1. $y' = \frac{(y^3+1)\cot g\, x}{3y^2}$, $y(\frac{\pi}{2}) = \sqrt[3]{1336}$; $\left[y \equiv -1 \quad \text{S.R.} \right]$ 1

$$\frac{3y^2 y'}{y^3+1} = \cot g\, x \qquad / \int \dots dx$$

4 $\int \frac{3y^2 dy}{y^3+1} = \left| \begin{matrix} z = y^3+1 \\ dz = 3y^2 dy \end{matrix} \right| = \dots \overset{c}{=} \ln|y^3+1|$

4 $\int \cot g\, x\, dx = \int \frac{\cos x}{\sin x} dx \overset{c}{=} \ln|\sin x|$

Tedy celkem:

$$\ln|y^3+1| = \ln|\sin x| + C \quad , \quad C \in \mathbb{R}$$

$$|y^3+1| = e^C |\sin x|$$

3 $y^3+1 = K \cdot \sin x \quad , \quad K \in \mathbb{R}$

1 $y(x) = \sqrt[3]{K \cdot \sin x - 1}$

$$\left[y(x) = \sqrt[3]{1337 \sin x - 1} \right]$$

$$\Uparrow$$

P.P.: $\sqrt[3]{1336} = y(\frac{\pi}{2}) = \sqrt[3]{K \cdot 1 - 1} \Rightarrow K = 1337$. 2

3. $y' - \frac{y}{x} = \sqrt[3]{x}$ ... $p(x) = -\frac{1}{x}$, $P(x) = -\ln|x|$, 5 I.F.: $\frac{1}{|x|}$.

$$\frac{y'}{|x|} - \frac{y}{|x| \cdot x} = \frac{\sqrt[3]{x}}{|x|} \qquad / \cdot \operatorname{sgn} x \quad 3$$

$$\frac{y'}{x} - \frac{y}{x^2} = x^{-2/3}$$

$$\left(\frac{1}{x} \cdot y\right)' = x^{-2/3} \qquad / \int \dots dx$$

$$\int x^{-2/3} \overset{c}{=} \frac{1}{\frac{1}{3}} x^{1/3} = 3x^{1/3}. \quad 3$$

$$\frac{1}{x} y = 3x^{1/3} + C$$

$y(x) = 3x^{4/3} + Cx$ 3, $x \in (-\infty, 0), (0, \infty)$ 2 ; $C \in \mathbb{R}$.

② $y' = \arcsin\left(\frac{1-y^2}{4}\right)\cdot\sqrt{4-y^2} =: g(y)$

3

Def. obor g: • $4-y^2 \geq 0 \iff y \in [-2,2]$

• $-1 \leq \frac{1-y^2}{4} \leq 1 \iff -4 \leq 1-y^2 \leq 4 \iff -5 \leq -y^2 \leq 3$

$\iff (-3 \leq) y^2 \leq 5 \iff y \in [-\sqrt{5}, \sqrt{5}]$.

Celkem: $D_g = [-2,2] \cap [-\sqrt{5}, \sqrt{5}] = [-2,2]$.

3

STAC. ŘEŠENÍ: $y \equiv \pm 2, \quad y \equiv \pm 1$

1

Monotonie:

$$\begin{array}{ccccccccc} & & - & & + & & - & & \\ & -2 & & -1 & & 1 & & 2 & \end{array}$$

LEPENÍČKO: „na 1": 3 $\quad \int_0^1 \frac{1}{g} = \int_0^1 \frac{dy}{\arcsin\left(\frac{1-y^2}{4}\right)\sqrt{4-y^2}}$

srovnáme s $\int_0^1 \frac{1}{1-y^2}\,dy$: LSK $\quad \lim_{y\to 1_-} \frac{\frac{1}{g(y)}}{\frac{1}{1-y^2}} = \lim_{y\to 1_-} \frac{1-y^2}{\arcsin\left(\frac{1-y^2}{4}\right)\sqrt{4-y^2}} =$

$= \lim_{y\to 1_-} \frac{1}{\sqrt{4-y^2}} \cdot \lim_{y\to 1} \frac{1-y^2}{\arcsin\ldots} \overset{\text{VOLSF}}{=} \frac{1}{\sqrt{3}} \cdot 1 \in (0,\infty)$.

VOLSF $\quad \lim_{y\to 1_-} 1-y^2 = 0$

$\lim_{z\to 0} \frac{z}{\arcsin z} \quad \left(\overset{l'H}{=} \lim_{z\to 0} \frac{1}{\frac{1}{\sqrt{1-z^2}}}\right) = 1$

(P) je OK, protože $1-y^2 = 0 \iff y = 1 \vee y = -1 \ldots$

Podle LSK: $\int_0^1 \frac{1}{g}$ K $\iff \int_0^1 \frac{dy}{1-y^2}$ K.

Dále $\int_0^1 \frac{dy}{1-y^2}$ srovnáme s $\underbrace{\int_0^1 \frac{dy}{1-y}}_{D}$. LSK: $\lim_{y\to 1_-} \frac{1-y}{1-y^2} = \frac{1}{2} \in (0,\infty)$.

Tedy $\int_0^1 \frac{1}{g}$ K $\iff \int_0^1 \frac{dy}{1-y^2}$ K $\iff \int_0^1 \frac{dy}{1-y}$ K., což neplatí.

Tedy $\int \frac{1}{g}$ D, a nelze lepit shora; ~~(zespoda)~~ na 1. Shora taktéž.

„na -1": Analogicky (stejný výpočet, pouze s $\lim\limits_{y\to -1_+}$ a druhou „srovnávací fcí" $\frac{1}{1+y}$ místo $\frac{1}{1-y}$. nelze lepit

„na 2" 2 Srovnáme s $\frac{1}{\sqrt{2-y}}$: (protože $\sqrt{4-y^2} = \sqrt{2-y}\cdot\sqrt{2+y}$)

LSK: $\lim\limits_{y\to 2_-} \dfrac{\frac{1}{g}}{\frac{1}{\sqrt{2-y}}} = \lim\limits_{y\to 2_-} \dfrac{\sqrt{2-y}}{\arcsin(\dots)\cdot\sqrt{4-y^2}} =$

$= \lim\limits_{y\to 2_-} \dfrac{1}{\arcsin\left(\frac{1-y^2}{4}\right)} \cdot \lim\limits_{y\to 2_-} \dfrac{1}{\sqrt{2+y}} = \dfrac{1}{\arcsin\left(-\frac{3}{4}\right)} \cdot \dfrac{1}{2} \in \mathbb{R}\setminus\{0\}.$

Tedy $\int_{3/2}^{2} \frac{1}{g}$ K $\iff \int_{3/2}^{2} \frac{1}{\sqrt{2-y}}\,dy$ K, což je pravda.

Tedy na 2 lze lepit (zespoda).

„na -2" Analogicky (sudost g). Lze lepit shora

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