CHARLES UNIVERSITY IN PRAGUE
Mathematical Institute
Department of Mathematical Analysis

Sixth School

MATHEMATICAL THEORY IN FLUID MECHANICS

PASEKY (Czech Republic)
September 19 – 26, 1999

First Announcement

MAIN LECTURES

Marco **CANNONE** (Paris, France):
Viscous flows in Besov spaces

Eduard **FEIREISL** (Prague, Czech Republic):
On the long-time behaviour of solutions to the Navier-Stokes equations of compressible flow

Michael **GRIEBEL** (Bonn, Germany):
Multilevel techniques for the fast numerical solution of the Navier-Stokes equations

Nader **MASMOUDI** (Paris, France):
Some asymptotic problems in fluid mechanics

Serguei A. **NAZAROV** (Saint Petersburg, Russia):
Weighted spaces with detached asymptotics in application to Navier-Stokes problems

Athanasios E. **TZAVARAS** (Madison, USA):
Topics in the theory of hyperbolic systems of conservation laws

PROGRAM

1. **Marco CANNONE** (Universitè Denis Diderot, Paris VII, France):
   Viscous flows in Besov spaces

   The aim of these lectures is to present an introductory survey to the existence and uniqueness results of strong solutions to the Navier-Stokes equations. These equations govern the time evolution of an incompressible viscous fluid which is assumed here to fill the whole three dimensional space.
   A topic which will be extensively treated concerns the space of initial conditions for which there exists a unique local (in time) solution or a global small solution. For sufficiently small Reynolds numbers, the conditional stability in the sense of Lyapunov will also be studied.
   The definition of a somewhat esoteric Besov space (in terms of the Littlewood-Paley dyadic decomposition) will play a crucial role throughout the lectures.

2. **Eduard FEIREISL** (Czech Academy of Sciences, Prague, Czech Republic):
   On the long-time behaviour of solutions to the Navier-Stokes equations of compressible flow

   We give a survey on recent development of the mathematical theory of viscous compressible fluids. The questions addressed will be: The global existence of weak solutions in several space dimensions, the existence of time-periodic solutions under the condition that the fluid is driven by a periodic external force and, last but not least, the long-time stabilization towards a stationary state for conservative external forces.
   We shall also discuss the role of integrability of the density for the isentropic model which is intimately related to the problem of existence.
   As for the mathematical tools employed, we shall only need the classical Sobolev spaces as well as some elements of the modern functional analysis, in particular, the Lebesgue spaces endowed with the weak topology.

3. **Michael GRIEBEL** (University of Bonn, Germany):
   Multilevel techniques for the fast numerical solution of the Navier-Stokes equations

   Multiscale techniques can help to obtain a numerical approximation to the solution of PDEs in various ways. First, they directly allow to use a fast solver of the linear equation systems which arise after linearization and time- and space-discretization. Second, they also allow to use an adaptive nonlinear approximation of the solution itself. Here, the discretization is locally refined to resolve steep gradients or singularities of the solution. Moreover, information obtained from a multilevel resolution of the actually computed approximation can guide further refinement or coarsening of the discretization. Third, multiscale techniques allow to use certain norm equivalences which, in turn, give a priori and a posteriori error estimates.
   These benefits of a multiscale approach which are well known for scalar elliptic PDEs can also be exploited for nonlinear PDE systems to some extent. Here we consider the Navier-Stokes equations. We discuss the advantages of certain multiscale bases (multigrid, wavelets) for the adaptive discretization, the multigrid solution and the multiscale error estimation for incompressible viscous flow problems with moderate Reynolds number. We also address the benefits of a multiscale technique for turbulence modeling.

Furthermore, we present the results of numerical experiments obtained with a multiscale wavelet discretization and solution method for various laminar and turbulent flow problems.

4. **Nader MASMOUDI** (Universitè de Paris-Dauphine, France):
   Some asymptotic problems in fluid mechanics

   We will discuss various asymptotic problems arising in fluid mechanics and specially the compressible-incompressible limit which corresponds to a Mach number going to zero. More precisely we will show various results establishing the convergence, as the density becomes constant and the Mach number goes to zero, towards solutions of incompressible models (Navier-Stokes or Euler).

   Other problems may be discussed as the Navier-Stokes-Euler limit corresponding to a viscosity going to zero as well as the the study of rotating fluids at high frequency.

5. **Serguei A. NAZAROV** (St. Petersburg University, Russia):
   Weighted spaces with detached asymptotics in application to Navier-Stokes problems.

   Linear elliptic problems in domains with cylindrical and conical outlets to infinity gain the Fredholm property if posed in function spaces with weighted norms of exponential and power types. With the help of such norms, asymptotics of solutions in the outlet can be described as well. A correct formulation of limiting conditions at infinity (like flux and pressure conditions) needs a certain generalization of the spaces mentioned above, and it gives rise to weighted spaces with detached asymptotics. These spaces consist of functions with a prescribed asymptotic form while their norms reflect properties of both, the detached terms and the remainder. However, the most applicable ability of those spaces becomes the direct maintenance of nonlinear problems in the case that linear and nonlinear differential expressions are of the same asymptotic power at infinity. The 3D exterior problem, the 2D aperture problem, the problem of quiet flows are to be presented as applications of the developed mathematical tools for Navier-Stokes equations.

6. **Athanasios E. TZAVARAS** (University of Wisconsin, Madison, USA):
   Topics in the theory of hyperbolic systems of conservation laws

   These lectures will discuss topics in the theory of hyperbolic systems of conservation laws focusing on the equations of gas dynamics. First, we outline the structure of the equations as dictated by the second law of thermodynamics (in the form of Clausius-Duhem inequality) and by the issue of fluid-mechanic limits of the Boltzman equation. Then we will survey certain topics in the theory of weak solutions for conservation laws: these are the kinetic formulation of conservation laws, and the approximation of conservation laws via relaxation or via discrete-velocity models.

It is supposed that every main lecturer will give comprehensive lectures of the total length of about 5 hours. The participants of the conference can present their results in the form of very short communications and/or they can exhibit their papers, preprints and books (the number of these is not limited, and they may be related to other scientific areas).

Graduate students and others beginning their mathematical career are encouraged to participate.

PLACE, TRAVEL INFORMATION

The village of Paseky lies on the slopes of the Krkonoše Mountains, in the North Bohemia. Accommodation consists of rooms for two or three people. There are good facilities and conditions for sporting activities, in particular hiking. Sauna and fitness room are available.

A special bus from Prague to Paseky will leave at 15^{00} on **September 19** (Sunday). The bus from Paseky will arrive at Prague at 11^{00}, on **September 26** (Sunday).

CONFERENCE FEE

The conference fee which covers full board, conference materials, accommodation and transport between Prague and Paseky amounts to $300 per participant. The fee is the same for accompanying persons.
The payment should be made either by cash at the registration desk at Paseky or by cheque, bearing the designation "Fluid Mechanics" and participant's name. This cheque should be made payable and sent to the organizers (see mailing address below) before July 15, 1999. However, the payment should not be made before receiving the Second Announcement, confirming the participation.

DEADLINE

In case of your interest, please, fill in the enclosed Registration Form and send it to the address below before **June 10, 1999**.

Due to limited capacity of accommodation facilities the organizers may be forced to decline registrations. For this reason we would appreciate if your Registration Form is delivered to us as soon as possible.

ORGANIZING COMMITTEE

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The First Announcement, Registration Form and further up-to-date information can be found also on `http://www.karlin.mff.cuni.cz/~rokyta/paseky/99/`