Goniometrické a hyperbolické funkce (porovnání)

$$\sin x = \frac{e^{ix} - e^{-ix}}{2i} \qquad\qquad \sinh x = \frac{e^x - e^{-x}}{2}$$

$$\cos x = \frac{e^{ix} + e^{-ix}}{2} \qquad\qquad \cosh x = \frac{e^x + e^{-x}}{2}$$

$$e^{ix} = \cos x + i \sin x \qquad\qquad e^x = \cosh x + \sinh x$$

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$$\sin^2 x + \cos^2 x = 1 \qquad\qquad \cosh^2 x - \sinh^2 x = 1$$

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$$\sin 2x = 2 \sin x \cos x \qquad\qquad \sinh 2x = 2 \sinh x \cosh x$$

$$\cos 2x = \cos^2 x - \sin^2 x \qquad\qquad \cosh 2x = \cosh^2 x + \sinh^2 x$$

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$$\sin^2 \frac{x}{2} = \frac{1 - \cos x}{2} \qquad\qquad \sinh^2 \frac{x}{2} = \frac{\cosh x - 1}{2}$$

$$\cos^2 \frac{x}{2} = \frac{1 + \cos x}{2} \qquad\qquad \cosh^2 \frac{x}{2} = \frac{\cosh x + 1}{2}$$

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$$\sin(x + y) = \sin x \cos y + \cos x \sin y \qquad\qquad \sinh(x + y) = \sinh x \cosh y + \cosh x \sinh y$$

$$\sin(x - y) = \sin x \cos y - \cos x \sin y \qquad\qquad \sinh(x - y) = \sinh x \cosh y - \cosh x \sinh y$$

$$\cos(x + y) = \cos x \cos y - \sin x \sin y \qquad\qquad \cosh(x + y) = \cosh x \cosh y + \sinh x \sinh y$$

$$\cos(x - y) = \cos x \cos y + \sin x \sin y \qquad\qquad \cosh(x - y) = \cosh x \cosh y - \sinh x \sinh y$$

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$$\sin x + \sin y = 2 \sin \frac{x + y}{2} \cos \frac{x - y}{2} \qquad\qquad \sinh x + \sinh y = 2 \sinh \frac{x + y}{2} \cosh \frac{x - y}{2}$$

$$\sin x - \sin y = 2 \cos \frac{x + y}{2} \sin \frac{x - y}{2} \qquad\qquad \sinh x - \sinh y = 2 \cosh \frac{x + y}{2} \sinh \frac{x - y}{2}$$

$$\cos x + \cos y = 2 \cos \frac{x + y}{2} \cos \frac{x - y}{2} \qquad\qquad \cosh x + \cosh y = 2 \cosh \frac{x + y}{2} \cosh \frac{x - y}{2}$$

$$\cos x - \cos y = -2 \sin \frac{x + y}{2} \sin \frac{x - y}{2} \qquad\qquad \cosh x - \cosh y = 2 \sinh \frac{x + y}{2} \sinh \frac{x - y}{2}$$

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$$\sin(ix) = i \sinh x \qquad \sinh(ix) = i \sin x$$

$$\cos(ix) = \cosh x \qquad \cosh(ix) = \cos x$$

$$\sin(x + iy) = \sin x \cosh y + i \cos x \sinh y$$

$$\cos(x + iy) = \cos x \cosh y - i \sin x \sinh y$$

$$\sinh(x + iy) = \sinh x \cos y + i \cosh x \sin y$$

$$\cosh(x + iy) = \cosh x \cos y + i \sinh x \sin y$$