Rubik cube: Introduction

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Rubik cube: The first approach

Names of faces of the Rubik cube: **U**p,**D**own,**F**ront,**B**ack,**R**ight and **L**eft.

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Names of faces of the Rubik cube: **U**p,**D**own,**F**ront,**B**ack,**R**ight and **L**eft.
The rotations of the U face is denoted as U.

Action of a group on a set

Definition

Let X be a set and G be a group. We say that G **acts on the set** X on the right provided by following conditions hold:

1. Each g belonging to G gives rise to a function

$$\phi_g : X \to X$$

2. The identity of G defines the identity function on X.
3. For $g, h \in G$ then $\phi_{gh}(x) = \phi_h(\phi_g(x))$.

Permutations

The relative position of the centers of the faces of the Rubik cube is not changed by any move.

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The Rubik cube group is

$$< U, D, F, B, R, L > \subset S_{54}$$

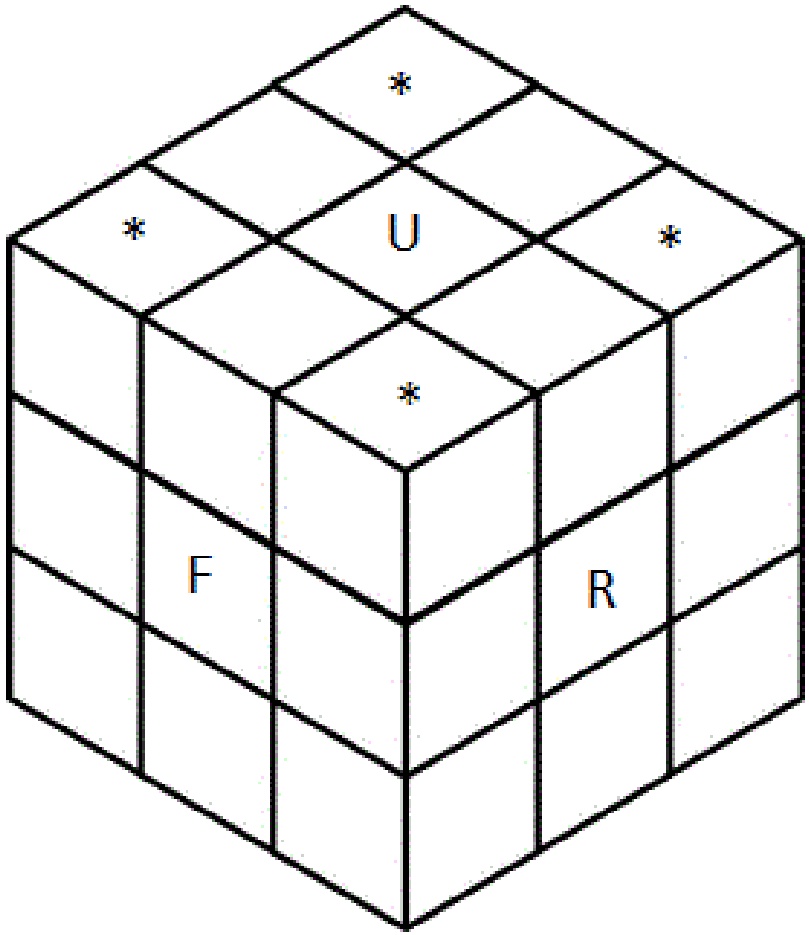
The second approach

Divide and conquer:

The centers are fixed. There remain edge and corner subcubes.

Orientation of vertices

Figure: Orientation of vertices the Rubik Cube



Semi-direct product

Definition

Let G, H are groups and ϕ a homomorphism

$$\phi : H \to Aut(G).$$

We define a multiplication on the set $G \times H$ by

$$(x_1, y_1) \cdot (x_2, y_2) = (x_1 \cdot \phi(y_1)(x_2), y_1 \cdot y_2).$$

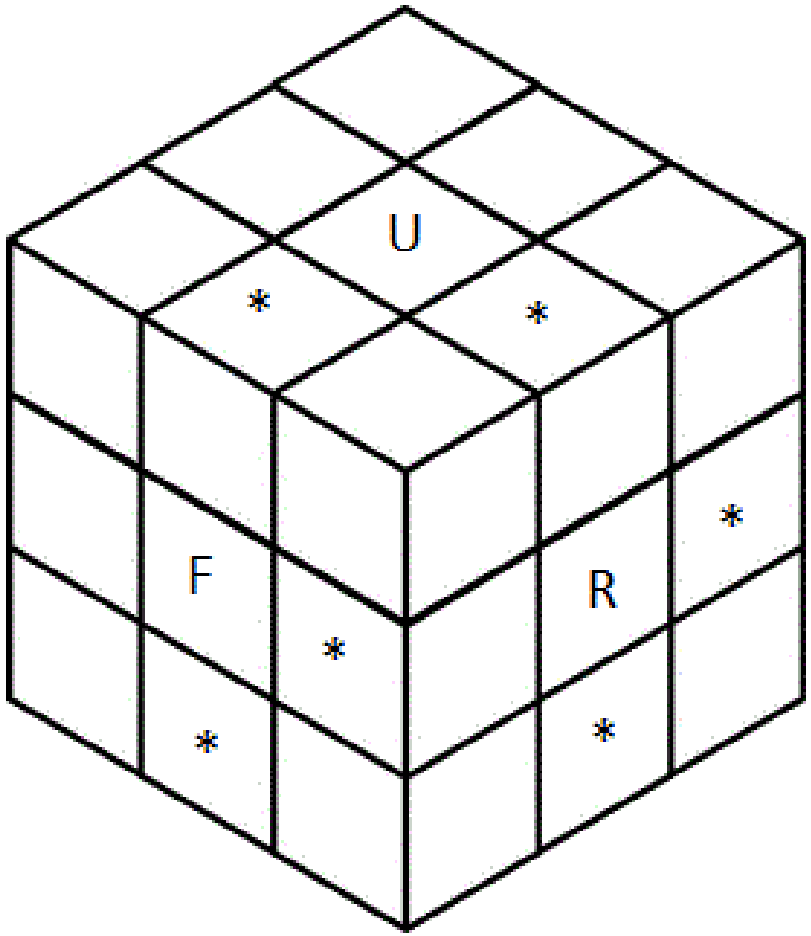
We get a group called semi-direct product, denoted $G \rtimes H$.

Group of Rubik corner subcubes

$$C_3^8 \rtimes S_8$$

Orientation of edges

Figure: Orientation of edges of the Rubik Cube



Group of Rubik edge subcubes

$$C_2^{12} \rtimes S_{12}$$

Direct Product

The direct product of two group S, T is the group

$$S \times T = \{(s, t) | s \in S, t \in T\}$$

with intuitive operations.

Moves change the orientation

Corner subcubes: $v : H \to C_3^8$.
$v(F) = (2, 0, 1, 0, 1, 0, 0, 2)$
$v(U) = (0, 0, 0, 0, 0, 0, 0, 0)$
$v(R) = (1, 2, 2, 1, 0, 0, 0, 0)$

Moves change the orientation

Edge subcubes: $v : H \to C_2^{12}$.

$v(F) = (1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0)$

$v(U) = (1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0)$

$v(R) = (0, 1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0)$

First Fundamental Theorem

A position of the Rubik cube is determined by

- permutation of center subcubes
- permutation of edge subcubes
- permutation of corner subcubes
- flips of corner markings
- flips of edge markings

Rubik Group

The Rubik cube group is

$$< U, D, F, B, R, L > \subset \left(C_3^8 \rtimes S_8\right) \times \left(C_2^{12} \rtimes S_{12}\right)$$

Thank you for your attention!
Any Questions? Or can I offer some interesting facts...

Interesting facts I

Ernő Rubik, born 13th July 1944 in Budapest, Hungarian inventor, architect and professor of architecture.
Inventions: Rubik cube, Rubik's tower, Rubik's magic,...

Interesting facts II

Really a Magic cube:
How many moves we have to do to solve the Rubik cube ?

Interesting facts II

Really a Magic cube:
How many moves we have to do to solve the Rubik cube ?
There is over $43 \times 10^{1}8$ possible positions, 43 quintillion.

Interesting facts III

Some numbers:
$5,55s$ solved a single cube by Mats Valk
$9,03s$ solved one-handed by Feliks Zemdegs
$23.19s$ blindfolded(including memorization) by Marcin Kowalczyk
41 of 41multiple blindfolded by Marcin Kowalczyk