

In[138]:=

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B[n_, i_] := Binomial[n, i] * (t^i) * (1 - t)^(n - i)
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In[139]:=

```
P = {{0, 0}, {2, -1}, {3, 3}, {-1, 4}, {-3, 0}, {-1, 0}}
n = Length[P] - 1
pl1 = Graphics[Line[P]];
c = Sum[P[[i + 1]] * B[n, i], {i, 0, n}]
pl2 = ParametricPlot[c, {t, 0, 1}];
Show[pl1, pl2]
rychlost = Simplify[Sqrt[(D[c, t].D[c, t])]]
kz = Simplify[Det[{D[c, t], D[c, {t, 2}]}] / rychlost^3]
```

Out[139]=

```
{{0, 0}, {2, -1}, {3, 3}, {-1, 4}, {-3, 0}, {-1, 0}}
```

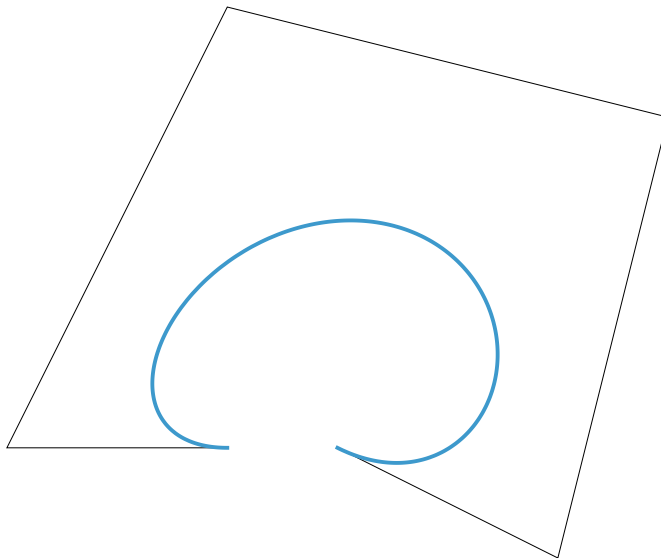
Out[140]=

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5
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Out[142]=

```
{10 (1 - t)^4 t + 30 (1 - t)^3 t^2 - 10 (1 - t)^2 t^3 - 15 (1 - t) t^4 - t^5,
-5 (1 - t)^4 t + 30 (1 - t)^3 t^2 + 40 (1 - t)^2 t^3}
```

Out[144]=



Out[145]=

$$5 \sqrt{5 - 56 t + 416 t^2 - 1600 t^3 + 3414 t^4 - 4088 t^5 + 2800 t^6 - 1168 t^7 + 281 t^8}$$

Out[146]=

$$\frac{4 (9 - 60 t + 237 t^2 - 494 t^3 + 609 t^4 - 444 t^5 + 151 t^6)}{5 (5 - 56 t + 416 t^2 - 1600 t^3 + 3414 t^4 - 4088 t^5 + 2800 t^6 - 1168 t^7 + 281 t^8)^{3/2}}$$

In[147]:=

```
t0 = 4 / 5
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Out[147]=

```
 $\frac{4}{5}$ 
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In[148]:=

kz0 = kz /. {t → t0}

Out[148]=

$$\frac{643\,012\,500}{882\,181 \sqrt{882\,181}}$$

In[149]:=

c0 = c /. {t → t0}**n0 = {{0, -1}, {1, 0}}.Normalize[D[c, t] /. {t → t0}]****kz0****R0 = 1 / kz0****S0 = c0 + n0 * R0****k = S0 + R0 * {Cos[s], Sin[s]}**

Out[149]=

$$\left\{ -\frac{4984}{3125}, \frac{604}{625} \right\}$$

Out[150]=

$$\left\{ \frac{865}{\sqrt{882\,181}}, -\frac{366}{\sqrt{882\,181}} \right\}$$

Out[151]=

$$\frac{643\,012\,500}{882\,181 \sqrt{882\,181}}$$

Out[152]=

$$\frac{882\,181 \sqrt{882\,181}}{643\,012\,500}$$

Out[153]=

$$\left\{ -\frac{262\,441\,211}{643\,012\,500}, \frac{49\,754\,839}{107\,168\,750} \right\}$$

Out[154]=

$$\left\{ -\frac{262\,441\,211}{643\,012\,500} + \frac{882\,181 \sqrt{882\,181} \cos[s]}{643\,012\,500}, \frac{49\,754\,839}{107\,168\,750} + \frac{882\,181 \sqrt{882\,181} \sin[s]}{643\,012\,500} \right\}$$

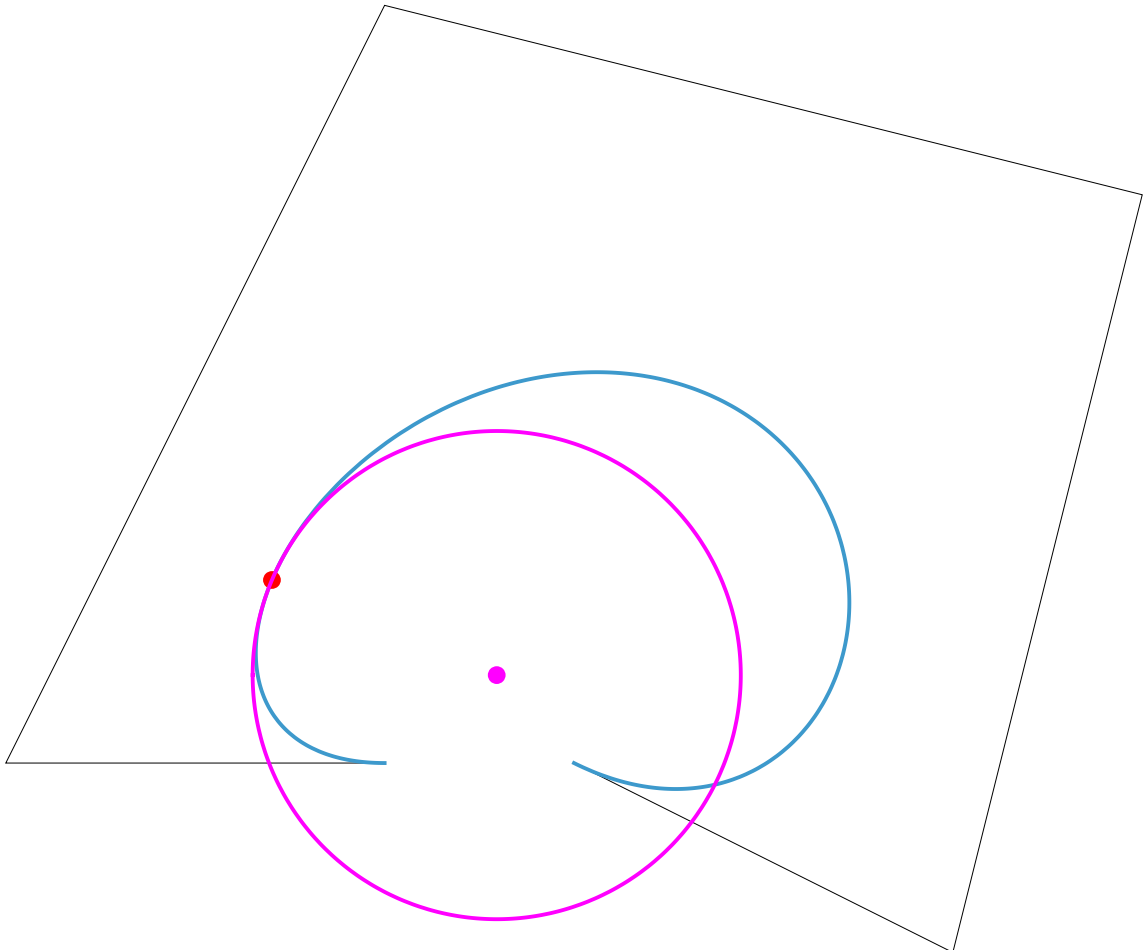
In[155]:=

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pl3 = Graphics[{PointSize[0.015], Red, Point[c0]}];
pl4 = ParametricPlot[k, {s, -Pi, Pi}, PlotStyle -> Magenta];
pl45 = Graphics[{Magenta, PointSize[0.015], Point[S0]}];
Show[pl1, pl2, pl3, pl4, pl45]

```

Out[158]=



In[159]:=

```

N0 = c0 + s * n0
pl5 = ParametricPlot[N0, {s, -0.1, 2}, PlotStyle -> Black];
Show[pl2, pl3, pl4, pl45, pl5, PlotRange -> All];
NN = c + s * {{0, -1}, {1, 0}}.(D[c, t] / rychlost)
Prusecik = FullSimplify[NN /. Solve[(N0 /. {s -> u}) == NN, {s, u}][[1]]]
Manipulate[Show[pl2, pl3, pl4, pl45, pl5,
  ParametricPlot[NN /. {t -> T}, {s, -0.1, 2}, PlotStyle -> Blue],
  Graphics[{Blue, PointSize[0.015], Point[Prusecik /. {t -> T}]}],
  PlotRange -> All], {T, t0 - 0.1, t0 + 0.1}]

```

Out[159]=

$$\left\{ -\frac{4984}{3125} + \frac{865 s}{\sqrt{882181}}, \frac{604}{625} - \frac{366 s}{\sqrt{882181}} \right\}$$

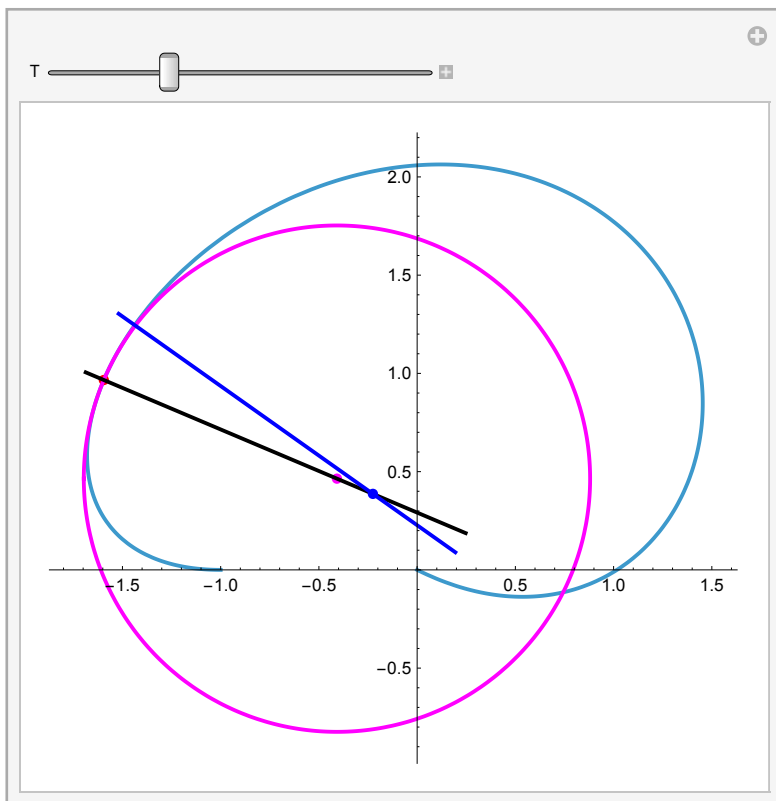
Out[162]=

$$\left\{ \frac{10(1-t)^4 t + 30(1-t)^3 t^2 - 10(1-t)^2 t^3 - 15(1-t)t^4 - t^5 - s(-5(1-t)^4 + 80(1-t)^3 t + 30(1-t)^2 t^2 - 80(1-t)t^3)}{5\sqrt{5-56t+416t^2-1600t^3+3414t^4-4088t^5+2800t^6-1168t^7+281t^8}}, \right. \\ \left. \frac{-5(1-t)^4 t + 30(1-t)^3 t^2 + 40(1-t)^2 t^3 + s(10(1-t)^4 + 20(1-t)^3 t - 120(1-t)^2 t^2 - 40(1-t)t^3 + 10t^4)}{5\sqrt{5-56t+416t^2-1600t^3+3414t^4-4088t^5+2800t^6-1168t^7+281t^8}} \right\}$$

Out[163]=

$$\left\{ (197039 + t(13200050 + t(-115956128 + t(553138404 - 865t(1701806 + 5t(-467986 + 5t(66316 + 5t(-5446 + 1405t)))))))) / (6250(262 + t(-1020 + t(-1674 + 1567t))))), (197039 + t(-3721998 + t(23006472 + t(-115594552 + 183t(1701806 + 5t(-467986 + 5t(66316 + 5t(-5446 + 1405t)))))))) / (3125(262 + t(-1020 + t(-1674 + 1567t)))) \right\}$$

Out[164]=



In[165]:=

Prusecik /. {t -> t0}

S0

Out[165]=

$$\left\{ -\frac{262441211}{643012500}, \frac{49754839}{107168750} \right\}$$

Out[166]=

$$\left\{ -\frac{262441211}{643012500}, \frac{49754839}{107168750} \right\}$$

In[167]:=

kz * rychlost

Out[167]=

$$\frac{4 (9 - 60 t + 237 t^2 - 494 t^3 + 609 t^4 - 444 t^5 + 151 t^6)}{5 - 56 t + 416 t^2 - 1600 t^3 + 3414 t^4 - 4088 t^5 + 2800 t^6 - 1168 t^7 + 281 t^8}$$

In[168]:=

uh = Integrate[kz * rychlost, {t, 0, 1}]**N[uh]**

Out[168]=

$$\frac{5 \pi}{2} - \text{ArcCot}\left[\frac{1}{2}\right]$$

Out[169]=

6.74683

In[170]:=

T0 = (D[c, t] / rychlost) /. {t -> 0}**T1 = (D[c, t] / rychlost) /. {t -> 1}****N[(2 Pi + ArcCos[T0.T1])]****FullSimplify[uh - (2 Pi + ArcCos[T0.T1])]**

Out[170]=

$$\left\{ \frac{2}{\sqrt{5}}, -\frac{1}{\sqrt{5}} \right\}$$

Out[171]=

{1, 0}

Out[172]=

6.74683

Out[173]=

0

In[174]:=

Q = Reverse[P]**cQ = Sum[Q[[i + 1]] * B[n, i], {i, 0, n}]****(c /. {t -> (1 - t)}) - cQ**

Out[174]=

{{-1, 0}, {-3, 0}, {-1, 4}, {3, 3}, {2, -1}, {0, 0}}

Out[175]=

$$\left\{ -(1-t)^5 - 15(1-t)^4 t - 10(1-t)^3 t^2 + 30(1-t)^2 t^3 + 10(1-t) t^4, \right. \\ \left. 40(1-t)^3 t^2 + 30(1-t)^2 t^3 - 5(1-t) t^4 \right\}$$

Out[176]=

{0, 0}

In[177]:=

In[178]:=

hodograph = D[c, t]

Out[178]=

$$\left\{ 10(1-t)^4 + 20(1-t)^3 t - 120(1-t)^2 t^2 - 40(1-t) t^3 + 10 t^4, \right. \\ \left. -5(1-t)^4 + 80(1-t)^3 t + 30(1-t)^2 t^2 - 80(1-t) t^3 \right\}$$

In[179]:=

```
Show[pl2, ParametricPlot[hodograph, {t, 0, 1}], PlotRange -> All]
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Out[179]=

