(1.) T.P. jmenovatele: $x\cdot(\sin x - x) =$

$$= x\cdot\left(x - \frac{x^3}{6} + o(x^4) - x\right),\ x\to 0 \quad = -\frac{x^4}{6} + o(x^5),\ x\to 0$$

čitatel: $e^y = 1 + y + \frac{y^2}{2} + o(y^2),_{y\to 0}$ $\quad \cos x - 1 = -\frac{x^2}{2} + \frac{x^4}{24} + o(x^5),_{x\to 0}$

$$e^{\cos x - 1} = 1 - \frac{x^2}{2} + \underbrace{\frac{x^4}{24} + \frac{1}{2}\left(\frac{x^4}{4}\right)}_{\frac{1+3}{24}x^4 = \frac{1}{6}x^4} + o(x^5),\ x\to 0$$

$$\lim_{x\to 0} \frac{e^{\cos x - 1} - \cos x}{x(\sin x - x)} =$$

$$= \lim_{x\to 0} \frac{\cancel{1} - \cancel{\frac{x^2}{2}} + \frac{1}{6}x^4 - \left(\cancel{1} - \cancel{\frac{x^2}{2}} + \frac{x^4}{24}\right) + o(x^5)}{-\frac{x^4}{6} + o(x^5)} =$$

$$= \lim_{x\to 0} \frac{\frac{x^4}{8} + o(x^5)}{-\frac{x^4}{6} + o(x^5)} = \frac{\frac{1}{8} + \lim \frac{o(x^5)}{x^4\cdot x}\cdot x}{-\frac{1}{6} + \lim \frac{o(x^5)}{x^4\cdot x}\cdot x} = \frac{\frac{1}{8} + 0\cdot 0}{-\frac{1}{6} + 0\cdot 0} = -\frac{3}{4}$$

1b

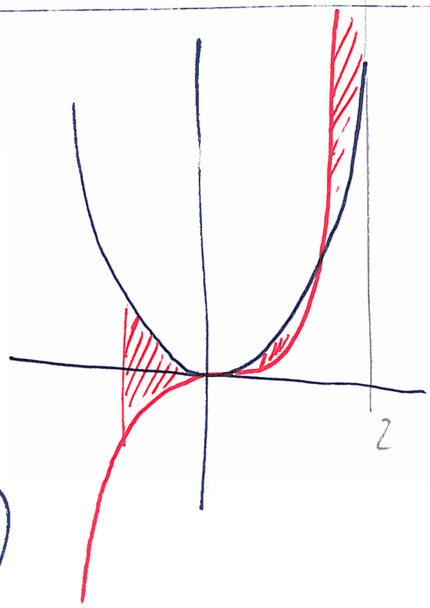
(3)

$$S = \int_{-1}^{1} (x^2 - x^3)\,dx + \int_{1}^{2} (x^3 - x^2)\,dx =$$

$$= \left[\frac{x^3}{3} - \frac{x^4}{4}\right]_{-1}^{1} + \left[\frac{x^4}{4} - \frac{x^3}{3}\right]_{1}^{2} =$$

$$= \frac{1}{3} - \frac{1}{4} - \left(-\frac{1}{3} - \frac{1}{4}\right) + \frac{16}{4} - \frac{8}{3} - \left(\frac{1}{4} - \frac{1}{3}\right)$$

$$= \frac{1}{12} + \frac{7}{12} + \frac{4}{3} + \frac{1}{12} = \frac{9}{12} + \frac{16}{12} = \frac{25}{12}$$



(2.) $$\int \frac{\mathrm{tg}\,x \cdot (2\cos x - \sin x)}{(\cos^2 x + 1)\cdot(\cos x + \sin x)}\,dx =$$

„kuchařka“ nám velí použít substituci $y = \mathrm{tg}\,x$

$$= \int \frac{\mathrm{tg}\,x \cdot (2\cos x - \sin x)\cdot \cos^2 x}{(\cos^2 x + 1)\cdot(\cos x + \sin x)} \cdot \frac{dx}{\cos^2 x} =$$

$$\frac{1}{\cos^2 x} = \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = 1 + \mathrm{tg}^2 x = 1 + y^2$$

$$\int \frac{2\sin x \cdot \cos^2 x - \sin^2 x \cos x}{(\cos^2 x + 1)\cdot(\cos x + \sin x)} \cdot \frac{dx}{\cos^2 x} =$$

$$= \int \frac{2\cdot \frac{\sin x}{\cos x} - \frac{\sin^2 x}{\cos^2 x}}{\left(1 + \frac{1}{\cos^2 x}\right)\cdot\left(1 + \frac{\sin x}{\cos x}\right)} \cdot \frac{dx}{\cos^2 x} \longleftarrow \left[\begin{array}{l} y = \mathrm{tg}\,x \\ dy = \frac{1}{\cos^2 x}\,dx \end{array}\right]$$

7

$$= \int \frac{2y - y^2}{(2+y^2)(1+y)}\,dy = \int \frac{2\,dy}{2+y^2} - \int \frac{dy}{1+y} = \int \frac{dy}{1+\left(\frac{y}{\sqrt{2}}\right)^2} - \ln|1+y|$$

$$\frac{2y - y^2}{(2+y^2)(1+y)} = \frac{Ay + B}{2+y^2} + \frac{C}{1+y} = \frac{Ay + B + Ay^2 + By + 2C + Cy^2}{(2+y^2)(1+y)}$$

6

$$= \frac{(A+C)y^2 + (A+B)y + B + 2C}{(2+y^2)(1+y)}$$

$$\left.\begin{array}{l} A + C = -1 \\ A + B = 2 \\ B + 2C = 0 \end{array}\right\} \quad \left.\begin{array}{l} B - C = 3 \end{array}\right\} \quad 3C = -3$$

$$\underline{C = -1} \qquad \underline{B = 2} \qquad \underline{A = 0}$$

4 3

$$\stackrel{c}{=} \sqrt{2}\,\mathrm{arctg}\left(\frac{y}{\sqrt{2}}\right) - \ln|1+y| = \sqrt{2}\,\mathrm{arctg}\left(\frac{\mathrm{tg}\,x}{\sqrt{2}}\right) - \ln|1 + \mathrm{tg}\,x|$$