(1.) $$\lim_{x\to 0} \frac{e^{x^2} - x\sin x - 1}{e^{x^4} - 1} =$$

- $e^{y} = 1 + y + \frac{y^2}{2} + o(y^2)$, $y \to 0$
- $e^{x^2} = 1 + x^2 + \frac{x^4}{2} + o(x^4)$, $x \to 0$
- $e^{x^4} = 1 + x^4 + o(x^4)$, $x \to 0$
- $\sin x = x - \frac{x^3}{6} + o(x^4)$, $x \to 0$
- $x \sin x = x^2 - \frac{x^4}{6} + o(x^5)$, $x \to 0$

$$= \lim_{x\to 0} \frac{1 + x^2 + \frac{x^4}{2} + o(x^4) - \left(x^2 - \frac{x^4}{6} + o(x^5)\right) - 1}{1 + x^4 + o(x^4) - 1} =$$

$$= \lim_{x\to 0} \frac{x^4 \cdot \left(\frac{1}{2} + \frac{1}{6}\right) + o(x^4)}{x^4 + o(x^4)} = \lim_{x\to 0} \frac{\frac{2}{3} + \frac{o(x^4)}{x^4}}{1 + \frac{o(x^4)}{x^4}} \overset{\text{VOAL}}{=}$$

$$= \frac{\frac{2}{3} + 0}{1 + 0} = \underline{\frac{2}{3}}$$

(3.) $$\int_1^4 \frac{\sqrt{x-1}}{x+2}\,dx = \int_0^{\sqrt{3}} \frac{y}{y^2+3} \cdot 2y\,dy = 2\int_0^{\sqrt{3}} \left(1 - \frac{3}{y^2+3}\right) dy =$$

$$\left[\begin{array}{l} y = \sqrt{x-1} \\ y^2 = x-1 \\ x = y^2+1 \\ dx = 2y\,dy \end{array}\right]$$

$$= 2\sqrt{3} - 6\int_0^{\sqrt{3}} \frac{dy}{3\left(\left(\frac{y}{\sqrt{3}}\right)^2 + 1\right)} =$$

$$= 2\sqrt{3} - 6 \cdot \frac{1}{3} \cdot \left[\sqrt{3}\,\mathrm{arctg}\left(\frac{y}{\sqrt{3}}\right)\right]_0^{\sqrt{3}} =$$

$$= 2\sqrt{3} - 2\sqrt{3} \cdot \left(\mathrm{arctg}\,1 - \mathrm{arctg}\,0\right) =$$

$$= 2\sqrt{3}\left(1 - \frac{\pi}{4}\right)$$

$$\text{(2.)}\quad \int \frac{2e^{3x}+3e^{2x}+2e^{x}}{(e^x+1)(e^{2x}+e^x+1)}\,dx \longleftarrow \begin{bmatrix} y=e^x \\ dy=e^x dx \end{bmatrix} =$$

$$= \int \frac{2y^2+3y+2}{\underbrace{(y+1)(y^2+y+1)}}\,dy = \dots \text{ Parciální zlomky:}$$

$$= \frac{A}{y+1} + \frac{By+C}{y^2+y+1} = \frac{1}{\dots}\left(Ay^2+Ay+A+By^2+By+Cy+C\right)$$

$$\left.\begin{aligned} \underline{y^2:}\quad & 2 = A+B \\ \underline{y:}\quad & 3 = A+B+C \\ \underline{1:}\quad & 2 = A+C \end{aligned}\right\} \overset{\text{TRIV.}}{\Longrightarrow} \quad A=B=C=1$$

$$= \underbrace{\int \frac{dy}{y+1}}_{I_1} + \underbrace{\int \frac{y+1}{y^2+y+1}}_{I_2}. \qquad I_1 \overset{c}{=} \ln|y+1| = \ln|e^x+1|$$

$$I_2 = \frac{1}{2}\left(\int \frac{2y+1}{y^2+y+1}\,dy + \int \frac{dy}{y^2+y+1}\right) = \frac{1}{2}\ln|y^2+y+1| +$$

$$+ \int \frac{dy}{(y+\frac{1}{2})^2+\frac{3}{4}} = \frac{1}{2}\ln(y^2+y+1) + \frac{4}{3}\int \frac{dy}{\left(\frac{2y+1}{\sqrt{3}}\right)^2+1} =$$

$$\overset{c}{=} \frac{1}{2}\ln(y^2+y+1) + \frac{4}{3}\cdot\frac{\sqrt{3}}{2}\cdot \operatorname{arctg}\left(\frac{2y+1}{\sqrt{3}}\right) =$$

$$= \frac{1}{2}\ln\left(e^{2x}+e^x+1\right) + \frac{2}{\sqrt{3}}\operatorname{arctg}\left(\frac{2e^x+1}{\sqrt{3}}\right)$$