EXISTENCE OF ANISOTROPIC OUTER AND CLASSICAL MINKOWSKI CONTENTS

In this talk, we introduce the notions of C-anisotropic outer and classical Minkowski contents.

First, denote by $\mathcal{C}_0^n$ the set of all convex bodies in $\mathbb{R}^n$ whose interiors contain the origin. For an open set Ω, a measurable set $E \subseteq \Omega$, and $C \in \mathcal{C}_0^n$, we define the *C-anisotropic outer Minkowski content of E in Ω* as

$$\mathcal{SM}_C(E;\Omega) \coloneqq \lim_{r\to 0_+} \frac{1}{r}\lambda^n\big((E \oplus rC) \cap (\Omega \setminus E)\big),$$

whenever this limit exists.

Similarly, for $S \subseteq \Omega$, we define the *C-anisotropic Minkowski content of S in Ω* as

$$\mathcal{M}_C(S;\Omega) \coloneqq \lim_{r\to 0_+} \frac{1}{2r}\lambda^n\big((S \oplus rC) \cap \Omega\big),$$

whenever this limit exists.

We discuss the conditions under which the limits $\mathcal{SM}_C(E;\Omega)$ and $\mathcal{M}_C(S;\Omega)$ exist, their interrelation, and their dependence on the choice of C.

For $C \in \mathcal{C}_0^n$, we define the *C-anisotropic perimeter of E in Ω* as

$$\mathrm{Per}_{h_C}(E;\Omega) \coloneqq \int_{\Omega\cap\partial^* E} h_C(\nu_E)\,\mathrm{d}\mathcal{H}^{n-1}.$$

It is known that if

$$\mathcal{SM}_{B(0,1)}(E;\Omega) = \mathrm{Per}_{h_{B(0,1)}}(E;\Omega),$$

then, for any $C \in \mathcal{C}_0^n$, it holds that

$$\mathcal{SM}_C(E;\Omega) = \mathrm{Per}_{h_C}(E;\Omega).$$

However, the converse question — whether

$$\mathcal{SM}_C(E;\Omega) = \mathrm{Per}_{h_C}(E;\Omega)$$

for some $C \in \mathcal{C}_0^n$ implies

$$\mathcal{SM}_{B(0,1)}(E;\Omega) = \mathrm{Per}_{h_{B(0,1)}}(E;\Omega)$$

— remains open.

We present a new result obtained for the anisotropic classical Minkowski content:

$$\mathcal{M}_C(\partial E;\Omega) = \tfrac{1}{2}\big(\mathrm{Per}_{h_C}(E;\Omega) + \mathrm{Per}_{h_C}(\Omega \setminus E;\Omega)\big)$$

if and only if an analogous equality holds for some $C' \in \mathcal{C}_0^n$.

We conclude by discussing recent developments and open problems in this field.