{9.}\begin{pmatrix}-3&-4&4&-3\\-4&-6&5&-6\\4&5&-6&2\\-3&-6&2&-11\end{pmatrix}.$$

Výsledky: **1.** ID, **2.** ID, **3.** PSD, **4.** PD, **5.** PD, **6.** ID, **7.** ND, **8.** ID, **9.** NSD.

V závislosti na parametru $a \in \mathbb{R}$ určete definitnost následujících matic:

$$\textbf{1.}\begin{pmatrix}337&338&400&398\\338&415&371&399\\400&371&333&343\\398&399&343&a\end{pmatrix},\quad \textbf{2.}\begin{pmatrix}3&3&1&2\\3&4&2&-1\\1&2&3&a\\2&-1&a&13\end{pmatrix}.$$

Výsledky: **1.** vždy ID, **2.** PD pro $a \in \left(-\frac{7}{3}-\frac{2}{3}\sqrt{10}, -\frac{7}{3}+\frac{2}{3}\sqrt{10}\right)$, ID pro $a \in \left(-\infty, -\frac{7}{3}-\frac{2}{3}\sqrt{10}\right) \cup \left(-\frac{7}{3}+\frac{2}{3}\sqrt{10}, +\infty\right)$, PSD pro $a = -\frac{7}{3} \pm \frac{2}{3}\sqrt{10}$.

Najděte vlastní čísla a jim příslušné vlastní vektory pro následující matice:

$$\textbf{1.}\begin{pmatrix}3&-1\\2&0\end{pmatrix},\quad \textbf{2.}\begin{pmatrix}1&3\\2&2\end{pmatrix},\quad \textbf{3.}\begin{pmatrix}1&1\\-1&1\end{pmatrix},\quad \textbf{4.}\begin{pmatrix}1&2\\-2&5\end{pmatrix},\quad \textbf{5.}\begin{pmatrix}0&a\\a&0\end{pmatrix},\quad \textbf{6.}\begin{pmatrix}-2&1&2\\-4&3&2\\-5&1&5\end{pmatrix}$$

$$\textbf{7.}\begin{pmatrix}5&0&-2\\2&3&-2\\4&0&-1\end{pmatrix},\quad \textbf{8.}\begin{pmatrix}8&-1&-3\\8&1&-4\\7&-1&-2\end{pmatrix},\quad \textbf{9.}\begin{pmatrix}3&-1&-1\\6&-2&-2\\3&-1&-1\end{pmatrix},\quad \textbf{10.}\begin{pmatrix}2&0&-1\\5&-1&-2\\1&1&-1\end{pmatrix},$$

$$\textbf{11.}\begin{pmatrix}-2&0&-2&3\\-2&1&-1&2\\-2&0&0&2\\-5&0&-3&6\end{pmatrix},\quad \textbf{12.}\begin{pmatrix}-23&21&3&-17\\-40&35&4&-31\\58&-50&-5&47\\-8&6&0&-7\end{pmatrix}.$$

Výsledky (vlastní číslo, násobnost, vlastní vektory):

1. $(1,1,\{[t,2t];\ t\in\mathbb{C}\setminus\{0\}\}), (2,1,\{[t,t];\ t\in\mathbb{C}\setminus\{0\}\})$, **2.** $(4,1,\{[t,t];\ t\in\mathbb{C}\setminus\{0\}\}), (-1,1,\{[3t,-2t];\ t\in\mathbb{C}\setminus\{0\}\})$, **3.** $(1+i,1,\{[t,ti];\ t\in\mathbb{C}\setminus\{0\}\}), (1-i,1,\{[t,-ti];\ t\in\mathbb{C}\setminus\{0\}\})$, **4.** $(3,2,\{[t,t];\ t\in\mathbb{C}\setminus\{0\}\})$, **5.** pro $a\neq 0$: $(a,1,\{[t,-t];\ t\in\mathbb{C}\setminus\{0\}\}), (-a,1,\{[t,t];\ t\in\mathbb{C}\setminus\{0\}\})$, pro $a=0$: $(0,2,\{[s,t];\ [s,t]\in\mathbb{C}^2\setminus\{[0,0]\}\})$, **6.** $(1,1,\{[t,t,t];\ t\in\mathbb{C}\setminus\{0\}\}), (2,1,\{[t,2t,t];\ t\in\mathbb{C}\setminus\{0\}\}), (3,1,\{[t,t,2t];\ t\in\mathbb{C}\setminus\{0\}\})$, **7.** $(1,1,\{[t,t,2t];\ t\in\mathbb{C}\setminus\{0\}\}), (3,2,\{[t,s,t];\ [s,t]\in\mathbb{C}^2\setminus\{[0,0]\}\})$, **8.** $(1,1,\{[t,t,2t];\ t\in\mathbb{C}\setminus\{0\}\}), (3,2,\{[t,2t,t];\ t\in\mathbb{C}\setminus\{0\}\})$, **9.** $(0,3,\{[t,3t-s,s];\ [s,t]\in\mathbb{C}^2\setminus\{[0,0]\}\})$, **10.** $(0,3,\{[t,t,2t];\ t\in\mathbb{C}\setminus\{0\}\})$, **11.** $(2,1,\{[t,t,t,2t];\ t\in\mathbb{C}\setminus\{0\}\}), (1,3,\{[s,t,0,s];\ [s,t]\in\mathbb{C}^2\setminus\{[0,0]\}\})$, **12.** $(i,2,\{[3t-(7+i)s,4t,(-5+i)t-(8+10i)s,8s];\ [s,t]\in\mathbb{C}^2\setminus\{[0,0]\}\}), (-i,2,\{[3t+(-7+i)s,4t,-(5+i)t+(-8+10i)s,8s];\ [s,t]\in\mathbb{C}^2\setminus\{[0,0]\}\})$.

Určete definitnost následujících matic:

$$\textbf{1.}\begin{pmatrix}1&2&1\\2&2&1\\1&1&1\end{pmatrix},\quad \textbf{2.}\begin{pmatrix}-1&-1&-1\\-1&-1&0\\-1&0&1\end{pmatrix},\quad \textbf{3.}\begin{pmatrix}2&1&1\\1&1&2\\1&2&5\end{pmatrix},\quad \textbf{4.}\begin{pmatrix}6&0&3\\0&3&3\\3&3&5\end{pmatrix},\quad \textbf{5.}\begin{pmatrix}7&0&4&2\\0&3&3&2\\4&3&6&3\\2&2&3&2\end{pmatrix}$$

$$\textbf{6.}\begin{pmatrix}2&-3&1&-1\\-3&8&3&0\\1&3&5&-2\\-1&0&-2&1\end{pmatrix},\quad \textbf{7.}\begin{pmatrix}2&1&1&-1\\1&-7&1&-2\\1&1&-10&0\\-1&-2&0&-2\end{pmatrix},\quad \textbf{8.}\begin{pmatrix}3&4&4&3\\4&6&5&6\\4&5&6&2\\3&6&2&11\end{pmatrix},\quad \textbf{9.}\begin{pmatrix}-3&-4&4&-3\\-4&-6&5&-6\\4&5&-6&2\\-3&-6&2&-11\end{pmatrix}.$$

Výsledky: **1.** ID, **2.** ID, **3.** PSD, **4.** PD, **5.** PD, **6.** ID, **7.** ND, **8.** ID, **9.** NSD.

V závislosti na parametru $a \in \mathbb{R}$ určete definitnost následujících matic:

$$\textbf{1.}\begin{pmatrix}337&338&400&398\\338&415&371&399\\400&371&333&343\\398&399&343&a\end{pmatrix},\qquad \textbf{2.}\begin{pmatrix}3&3&1&2\\3&4&2&-1\\1&2&3&a\\2&-1&a&13\end{pmatrix}.$$

Výsledky: **1.** vždy ID, **2.** PD pro $a \in \left(-\frac{7}{3}-\frac{2}{3}\sqrt{10}, -\frac{7}{3}+\frac{2}{3}\sqrt{10}\right)$, ID pro $a \in \left(-\infty, -\frac{7}{3}-\frac{2}{3}\sqrt{10}\right) \cup \left(-\frac{7}{3}+\frac{2}{3}\sqrt{10}, +\infty\right)$, PSD pro $a = -\frac{7}{3} \pm \frac{2}{3}\sqrt{10}$.

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$$\textbf{1.}\begin{pmatrix}3&-1\\2&0\end{pmatrix},\quad \textbf{2.}\begin{pmatrix}1&3\\2&2\end{pmatrix},\quad \textbf{3.}\begin{pmatrix}1&1\\-1&1\end{pmatrix},\quad \textbf{4.}\begin{pmatrix}1&2\\-2&5\end{pmatrix},\quad \textbf{5.}\begin{pmatrix}0&a\\a&0\end{pmatrix},\quad \textbf{6.}\begin{pmatrix}-2&1&2\\-4&3&2\\-5&1&5\end{pmatrix}$$

$$\textbf{7.}\begin{pmatrix}5&0&-2\\2&3&-2\\4&0&-1\end{pmatrix},\quad \textbf{8.}\begin{pmatrix}8&-1&-3\\8&1&-4\\7&-1&-2\end{pmatrix},\quad \textbf{9.}\begin{pmatrix}3&-1&-1\\6&-2&-2\\3&-1&-1\end{pmatrix},\quad \textbf{10.}\begin{pmatrix}2&0&-1\\5&-1&-2\\1&1&-1\end{pmatrix},$$

$$\textbf{11.}\begin{pmatrix}-2&0&-2&3\\-2&1&-1&2\\-2&0&0&2\\-5&0&-3&6\end{pmatrix},\qquad \textbf{12.}\begin{pmatrix}-23&21&3&-17\\-40&35&4&-31\\58&-50&-5&47\\-8&6&0&-7\end{pmatrix}.$$

Výsledky (vlastní číslo, násobnost, vlastní vektory):

1. $(1, 1, \{[t, 2t];\ t \in \mathbb{C}\setminus\{0\}\}), (2, 1, \{[t, t];\ t \in \mathbb{C}\setminus\{0\}\})$, **2.** $(4, 1, \{[t, t];\ t \in \mathbb{C}\setminus\{0\}\}), (-1, 1, \{[3t, -2t];\ t \in \mathbb{C}\setminus\{0\}\})$, **3.** $(1+i, 1, \{[t, ti];\ t \in \mathbb{C}\setminus\{0\}\}), (1-i, 1, \{[t, -ti];\ t \in \mathbb{C}\setminus\{0\}\})$, **4.** $(3, 2, \{[t, t];\ t \in \mathbb{C}\setminus\{0\}\})$, **5.** pro $a \neq 0$: $(a, 1, \{[t, -t];\ t \in \mathbb{C}\setminus\{0\}\}), (-a, 1, \{[t, t];\ t \in \mathbb{C}\setminus\{0\}\})$, pro $a = 0$: $(0, 2, \{[s, t];\ [s, t] \in \mathbb{C}^2\setminus\{[0, 0]\}\})$, **6.** $(1, 1, \{[t, t, t];\ t \in \mathbb{C}\setminus\{0\}\}), (2, 1, \{[t, 2t, t];\ t \in \mathbb{C}\setminus\{0\}\}), (3, 1, \{[t, t, 2t];\ t \in \mathbb{C}\setminus\{0\}\})$, **7.** $(1, 1, \{[t, t, 2t];\ t \in \mathbb{C}\setminus\{0\}\}), (3, 2, \{[t, s, t];\ [s, t] \in \mathbb{C}^2\setminus\{[0, 0]\}\})$, **8.** $(1, 1, \{[t, t, 2t];\ t \in \mathbb{C}\setminus\{0\}\}), (3, 2, \{[t, 2t, t];\ t \in \mathbb{C}\setminus\{0\}\})$, **9.** $(0, 3, \{[t, 3t-s, s];\ [s, t] \in \mathbb{C}^2\setminus\{[0, 0]\}\})$, **10.** $(0, 3, \{[t, t, 2t];\ t \in \mathbb{C}\setminus\{0\}\})$, **11.** $(2, 1, \{[t, t, t, 2t];\ t \in \mathbb{C}\setminus\{0\}\}), (1, 3, \{[s, t, 0, s];\ [s, t] \in \mathbb{C}^2\setminus\{[0, 0]\}\})$, **12.** $(i, 2, \{[3t-(7+i)s, 4t, (-5+i)t-(8+10i)s, 8s];\ [s, t] \in \mathbb{C}^2\setminus\{[0, 0]\}\}), (-i, 2, \{[3t+(-7+i)s, 4t, -(5+i)t+(-8+10i)s, 8s];\ [s, t] \in \mathbb{C}^2\setminus\{[0, 0]\}\})$.