On the Dependence of Solutions and Eigenvalues on Potentials, Weights and Measures

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Abstract

Differential equations usually contain finitely or infinitely dimensional parameters. In this talk I will address how solutions and eigenvalues will depend on infinitely dimensional parameters like potentials, weights, densities or distributions (measures). By considering the second-order ordinary differential equations and their generalizations, I will show that solutions and eigenvalues will continuously depend on these parameters in a very strong way, i.e. even when the weak topologies are considered, solutions and eigenvalues are continuous in these infinitely dimensional parameters. As an application, I will explain how such a strong continuity can be used to solve some infinitely dimensional extremal problems on eigenvalues which will yield optimal estimation of eigenvalues.

The contents of the talk are: 1. Introduction of the problems; 2. Weak topologies; 3. Dependence of solutions; 4. Dependence of eigenvalues; 5. An application to extremal problems of eigenvalues; 6. Extension to measure differential equations.