Cylindrical Coordinates

Question

What does the following integral compute?

$$\int_0^{2\pi} \int_0^{\sqrt{3}} \int_0^{3-r^2} r \, dz \, dr \, d\theta$$

A. The volume under the paraboloid $z = 3 - x^2 - y^2$ above the xy-plane.

B. The volume enclosed by the upper half-ball $x^2 + y^2 + z^2 \leq 3$ and $z \geq 0$.

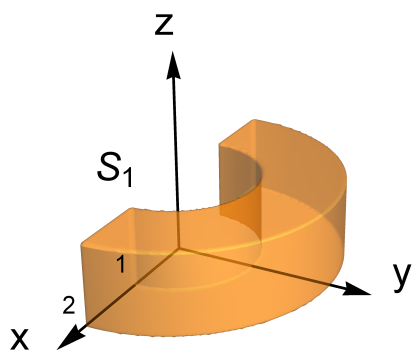
C. The mass of the solid described by $0 \leq z \leq 3 - r^2$, $0 \leq r \leq 3$, with density $f(r, \theta, z) = r$.

D. More than one of the above.

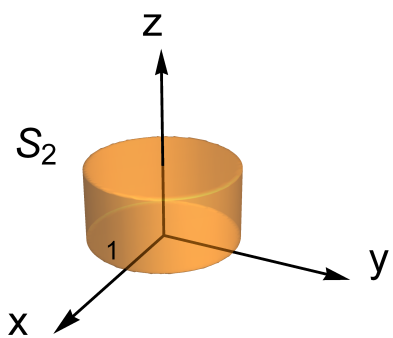
Cylindrical Coordinates

Question

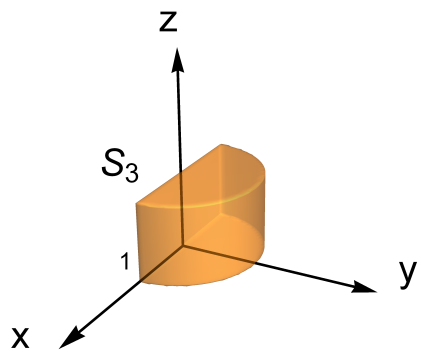
For the solids S_k $(k = 1, 2, 3, 4)$ shown, rank the triple integrals $I_k = \iiint_{S_k} (1 - r)\, dV$ from smallest to largest.



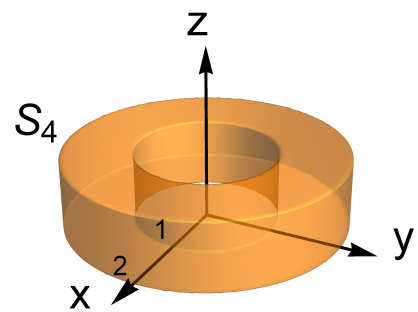
A



B



C

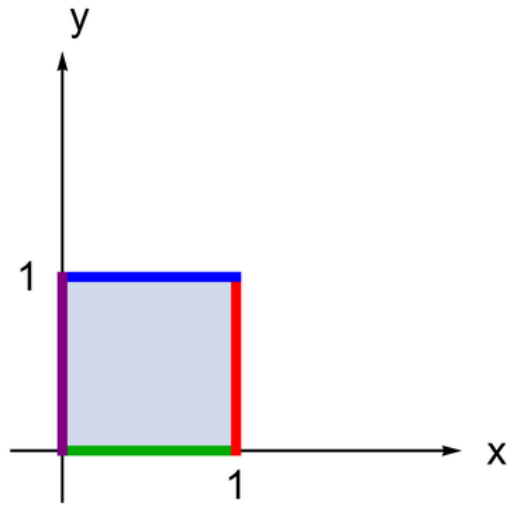


D

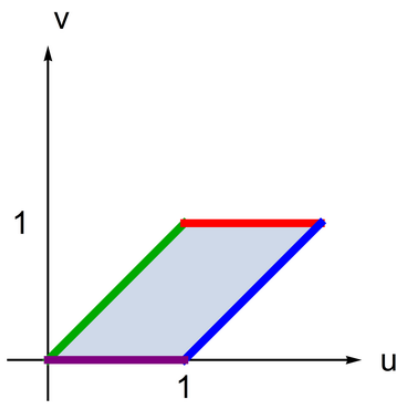
Change of Variables

Question

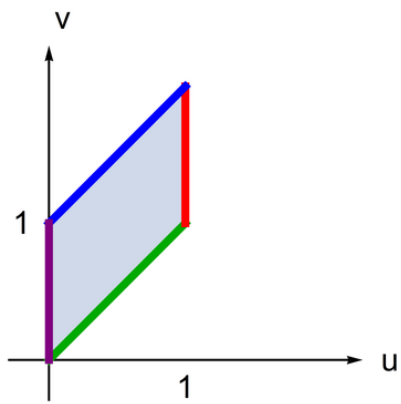
Which picture is the image of the region shown at right under the transformation $u = x + y$, $v = x$?



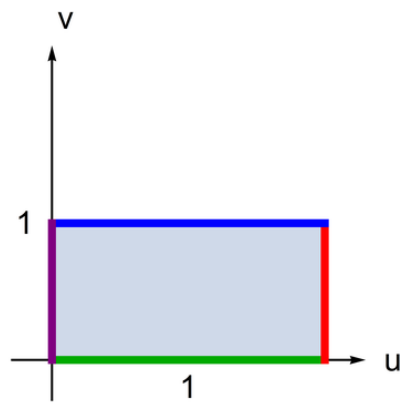
A.



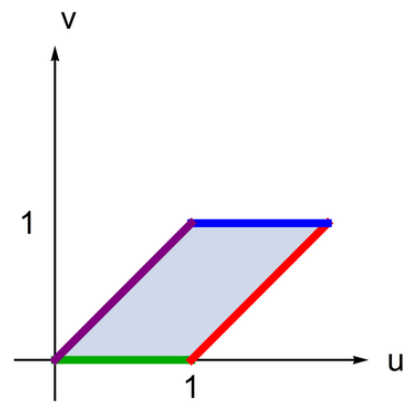
B.



C.



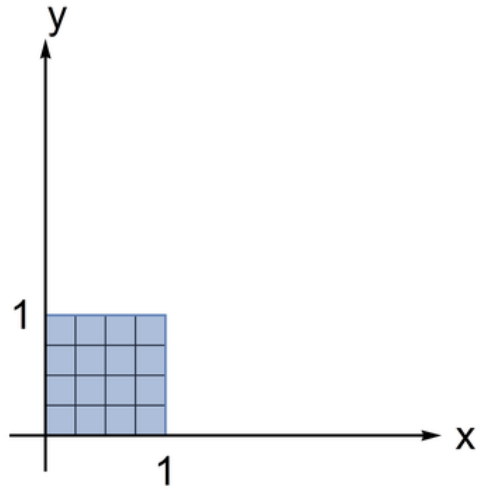
D.



Change of Variables

Question

Which picture is the image of the region shown at right under the transformation $u = e^x \cos(y)$, $v = e^x \sin(y)$?



DRAW THE PICTURE.

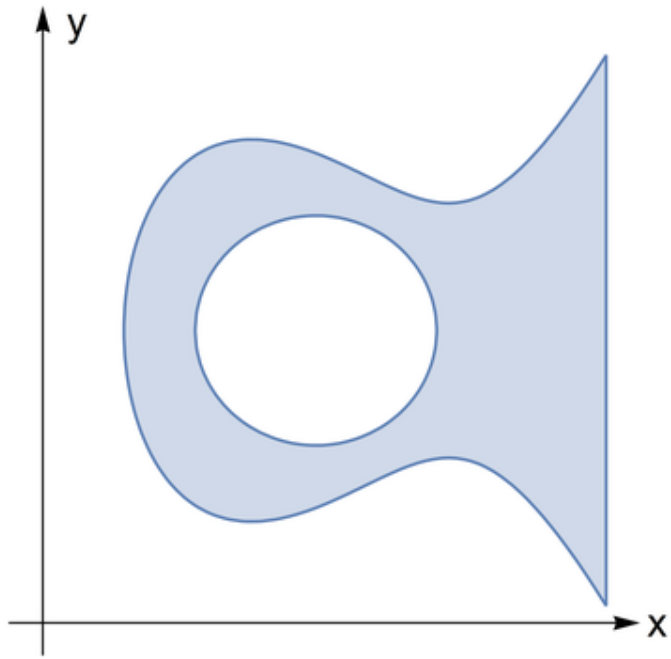
Double Integrals over General Regions

Question

To compute $\iint_{\mathcal{R}} 1 \, dA$ using iterated integrals of the form

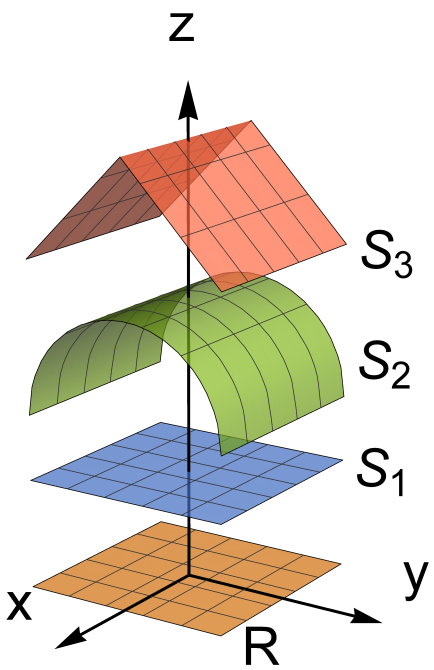
$$\int_a^b \int_{f(x)}^{g(x)} 1 \, dy dx,$$

how many sub-regions must the region $\mathcal{R}$ shown be broken into?



Write the smallest possible number of sub-regions.

Surface Area



Question

The figure shows the graphs of 3 functions over the same domain R in the xy-plane. Rank the surface areas from smallest to largest.

A. $\text{Area}(S_1) < \text{Area}(S_2) < \text{Area}(S_3)$

B. $\text{Area}(S_1) < \text{Area}(S_3) < \text{Area}(S_2)$

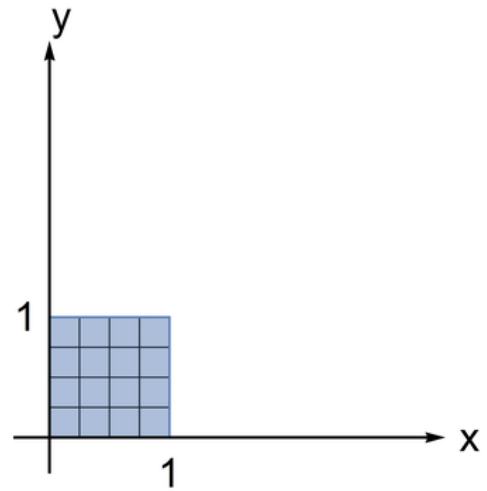
C. $\text{Area}(S_2) < \text{Area}(S_3) < \text{Area}(S_1)$

D. $\text{Area}(S_3) < \text{Area}(S_2) < \text{Area}(S_1)$

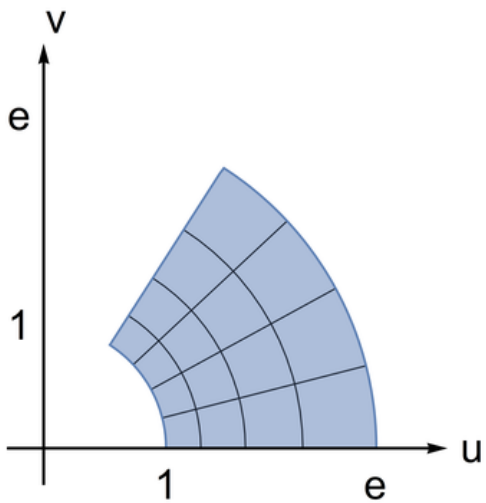
Change of Variables

Question

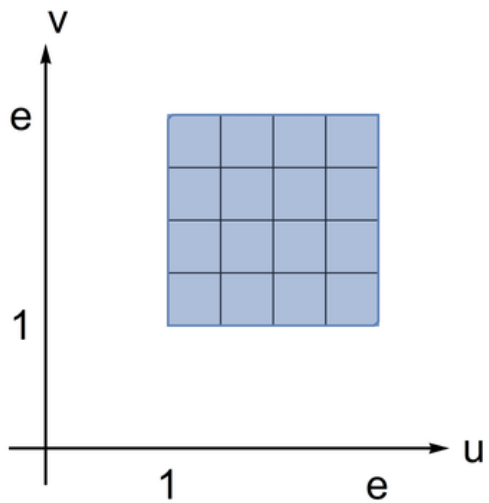
Which picture is the image of the region shown at right under the transformation $u = e^x \cos(y)$, $v = e^x \sin(y)$?



A.



B.



C.