

5

$$1) \quad X \begin{pmatrix} 1 & -1 & 1 \\ 1 & 0 & -1 \\ 2 & -1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & -1 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 & 2 \\ -1 & 0 & -1 \\ 1 & -1 & 0 \end{pmatrix} X^T = \begin{pmatrix} 0 & 0 & 0 \\ -1 & 0 & 0 \\ 2 & 0 & 0 \end{pmatrix}$$

$$\left(\begin{array}{ccc|ccc} 1 & 1 & 2 & 0 & 0 & 0 \\ -1 & 0 & -1 & -1 & 0 & 0 \\ 1 & -1 & 0 & 2 & 0 & 0 \end{array} \right) \sim \left(\begin{array}{ccc|ccc} 1 & 1 & 2 & 0 & 0 & 0 \\ 0 & 1 & 1 & -1 & 0 & 0 \\ 0 & -2 & -2 & 2 & 0 & 0 \end{array} \right) \sim \left(\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & -1 & 0 & 0 \end{array} \right)$$

$$X^T = \begin{pmatrix} 1 & 0 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} s & s & -s \\ \pi & \pi & -\pi \\ \epsilon & \epsilon & -\epsilon \end{pmatrix}^T$$

$$X = \begin{pmatrix} s+1 & s-1 & -s \\ \pi & \pi & -\pi \\ \epsilon & \epsilon & -\epsilon \end{pmatrix} \quad \text{non 'régulier'}$$

$$2) \quad \begin{pmatrix} 1 & -1 & -1 & 2 \\ 0 & 1 & 3 & -1 \\ 1 & 1 & 1 & 0 \end{pmatrix} \sim \begin{pmatrix} 1 & -1 & -1 & 2 \\ 0 & 1 & 3 & -1 \\ 0 & 2 & 2 & -2 \end{pmatrix} \quad \dim = 3$$

$$\left(\begin{array}{cccc} 1 & -1 & -1 & 2 \\ 0 & 1 & 3 & -1 \\ 0 & 1 & 1 & -1 \\ 1 & 0 & 2 & 1 \end{array} \right) \sim \left(\begin{array}{cccc} 1 & -1 & -1 & 2 \\ 0 & 1 & 3 & -1 \\ 0 & 0 & 2 & 0 \\ 0 & 1 & 3 & -1 \end{array} \right)$$

$$\Psi = \langle (1, 0, 2, 1), (0, 1, 3, -1), (0, 0, 1, 0) \rangle$$

$$\left(\begin{array}{cccc} 1 & 0 & 2 & 1 \\ 0 & 1 & 3 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & -1 \end{array} \right) \sim \left(\begin{array}{cccc} 1 & 0 & 2 & 1 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & 1 & 0 \end{array} \right)$$

$$\{x^3 + 2x + 1, x^2 + x - 1, x\}$$

$$3) \quad \begin{pmatrix} 1 & 1 & 1 & 2 \\ -1 & 0 & 1 & 1 \\ 2 & 1 & 0 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 1 & 2 \\ 0 & 1 & 2 & 3 \\ 0 & -1 & -2 & -3 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 1 & 2 \\ 0 & 1 & 2 & 3 \\ 0 & 1 & 2 & 3 \end{pmatrix} \quad \dim V = 2$$

$$\left(\begin{array}{cccc} 1 & 1 & -1 & -1 \\ 3 & 1 & -1 & 0 \\ -1 & a & 3 & 4 \end{array} \right) \sim \left(\begin{array}{cccc} 1 & 1 & -1 & -1 \\ 0 & -2 & 2 & 3 \\ 0 & a+1 & 2 & 3 \end{array} \right)$$

$$a = -3 \Rightarrow \dim W = 2$$

$$a \neq -3 \Rightarrow \dim W = 3$$

$$\left(\begin{array}{cccc} 1 & 1 & -1 & -1 \\ 1 & 1 & 1 & 2 \\ 0 & 1 & 2 & 3 \\ 0 & -2 & 2 & 3 \\ 0 & a+1 & 2 & 3 \end{array} \right) \sim \left(\begin{array}{cccc} 1 & 1 & -1 & -1 \\ 0 & 0 & 2 & 3 \\ 0 & 1 & 0 & 0 \\ 0 & a+1 & 0 & 0 \end{array} \right)$$

$$\dim V \cup W = 3$$

$$\dim V \cap W = 1 \text{ pro } a = -3$$

$$= 2 \text{ pro } a \in \mathbb{R} - \{-3\}$$