$\boxed{\text{VAR F., 27.3.17}}$

1

1. $$\sin x - x\cos x = x - \frac{x^3}{3!} + o(x^3) - x\left(1 - \frac{x^2}{2!} + o(x^2)\right)$$
$$= x^3\left(-\frac{1}{6} + \frac{1}{2}\right) + o(x^3)$$

15b

$$\frac{\sin x - x\cos x}{x^3} = \frac{1}{2} - \frac{1}{6} + \frac{o(x^3)}{x^3} \longrightarrow \frac{1}{3}$$

2. $$\int \frac{5x^2+14x+13}{(2x+3)(x^2+2x+4)} = \int \frac{A}{2x+3} + \frac{Bx+C}{x^2+2x+4}$$

5b

$$5x^2+14x+13 = A(x^2+2x+4) + (2x+3)(Bx+C)$$

x^2: $5 = A + 2B$ $\qquad A = 5-2B$

5b

x^1: $14 = 2A + 2C + 3B$

x^0: $13 = 4A + 3C$

$14 = 10 - 4B + 2C + 3B \Rightarrow 4 = 2C - B$ $\qquad B = 2C-4$

$13 = 20 - 8B + 3C \Rightarrow -7 = 3C - 8B$

$-7 = 3C - 16C + 32$

3b

$13C = 39$

$\boxed{C=3}$ $\boxed{B=2}$ $\boxed{A=1}$

$$\int \dots = \int \frac{1}{2x+3} + \int \frac{2x+3}{x^2+2x+4} = \frac{1}{2}\int \frac{1}{x+\frac{3}{2}} + \int \frac{2x+2}{x^2+2x+4} + \int \frac{1}{x^2+2x+4}$$

$$= \frac{1}{2}\ln\left|x+\frac{3}{2}\right| + \ln(x^2+2x+4) + \underbrace{\int \frac{1}{(x+1)^2+3}}$$

7b

$$= \frac{1}{2}\ln\left|x+\frac{3}{2}\right| + \ln(x^2+2x+4) + \frac{1}{\sqrt{3}}\operatorname{arctg}\frac{x+1}{\sqrt{3}}$$

$x \in \left(-\infty, -\frac{3}{2}\right) \cup \left(-\frac{3}{2}, \infty\right)$

2

(3) $y'' - y' = xe^{2x}$

$y'' - y' = 0$

$\lambda^2 - \lambda = 0$

$\lambda(\lambda - 1) = 0 \quad \dots \quad \lambda = 0,\ \lambda = 1$

$y_H = c_1 + c_2 e^x$ 5b

$y_p = (ax+b)e^{2x}$ 5b

$$\left.\begin{aligned} y_p' &= ae^{2x} + 2(ax+b)e^{2x} \\ y_p'' &= 2ae^{2x} + 2ae^{2x} + 4(ax+b)e^{2x} \end{aligned}\right\} -$$

$$y_p'' - y_p' = e^{2x}(2a + 2a + 4ax + 4b - a - 2ax - 2b)$$

$$= e^{2x}(3a + 2ax + 2b) \stackrel{?}{=} xe^{2x}$$

$$\left.\begin{aligned} &2a = 1 \qquad a = \tfrac{1}{2} \\ &3a + 2b = 0 \\ &\tfrac{3}{2} + 2b = 0 \qquad \underline{b = -\tfrac{3}{4}} \end{aligned}\right\} \text{3b}$$

$$\boxed{y = -\tfrac{3}{4}e^{2x} + \tfrac{1}{2}xe^{2x} + c_1 + c_2 e^x}$$ 2b

(4) $\underbrace{\arcsin \frac{x}{x+1}}_{f(x)}$ $\quad D(f): \ -1 \le \frac{x}{x+1} \le 1 \quad ; x \ne -1$

$$\left.\begin{aligned} x+1>0: &\ \underbrace{-x-1 \le x}\ \underbrace{\le x+1} \\ &-1 \le 2x \qquad 0 \le 1 \checkmark \\ &-\tfrac{1}{2} \le x \end{aligned}\right\} \langle -\tfrac{1}{2}, \infty)$$ 3b

$x+1<0: \ -x-1 \ge x \ge x+1$ ⓧ

$\underline{D(f) = \langle -\tfrac{1}{2}, \infty) \quad \text{spojitá na } D(f)}$ 2b

- $\lim\limits_{x\to -\frac{1}{2}+} f(x) = f(-\tfrac{1}{2}) = \arcsin \dfrac{-\frac{1}{2}}{\frac{1}{2}} = \arcsin(-1) = -\dfrac{\pi}{2}$ 1b

- $\lim\limits_{x\to\infty} f(x) = \arcsin 1 = \dfrac{\pi}{2}$ 1b

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$$f'(x) = \frac{1}{\sqrt{1-\left(\frac{x}{x+1}\right)^2}} \cdot \frac{x+1-x}{(x+1)^2} = \frac{1}{\sqrt{\frac{2x+1}{(x+1)^2}}} \cdot \frac{1}{(x+1)^2} = \frac{1}{\sqrt{2x+1}\cdot(x+1)} > 0$$

3b

pro $x > -\frac{1}{2}$

$$f''(x) = \frac{-\frac{1}{2}\cdot\frac{2}{\sqrt{2x+1}}\cdot(x+1) - \sqrt{2x+1}}{(2x+1)(x+1)^2} =$$

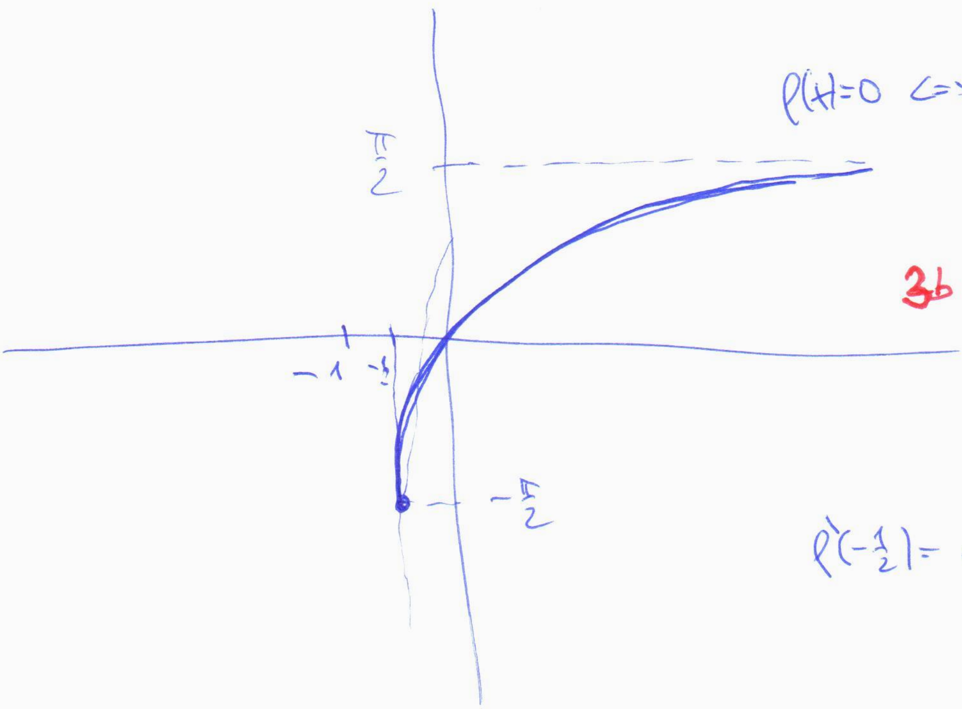
3b

$$= \frac{-(x+1)-(2x+1)}{(2x+1)\sqrt{2x+1}\,(x+1)^2} = -\frac{3x+2}{(x+1)^2(2x+1)^{3/2}} < 0$$

pro $x < -\frac{1}{2}$

1b 1b

vrchol a je konkávní na $\mathcal{D}(f)$



$f(x) = 0 \iff \frac{x}{x+1} = 0 \iff x = 0$

3b

$f'(-\tfrac{1}{2}) = +\infty \quad \left(\lim\limits_{x\to -\frac{1}{2}} f'\right)$

$\mathcal{H}(f) = \langle -\frac{\pi}{2}, \frac{\pi}{2})$ 2b