On Lyapunov stability/instability of equilibria of free damped pendulum with vertically oscillating suspension point

Jiří Šremr

We consider the free damped pendulum with vertically oscillating suspension point, whose equation of motion is of the form

$$\varphi'' + \frac{b}{m}\,\varphi' + \left(\frac{\mathrm{g}}{\ell} + \frac{\psi''(t)}{\ell}\right)\sin\varphi = 0. \tag{1}$$

Here, $\psi \in C^2(\mathbb{R})$ is a T-periodic function determining the oscillations of the suspension point.

This system is also known as Kaptiza's pendulum or the inverted pendulum. We first recall the results of Bogolyubov and Kapitza, then we present new effective criteria for stability/instability of the equilibria of (1) with their exact and complete proofs. Furthermore, we show that if both lower and upper equilibria are asymptotically stable, then the pendulum considered may possess a periodic motion that corresponds to the "quasistatic solution" of Bogolyubov as well as to the "quasistatic balance" of Kapitza.